Quantum annealing applications in high-energy physics

ACAT 2022

S.A. Abel, JCC, M. Spannowsky, 2202.11727

JCC, M. Spannowsky, 2204.03657

JCC, R. Kogler, M. Spannowsky, 2207.10088

Juan Carlos Criado (IPPP Durham)

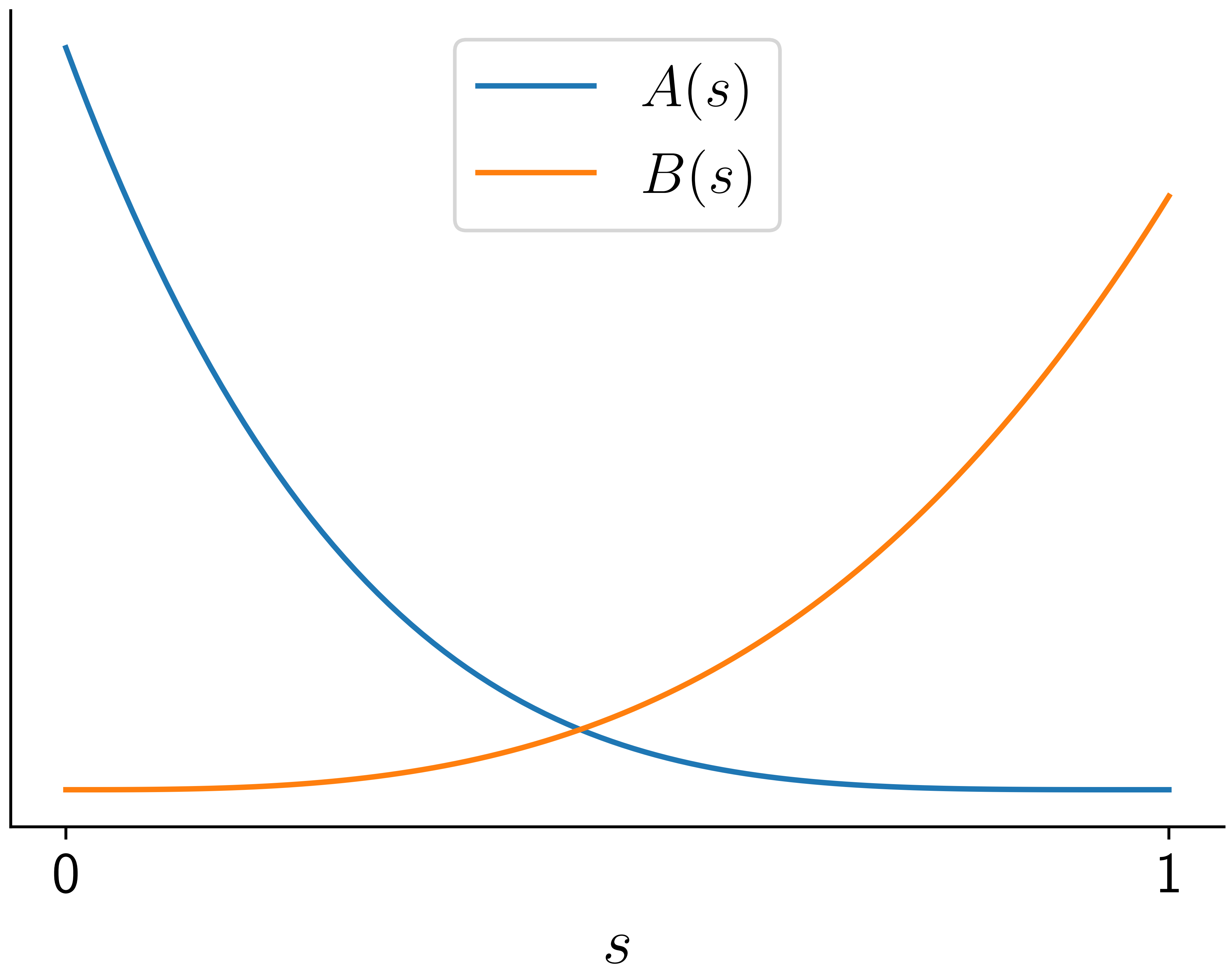
Classical and quantum optimisation

Classical	Minimize the loss function $\mathscr{L}(x)$
Quantum	Find the ground state of the Hamiltonian H

Machine learning, fits, …

Quantum annealing

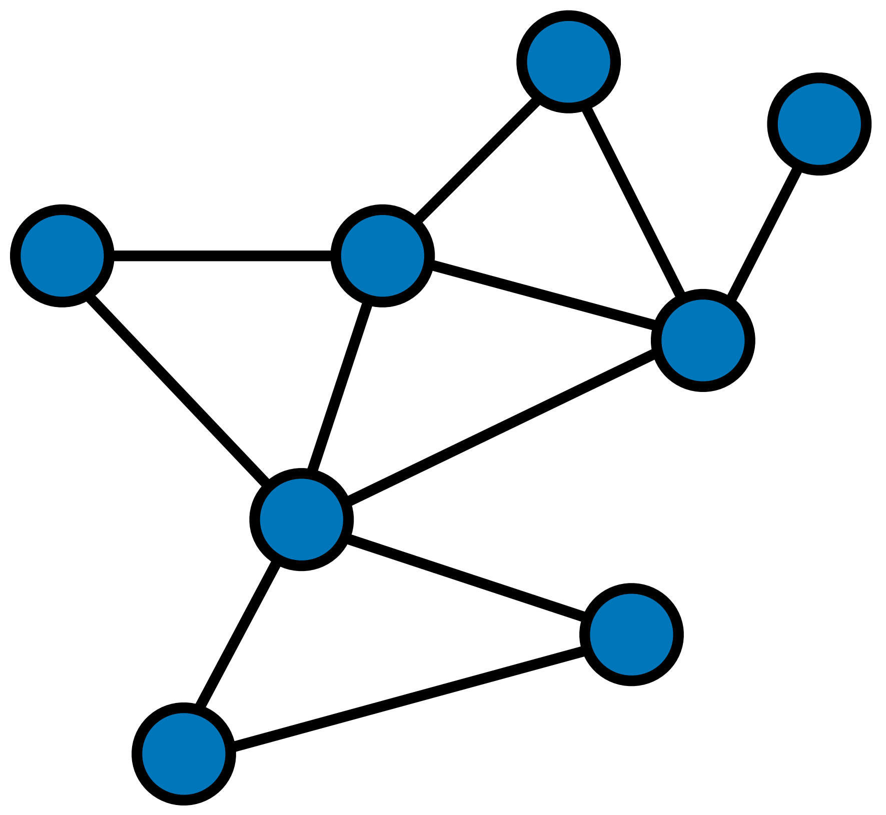
$$H(s) = A(s)H_0 + B(s)H_1$$



1. Prepare the system in the ground state of H_0
2. Change s slowly from 0 to 1
3. Measure: **obtain the ground state of H_1**

