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Prospects for Probing the Quartic Higgs Self-coupling at the Future Colliders

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Based on:

Benjamin Fuks, **J. H. Kim**, and Seung J. Lee [Phys. Lett. B 771 (2017)]

Benjamin Fuks, **J. H. Kim**, and Seung J. Lee [Phys. Rev. D 3 (2016)]

Outline

- Motivation & Overview
- Effective Field Theory (EFT) Approach
- Parameterizations of Higgs couplings
- Probing the quartic Higgs self-coupling at
    1. FCC-hh 100 TeV Collider
    2. Future e^+e^- Colliders
    3. Future Multi-TeV Muon Colliders
- Summary

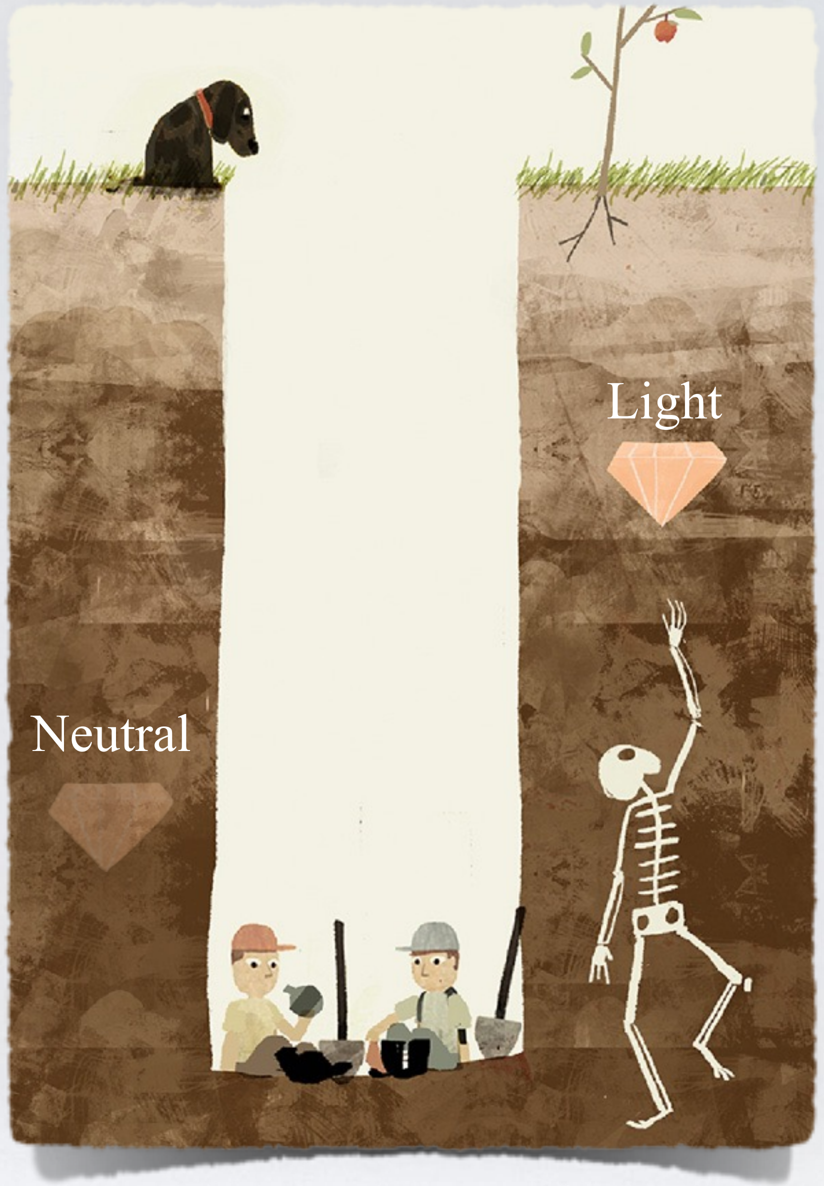
What We Have Found So Far

- After a long period of direct searches in the LHC, no tantalizing evidence of new physics has been found.
- What are we going to do about it?
- How are we going to make a progress to understand the next layer of physics?

What Are We Going to Do?

- Hidden in exotic places

- The null result opens up various possibilities…



- Very light

- Completely neutral under the SM gauge group

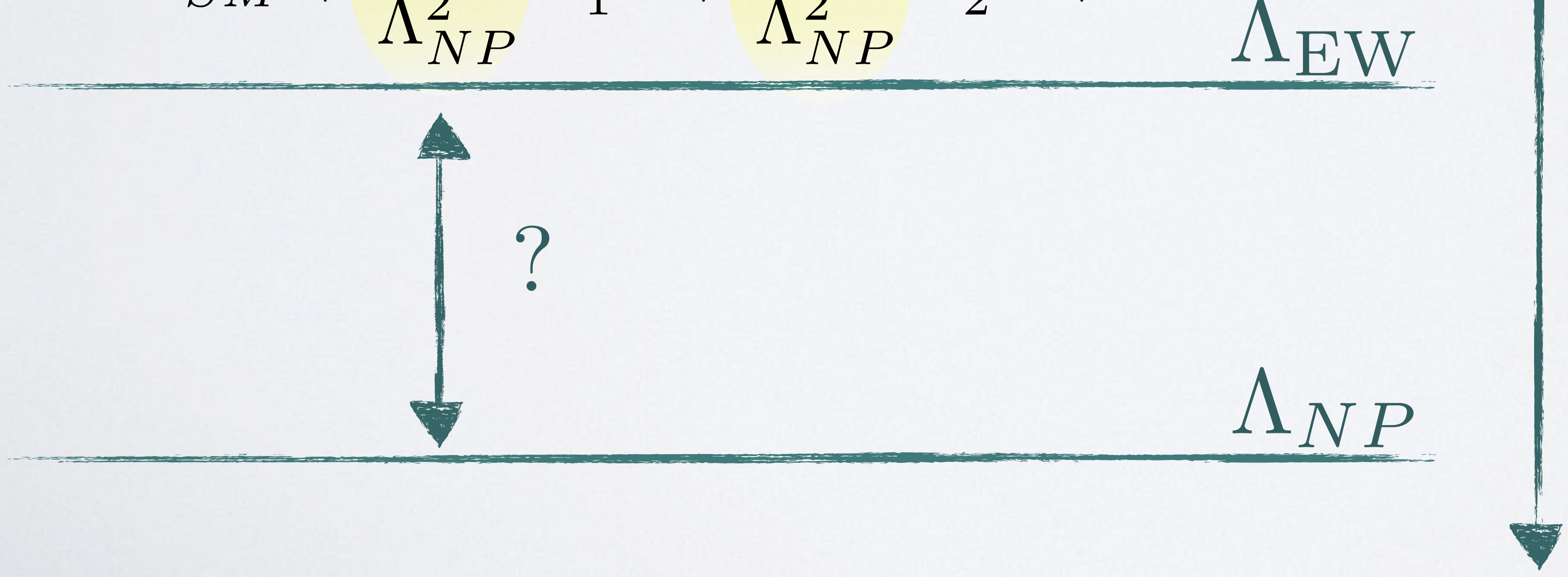
- Too heavy to directly probe

Effective Field Theory (EFT) Approach

- If the scale of new physics is too heavy to reach.

- The EFT provides a framework to encode new physics effects in higher dimensional operators.

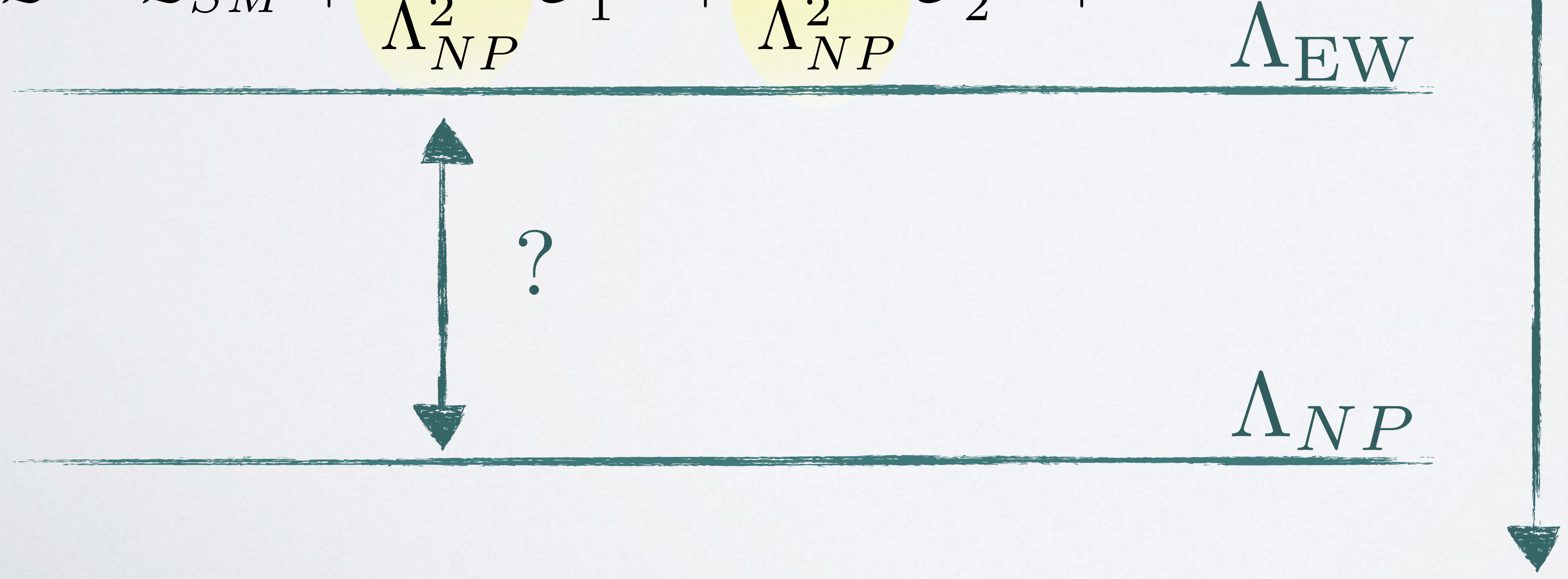
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_1^{(6)}}{\Lambda_{NP}^2}\mathcal{O}_1^{(6)} + \frac{c_2^{(6)}}{\Lambda_{NP}^2}\mathcal{O}_2^{(6)} + \cdots$$



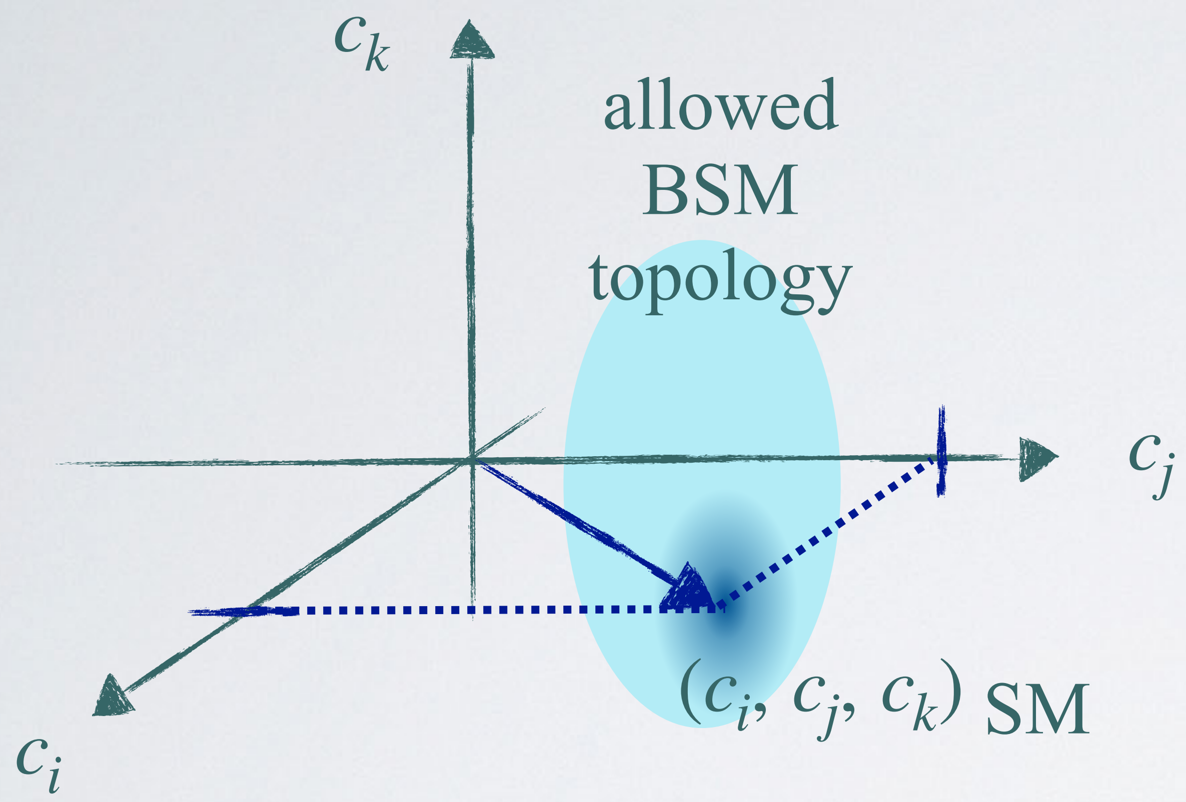
Effective Field Theory (EFT) Approach

- Precision measurements of these coefficients will tell us two things:
- What would be a rough scale of new physics.
- What would be a possible structure of the underlying new physics.

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_1^{(6)}}{\Lambda_{NP}^2}\mathcal{O}_1^{(6)} + \frac{c_2^{(6)}}{\Lambda_{NP}^2}\mathcal{O}_2^{(6)} + \cdots$$



Precision Higgs Physics



- The Higgs boson can be used as a BSM indicator.
- To see deviations in the Higgs couplings that give a hint of the new physics.

- Typical modifications of the Higgs couplings induced by new physics:

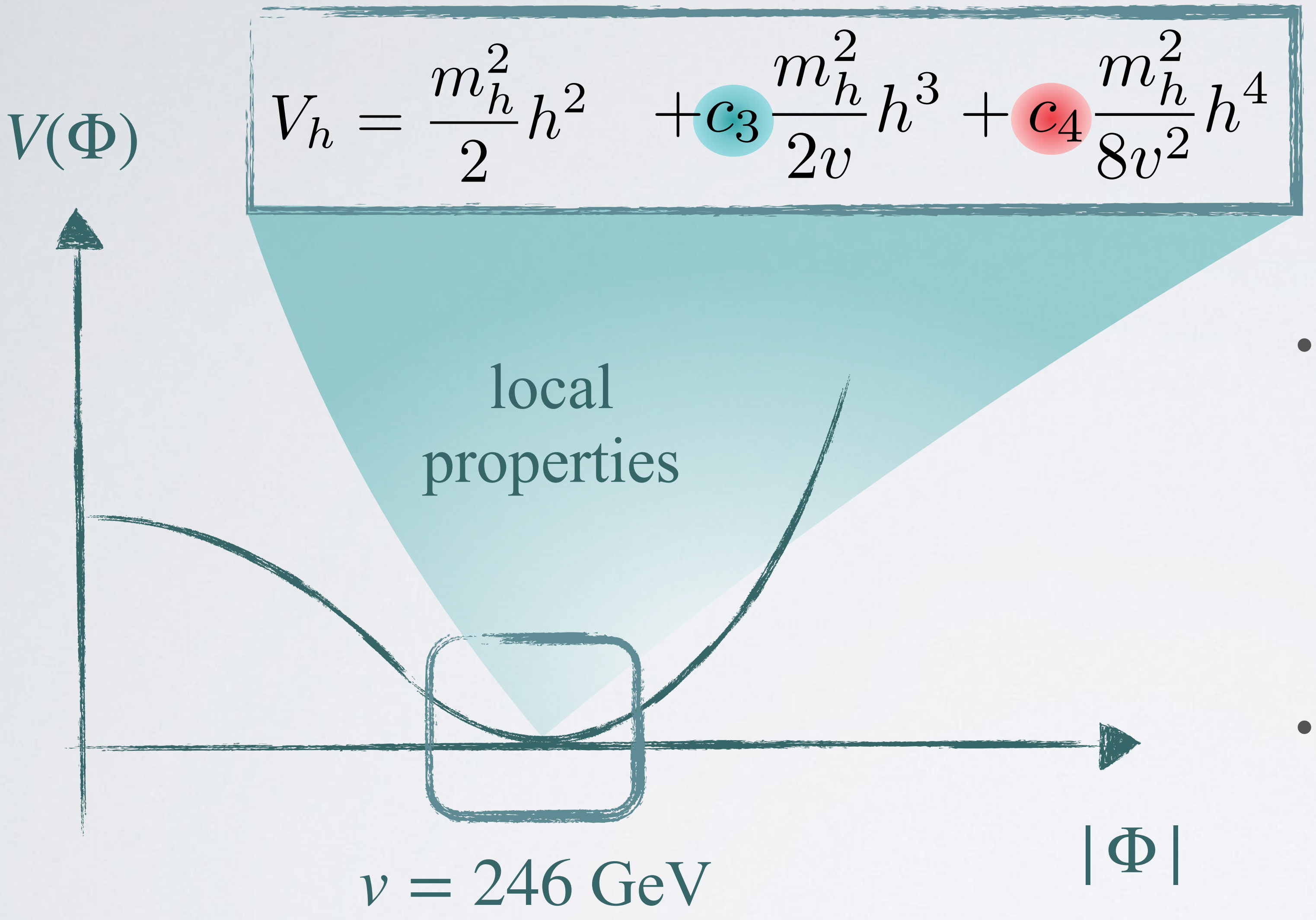
$2 \to 2$ process :
$$\frac{\delta c}{c} \sim \frac{g_*}{g_{SM}} \frac{E^2}{m_*^2}$$

their coupling strength to the Higgs

mass of new states

- Typically growing with E (large corrections to Higgs couplings).

The Shape of the Higgs Potential

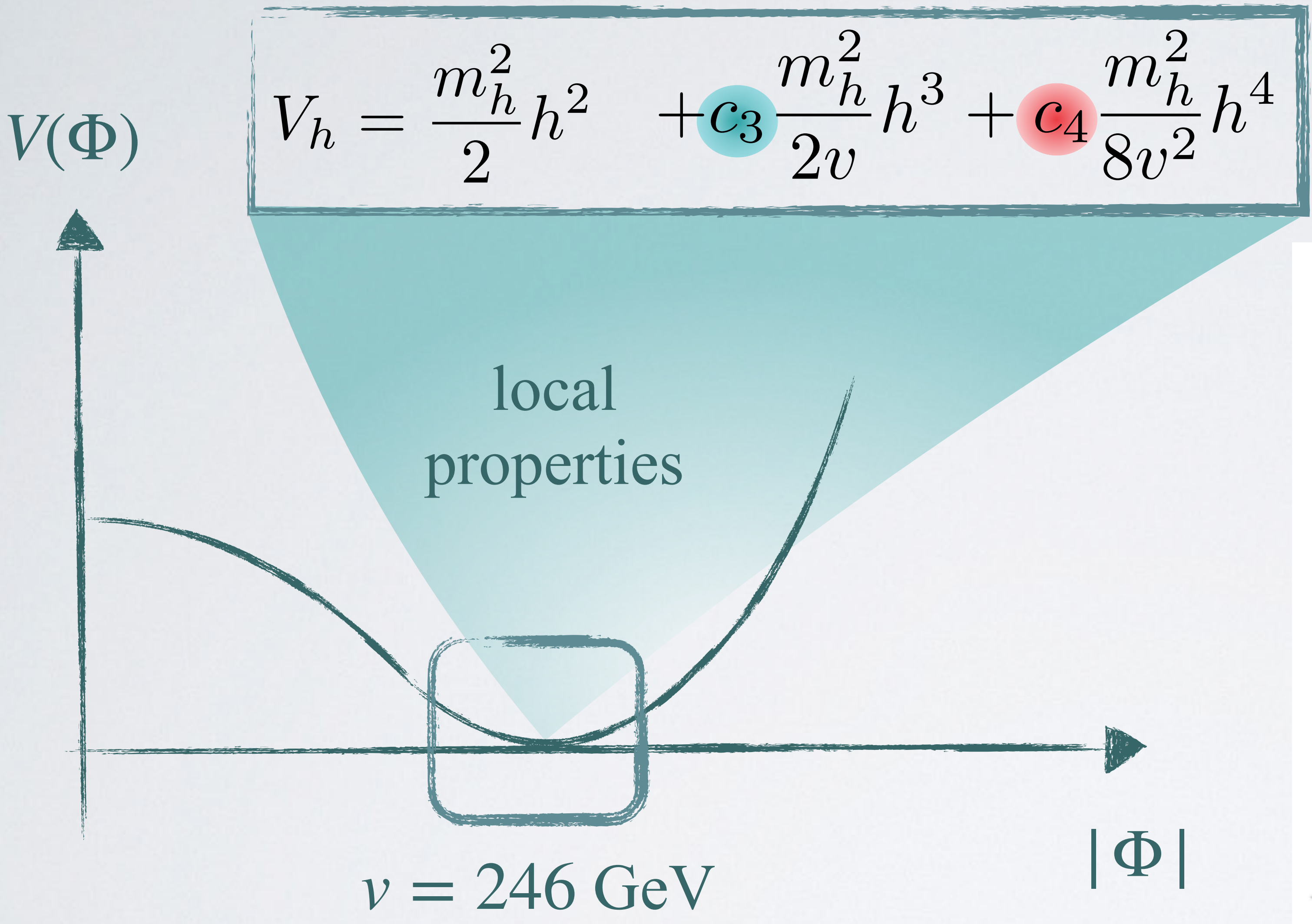


- A precise knowledge on the Higgs self-couplings is key to reconstruct the Higgs potential.
- Only the first term has been probed.

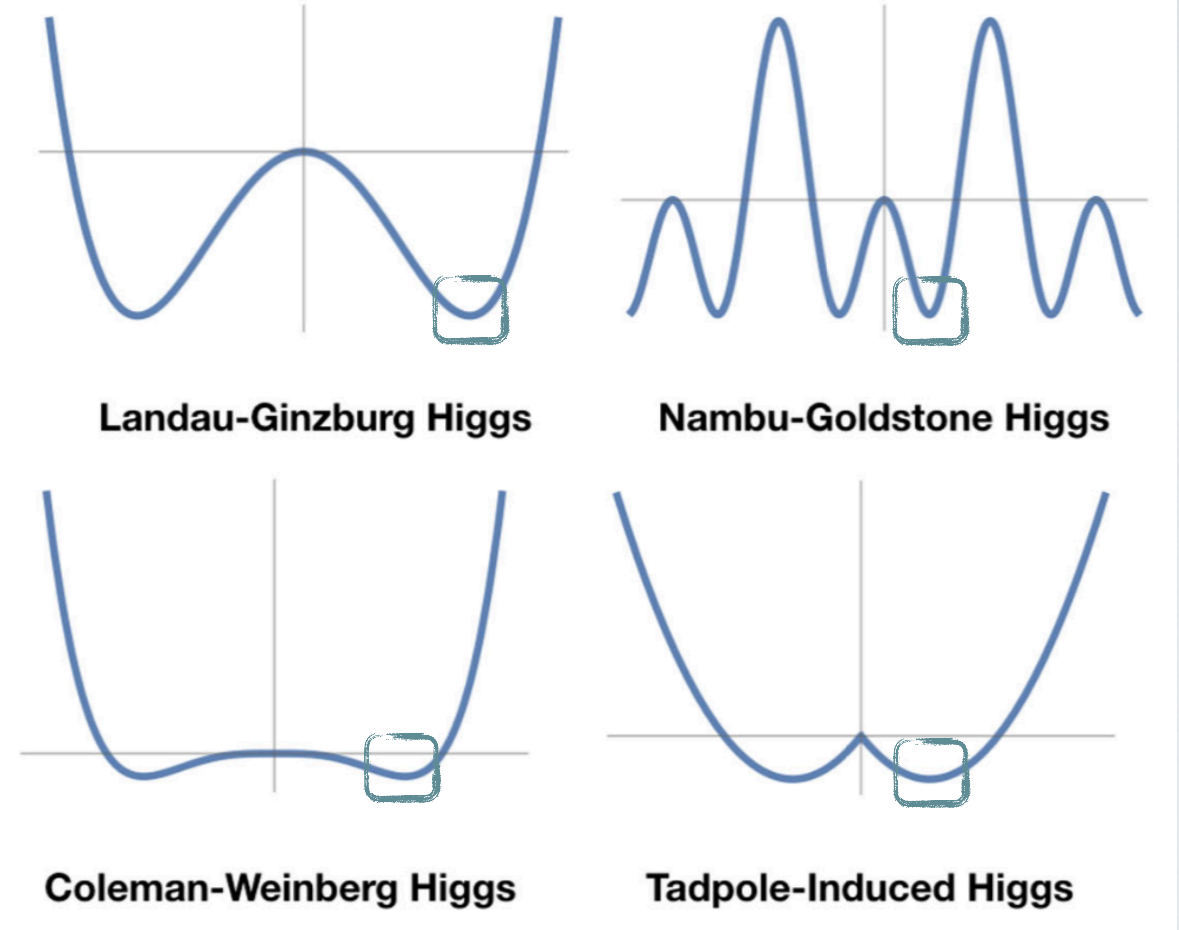
- Probing the c_3 and c_4 couplings provides a crucial information.
- We might be able to confirm the SM EWSB, or discover a new mechanism.

The Shape of the Higgs Potential

$$V_h = \frac{m_h^2}{2}h^2 + c_3 \frac{m_h^2}{2v}h^3 + c_4 \frac{m_h^2}{8v^2}h^4$$



Taken from Agrawal, Saha, Xu, Yu, Yuan [2020]



- Probing the c_3 and c_4 couplings provides a crucial information.
- We might be able to confirm the SM EWSB, or discover a new mechanism.

Parameterizations of Higgs Self-Couplings (Linear Basis)

Use c_3 and c_4 to denote Higgs self-couplings

- All the above scenarios can be described in the EFT famework.
- Gauge-invariant dimension-6 operators relevant to multi-Higgs productions.

