DMG4: a fully GEANT4-compatible package for the simulation of Dark Matter

21st International Workshop on Advanced Computing and Analysis Techniques in Physics Research (ACAT 2022)

Henri Sieber on behalf of the DMG4 team

(prologue) Light Dark Matter (LDM): Hidden sectors and the portal formalism

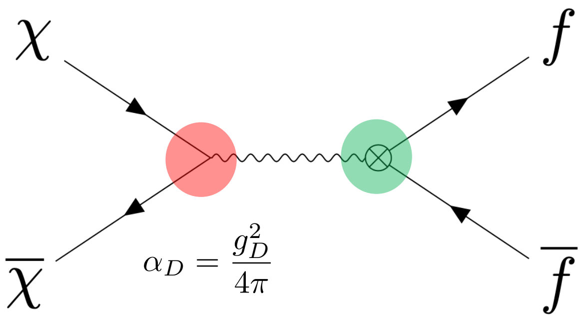
Particle model: interaction of DM with Standard Model (SM) through a **new force** carried by a light mediator

$$\mathcal{L}_{\mathrm{total}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{DS}} + \mathcal{L}_{\mathrm{Portal}}$$

Vector portal (Dark photon, A')
Scalar portal (Dark Higgs, S)
Fermion portal (Heavy Neutral Lepton, N)
Pseudo-scalar portal (ALPs, a)

E.g. the **vector** portal: a massive U(1)’ vector mediator, A’, interacts with SM particles through kinetic mixing Ɛ with SM photons

$$\mathcal{L}_V \supset g_D A'_\mu \bar{\chi}\gamma_\mu \chi + m_\chi \bar{\chi}\chi + \frac{1}{2} m^2_{A'} (A'_\mu)^2 + \frac{\epsilon}{2} F'_{\mu\nu} F^{\mu\nu}$$

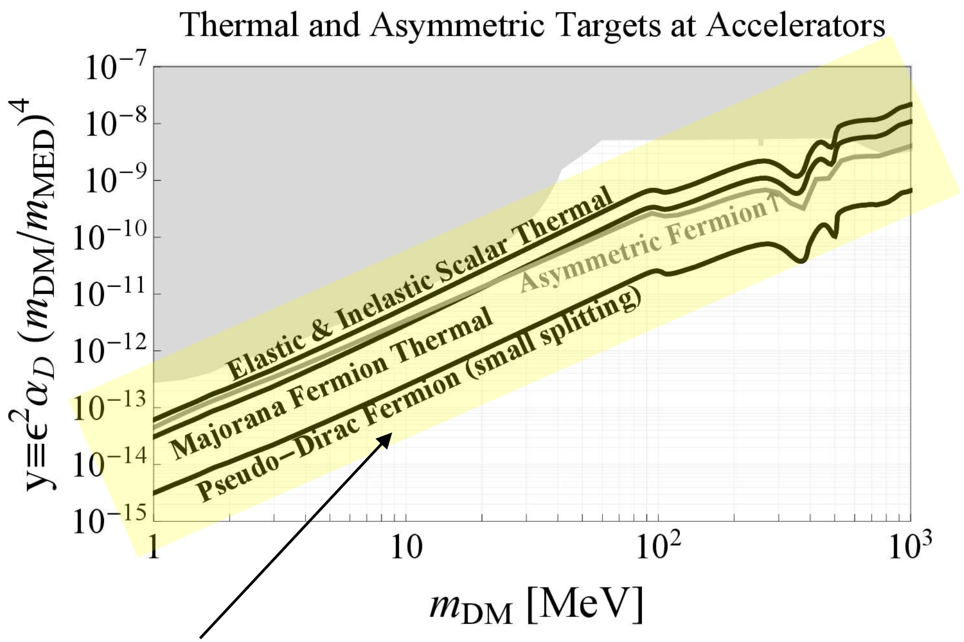


The relic abundance can be expressed in terms of the model **4 parameters**

$$y = \alpha_D \epsilon^2 \left(\frac{m_\chi}{m_{A'}}\right)^4$$

$$\Rightarrow \Omega_\chi \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_\chi^2}{y}$$

for which the correct abundance is obtained through fine –tuning of the **y parameter**.



”thermal target” in {m, y} plane

M. Battaglieri et al., arXiv:1707.04591 **[hep-ph]**

→ In this talk, production of DM at **fixed-target experiments**

(prologue) Events simulations: the GEANT4 toolkit

- Simulations toolkit for the **passage of particles through matter** (Geometry And Track) based on Monte Carlo (MC) methods
- First release (version 0) in December 1998, current version 11.0 (September 2022)
- Widely used in the High Energy Physics (HEP) community

