DMG4: a fully GEANT4-compatible package for the simulation of Dark Matter

H H Sieber

ETH Zürich, Institute for Particle Physics and Astrophysics, CH-8093 Zürich, Switzerland

E-mail: `henri.hugo.sieber@cern.ch`

Abstract. The search of New Physics through Dark Sectors is an exciting possibility to explain, among others, the origin of Dark Matter. Within this context, the sensitivity study of a given experiment and its design optimization are key points in estimating its potential for discovery. In this proceeding we present the fully `GEANT4`-compatible Monte Carlo simulation package for production and propagation of Dark Matter particles, `DMG4`. In particular, we discuss the implementation of production cross-sections in its `GEANT4`-independent sub-package, DarkMatter, and the latest `DMG4` release, including a finer application programming interface to `GEANT4`. We also cover its recent developments with faster and more accurate cross-section computations, sampling methods, extended energy range, as well as the expansion of the package to $B - L$ and semi-visible models. We finally discuss the improvements in the simulations of New Physics processes specific to muon beams.

1. Introduction

Dark Sectors (DS) are an attractive framework in the search for Physics beyond the Standard Model (SM). In particular, light Dark Matter (LDM) is a popular candidate for DM in the sub-GeV mass range. Within this framework, the benchmark model assumes interactions between DM and SM particles through a $U'(1)$ light vector boson mediator, A', also called *dark photons*, through the kinetic mixing with SM photons, with mixing strength parameter ϵ (see e.g. [1] for a review). To this extent, a broader class of mediators flavors have also been suggested, such as scalar or pseudo-scalar particles. Such models suggest the existence of DS in Nature and provide a convenient way to explain the observed thermal DM relic abundance.

Accelerator-based experiments provide a suitable tool to probe DM candidates at typical energy scales comparable to the DM thermal freeze-out [2]. At moderate beam energy $\sim \mathcal{O}(10-100)$ GeV a wide region of the parameter space is in reach of a potential discovery in the mass range MeV-GeV. In particular, fixed-target and beam dump experiments attempt to produce DM in the interactions of protons, electrons or muons with an (active) beam dump.

To infer the sensitivity of those experiments to LDM, we propose the `DMG4` package for the simulation of DM with the particularity of being fully `GEANT4`-compatible [3]. In this manner, is not only provided a Monte Carlo (MC) DM event generator, but also the propagation of the later through the experiment detectors. In the following, we cover both the package structure and its recent developments. We finally show typical sensitivities at beam-dump experiments.

2. Structure of the DMG4 package

The DMG4 package [4] aims at combining both a MC event generator for DM production in fixed-target experiments and the propagation of its products in an experimental set-up. It has been written in C++ and built to be compatible with the well-established simulations toolkit GEANT4. This consists in following closely the GEANT4 application programming interface (API) conventions to construct a user physics list. As such, all DM particles are implemented in classes derived from `G4ParticleDefinition` and their production mechanisms in classes overwriting `G4VDiscreteProcess` base methods [3]. The three DM mediators emission processes included in DMG4 are (i) bremsstrahlung-like processes with an initial-state lepton $lN \to lNX$ with $l = e^-$, μ^-, $X =$ DM, (ii) resonant in-flight annihilation of positrons against atomic electrons, $e^+e^- \to X$ and (iii) production of axion-like particles (ALPs, a) in the Primakoff process, $\gamma N \to Na$. More details can be found in [4].

2.1. The DarkMatter sub-package

Historically, cross-section computations have been implemented to be GEANT4-independent, and thus included in any other user-defined program. As such, DMG4 comprises a sub-package `DarkMatter`. All models are built from the sub-package base class `DarkMatter` and the methods for double-differential and total cross-section computations are accordingly overridden (see figure 1). The latter are performed at exact tree-level (ETL) and because of the complicated expressions and need of acceptable run-time performance, implemented with a phase-space approximation [5]. For bremsstrahlung-like processes, the Weizsäcker-William (WW) approach is adopted for muon beams [6], whereas a further simplification of the method through the improved WW (IWW) is chosen for electrons [7]. If in the former case, the relative error with respect to the ETL is within a few percent, the latter case overestimates by $\mathcal{O}(10\%)$ the ETL. This is corrected at run-time by means of tabulated K-factors, $K = \sigma_{IWW}/\sigma_{ETL}$, (see figure 2) over the full energy and mass ranges. The production cross-sections for resonant in-flight DM and ALPs are discussed in [8, 9].

