

Boost-Invariant Polynomials: an efficient and interpretable approach to jet tagging

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Personal Introduction



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Personal Introduction



Christoph Ortner



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- Undergrad Physics Student
- ML Engineer at Guane
- Focus on Geometric-DL and ML for HEP
- UBC Intern
- CERN Summer Student

Jose M Munoz



Jet Tagging

Brief Introduction

ML for Jet Tagging

State of the art

Our Proposal

The Boost Invariant Polynomials

Results

Of the approach

Future Outlook

& Conclusions

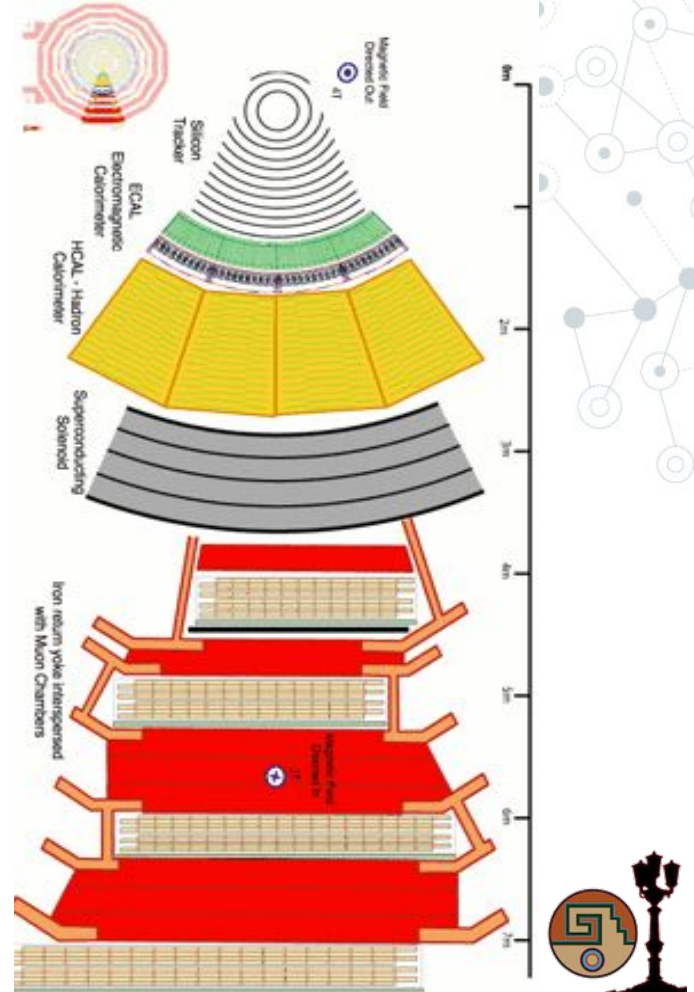
1.

2.

3

4.

5.



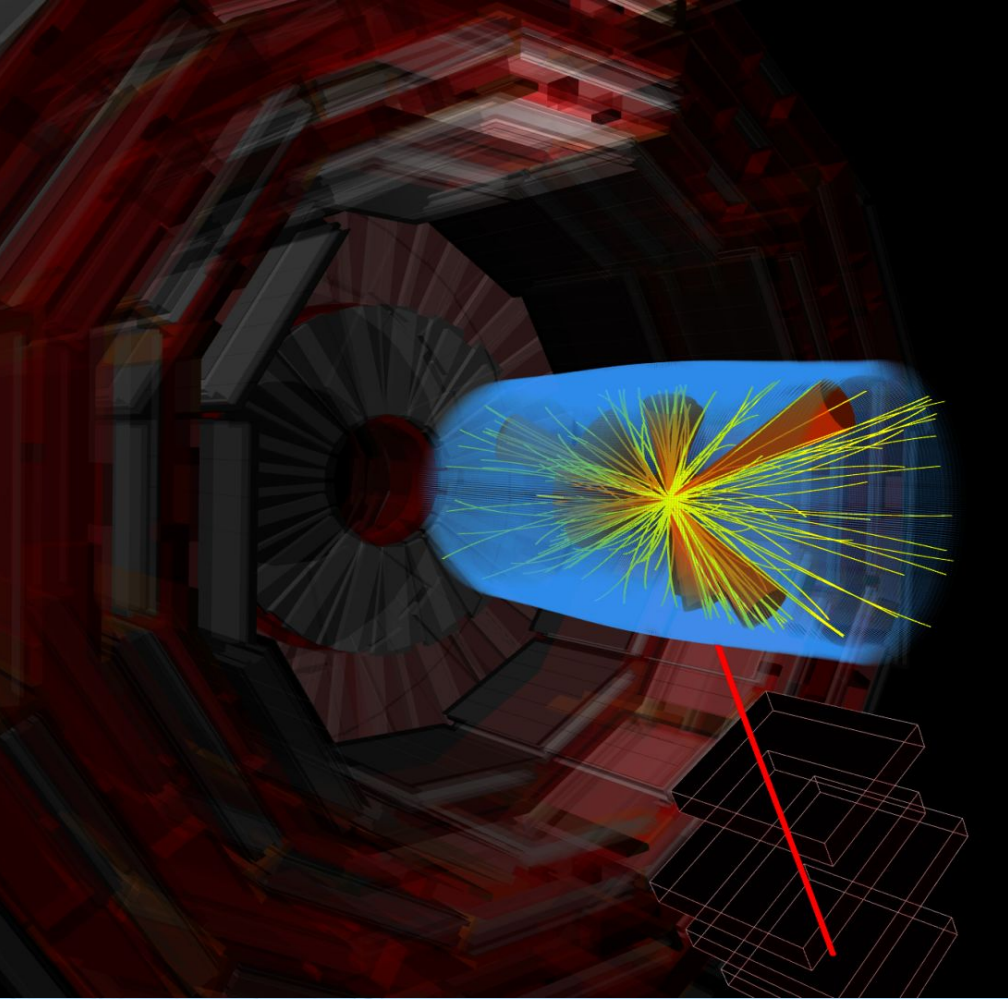


1.

Jet Tagging

A Brief Introduction



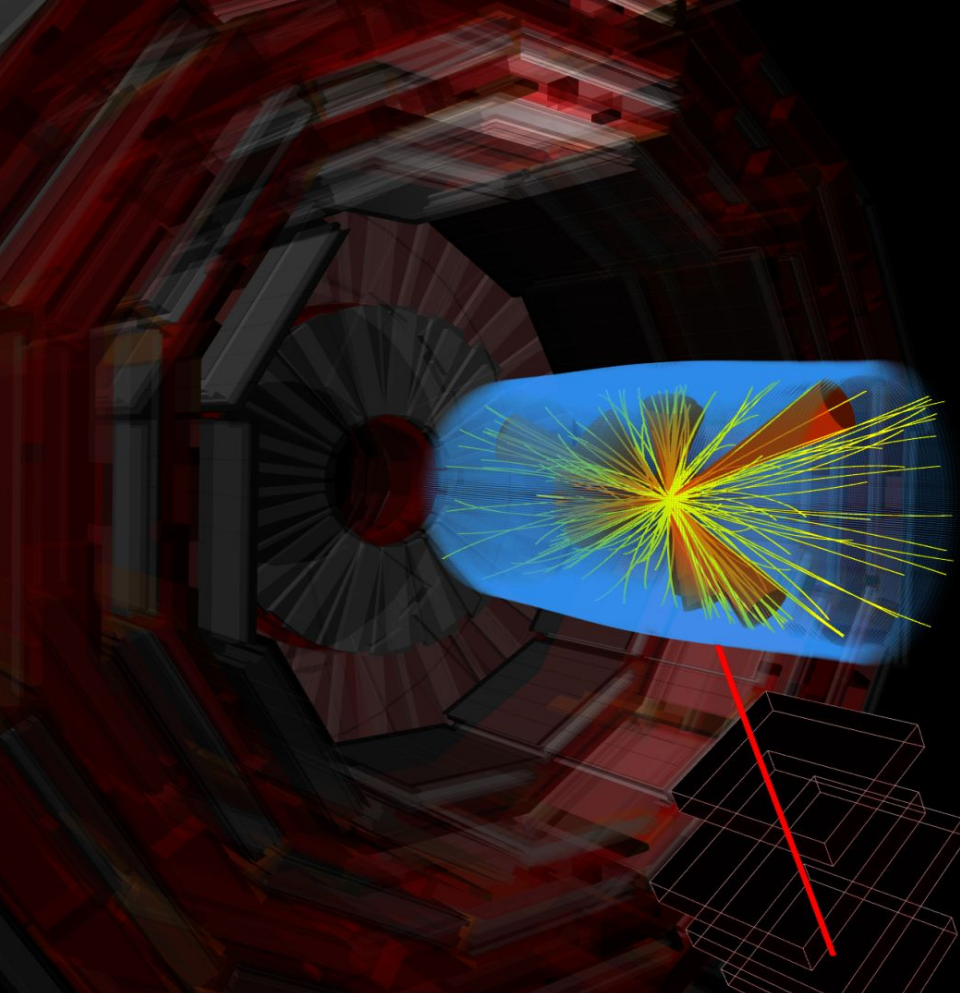


Jet Tagging

Identify signal vs background in particles coming from different processes

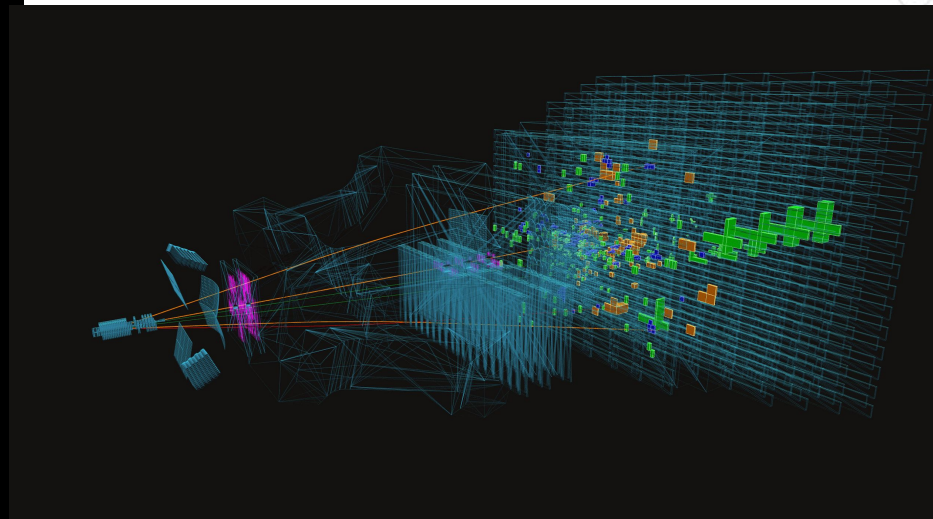
The particles are detected and reconstructed as point clouds





