Achievements and challenges of high-precision Standard Model physics at future e+e-colliders

ACAT 2022

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26. October 2022

Outline

Precision tests of the Standard Model

[Discovery machine] Today the Large Hadron Collider (LHC) probes the Standard Model at high energies

The ATLAS and CMS collaborations at the LHC discovered the Higgs boson in 2012

[Precision machine] LEP an electron positron collider and the first linear collider at Stanford

- ALEPH, DELPHI, L3, OPAL and SLD collaborations analyzed the data taken at the Z-boson resonance
- Measured Z-boson width and mass up to a precision of per-mil level
- Effectively testing 1-loop and 2-loop higher order corrections in the Standard Model, which are at sub per-mil level precision

Example measurement at the Z pole

We study the process $e^+e^- \to (Z) \to f\overline{f}$

Pseudo-observables (QED effects subtracted), unfolded at the Z peak

forward-backward asymmetry $A_{\mathrm{FB}}^{\mathrm{f\bar{f}},0} = \frac{3}{4} A_{\mathrm{e}} A_{\mathrm{f}}$

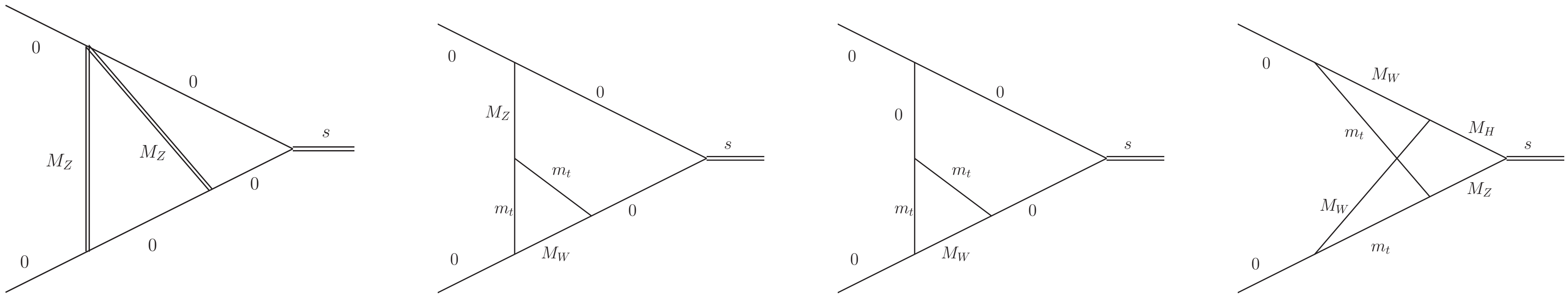
$$A_{\mathrm{f}} = \frac{2\Re e \frac{v_f}{a_f}}{1 + \left(\Re e \frac{v_f}{a_f}\right)^2} = \frac{1 - 4|Q_f| \sin^2 \theta_{\mathrm{eff}}^{\mathrm{f}}}{1 - 4|Q_f| \sin^2 \theta_{\mathrm{eff}}^{\mathrm{f}} + 8Q_f^2 (\sin^2 \theta_{\mathrm{eff}}^{\mathrm{f}})^2}$$

Definition of the effective weak mixing angle

$$\sin^2 \theta_{\mathrm{eff}}^{\mathrm{f}} = \frac{1}{4|Q_f|}\left(1 - \Re e \frac{v_f}{a_f}\right) = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_Z^f(M_Z^2))$$

v_f and a_f are effective vector coupling and axial-vector coupling of the $Zf\overline{f}$ vertex, $\Delta\kappa_Z^f(M_Z^2)$ contain the perturbative corrections

Samples of Feynman integrals for the $Z\bar{b}b$ vertex



- Number of closed loops grows with the perturbative order
- From Feynman diagrams we can project to scalar integrals
- Feynman integrals are UV and infrared divergent
- Regularized in dimensional regularization with $\epsilon = (4 - D)/2$, D the space time dimension

Historical time stamps for Electroweak $\sin^2\theta_{\rm eff}^{\rm b}$

- One-loop corrections to the $\sin^2\theta_{\rm eff}^{\rm b}$ [A. Akhundov, D. Bardin, T. Riemann, Electroweak one loop corrections to the decay of the neutral vector boson, Nucl. Phys. B276 (1986) 1.] [W. Beenakker, W. Hollik, The width of the Z boson, Z. Phys. C40 (1988) 141.]

- Two-loop electroweak corrections to the $\sin^2\theta_{\rm eff}^{\rm b}$ [Awramik, M. Czakon, A. Freitas, B. Kniehl, Two-loop electroweak fermionic corrections to $\sin^2\theta_{\rm eff}^{\rm b}$, Nucl. Phys. B813 (2009) 174-187.] [I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, The two-loop electroweak bosonic corrections to $\sin^2\theta_{\rm eff}^{\rm b}$, Phys. Lett. B762 (2016) 184-189.]

Electroweak precision physics

	Experiment	Theory uncertainty	Main source
$M_W\,[\mathrm{MeV}]$	80385 ± 15	4	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_\mathrm{s}$
$\sin^2\theta_\mathrm{eff}^\mathrm{l}\,[10^{-5}]$	23153 ± 16	4.5	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_\mathrm{s}$
$\Gamma_Z\,[\mathrm{MeV}]$	2495.2 ± 2.3	0.4	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_\mathrm{s},\ \alpha\alpha_\mathrm{s}^2$
$\sigma_\mathrm{had}^0\,[\mathrm{pb}]$	41540 ± 37	6	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_\mathrm{s}$
$R_f = \Gamma_Z^f/\Gamma_Z^\mathrm{had}\,[10^{-5}]$	21629 ± 66	15	$N_f^2\alpha^3,\ N_f\alpha^2\alpha_\mathrm{s}$

- The number of Z-bosons collected at LEP is 1.7×10^7
- New results fermionic $\mathcal{O}(\alpha^2\alpha_\mathrm{s})$ [Lisong Chen and Ayres Freitas, JHEP 03 (2021) 215] are not included, yet

Overview of future experiments as of 2021

	Experiment uncertainty			Theory uncertainty
	ILC	**CEPC**	**FCC-ee**	**Current**
M_W [MeV]	3-4	3	1	4
$\sin^2\theta^{\mathrm{l}}_{\mathrm{eff}}$ [10^{-5}]	1	2.3	0.6	4.5
Γ_Z [MeV]	0.8	0.5	0.1	0.4
R_f [10^{-5}]	14	17	6	15

- FCC-ee Tera-Z operating at 88-95 GeV producing 5×10^{12} visible Z decays, 5 orders of magnitude more events than at LEP
- FCC-ee Tera-Z reproduces the LEP data in 23 hours and is planned to operate for 5 years
- To match the precision of the experiment we compute 3-loop and 4-loop Standard Model predictions

Overview of future experiments as of 2022

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M_W [MeV]	3-4	3	~~1~~ 0.3	4
$\sin^2\theta^{l}_{\text{eff}}$ [10^{-5}]	1	2.3	?0.6	4.5
Γ_Z [MeV]	0.8	0.5	~~0.1~~ 0.025	0.4
R_f [10^{-5}]	14	17	~~6~~ 1	15

- Recent update from [Alain Blondel, Patrick Janot, Eur.Phys.J.Plus 137 (2022) 1]
- To match the precision of the experiment we compute 3-loop and 4-loop Standard Model predictions

Z-boson form factors at two-loop accuracy

[I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy, JHEP 08 (2019) 113.]