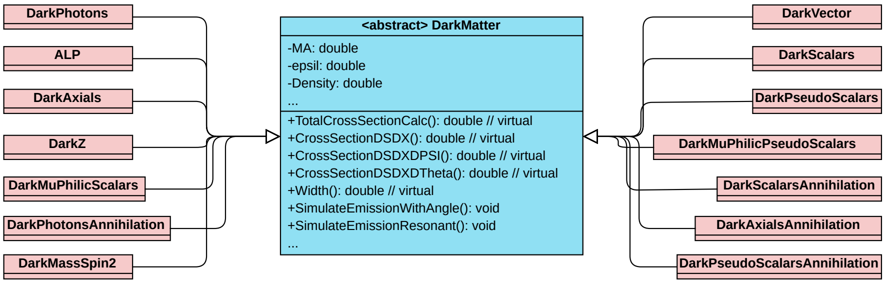

Figure 1. Unified Modeling Language (UML) class diagram for the `DarkMatter` sub-package main components.

3. Recent developments in DMG4

In an attempt to broaden its simulations horizon, DMG4 has been enriched with a wider class of models. In particular this includes an extension to non-minimal portals with more experimental signatures and the possibility to accommodate some anomalies. Models with gauged SM accidental symmetries such as the $B - L$ gauge boson [10], or asymmetric DM particles production are added (see figure 3). Additionally, effort is driven towards improving the computational performance of the underlying cross-section calculations. This embraces both speeding-up the integration of the differential cross-sections and enlarging the tabulated K-factors to a wider mass range for run-time computations.

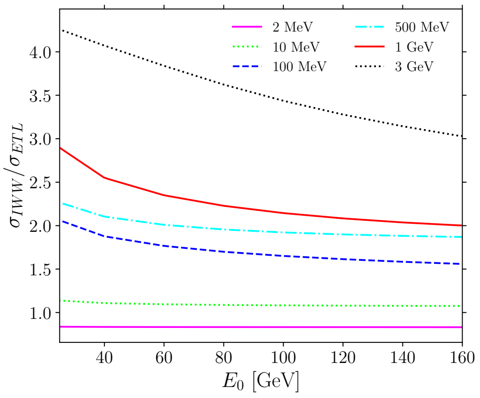

Figure 2. K-factors for IWW and ETL in electron bremsstrahlung of the A' over an extended mass and energy range. The different linestyles and colors correspond to different masses with $m_{A'} = 3000, 1000, 500, 100, 10, 2$ MeV from top to bottom.

3.1. *Inelastic DM*

As an extension to the minimal A' dark photon model [1], we consider the inelastic coupling of the A' to both stable and unstable DM particles, χ_1 and χ_2, with Lagrangian

$$\mathcal{L} \supset \sum_{i=1,2} \bar{\chi}_n(i\gamma^\mu\partial_\mu - m_{\chi_n})\chi_n - g_D(\bar{\chi}_2\gamma^\mu A'_\mu\chi_1 + h.c), \tag{1}$$

with g_D the dark coupling. Within the inelastic Dark Matter (iDM) model, the decay of the dark photon is $A' \to \chi_1(\chi_2 \to \chi_1 e^+e^-)$. Both decays are implemented in `DMG4` with widths expressions according to [11] and [12] respectively. The latter 3-body decay is implemented in `DMG4` in a derived class from `G4VDecayChannel` to accurately sample the final state particles phase-space (see figure 4).

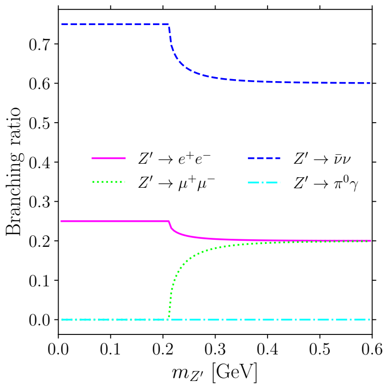

Figure 3. Branching ratios in $B - L$ Z' model, with decay modes from top to bottom $Z' \to \bar{\nu}\nu$, $Z' \to e^+e^-$, $Z' \to \mu^+\mu-$ and $Z' \to \pi^0\gamma$.