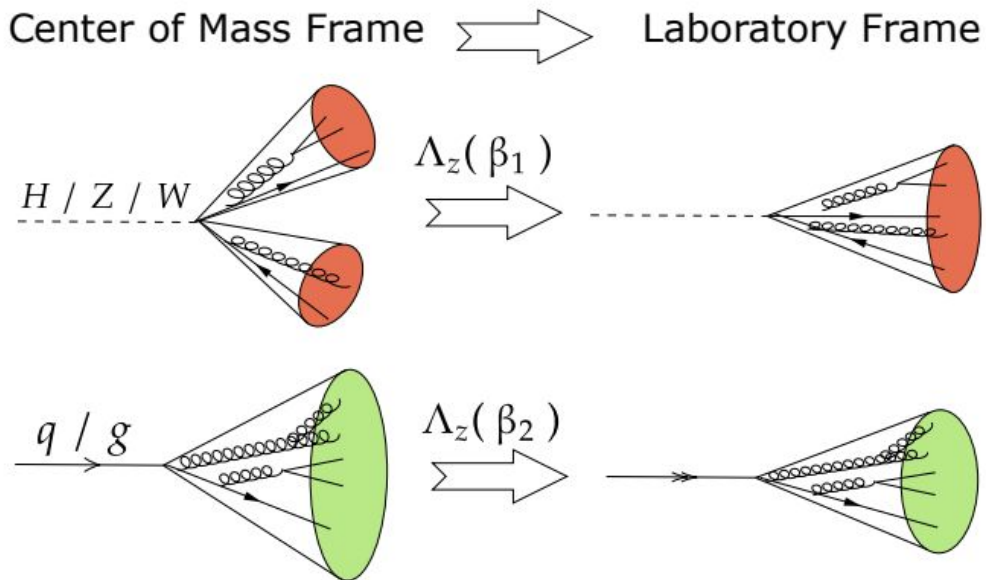
Jet Tagging

Identify signal vs background in particles coming from different processes



The issue of Boosted Jet Tagging

The angular resolution depends on the p_T of the jets...





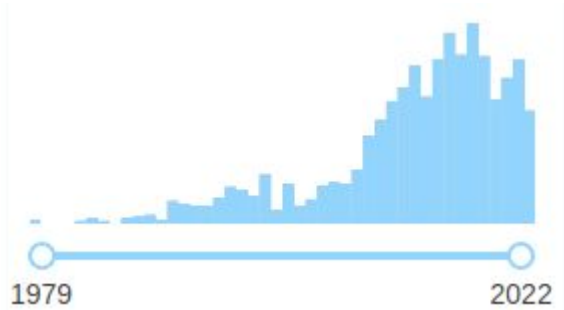
2.

ML for Jet Tagging

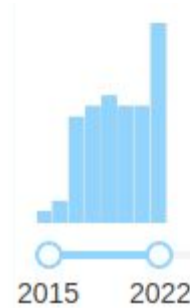
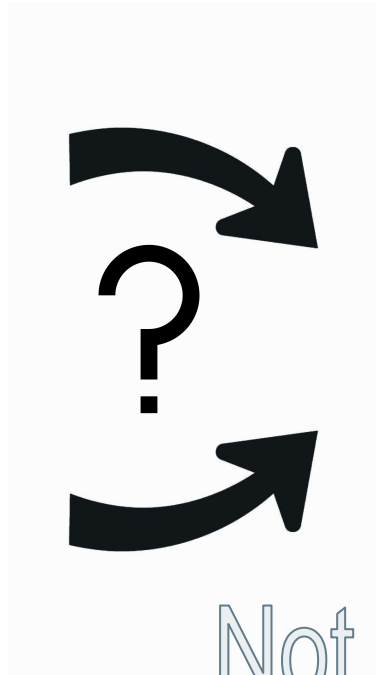
A Brief Introduction

Why Machine Learning?

Jet-Tagging Papers:



ML papers:

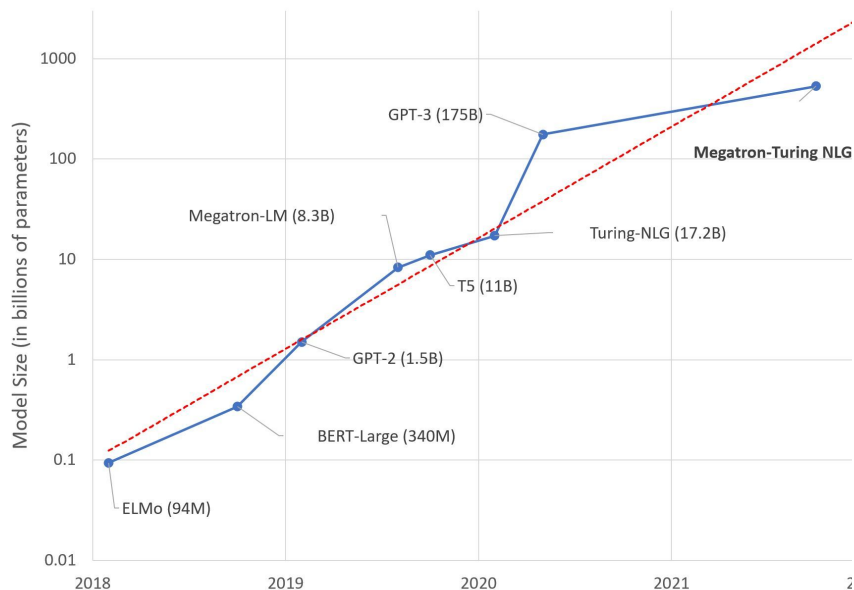


Not a convolution!



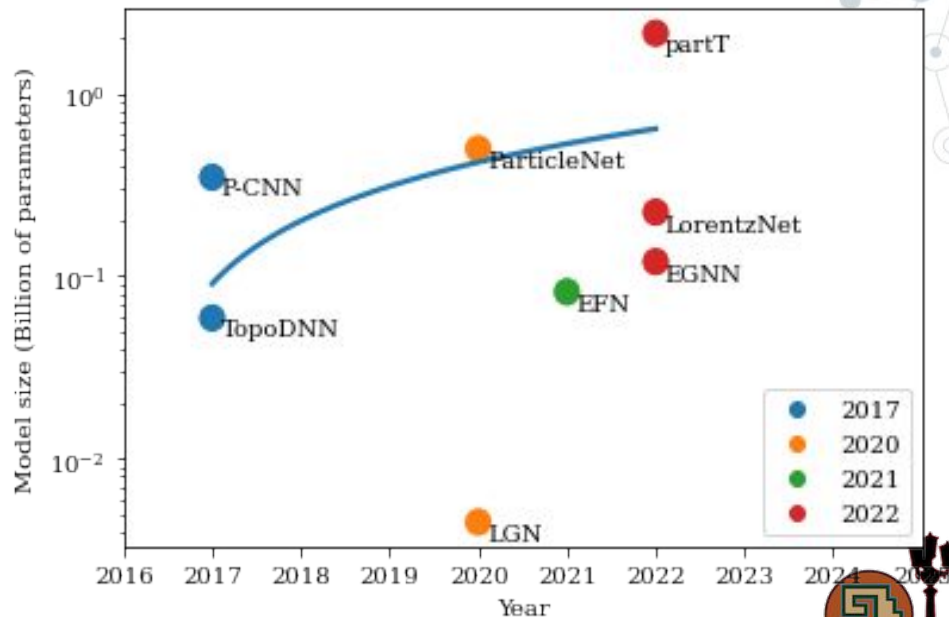
The model size is a determining factor

In other fields: (LLMs)



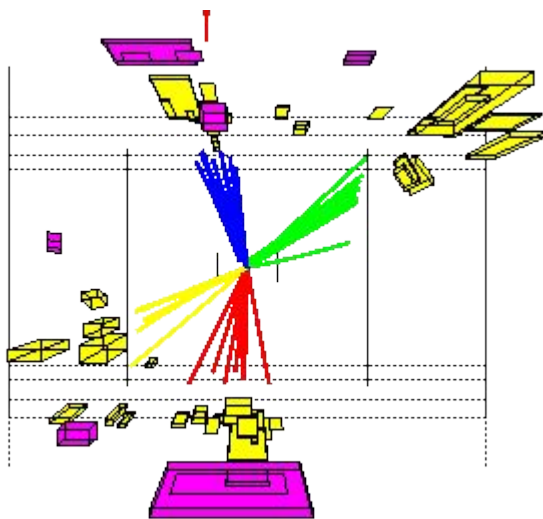
<https://huggingface.co/blog/large-language-models>

Particle physics

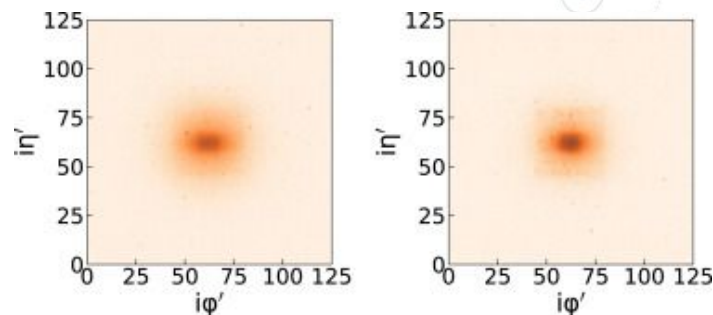


What information should we give the models?