Form fact.	Born	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha\alpha_s)$ non-fact.	$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	$\mathcal{O}(N_f^2\alpha^2)$	$\mathcal{O}(N_f\alpha^2)$	$\mathcal{O}(\alpha_{bos}^2)$
F_V^ℓ $[10^{-5}]$	39.07	−24.86	2.41	–	0.35	1.47	2.37	0.27
F_A^ℓ $[10^{-5}]$	3309.44	118.59	9.46	–	1.22	8.60	2.60	0.45
$F_{V,A}^\nu$ $[10^{-5}]$	3309.44	127.56	9.46	–	1.22	8.60	3.83	0.39
$F_V^{u,c}$ $[10^{-5}]$	544.88	−44.80	7.29	−0.39	1.02	−1.67	3.25	0.33
$F_A^{u,c}$ $[10^{-5}]$	3309.44	120.79	9.46	−0.98	1.22	8.60	3.27	0.44
$F_V^{d,s}$ $[10^{-5}]$	1635.01	5.84	9.64	−0.80	1.32	0.71	3.45	0.37
$F_A^{d,s}$ $[10^{-5}]$	3309.44	123.78	9.46	−1.14	1.22	8.60	3.11	0.42
F_V^b $[10^{-5}]$	1635.01	−26.16	9.64	3.13	1.32	0.71	1.77	1.05
F_A^b $[10^{-5}]$	3309.44	78.26	9.46	4.45	1.22	8.60	0.13	1.18

Table: *Contributions of different perturbative orders to the Z vertex form factors. A fixed value of M_W has been used as input, instead of G_μ. N_f^n refers to corrections with n closed fermions loops, whereas α_{bos}^2 denotes corrections without closed fermions loops. Furthermore, $\alpha_t = y_t/(4\pi)$ where y_t is the top Yukawa coupling.*

- Some progress towards three-loop Electroweak with fermionic three-loop corrections at $\mathcal{O}(\alpha^2\alpha_s)$ [Lisong Chen and Ayres Freitas, JHEP 03 (2021) 215]

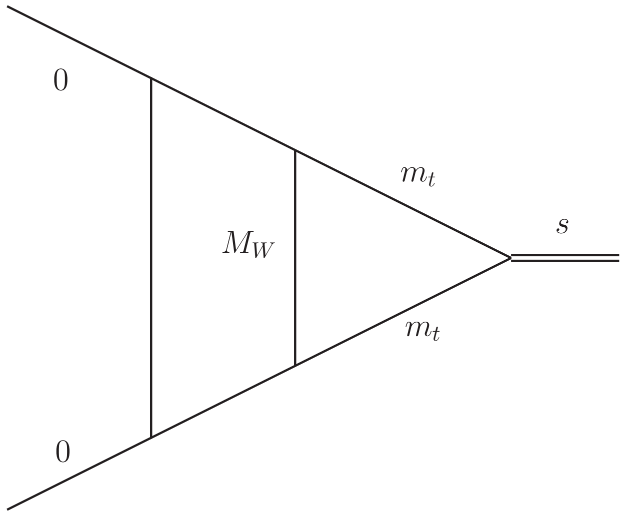
How well goes the calculation

- We generate systematically Feynman diagrams at 3-loop order and 4-loop order
- Public codes are FeynArts [T. Hahn, 2001] and QGRAF [P. Nogueira, 1993]
- Number of Feynman diagrams grows factorially
- 3-loop full Electroweak order $\sim 400\,000$ Feynman diagrams
- 4-loop involves more than 1 million Feynman diagrams [work in progress]

Numerical evaluation

- Integrals are divergent like $1/\epsilon^{2L}$, L the loop-number
- A cancellation of all divergences is required
- Large cancellations between the terms; require high numerical precision
- General methods for Feynman integral computation: sector decomposition [T. Binoth, G. Heinrich, 2000, G. Heinrich, 2008], Mellin-Barnes approach [V. A. Smirnov:1999, B. Tausk,1999], system of differential equations [Kotikov, 1991, Remiddi, 1997, Gehrmann, Remiddi, 2000]
- General tools to compute Feynman integrals numerically for precision physics: pySecDec [G. Heinrich, S. Jahn,S.P. Jones,M. Kerner,F. Langer,2022], FIESTA5 [A.V. Smirnov, , N. D. Shapurov, L. I. Vysotsky, 2021], DiffExp [M. Hidding, 2006.05510] and AMFlow [Liu, Xiao and Ma, Yan-Qing,2201.11669], SeaSyde [T. Armadillo, R. Bonciani, S. Devoto, N. Rana, A. Vicini]

State of the art 6 years ago

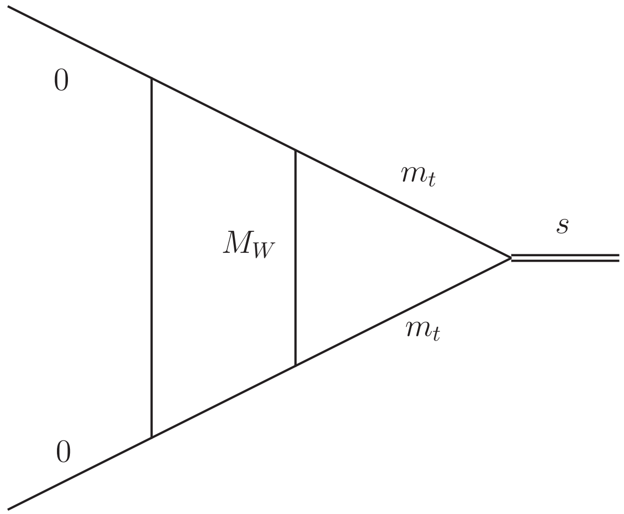


- In physical regions $(s, M_W^2, m_t^2) = (1, (\frac{401925}{4559382})^2, (\frac{433000}{227969})^2)$
- Arbitrary kinematic point, but with restricted accuracy
- A complementary mixture of Mellin-Barnes integral and sector decomposition methods

$$\text{soft13}^{d=4-2\epsilon}[1,1,1,1,1,1,0] = 0.93453624 + 0.54089756\, i$$
$$+(0.1901137256 - 0.6583157563\, i)/\epsilon - 0.2095484134808370/\epsilon^2$$