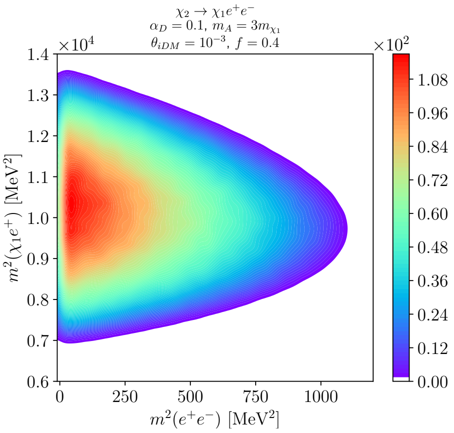

Figure 4. Dalitz plot for $\chi_2 \to \chi_1 e^+e^-$.

3.2. Muon-philic processes

Production of DM through interactions of muons with an (active) target is well motivated through the increased yield at low mass due to the cross-section behaviour, $\sigma \sim \epsilon^2/m_\mu^2$, and production $N_{DM} \propto nX_0$, with n the number of interaction length X_0 of the target. Additionally, muon-philic DM candidates such as $L_\mu - L_\tau$ models [13, 14, 15] provide a possible solution to the $(g-2)_\mu$ anomaly with an additional loop contribution. In its new version, `DMG4` includes in addition to a vector mediator (Z' or more generically V), both scalar (S) and pseudo-scalar (P) particles with Lagrangians

$$\mathcal{L} \supset \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 + \epsilon_S e S \bar{\mu}\mu, \tag{2}$$

$$\mathcal{L} \supset \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu P)^2 - \frac{1}{2}m_P^2 P^2 + i\epsilon_P e P \bar{\mu}\gamma_5\mu, \tag{3}$$

with mixing strengths ϵ_S, ϵ_P and masses m_S, m_P. The ETL computations are performed in [16]. In order to reduce the run-time computation time, further developments have been made in obtaining an analytical expression of the single-differential cross-section $d\sigma/dx$ in the WW approach [16], as shown in figure 5. The later result is obtained through analytical integration of the photon flux χ over the angle of emission θ of the DM mediator such that

$$\left(\frac{d\sigma}{dx}\right)_{WW} = \epsilon^2\alpha^3\sqrt{1-\frac{m_{Z'}^2}{E_\mu^2}}\,\frac{1-x}{x}\sum_{i=1}^{6} I_i(x,u)\Bigg|_{u=u_{min}}^{u=u_{max}}, \tag{4}$$

where the six functions I_i, $i = 1, 2, \ldots 6$ can be found in [16].

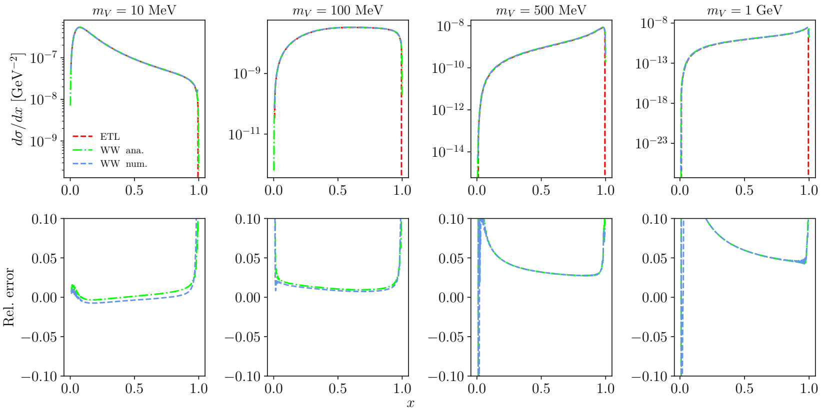

Figure 5. (*top row*) Analytical (green dash-dot), numerical (light blue dashed) and ETL (red dashed) single-differential cross-sections and (*bottom row*) their relative errors. From left to right the masses of the mediator are $m_V = 10, 100, 500, 1000$ MeV.

