Constructing the covariant three-point vertices systematically

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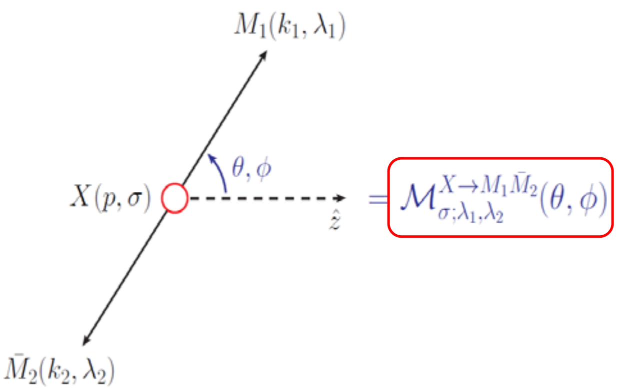
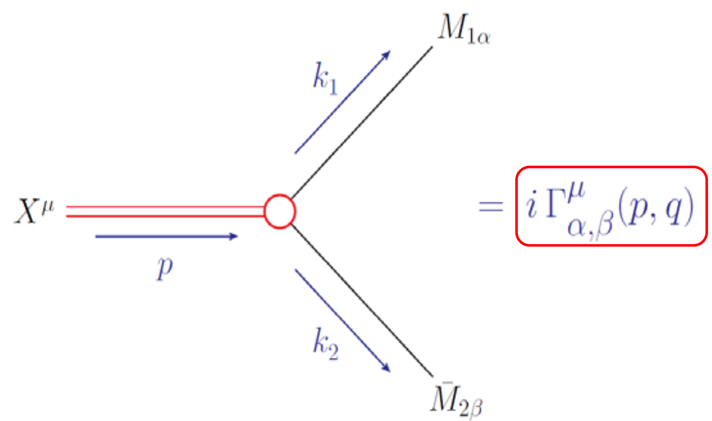
Helicity formalism



Covariant formulation



[SYC and Jae Hoon Jeong, PRD 103 (2021) 096013 & 104 (2021) 055046, arXiv: 2111.0836 (to appear in PRD)]

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Introduction

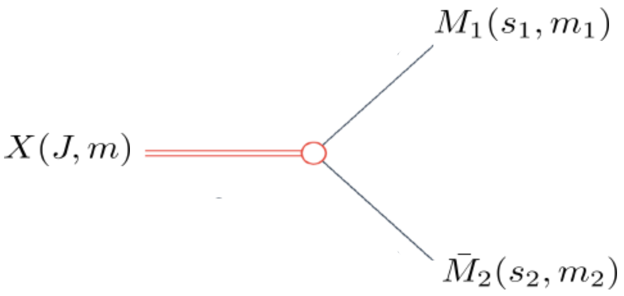
No BSM particles and phenomena have been found so far at the LHC
Many unsolved issues and problems : DM, DE, Matter asymmetry, …

One powerful strategy is to set up a platform for describing particles of any spin and their interactions generically

Composite high-spin particles in hadron physics
[PTEP (2020) 083C01]
Gravitational wave discovery → spin-2 massless graviton
[PRL (2016) 061102]
Spin-3/2 gravitino in supergravity
[PRL (1975) 177; JTEP Lett. 18 (1973) 312; PRD 13 (1976) 3214; PLB 62 (1976) 335]
Massive spin-2 KK gravitons
[PLB 436 (1998) 257; PLB 429 (1998) 263; PRL 83 (1999) 3370]
High-spin DM particles
[PRD 94 (2016) 084055; JCAP 09 (2016) 016; PRD 97 (2018) 024010; PRD 104 (2021) 063017]

Our Work

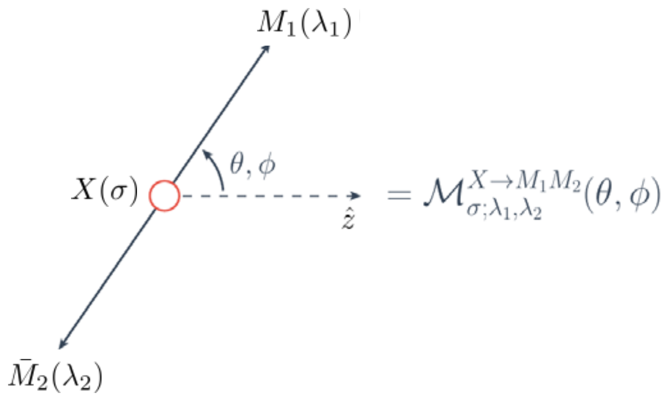
Develop an efficient algorithm for constructing all the effective covariant three-point vertices systematically