Initialization

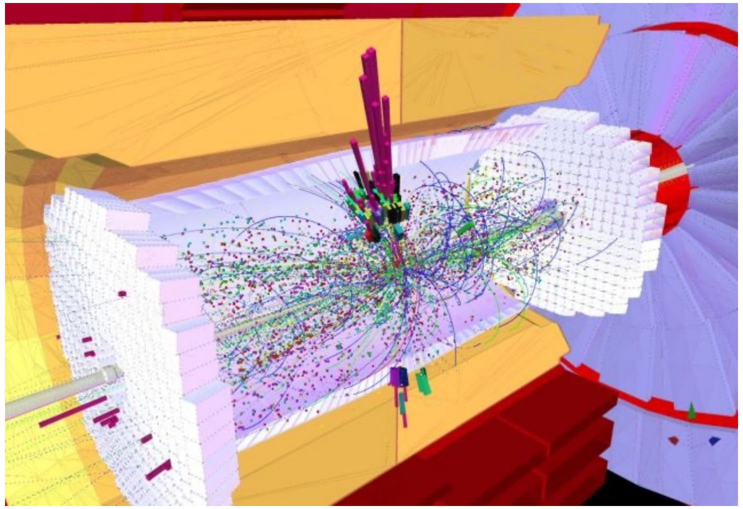
G4VUserDetector Construction

G4VUserPhysics List

Action

G4VUserPrimary GeneratorAction

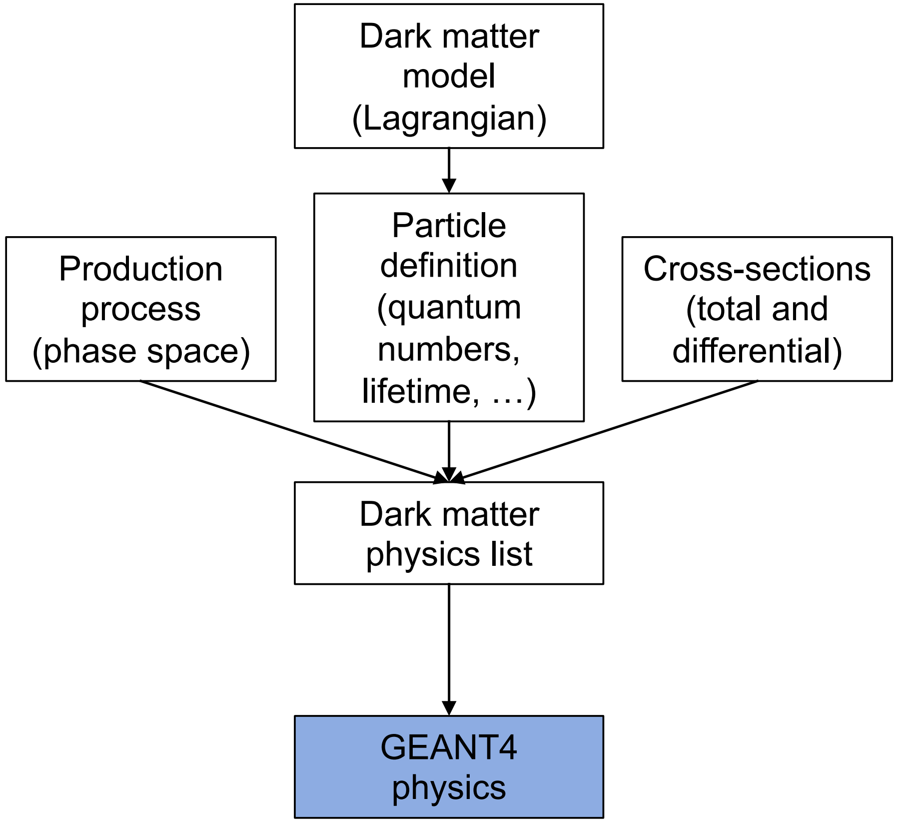
G4UserRunAction …



- Main reference: S. Agostinelli et al., Nucl. Instrum. Methods Phys. Res. A 506 (2003) 250-303

The DMG4 package: overview

- Combine both the **DM event generator** in fixed-target experiments and the propagation of the particle in the geometry (experimental set-up) → natural choice of simulation framework is GEANT4 (well established as HEP software)
- Implementation based on **C++ language**, initially release in its beta version 1.0 in February 2021, latest version dating from October 2022 (**version 2.2**)
- The **main reference** is M. Bondi et al., Comput. Phys. Commun. 269 (2021) 108129
- Open source **code** available for download at http://mkirsano.web.cern.ch/mkirsano/DMG4.tar.gz (to be compiled against GEANT4)



Dark matter model (Lagrangian)

Production process (phase space)

Particle definition (quantum numbers, lifetime, …)

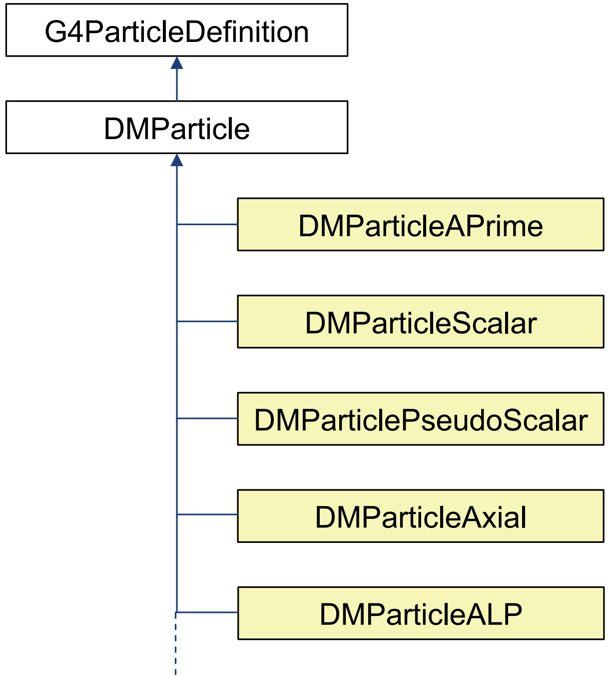
Cross-sections (total and differential)