By the **adiabatic theorem**

Transverse-field Ising model QA



$$H(s) = A(s)H_0 + B(s)H_1$$

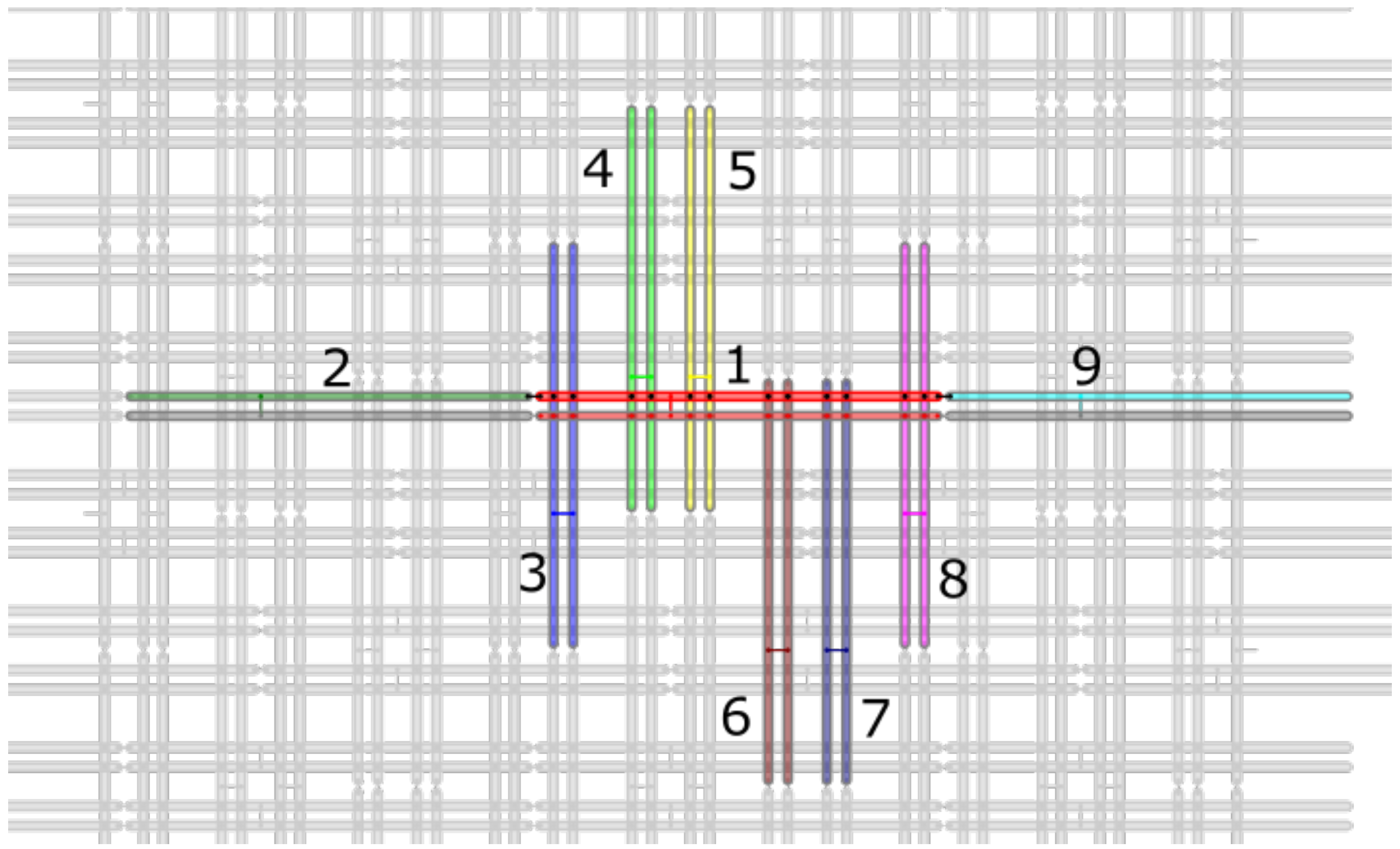
$$H_0 = \sum_i \sigma_i^x \qquad H_1 = \sum_{ij} J_{ij}\sigma_i^z\sigma_j^z + \sum_i h_i\sigma_i^z$$

Measuring the eigenvalue $(s_1, s_2, \ldots)$ of $(\sigma_1^z, \sigma_2^z, \ldots)$ solves the problem

Ising	$\Leftrightarrow$	QUBO
$\min\limits_{s_i=\pm 1} \sum_{ij} J_{ij}\, s_i\, s_j + \sum_i h_i\, s_i$		$\min\limits_{q_i\in\{0,1\}} \sum_{ij} Q_{ij}\, q_i\, q_j$