- One kinematic point in 1 day

The two-loop example

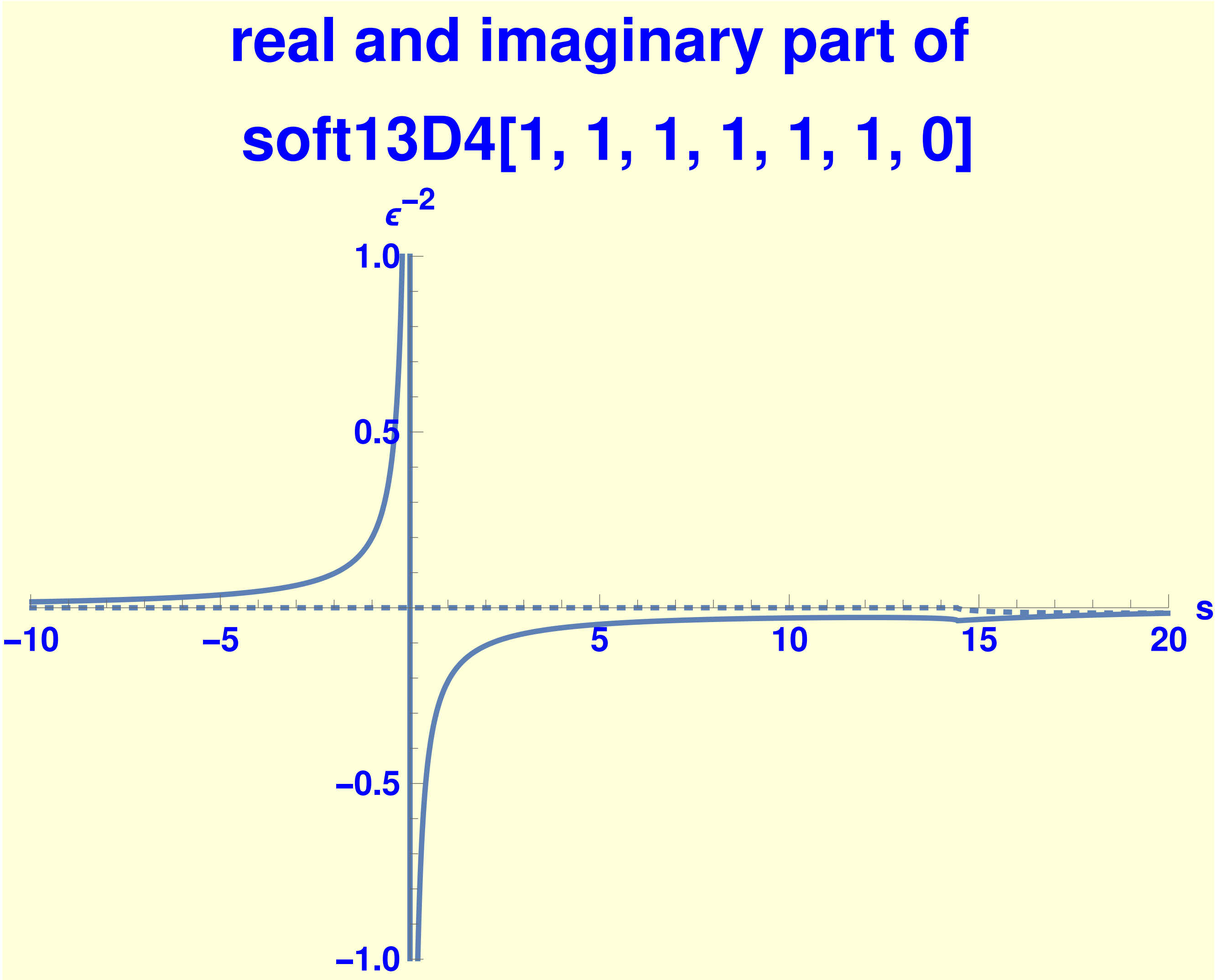


- With the program AMFlow

$$\text{soft13}^{d=4-2\epsilon}[1,1,1,1,1,1,0]$$
$$= (0.934536247523241 + 0.540897568924577\, i)$$
$$+(0.190113725674667 - 0.658315756362794\, i)1/\epsilon$$
$$-0.209548413480837 0/\epsilon^2$$

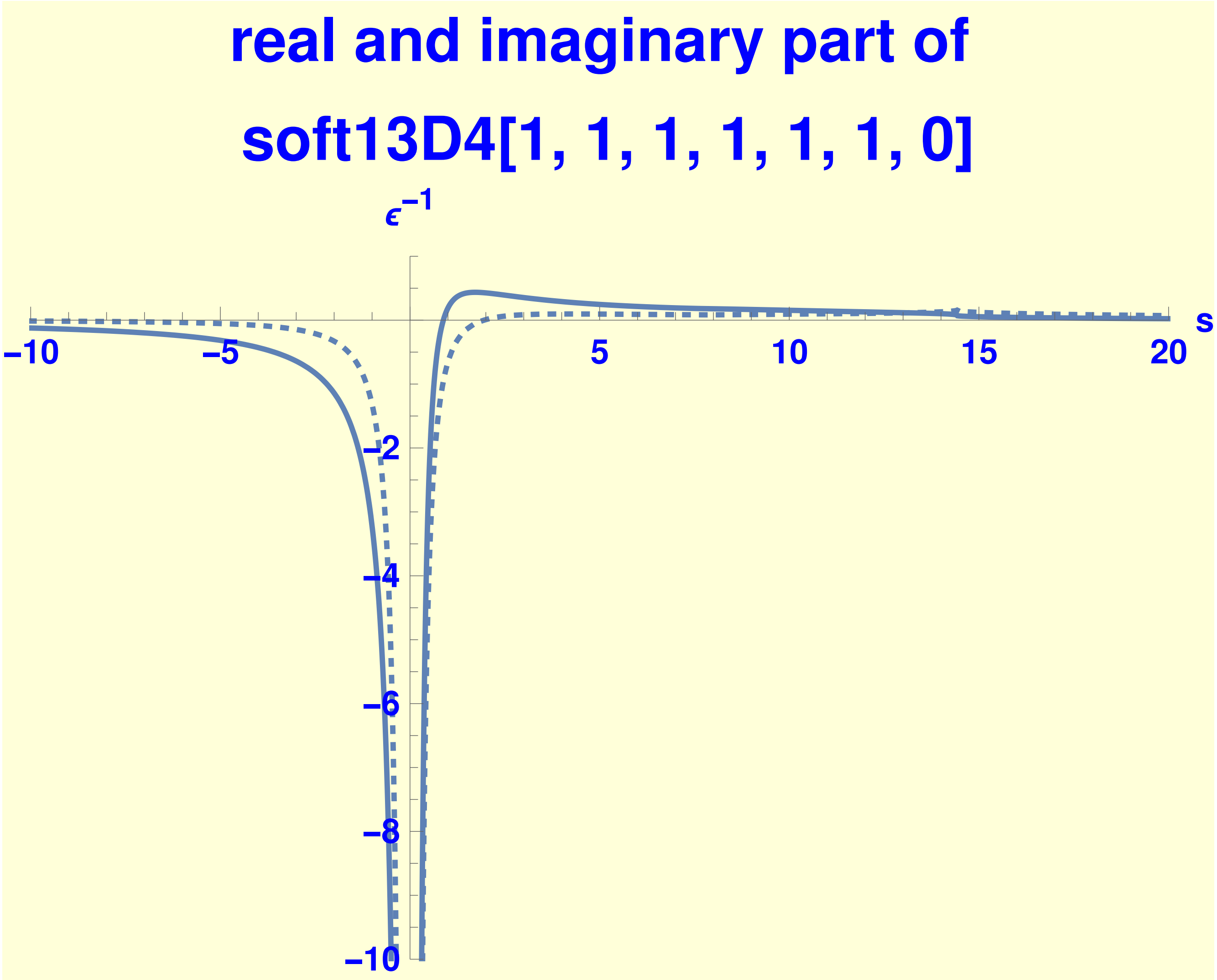
- Arbitrary kinematic point in 5 minutes