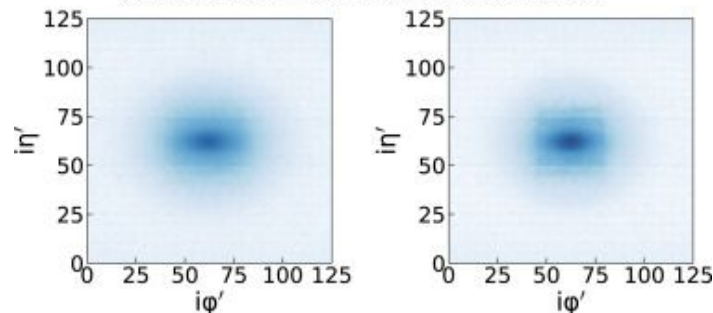
Low level variables



High Level variables



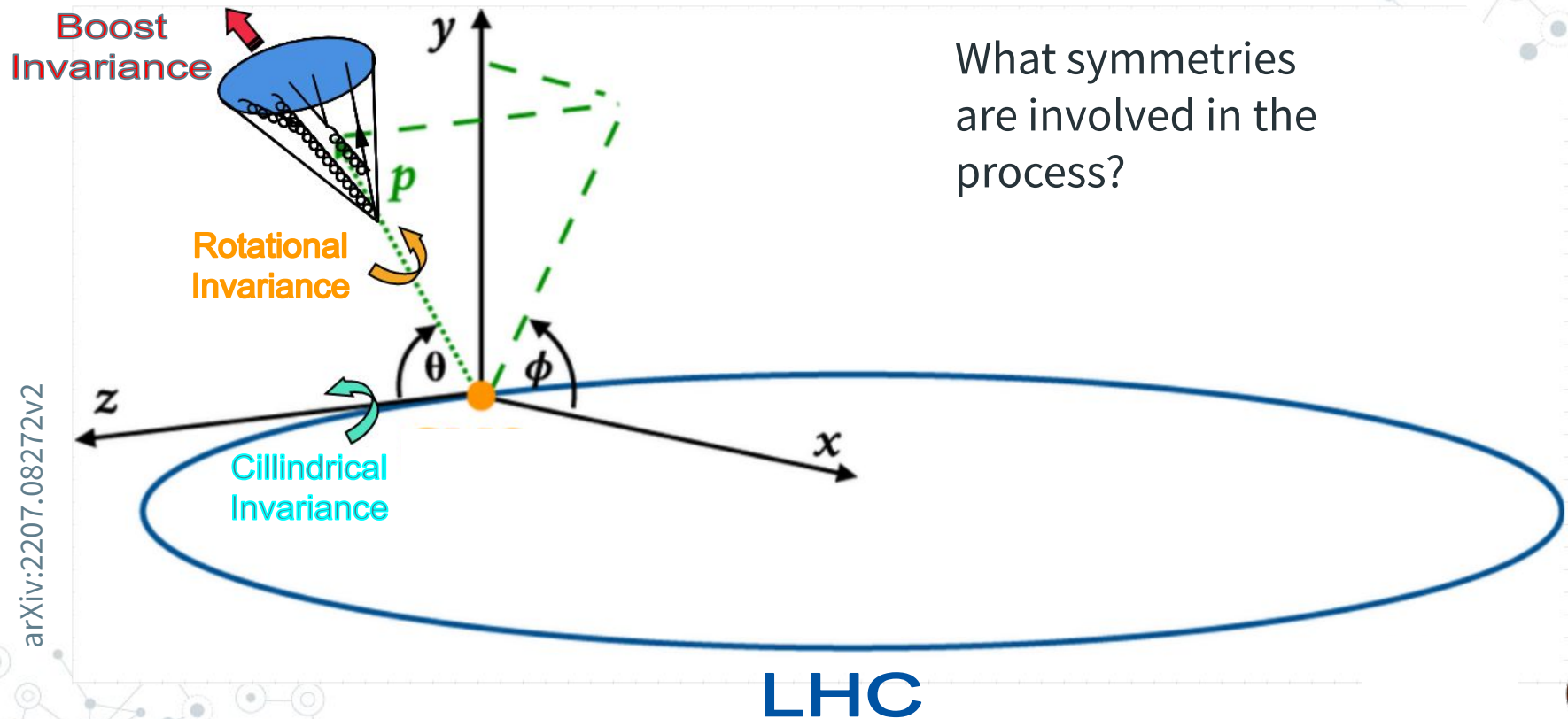
(a) Tracks overlay. Left: gluon jet, Right: quark jet.



End-to-end jet classification of quarks and gluons with the CMS Open Data. Andrew et Al.



What about inductive biases?



arXiv:2207.08272v2

What Lorentz group equivariant message passing networks achieve?

Equivariant Energy Flow Networks for jet tagging

Matthew J. Dolan^{1,*} and Ayodele Ore^{1,†}

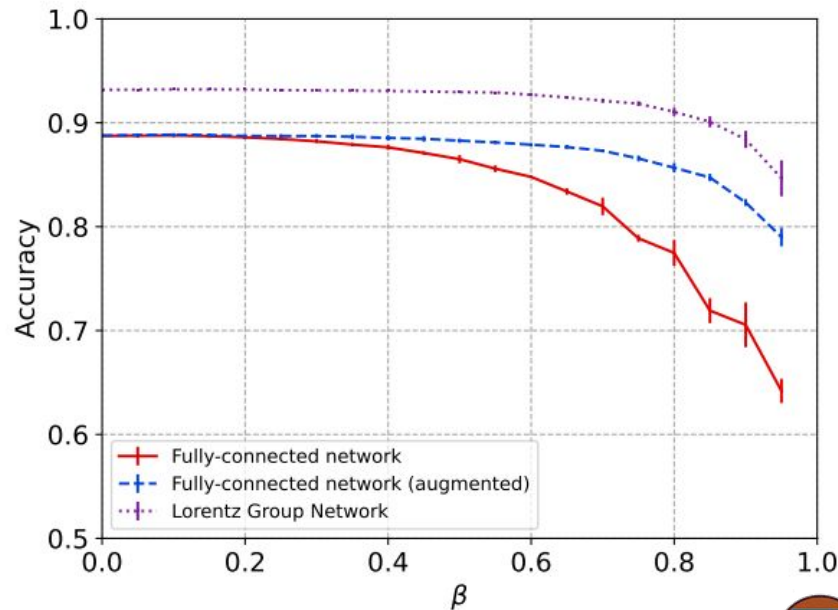
¹ARC Centre of Excellence for Dark Matter Particle Physics,
School of Physics, The University of Melbourne, Victoria 3010, Australia

An Efficient Lorentz Equivariant Graph Neural Network for Jet Tagging

Shiqi Gong^{a,e,1} Qi Meng^b Jue Zhang^b Huilin Qu^c Congqiao Li^d Sitian Qian^d Weitao Du^a Zhi-Ming Ma^a Tie-Yan Liu^b

Semi-Equivariant GNN Architectures for Jet Tagging

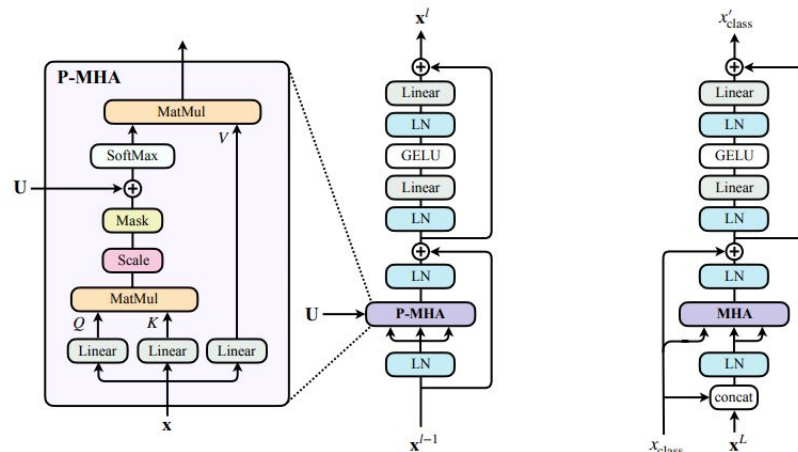
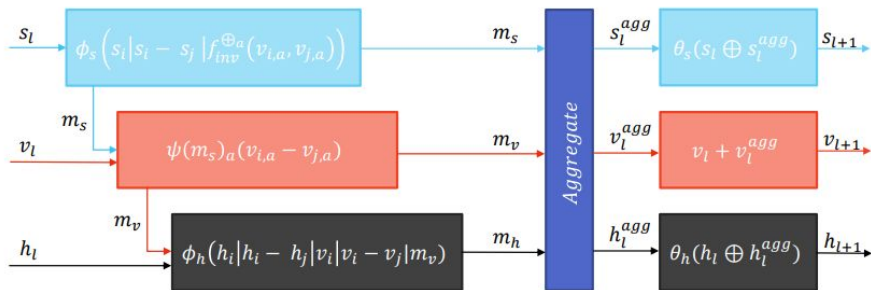
Daniel Murnane¹, Savannah Thais², Jason Wong³



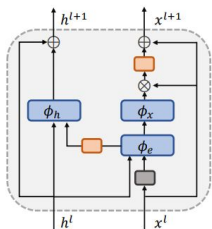
Bogatskiy et al.



