Two-loop five-point amplitudes in massless QCD with finite fields

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ACAT

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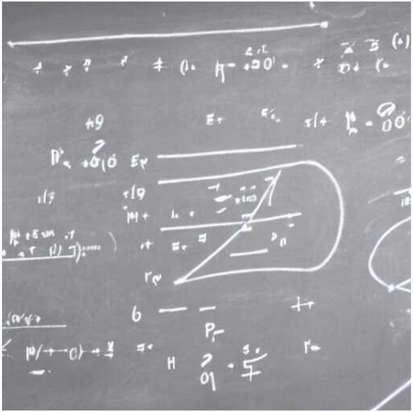
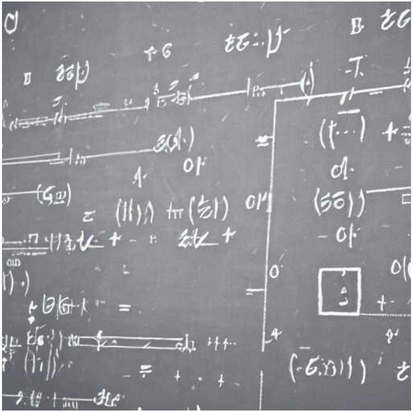
Outline



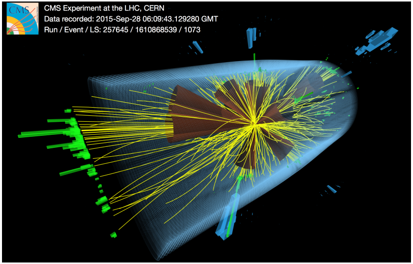


Precision frontier

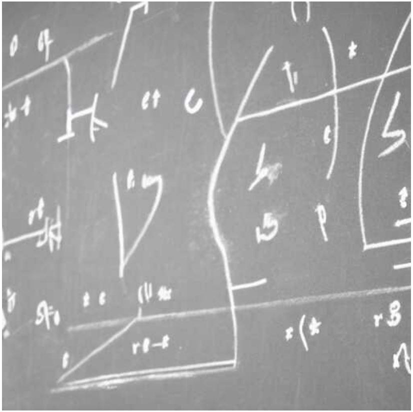
- Better understand properties of SM
- Indirect probe of new physics by small deviations
- LHC demanding increasing precision
- Theory predictions at 1%
- Requires $\geq$ NNLO QCD

$$\alpha_s \approx 0.1$$

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_s \, \mathrm{d}\sigma_{\mathrm{NLO}} + \alpha_s^2 \, \mathrm{d}\sigma_{\mathrm{NNLO}} + \ldots$$



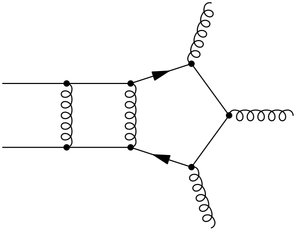
[McCauley et al. 2015]

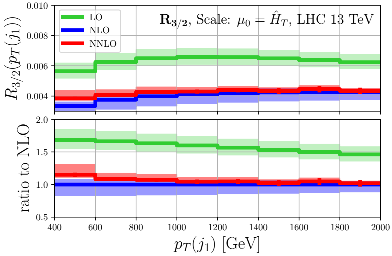


Trijet production

- NNLO leading-colour $pp \to jjj$

[Badger, Brønnum-Hansen, Hartanto, **RM**, Peraro, Zoia n.d.]





[Czakon, Mitov, Poncelet 2021]

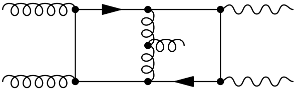
- Probe fundamental parameters of SM, $R_{3/2} \to \alpha_s$
- Significant theory uncertainty reduction

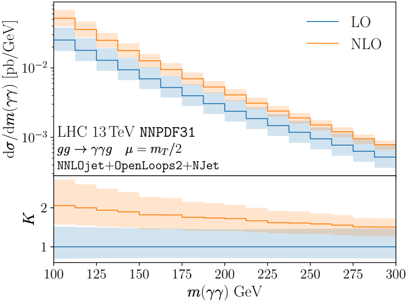
[Abreu, Febres Cordero, Ita, Page, Sotnikov 2021] [Czakon, Mitov, Poncelet 2021]

