Algebra, Topology and the Grothendieck-Teichmüller group

Report of Contributions

https://indico.cern.ch/e/1106625

Contribution ID: **1** Type: **not specified**

Stabilizer bitorsors in double shuffle theory

Tuesday, August 30, 2022 9:00 AM (50 minutes)

We explain the construction of a pair of "Betti" and "de Rham" Hopf algebras and a pair of module-coalgebras over this pair, as well as the bitorsors related to both structures (which will be called the "module" and "algebra" stabilizer bitorsors). We show that Racinet's DMR torsor constructed out of the double shuffle and regularization relations between multiple zeta values is essentially equal to the "module" stabilizer bitorsor and that it is also equal to the "algebra" stabilizer bitorsor. We explain why this is a step in the construction of an "intermediate" group between GRT and DMR.
(joint w H Furusho)

Presenter: ENRIQUEZ, Benjamin (Strasbourg)

Contribution ID: **2** Type: **not specified**

Multizeta values and associators in genus zero and one

Monday, August 29, 2022 10:10 AM (50 minutes)

In the first part of this talk, we will recall the construction of the elliptic associator by Enriquez, and underline its properties in analogy to the well-known properties of the Drinfeld associator ΦKZ. In the second part, we will give a theorem showing that what is going here is much more than a mere analogy: in fact the elliptic associator can be constructed directly from the Drinfeld associator, and its main properties can then be derived directly from those of ΦKZ.

Presenter: SCHNEPS, Leila (Paris)

Contribution ID: **3** Type: **not specified**

A topological characterisation on the Kashiwara-Vergne groups

Monday, August 29, 2022 11:00 AM (50 minutes)

This talk is based on joint work with Iva Halacheva and Marcy Robertson (https://arxiv.org/pdf/2106.02373.pdf) and ongoing joint work with Tamara Hogan and Marcy Robertson. I will present a topological characterisation of the Kashiwara-Vergne groups KV and KRV as automorphism groups of wheeled PROPs (aka circuit algebras) of certain four-dimensional tangles, and their associated graded space of "arrow diagrams". I'll also explain how the Alekseev-Torossian map GRT -> KRV arises in this context as a map from classical chord diagrams to arrow diagrams.

Presenter: DANCSO, Zsuzsi (Sydney)

Contribution ID: **4** Type: **not specified**

Towards a KRV_2 action in the derived category

Monday, August 29, 2022 5:00 PM (50 minutes)

Let X be a smooth complex variety. In previous joint work with V. Dolgushev and T. Willwacher, we exhibited an explicit action of GRT_1 on the Gerstenhaber algebra $H^*(X, \mathcal{T}_{\mathrm{poly}})$. This action produces a large supply of nontrivial GRT_1 representations. After reviewing this construction, I will discuss extending it to an action of KRV_2 on the set of those isomorphisms $H^*(X, \mathcal{T}_{\mathrm{poly}}) \cong HH^*(X)$ which correct the HKR map between the harmonic and Hochschild structures of X. A key ingredient is the recent characterization of KRV_2 by Z. Dancso, I. Halacheva, and M. Robertson as automorphisms of a wheeled PROP. This is joint work in progress with M. Robertson.

Presenter: ROGERS, Chris (Reno)

Contribution ID: **5** Type: **not specified**

Modular Inverters

Monday, August 29, 2022 6:00 PM (50 minutes)

This is a report on a project with Francis Brown and its relation to some more recent work on Hecke actions on periods of iterated integrals of classical modular forms.

The cross ratio identifies the Riemann sphere minus 0, 1 and infinity with the moduli space $M_{0,4}$ of ordered 4 tuples of distinct points on P^1 mod projective equivalence. As observed by Deligne and Ihara in the 1980s, to understand the Galois action or the mixed Hodge structure on the fundamental group of $M_{0,4}$ with base point the tangent vector d/dx at $x = 0$, it suffices to understand the Galois action, respectively, the periods, of the (etale, resp. de Rham) straight line path in $M_{0,4}$ from the tangent vector d/dx at $x = 0$ to the tangent vector $-d/dx$ at $x = 1$. This path is the prototypical associator and is sometimes called "le droit chemin".

Many (perhaps all?) relations satisfied by associators come from the action of the symmetric group S_3 on $M_{0,4}$, the topology of $M_{0,4}$ and the embeddings of $M_{0,4}$ as open strata in the boundary of the natural compactification of $M_{0,5}$.

One can play the same game with $M_{0,4}$ replaced by $M_{1,1}$, the moduli space of smooth elliptic curves. In this case, to understand the Galois action and mixed Hodge structure on the fundamental group (with base point d/dq) of $M_{1,1}$, it suffices to understand the Galois action on the loop in $M_{1,1}$ corresponding to the imaginary axis in the upper half plane. This loop corresponds to an element of order 4 in the topological fundamental group $SL_2(Z)$ and is the prototypical inverter. Relations satisfied by inverters arise from the embedding of $M_{1,1}$ as a boundary stratum of the natural compactification of $M_{1,2}$, the universal elliptic curve with its identity section removed.