D-Wave quantum annealers

Pegasus architecture



Advantage_system6.1

5616 qubits

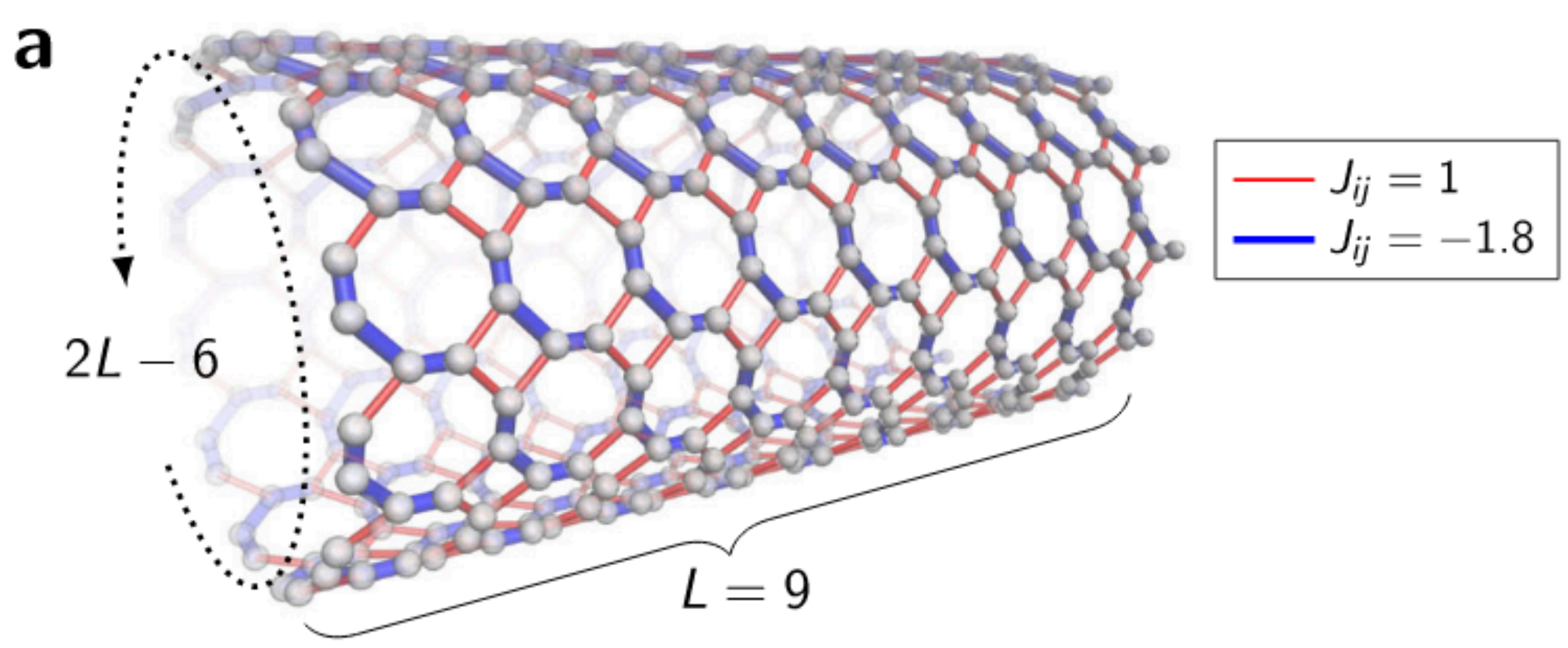
40135 couplings

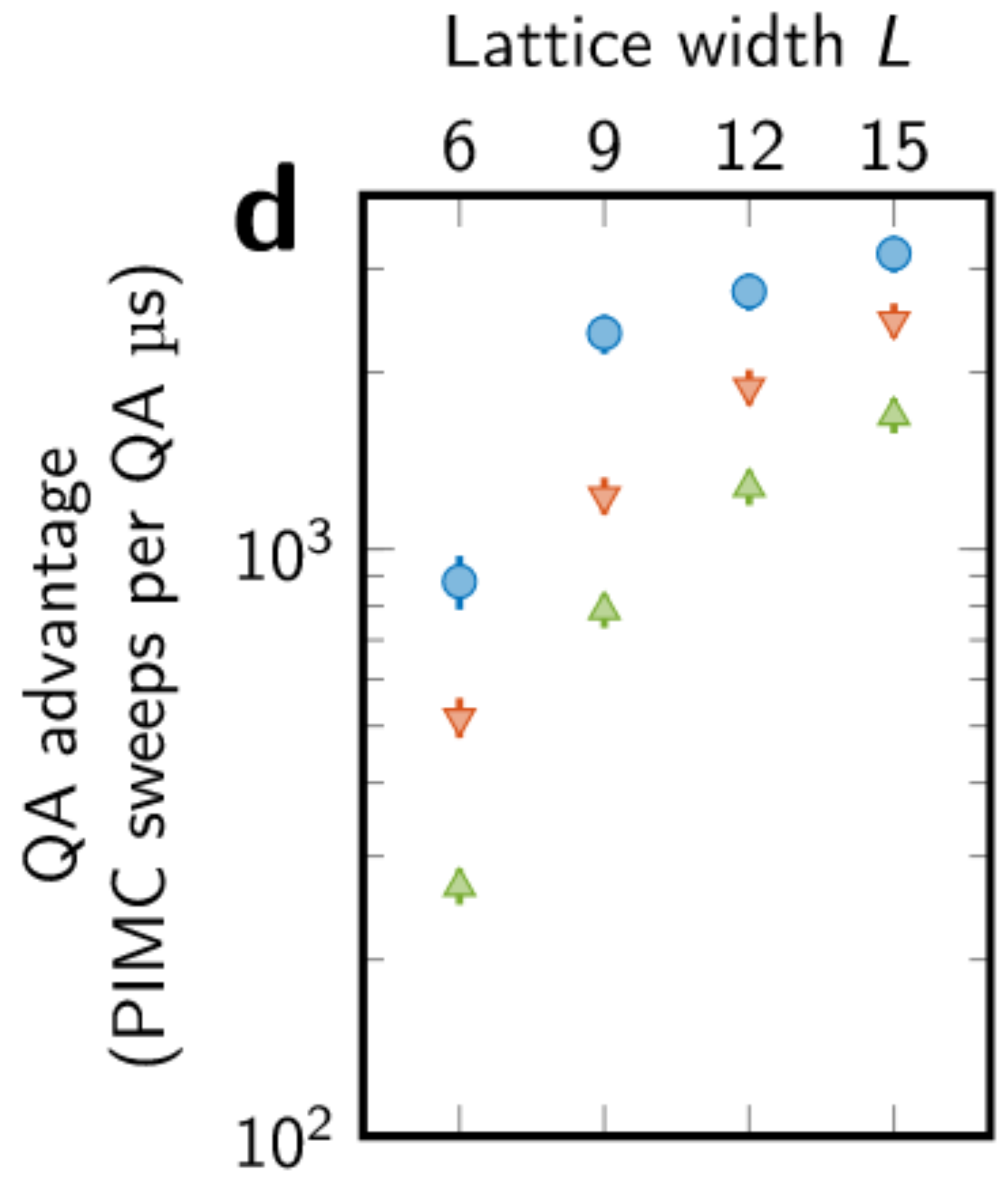
Max. 16 couplings per qubit

If the Ising model is more densely connected $\Longrightarrow$ **chain** several qubits together

Quantum annealing advantage

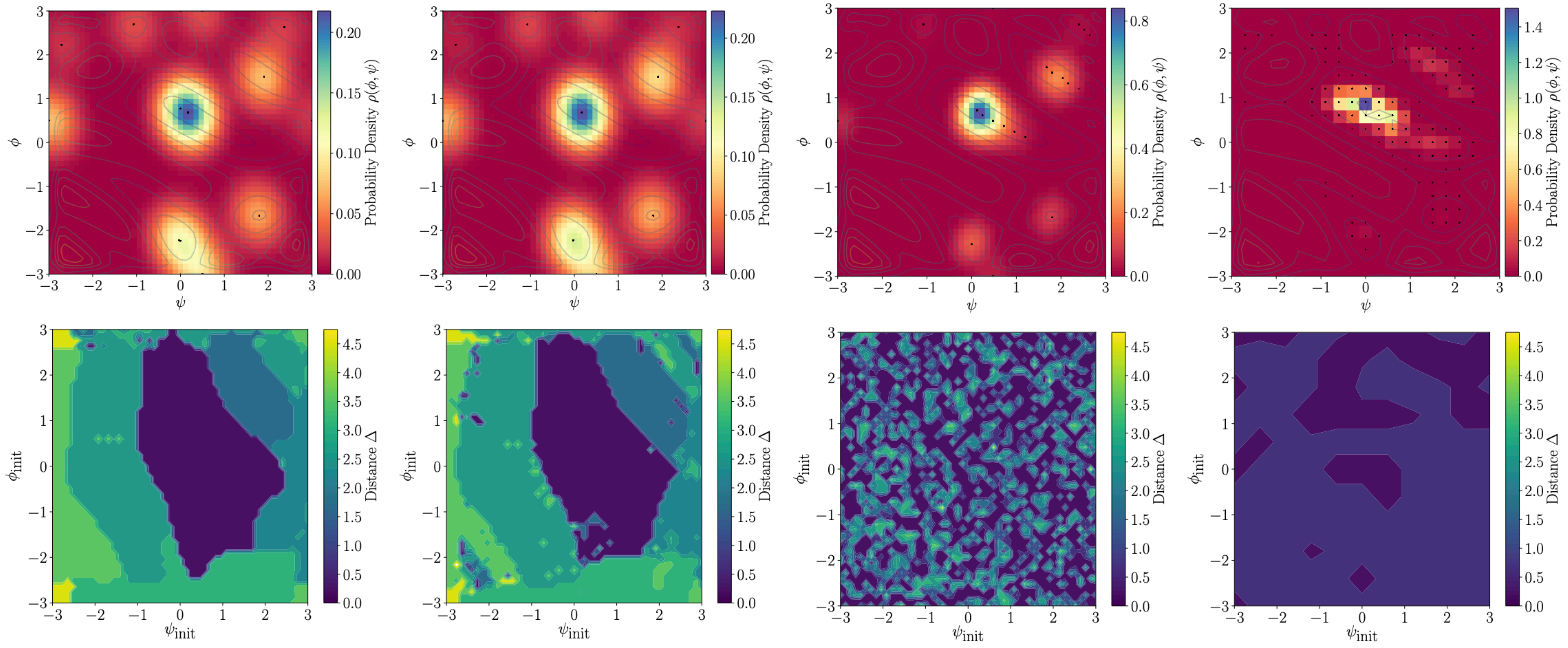
A.D. King, Nature Communications (2021)12:1113