Diphoton-plus-jet production via gluon fusion

- NLO full-colour $gg \to g\gamma\gamma$ (N^{3}LO $pp \to g\gamma\gamma$)

[Badger, Brønnum-Hansen, Chicherin, Gehrmann, Hartanto, Henn, Marcoli, **RM**, Peraro, Zoia 2021]

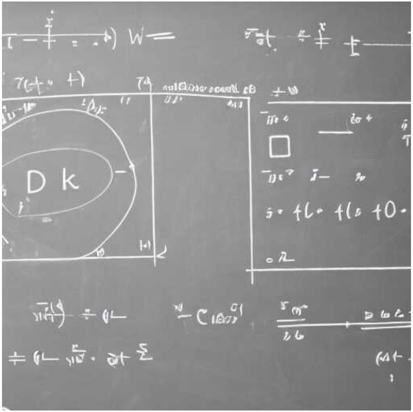




[Badger, Gehrmann, Marcoli, **RM** 2022]

- Important background for measuring Higgs properties
- Significant NLO corrections, enhances NNLO $pp \to g\gamma\gamma$

[Chawdhry, Czakon, Mitov, Poncelet 2021]



Five-point two-loop computation

- Major theoretical challenge
  - New methods
- Reconstruct over finite fields

  [Peraro 2019]
- Pentagon function basis

  [Gehrmann, Henn, Lo Presti 2018]

  [Chicherin, Sotnikov 2020]

  [Chicherin, Sotnikov, Zoia 2022]
- Fast and stable `C++` implementation in `NJet`

  [Badger, Biedermann, **RM**, Uwer, Yundin 2021]

Method overview

Helicity amplitudes

$\sum_i \mathrm{diagram}_i(\boldsymbol{p}, \epsilon)$

qgraf [Nogueira 1993]

FORM [Ruijl, Ueda, Vermaseren 2017]

FiniteFlow [Peraro 2019]

Finite fields

Algebra → $\sum_i \boldsymbol{a}_i(\boldsymbol{p}, \epsilon) \cdot \boldsymbol{F}_i(\boldsymbol{p})$

Global integrand map ↓

$\sum_i b_i(\boldsymbol{p}, \epsilon)\, \mathcal{F}_i(\boldsymbol{p})$

LiteRED [Lee 2012]

IBP identities ↓

Laporta [Laporta 2000]

$\sum_i c_i(\boldsymbol{p}, \epsilon)\, \mathrm{MI}_i(\boldsymbol{p})$

Special function basis ↓

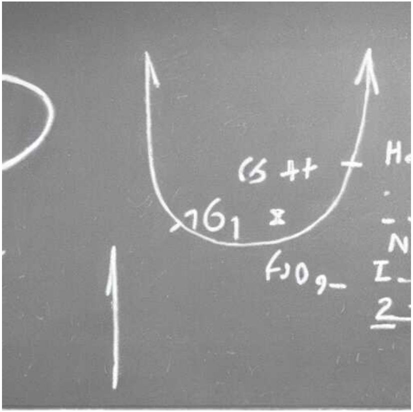
PentagonFunctions++ [Chicherin, Sotnikov, Zoia 2022]

$\sum_i d_i(\boldsymbol{p}, \epsilon)\, \mathrm{mon}_i(\boldsymbol{f}\,(\boldsymbol{p}))$

Subtract poles ↓

Dipole scheme [Catani, Seymour 1997]

$\sum_i e_i(\boldsymbol{p})\, \mathrm{mon}_i(\boldsymbol{f}\,(\boldsymbol{p}))$



Finite field arithmetic

- Set of $n \in \mathbb{P}^p$ non-negative integers

$$\mathbb{F}_n = \{0, \ldots, n-1\}$$

- Arithmetic operations modulo n

$$+ \quad - \quad \times \quad \div$$

- Modular multiplicative inverse $x = a^{-1} \mod n$

$$ax = 1 \mod n \qquad\qquad a \neq 0$$

- One-to-many map $\mathbb{Q} \to \mathbb{F}_n$

$$\frac{a}{b} \to a\, b^{-1} \mod n$$

Numeric representation for computation

- `float` Catastrophic cancellation
- `int` Overflow
- $\mathbb{Q}$ Slow
- $\mathbb{F}_n$
  - No precision loss (n large)
  - Fast: fixed size integer
  - Recover $\mathbb{F}_n \to \mathbb{Q}$
  - Chinese Remainder Theorem: $\{\mathbb{F}_n\} \to \mathbb{F}_{\prod n}$