The KZ associator is the generating series of iterated integrals of the 1-forms dx/x and $dx/(1-x)$ on $M_{0,4}$. This has a whole hierarchy of analogues in the modular case. In all cases, these are power series that are generating functions of iterated integrals of modular forms. The series one gets depend on which proalgebraic completion of $SL_2(Z)$ one chooses. In the most general version, the coefficients are all iterated integrals of classical modular forms of all levels. In another, they are only iterated integrals of modular forms of weight 2, which are just 1-forms on modular curves.

In addition to expanding on the comments above, I will construct the various completions of $SL_2(Z)$ and the corresponding de Rham inverter. I will explain how, when one pulls back the "weight 2" inverter to level 2, one gets a generalization of the usual KZ associator.

Presenter: HAIN, Richard (Duke)

Contribution ID: **6** Type: **not specified**

Graph Complexes, GRT and cohomology of GL_n

Monday, August 29, 2022 9:00 AM (50 minutes)

I will give a tour of some recent ideas which relate: the commutative even graph complex, the moduli space of tropical curves, the cohomology of the general linear group, the Grothendieck-Teichmüller group, MZV's and Feynman integrals.

Presenter: BROWN, Francis (Oxford)

Contribution ID: **7** Type: **not specified**

An integral version of the Grothendieck-Teichmüller group

Tuesday, August 30, 2022 10:10 AM (50 minutes)

I will explain an integral generalization of rational homotopy theory based on binomial commutative rings. I will then explain the construction of a derived binomial algebraic group over the integers which specializes to the pro-algebraic Grothendieck-Teichmüller group as well as the pro-p version for all prime p. I will finally describe the conjectural relationship between this group and the Tannakian Galois group of Nori motives with integral coefficients.

Presenter: HOREL, Geoffroy (Paris)

Contribution ID: **8** Type: **not specified**

Asymptotics for graph complex Euler characteristics

Tuesday, August 30, 2022 11:10 AM (50 minutes)

I will report on a work on the asymptotic growth rate of the top-weight Euler characteristic of the moduli space of curves and on an ongoing joint work with Karen Vogtmann on the topology of Out(Fn). In both cases, graph complexes, which compute the cohomology of the respective spaces, are instrumental. Proofs for the super-exponential asymptotic growth rate of the Euler characteristics in each case establish the existence of large amounts of unexplained cohomology both in Out(Fn) and the top-weight cohomology of the moduli space of curves.

Presenter: BORINSKY, Michael (ETH Zurich)

Contribution ID: **9** Type: **not specified**

Graph complexes, operadic mapping spaces and embedding calculus - a survey

Thursday, September 1, 2022 5:00 PM (50 minutes)

I propose to give an account on results of a collaboration with Victor Turchin and Thomas Willwacher about rational models of operads operads and their applications to the study of the rational homotopy type of embedding spaces.

In a preliminary part, I will give a brief review of the rational homotopy theory of operads. Then I will explain a graph complex description of the rational homotopy of mapping spaces of E_n-operads, and applications of results of the Goodwillie-Weiss calculus of embeddings to check that this computation gives a description of a delooping of embedding spaces of Euclidean spaces. If time permit, I will also explain a generalization of our constructions for the computation of the rational homotopy of the embedding spaces of manifolds into Euclidean spaces.

The homotopy automorphism spaces of E_n-operads represent generalizations of the Grothendieck-Teichmüller, which concern the case $n = 2$. These spaces have a description in terms of graph complexes too, and another option (depending on the interests of the audience) is to explain this result in detail, giving in particular some precision on the computation of the monoid structure associated to these spaces.

Presenter: FRESSE, Benoit (Lille)

Contribution ID: **10** Type: **not specified**

Rational homotopy of embedding spaces: grt(Q) and unitrivalent graphs

Thursday, September 1, 2022 6:00 PM (50 minutes)

I will speak about two distantly related topics. The first one is grt(Q) as subspace of the rational homotopy of spaces of long embeddings $R^m -> R^{2n}$, $2n - m > 2$.
The second one is invariants of embeddings $f : M^m - - > R^n$,$n - m > 2$, in terms of unitrivalent trees that encode the rational homotopy type of the path-component $Emb(M, R^n)_f$ of the embedding space $Emb(M, N)$. The first topic is a joint work with T. Willwacher, while the second one is a joint work with B. Fresse and T. Willwacher.