Dark matter physics list

GEANT4 physics

The DMG4 package: structure

Closely following GEANT application programming interface (API) conventions

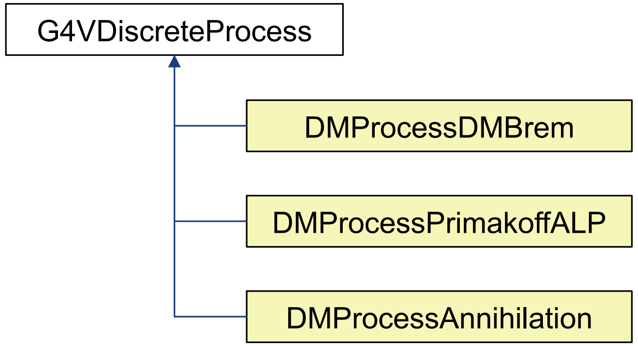


G4ParticleDefinition

DMParticle

- DMParticleAPrime
- DMParticleScalar
- DMParticlePseudoScalar
- DMParticleAxial
- DMParticleALP

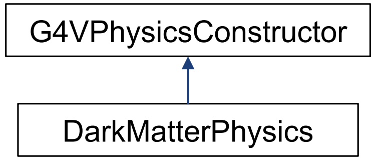
All DM particles provide the methods of the **G4ParticleDefinition** class, among which are available their **quantum numbers** and **decay channels**.



G4VDiscreteProcess

- DMProcessDMBrem
- DMProcessPrimakoffALP
- DMProcessAnnihilation

Three main processes are implemented in DMG4. In particular, the classes methods return the **mean free path** and process with the **phase space** of the final states.



G4VPhysicsConstructor

DarkMatterPhysics

DMG4 provides a **physics list** to be registered in the user initialisation phase of GEANT4. The input parameters of the model can be configured in its method ***DarkMatterPhysicsConfigure***.

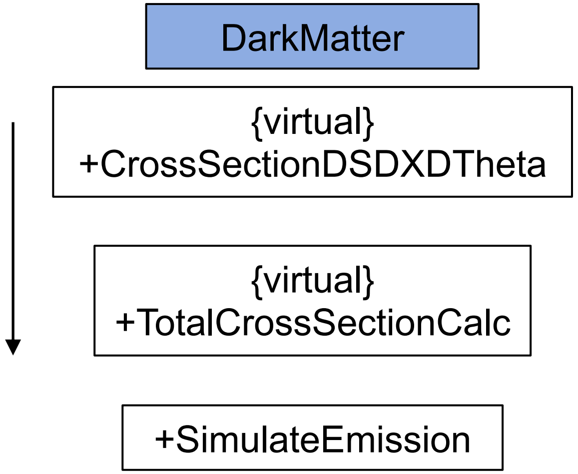
Cross-sections implementations: the DarkMatter sub-package

- The needed formulas (total and differential cross-sections) are derived from the DM model's **Lagrangian** (e.g. vector case)

S. N. Gninenko et al., Phys. Lett. B 782 (2018) 406-411,
Y. Sheng and G. A. Miller, Phys. Rev. D 96 (2017) 016004

$$\mathcal{L}_V \supset \mathcal{L}_{SM} - \frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}m_V^2 V_\mu^2 + \sum_\psi e\epsilon_V V_\mu \bar{\psi}\gamma^\mu\psi + g_V^D V_\mu \bar{\chi}\gamma^\mu\chi + \bar{\chi}(i\gamma^\mu\partial_\mu - m_\chi)\chi$$

$$\left(\frac{d\sigma}{dx\,d\cos\theta}\right)_{ETL} = \frac{\epsilon^2\alpha^3|\mathbf{k}|E_0}{|\mathbf{p}||\mathbf{k}-\mathbf{p}|}\cdot\int_{t_{min}}^{t_{max}}\frac{dt}{t^2}G_2^{elec}(t)\cdot\int_0^{2\pi}\frac{d\phi_q}{2\pi}\frac{|A_X^{2\to 3}|^2}{8M^2}$$

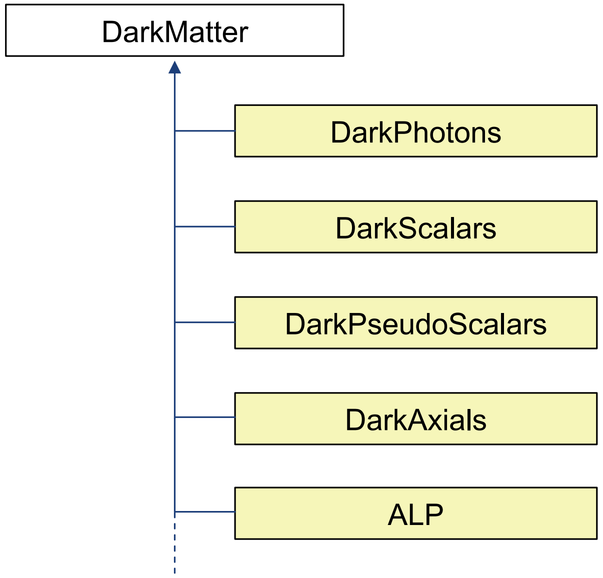


DarkMatter

{virtual}
+CrossSectionDSDXDTheta

{virtual}
+TotalCrossSectionCalc

+SimulateEmission



DarkMatter

DarkPhotons

DarkScalars

DarkPseudoScalars

DarkAxials

ALP

Each DM particle inherits from the DarkMatter methods, among which the computations of the **tabulated cross-sections** needed for GEANT4 physics

- **GEANT4-independent**, results can be used with any user-program in the **mass range** < 1 MeV up to few GeV, **energy range** O(10-100 GeV)

A phase-space approximation: the Weizsäcker-William (WW) approach

- ETL-based computations are **computationally expensive** (four integrals)