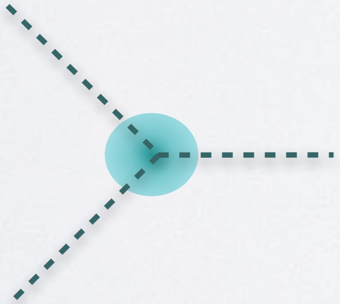
$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \boxed{\Delta\mathcal{L}_6} + \Delta\mathcal{L}_8 + \cdots$$

$$\frac{\bar{c}_H}{2v^2}(\partial^\mu|H|^2)^2 - \frac{\bar{c}_6}{v^2}\left(\frac{m_h^2}{2v^2}\right)|H|^6 + \frac{\bar{c}_u}{v^2}\mathrm{y}_t(|H|^2\overline{Q}_L H^c t_R + \text{h.c.}) + \cdots$$

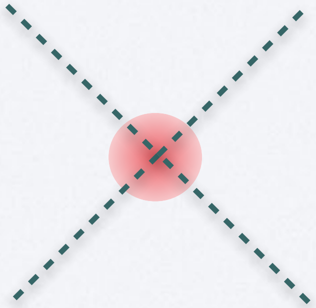
After EWSB…

Giudice, Grojean, Pomarol, Rattazzi [2007], …

$$\mathcal{L} = -m_t t\bar{t}\left(c_t\frac{h}{v} + c_{2t}\frac{h^2}{v^2}\right) - c_3\left(\frac{m_h^2}{2v}\right)h^3 - c_4\left(\frac{m_h^2}{8v}\right)h^4 + \ldots$$



$$c_3 \simeq 1 - \frac{3}{2}\bar{c}_H + \bar{c}_6$$



$$c_4 \simeq 1 - \frac{25}{3}\bar{c}_H + 6\bar{c}_6$$

- Assume $\bar{c}_H = 0$.
- c_3 and c_4 are related to each other.

$$c_4 = 6c_3 - 5$$

Parameterizations of Higgs Self-Couplings (Non-Linear Basis)

Use κ_3 and κ_4 to denote Higgs self-couplings

- In the non-linear basis, however, all couplings are assumed to be free parameters.

- The Higgs is not a part of *SU(2)*$_L$ doublet, but it can be more generic.

$$\mathcal{L} = -m_t t\bar{t}\left(\kappa_t \frac{h}{v} + \kappa_{2t}\frac{h^2}{v^2}\right) - \kappa_3\left(\frac{m_h^2}{2v}\right)h^3 - \kappa_4\left(\frac{m_h^2}{8v}\right)h^4 + \ldots$$

(κ framework)

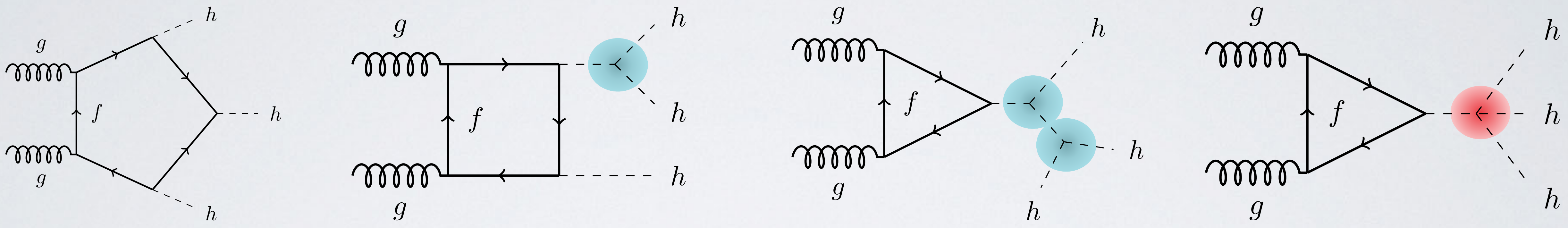
- Simply rescaling the SM interactions with simplied EFT to encapsulate NP effects.

- We will assume the SM top Yukawa, and vary κ_3 and κ_4 independently…

$(\kappa_t = 1, \kappa_{2t} = 0\ )$

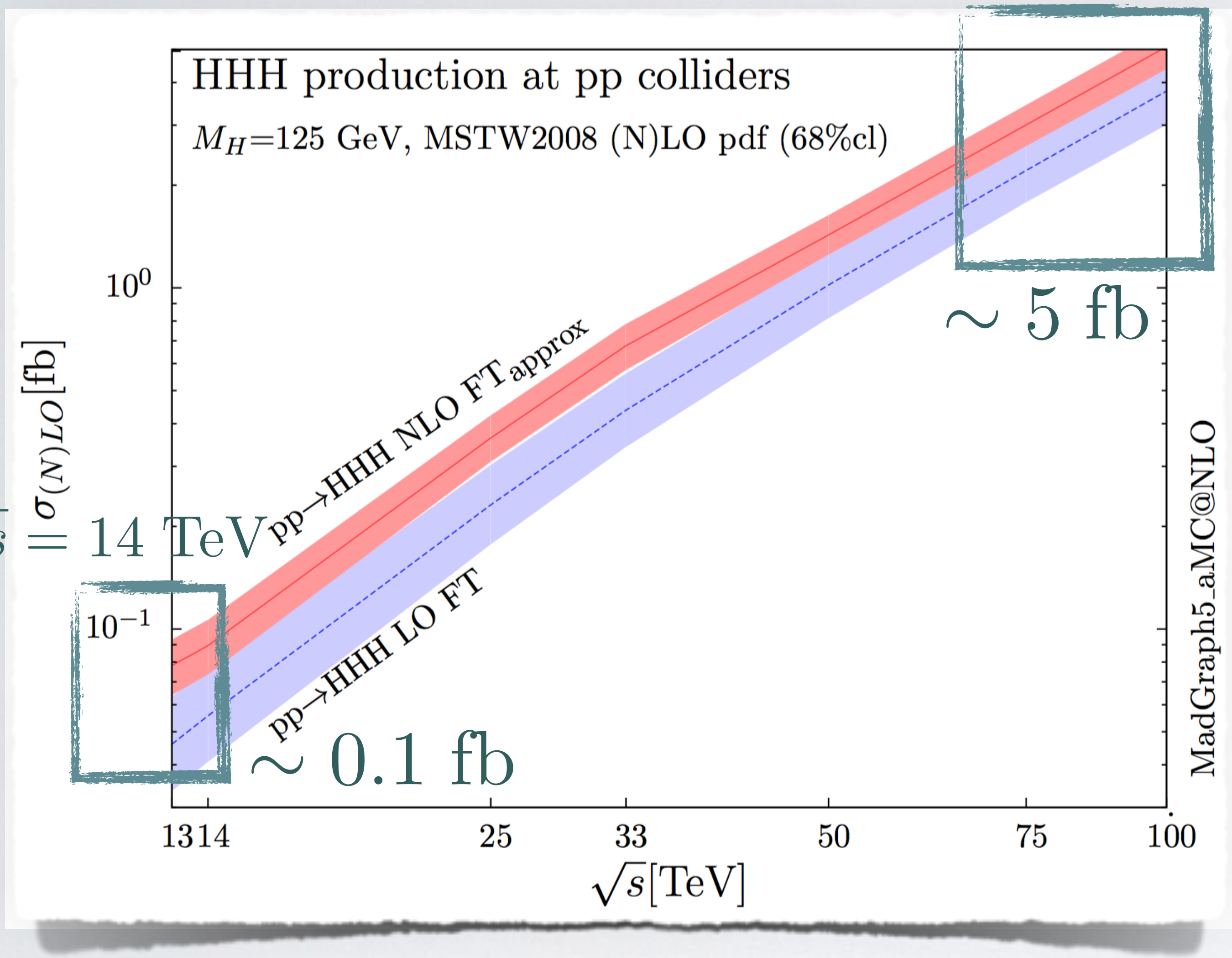
Tiple Higgs production

- The first avenue to scrutinize these Higgs couplings is *hh* production.



- The *hhh* production was the first avenue to probe κ_4.
- It has a mild dependence on κ_4, and strong dependence on and κ_3.
- It receives a phase space suppression (*cf*. double Higgs production).
- So the signal rate is expected to be small.

Tiple Higgs production



$\sqrt{s} = 100$ TeV

~ 5 fb

$\sqrt{s} = 14$ TeV

~ 0.1 fb

F. Maltoni, E. Vryonidou, M. Zaro [2014]

Impossible to probe at LHC

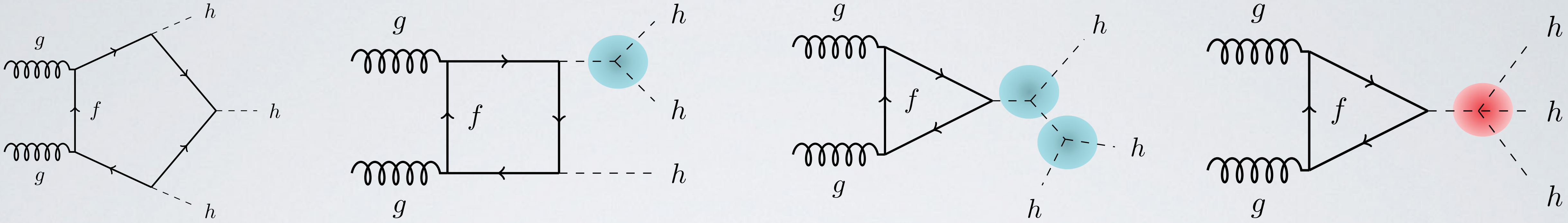
- Signal rate is 0.1 fb …

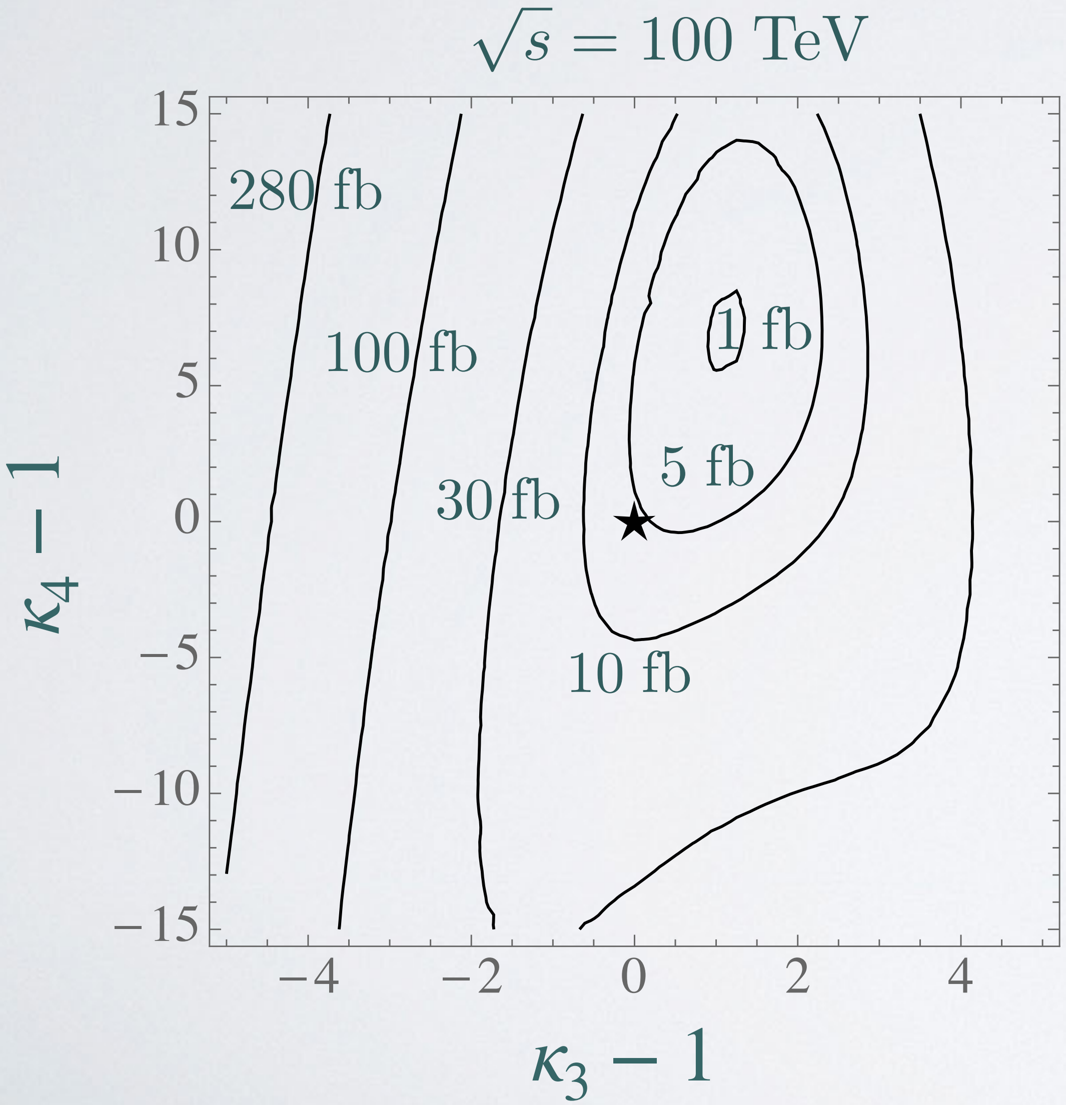
FCC-hh at 100 TeV

- In this stage, at 100 TeV FCC, signal rate can reach to 5 fb…

SM prediction

Cross Cections in the $\kappa_3 - \kappa_4$ Plane





B. Fuks, **J. H. Kim**, S. J. Lee [2015]

- Strong dependence on κ_3
- Mild dependence on κ_4
- Destructive interference in the region of

$$1 < \kappa_3 < 3$$

$$1 < \kappa_4 < 15$$

- Even at the 100 TeV, the cross section is small.

Rich Final States

Number of events (30ab^{-1})

- $(b\bar{b})(b\bar{b})(\gamma\gamma)$ channel

Papaefstathiou, Sakurai [2015]

B. Fuks, **J. H. Kim**, S. J. Lee [2015]

$\cdots$ Clean signature and low backgrounds

- $(b\bar{b})(WW^*)_{1\ell}(WW^*)_{1\ell}$ channel

Kilian, Sun, Yan, Zhao, Zhao [2017]

$\cdots$ The same-sign dilepton $\ell^+\ell^+$or $\ell^-\ell^-$.

- $(b\bar{b})(b\bar{b})(\tau\bar{\tau})$ channel

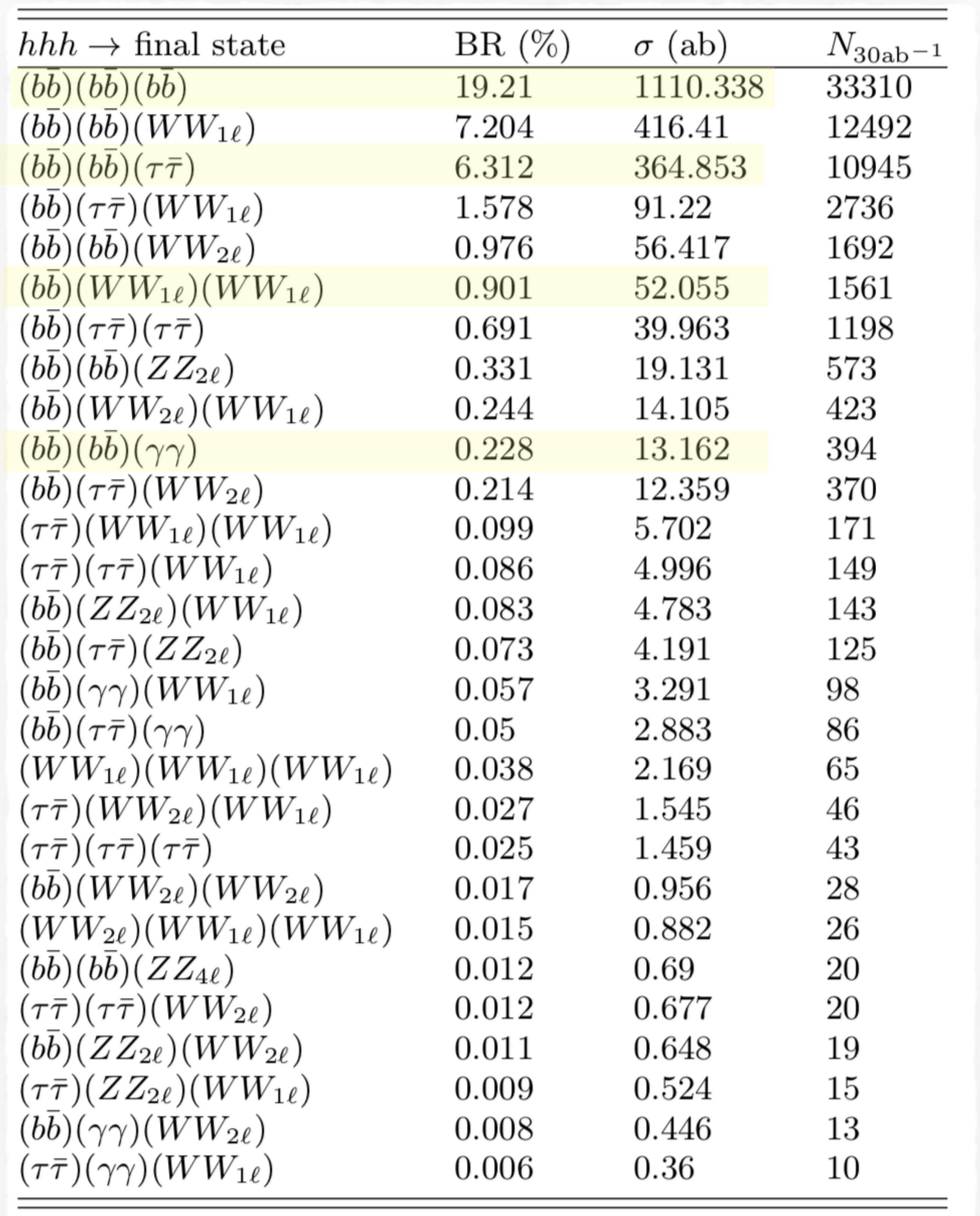
B. Fuks, **J. H. Kim**, S. J. Lee [2018]

$\cdots$ With an advantage of $4b + 2\tau$ tagging

- $(b\bar{b})(b\bar{b})(b\bar{b})$ channel

Papaefstathiou, Tetlalmatzi, Zaro [2019]

$\cdots$ Large branching fraction (19%)

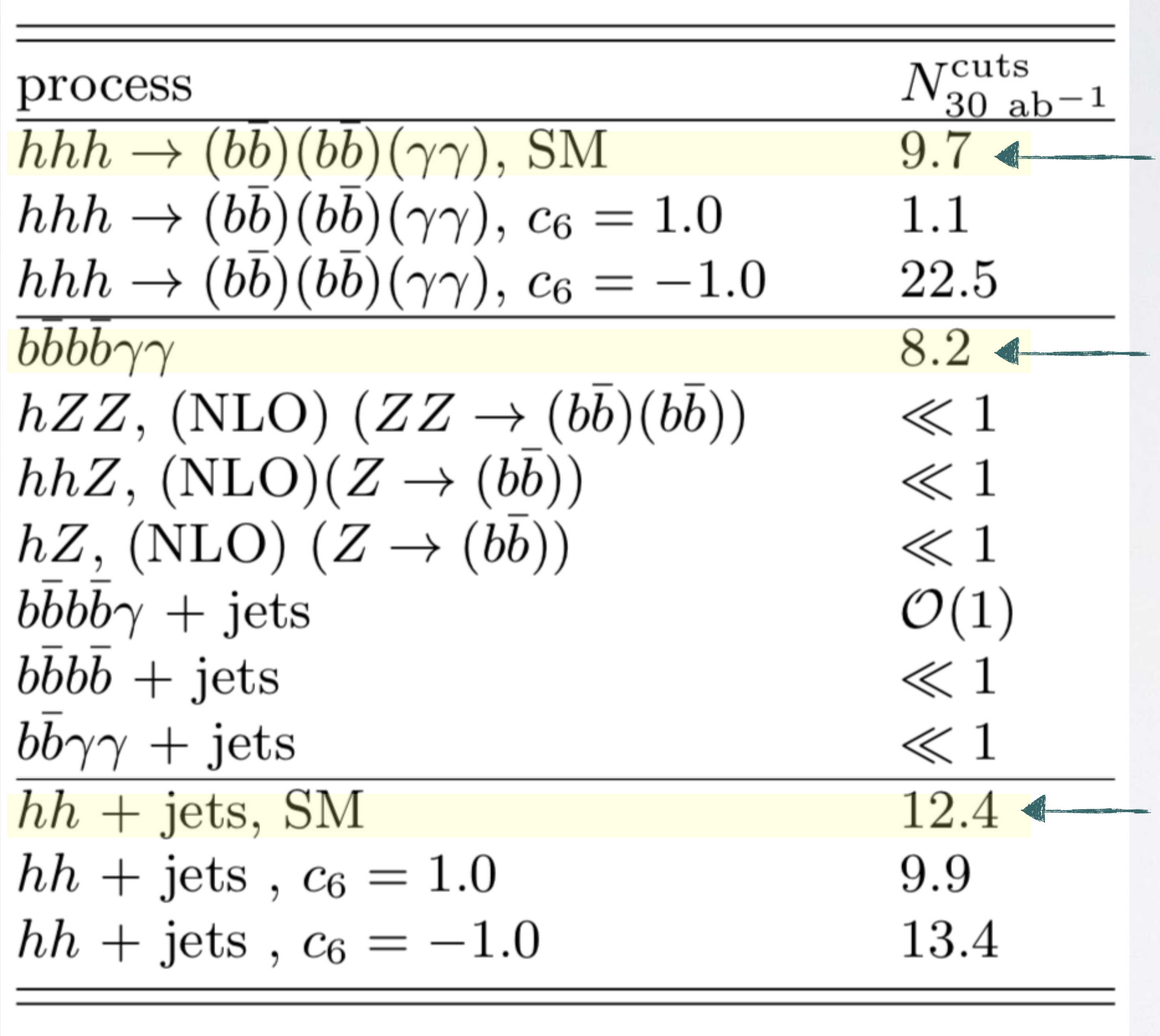


$hhh \to$ final state	BR (%)	σ (ab)	$N_{30\text{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
$(b\bar{b})(WW_{2\ell})(WW_{1\ell})$	0.244	14.105	423
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.228	13.162	394
$(b\bar{b})(\tau\bar{\tau})(WW_{2\ell})$	0.214	12.359	370
$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
$(b\bar{b})(\tau\bar{\tau})(ZZ_{2\ell})$	0.073	4.191	125
$(b\bar{b})(\gamma\gamma)(WW_{1\ell})$	0.057	3.291	98
$(b\bar{b})(\tau\bar{\tau})(\gamma\gamma)$	0.05	2.883	86
$(WW_{1\ell})(WW_{1\ell})(WW_{1\ell})$	0.038	2.169	65
$(\tau\bar{\tau})(WW_{2\ell})(WW_{1\ell})$	0.027	1.545	46
$(\tau\bar{\tau})(\tau\bar{\tau})(\tau\bar{\tau})$	0.025	1.459	43
$(b\bar{b})(WW_{2\ell})(WW_{2\ell})$	0.017	0.956	28
$(WW_{2\ell})(WW_{1\ell})(WW_{1\ell})$	0.015	0.882	26
$(b\bar{b})(b\bar{b})(ZZ_{4\ell})$	0.012	0.69	20
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{2\ell})$	0.012	0.677	20
$(b\bar{b})(ZZ_{2\ell})(WW_{2\ell})$	0.011	0.648	19
$(\tau\bar{\tau})(ZZ_{2\ell})(WW_{1\ell})$	0.009	0.524	15
$(b\bar{b})(\gamma\gamma)(WW_{2\ell})$	0.008	0.446	13
$(\tau\bar{\tau})(\gamma\gamma)(WW_{1\ell})$	0.006	0.36	10

Papaefstathiou, Sakurai [2015]

Constraints from $hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$

Papaefstathiou, Sakurai [2015]



process	$N^{\text{cuts}}_{30\ \text{ab}^{-1}}$
$hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$, SM	9.7
$hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$, $c_6 = 1.0$	1.1
$hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$, $c_6 = -1.0$	22.5
$b\bar{b}b\bar{b}\gamma\gamma$	8.2
hZZ, (NLO) $(ZZ \to (b\bar{b})(b\bar{b}))$	$\ll 1$
hhZ, (NLO)$(Z \to (b\bar{b}))$	$\ll 1$
hZ, (NLO) $(Z \to (b\bar{b}))$	$\ll 1$
$b\bar{b}b\bar{b}\gamma$ + jets	$\mathcal{O}(1)$
$b\bar{b}b\bar{b}$ + jets	$\ll 1$
$b\bar{b}\gamma\gamma$ + jets	$\ll 1$
hh + jets, SM	12.4
hh + jets , $c_6 = 1.0$	9.9
hh + jets , $c_6 = -1.0$	13.4

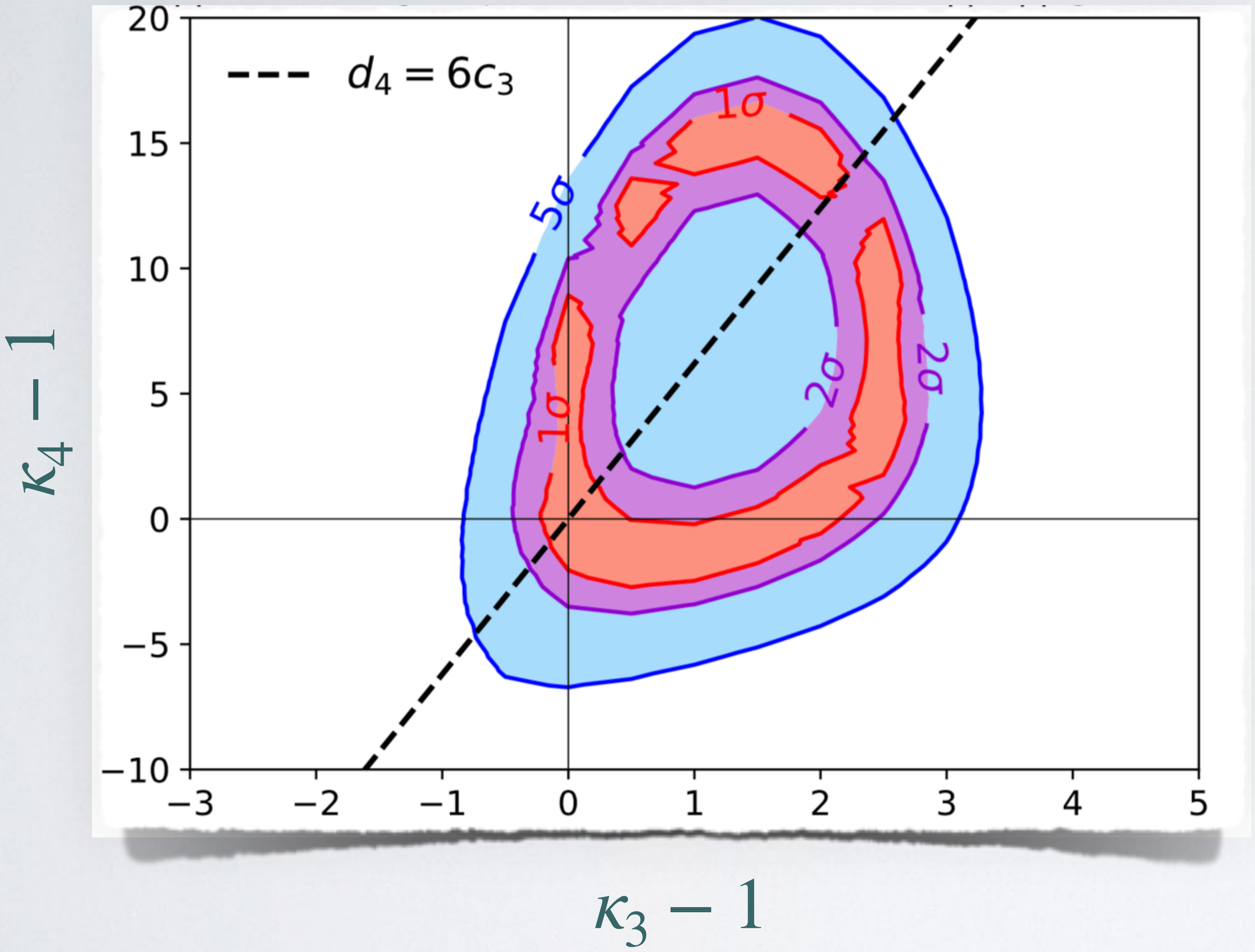
Parton-level analysis

- In the end of days, only 10 events are expected.
- A major background (including light jets faking b-jets and γ's)
- Double Higgs production is a part of backgrounds.