- Amplitudes
  - Large intermediate expressions
  - Bypass with numerical evaluation over $\mathbb{F}_n$
  - Reconstruct analytic expression

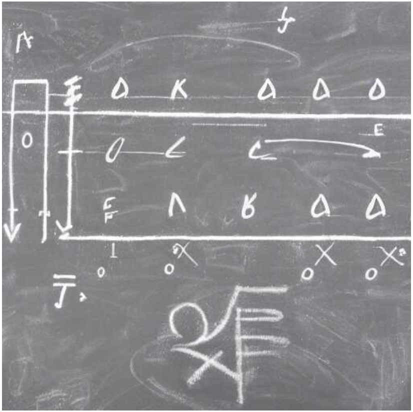
Rational on-shell parametrisation

- Momentum twistor variables

$$\langle ij \rangle, [ij] \to x_i$$

$$Z = \begin{pmatrix} \lambda_1 & \cdots & \lambda_n \\ \mu_1(\tilde{\boldsymbol{\lambda}}) & \cdots & \mu_n(\tilde{\boldsymbol{\lambda}}) \end{pmatrix}$$

- Rational functions
  - $p_i(\boldsymbol{x}), s_{ij}(\boldsymbol{x}), \mathrm{tr}_5(\boldsymbol{x}), \ldots$
- $\boldsymbol{x}$ unconstrained
  - On-shell
  - Momentum conserving

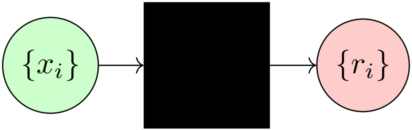


Reconstruction

- Amplitude components

$$F(\boldsymbol{x}) = \sum_i r_i(\boldsymbol{x}) \, \mathrm{mon}_i(f)$$

- Have numerical algorithm for r_i
- Reconstruct expression from sufficient evaluations
- `FiniteFlow` [Peraro 2019]



- Strategies to optimise and compactify

Linear relations in the coefficients

- Linearise the r_i

$$\{r_i\}_{i\in S} \to \{r_i\}_{i\in T\subseteq S}$$

- Numerically solve:

$$\sum_i a_i\, r_i(\boldsymbol{x}) = 0$$

and choose lowest degrees

Matching factors

- Coefficient ansatz

$$r(\boldsymbol{x}) = \frac{n(\boldsymbol{x})}{\prod_k {\ell_k}^{e_k}(\boldsymbol{x})} \qquad \ell_k \in \text{pentagon alphabet}$$

- Determine e_k by reconstructing r on univariate slice

$$\boldsymbol{x} = \boldsymbol{c}_0 + \boldsymbol{c}_1 t \qquad \rightsquigarrow \qquad r(t)$$

  and matching RHS
- Fix denominators

$$\{r_i\} \to \{n_i\}$$

[Abreu, Dormans, Febres Cordero, Ita, Page 2019]

Univariate partial fraction decomposition

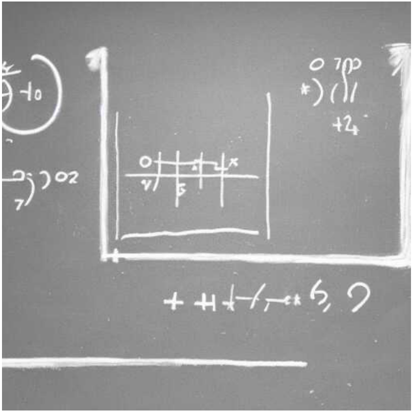
- Example in y

$$\frac{n(x,y)}{x^2y^2(x^2+y^2)} = q_0(x)y^{d_n-d_d} + \frac{q_1(x)}{y} + \frac{q_2(x)+q_3(x)y}{y^2} + \frac{q_4(x)+q_5(x)y}{x^2+y^2}$$