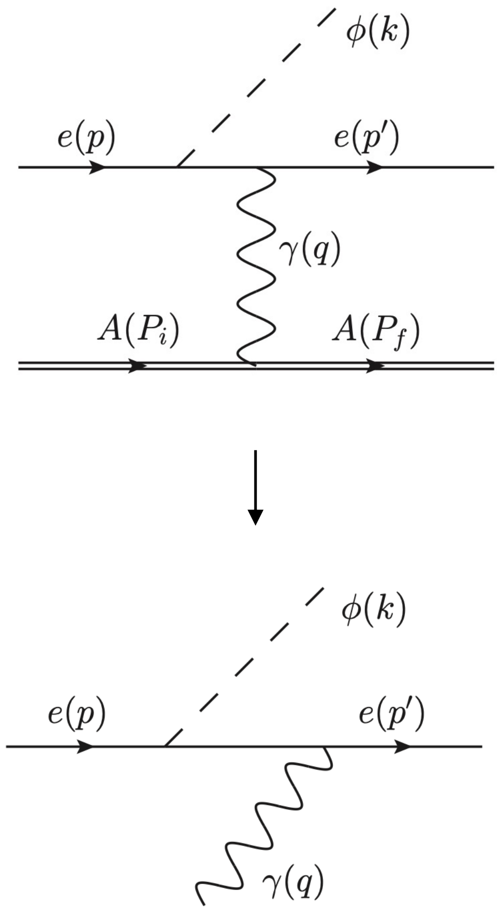
- In the regime where the initial energy $\mathrm{E_0} \gg \mathrm{m_X}, \mathrm{m_l}$, the virtual photon flux can be approximate by a **real photon** (Weizsäcker-William approximation), and the process transforms from **2→3 to 2→2**

$$\left.\frac{d\sigma(p + P_i \to k + p' + P_f)}{d(pk)d(kP_i)}\right|_{WW} = \frac{\alpha\chi^{WW}}{\pi(p'P_i)} \cdot \left.\frac{d\sigma(p + q \to k + p')}{d(pk)}\right|_{t=t_{min}}$$

$$\chi^{WW} = \int\limits_{t_{min}}^{t_{max}} dt \frac{t - t_{min}}{t^2} F^2(t)$$

- The flux can be **analytically integrated** for faster run-time computations of the tabulated cross-sections and sampling of the final particles phase-space

- This approach is adopted for **muon-beam based processes**, with uncertainties w.r.t the ETL of a **few percents**. Work in progress for electron-beam



D. V. Kirpichnikov et al., Phys. Rev. D 104 (2021) 076012, Y. Sheng and G. A. Miller, Phys. Rev. D 96 (2017) 016004, …

The improved WW (IWW) method and K-factors

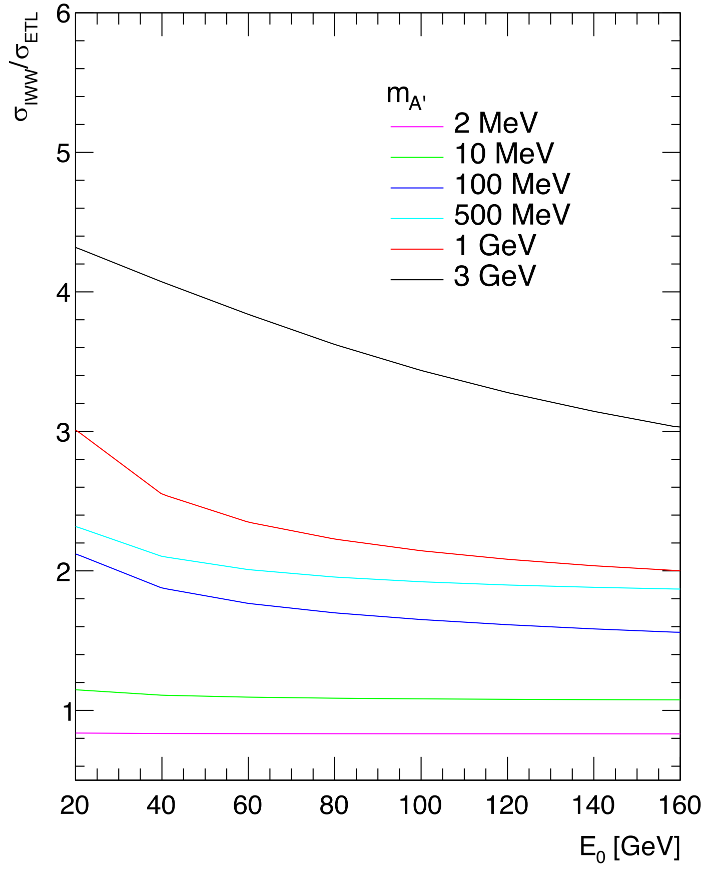
- Further approximation of the WW approach: t_{min} in the bound of the flux integral is **independent** of the fractional energy and emission angle of the DM particle

- IWW approximation introduces large uncertainties w.r.t the ETL calculations. Those can be corrected using **K-factors** (in electron-beam bremsstrahlung processes)

$$K(m_{A'}, E_0, Z, A) = \frac{\sigma_{IWW}}{\sigma_{ETL}}$$

- In DMG4, K-factors are **tabulated** over the mass range of a few MeV up to few GeV, and beam energy few GeV up to O(100 GeV). ETL is calculated through the Mathematica symbolic language

- At run-time, the total cross-section is obtained through **interpolation** of the reference tables



S. N. Gninenko et al., Phys. Lett. B 782 (2018) 406-411

Particle decays

- DM particles decays cover **invisible**, **semi-visible** and **visible** modes

- All decay widths are computed at **ETL** for vector, axial, scalar and pseudo-scalar