Presenter: TURCHIN, Victor (Kansas State University)

Contribution ID: **11** Type: **not specified**

Extensions of 1-cocycles of the mapping class group to the Ptolemy groupoid

Wednesday, August 31, 2022 9:00 AM (50 minutes)

For a punctured oriented surface, the isotopy classes of ribbon graph spines on it constitute a cell complex. It can be thought of as a combinatorial model of the Teichmuller space of the surface. The fundamental path groupoid of this cell complex is called the Ptolemy groupoid. In this talk, we discuss various 1-cocycles on the Ptolemy groupoid and the corresponding twisted first cohomology of the mapping class group of the surface. In particular, we focus on a 1-cocycle introduced by Penner in 2012. (Joint work with Kae Takezawa.)

Presenter: KUNO, Yusuke (Tsuda University)

Contribution ID: **12** Type: **not specified**

Poincare duality and TQFT structures for loop spaces

Wednesday, August 31, 2022 10:10 AM (50 minutes)

This talk is about ongoing joint work with Nancy Hingston and Alexandru Oancea.
I will explain how various puzzles in string topology get resolved in terms of symplectic geometry: Loop space homology and cohomology are merged into a larger space, Rabinowitz Floer homology, which carries the structure of a graded TQFT and satisfies Poincare duality.

Presenter: CIELIEBAK, Kai (Augsburg)

Contribution ID: **13** Type: **not specified**

Gravity properad, moduli spaces M_g,n, and string topology

Wednesday, August 31, 2022 11:00 AM (50 minutes)

Using Thomas Willwacher's twisting endofunctor, and Kevin Costello's theory of partially compactified moduli spaces of algebraic curves of arbitrary genus with marked points, we introduce a new dg properad which contains Ezra Getzler's operad controlling genus zero moduli spaces. We discuss its applications in the theory of moduli spaces M_g, n, and in string topology

Presenter: MERKULOV, Sergei (Luxembourg)

Contribution ID: **14** Type: **not specified**

Top weight cohomology of M_{g,n} and the handlebody group

Thursday, September 1, 2022 9:00 AM (50 minutes)

Chan-Galatius-Payne have shown that the homology of Kontsevich's commutative graph complex injects into the compact support cohomology of moduli spaces of curves, in such a way that the image is the weight zero part of the mixed Hodge structure on the target. We explain that this map factors through the homology of the handlebody mapping class groups. This is joint work in progress with my PhD student Louis Hainaut.

Presenter: PETERSEN, Dan (Stockholm)

Contribution ID: **15** Type: **not specified**

Algebraic models for classifying spaces of fibrations

Thursday, September 1, 2022 10:10 AM (50 minutes)

We construct an algebraic model for the rational homotopy type of Baut(X), the classifying space of fibrations with fiber X, for arbitrary simply connected CW-complexes X. As an application, we express the rational cohomology ring of Baut(X) in terms of cohomology of arithmetic groups and dg Lie algebras. In special cases, this leads to connections to modular forms and to graph complexes in the sense of Kontsevich. Another corollary is an algebraicity result for the representations of the homotopy mapping class group of X in the higher rational homotopy groups of Baut(X), which extends a classical result of Sullivan and Wilkerson. This is joint work with Tomas Zeman.

Presenter: BERGLUND, Alexander (Stockholm)

Contribution ID: **16** Type: **not specified**

Automorphisms of seemed surfaces, modular operads and Galois actions

Thursday, September 1, 2022 11:10 AM (50 minutes)

The idea behind Grothendieck-Teichmüller theory is to study the absolute Galois group via its actions on (the collection of all) moduli spaces of genus curves. In practice, this is often done by studying an intermediate object: The Grothendieck-Teichmüller group, GT. In this talk, I'll describe an algebraic gadget built from simple decomposition data of Riemann surfaces. This gadget, called an infinity modular operad, provides a model for the collection of all moduli spaces of genus curves with boundaries, which we justify by showing that the automorphisms of this algebraic object is isomorphic to a subgroup of Grothendieck-Teichmüller group. This is joint work with L. Bonatto.

Presenter: ROBERTSON, Marcy (Melbourne)

Contribution ID: **17** Type: **not specified**

Torelli groups of high-dimensional manifolds

Tuesday, August 30, 2022 5:00 PM (50 minutes)

I will explain joint work with Oscar Randal-Williams, in which we study Torelli groups of the higher-dimensional analogues of surfaces. This is done by combining the work of Galatius–Randal-Williams on stable moduli spaces of manifolds with Goodwillie–Klein–Weiss embedding calculus. Particular attention will be given to the relationship between our results and graph complexes.