Utilize the equivalent helicity formalism and covariant formulation
Adopt the conventional integer and half-integer spin wave tensors

Identify all the basic bosonic and fermionic basic vertex operators
Assemble them for constructing the general three-point vertices

Characterization in the Helicity Formalism



X : spin J and mass m

M_1 : spin s_1 and mass m_1

$\bar{M}_2$: spin s_2 and mass m_2

[Jacob + Wick, Annals Phys. 7 (1959) 404]

$$\mathcal{M}^{X \to M_1 \bar{M}_2}_{\sigma;\lambda_1,\lambda_2}(\theta, \phi) \; = \; \mathcal{C}^J_{\lambda_1,\lambda_2} \, d^J_{\sigma,\, \lambda_1 - \lambda_2}(\theta)\, e^{i(\sigma - \lambda_1 + \lambda_2)\phi} \quad \text{with} \quad |\lambda_1 - \lambda_2| \leq J$$

Number of Independent Terms

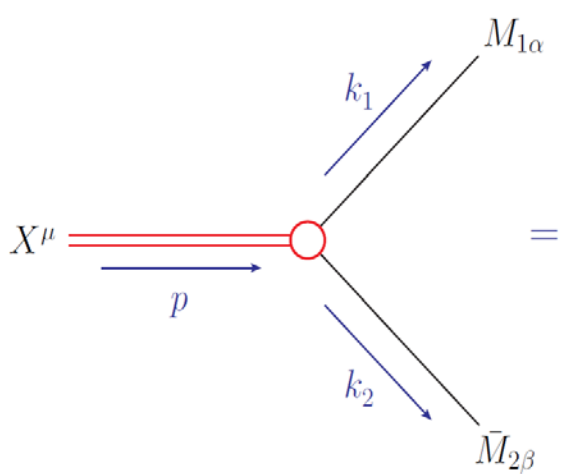
$$n\,[J, s_1, s_2] \; = \; \begin{cases} (2s_1+1)(2s_2+1) & \text{for} \quad J \geq s_1 + s_2 \\ (2s_1+1)(2s_2+1) & \text{for} \quad |s_1 - s_2| \leq J < s_1 + s_2 \\ -(s_1+s_2-J)(s_1+s_2-J+1) & \\ (2J+1)(s_1+s_2-|s_1-s_2|+1) & \text{for} \quad J < |s_1 - s_2| \end{cases}$$

Helicity Formalism ⇔ Covariant Formulation

$$\mathcal{M}^{X\to M_1\bar{M}_2}_{\sigma;\lambda_1,\lambda_2}(\theta,\phi) \;=\; \mathcal{C}^J_{\lambda_1,\lambda_2}\; d^J_{\sigma,\,\lambda_1-\lambda_2}(\theta)\, e^{i(\sigma-\lambda_1+\lambda_2)\phi} \quad \text{with} \quad |\lambda_1-\lambda_2|\le J$$

⇕

$$\mathcal{M}^{X\to M_1\bar{M}_2}_{\sigma;\lambda_1,\lambda_2} \;=\; \bar{\psi}_1^{\alpha_1\cdots\alpha_{n_1}}(k_1,\lambda_1)\;\Gamma^{\mu_1\cdots\mu_n}_{\alpha_1\cdots\alpha_{n_1},\beta_1\cdots\beta_{n_2}}(p,q)\;\psi_2^{\beta_1\cdots\beta_{n_2}}(k_2,\lambda_2)\;\psi_{\mu_1\cdots\mu_n}(p,\sigma)$$



$$= \; i\,\Gamma^{\mu}_{\alpha,\beta}(p,q)$$

$$p = k_1 + k_2$$

$$q = k_1 - k_2$$

Bosonic and Fermionic Wave Tensors

Integer J = n

$$\psi^{\mu_1\cdots\mu_n}(p,\sigma) = \epsilon^{\mu_1\cdots\mu_n}(p,\sigma) = \sqrt{\frac{2^n\,(n+\sigma)!\,(n-\sigma)!}{(2n)!}} \sum_{\{\tau\}=-1}^{1} \delta_{\tau_1+\cdots+\tau_n,\,\sigma} \prod_{i=1}^{n} \frac{\epsilon^{\mu_i}(p,\tau_i)}{\sqrt{2}^{|\tau_i|}}$$