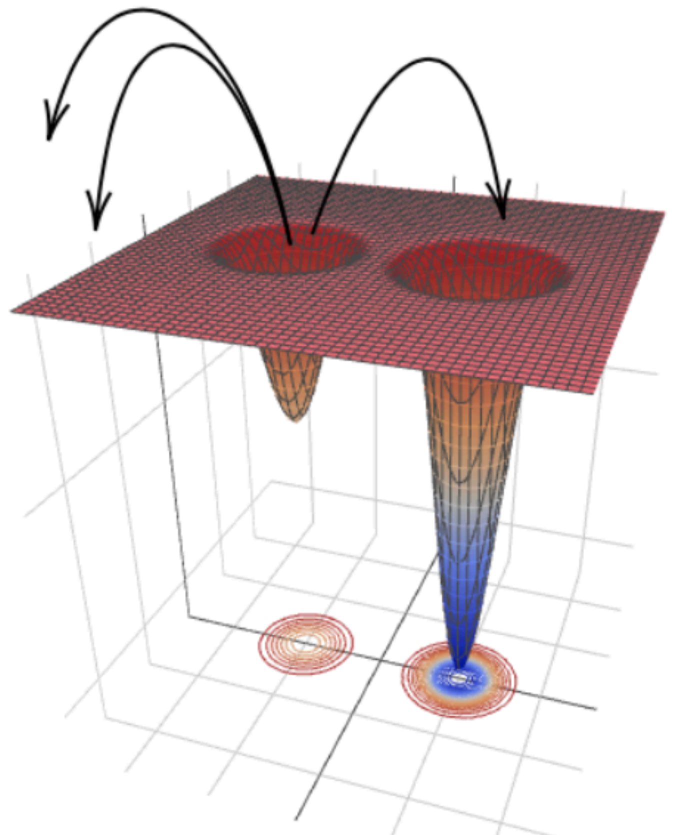




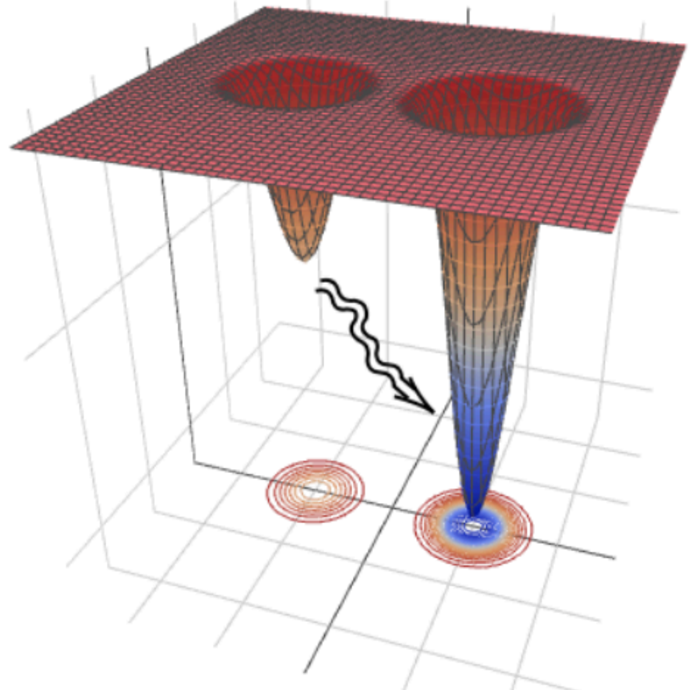
Tunneling

S.A. Abel, A. Blance, M. Spannowsky, 2105.13945





Thermal excitations over top of barrier at large T



Quantum excitations through barrier at small s

Encoding

Binary encoding

Minimize a loss function $\mathscr{L}(x_1, x_2, \ldots)$

in a box $[L_1, U_1] \times [L_2, U_2] \times \ldots$

up to a finite precision $1/2^p$

$$x_i = L_i + \frac{U_i - L_i}{1 - 2^{-n-1}} \sum_{n=1}^{p} \frac{q_{i\alpha}}{2^\alpha} \qquad (q_{i\alpha} \in \{0,1\})$$

Reduction to quadratic

What if $\mathscr{L}(x)$ has higher-degree terms in x?

It becomes a higher-degree polynomial in the qubits q_a

1. Introduce **auxiliary** qubits p_{ab}, representing the products $p_{ab} = q_a q_b$
2. New **loss function**: $\mathscr{L}'(q,p) = \mathscr{L}(q,p) + \lambda \sum_{ab} C(q_a, q_b, p_{ab})$