Constraints from $hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$

Papaefstathiou, Sakurai [2015]

$30\ \mathrm{ab}^{-1}$



- Assuming $\kappa_3 = 1$, constraints are

$$-3 < \kappa_4 < 11 \qquad [2\sigma]$$

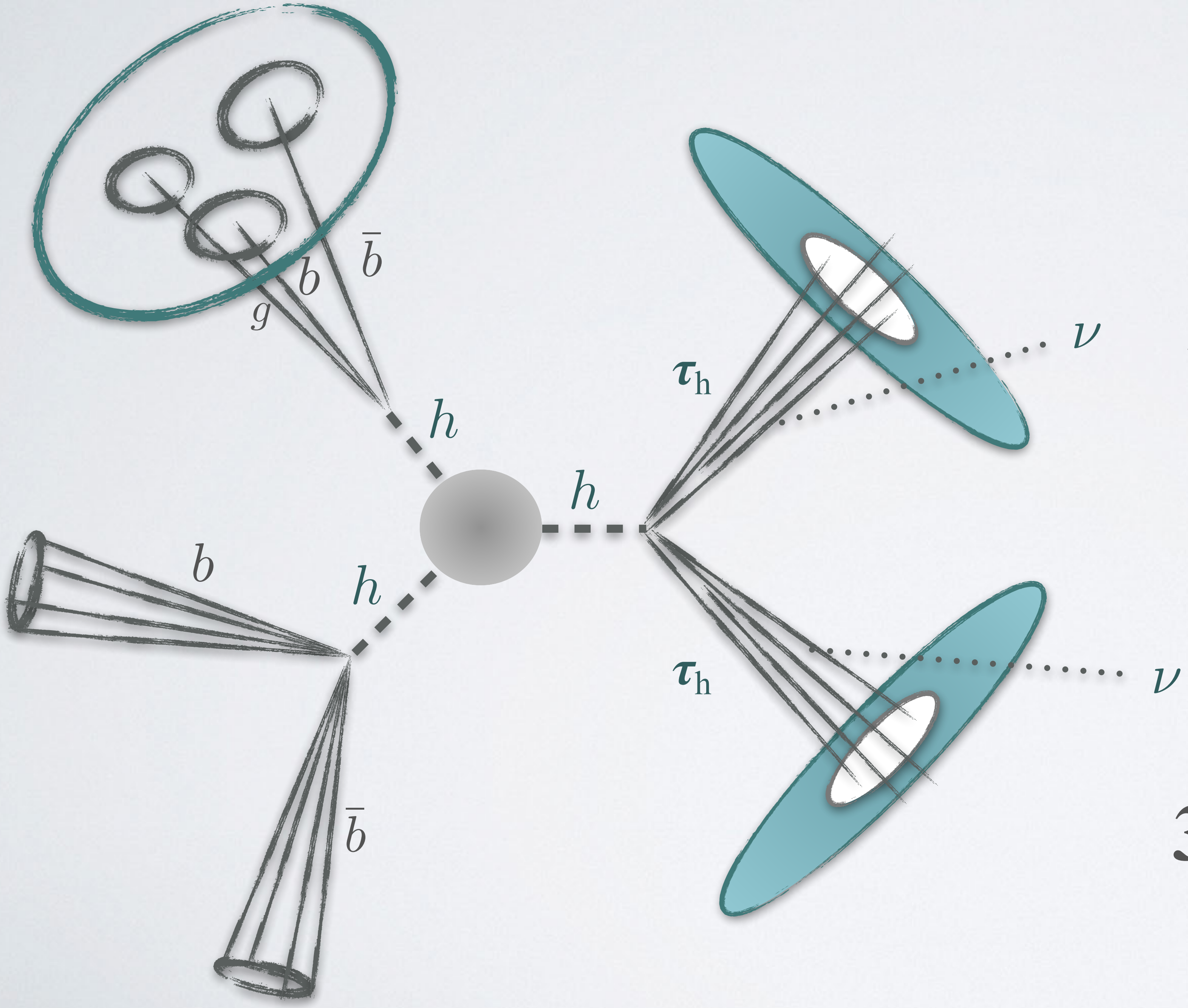
$$-1 < \kappa_4 < 10 \qquad [1\sigma]$$

- The κ_4 will be poorly constrained.
- The bound largely depends on κ_3.

Constraints from $hhh \to (b\bar{b})(b\bar{b})(\tau\bar{\tau})$

B. Fuks, **J. H. Kim**, S. J. Lee [2018]

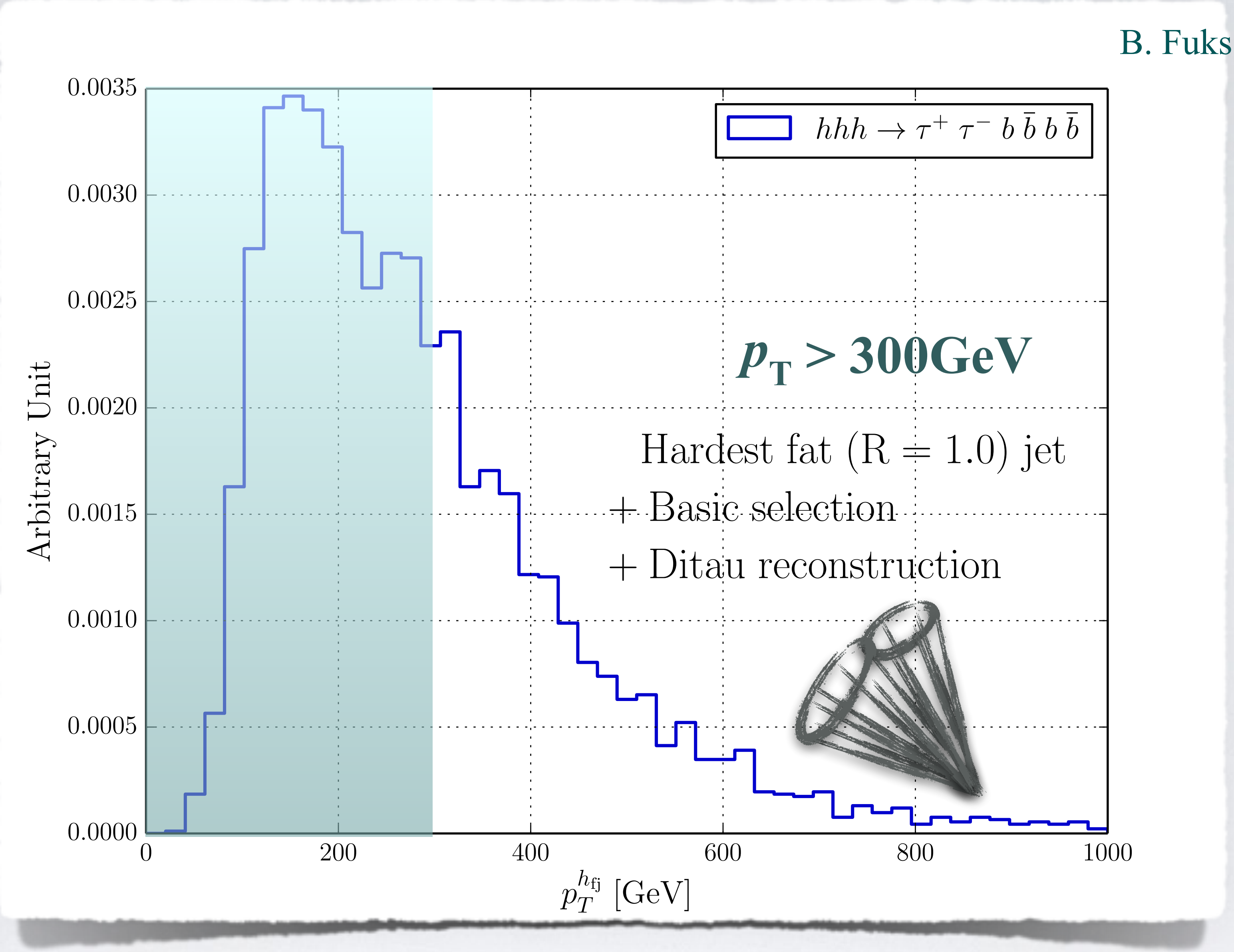
1. Boosted phase space



2. $h_{\tau\tau}$ reconstruction

3. M_{T2} reinforcement

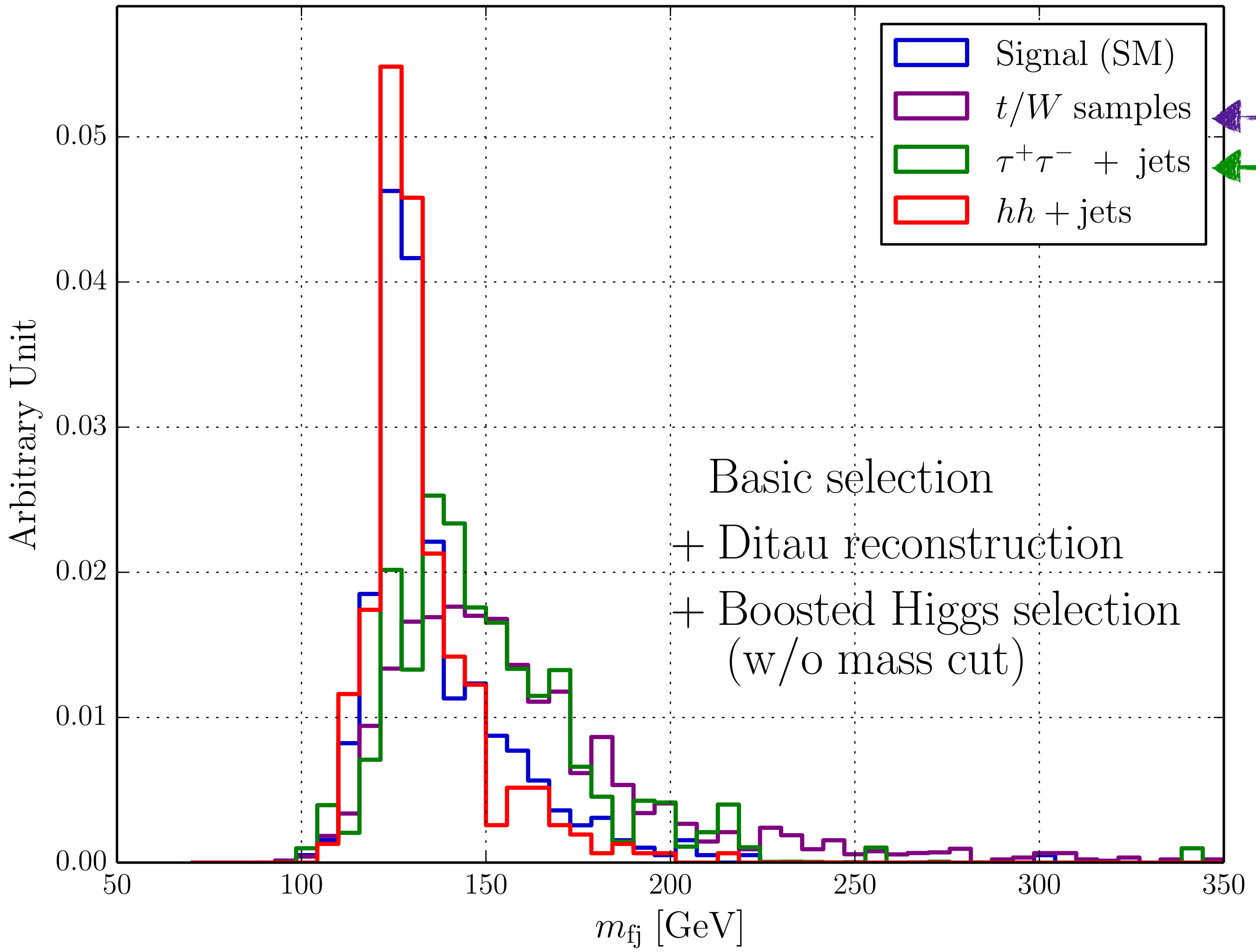
Boosted phase space



- There is a reasonable boosted phase space, at the cost of $\sim 50\,\%$ signal loss.
- Two *b* jets from a Higgs are collimated.
- The jet-substructure technique may be able to reduce backgrouds.

Boosted Higgs Reconstruction

B. Fuks, **J. H. Kim**, S. J. Lee [2018]



$t_\tau \bar{t}_\tau h_{b\bar{b}}$

$t_\tau \bar{t}_\tau Z_{b\bar{b}}$

$t_\tau \bar{t}_\tau b\bar{b}$

$W^+_\tau W^-_\tau b\bar{b}b\bar{b}$

$t\bar{t}t\bar{t}$

t/*W* backgrounds

$X_{\tau\tau} b\bar{b}b\bar{b}$

$X_{\tau\tau} b\bar{b}jj$

$X_{\tau\tau} t_h \bar{t}_h$

$X_{\tau\tau} Z_{b\bar{b}} b\bar{b}$

$Z_{\tau\tau} h_{b\bar{b}} b\bar{b}$

$X_{\tau\tau} Z_{b\bar{b}} Z_{b\bar{b}}$

$Z_{\tau\tau} h_{b\bar{b}} Z_{b\bar{b}}$

$h_{\tau\tau} h_{b\bar{b}} Z_{b\bar{b}}$

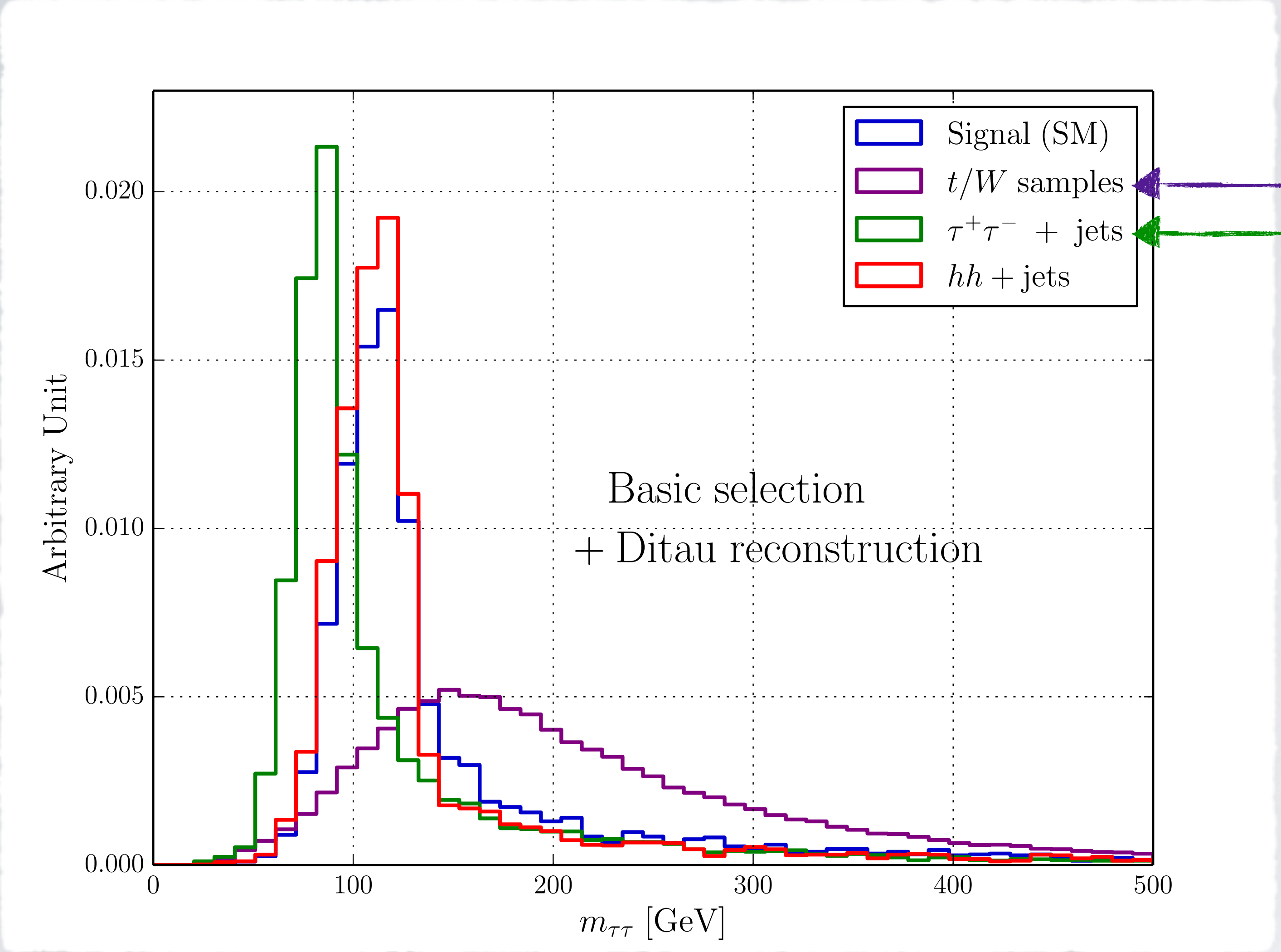
$h_{b\bar{b}} h_{b\bar{b}} Z_{\tau\tau}$

Drell-Yan

- Reconstructed boosted Higgs mass (before b-tagging).
- The use of boosted phase turns out to be useful to suppress the backgrounds.

$h_{\tau\tau}$ Reconstruction

B. Fuks, **J. H. Kim**, S. J. Lee [2018]



$X_{\tau\tau}b\bar{b}b\bar{b}$
$X_{\tau\tau}b\bar{b}jj$

$X_{\tau\tau}t_h\bar{t}_h$

$X_{\tau\tau}Z_{b\bar{b}}b\bar{b}$
$Z_{\tau\tau}h_{b\bar{b}}b\bar{b}$

$X_{\tau\tau}Z_{b\bar{b}}Z_{b\bar{b}}$
$Z_{\tau\tau}h_{b\bar{b}}Z_{b\bar{b}}$

$h_{\tau\tau}h_{b\bar{b}}Z_{b\bar{b}}$
$h_{b\bar{b}}h_{b\bar{b}}Z_{\tau\tau}$

Drell-Yan

$t_\tau\bar{t}_\tau h_{b\bar{b}}$
$t_\tau\bar{t}_\tau Z_{b\bar{b}}$
$t_\tau\bar{t}_\tau b\bar{b}$

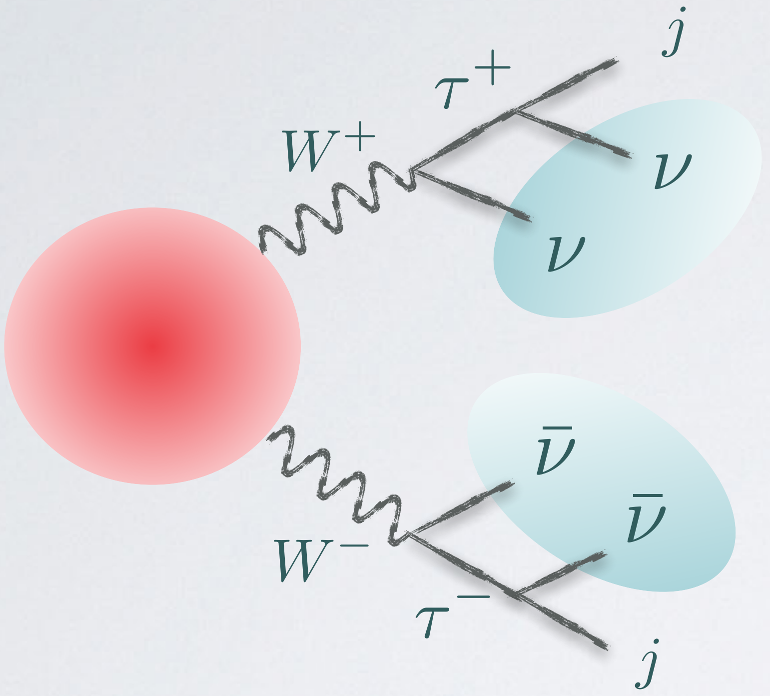
$W^+_\tau W^-_\tau b\bar{b}b\bar{b}$

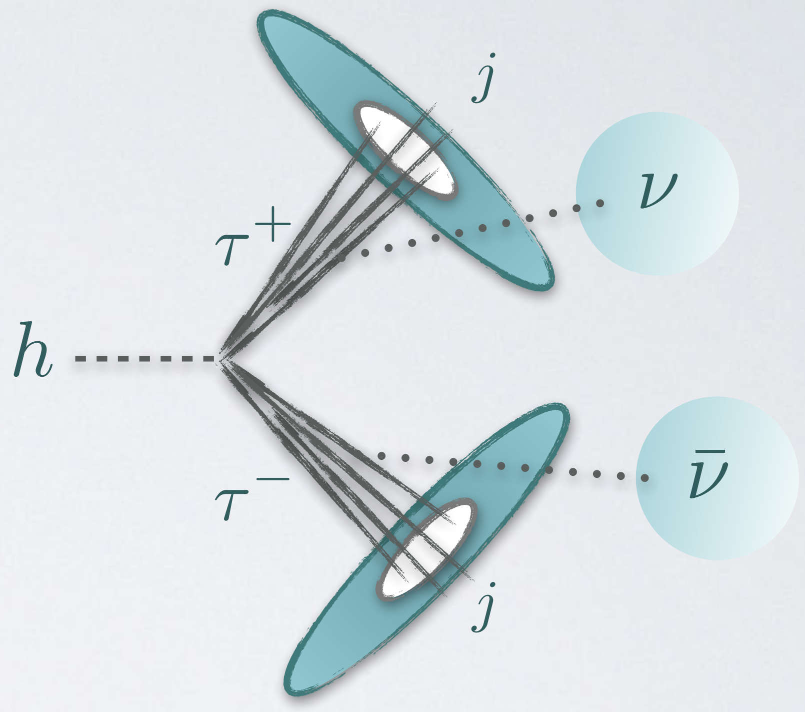
$t\bar{t}t\bar{t}$

t/W backgrounds

- The $h \rightarrow \tau\tau$ reconstruction can reduce potentially large Drell-Yan backgrounds.
- The continuum t/W-containing backgrounds can be also effectively removed.

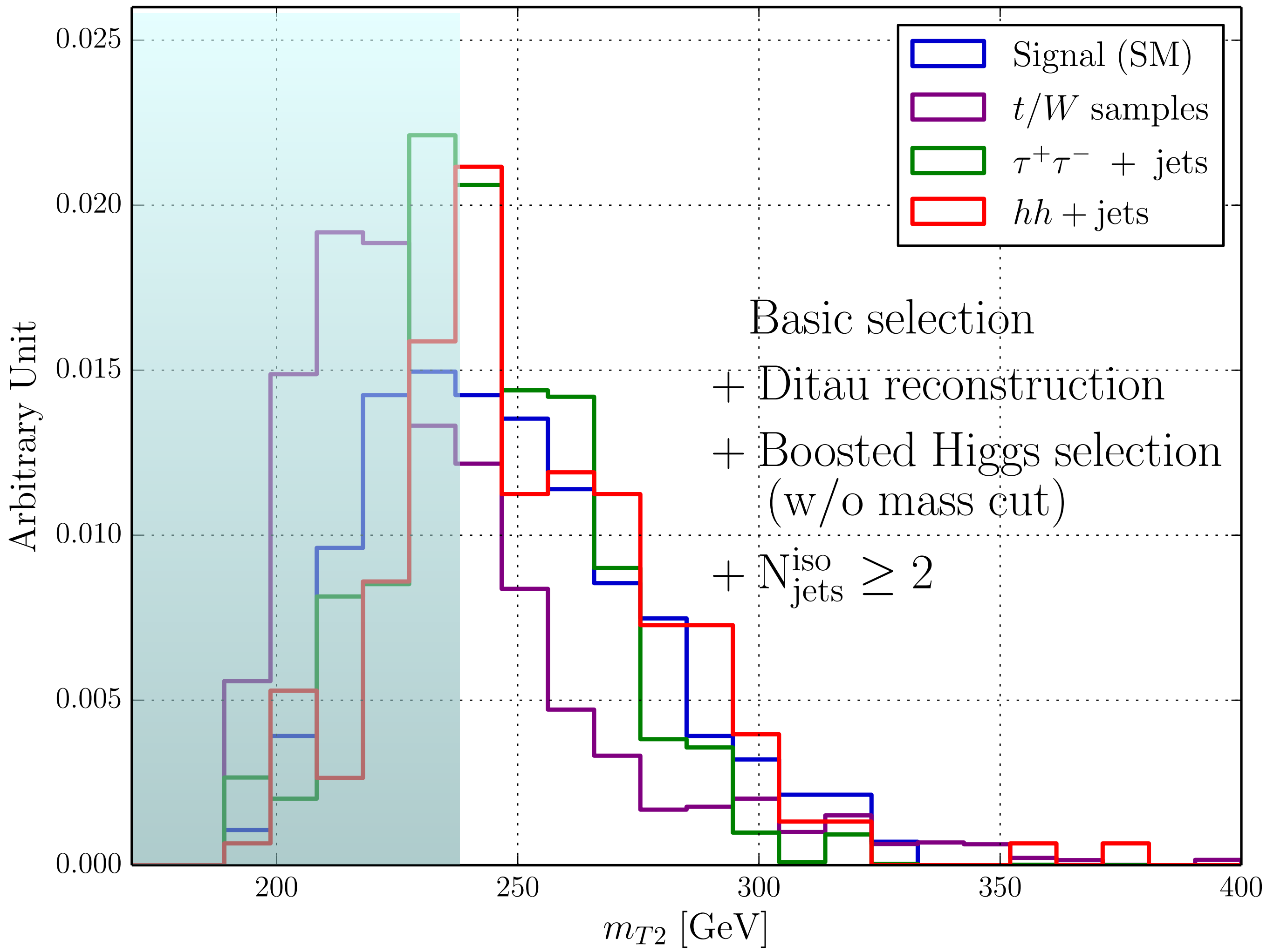
M_{T2} Variable Constructions





- To eliminate the WW-containing backgrounds using the end-point feature of M_{T2} varible.
- It is tricky because there are two neutrinos on each branch in WW-containing backgrounds.
- What test mass m_χ^{test} should we use?
- Our Ad-hoc choice is $m_\chi^{\text{test}} = 190$ GeV.