$$\varepsilon_{\alpha\beta\mu_i\mu_j}\,\epsilon^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_n}(p,\sigma) = 0$$ totally symmetric

$$g_{\mu_i\mu_j}\,\epsilon^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_n}(p,\sigma) = 0$$ traceless

$$p_{\mu_i}\,\epsilon^{\mu_1\cdots\mu_i\cdots\mu_n}(p,\sigma) = 0$$ divergence-free

Half-integer J = n + 1/2

$$\psi^{\mu}(p,\sigma) = u^{\mu_1\cdots\mu_n}(p,\sigma) = \sum_{\tau=\pm 1/2} \sqrt{\frac{J+2\tau\sigma}{2J}}\,\epsilon^{\mu_1\cdots\mu_n}(p,\sigma-\tau)\,u(p,\tau)$$

$$\psi^{\mu}(p,\sigma) = v^{\mu_1\cdots\mu_n}(p,\sigma) = \sum_{\tau=\pm 1/2} \sqrt{\frac{J+2\tau\sigma}{2J}}\,\epsilon^{*\mu_1\cdots\mu_n}(p,\sigma-\tau)\,v(p,\sigma)$$

$$\gamma_{\mu_i} u^{\mu_1\cdots\mu_i\cdots\mu_n}(p,\sigma) = \gamma_{\mu_i} v^{\mu_1\cdots\mu_i\cdots\mu_n}(p,\sigma) = 0$$

The master key to constructing the covariant three-point vertices is to find every helicity-specific covariant operator generating a reduced helicity amplitude nonzero only for each specific helicity combination.

Kinematics in the X rest frame (XRF)

$$p = m(1, \vec{0})$$

$$q = m(\omega_1^2 - \omega_2^2, \kappa\hat{n})$$

$$\omega_{1,2} = m_{1,2}/m$$

$$\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\kappa = \eta^+\eta^- \quad \text{with} \quad \eta^\pm = \sqrt{1-(\omega_1 \pm \omega_2)^2}$$

$$\hat{p} = p/m = (1, \vec{0})$$

$$\hat{k} = \frac{q - (\omega_1^2 - \omega_2^2)p}{m\kappa} = (0, \hat{n})$$

$$\hat{p}_{1,2} = 2\,\omega_{1,2}\,\hat{p}$$

Basic bosonic operators : 1 → 1 + 1

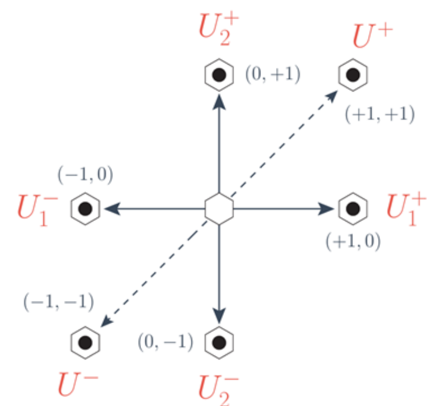
$$U^0_{\alpha\beta}\,\hat{k}_\mu = \hat{p}_{1\alpha}\hat{p}_{2\beta}\,\hat{k}_\mu \quad \leftrightarrow \quad \mathcal{C}^1_{0,0} = +\kappa^2$$

$$U^\pm_{1\alpha\mu}\,\hat{p}_{2\beta} = \frac{1}{2}\left[g_{\perp\alpha\mu} \pm i\langle\alpha\mu\hat{p}\hat{k}\rangle\right]\hat{p}_{2\beta} \quad \leftrightarrow \quad \mathcal{C}^1_{\pm1,0} = +\kappa$$

$$U^\pm_{2\beta\mu}\,\hat{p}_{1\alpha} = \frac{1}{2}\left[g_{\perp\beta\mu} \mp i\langle\beta\mu\hat{p}\hat{k}\rangle\right]\hat{p}_{1\alpha} \quad \leftrightarrow \quad \mathcal{C}^1_{0,\pm1} = -\kappa$$

$$U^\pm_{\alpha\beta}\,\hat{k}_\mu \equiv g^{\mu_1\mu_2}U_{1\alpha\mu_1}U_{2\beta\mu_2}\,\hat{k}_\mu$$

$$= \frac{1}{2}\left[g_{\perp\alpha\beta} \pm i\langle\alpha\beta\hat{p}\hat{k}\rangle\right]\hat{k}_\mu \quad \leftrightarrow \quad \mathcal{C}^1_{\pm1,\pm1} = -1$$