The state of the art - automatic computations



- With DiffExp
- Arbitrary line in the phase space with arbitrary accuracy

The state of the art - automatic computations



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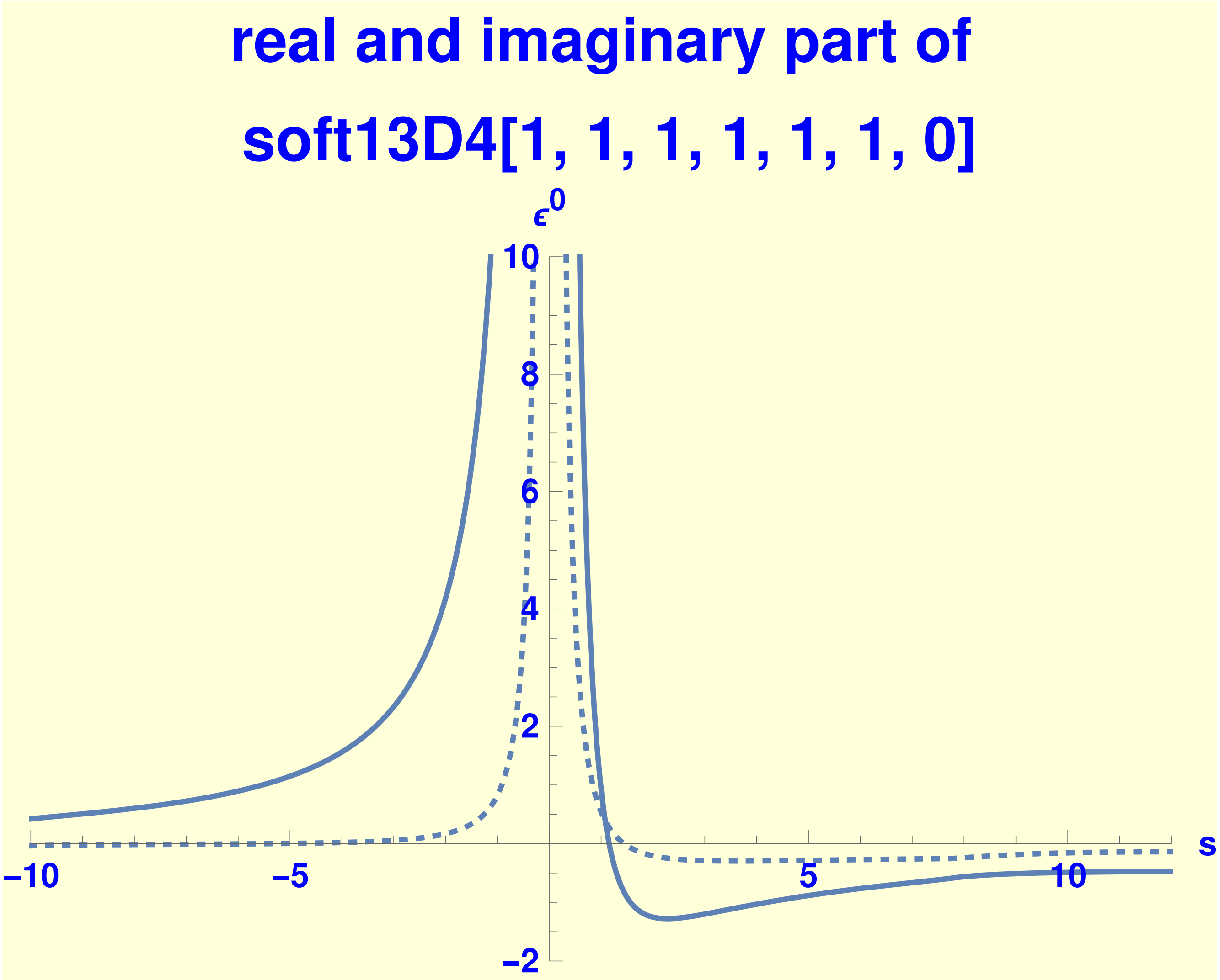

- With DiffExp
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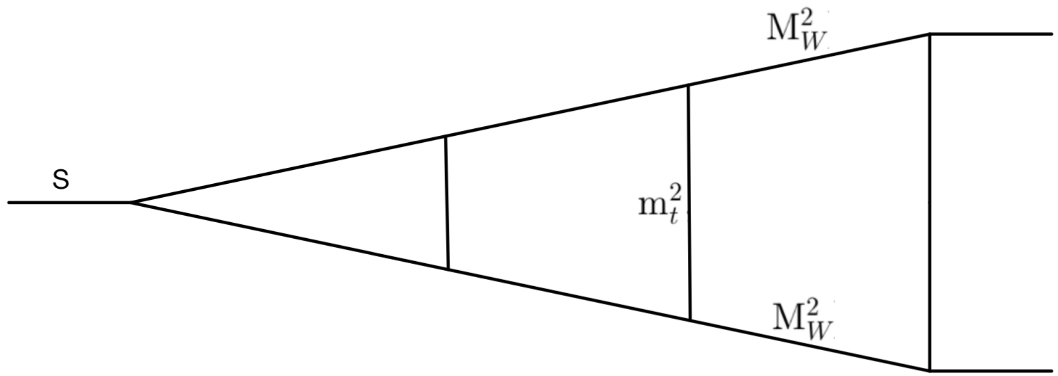
The state of the art 2021 - automatic computations