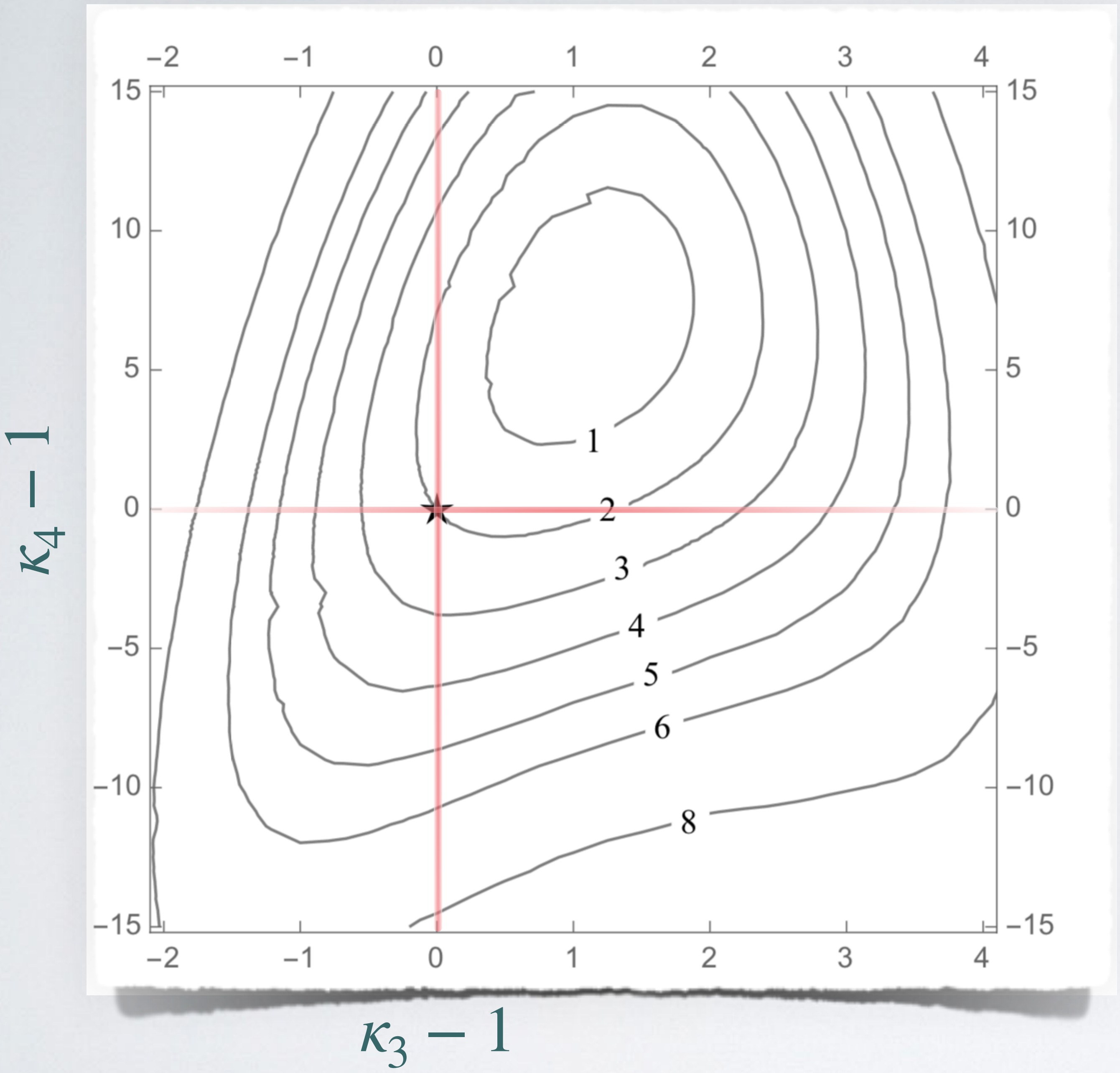
M_{T2} construction



- Signal M_{T2} distribution is slightly broader than WW-containing backgrounds.
- M_{T2} cut helps to disentangle WW-containing backgrounds.

Constraints from $hhh \to (b\bar{b})(b\bar{b})(\tau\bar{\tau})$

B. Fuks, **J. H. Kim**, S. J. Lee [2018] $30\ \text{ab}^{-1}$



Pythia-level analysis
(no-detector effects)

- Significance contours for $30\ \text{ab}^{-1}$.
- Assuming $\kappa_3 = 1$, the constraint turns out to be

$$1 < \kappa_4 < 8 \qquad [2\sigma] \quad (b\bar{b})(b\bar{b})(\tau\bar{\tau})$$

$$\updownarrow$$

$$-3 < \kappa_4 < 11 \qquad [2\sigma] \quad (b\bar{b})(b\bar{b})(\gamma\gamma)$$

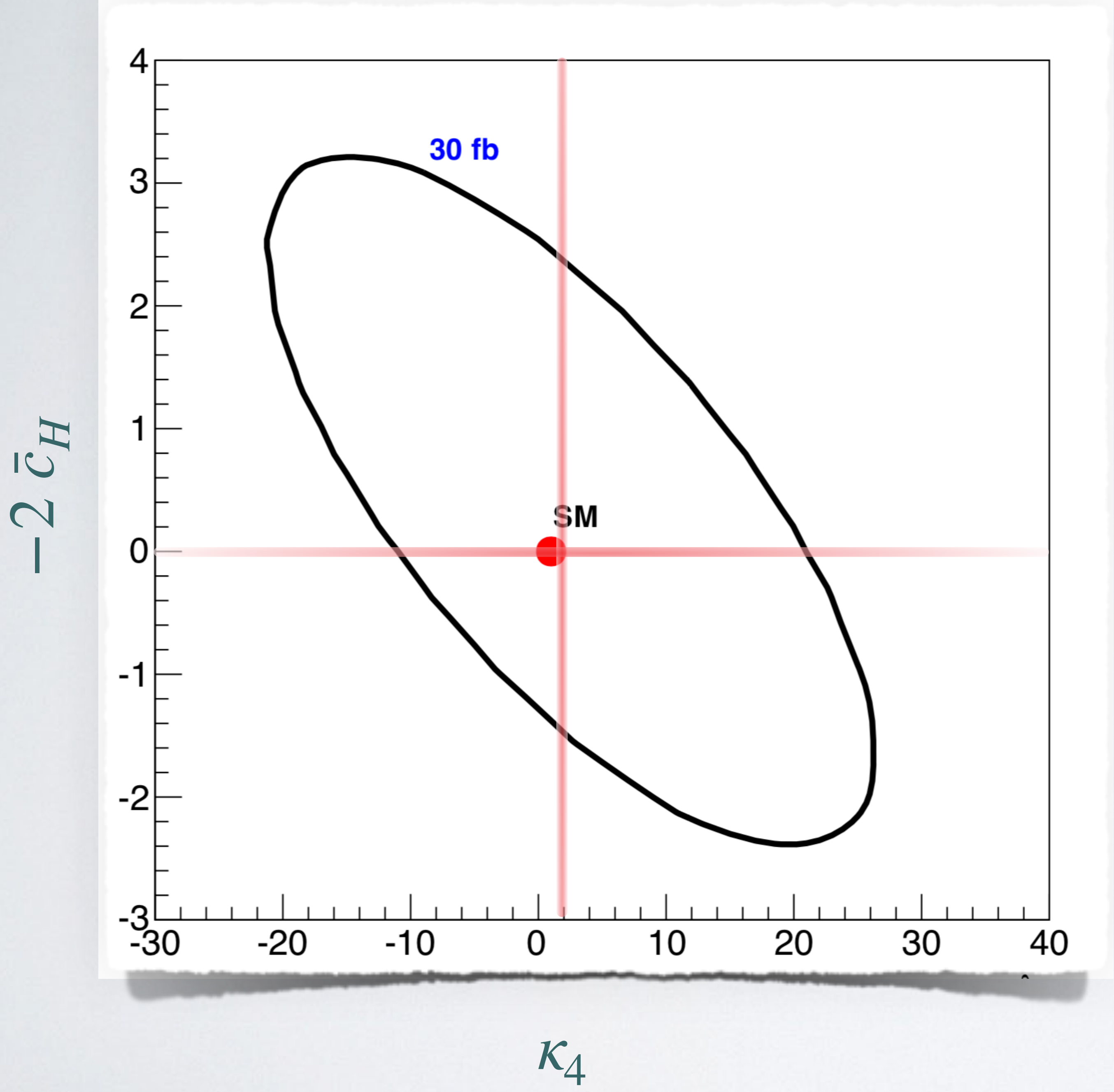
Papaefstathiou, Sakurai [2015]

- The bound is at least comparable with the $(b\bar{b})(b\bar{b})(\gamma\gamma)$ channel.

Constraints from $hhh \to (b\bar{b})(WW^*)_{1\ell}(WW^*)_{1\ell}$

Kilian, Sun, Yan, Zhao, Zhao [2017]

$30\ \mathrm{ab}^{-1}$



Detector-level analysis

- Requiring the same-sign dilepton $\ell^+\ell^+$ or $\ell^-\ell^-$ suppresses backgrounds.
- The bound turns out to be

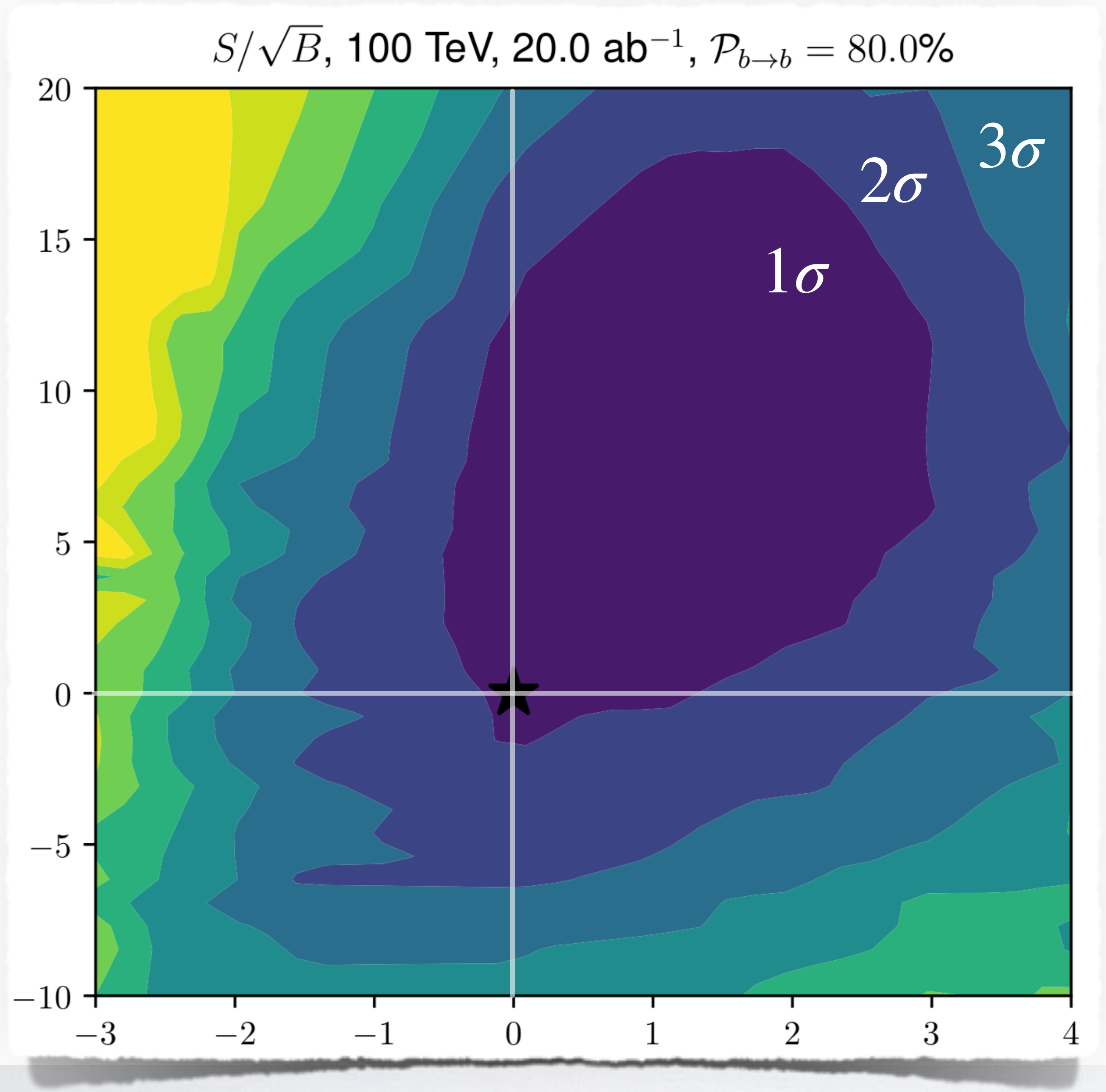
$$-10 < \kappa_4 < 20 \qquad [2\sigma]$$

- It is weaker than any other channels considered above.

Constraints from $hhh \to (b\bar{b})(b\bar{b})(b\bar{b})$

Papaefstathiou, Tetlalmatzi, Zaro [2019]

$20\ \mathrm{ab}^{-1}$



$\kappa_4 - 1$

$\kappa_3 - 1$

Parton-level analysis

- The $(b\bar{b})(b\bar{b})(b\bar{b})$ channel takes an advantage of the largest branching ratio.

- The bound turns out to be

$$-5 < \kappa_4 < 18 \qquad [2\sigma]$$

Constraints from *hhh*

- $(b\bar{b})(b\bar{b})(\gamma\gamma)$ channel

Papaefstathiou, Sakurai [2015]

B. Fuks, **J. H. Kim**, S. J. Lee [2015]

$$-3 < \kappa_4 < 11 \qquad [2\sigma]$$

- $(b\bar{b})(WW^*)_{1\ell}(WW^*)_{1\ell}$ channel

Kilian, Sun, Yan, Zhao, Zhao [2017]

$$-10 < \kappa_4 < 20 \qquad [2\sigma]$$

- $(b\bar{b})(b\bar{b})(\tau\bar{\tau})$ channel

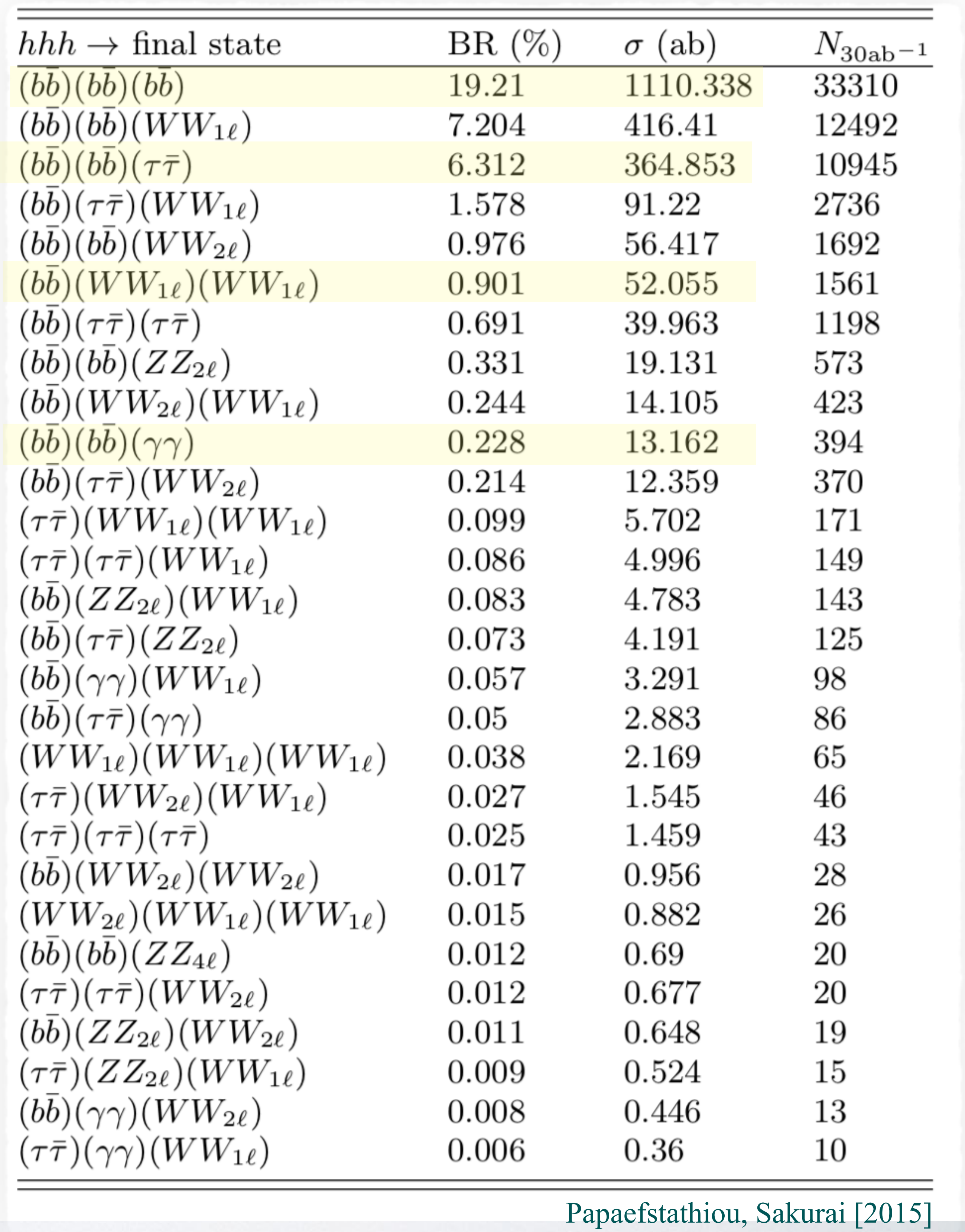
B. Fuks, **J. H. Kim**, S. J. Lee [2018]

$$1 < \kappa_4 < 8 \qquad [2\sigma]$$

- $(b\bar{b})(b\bar{b})(b\bar{b})$ channel

Papaefstathiou, Tetlalmatzi, Zaro [2019]

$$-5 < \kappa_4 < 18 \qquad [2\sigma]$$



$hhh \rightarrow$ final state	BR (%)	σ (ab)	$N_{30\text{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
$(b\bar{b})(WW_{2\ell})(WW_{1\ell})$	0.244	14.105	423
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.228	13.162	394
$(b\bar{b})(\tau\bar{\tau})(WW_{2\ell})$	0.214	12.359	370
$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
$(b\bar{b})(\tau\bar{\tau})(ZZ_{2\ell})$	0.073	4.191	125
$(b\bar{b})(\gamma\gamma)(WW_{1\ell})$	0.057	3.291	98
$(b\bar{b})(\tau\bar{\tau})(\gamma\gamma)$	0.05	2.883	86
$(WW_{1\ell})(WW_{1\ell})(WW_{1\ell})$	0.038	2.169	65
$(\tau\bar{\tau})(WW_{2\ell})(WW_{1\ell})$	0.027	1.545	46
$(\tau\bar{\tau})(\tau\bar{\tau})(\tau\bar{\tau})$	0.025	1.459	43
$(b\bar{b})(WW_{2\ell})(WW_{2\ell})$	0.017	0.956	28
$(WW_{2\ell})(WW_{1\ell})(WW_{1\ell})$	0.015	0.882	26
$(b\bar{b})(b\bar{b})(ZZ_{4\ell})$	0.012	0.69	20
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{2\ell})$	0.012	0.677	20
$(b\bar{b})(ZZ_{2\ell})(WW_{2\ell})$	0.011	0.648	19
$(\tau\bar{\tau})(ZZ_{2\ell})(WW_{1\ell})$	0.009	0.524	15
$(b\bar{b})(\gamma\gamma)(WW_{2\ell})$	0.008	0.446	13
$(\tau\bar{\tau})(\gamma\gamma)(WW_{1\ell})$	0.006	0.36	10

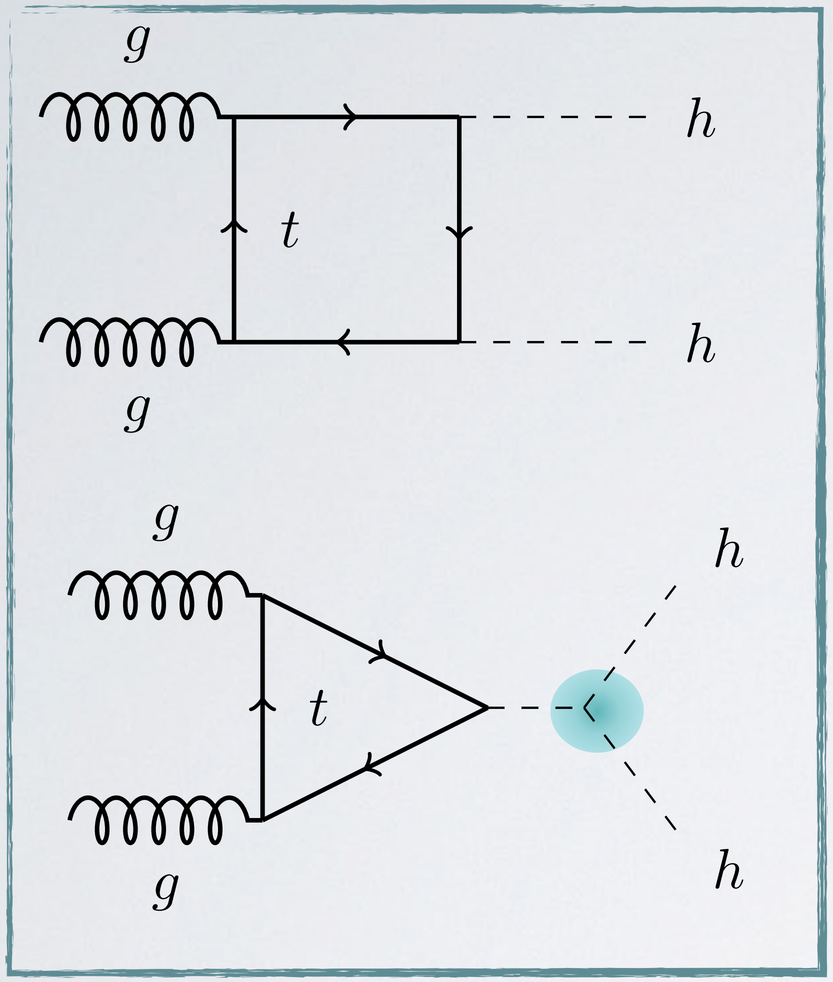
Papaefstathiou, Sakurai [2015]

The Quartic Coupling from Double Higgs

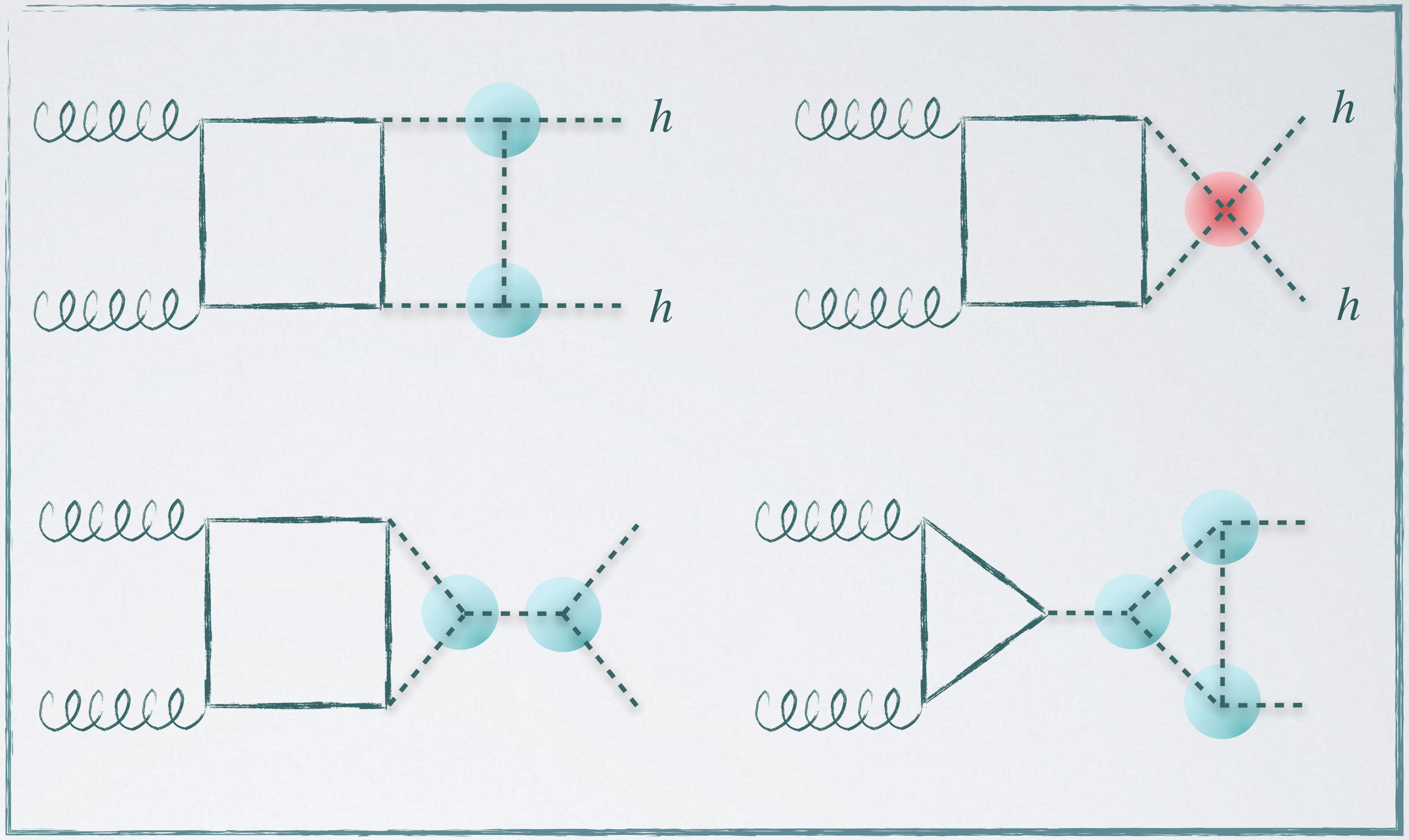
Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao [2018]

Bizon, Haisch, Rottoli [2019]

LO (one-loop)



NLO (two-loop)

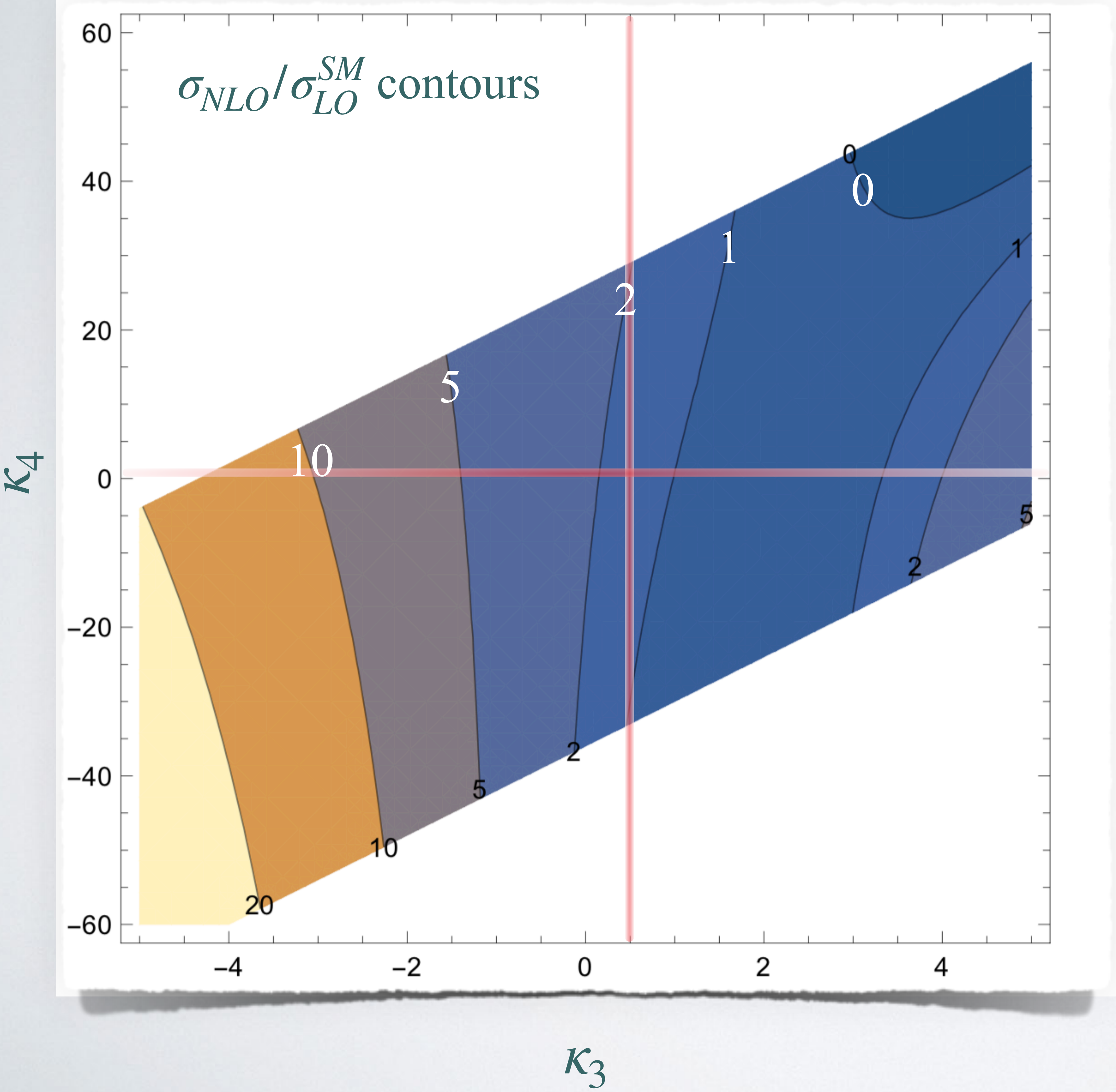


- The hh production is a major avenue to probe κ_3 coupling at LO.
- However, the hh production also contains κ_4 at NLO.
- This may provide a chance probe κ_4.