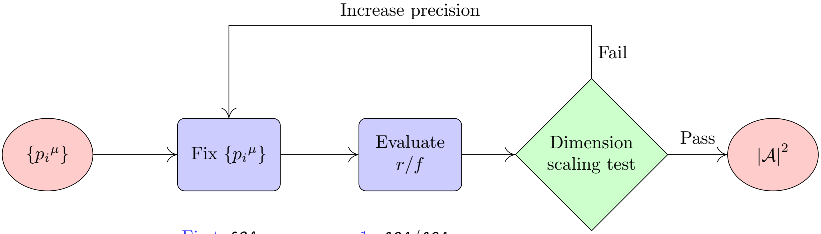
  - Only need to know y degree of n, d_n
  - Choose y by studying one-loop
- Reconstruct r_i directly in decomposed form

$$\{r_i(\overline{\boldsymbol{x}},y)\} \to \{q_i(\overline{\boldsymbol{x}})\}$$

  - Reduce variables by one
  - Lower degrees



Evaluation strategy



First `f64`

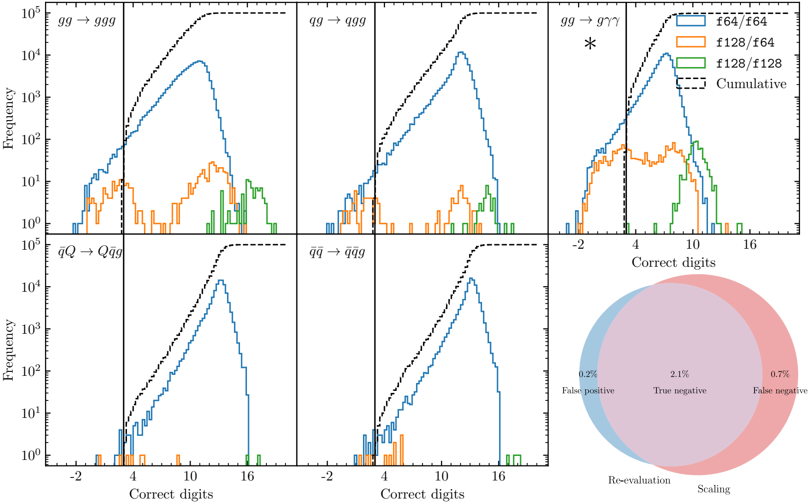
Else `f128`

1. `f64/f64`
2. `f128/f64`
3. `f128/f128`

Timing

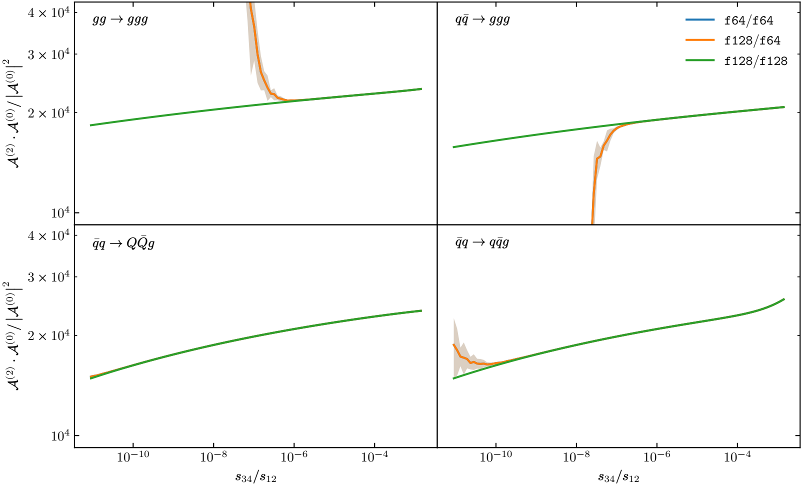
Channel	`f64/f64`		Evaluation strategy	
	Time (s)	f (%)	Time (s)	f (%)
$gg \to ggg$	1.39	69	1.89	77
$gg \to \bar{q}qg$	1.35	91	1.37	91
$qg \to qgg$	1.34	92	1.57	93
$q\bar{q} \to ggg$	1.34	93	1.38	93
$\bar{q}Q \to Q\bar{q}g$	1.14	99	1.16	99
$\bar{q}\bar{Q} \to \bar{q}\bar{Q}g$	1.36	99	1.39	99
$\bar{q}g \to \bar{q}Q\bar{Q}$	1.36	99	1.39	99
$\bar{q}q \to Q\bar{Q}g$	1.14	99	1.14	99
$\bar{q}g \to \bar{q}q\bar{q}$	1.84	99	1.90	99
$\bar{q}\bar{q} \to \bar{q}\bar{q}g$	1.82	99	1.94	99
$\bar{q}q \to q\bar{q}g$	1.71	99	1.77	99
$gg \to \gamma\gamma g$ *	9	99	26	99