A problem...

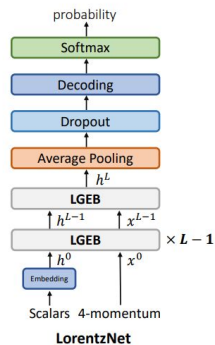


Murnane et al.



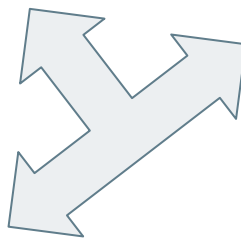
MLP Sum Pooling Minkowski Norm & Inner Product

Lorentz Group Equivariant Block (LGEb)



LorentzNet

Qu et al.



Very expensive tensorial computation



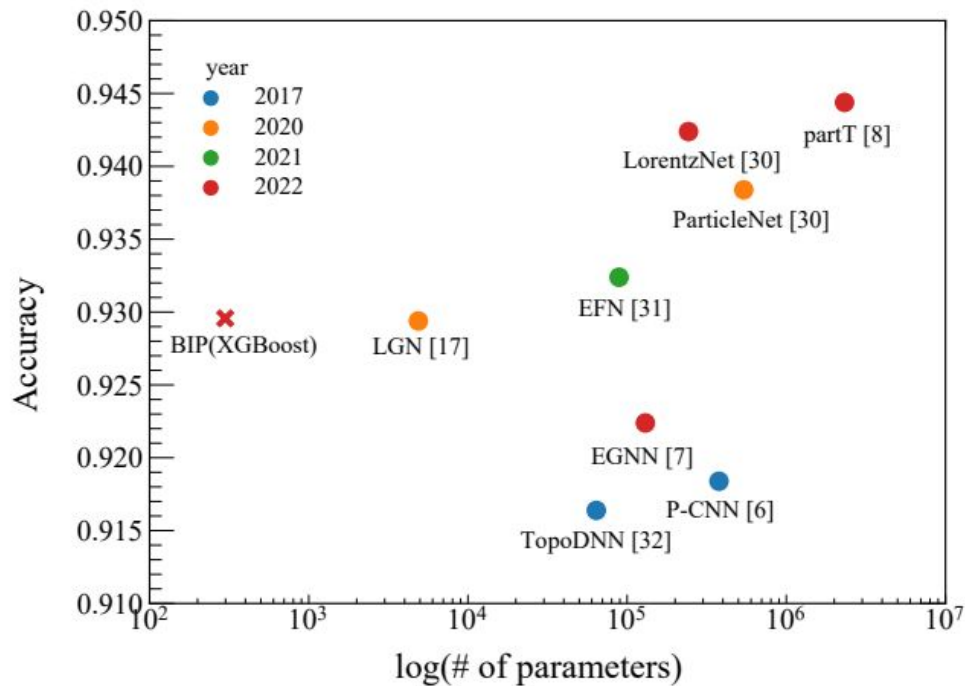


3.

The Boost Invariant Polynomials

A Brief Introduction

SOTA Accuracy with a fraction of the parameters



arXiv:2207.08272v2



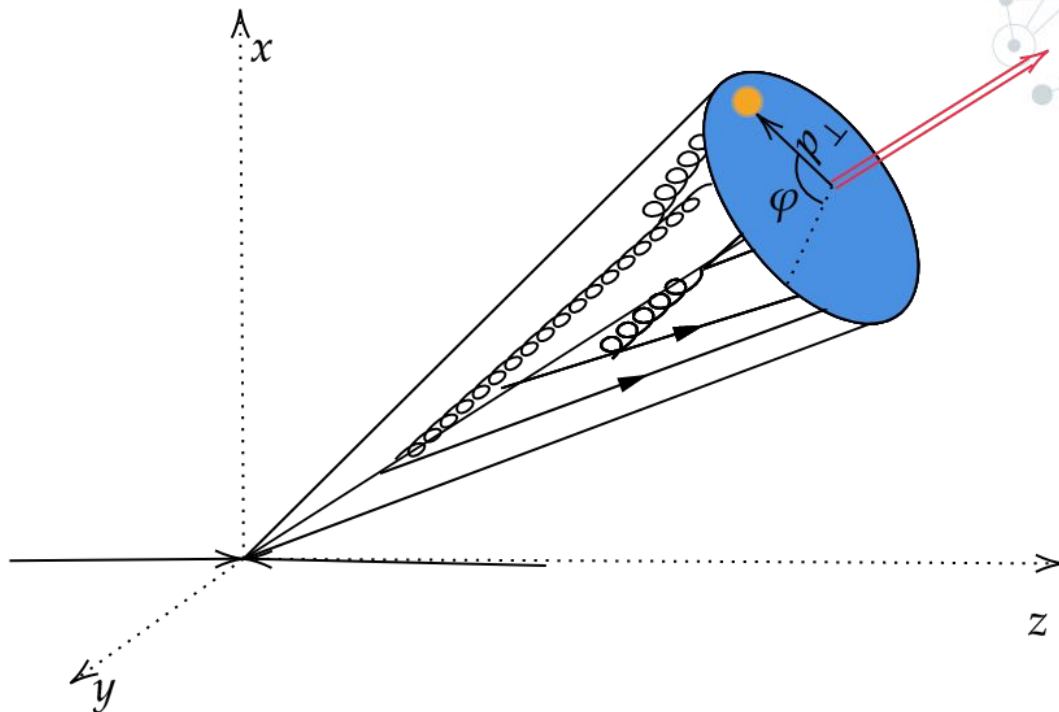
A basis change

$$y_i = \frac{1}{2} \log \left(\frac{\delta_1 + E_i + p_{\parallel i}}{\delta_1 + E_i - p_{\parallel i}} \right)$$

p_{\perp}

$$E_{\perp, i} = \sqrt{m_i^2 + p_{\perp, i}^2}$$

φ_i



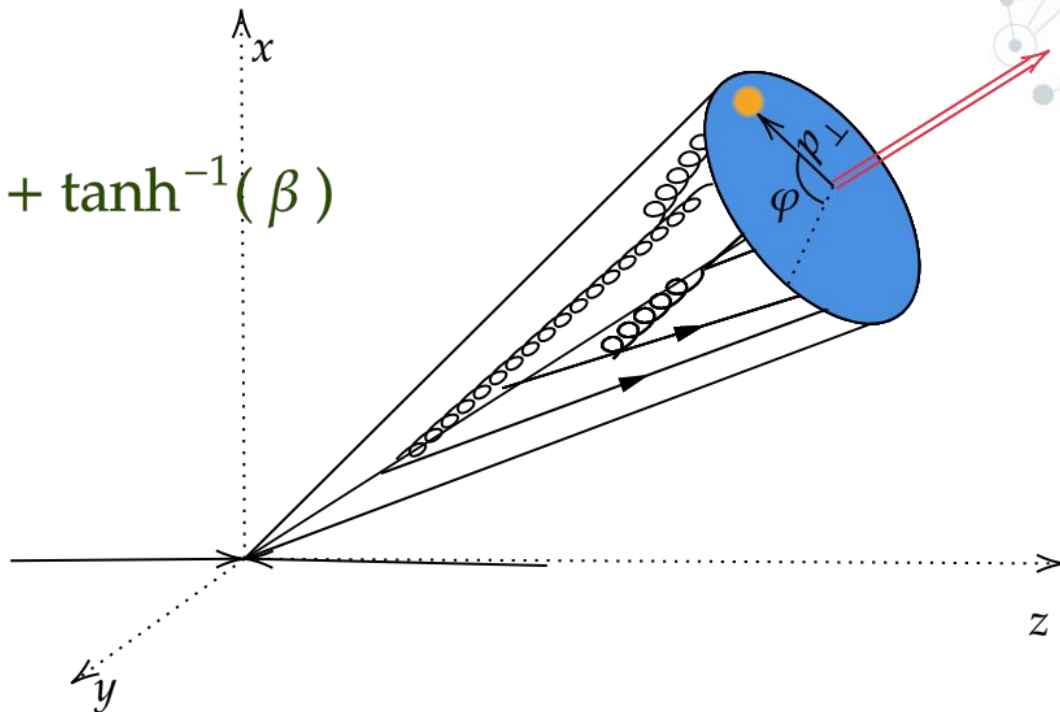
A basis change... such that

$$y_i = \frac{1}{2} \log \left(\frac{\delta_1 + E_i + p_{\parallel i}}{\delta_1 + E_i - p_{\parallel i}} \right) + \tanh^{-1}(\beta)$$