The Quartic Coupling from Double Higgs

Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao [2018]

$\sqrt{s} = 14$ TeV



- The NLO cross ection is shown as a ratio between:

$$\sigma_{NLO} = \sigma_{NLO}(\kappa_3, \kappa_4)$$

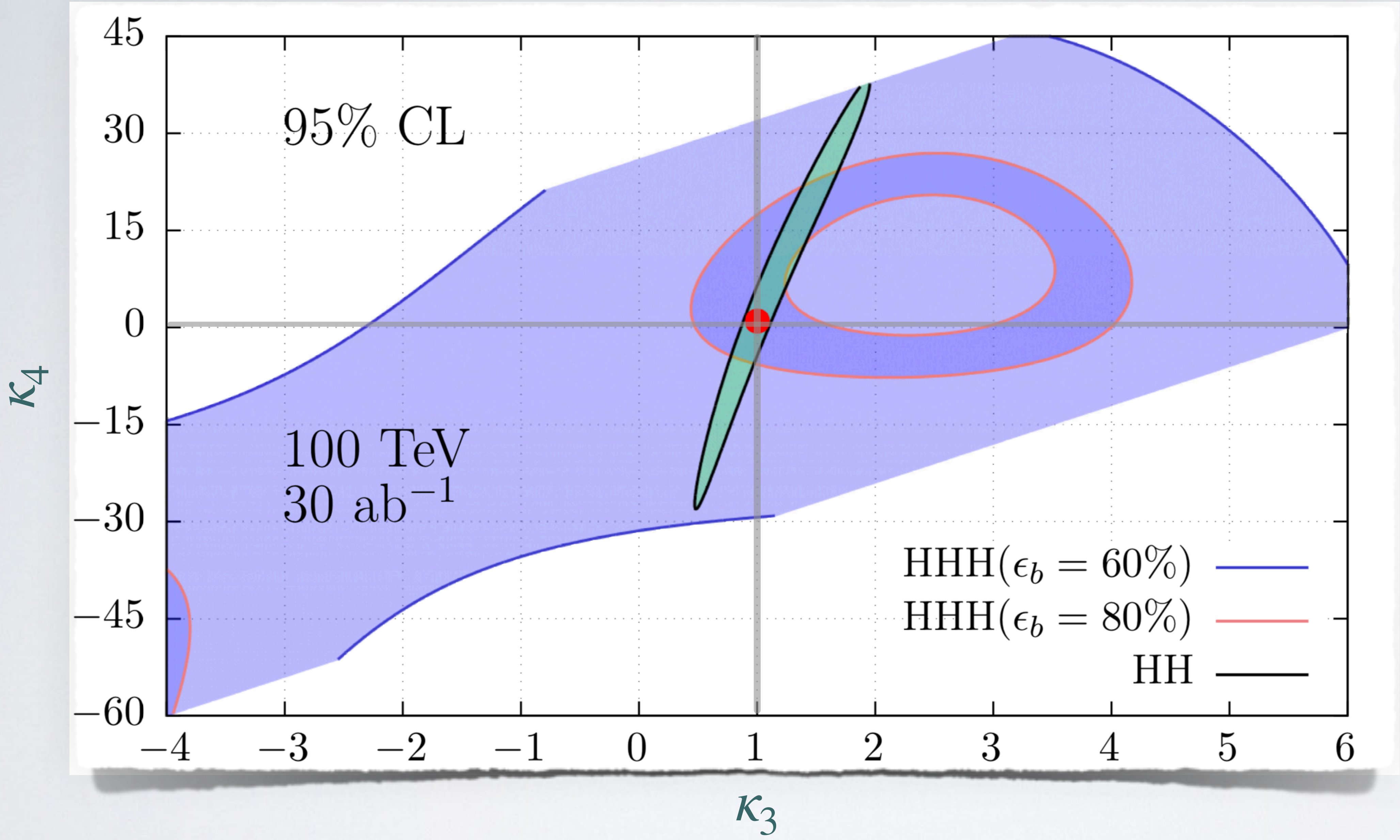
$$\sigma_{LO}^{SM} \sim 20 \text{ fb}$$

- Strong dependence on κ_3 .

- Mild dependence on κ_4 .

$hh\ (b\bar{b}\gamma\gamma)$ vs $hhh\ (b\bar{b}b\bar{b}\gamma\gamma)$

Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao [2018]



- Assuming $\kappa_3 = 1$, the constraint turns out to be

$$\boxed{-4 < \kappa_4 < 6} \quad [2\sigma]$$

$\longleftrightarrow$

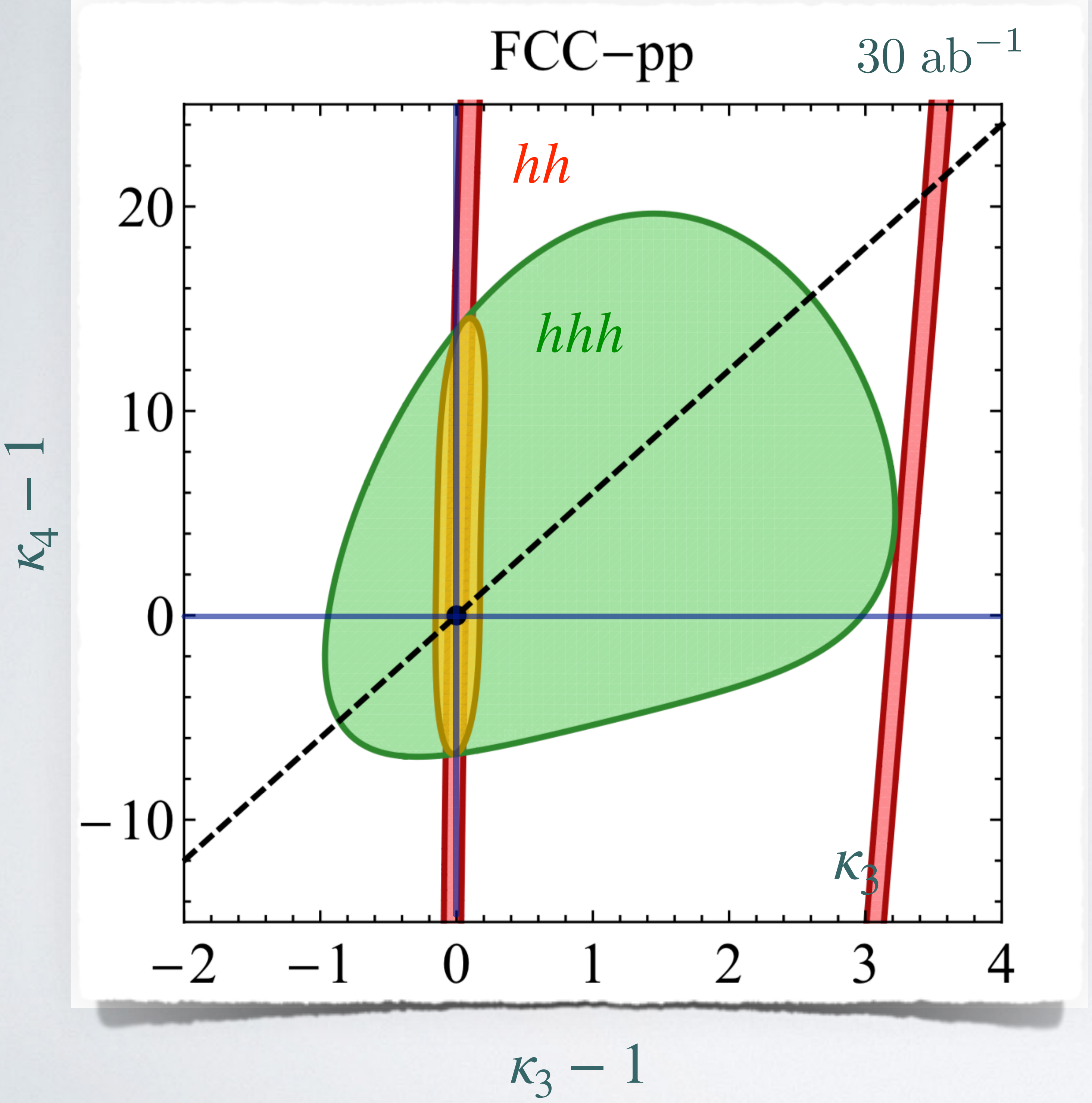
- The bound is stronger than *hhh*.

$$-3 < \kappa_4 < 11 \quad [2\sigma]$$

Papaefstathiou, Sakurai [2015]

$hh\ (b\bar{b}\gamma\gamma)$ vs $hhh\ (b\bar{b}b\bar{b}\gamma\gamma)$

Bizon, Haisch, Rottoli [2019]



- The same final state is analyzed by Bizon et. al.

- Assuming $\kappa_3 = 1$, the bound turns out to be weaker

hh : $\boxed{-5 < \kappa_4 < 14}$ $[2\sigma]$

Bizon, Haisch, Rottoli [2019]

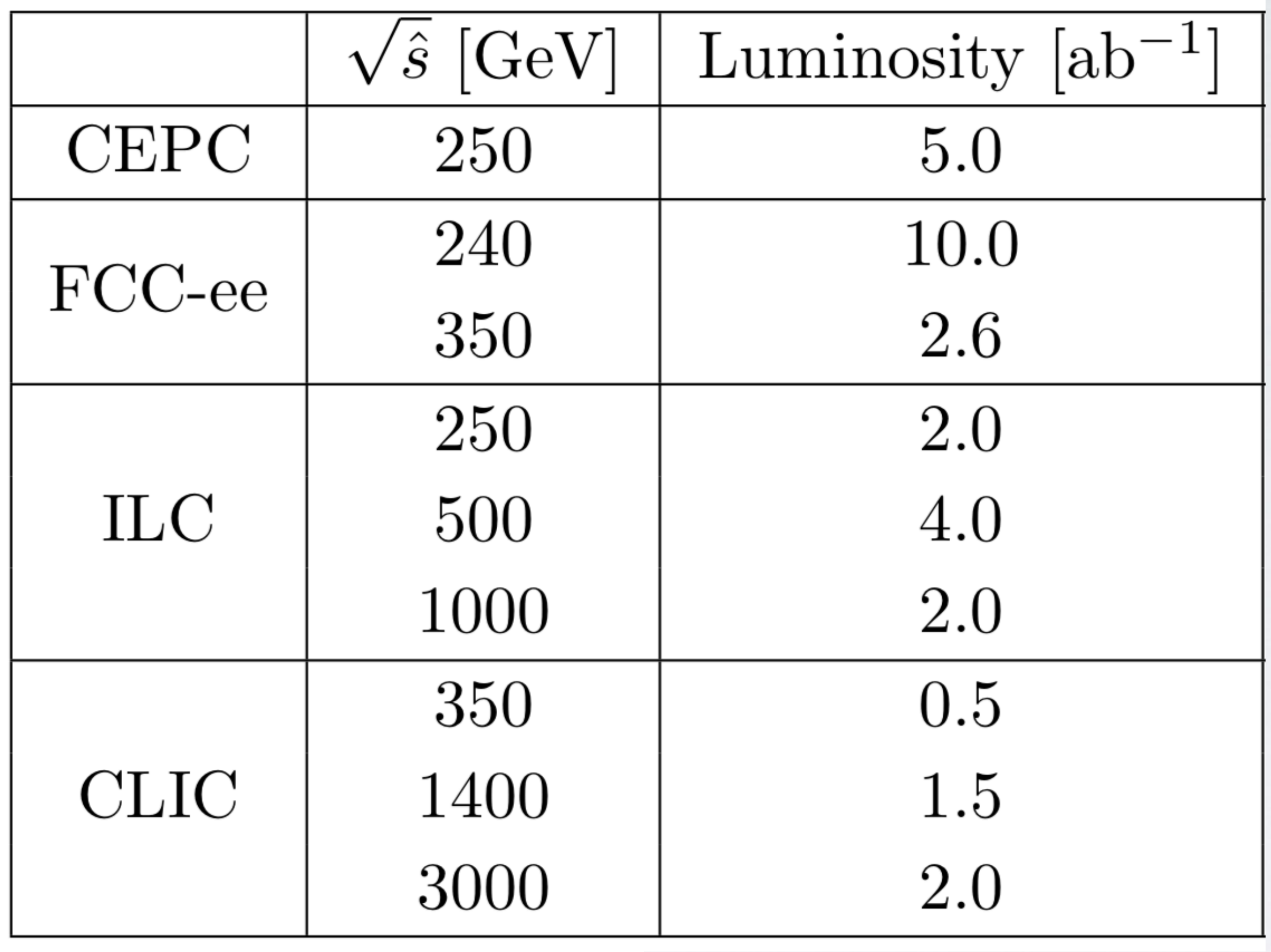
↕

hh : $-4 < \kappa_4 < 6$ $[2\sigma]$

Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao [2018]

- In general, the bound from the *hh* is comparable with the one from *hhh*.

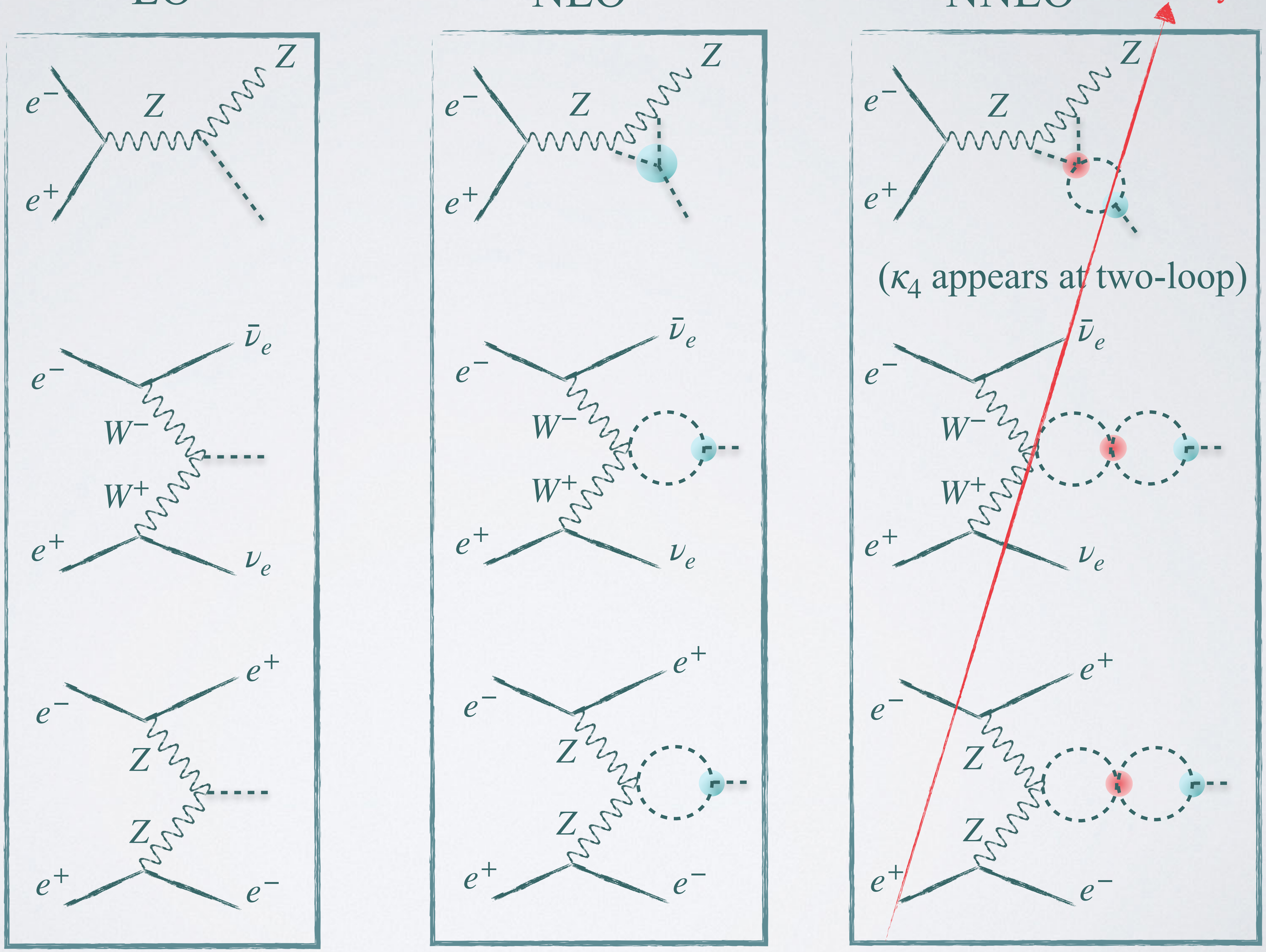
Benchmark Future e^-e^+ Colliders



	$\sqrt{\hat{s}}$ [GeV]	Luminosity [ab^{-1}]
CEPC	250	5.0
FCC-ee	240	10.0
	350	2.6
ILC	250	2.0
	500	4.0
	1000	2.0
CLIC	350	0.5
	1400	1.5
	3000	2.0

- Future e^-e^+ colliders with target energies and luminosities are shown here.
- Can κ_4 be better constrained at e^-e^+ colliders?

Single Higgs Production at e^-e^+ Colliders



LO

NLO

NNLO

cross section is very small

Zh :

s-channel

(κ_4 appears at two-loop)

$\nu\nu h$:

W-boson fusion (WBF)

~~eeh :~~

~~Z-boson fusion (ZBF)~~

cross section is very small

Single Higgs Production at e^-e^+ Colliders

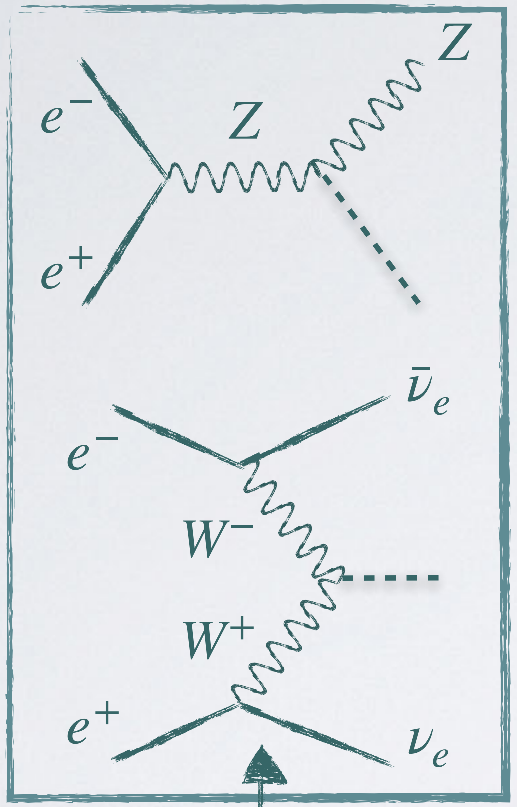
Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]

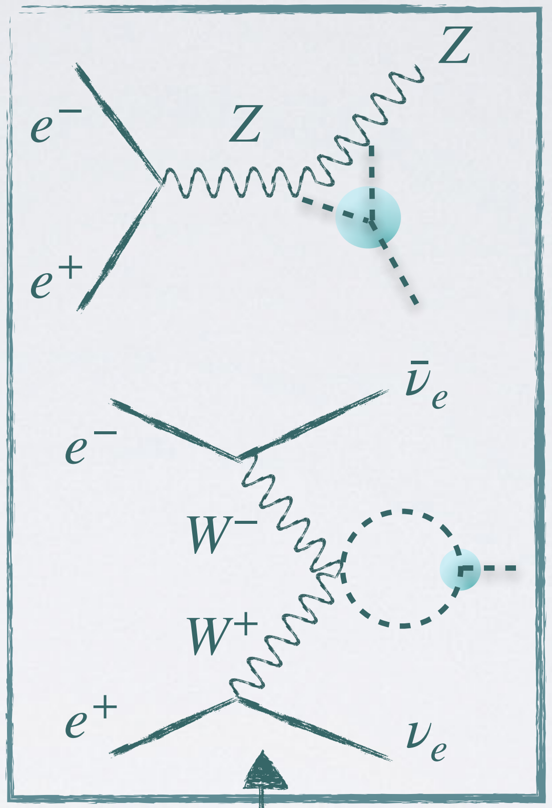
Zh :

s-channel

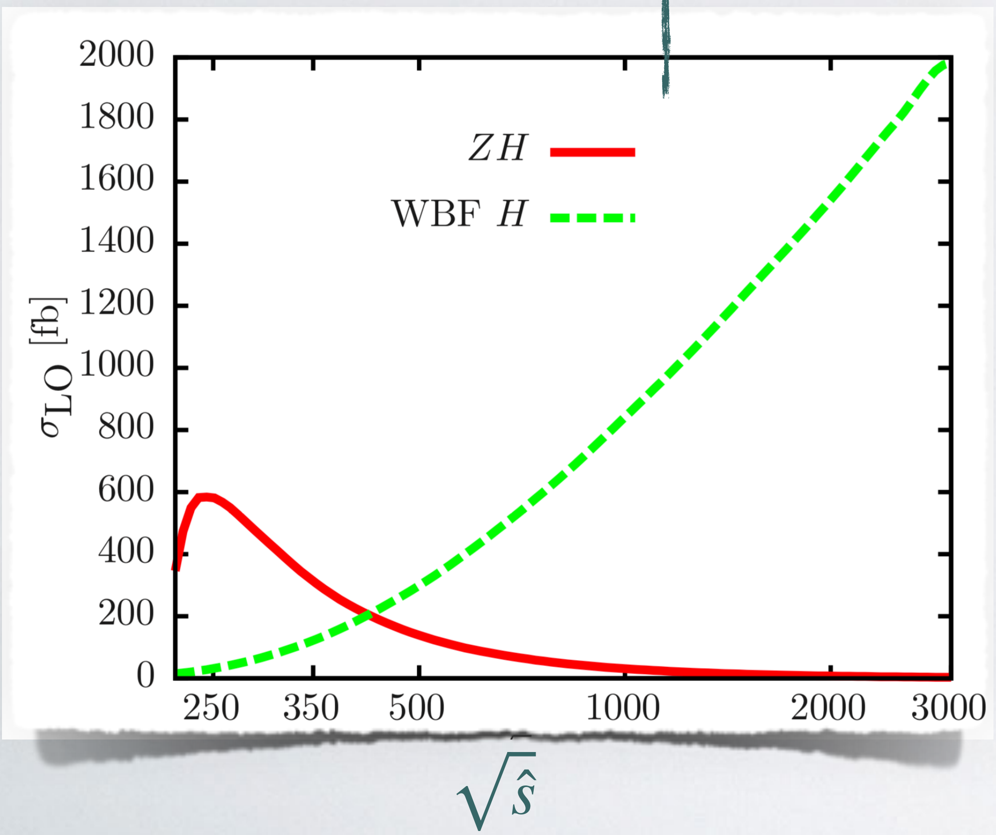
$\nu\nu h$:

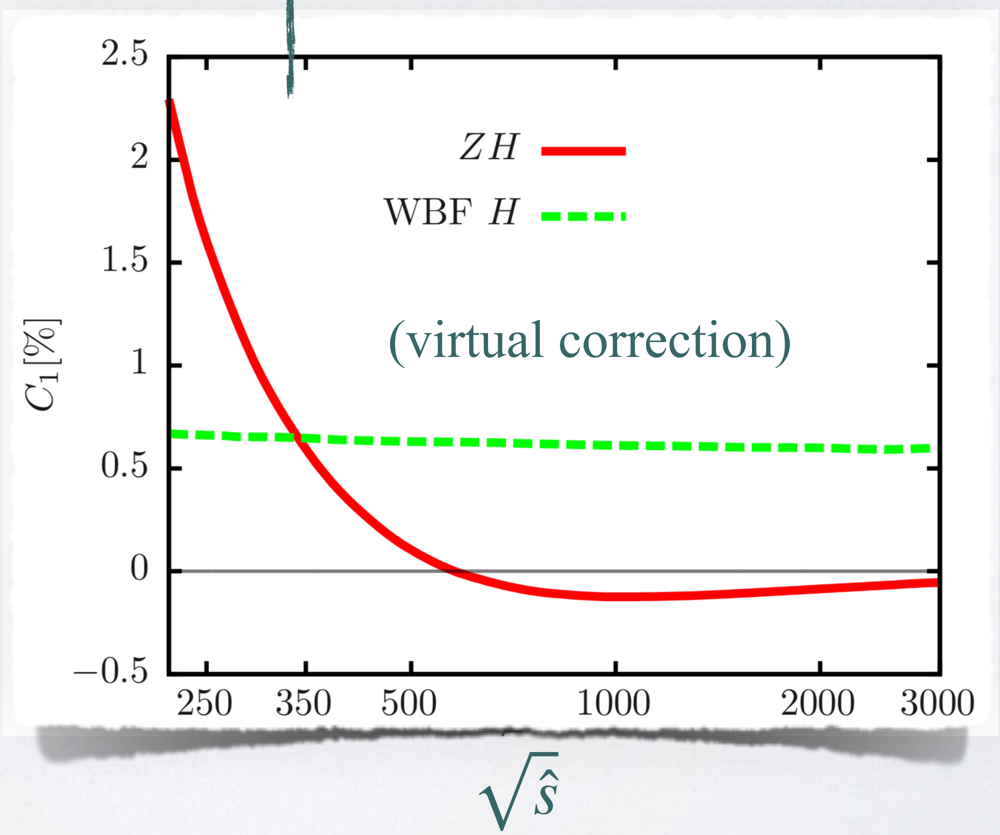
W-boson fusion (WBF)





- Two major channels to probe κ_3 are shown.
- However, no sensitivity on κ_4.
- Virtual loop correction is 1 ~ 2% at most.