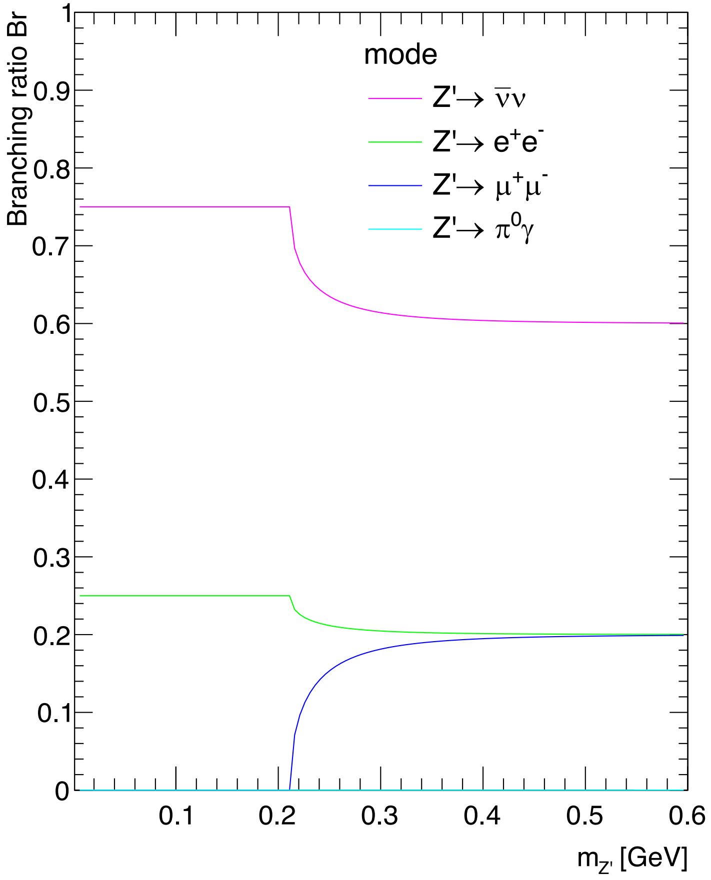
$$\Gamma(V \to e^+e^-) = \frac{\alpha\epsilon^2}{3} m_V \left(1 + \frac{2m_e^2}{m_V^2}\right)\sqrt{1 - \frac{4m_e^2}{m_V^2}},\ \Gamma(V \to \bar{\chi}\chi) = \frac{\alpha_D}{3} m_V \left(1 + \frac{2m_\chi^2}{m_V^2}\right)\sqrt{1 - \frac{4m_\chi^2}{m_V^2}}, \ldots$$

- Decay channels are integrated following the GEANT4 conventions **G4VDecayChannel** in the particle definition, and the corresponding branching ratio computed accordingly

- The type of decay for a given model is handled by the user in the parameters configuration. In practice, decays can also be **forced** by the user to happen in a given region (not part of DMG4)

- New B-L models, as well as (Dirac) **inelastic DM** (iDM), spin-2 boson have recently been implemented

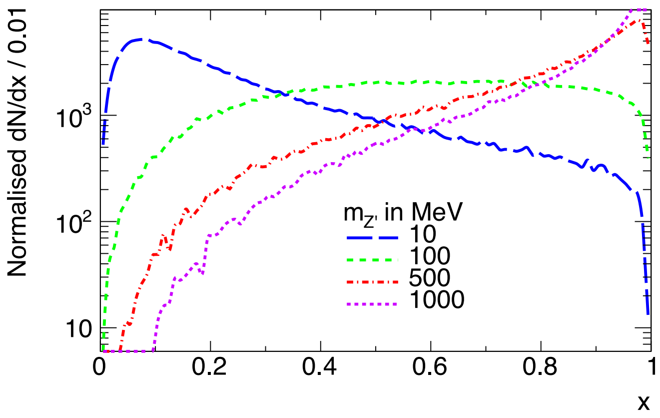
Y. M. Andreev et al., Phys. Rev. Lett. 129 (2022) 161801

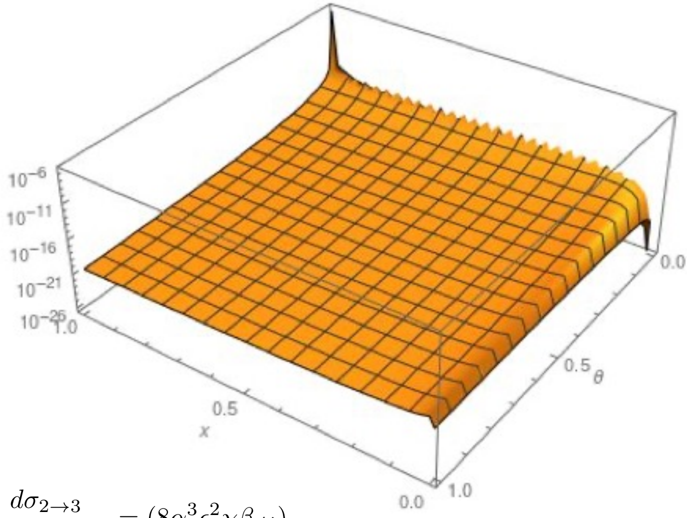


Sampling of the phase-space

- Final-state particles phase-space is sampled using **direct Von-Neuman** accept/reject method
- For electron-beam based processes, cross-sections are **sharply peaked**, especially in the heavy DM scenarios → lots of rejected samples (sub-optimal)
- Implementation of modified Von-Neuman method based on **majorant functions** (closely following the target PDF in the maximum vicinity, and greater than the PDF in the full domain) → 2-step sampling
- Sampling performed over the different **kinematical variables** of the DM particle (x, θ) or final state lepton (y, ϕ) in the muon-based processes