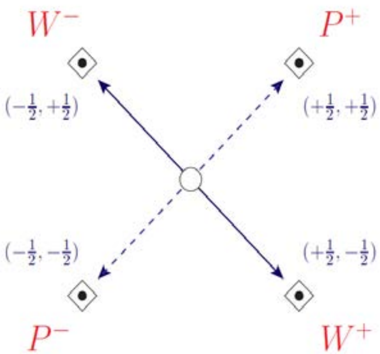


[JW convention]

$$g_{\perp\mu\nu} = g_{\mu\nu} - \hat{p}_\mu\hat{p}_\nu + \hat{k}_\mu\hat{k}_\nu \quad \text{and} \quad \langle\mu\nu\hat{p}\hat{k}\rangle = \varepsilon_{\mu\nu\rho\sigma}\hat{p}^\rho\hat{k}^\sigma$$

Basic fermionic operators

$1 \to 1/2 + 1/2$

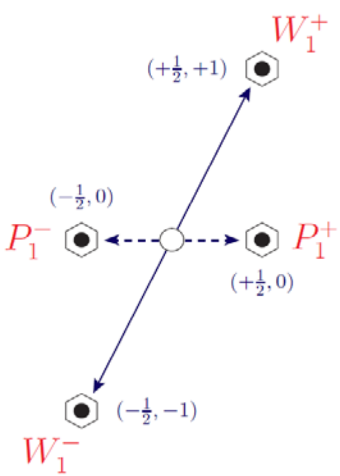


$$P^{\pm}\,\hat{k}_\mu = \frac{1}{2m}(\eta^- \mp \eta^+\gamma_5)\,\hat{k}_\mu \qquad \leftrightarrow \qquad \mathcal{C}^{1}_{\pm 1/2, \pm 1/2} = -\kappa$$

$$W^{\pm}_\mu = \frac{1}{2\sqrt{2}m}\,(\eta^+\gamma^+_\mu \pm \eta^-\gamma^-_\mu\gamma_5) \qquad \leftrightarrow \qquad \mathcal{C}^{1}_{\pm 1/2, \mp 1/2} = +\kappa$$

$$\eta^{\pm} = \sqrt{1-(\omega_1 \pm \omega_2)^2} \quad \text{and} \quad \gamma^{\pm}_\mu = \gamma_\mu + \frac{(\omega_1 \pm \omega_2)\,\kappa}{1-(\omega_1 \pm \omega_2)^2}\,\hat{k}_\mu$$

$1/2 \to 1/2 + 1$



$$P_1^{\pm}\,\hat{p}_{2\beta} = \frac{1}{2m}\,(\eta_1^- \mp \eta_1^+\gamma_5)\,\hat{p}_{2\beta} \qquad \leftrightarrow \qquad \mathcal{C}^{1/2}_{\pm 1/2, 0} = -\kappa^2$$

$$W^{\pm}_{1\beta} = \frac{1}{2\sqrt{2}m}\,(\eta_1^+\gamma^+_{1\beta} \pm \eta_1^-\gamma^-_{1\beta}\gamma_5) \qquad \leftrightarrow \qquad \mathcal{C}^{1/2}_{\pm 1/2, \pm 1} = -\kappa$$

$$\eta_1^{\pm} = \sqrt{(1 \pm \omega_1)^2 - \omega_2^2} \quad \text{and} \quad \gamma^{\pm}_{1\beta} = \gamma_\beta \mp \frac{2(1 \pm \omega_1)}{(1 \pm \omega_1)^2 - \omega_2^2}\,\hat{p}_\beta$$