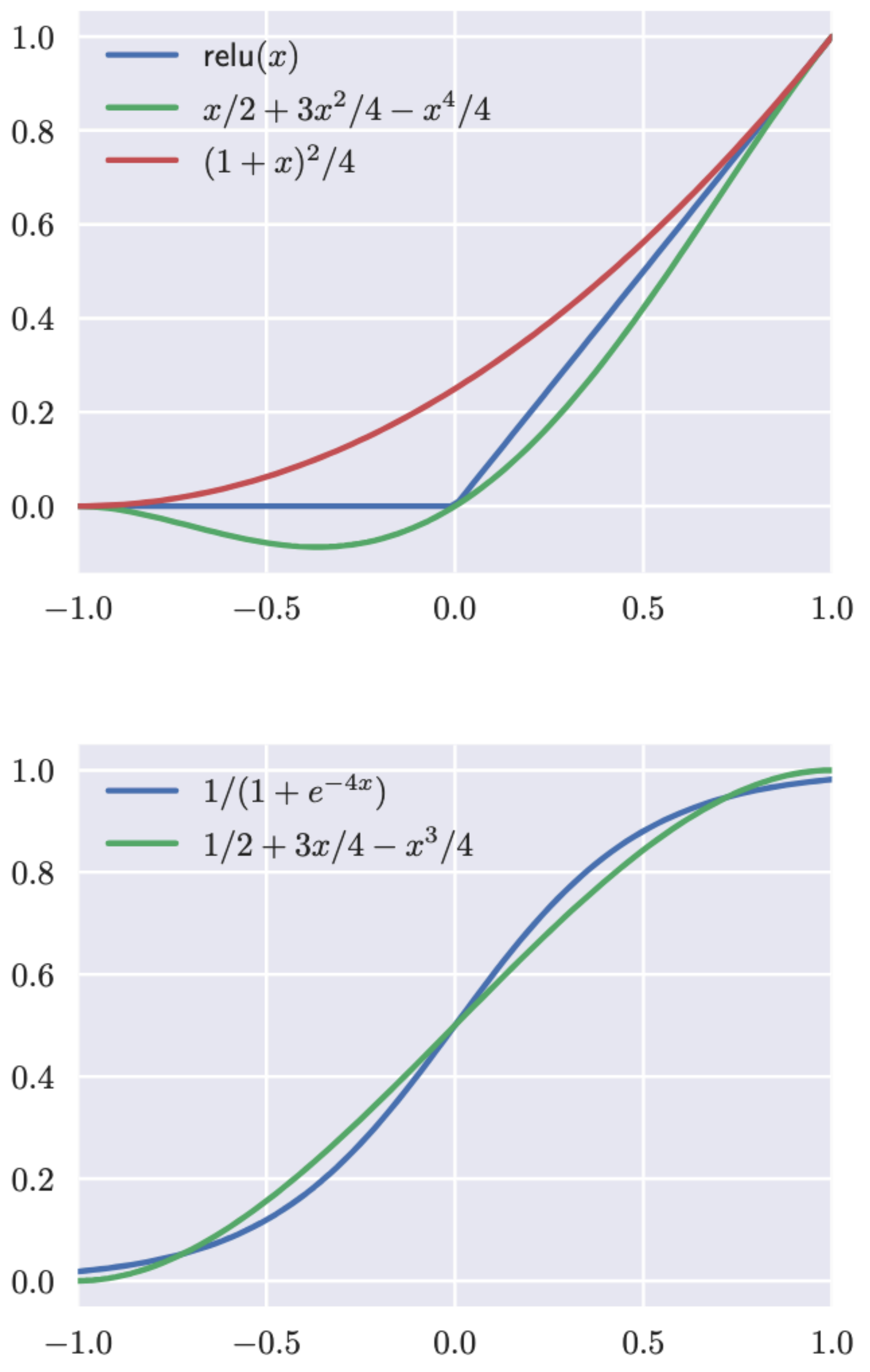
$C(x,y,z) = xy - 2z(x+y) + 3z$ minimised $\iff$ $xy = z$