H. Sieber et al., Phys. Rev. D 105 (2022) 052006





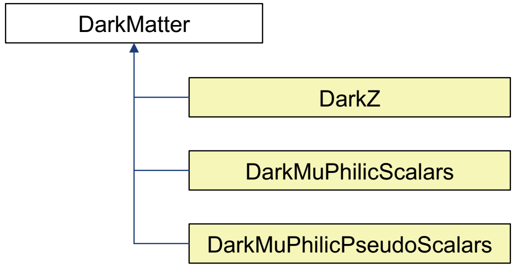
$$\frac{1}{E_0^2 x}\frac{d\sigma_{2\to3}}{dx\,d\cos\theta_{A'}} = (8\alpha^3\epsilon^2\chi\beta_{A'}) \times\left(\frac{1-x+x^2}{U^2}+\frac{(1-x)^2 m_{A'}^2}{U^4}(m_{A'}^2-\frac{Ux}{1-x})\right)$$

Recent developments in muon-philic DM production

- In addition to vector model (version 1.0), implementation of **scalar** and **pseudo-scalar** DM production

$$\mathcal{L} \supset \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 + \epsilon_S e S \bar{\mu}\mu$$

$$\mathcal{L} \supset \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu P)^2 - \frac{1}{2}m_P^2 P^2 + i\epsilon_P e P \bar{\mu}\gamma_5\mu$$



DarkMatter

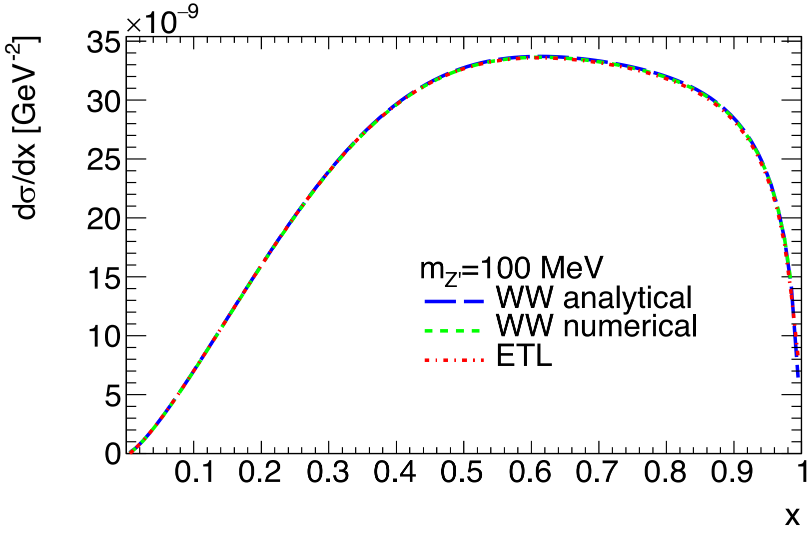
DarkZ

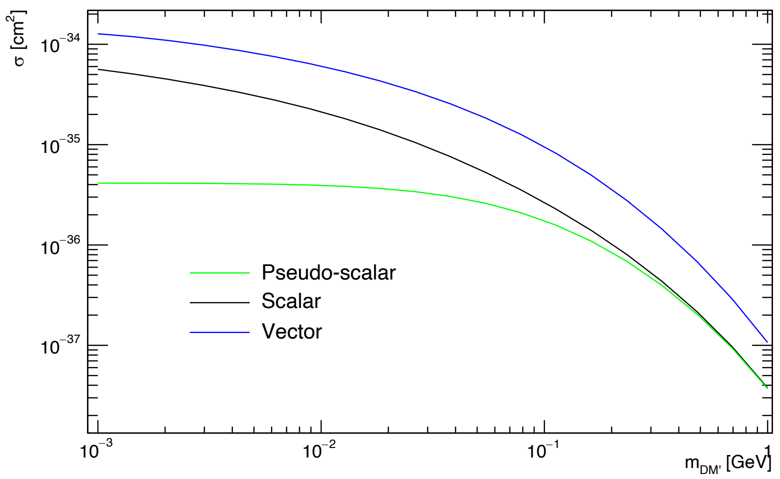
DarkMuPhilicScalars

DarkMuPhilicPseudoScalars

- Improvement of WW computations with an **analytical expression** for the differential cross-section dσ/dx (integration of the flux w.r.t θ) → faster run-time cross-section computations

$$\left.\frac{d\sigma_{2\to3}}{dx\,d\cos\theta}\right|_{WW} = \frac{\alpha\chi^{WW}}{\pi(1-x)}E_0^2 x\beta \cdot \left.\frac{d\sigma_{2\to2}}{d(pk)}\right|_{t=t_{min}} \to \left.\frac{d\sigma}{dx}\right|_{WW}$$





D. V. Kirpichnikov et al., *to be published*

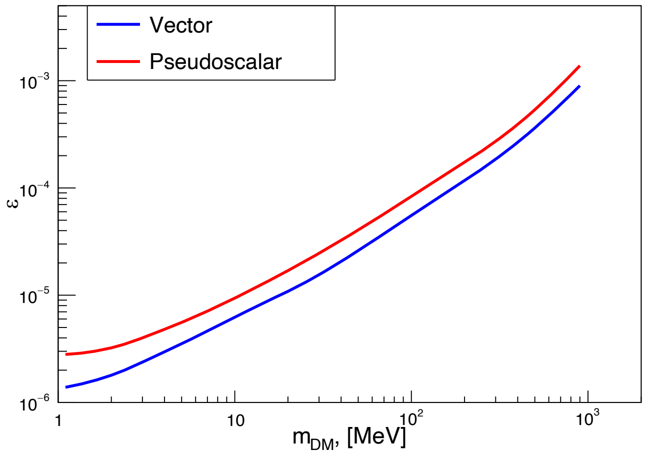
Beam-dump experiments sensitivity

- Given an experimental set-up and selection cuts, DMG4 enables a realistic study of the **sensitivity** of an experiment with a full set of simulations