p_{\perp}

$$E_{\perp, i} = \sqrt{m_i^2 + p_{\perp, i}^2}$$

φ_i



Boost

\vec{p}

z

y



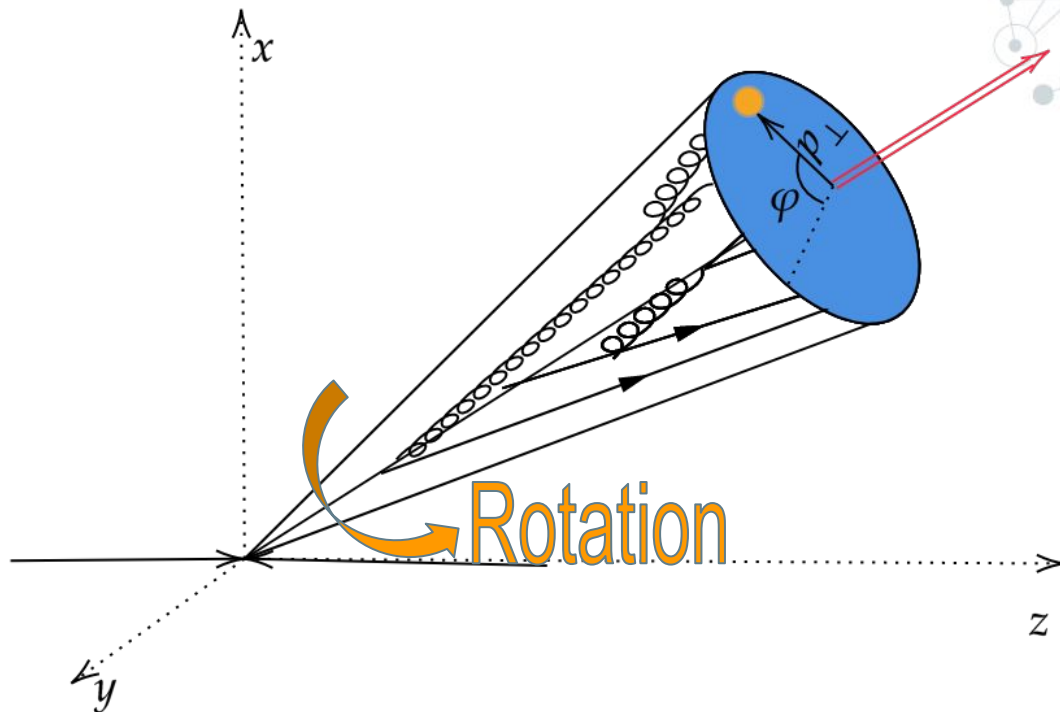
A basis change... such that

$$y_i = \frac{1}{2} \log \left(\frac{\delta_1 + E_i + p_{\parallel i}}{\delta_1 + E_i - p_{\parallel i}} \right)$$

p_{\perp}

$$E_{\perp, i} = \sqrt{m_i^2 + p_{\perp, i}^2}$$

$$\varphi_i + \Delta\varphi$$



A basis change... such that

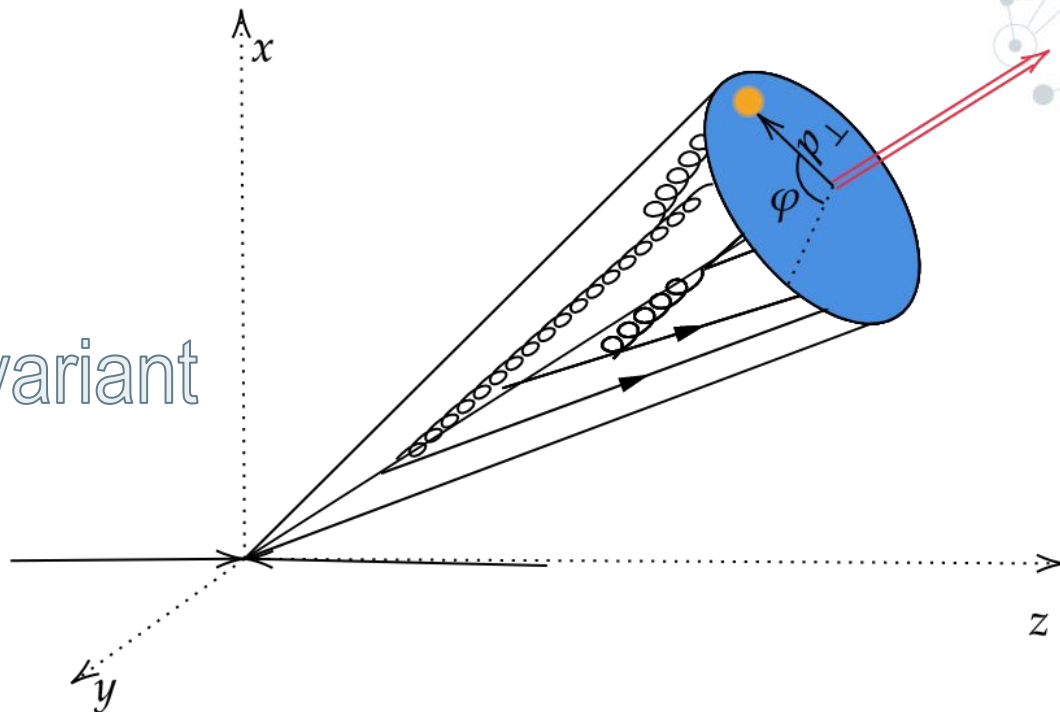
$$y_i = \frac{1}{2} \log \left(\frac{\delta_1 + E_i + p_{\parallel i}}{\delta_1 + E_i - p_{\parallel i}} \right)$$

p_{\perp}

$$E_{\perp, i} = \sqrt{m_i^2 + p_{\perp, i}^2}$$

Invariant

φ_i



arXiv:2207.08272v2



The Atomic Cluster Expansion Recipe:

1. Build a one particle basis:

$$A_{nlk} = \sum_{i=1}^N Q_n(p_{\perp,i}, E_{\perp,i}, \xi_i) e^{il\varphi_i} e^{-\lambda ky_i}$$



The Atomic Cluster Expansion Recipe:

1. Build a one particle basis:

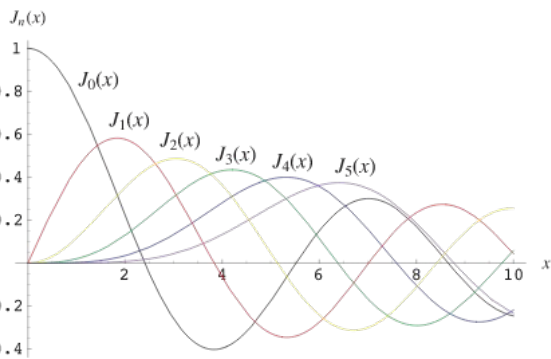
$$A_{nlk} = \sum_{i=1}^N Q_n(p_{\perp,i}, E_{\perp,i}, \xi_i) e^{il\varphi_i} e^{-\lambda ky_i}$$



A general basis allows to introduce additional features... e.g

$$Q_n(E_{\perp,i}, p_{\perp,i}) = B_n(\tilde{p}_{\perp,i}) \log(1 + E_{\perp,i})$$

Bessel
polynomials



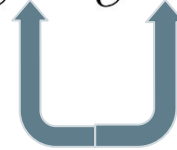
arXiv:2207.08272v2



The Atomic Cluster Expansion Recipe:

1. Build a one particle basis:

$$A_{nlk} = \sum_{i=1}^N Q_n(p_{\perp,i}, E_{\perp,i}, \xi_i) e^{il\varphi_i} e^{-\lambda ky_i}$$



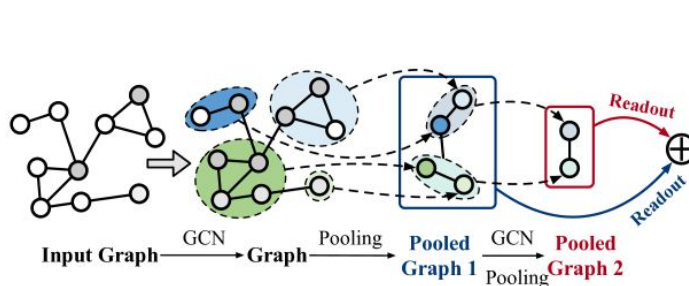
Decoupled generators
(rotations + boosts)



The Atomic Cluster Expansion Recipe:

1. Build a one particle basis:

$$A_{nlk} = \sum_{i=1}^N Q_n(p_{\perp,i}, E_{\perp,i}, \xi_i) e^{il\varphi_i} e^{-\lambda ky_i}$$



Liu et al.



The Atomic Cluster Expansion Recipe:

1. Build a one particle basis:

$$A_{nlk} = \sum_{i=1}^N Q_n(p_{\perp,i}, E_{\perp,i}, \xi_i) e^{il\varphi_i} e^{-\lambda ky_i}$$



NOT an Invariant jet
descriptor



The Atomic Cluster Expansion Recipe:

1. Build a one particle basis.
2. Symmetrize the basis:

$$\mathbf{A}_{nlk} = \prod_{t=1}^{\nu} A_{n_t l_t k_t}$$

↑
($n_1 l_1 k_1, \dots, n_\nu l_\nu k_\nu$)



The Atomic Cluster Expansion Recipe:

1. Build a one particle basis.
2. Symmetrize the basis.
3. Keep Invariant features:

$$\mathbf{A}_{nlk} = \prod_{t=1}^{\nu} \sum_{i=1}^N Q_n(p_{\perp,i}, E_{\perp,i}, \xi_i) e^{il\varphi_i} e^{-\lambda k y_i}$$



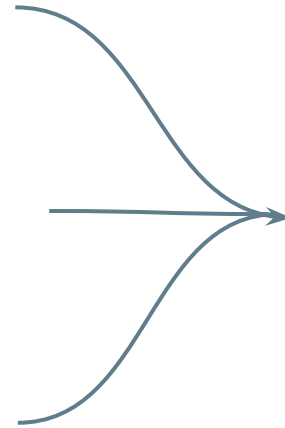
$$\sum_t l_t = \sum_t k_t = 0$$

$$\sum_{t=1}^{\nu} |l_t| + |k_t| + n_t \leq \Gamma$$



The Atomic Cluster Expansion Recipe:

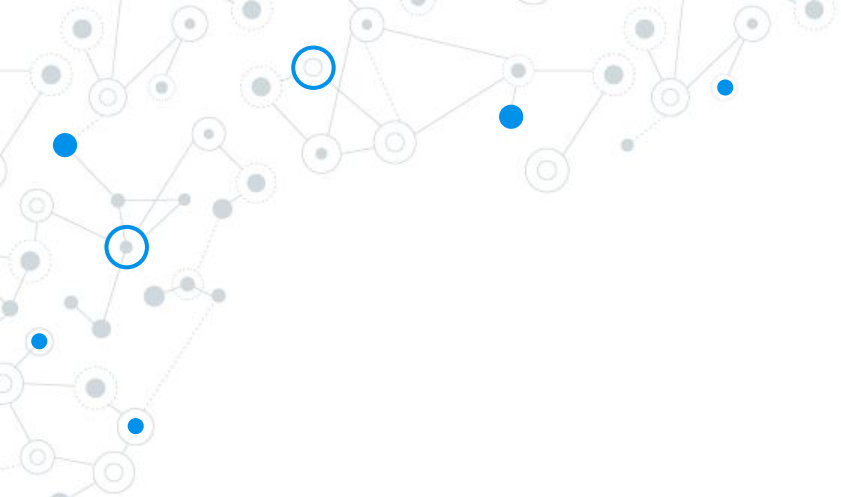
1. Build a one particle basis.
2. Symmetrize the basis.
3. Keep Invariant features.



et Voilà

A highly expressive
and invariant
jet representation





4.