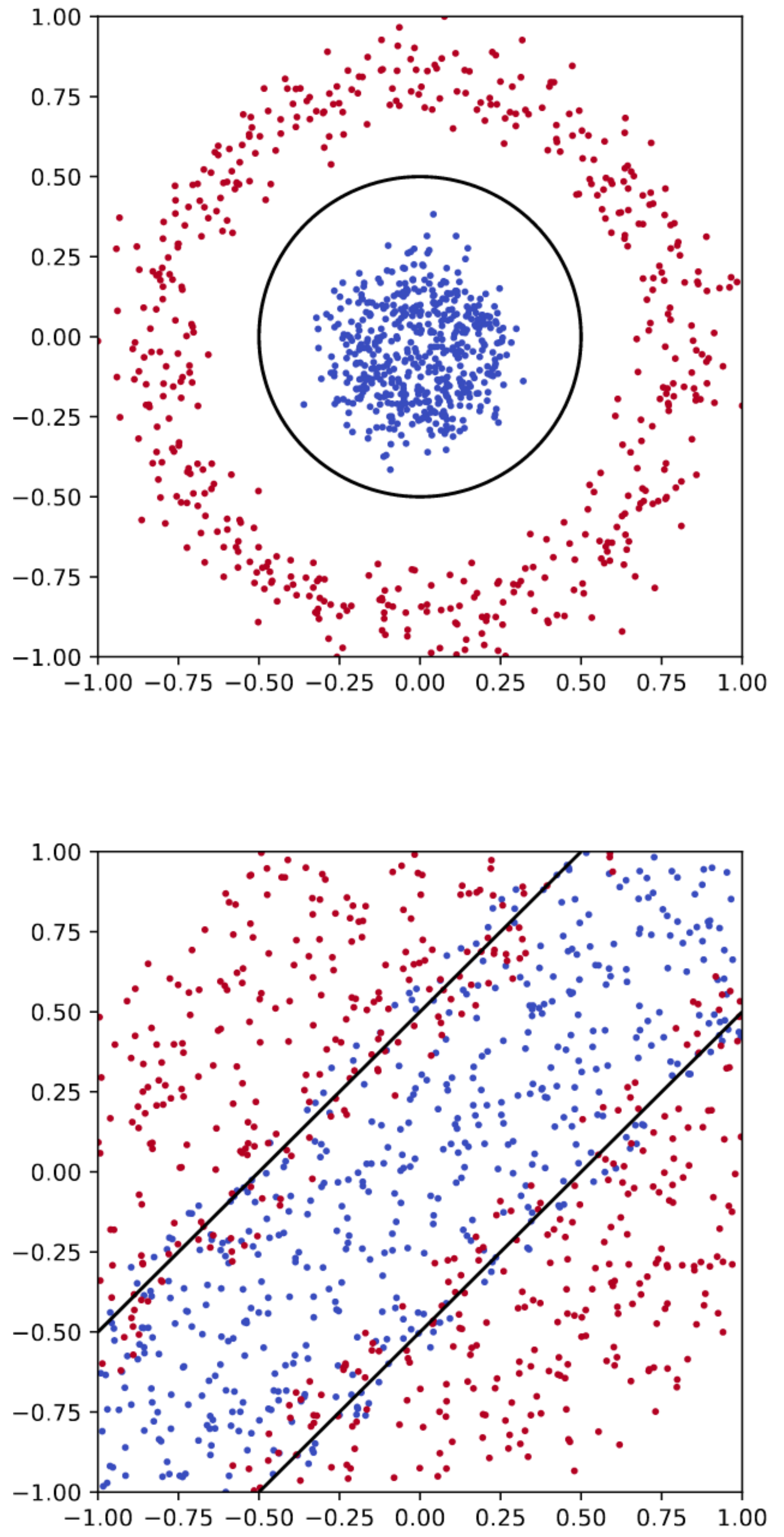
Applications

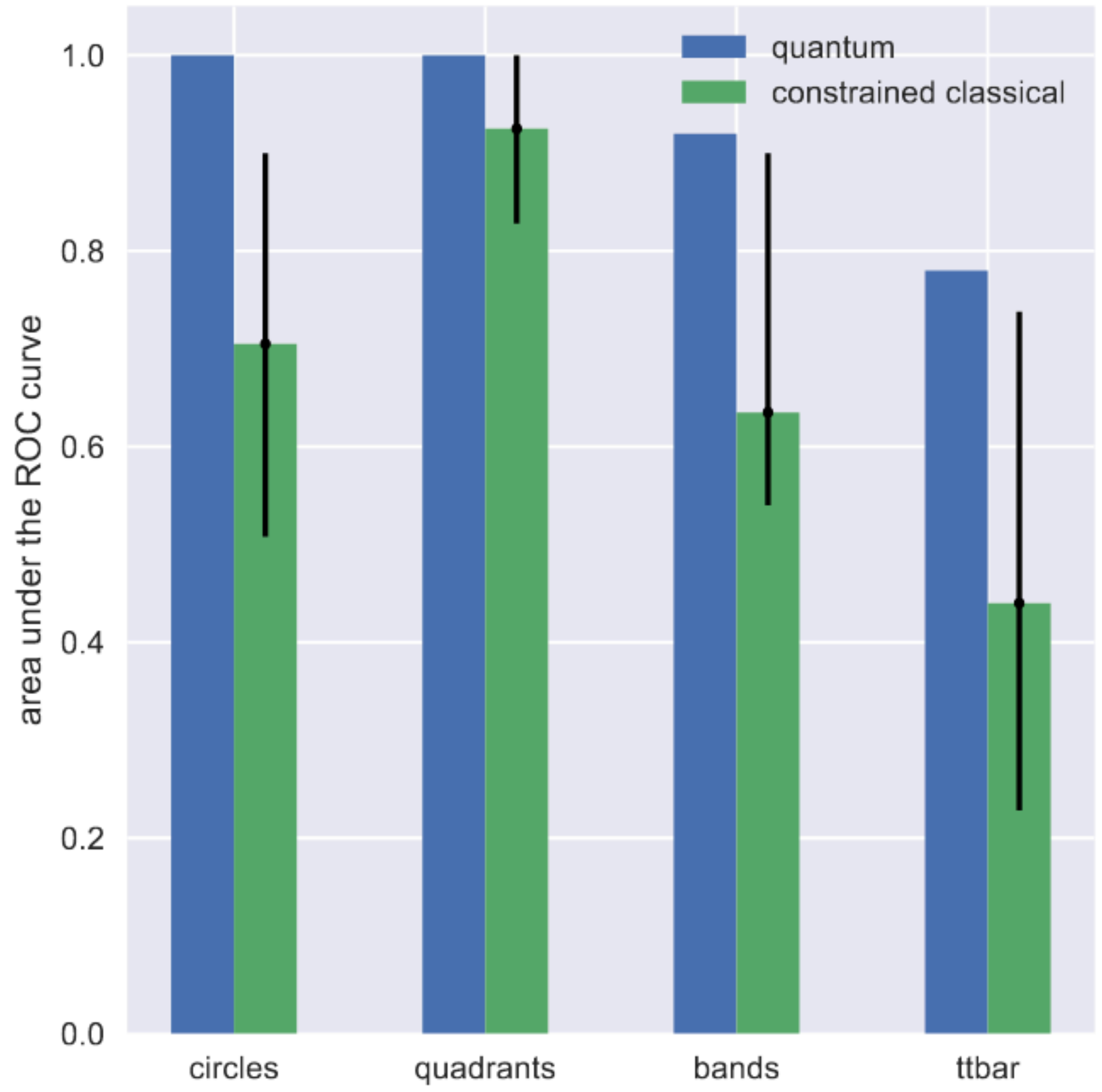
Training neural networks

2202.11727

Approximate activation functions with polynomials



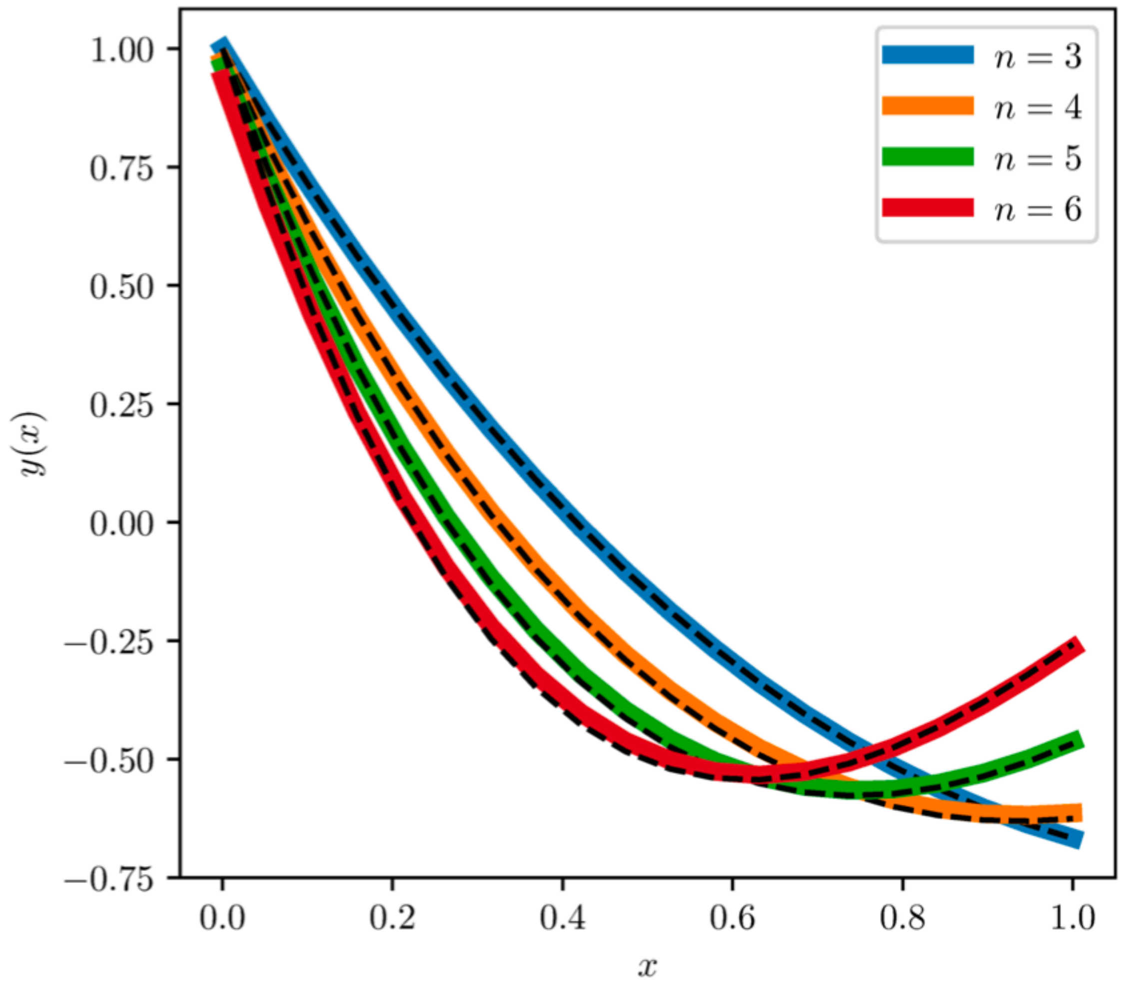




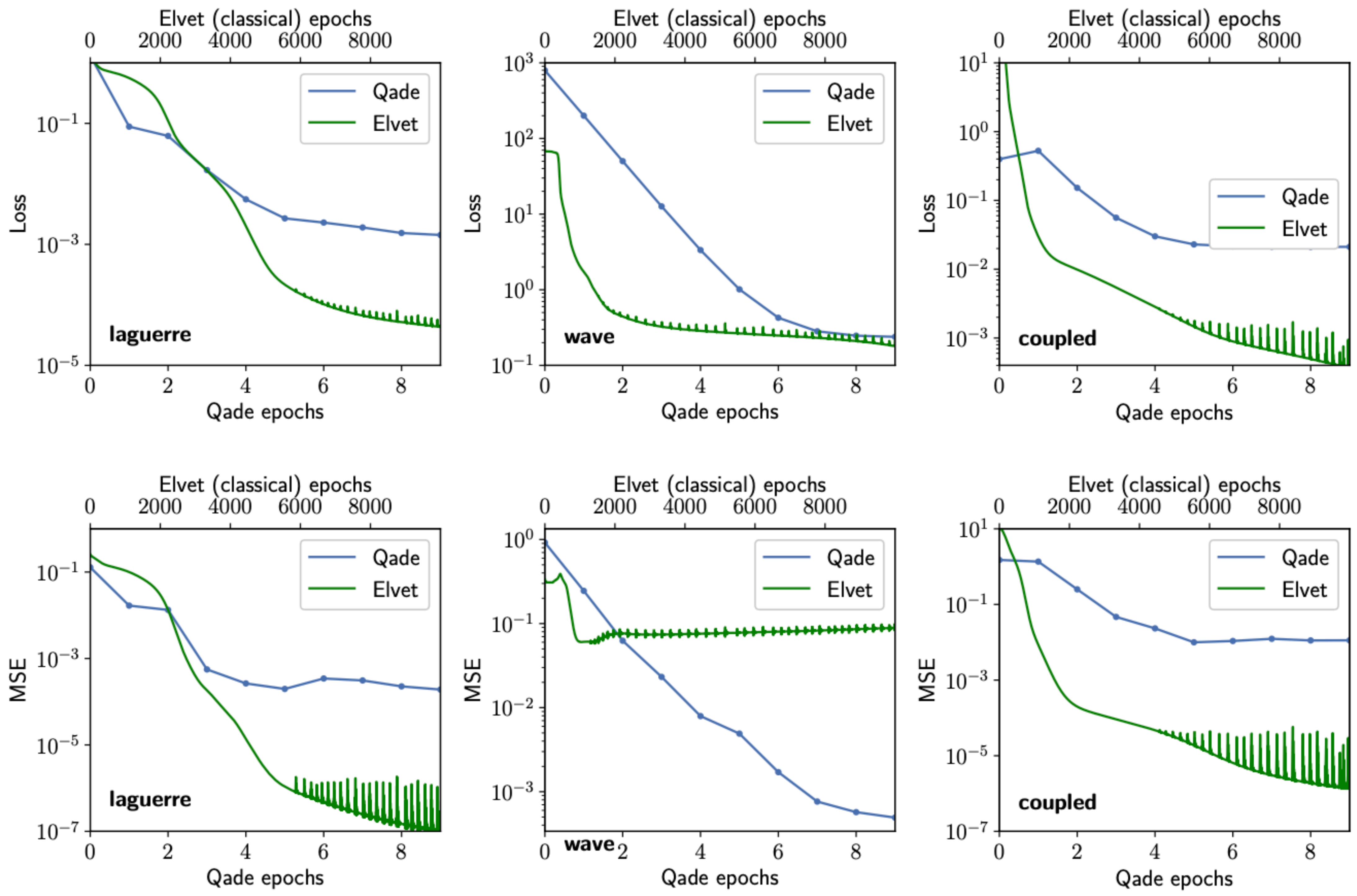
Qade: solving differential equations

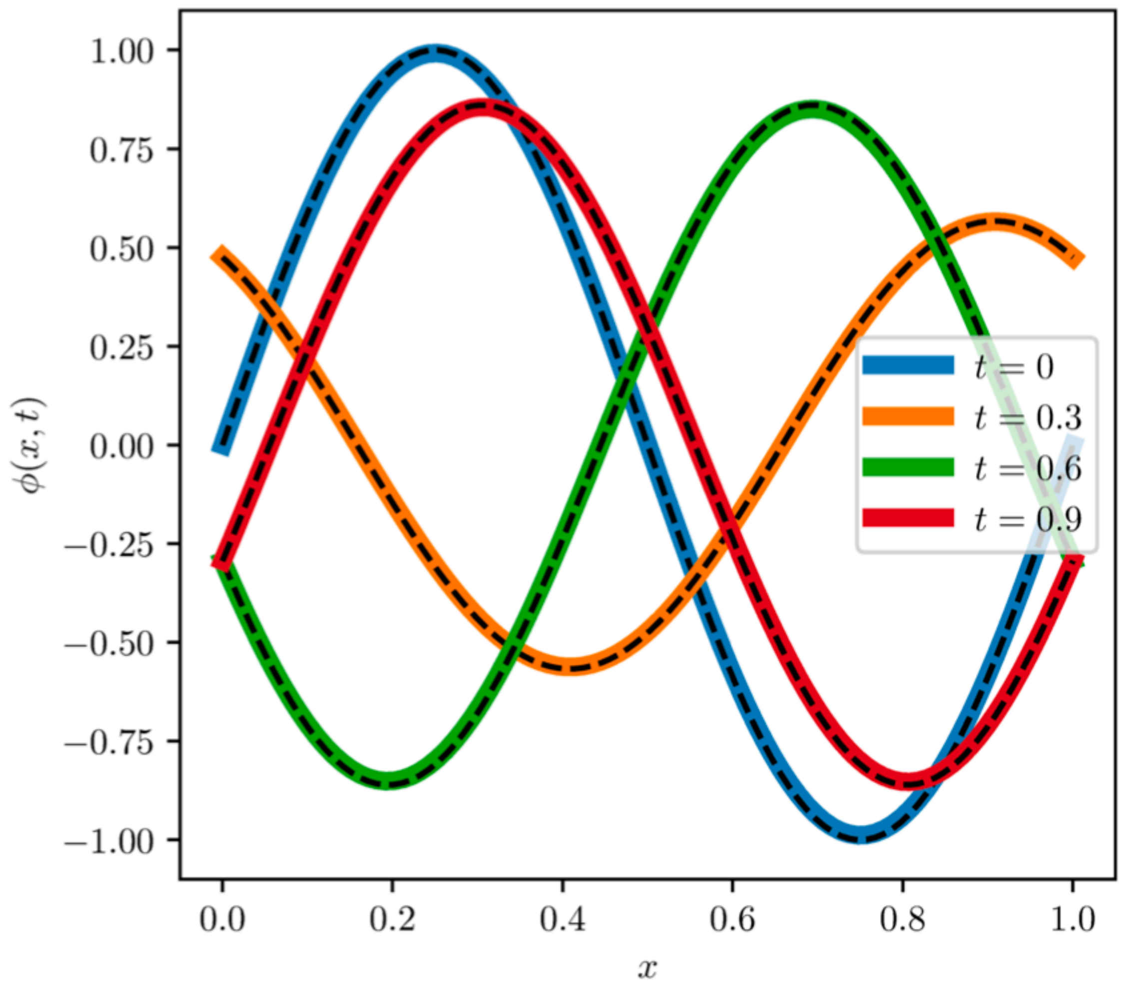
2204.03657

gitlab.com/jccriado/qade



$$\mathscr{L} = \sum_i \mathrm{E}_i(f, \partial f, \ldots)^2 + \sum_j \mathrm{BC}_j(f, \ldots)^2$$





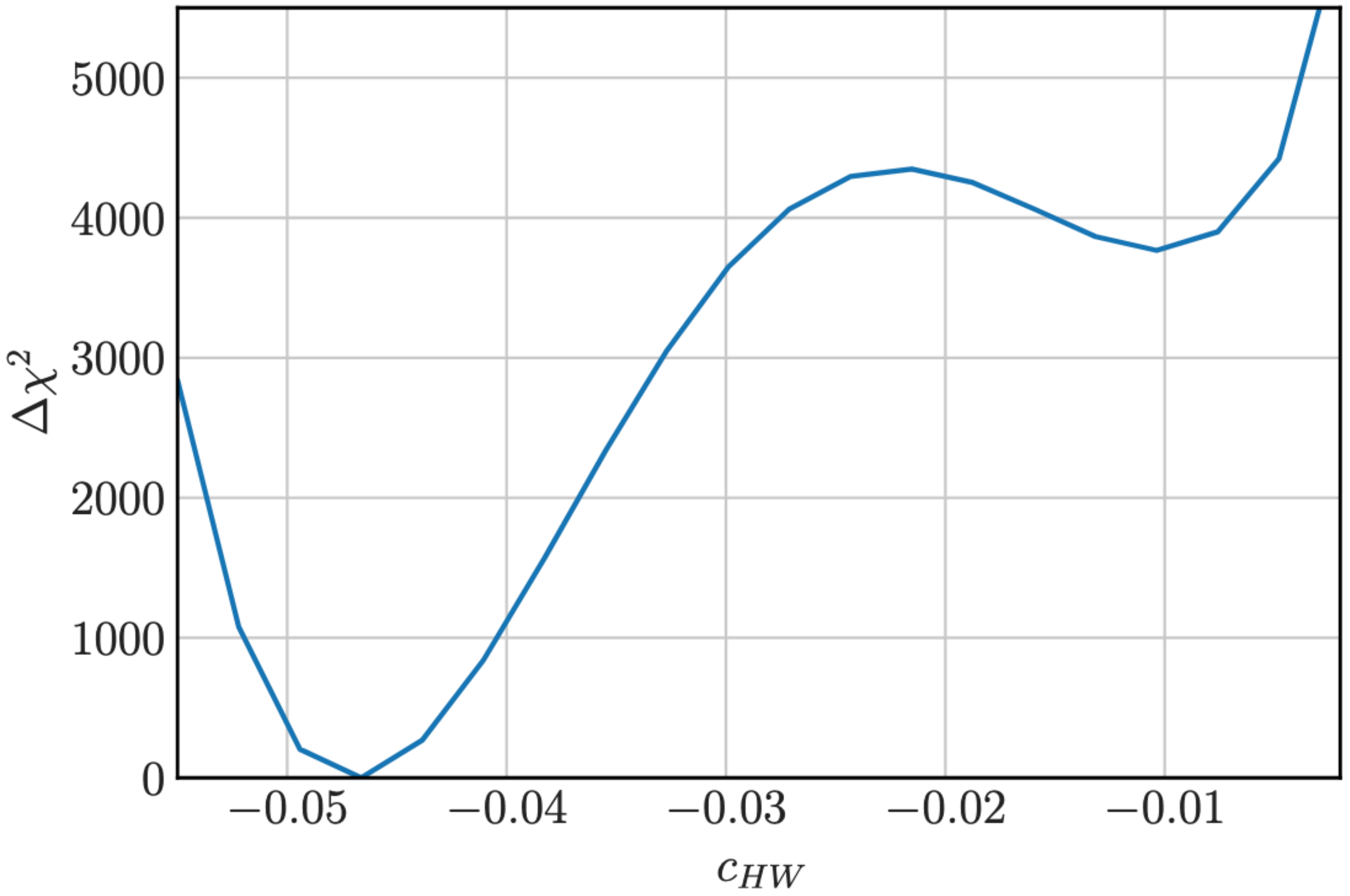
QFitter: EFT Wilson coefficient fits

2207.10088

$$\chi^2 = \sum_{ij} V_a C^{-1}_{ab} V_b, \qquad V_a = O_a^{(\mathrm{exp})} - O_a^{(\mathrm{th})}(c)$$