Single Higgs Production at e^-e^+ Colliders

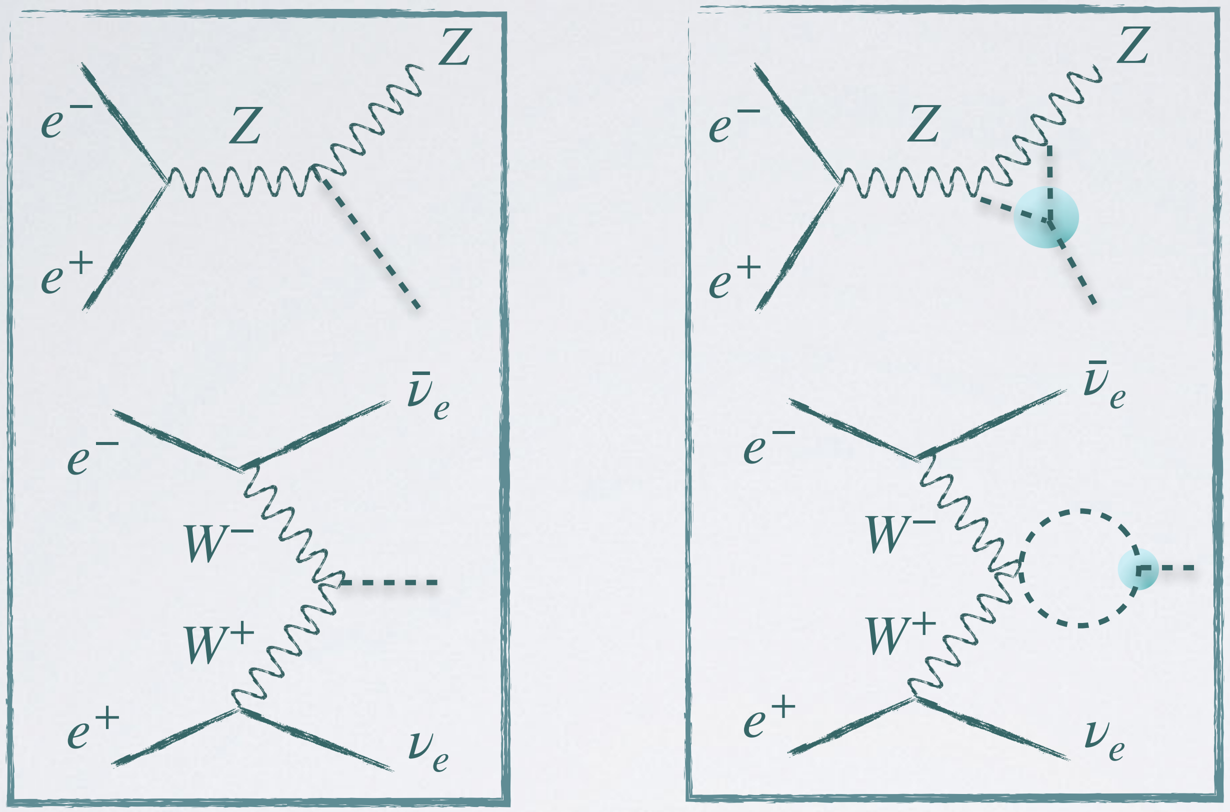
Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]

Zh :

s-channel

$\nu\nu h$:

W-boson fusion (WBF)

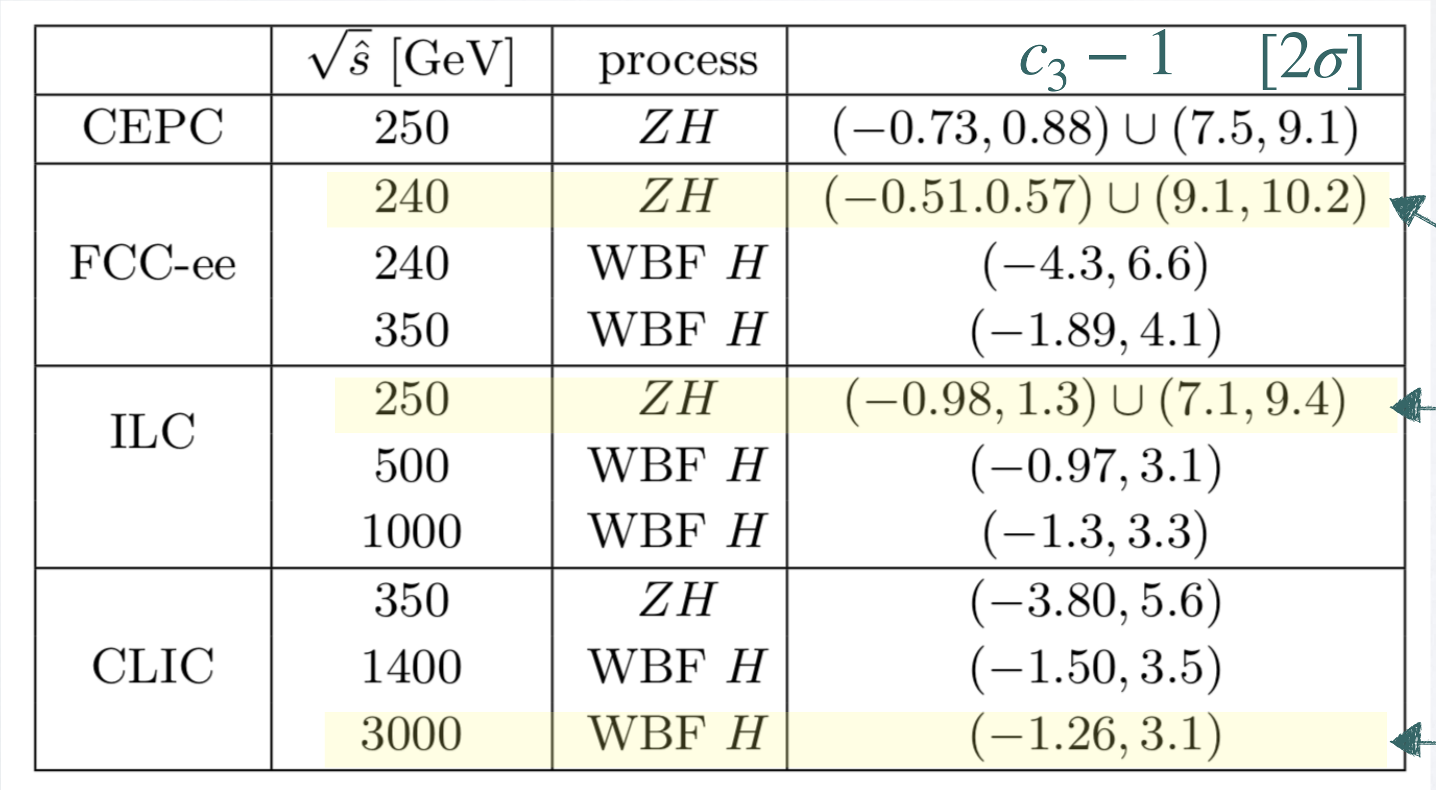


- Bounds are presented in the linear basis where the triple and quartic couplings are related by

$$c_4 = 6c_3 - 5$$

- With this assumption, the quartic coupling can be constrained by

$$-2.1 < c_4 < 4.4$$



	$\sqrt{\hat{s}}$ [GeV]	process	$c_3 - 1$ [2σ]
CEPC	250	ZH	$(-0.73, 0.88) \cup (7.5, 9.1)$
FCC-ee	240	ZH	$(-0.51.0.57) \cup (9.1, 10.2)$
	240	WBF H	$(-4.3, 6.6)$
	350	WBF H	$(-1.89, 4.1)$
ILC	250	ZH	$(-0.98, 1.3) \cup (7.1, 9.4)$
	500	WBF H	$(-0.97, 3.1)$
	1000	WBF H	$(-1.3, 3.3)$
CLIC	350	ZH	$(-3.80, 5.6)$
	1400	WBF H	$(-1.50, 3.5)$
	3000	WBF H	$(-1.26, 3.1)$

$0.49 < c_3 < 1.57$

$0.02 < c_3 < 2.3$

$-0.26 < c_3 < 4.1$

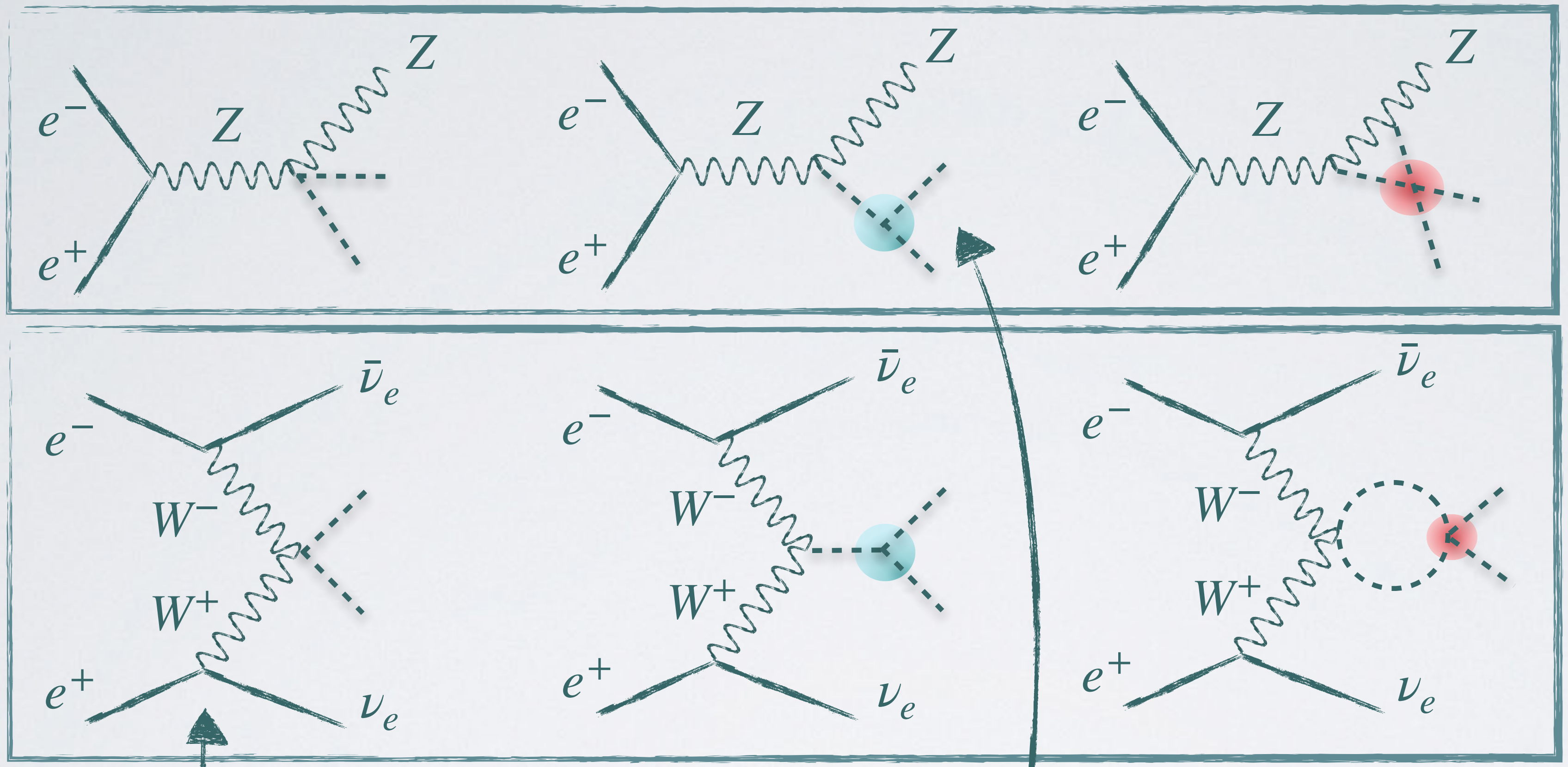
Double Higgs Production at e^-e^+ Colliders

Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]

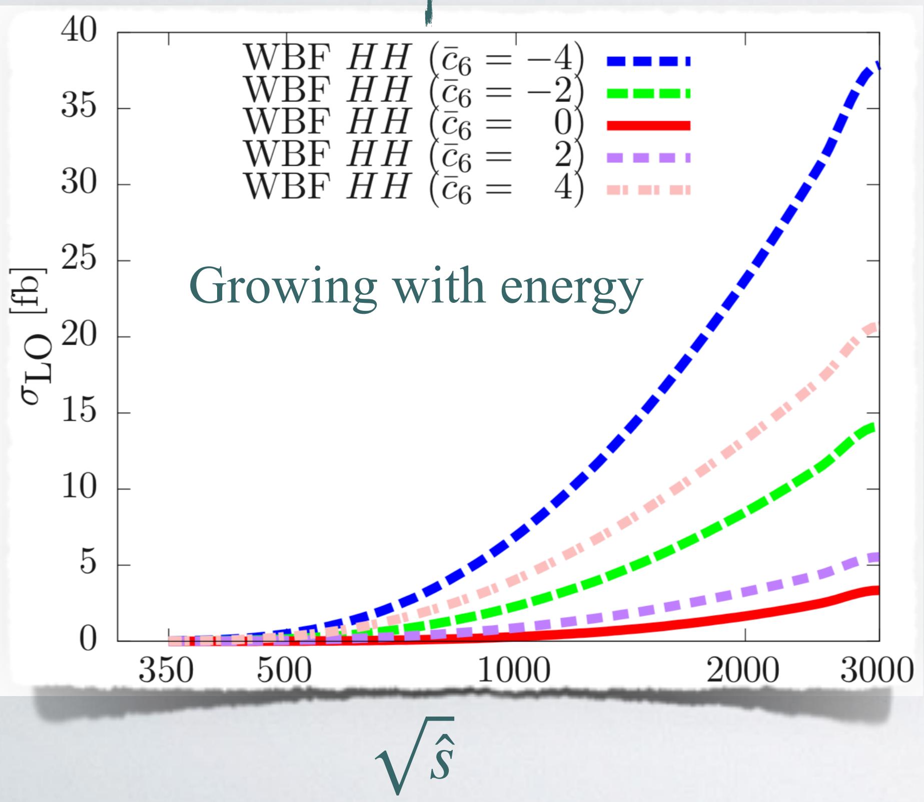
LO NLO

Zhh :
s-channel

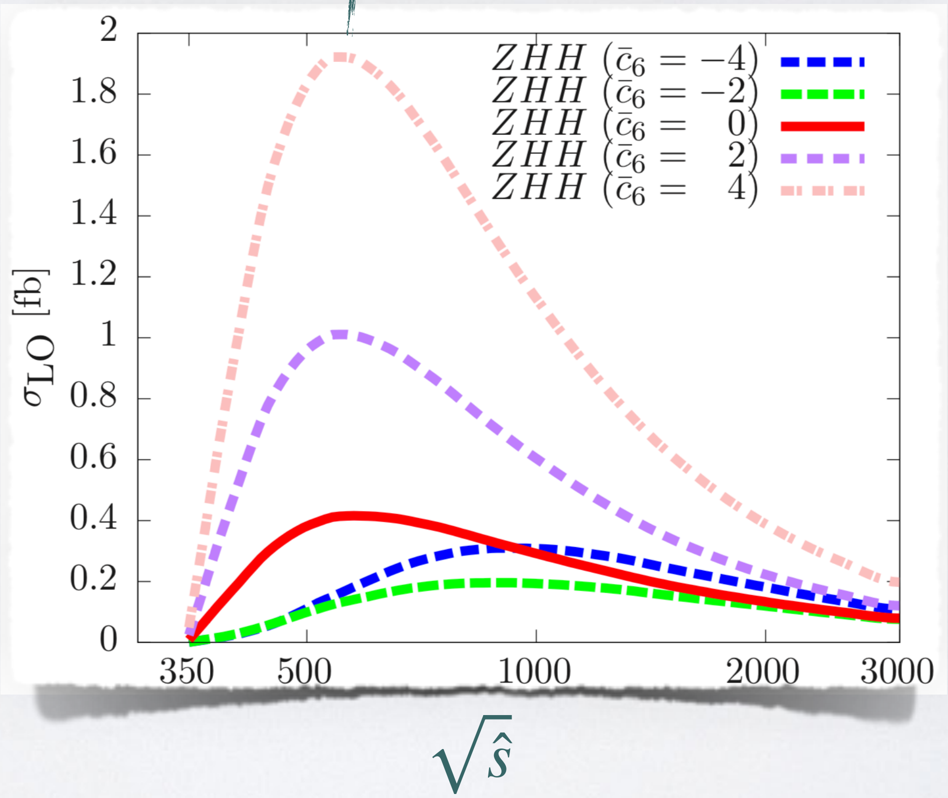
$\nu\nu hh$:
W-boson fusion (WBF)



- κ_4 appears at one-loop.
- The $\nu\nu hh$ process grows with energy

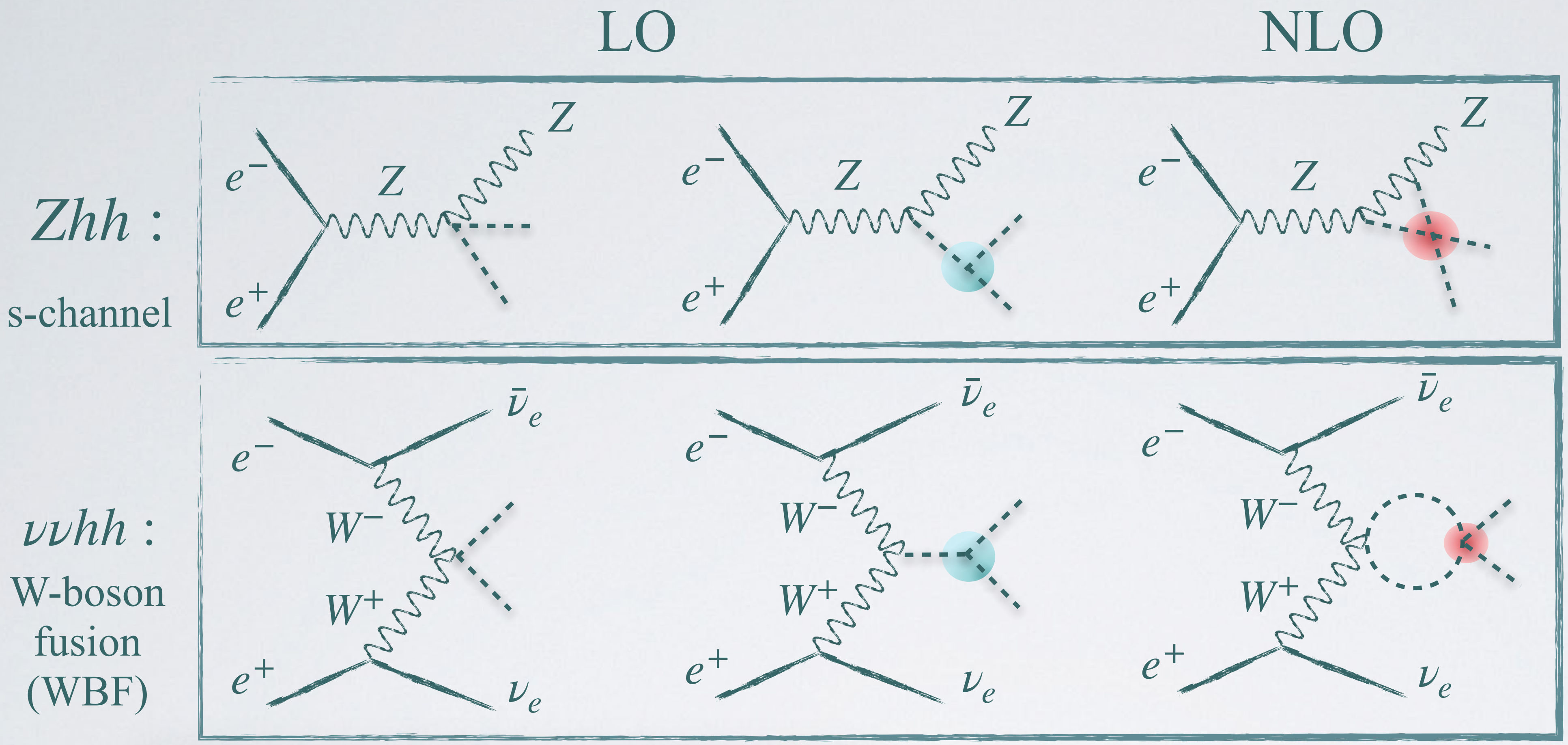


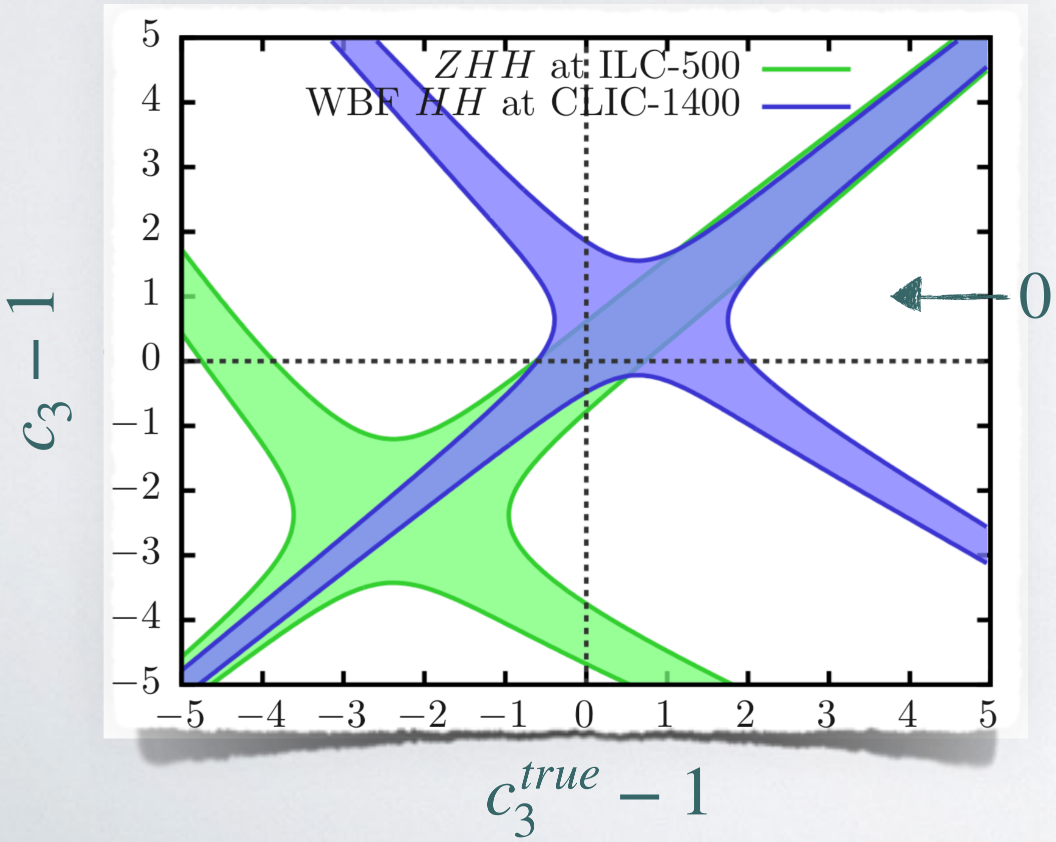
Growing with energy



Double Higgs Production at e^-e^+ Colliders

Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]





$0.5 < c_3 < 1.5$

- In the linear-basis, the quartic coupling can be constrained by

$$-2 < c_4 < 4$$

$$(c_4 = 6c_3 - 5 \text{ is assumed})$$

- This bound is comparable with the single Higgs search.