3.3. Spin-2 mediator

Gravity-mediated Dark Matter (GMDM) is a class of models based on the gravitational nature of DM interactions [17]. Within this model, the massive spin-2 mediator couples to DM and SM through the 5-dimensional energy-momentum tensor. The simplified Lagrangian reads [18]

$$\mathcal{L} \supset \frac{c_\gamma}{\Lambda}G^{\mu\nu}\left(\frac{1}{4}\eta_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho} + F_{\mu\lambda}F_\nu{}^\lambda\right) - \sum_l \frac{ic_l}{2\Lambda}G^{\mu\nu}\left(\bar{l}\gamma_\mu D_\nu l - \eta_{\mu\nu}\bar{l}\gamma_\rho D^\rho l\right) + \frac{c_\chi}{\Lambda}G^{\mu\nu}T^\chi_{\mu\nu}, \tag{5}$$

with $T^{\chi}_{\mu\nu}$ the energy-momentum tensor as described in [19], $G^{\mu\nu}$ the massive spin-2 field, Λ a dimensional parameter for massive spin-2 interactions, c_i dimensionless couplings. The latest version of `DMG4` includes both production in electron bremsstrahlung-like process, $e^-N \to e^-NG$, and in resonnant annihilation $e^+e^- \to G(\to \bar{\chi}\chi)$. Whereas for the later the production cross-section is computed at ETL, the emission process implementation relies on the WW approximation with an agreement of $\mathcal{O}(1\ \%)$ with respect to the ETL [18]. In this new release, the dominant branching ratio is $G \to \textit{invisible} \simeq 1$ with decay width

$$\Gamma(G \to \bar{\chi}\chi) = \frac{c_\chi^2 m_G}{160\pi}\left(\frac{m_G}{\Lambda}\right)^2\left(1-\frac{4m_\chi^2}{m_G^2}\right)^{\frac{3}{2}}\left(1+\frac{8m_\chi^2}{3m_G^2}\right). \qquad (6)$$

4. Sensitivity at beam-dump experiments

As a final figure of merit, `DMG4` aims at providing a realistic estimation of a given experiment's sensitivity to New Physics models through a complete `GEANT4` simulation. Because of the extremely small DM production rate, a bias in the total cross-section can be introduced through a multiplicative factor, `BiasSigmaFactor`. As a consequence, a reasonable fraction of DM is produced and propagated through the experimental set-up. This enables appropriate optimization of the event selection flow, and thus signal-to-noise. The final sensitivity is corrected given a bias-dependent normalisation factor. Typically in an electron-beam-based experiment, the sensitivity for a given number of electrons on target (EOT) is computed according to [7]

$$N_{DM} = N_{EOT}\frac{\rho\mathcal{N}_A}{A}\sum_i \sigma_{DM}(E_e^i)\Delta L_i, \qquad (7)$$

where the target properties are taken into account through its density, ρ, atomic weight A, and target differential length, ΔL_i.

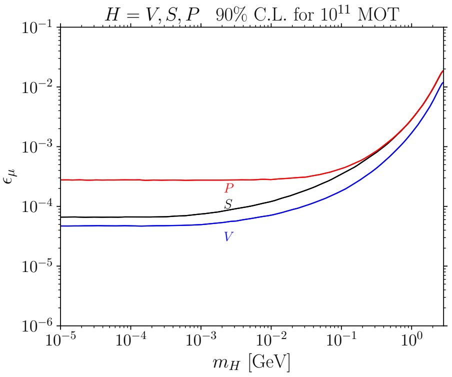

Figure 6. NA64-like experimental sensitivities for muon-based beam. From bottom to top, the sensitivity lines are given for the vector, scalar and pseudo-scalar case respectively. Electron- and positron-based beam dump experiment sensitivities can be found in [4].

In figure 6, the sensitivities are shown for different muon-philic DM mediator flavors through a realistic set of simulations performed with `GEANT4` for a beam dump experiment such as NA64μ [20, 21]. The efficiency is assumed to be 100% with no background.

5. Conclusion

`DMG4` is a package that combines both MC event generation for DM particles and propagation into matter through a `GEANT4` API. It includes a broad range of DM mediators produced in lepton- and photon-based processes. Given the available models, `DMG4` aims at providing the necessary tools for a fully realistic experimental sensitivity study.

In the next `DMG4` release, the further extension of the benchmark A' and Z' models coupling indirectly and directly to electron and muon respectively is considered. This mainly consists in adding the massless mediator scenario where milli-charged particles are produced. The inclusion of New Physics models such as lepton flavour conversion in $e \to \tau$ or $\mu \to \tau$ is also being studied. Additionally, significant effort is driven towards the implementation of the Weiszäcker-Williams approach for electrons. New physics in hadron-induced interactions is under investigation.

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