$$O_a^{(\mathrm{th})}(c) = A_a + \sum_i B_{ai} c_i + \sum_{ij} C_{aij} c_i c_j$$

$$\begin{aligned}
\mathcal{L} = & \frac{c_{u3} y_t}{v^2} (\phi^\dagger \phi)(\bar{q}_L \tilde{\phi} u_R) + \frac{c_{d3} y_b}{v^2} (\phi^\dagger \phi)(\bar{q}_L \phi d_R) \\
& + \frac{i c_W g}{2 m_W^2} (\phi^\dagger \sigma^a D^\mu \phi) D^\nu W^a_{\mu\nu} + \frac{c_H}{4 v^2} \left(\partial_\mu (\phi^\dagger \phi)\right)^2 \\
& + \frac{c_\gamma (g')^2}{2 m_W^2} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu} + \frac{c_g g_S^2}{2 m_W^2} (\phi^\dagger \phi) G^a_{\mu\nu} G^{a\mu\nu} \\
& + \frac{i c_{HW} g}{4 m_W^2} (\phi^\dagger \sigma^a D^\mu \phi) D^\nu W^a_{\mu\nu} \\
& + \frac{i c_{HB} g'}{4 m_W^2} (\phi^\dagger D^\mu \phi) D^\nu B_{\mu\nu} + \mathrm{h.c.}
\end{aligned}$$



Conclusions

- **Quantum annealing** paradigm: find the ground state of the Ising model
  - Scaling advantage in performance over classical methods
  - Robust global-minimum finding in non-convex loss functions
- Minimisation of general functions of continuous variables:
  - **Binary encoding**
  - **Reduction** to quadratic
- Applications:
  - Purely **quantum-trained neural networks**
  - Solving **partial differential equations**
  - **Fitting** EFT coefficients