Triple Higgs Production at e^-e^+ Colliders

Chiesa, Maltoni, Mantani, Mele,
Piccinini, Zhao [2020]

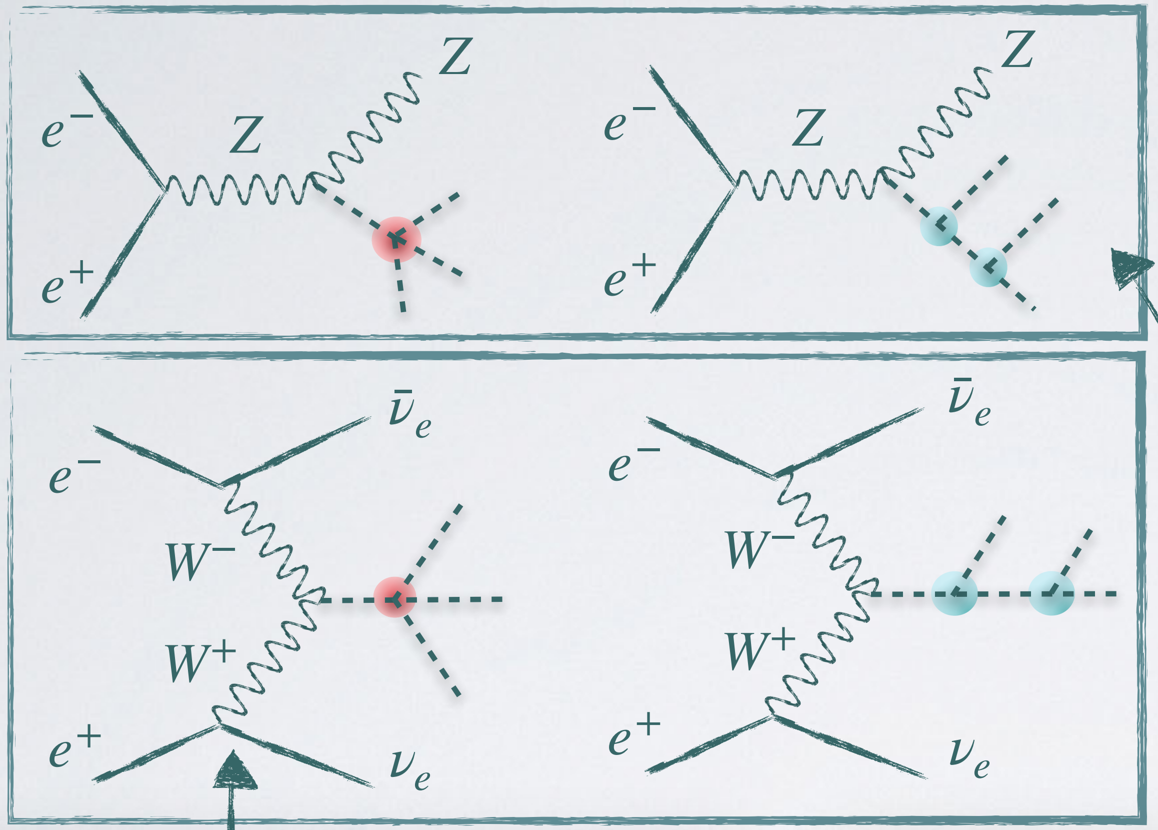
LO

$Zhhh$:

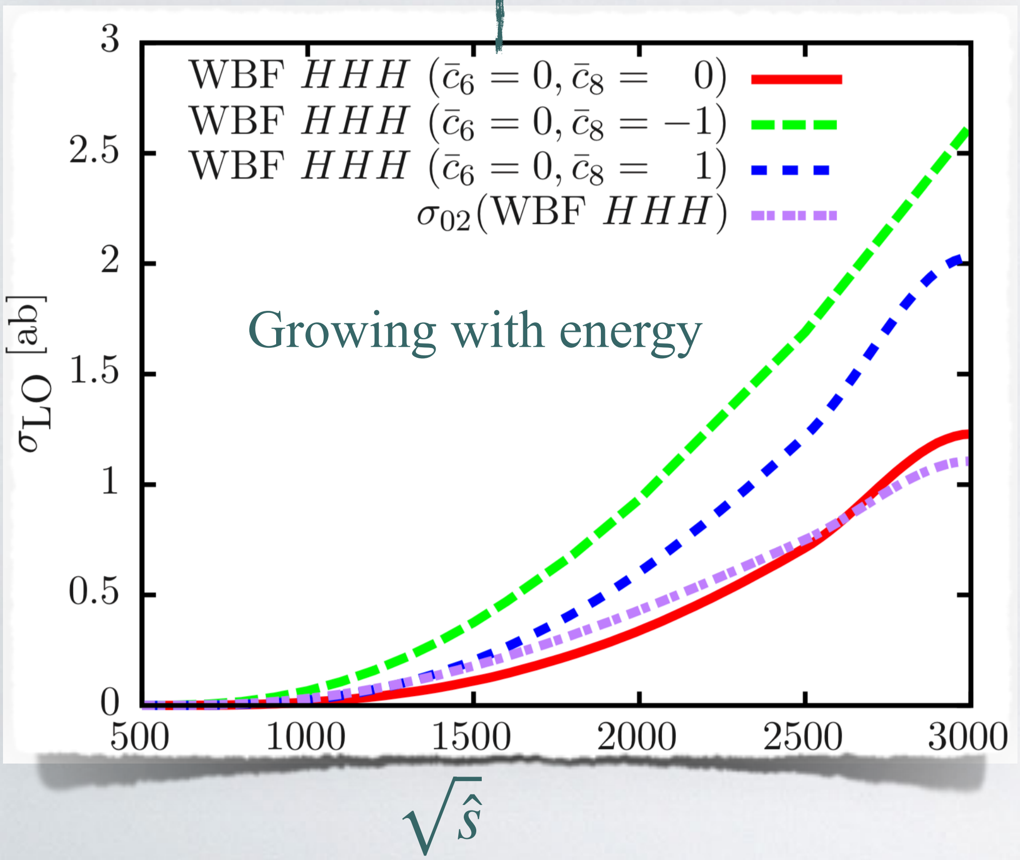
s-channel

$\nu\nu hhh$:

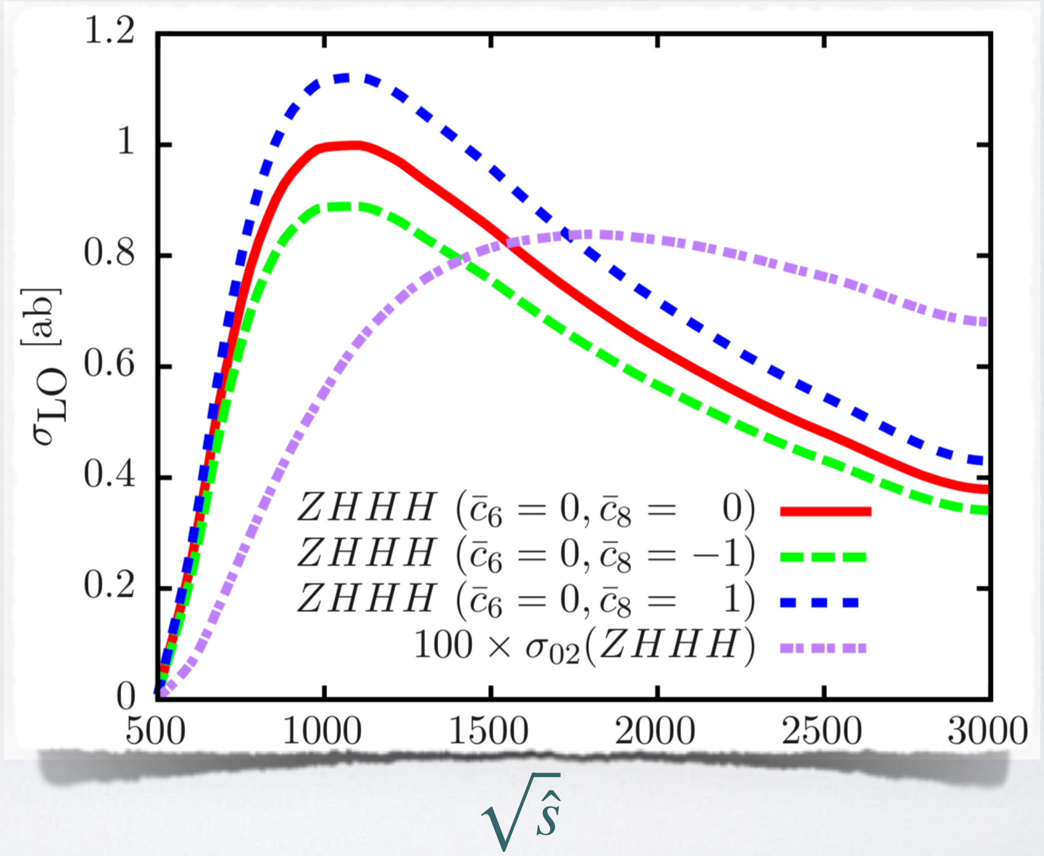
W-boson fusion (WBF)



- κ_4 appears at tree-level.
- Cross sections are very small.
- However, the $\nu\nu hhh$ process grows with energy.



Growing with energy



Triple Higgs Production at e^-e^+ Colliders

Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]

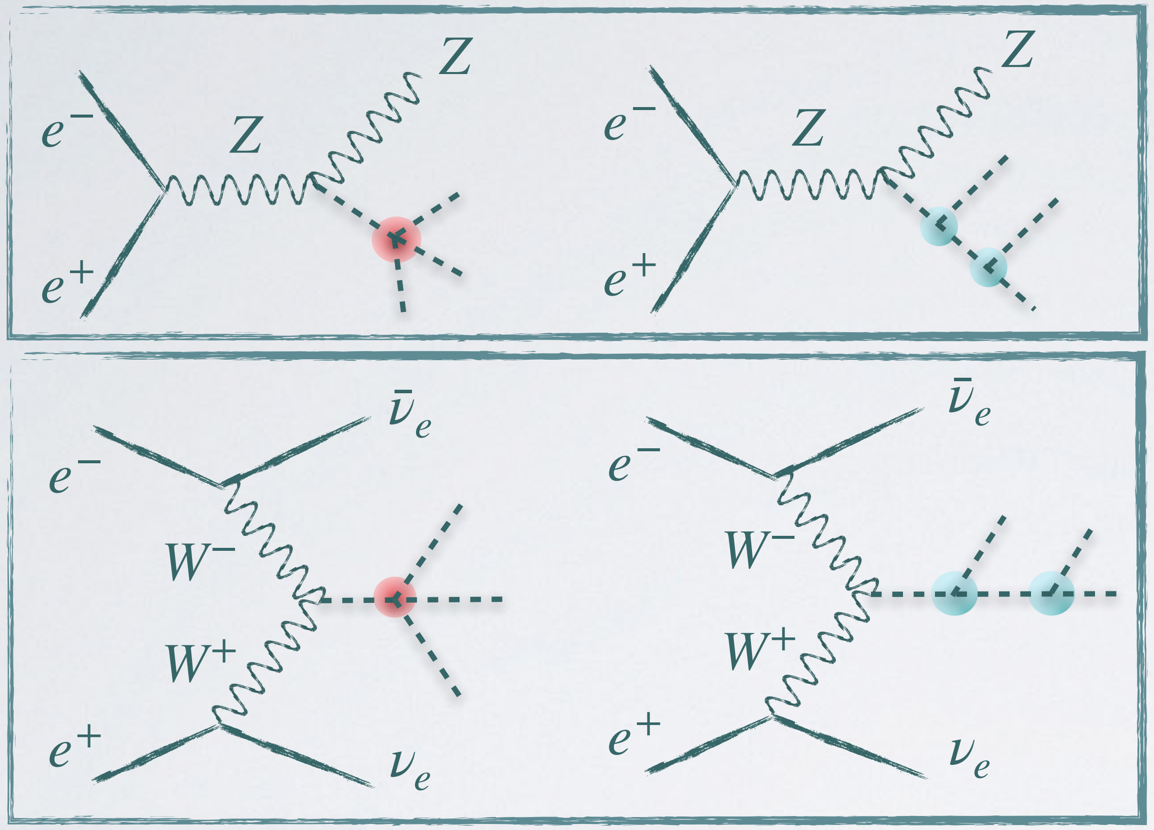
LO

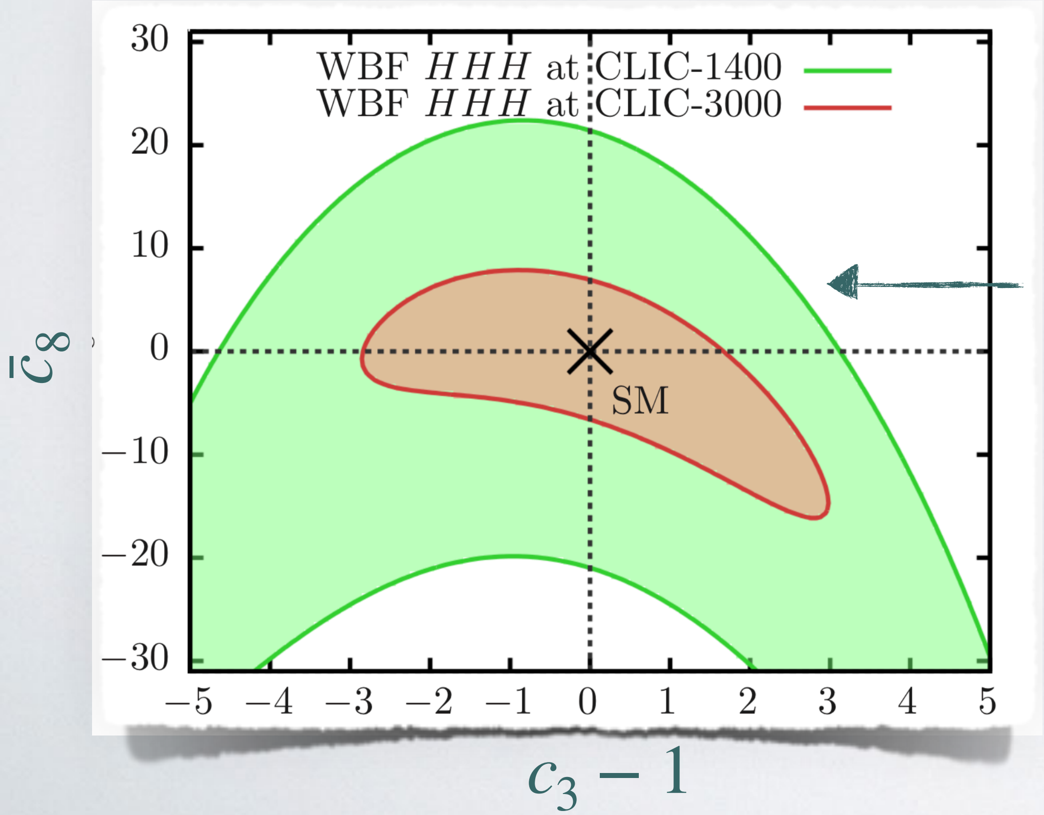
$Zhhh$:

s-channel

$\nu\nu hhh$:

W-boson fusion (WBF)





$-2 < c_3 < 4$

- Bounds are presented in the linear basis.
- In the linear-basis, the quartic coupling can be constrained by

$$-23 < c_4 < 19$$

$$(c_4 = 6c_3 - 5 \text{ is assumed})$$

- No meaningful bound can be obtained.

Motivation for a Multi-TeV Muon Collider

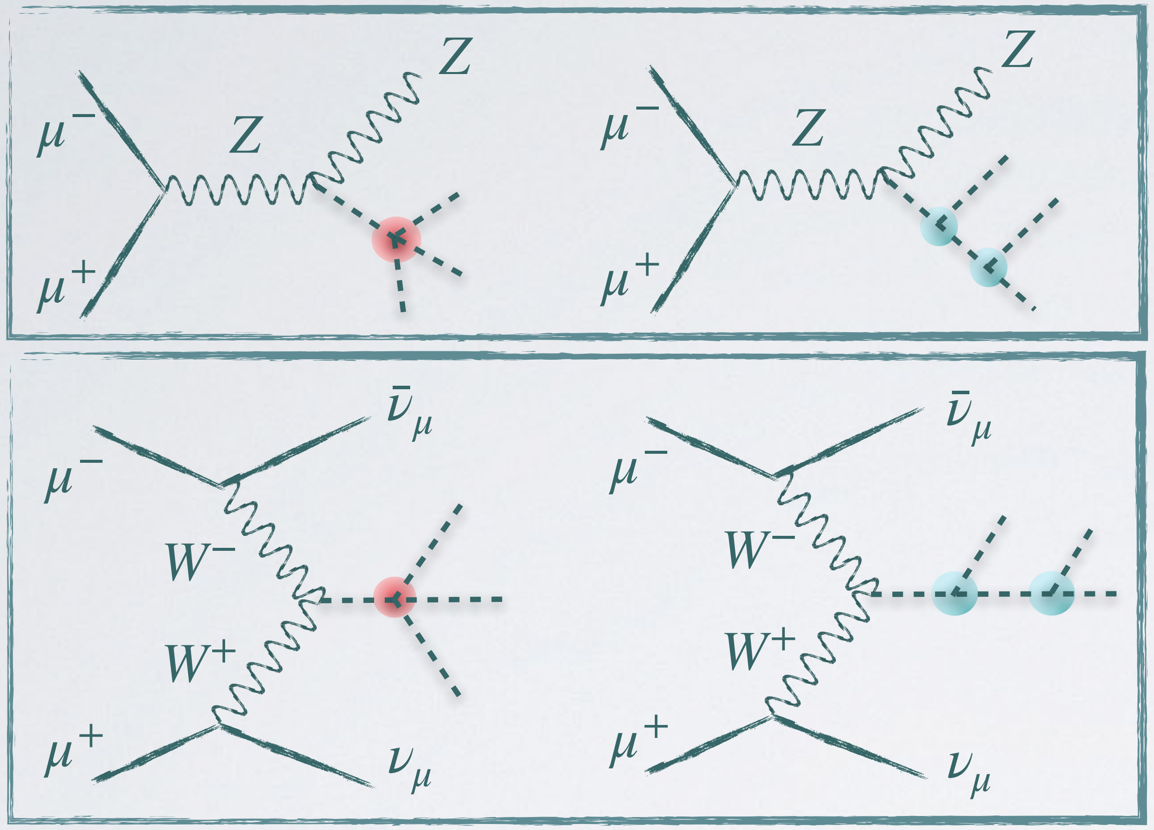
LO

$Zhhh$:

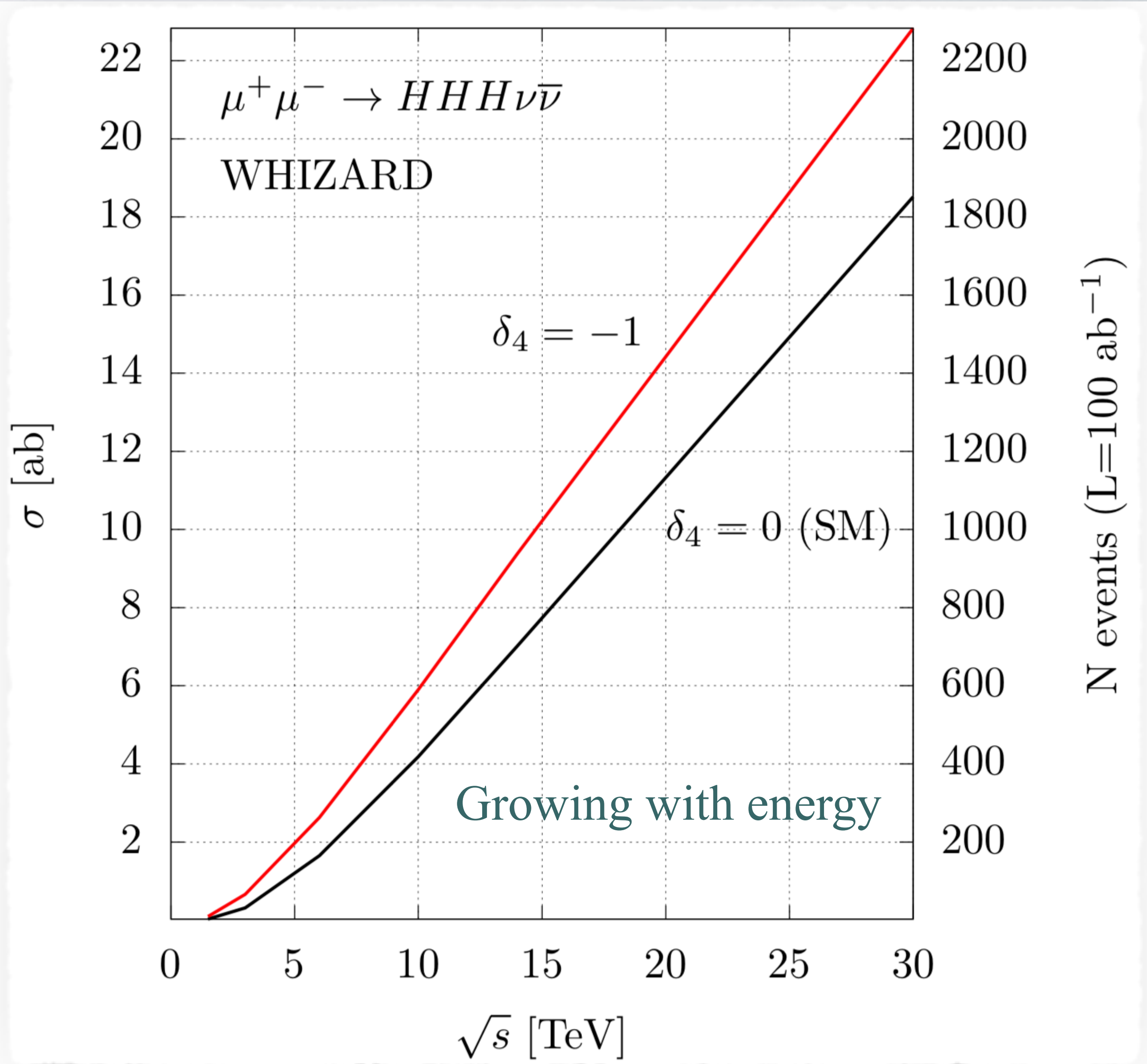
s-channel

$\nu\nu hhh$:

W-boson fusion (WBF)



Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]

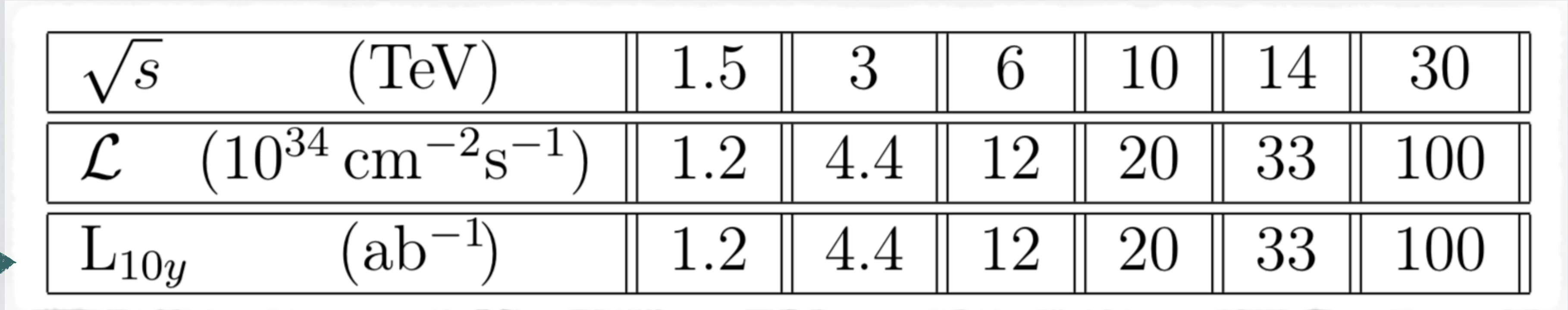


Growing with energy

- The triple Higgs production cross section grows with energy.
- κ_4 appears at tree-level.
- At high energy multi-TeV muon collider, we might be able to probe κ_4.

Benchmarks for a Future Muon Collider

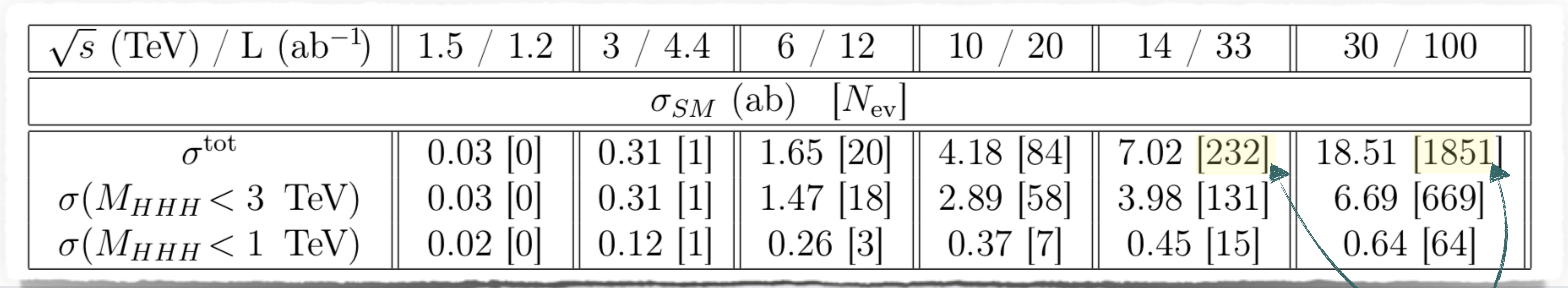
Muon Collider Working Group [1901.06150]



$\sqrt{s}$ (TeV)	1.5	3	6	10	14	30
$\mathcal{L}$ $(10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1})$	1.2	4.4	12	20	33	100
L_{10y} (ab^{-1})	1.2	4.4	12	20	33	100

Integrated luminosity

- Future muon colliders with target energies and luminosities are shown here.
- A large muon mass suppresses the synchrotron radiation by a factor of 10^9 (cf. e^-e^+ colliders).



$\sqrt{s}$ (TeV) / L (ab^{-1})	1.5 / 1.2	3 / 4.4	6 / 12	10 / 20	14 / 33	30 / 100
σ_{SM} (ab) $[N_{\mathrm{ev}}]$						
σ^{tot}	0.03 [0]	0.31 [1]	1.65 [20]	4.18 [84]	7.02 [232]	18.51 [1851]
$\sigma(M_{HHH} < 3$ TeV)	0.03 [0]	0.31 [1]	1.47 [18]	2.89 [58]	3.98 [131]	6.69 [669]
$\sigma(M_{HHH} < 1$ TeV)	0.02 [0]	0.12 [1]	0.26 [3]	0.37 [7]	0.45 [15]	0.64 [64]

Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]

A number of events

- At the 30 TeV muon collider, there are reasonable amount of a number of events to play with.