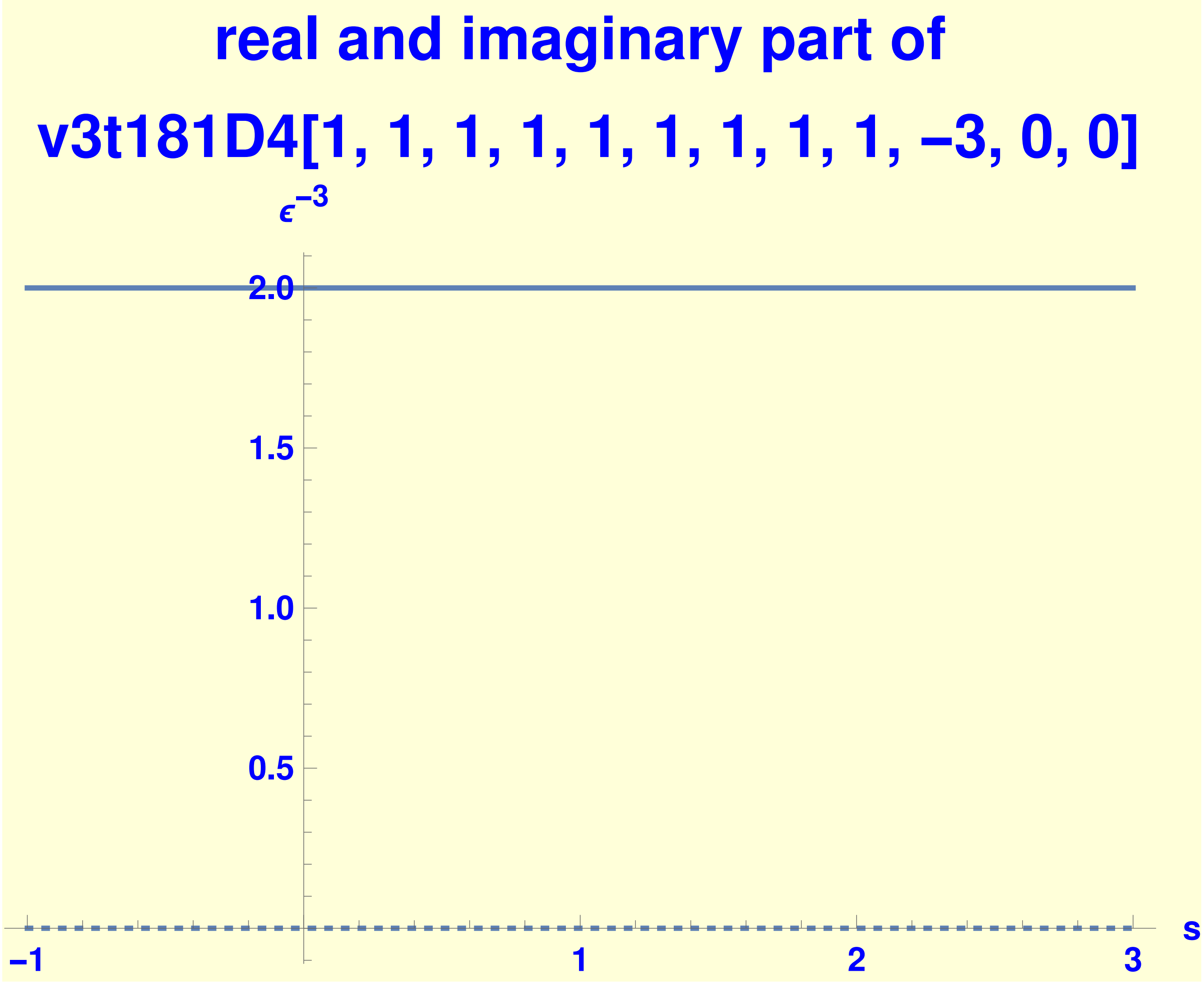
- In physical regions $(s, M_W^2, m_t^2)=(1,(\frac{401925}{4559382})^2, (\frac{433000}{227969})^2)$

-> v3t181$^{d=4-2\epsilon}$[1, 1, 1, 1, 1, 1, 1, 1, 1, -3, 0, 0] $=$

$$\frac{2.00000000000}{\epsilon^3} + \frac{9.8700393436 + 18.8495559213\, i}{\epsilon^2} - \frac{26.507336797 - 41.196707081\, i}{\epsilon} + (2.29574523 + 201.06880207\, i) + O(\epsilon)$$

- Fully automated with DiffExp[pySecDec]

The state of the art - automatic computations



- Fully automated with DiffExp[AMFlow]

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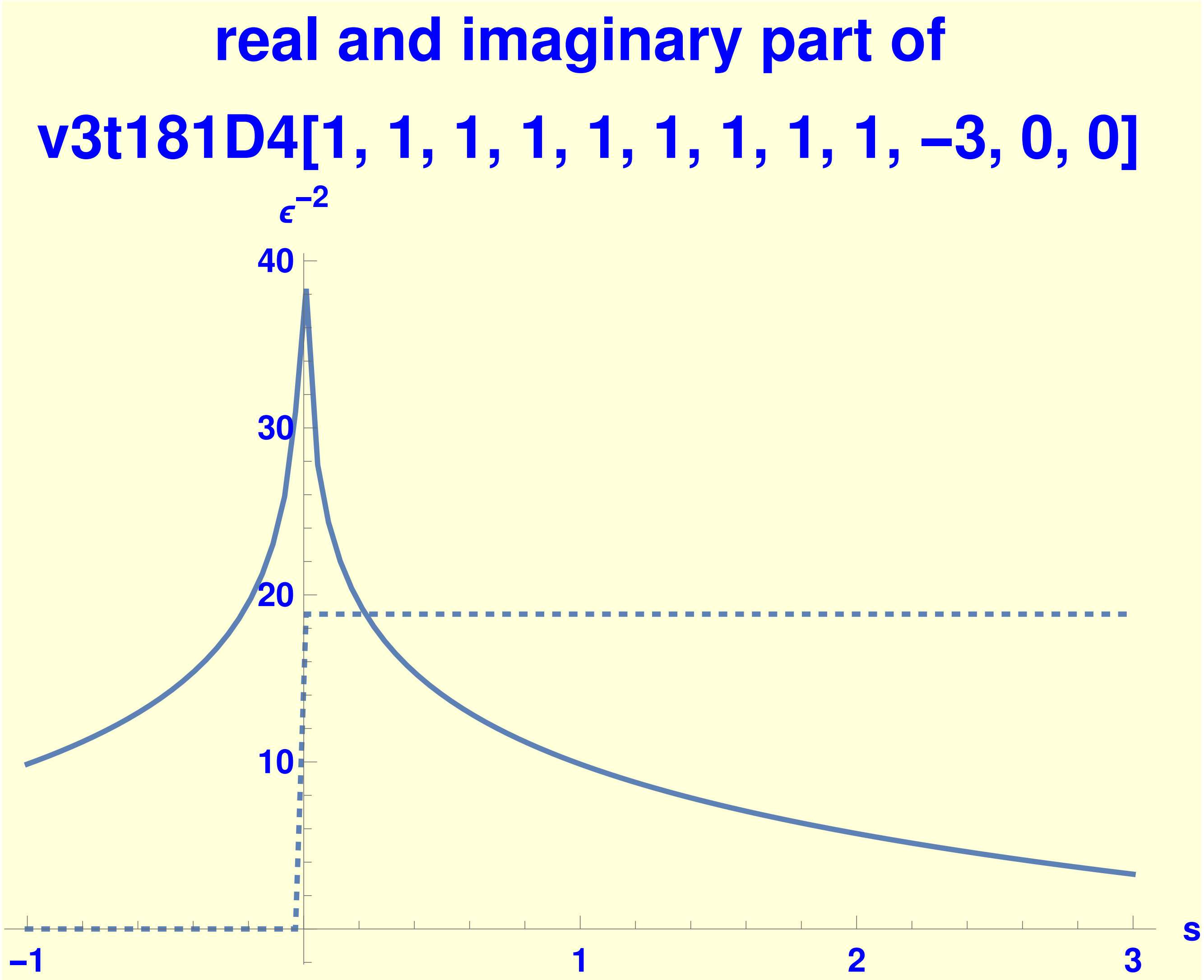