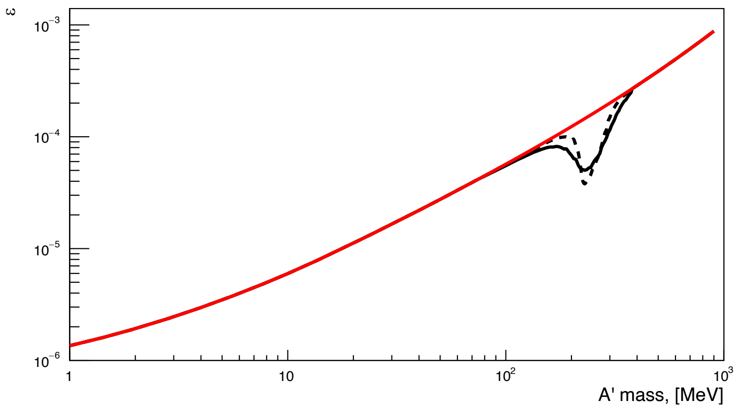
- To cope with the extremely low DM production rate, **biasing of the cross-section** can be introduced to generate a reasonable fraction of DM events in the set-up (*BiasSigmaFactor* parameter) → **observe DM** propagation in the set-up and optimize **event selection** (signal to background)

- Final sensitivity is **normalised** w.r.t the biasing factor to retrieve the expected number of DM events

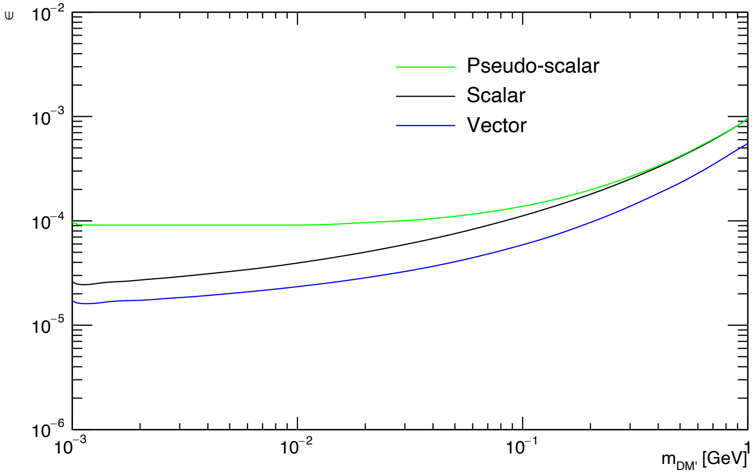
Electrons



Positrons



Muons



Summary and outlook

- DMG4 is a simulation package for DM production at **fixed-target experiments**
- The package can be easily integrated in the **GEANT4 simulation** toolkit and is conveniently built for adding **new models** (recent study of e → μ conversion)
- The DarkMatter sub-package is **fully-independent** from GEANT4, and can be used in other user-defined applications

Version 2.2	Future developments
- Vector, axial, scalar and pseudo-scalar	- Millicharged particles
- ALP	- $\mu \to \tau$, $e \to \tau$ conversions
- Electron-, muon- and positron-based processes	- WW for electrons
- $B - L$ Z' model, inelastic DM decay (iDM)	- Spin-2 dark boson mediator
	- hadronic processes ?

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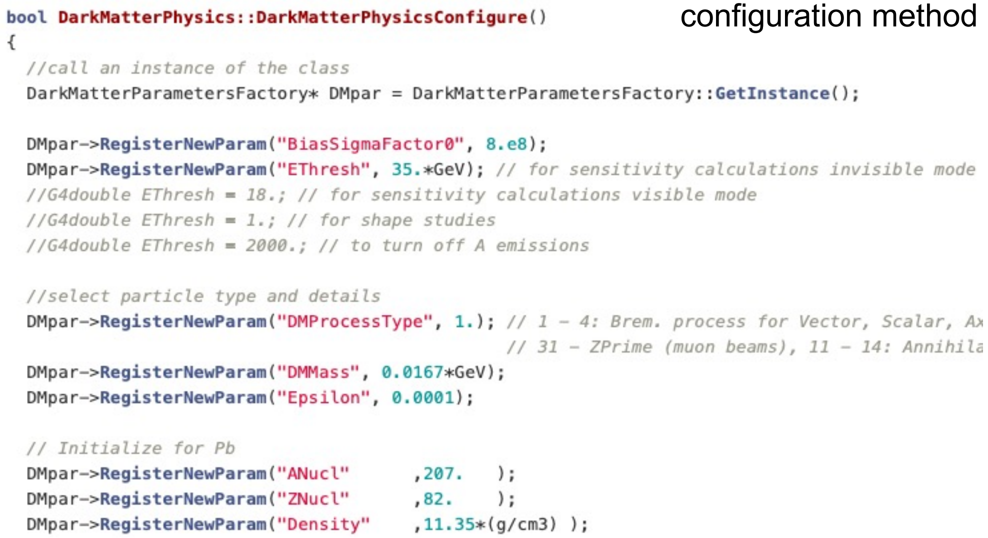
Back-up