Weaving the Covariant Three-point Vertices

$$
\begin{aligned}
[\,\hat{k}\,]^n \quad &\rightarrow \quad (\hat{k}^n)_{\mu_1\cdots\mu_n} = \hat{k}_{\mu_1}\cdots\hat{k}_{\mu_n} \\
[\,\hat{p}_1\,]^n \quad &\rightarrow \quad (\hat{p}_1^n)_{\alpha_1\cdots\alpha_n} = \hat{p}_{1\alpha_1}\cdots\hat{p}_{1\alpha_n} \\
[\,\hat{p}_2\,]^n \quad &\rightarrow \quad (\hat{p}_2^n)_{\beta_1\cdots\beta_n} = \hat{p}_{2\beta_1}\cdots\hat{p}_{2\beta_n} \\
[U_1^\pm]^n \quad &\rightarrow \quad (U_1^\pm)^n_{\alpha_1\cdots\alpha_n\mu_1\cdots\mu_n} = U^\pm_{1\alpha_1\mu_1}\cdots U^\pm_{1\alpha_n\mu_n} \\
[U_2^\pm]^n \quad &\rightarrow \quad (U_2^\pm)^n_{\beta_1\cdots\beta_n\mu_1\cdots\mu_n} = U^\pm_{2\beta_1\mu_1}\cdots U^\pm_{2\beta_n\mu_n} \\
[U^\pm]^n \quad &\rightarrow \quad (U^\pm)^n_{\alpha_1\cdots\alpha_n\beta_1\cdots\beta_n} = U^\pm_{\alpha_1\beta_1}\cdots U^\pm_{\alpha_n\beta_n}
\end{aligned}
$$

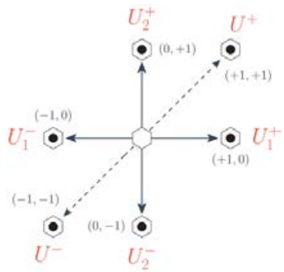
Helicity-specific Operators

$A = iii,\ ihh,\ hhi$

$$
[\,\mathcal{H}^{J,s_1,s_2}_{A[\lambda_1,\lambda_2]}\,] = [\,\hat{k}\,]^{J-|\lambda_1-\lambda_2|}\,[\,\hat{p}_1\,]^{s_1-|\lambda_1|}\,[\,\hat{p}_2\,]^{s_2-|\lambda_2|}\,[\,\mathcal{T}^{J,s_1,s_2}_{A[\lambda_1,\lambda_2]}\,]
$$

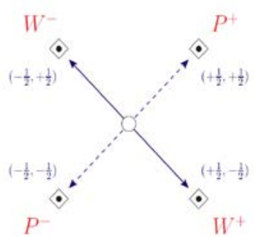
The helicity-specific operators can be applied even to the $m_{1,2} = 0$ case.
Only the maximal helicities identical to the spin in size are allowed
for a massless particle, leading to $[\hat{p}_{1,2}]^{s_{1.2}-|\lambda_{1,2}|} = 1$ for $m_{1,2} = 0$.

Integer Integer Integer (iii)



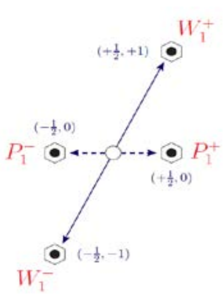
$$[\mathcal{T}^{J,s_1,s_2}_{iii[\lambda_1,\lambda_2]}] = \begin{cases} [U^{\pm}]^{|\lambda_2|}\,[U_1^{\pm}]^{|\lambda_1-\lambda_2|} & \text{for } \lambda_{1,2}=\pm|\lambda_{1,2}| \text{ and } 0<|\lambda_2|\le|\lambda_1| \\ [U^{\pm}]^{|\lambda_1|}\,[U_2^{\pm}]^{|\lambda_1-\lambda_2|} & \text{for } \lambda_{1,2}=\pm|\lambda_{1,2}| \text{ and } 0<|\lambda_1|<|\lambda_2| \\ [U_1^{\pm}]^{|\lambda_1|}\,[U_2^{\mp}]^{|\lambda_2|} & \text{for } \lambda_1=\pm|\lambda_1| \text{ and } \lambda_2=\mp|\lambda_2| \end{cases}$$