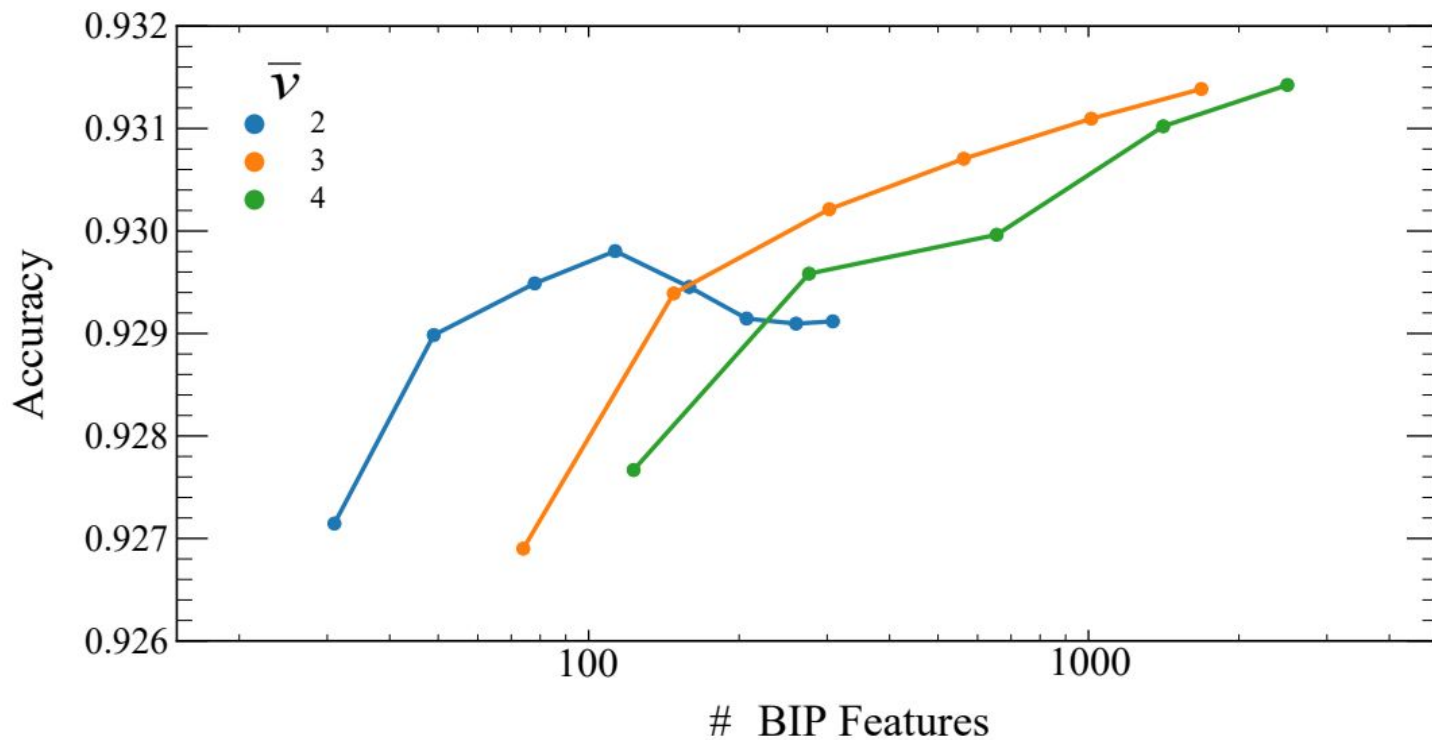
Results

Of the approach



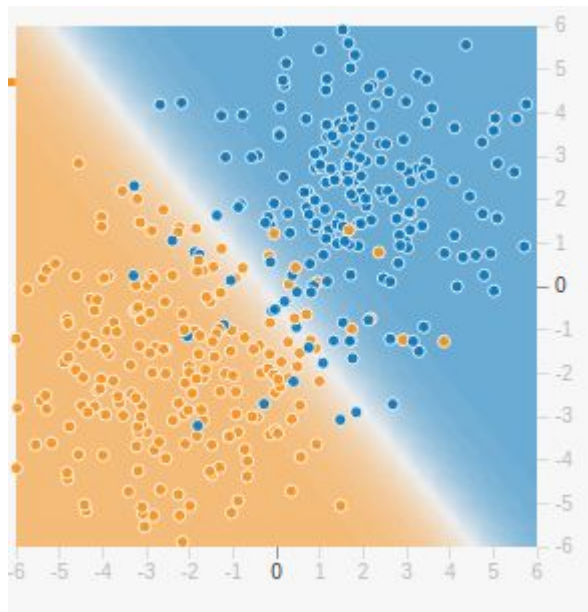
arXiv:2207.08272v2

Versatility on the basis size

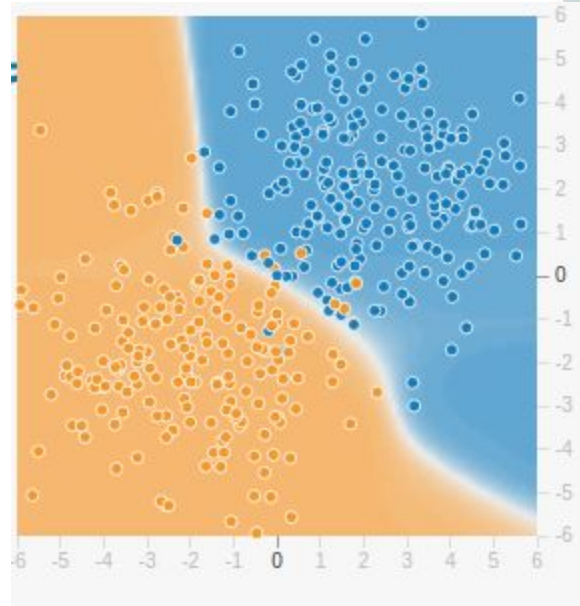


Versatility on the classifier algorithm

From logistic regressions



To DL and GBT



arXiv:2207.08272v2

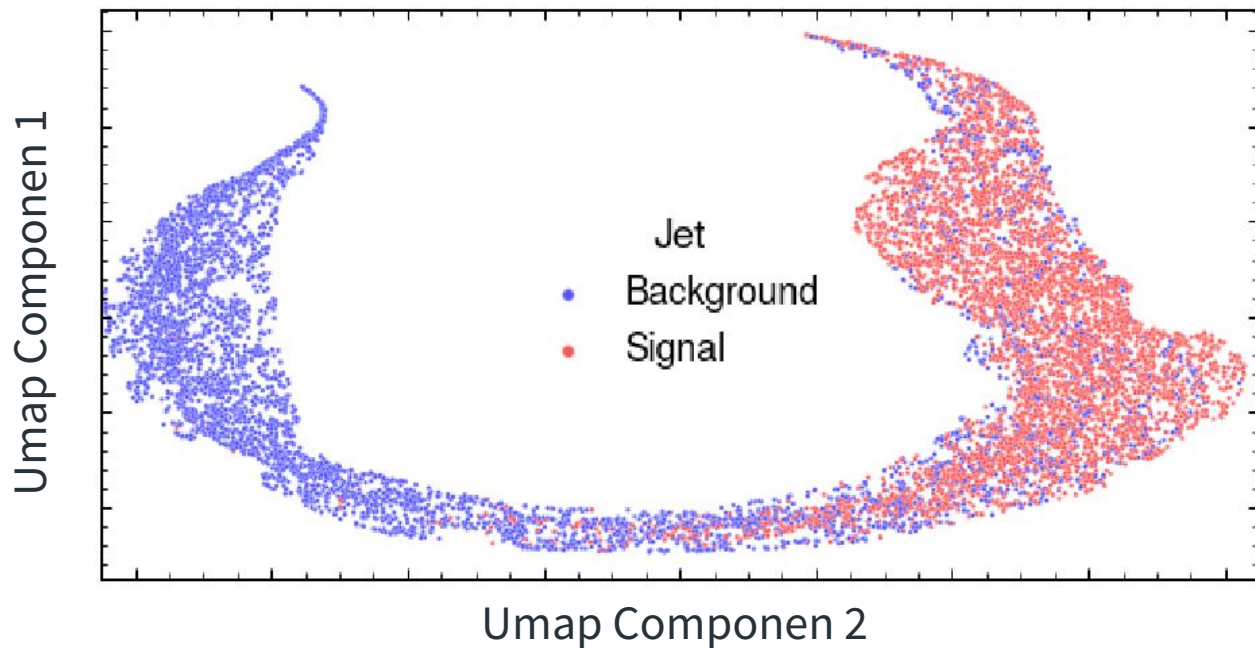


Versatility on the classifier algorithm

Architecture	#Param	Accuracy	AUC
*partT (2022) [8]	2.14M	0.944	0.988
EGNN (2022) [7]	120k	0.922	0.970
PCT (2021) [30]	139.3k	0.940	0.986
EFN (2021) [31]	82k	0.927	0.979
ParticleNet (2020) [30]	498k	0.938	0.985
LGN (2020) [17]	4.5k	0.929	0.964
P-CNN (2017) [6]	348k	0.918	0.980
TopoDNN (2017) [32]	59k	0.916	0.972
Supervised			
BIP(3, 6, MLP)	4k	0.931	0.981
BIP(3, 6, XGBoost)	300	0.929	0.978
BIP(3, 6, LogReg)	300	0.927	0.977
BIP(3, 6, SVM)	300	0.927	0.976