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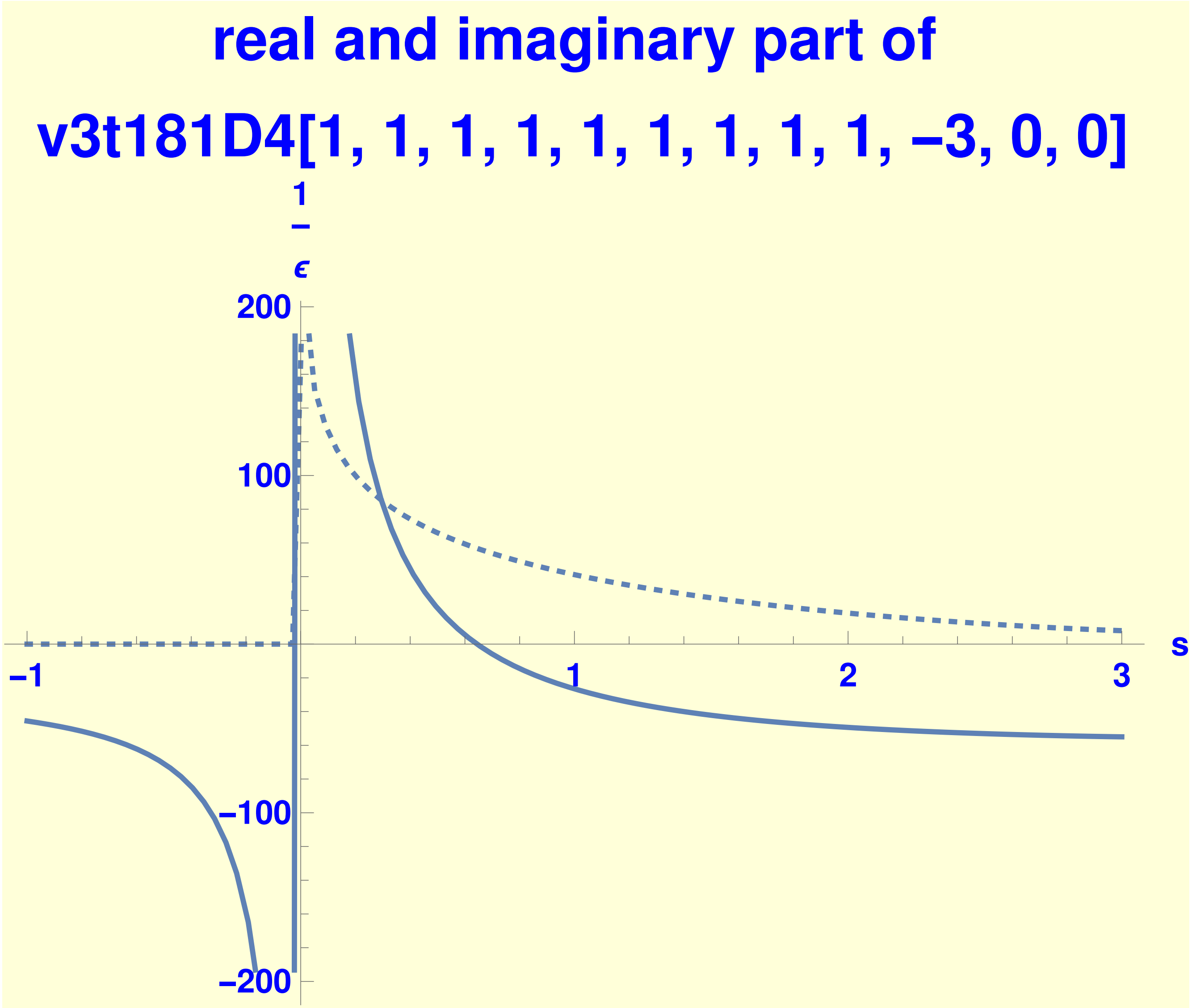

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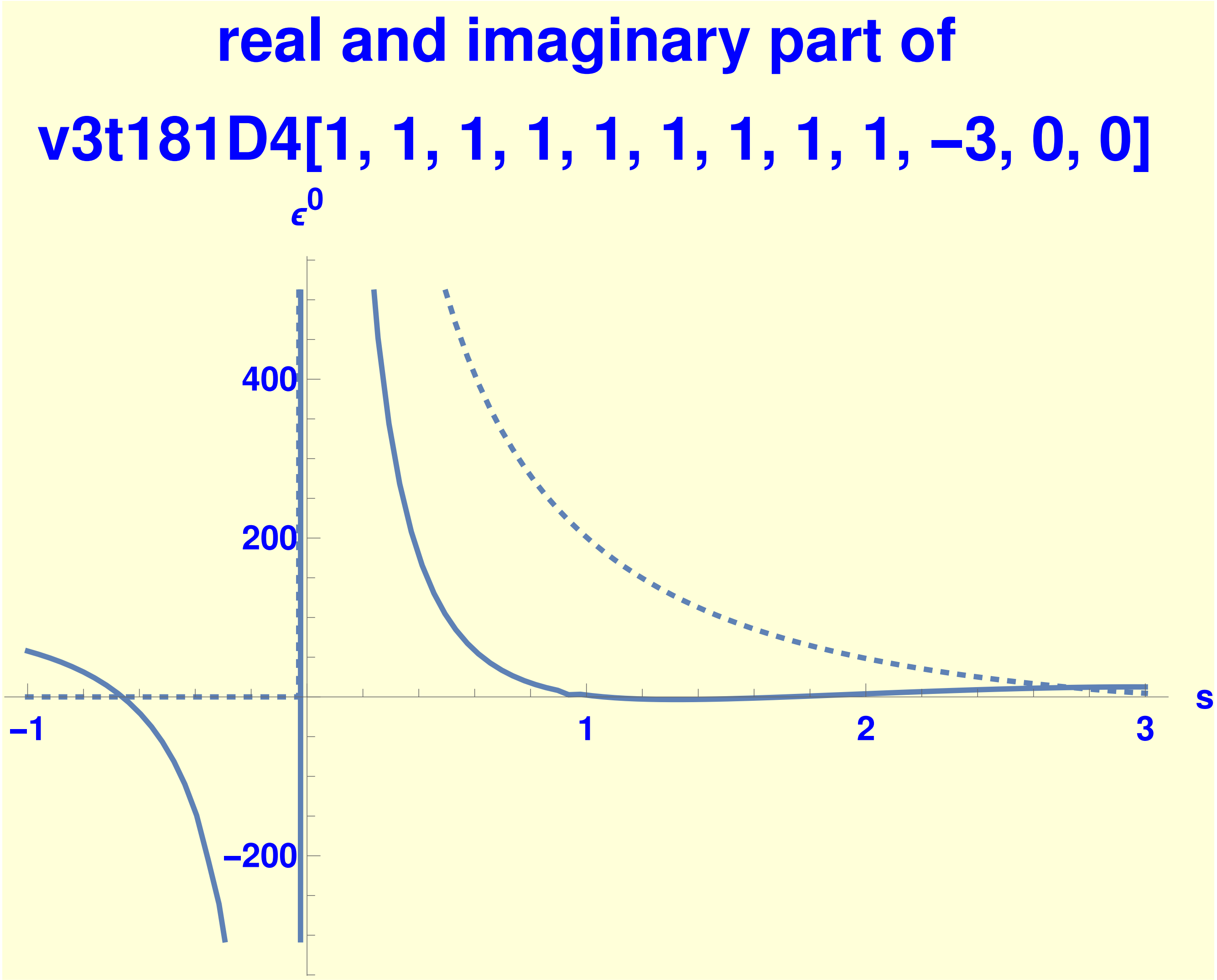