A simple example

- DMG4 repository contains a non-exhaustive lists of short **examples**

configuration method



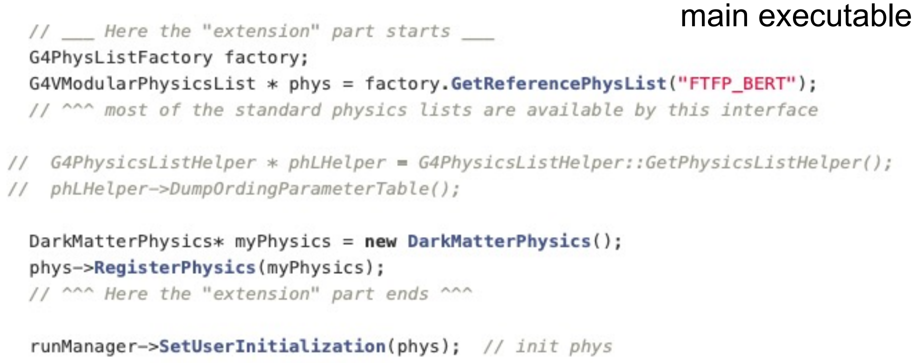
```
bool DarkMatterPhysics::DarkMatterPhysicsConfigure()
{
  //call an instance of the class
  DarkMatterParametersFactory* DMpar = DarkMatterParametersFactory::GetInstance();

  DMpar->RegisterNewParam("BiasSigmaFactor0", 8.e8);
  DMpar->RegisterNewParam("EThresh", 35.*GeV); // for sensitivity calculations invisible mode
  //G4double EThresh = 18.; // for sensitivity calculations visible mode
  //G4double EThresh = 1.; // for shape studies
  //G4double EThresh = 2000.; // to turn off A emissions

  //select particle type and details
  DMpar->RegisterNewParam("DMProcessType", 1.); // 1 - 4: Brem. process for Vector, Scalar, Ax
                                                // 31 - ZPrime (muon beams), 11 - 14: Annihila
  DMpar->RegisterNewParam("DMMass", 0.0167*GeV);
  DMpar->RegisterNewParam("Epsilon", 0.0001);

  // Initialize for Pb
  DMpar->RegisterNewParam("ANucl"     ,207.   );
  DMpar->RegisterNewParam("ZNucl"     ,82.    );
  DMpar->RegisterNewParam("Density"   ,11.35*(g/cm3) );
```

main executable



```
  // ___ Here the "extension" part starts ___
  G4PhysListFactory factory;
  G4VModularPhysicsList * phys = factory.GetReferencePhysList("FTFP_BERT");
  // ^^^ most of the standard physics lists are available by this interface

//  G4PhysicsListHelper * phLHelper = G4PhysicsListHelper::GetPhysicsListHelper();
//  phLHelper->DumpOrdingParameterTable();

  DarkMatterPhysics* myPhysics = new DarkMatterPhysics();
  phys->RegisterPhysics(myPhysics);
  // ^^^ Here the "extension" part ends ^^^

  runManager->SetUserInitialization(phys);  // init phys
```

physics list

```
#pragma once

#include <G4VPhysicsConstructor.hh>

class DarkMatter;

class DarkMatterPhysics : public G4VPhysicsConstructor {
  public:

    DarkMatterPhysics();
    ~DarkMatterPhysics();
    bool DarkMatterPhysicsConfigure();

    //A.C. I introduced this constructor to pass any data to DarkMatterP
    DarkMatterPhysics(void *ptr);
    bool DarkMatterPhysicsConfigure(void *ptr);


    void Init();



    // Should call initial constructor of particle singletons
    virtual void ConstructParticle() override;
    virtual void ConstructProcess() override;
    DarkMatter* GetDarkMatterPointer() {return myDarkMatter;}
    G4double GetBiasSigmaFactor() {return BiasSigmaFactor;}
  private:
    DarkMatterPhysics(const DarkMatterPhysics &) = delete;
    DarkMatterPhysics & operator=(const DarkMatterPhysics &) = delete;
  private:
    DarkMatter* myDarkMatter;
    G4double BiasSigmaFactor;
};
```

Dark Matter particles in the DMG4 package

- Overview of the available **DM particles** in DMG4

Name	PDG ID	emitted by	spin	parity	stable?	decay
DMParticleAPrime	5500022	e^+, e^-	1	1	true/false	$\bar{\nu}\nu$, e^+e^-, $\mu^+\mu^-$, $\pi^0\gamma$, $\chi_1\chi_2$, $\chi_1\chi_1$, $\chi_2\chi_2$
DMParticleScalar	5400022	e^+, e^-	0	1	true/false	e^+e^-
DMParticlePseudoScalar	5410022	e^+, e^-	0	-1	true/false	e^+e^-
DMParticleAxial	5510022	e^+, e^-	1	-1	true/false	e^+e^-
DMParticleZPrime	5500023	μ	1	1	true/false	$\bar{\nu}\nu$, $\mu^+\mu^-$
DMParticleALP	5300122	γ	0	-1	false	$\gamma\gamma$
DMParticleChi	5200012	A'	1/2	1	true	-
DMParticleChi1	5200014	A'	1/2	1	true	-
DMParticleChi2	5200013	A'	1/2	1	false	$\chi_1 e^+e^-$

Dark Matter models in the DarkMatter sub-package

- Overview of the available models (cross-sections, …) in the DarkMatter sub-package

Model	Parent PDG
DarkPhotons(Annihilation)	$e^-(e^+)$
DarkScalars(Annihilation)	$e^-(e^+)$
DarkPseudoScalars(Annihilation)	$e^-(e^+)$
DarkAxials(Annihilation)	$e^-(e^+)$
DarkMassSpin2(Annihilation)	$e^-(e^+)$
ALP	γ
DarkVector	e^-
DarkZ	μ
DarkMuPhilicScalars	μ
DarkMuPhilicPseudoScalars	μ

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