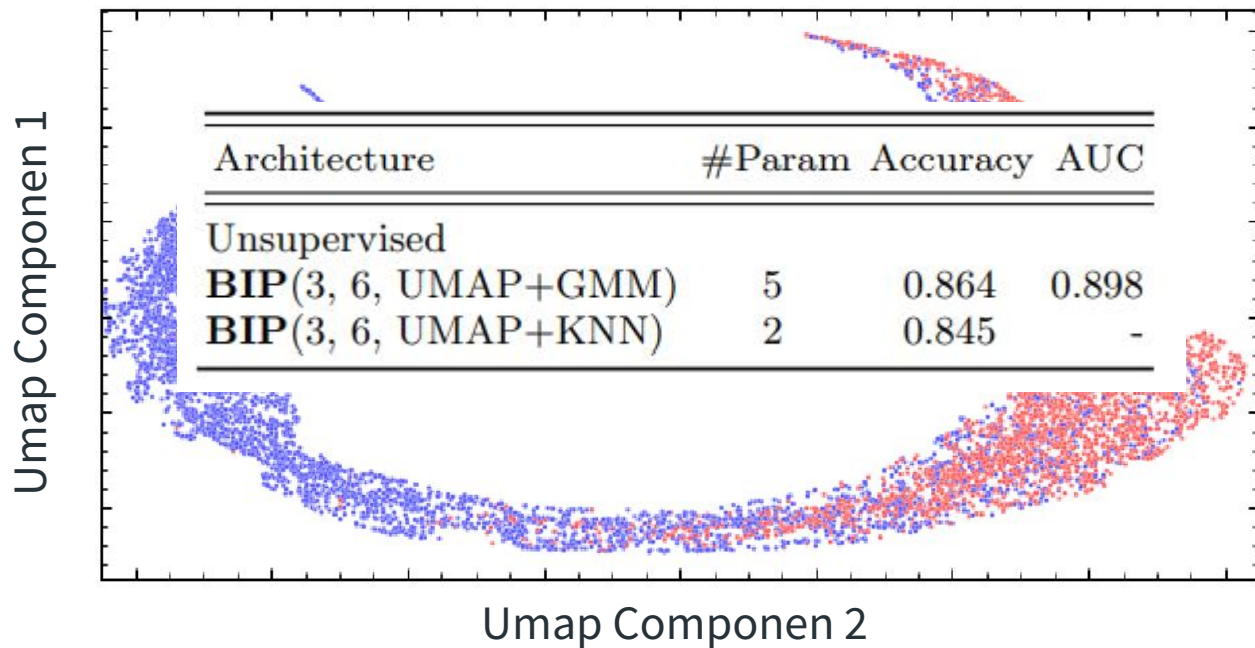
Expressive representation



arXiv:2207.08272v2



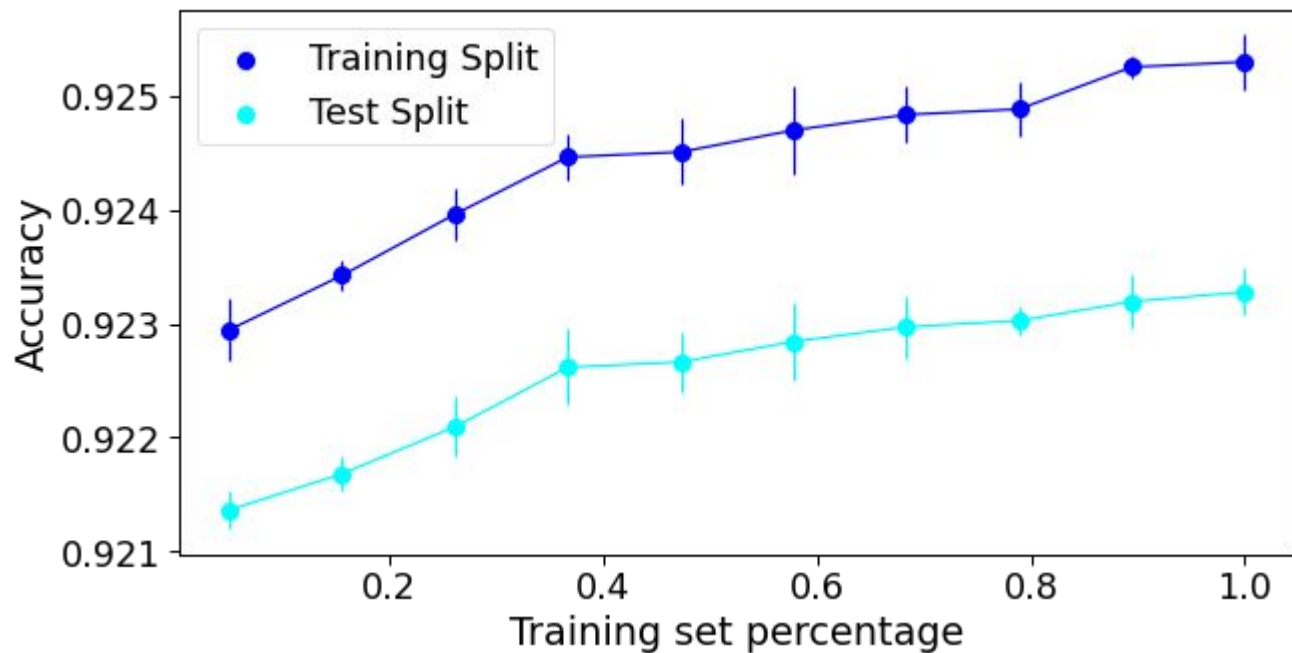
Expressive representation



arXiv:2207.08272v2



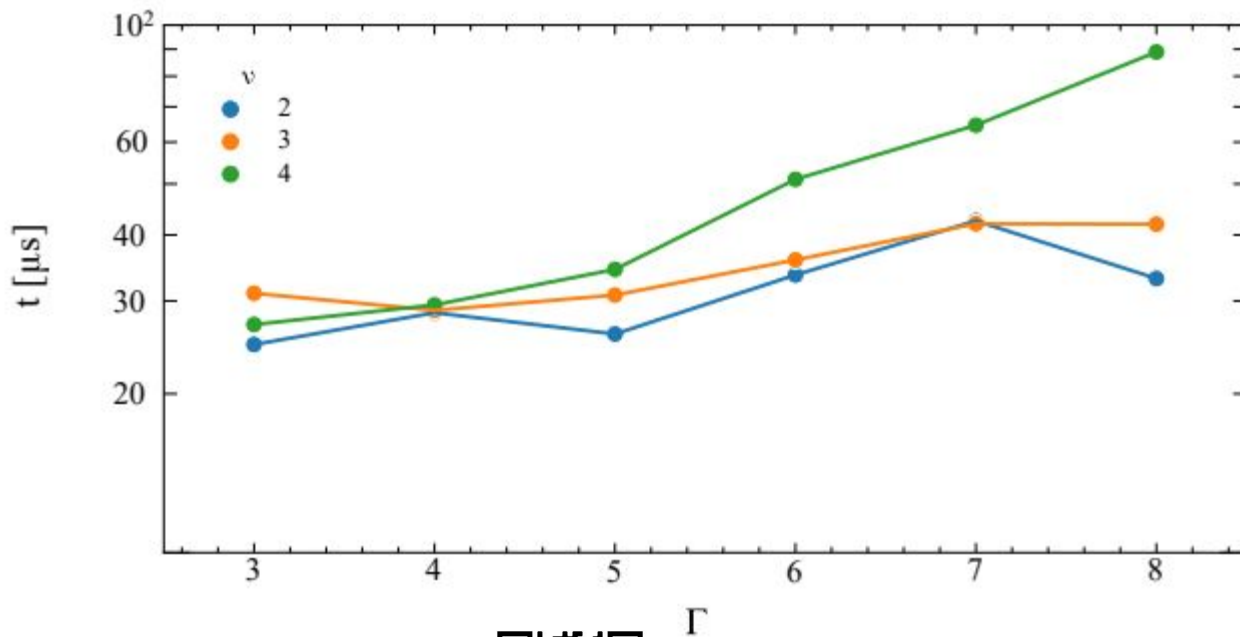
Data Efficiency



arXiv:2207.08272v2



High efficiency



arXiv:2207.08272v2

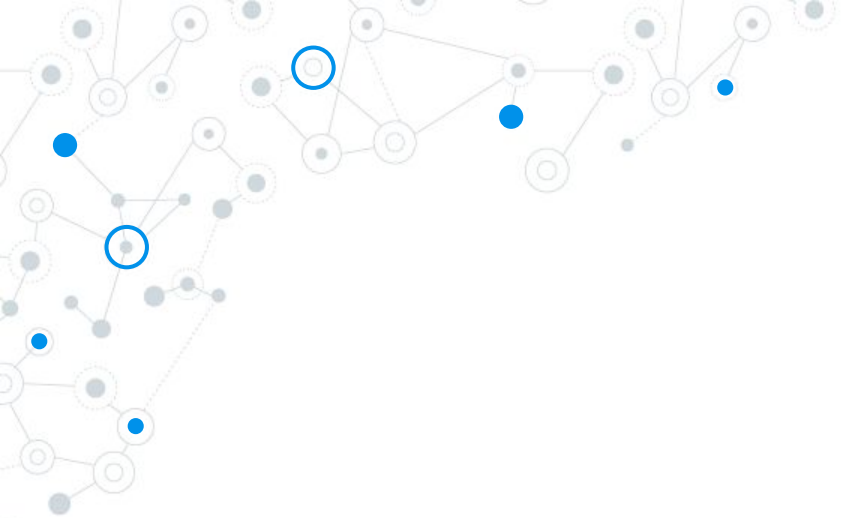
Julia logo



Γ

Also on... 
same performance





5.

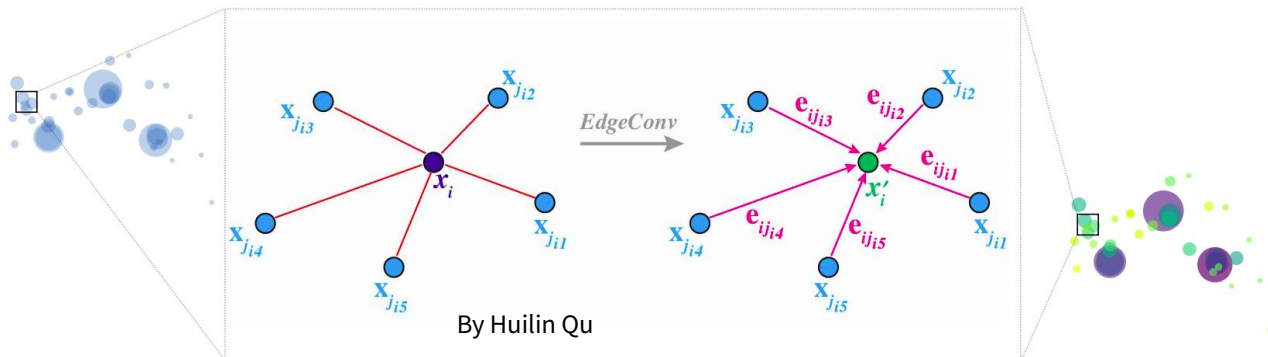
Outlooks

& Conclusions

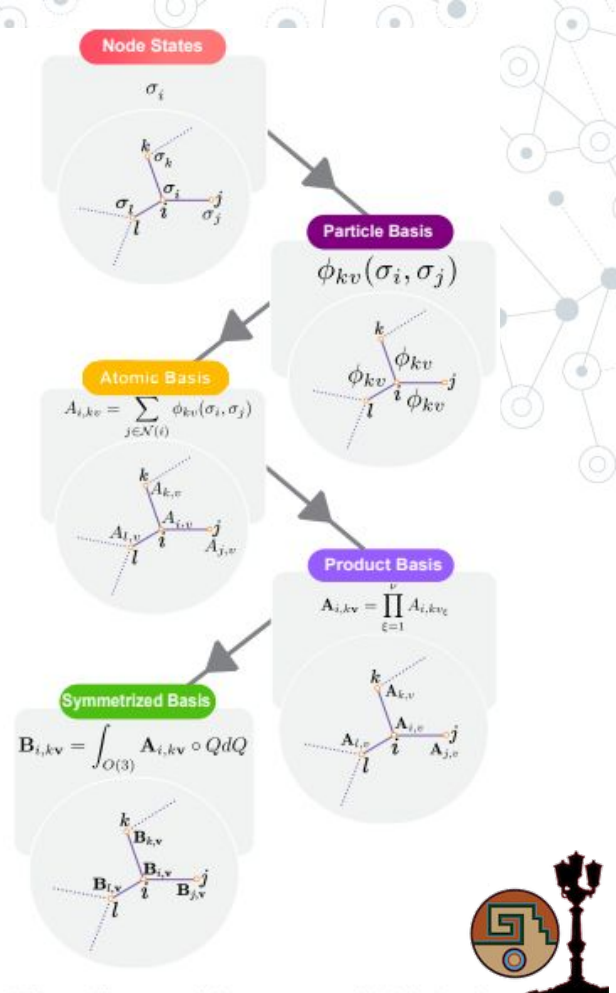


arXiv:2207.08272v2

Now... go deep



+ Learnable embeddings



Batatia et. al



arXiv:2207.08272v2

Play with the model



Paper



arXiv:2207.08272v2

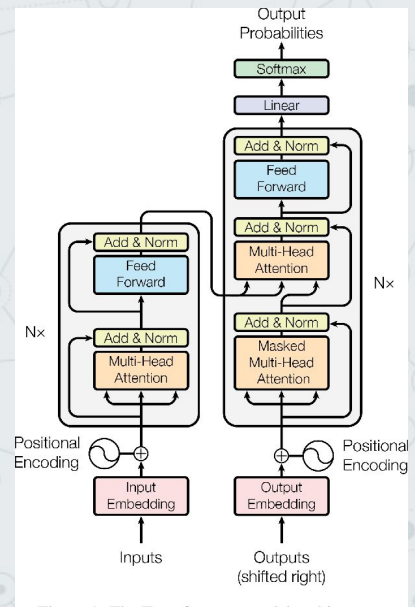


Thanks for the ~~attention!~~



profile

Please contact me for anything related

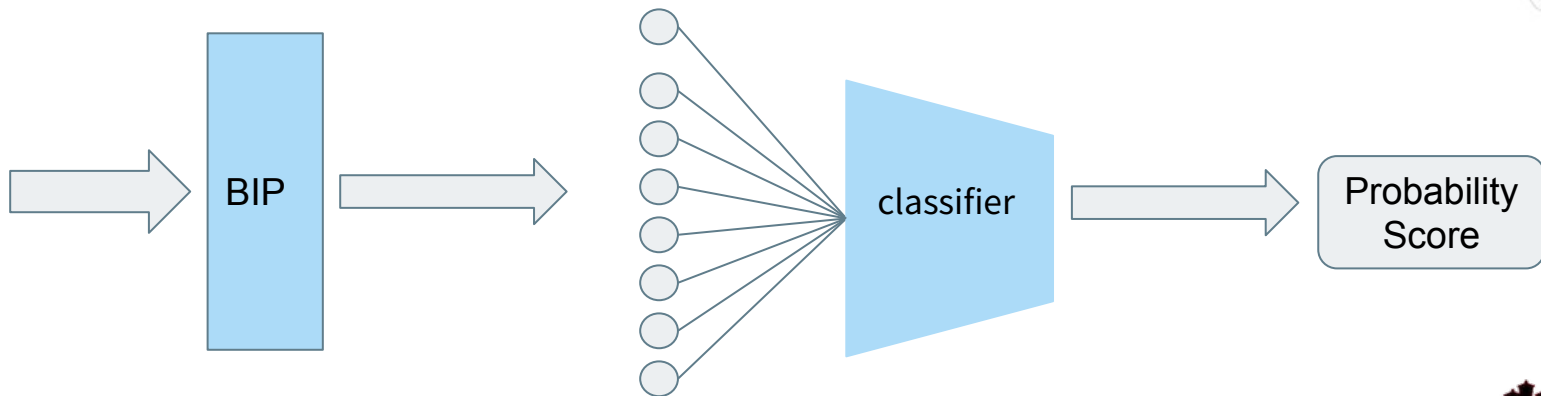


The image features a light gray background with a complex network of interconnected nodes and lines, representing a data network or cloud infrastructure. In the center, there is a dark blue rounded rectangle containing the word "BACKUP" in white, stylized, cloud-like letters. The overall theme is digital data protection and cloud storage.

BACKUP

The classifier setup:

$$f(\{E_i, \mathbf{p}_i, \xi_i\}_i) = \sum_{nlk} w_{nlk} A_{nlk},$$



Jet representation




A many body expansion

$$f(\{x_i\}_i) = f_0 + \sum_i f_1(x_i) + \sum_{i_1, i_2} f_2(x_{i_1}, x_{i_2}) \\ + \dots + \sum_{i_1, \dots, i_{\bar{v}}} f_{\bar{v}}(x_{i_1}, \dots, x_{i_{\bar{v}}}).$$

Or a density

$$\rho(x) = \sum_{i=1}^N \delta(x - x_i) \quad A_v = \langle Q_n(p_{\perp}, E_{\perp}, \bar{\xi}) e^{il\varphi} e^{-ky} | \rho \rangle$$



$$\langle \phi_{v_1} \otimes \dots \otimes \phi_{v_{\bar{v}}} | \rho \otimes \dots \otimes \rho \rangle \\ = \prod_t \langle \phi_{v_t} | \rho \rangle = \prod_t A_{v_t} = \mathbf{A}_v,$$