- Fully automated with DiffExp[AMFlow]

The state of the art - automatic computations

- System of differential equations with [Moriello, 1907.13234] approach scales linear with precision to computing time
- Linear scaling is already implemented at least in [Liu, Xiao and Ma, Yan-Qing,2201.11669]
- Great incentive to develop C++ code for better main memory and disk usage
- Great potential for parallel computing with MPI
- To hard to run calculations on a GPU for now; to much main memory consumption

Feynman parameter integration through differential equations

Novel idea in computing multi-loop Feynman integrals

Feynman parameter integration through differential equations

$$\int d^D k_1 \frac{1}{[k_1{}^2][(p_1+k_1)^2]} = \int_0^1 dx \underbrace{\int \frac{d^D k_1}{[k_1{}^2 x + (1-x)(p_1+k_1)^2]^2}}_{\vec{I}(x)}$$

- Methods solving system of differential equations [Kotikov, 1991, Remiddi, 1997, Gehrmann, Remiddi, 2000] are applicable

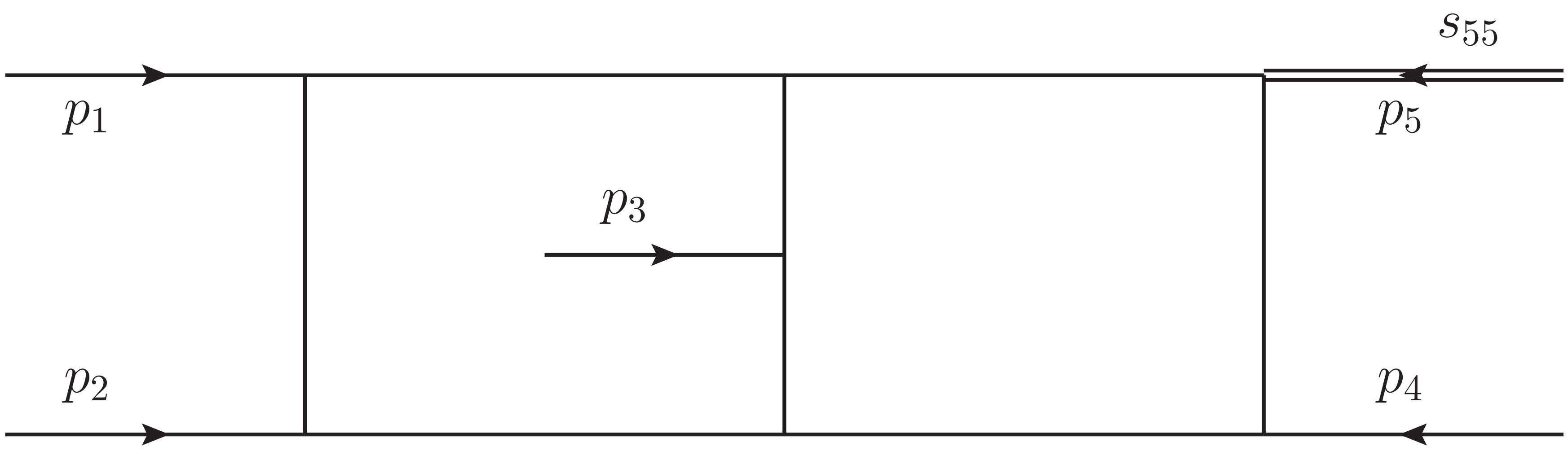
$$\partial_x \vec{I}(x) = M_x \vec{I}(x)$$

- We understand how to compute systematically a piece wise function for arbitrary $\vec{I}(x)$ [Moriello, 1907.13234][Hidding, 2006.05510]
- Integrating the $\vec{I}(x)$ in x gives numerical result of possibly arbitrary Feynman diagram

X-Feynman integrals

- Development of a numerical algorithm in C++ is desired to reach 3-loop and 4-loop goals in Electroweak Z-boson description
- Bottleneck of Feynman integral calculation is reduced by 2 more orders of magnitude
- Fast one-dimensional integration in the variable x
- Define X-Feynman diagrams