Presenter: KUPERS, Alexander (Toronto)

Contribution ID: **18** Type: **not specified**

TBA

Presenter: CHAN, Melody (Brown)

Contribution ID: **19** Type: **not specified**

Failure of invariance of the string topology coproduct

Friday, September 2, 2022 9:00 AM (50 minutes)

Florian Naef showed that the string topology coproduct is not in general invariant under homotopy equivalences, through a lens spaces computation. I will give one point of view on this failure of invariance

Presenter: WAHL, Nathalie (Copenhagen)

Contribution ID: **20** Type: **not specified**

Torsion in string topology

Friday, September 2, 2022 10:10 AM (50 minutes)

I will explain why a particularly simple rational model for string topology (more precisely, the S^1-equivariant version) whose construction was sketched by Cieliebak-Fukaya-Latchev does indeed exist. From this model one can expect that the string coproduct is not a homotopy invariant in general using a connection to the Kashiwara-Vergne problem. This begs the question, what kind of manifold invariant the string coproduct (and string topology in general) is. I will explain how the string coproduct is essentially the Dennis trace of Reidemeister/Whitehead-torsion. This relationship goes through the configuration space of two points.
This is based on joint works with Thomas Willwacher and Pavel Safronov.

Presenter: NAEF, Florian (Dublin)

Contribution ID: **21** Type: **not specified**

Tangles in a Pole Dance Studio: A Reading of Massuyeau, Alekseev, and Naef

Friday, September 2, 2022 11:10 AM (50 minutes)

I will report on joint work with Zsuzsanna Dancso, Tamara Hogan, Jessica Liu, and Nancy Scherich. Little of what we do is original, and much of it is simply a reading of Massuyeau's arXiv:1511.03974 and Alekseev and Naef's arXiv:1708.03119.

We study the pole-strand and strand-strand double filtration on the space of tangles in a pole dance studio (a punctured disk cross an interval), the corresponding homomorphic expansions, and a strand-only HOMFLY-PT relation. When the strands are transparent or nearly transparent to each other we recover and perhaps simplify substantial parts of the work of the aforementioned authors on expansions for the Goldman-Turaev Lie bi-algebra.

Presenter: BAR-NATAN, Dror (Toronto)

Contribution ID: **23** Type: **not specified**

Deformations of the wheeled Lie bialgebra properad

Tuesday, August 30, 2022 6:00 PM (15 minutes)

I give a short description of a recent result of mine. I compute the homotopy deformations of the wheeled Lie bialgebra, and show that it is quasi isomorphic to a subcomplex of the directed kontsevich graph complex consisting of graphs with at least one source and one target vertex. Most interestingly is that this complex is (probably) not quasi-isomorphic to the original Kontsevich graphs complex, which was otherwise suspected.

Presenter: FROST, Oskar (Luxembourg)

Contribution ID: **24** Type: **not specified**

Admissible Integrals and Noncommutative Geometry

Tuesday, August 30, 2022 6:15 PM (15 minutes)

In this short talk I will briefly present admissible integrals, a noncommutative version of the usual integrals on an euclidean space that have to satisfy a property related to integration by part. I will explain all the ingredients in the formula and discuss quickly the existence of such integrals.

Presenter: MONTESSUIT, Valérian (Geneva)

Contribution ID: **25** Type: **not specified**

Generalized Pentagon Equations

Tuesday, August 30, 2022 6:30 PM (15 minutes)

Vladimir Drinfeld defined his KZ associator by considering the monodromy of KZ equation along the real interval from 0 to 1 and proved that it satisfies the Pentagon equation. A natural question is that what kinds of equations will appear if we consider general curves with self intersections. I will try to talk about these equations for general curves, based on joint work with Anton Alekseev and Florian Naef.

Presenter: REN, Muze (Geneva)

Contribution ID: **26** Type: **not specified**

The stabilizer Lie algebra of the harmonic coproduct

Tuesday, August 30, 2022 6:45 PM (15 minutes)

For a finite abelian group G, Racinet constructed a Lie algebra $\mathfrak{dmr}_0^G$, which for $G = \mu_N$ describes double shuffle and regularisation relations between multiple polylogarithm values specialized to N^{th} roots of unity. Enriquez and Furusho then identified this Lie algebra with the stabilizer Lie algebra $\mathfrak{stab}(Delta^M)$ of a coalgebra $(M, Delta^M)$ appearing in Racinet's formalism. On the other hand, Racinet's formalism provides a Hopf algebra (

Presenter: YADDADEN, Khalef (Strasbourg)

Contribution ID: **27** Type: **not specified**

Profinite Grothendieck-Teichmüller theory and completed curve complexes

Thursday, September 1, 2022 4:00 PM (50 minutes)

After surveying the early developments in profinite Grothendieck-Teichmüller theory, I will explain how the introduction of completed curve complexes (as well as other types of complexes) led to some more recent results, including an important rigidity property which enables one to define the profinite version of associators.

Presenter: LOCHAK, Pierre (Paris Jussieu)