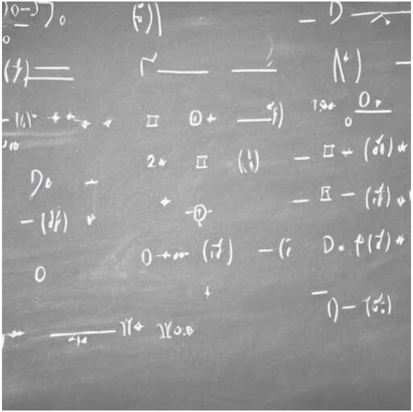
Stability



$gg \to ggg$
$gg \to q\bar{q}g$
$gg \to g\gamma\gamma$
*
f64/f64
f128/f64
f128/f128
Cumulative
$\bar{q}Q \to Q\bar{q}g$
$\bar{q}\bar{q} \to \bar{q}\bar{q}g$
Frequency
Correct digits
-2
4
10
16
10^0
10^1
10^2
10^3
10^4
10^5
0.2%
False positive
2.1%
True negative
0.7%
False negative
Re-evaluation
Scaling

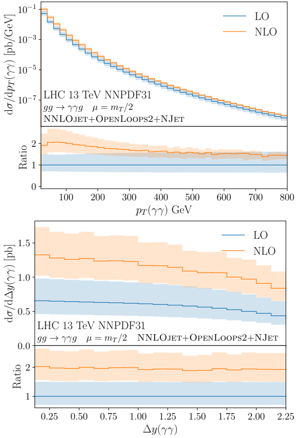
IR performance





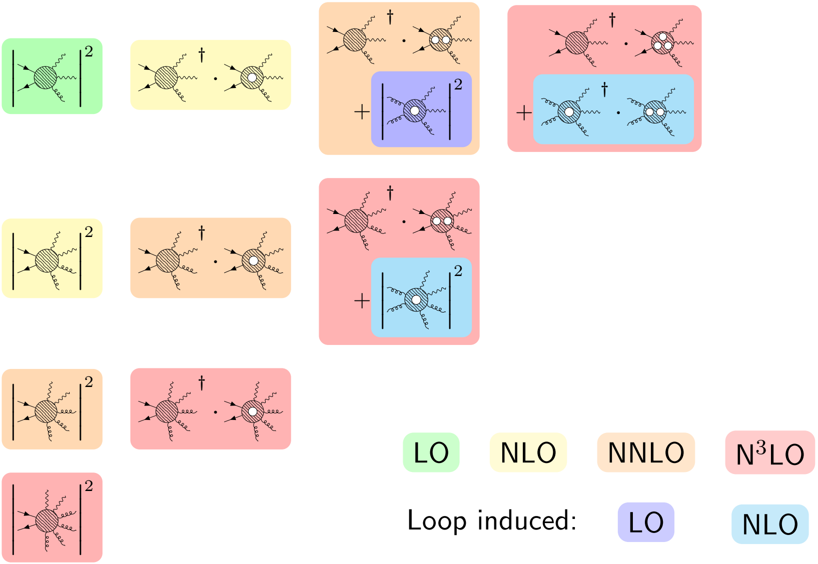
Conclusion

- Huge success of technical advancements in precision QCD
  - Numerical evaluation over finite fields
  - Bypass intermediate complexity
  - Reconstruct compact analytic forms
- Efficient and stable evaluation in `NJet`
  - $pp \to jjj$ (LC)
  - $gg \to g\gamma\gamma$
- Driving pheno predictions towards 1% precision



[Badger, Gehrmann, Marcoli, **RM** 2022]

$pp \to g\gamma\gamma$



Implementation

- Finite remainders coded up in `C++` as analytic expressions
- Construct partial amplitudes as

$$F^h = r_i^h(\boldsymbol{x})\, M_{ij}^h \; f_j^h$$

f_j^h	special function monomials	global
M_{ij}^h	rational sparse matrices	partials
r_i^h	independent rational coefficients	helicities
$\boldsymbol{x}$	momentum twistor variables	global

- Independent helicities permuted to all mostly-plus
  - Mostly-minus: ${r_i}^* \, M_{ij} \; P(f_j)$

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