Non trivial example double pentagon

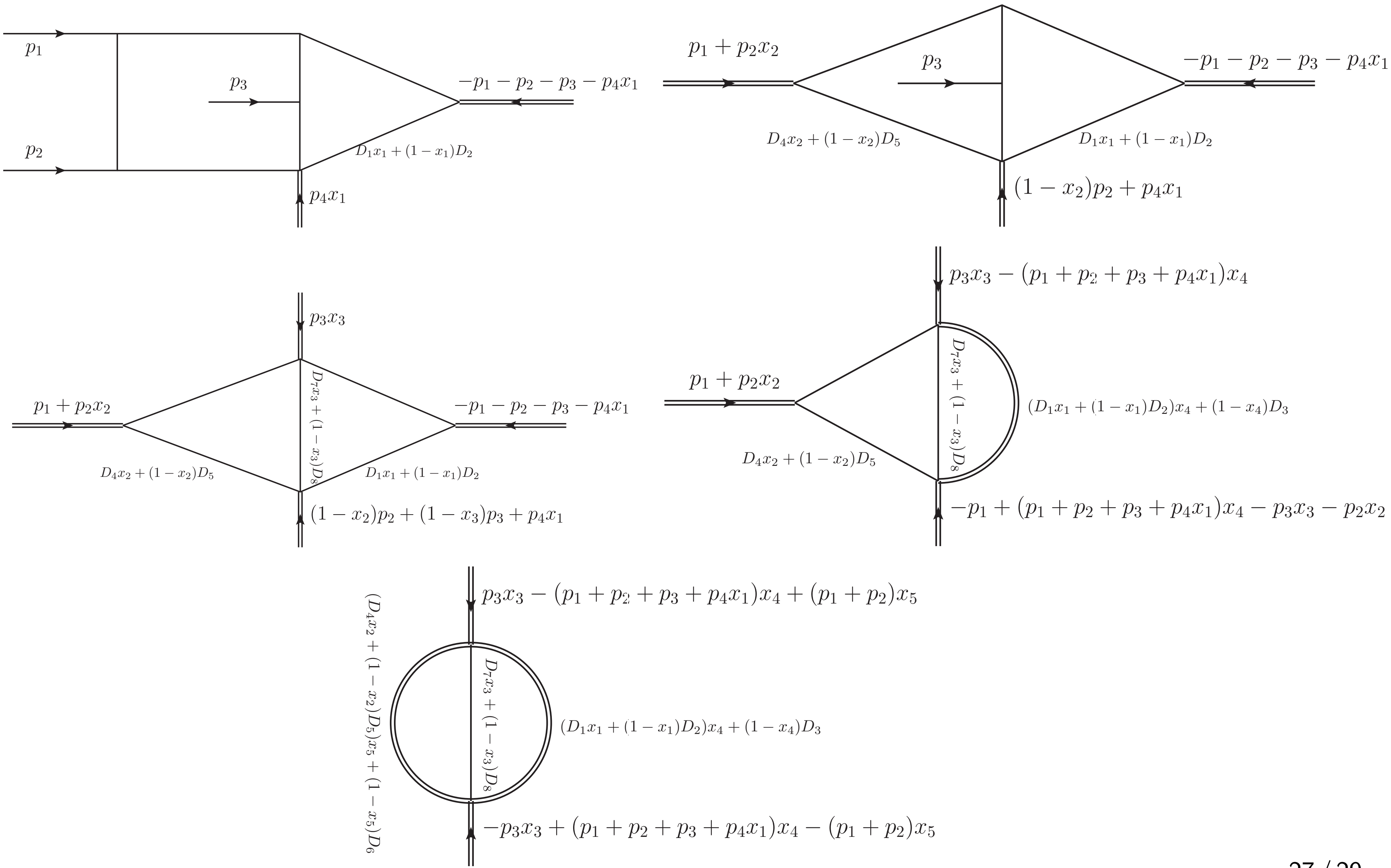


The kinematics is given by:

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0 \qquad p_1 \cdot p_2 = s_{12}/2 \qquad p_1 \cdot p_3 = s_{13}/2 \qquad p_1 \cdot p_4 = s_{14}/2$$

$$p_2 \cdot p_3 = s_{23}/2 \qquad p_2 \cdot p_4 = -(s_{12} + s_{13} + s_{14} + s_{23} + s_{34} - s_{55})/2 \qquad p_3 \cdot p_4 = s_{34}/2$$

Non trivial example double pentagon



Non trivial example double pentagon

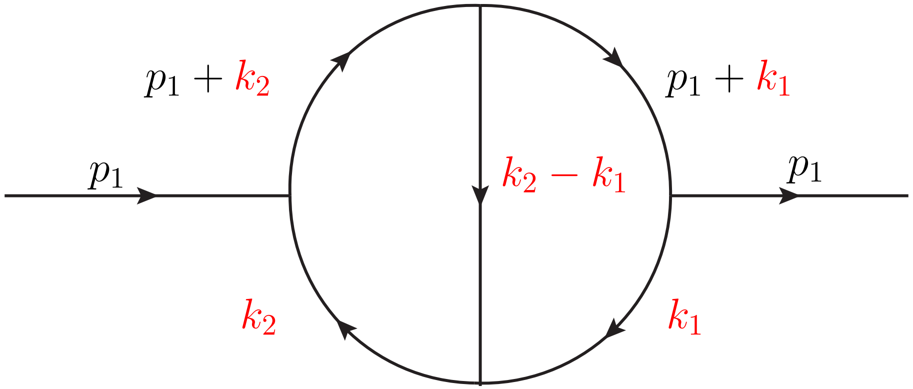
$$
\begin{aligned}
I^{\text{doublePentagon}}_{13111111000} = \frac{1}{\varepsilon^4}\Big(& -80991.4463494183281585513495668633013424445 9\Big) \\
+\frac{1}{\varepsilon^3}\Big(& -1176854.140501650857516200908950071824160111- \\
& 303701.8453350029342400125918254935316349429i\Big) \\
+\frac{1}{\varepsilon^2}\Big(& -13432835.847769296218563739493160489179767 4- \\
& 4251651.649659801661147742722015336765805 80i\Big) \\
+\frac{1}{\varepsilon^1}\Big(& -111346171.6370450328807043552785900423292 1- \\
& 32927342.395688330300021665788556801968176i\Big) \\
+\Big(& -763045644.5561305442093867867513427731742 - 183231121.4048774146788661490531205282119i\Big) \\
+\varepsilon\Big(& -4428755434.161197546975559276527347917 19 - 816059490.912195429388068459166197648719i\Big) \\
+\varepsilon^2\Big(& -23085640630.259889520777994526537639199 - 3082908606.755129481150421547364262960 5i\Big) \\
+\varepsilon^3\Big(& -110164352209.709241265245125661094393 8 - 10252510409.421856915506877661523536 40i\Big) \\
+\varepsilon^4\Big(& -497649560130.015209279192098631531920 - 30796992268.351608687056655955075410 4i\Big)
\end{aligned}
$$

Minkowskien point: $s_{14} = 3$, $s_{13} = -11/17$, $s_{23} = -13/17$, $s_{12} = -7/17$, $s_{34} = -7/13$, $s_{55} = -1$

Outlook

- State of the art theory and experiment for the Z-boson resonance physics are in a very good shape
- Future colliders push the precision state of the art in experimental measurements
- These measurements test the Electroweak Standard Model by 1 to 2 orders of magnitude more in precision; ~ 2 more loop orders required
- Modern techniques in numerical calculation of Feynman integrals scale linear with precision to run time
- Novel techniques are highly desirable

Feynman integral



$$I(a_1, \ldots, a_5) = \int \frac{d^D k_1 d^D k_2}{[k_1{}^2]^{a_1}[(p_1 + k_1)^2]^{a_2}[k_2{}^2]^{a_3}[(p_1 + k_2)^2]^{a_4}[(k_2 - k_1)^2]^{a_5}}$$

- To make Standard Model predictions we compute several thousand different integrals with different values for $\{a_f\}$
- Calculating each Feynman integral individually for every new choice of $\{a_f\}$ is inefficient