Triple Higgs Production at a Muon Collider

LO

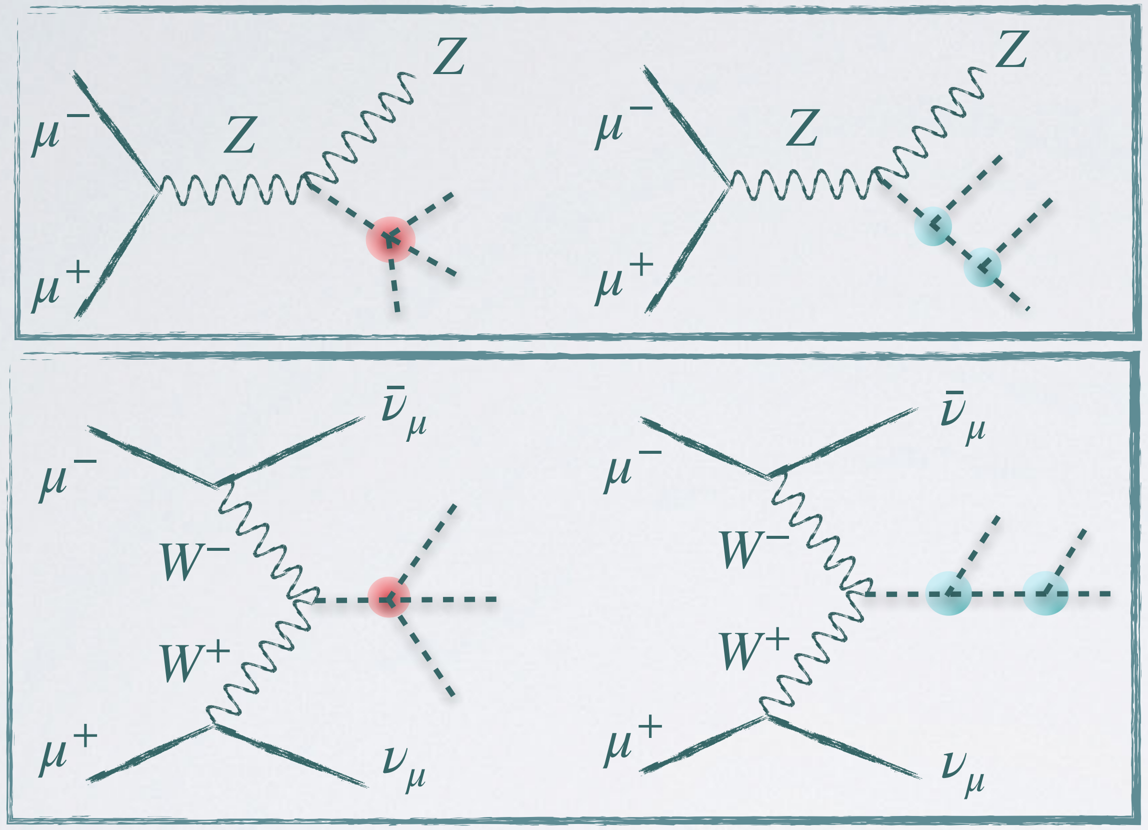
Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao [2020]

$Z(\to \nu\nu)hhh$:

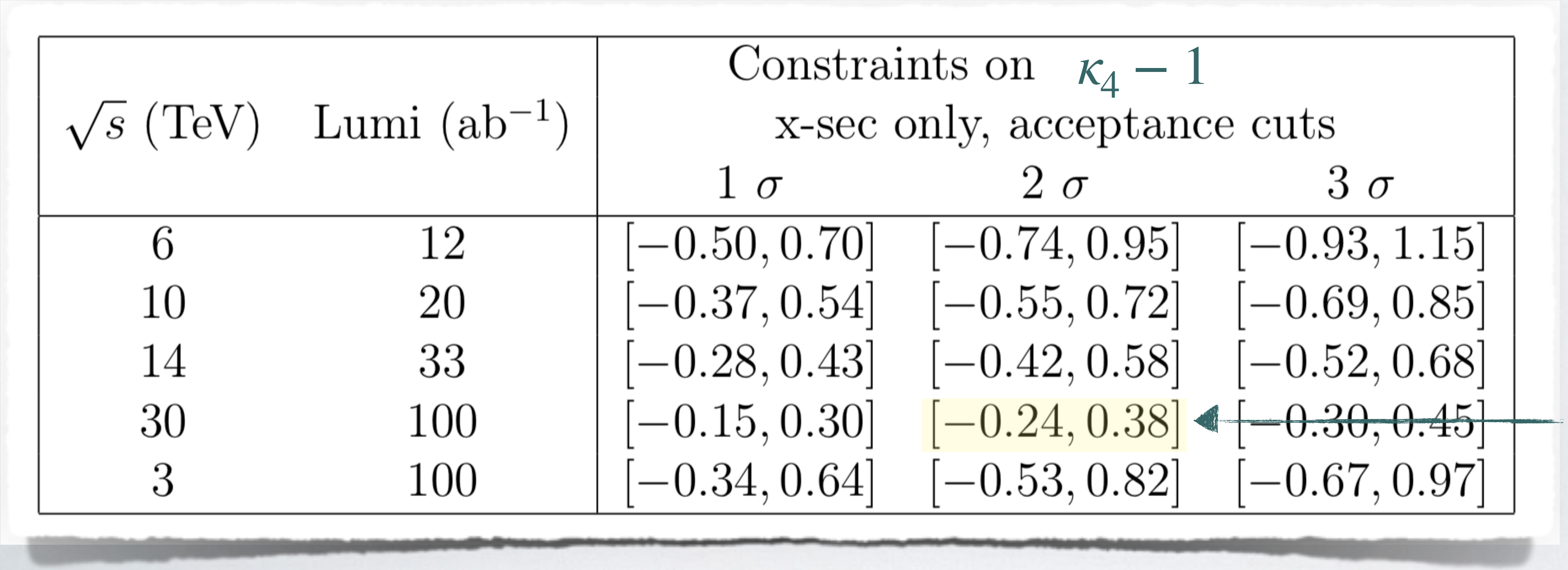
s-channel

$\nu\nu hhh$:

W-boson
fusion
(WBF)



- Assuming all Higgses decay into $b\bar{b}$.
- No $BR(h \to b\bar{b})$ is taken into account.
- Assuming a perfect b-tag.
- Only $(\nu\nu hhh)_{\kappa_3=1,\kappa_4=1}$ is taken to be a background.



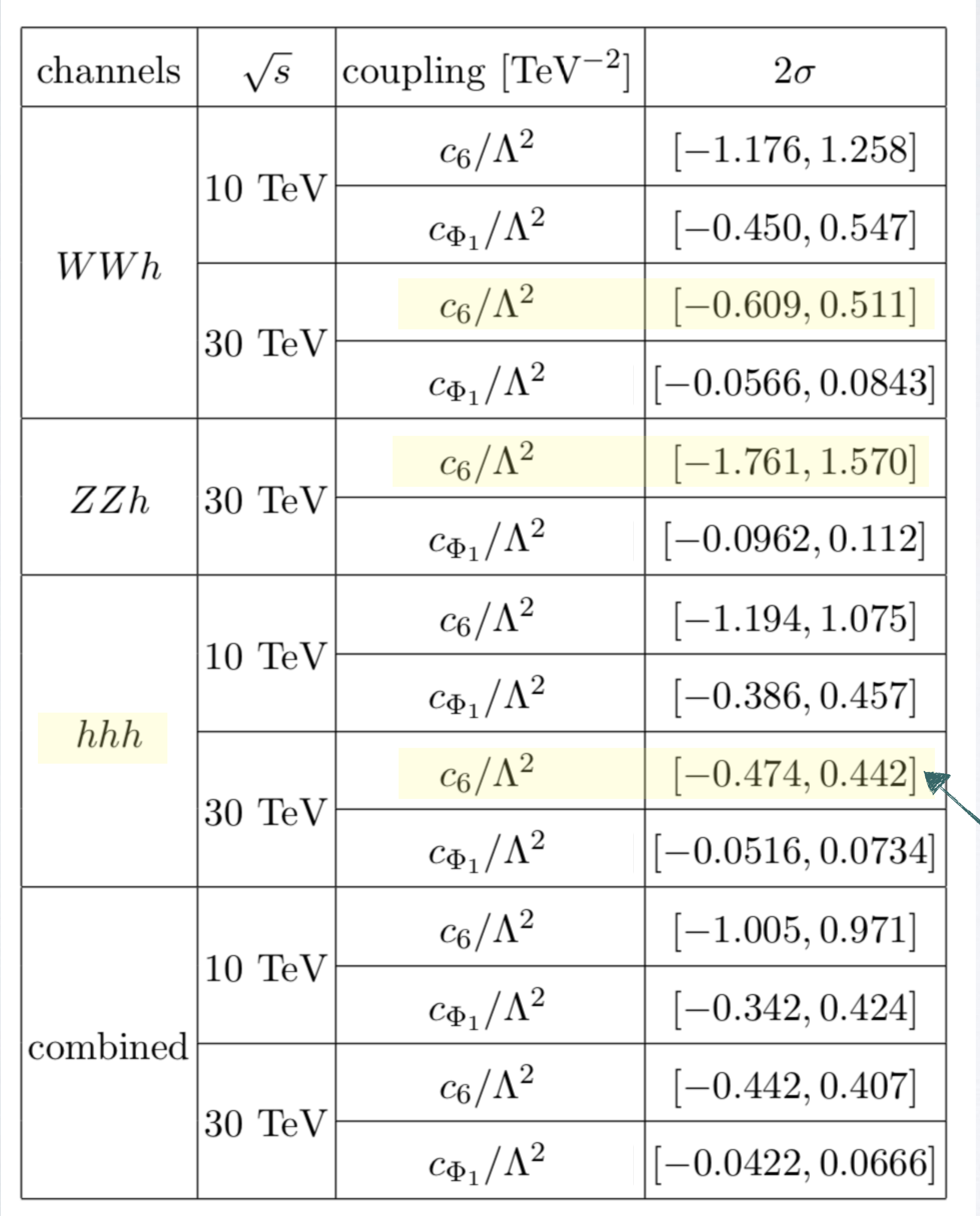
$\sqrt{s}$ (TeV)	Lumi (ab^{-1})	Constraints on $\kappa_4 - 1$ x-sec only, acceptance cuts		
		$1\ \sigma$	$2\ \sigma$	$3\ \sigma$
6	12	$[-0.50, 0.70]$	$[-0.74, 0.95]$	$[-0.93, 1.15]$
10	20	$[-0.37, 0.54]$	$[-0.55, 0.72]$	$[-0.69, 0.85]$
14	33	$[-0.28, 0.43]$	$[-0.42, 0.58]$	$[-0.52, 0.68]$
30	100	$[-0.15, 0.30]$	$[-0.24, 0.38]$	$[-0.30, 0.45]$
3	100	$[-0.34, 0.64]$	$[-0.53, 0.82]$	$[-0.67, 0.97]$

$0.76 < \kappa_4 < 1.38$

- The quartic coupling can be constrained with 40% precision.

Triple Higgs Production at a Muon Collider

Chen, Li, Lu, We, Yao[2021]



channels	$\sqrt{s}$	coupling [TeV^{-2}]	2σ
WWh	10 TeV	c_6/Λ^2	$[-1.176, 1.258]$
		c_{Φ_1}/Λ^2	$[-0.450, 0.547]$
	30 TeV	c_6/Λ^2	$[-0.609, 0.511]$
		c_{Φ_1}/Λ^2	$[-0.0566, 0.0843]$
ZZh	30 TeV	c_6/Λ^2	$[-1.761, 1.570]$
		c_{Φ_1}/Λ^2	$[-0.0962, 0.112]$
hhh	10 TeV	c_6/Λ^2	$[-1.194, 1.075]$
		c_{Φ_1}/Λ^2	$[-0.386, 0.457]$
	30 TeV	c_6/Λ^2	$[-0.474, 0.442]$
		c_{Φ_1}/Λ^2	$[-0.0516, 0.0734]$
combined	10 TeV	c_6/Λ^2	$[-1.005, 0.971]$
		c_{Φ_1}/Λ^2	$[-0.342, 0.424]$
	30 TeV	c_6/Λ^2	$[-0.442, 0.407]$
		c_{Φ_1}/Λ^2	$[-0.0422, 0.0666]$

- The same final state is analyzed by Chen et. al. including realistic backgrounds.
- With including a simple detector effect.
- The bounds are presented in the linear basis (difficult to compare apple-to-apple).

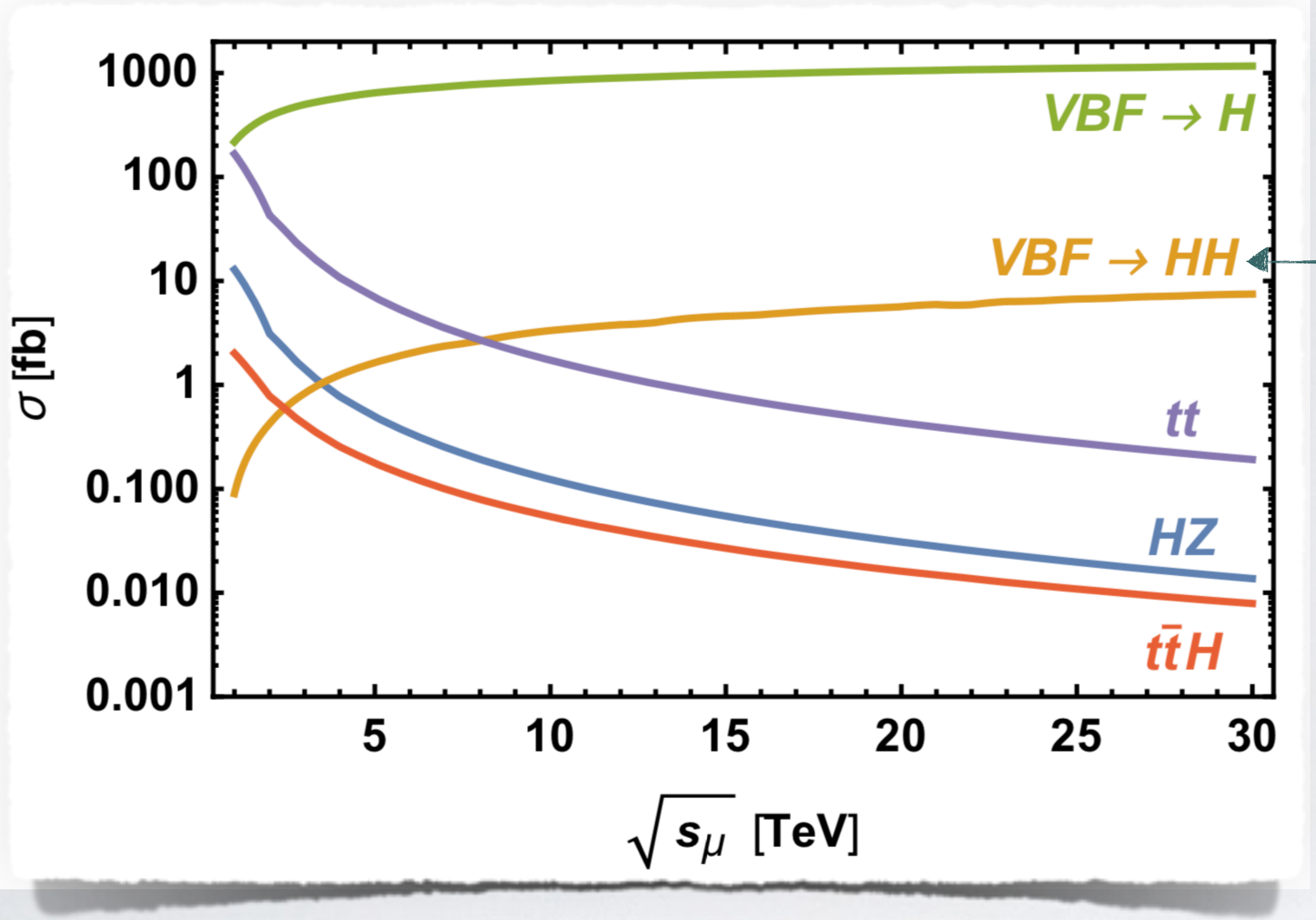
$$-0.56 < c_4 < 2.44$$

$$(c_4 = 6c_3 - 5 \text{ is assumed})$$

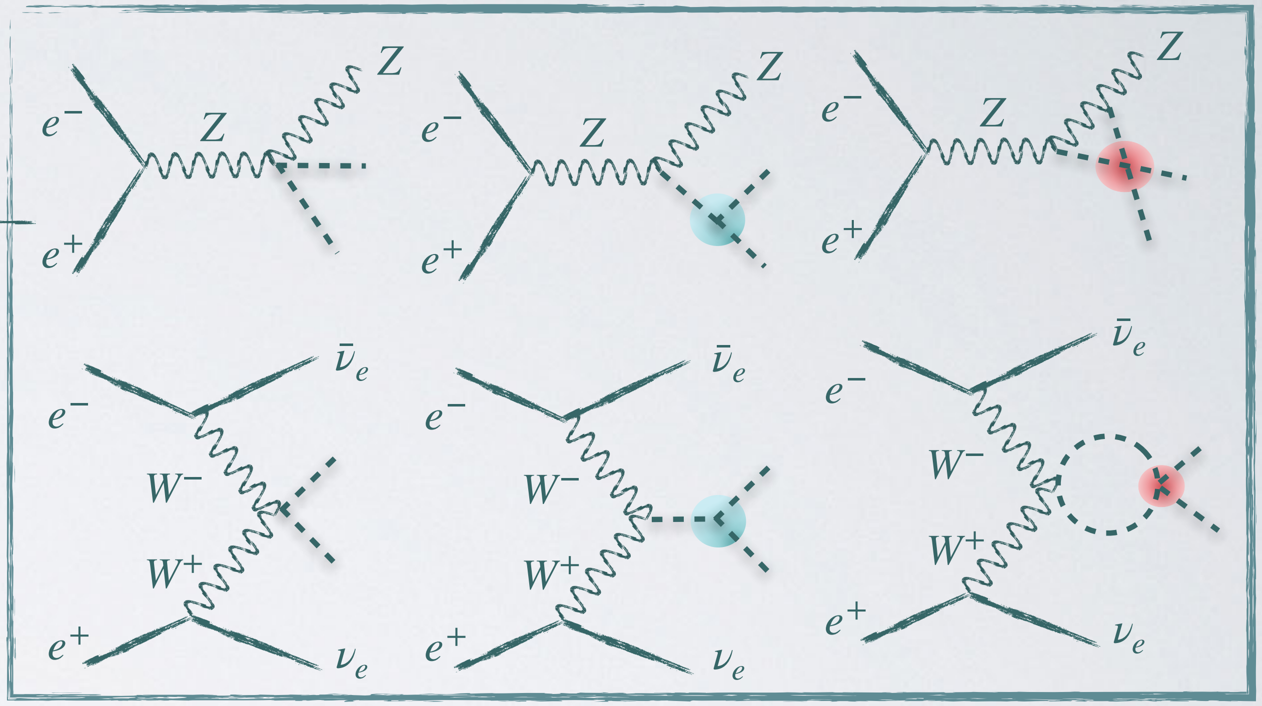
$$0.74 < c_3 < 1.24$$

Opportunities at a Muon Collider

Muon Collider Working Group [1901.06150]

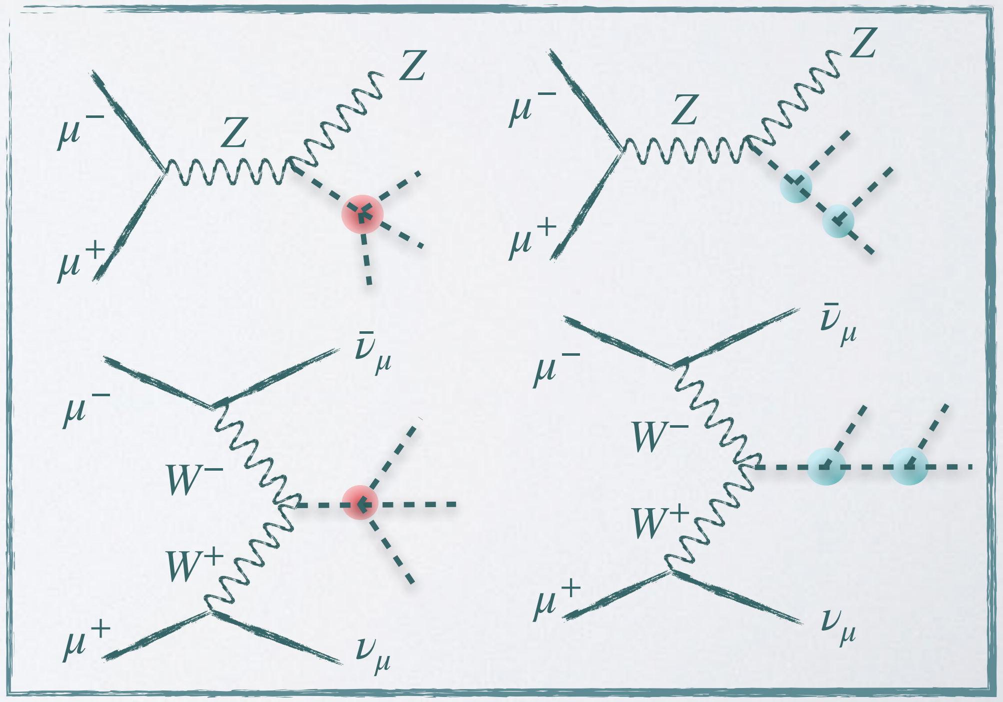


1. Quartic Coupling at Double Higgs



2. Combining Different Final States

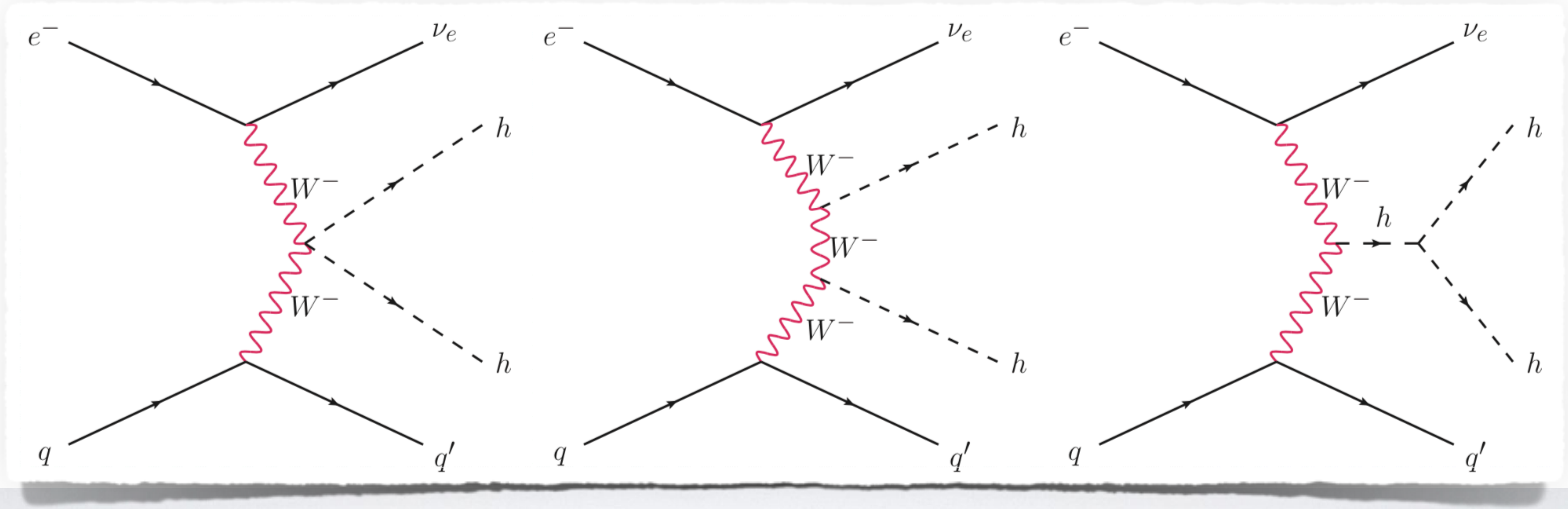
- $(b\bar{b})(b\bar{b})(\gamma\gamma)$ channel
- $(b\bar{b})(b\bar{b})(\tau\bar{\tau})$ channel
- …



Other Benchmark Future Colliders?

- A future *ep* collider with $E_e = 60$ GeV and $E_p = 50$ TeV?

Kumar, Ruan, Cornell, Islam,
Klein, Klein, Mellado [2015]



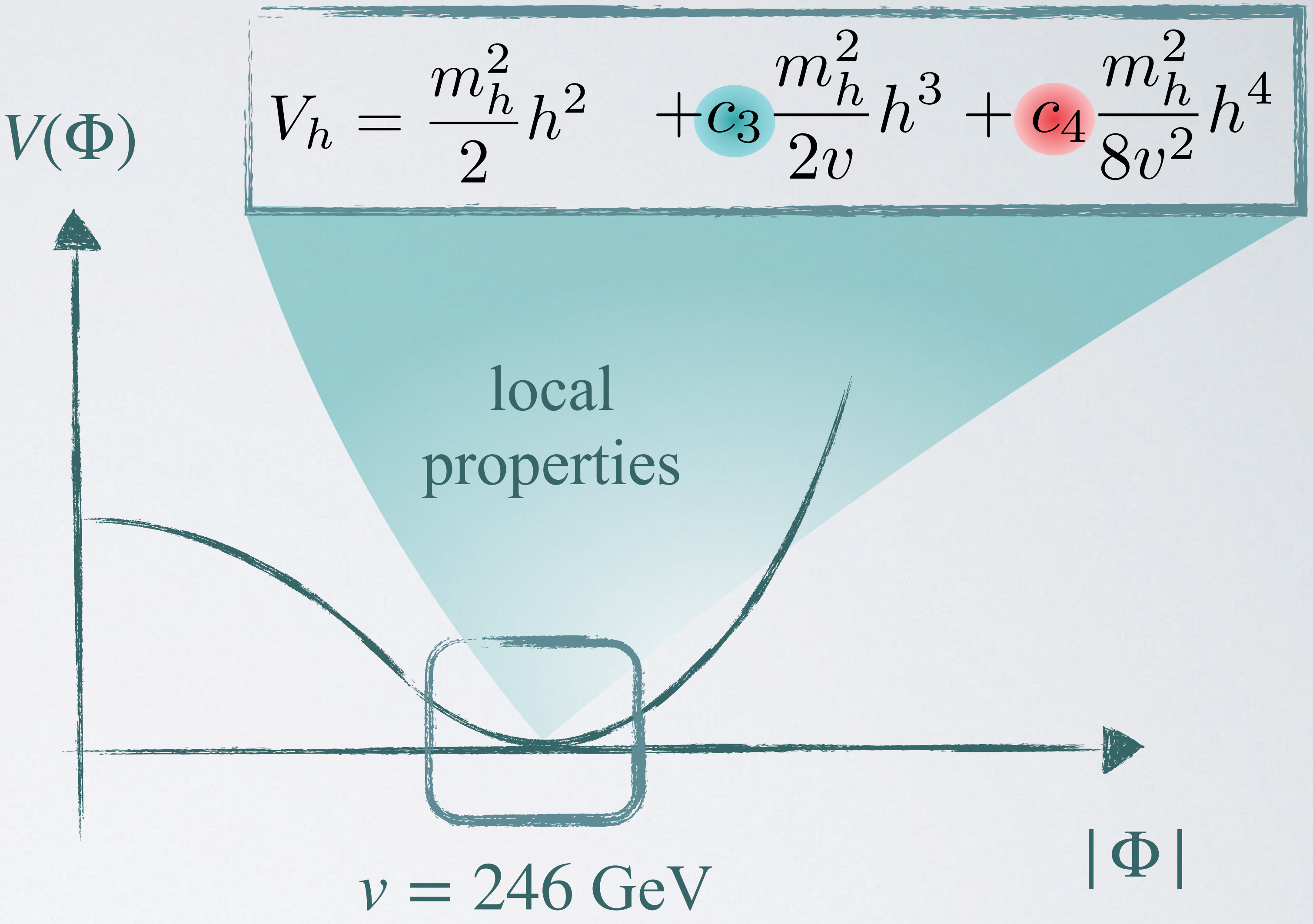
- For more about the *ep* collider, See the talk by Prof. Jeong Hyeon Song.

Summary

- Probing the Higgs self-couplings provides a crucial information on the EWSB.

- So far, no meaningful precision on the quartic coupling can be obtained at FCC-hh 100 TeV.

- Future e^-e^+ colliders have limited sensitivities on the quartic coupling.



- A future 30 TeV muon collider with 100 ab^{-1} luminosity may be able to probe the quartic coupling.

- Here a dedicated analysis on combining various final states makes a sense to try.

back-up