Integer Half-integer Half-integer (ihh)



$$[\mathcal{T}^{J,s_1,s_2}_{ihh[\lambda_1,\lambda_2]}] = \begin{cases} [P^{\pm}]\,[U^{\pm}]^{|\lambda_2|-1/2}\,[U_1^{\pm}]^{|\lambda_1-\lambda_2|} & \text{for } \lambda_{1,2}=\pm|\lambda_{1,2}| \text{ and } |\lambda_2|\le|\lambda_1| \\ [P^{\pm}]\,[U^{\pm}]^{|\lambda_1|-1/2}\,[U_2^{\pm}]^{|\lambda_1-\lambda_2|} & \text{for } \lambda_{1,2}=\pm|\lambda_{1,2}| \text{ and } |\lambda_1|<|\lambda_2| \\ [W^{\pm}]\,[U_1^{\pm}]^{|\lambda_1|-1/2}\,[U_2^{\mp}]^{|\lambda_2|-1/2} & \text{for } \lambda_1=\pm|\lambda_1| \text{ and } \lambda_2=\mp|\lambda_2| \end{cases}$$

Half-integer Half-integer Integer (hhi)



$$[\mathcal{T}^{J,s_1,s_2}_{hhi[\lambda_1,\lambda_2]}] = \begin{cases} [P_1^{\pm}]\,[U^{\pm}]^{|\lambda_2|}\,[U_1^{\pm}]^{|\lambda_1-\lambda_2|-1/2} & \text{for } \lambda_{1,2}=\pm|\lambda_{1,2}| \text{ and } 1<|\lambda_2|<|\lambda_1| \\ [W_1^{\pm}]\,[U^{\pm}]^{|\lambda_1|-1/2}\,[U_2^{\pm}]^{|\lambda_1-\lambda_2|-1/2} & \text{for } \lambda_{1,2}=\pm|\lambda_{1,2}| \text{ and } |\lambda_1|<|\lambda_2| \\ [P_1^{\pm}]\,[U_1^{\pm}]^{|\lambda_1|-1/2}\,[U_2^{\mp}]^{|\lambda_2|} & \text{for } \lambda_1=\pm|\lambda_1| \text{ and } \lambda_2=\mp|\lambda_2| \end{cases}$$

Half-integer Integer Half-integer (hih)

$$(\mathcal{H}^{J,s_2,s_1}_{hih[\lambda_2,\lambda_1]})^{\mu_1\cdots\mu_n}_{\alpha_1\cdots\alpha_{n_2},\beta_1\cdots\beta_{n_1}}(p,q) = (\mathcal{H}^{J,s_1,s_2}_{hhi[\lambda_1,\lambda_2]})^{\mu_1\cdots\mu_n}_{\beta_1\cdots\beta_{n_1},\alpha_1\cdots\alpha_{n_2}}(p,-q)$$

General form of covariant three-point vertices

$$[\Gamma_A] = \sum_{\lambda_1=-s_1}^{s_1}\sum_{\lambda_2=-s_2}^{s_2} c^{J,s_1,s_2}_{A[\lambda_1,\lambda_2]}[\mathcal{H}^{J,s_1,s_2}_{A[\lambda_1,\lambda_2]}] \quad \text{with} \quad A = iii,\, ihh,\, hhi,\, hih$$

Charge-conjugated process $\bar{X} \rightarrow \bar{M}_1 M_2$

$$[\bar{\mathcal{H}}^{J,s_1,s_2}_{iii[\lambda_1,\lambda_2]}] = [\mathcal{H}^{J,s_1,s_2}_{iii[\lambda_1,\lambda_2]}]$$

$$[\bar{\mathcal{H}}^{J,s_1,s_2}_{ihh[\lambda_1,\lambda_2]}] = -[C\,\mathcal{H}^{J,s_1,s_2}_{ihh[\lambda_1,\lambda_2]}\,C^{-1}]$$

$$[\bar{\mathcal{H}}^{J,s_1,s_2}_{hhi[\lambda_1,\lambda_2]}] = -[C\,\mathcal{H}^{J,s_1,s_2}_{hhi[\lambda_1,\lambda_2]}\,C^{-1}],$$

Conclusions

We have developed an efficient algorithm for weaving the effective covariant three-point vertices compactly.

This general algorithm is useful in studying various theoretical and phenomenological aspects systematically.

Local gauge invariance or Bose/Fermi symmetries can be imposed straightforwardly.

Can the bosonic and fermion cases be synthesized in a more compact way?

Extendable to 4-point contact terms?

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