Tangles in a Pole Dance Studio: A Reading of Massuyeau, Alekseev, and Naef

Abstract. I will report on joint work with Zsuzsanna Dancso, Tamara Hogan, Jessica Liu, and Nancy Scherich. Little of what we do is original, and much of it is simply a reading of Massuyeau [Ma] and Alekseev and Naef [AN1].

We study the pole-strand and strand-strand double filtration on the space of tangles in a pole dance studio (a punctured disk cross an interval), the corresponding homomorphic expansions, and a strand-only HOMFLY-PT relation. When the strands are transparent or nearly transparent to each other we recover and perhaps simplify substantial parts of the work of the aforementioned authors on expansions for the Goldman-Turaev Lie bi-algebra.

Kontsevich in Pole Dance Studio.   (w/o poles? See [])

$$\longrightarrow Z = \left(\sum_{m=0}^{\infty} \frac{1}{(2\pi i)^m} \oint_{\substack{t_1 < \ldots < t_m \\ P=\{(z_i, z_i')\}}} (-1)^{\#P_\downarrow} D_P \bigwedge_{i=1}^{m} \frac{dz_i - dz_i'}{z_i - z_i'} \right)^{\sim} \in \mathcal{A}_n$$

I'm Off-Passion!

Comments on the Kontsevich Integral.

1. In our example, $D_P = \ldots$.
2. The $(\cdots)^{\sim}$ means "a correction is needed near the caps and the cups" [].

Homework.

1. What more do we get if we don't mod out by the HOMFLY-PT relation?
2. What more do we get if we allow more than one strand-strand interaction?
3. In this language, recover the double bracket.
4. In this language, recover Kashiwara-Vergne.
5. How is all this related to w-knots?
6. Do the same with associators. Use that to derive formulas for solutions of Kashiwara-Vergne.
7. What's the relationship with the Habiro-Massuyeau invariants of links in handlebodies (same things, different filtration).

References.

[AKKN1] A. Alekseev, N. Kawazumi, Y. Kuno, and F. Naef, *The Goldman-Turaev Lie Bialgebra in Genus Zero and the Kashiwara-Vergne Problem,* Adv. Math. **326** (2018) 1–53, arXiv:1703.05813.

[AKKN2] A. Alekseev, N. Kawazumi, Y. Kuno, and F. Naef, *Goldman-Turaev formality implies Kashiwara-Vergne,* Quant. Topol. **11-4** (2020) 657-–689, arXiv:1812.01159.

[AN1] A. Alekseev and F. Naef, *Goldman-Turaev Formality from the Knizhnik-Zamolodchikov Connection,* Comp. Rend. Math. **355-11** (2017) 1138–1147, arXiv:1708.03119.

[Ma] G. Massuyeau, *Formal Descriptions of Turaev's Loop Operations,* Quant. Topol. **9-1** (2018) 39-–117, arXiv:1511.03974.