Color singlet decays at NNLO in MCFM

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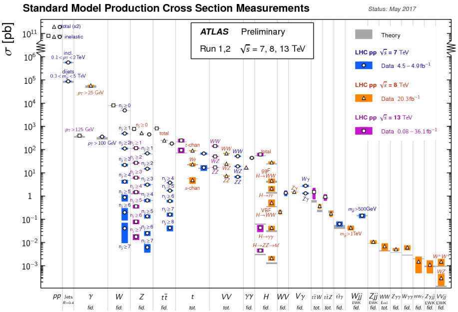
LHC physics in the precision era



So far no **direct** hints for BSM

- Deviations from SM small and featureless?
- Percent-level theory predictions ⇒ N^{x}LO precision

NNLO parton-level MCs crucial for data analysis and phenomenology!

- (Subtracted) real emission corrections typically cause biggest numerical convergence problems
- *NLO revolution* aided by completely generic implementations of universal subtraction schemes [[Catani, Seymour '96; Frixione, Kunszt, Signer '96-'97]]
- Despite many dedicated calculations, no such implementation available yet at NNLO
- **This talk**: First steps towards including nested soft-collinear subtraction in MCFM

Methods for NNLO calculations

General aim: Extract singularities analytically, compute remainder with MC

1. **Slicing:**
   - Find observable sensitive to IR real radiation, r_{cut}
   - Slice phase space into *resolved* and *unresolved* parts:

$$\int |\mathcal{M}|^2 \mathcal{F} d\phi_d = \int_0^{r_{\rm cut}} \left(|\mathcal{M}|^2 \mathcal{F} d\phi_d\right)_{C,S} + \int_{r_{\rm cut}}^1 |\mathcal{M}|^2 \mathcal{F} d\phi_4 + \mathcal{O}(r_{cut})$$

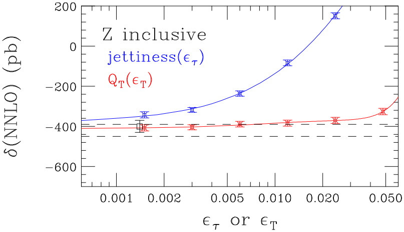
   - q_T [Catani,Grazzini '07] or N-jettiness [Gaunt et al '15; Boughezal et al '15]
   - Implemented at NNLO in MCFM (N-jettiness, q_T) and MATRIX (q_T). [Boughezal et al '17] [Kallweit,Grazzini,Wiesemann, '17]
   - **Pros**: straightforward to implement, recycles existing NLO calculations
   - **Cons**: slow convergence, large power corrections.
2. **Subtraction:**
   - Find set of subtraction terms, $\mathcal{D}$, which reproduce $|\mathcal{M}|^2$ in unresolved limits,

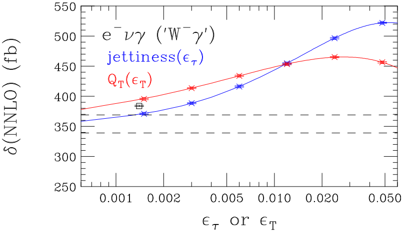
$$\int |\mathcal{M}|^2 \mathcal{F} d\phi_d = \int \left(|\mathcal{M}|^2 \mathcal{F} - \mathcal{D}\right) d\phi_4 + \int \mathcal{D} d\phi_4$$

   - $\mathcal{D}$ difficult to identify as it must be simple and integrable over unresolved PS.
   - Integrating counter-term non-trivial if singularities are overlapping.
   - *NLO fully solved* [Catani,Seymour '96], [Frixione,Kunszt,Signer '96].

Numerical performance of slicing techniques

- Comparative study with jettiness and q_T slicing method [Campbell,Ellis,Seth '22]





- Rough estimate of computing power required:

$\epsilon_T = q_T/M_X$	0.0015	0.003	0.006	0.012	0.024	0.048
t(CPU days)	200	100	50	15	3	1

Fermilab

Subtraction methods

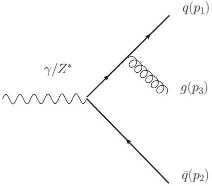
Commonly used techniques:

- Antenna subtraction [Gehrmann-DeRidder,Gehrmann,Glover ’05, ...]
- Stripper [Czakon ‘10, ...]
- ColorfulNNLO [delDuca,Somogyi,Trócsányi, ’05, ...]
- Geometric [Herzog ‘18]
- Local analytic sector [Magnea,Maina,Pellicioli,Signorile,Torrielli ‘18]
- Nested soft collinear subtraction [Czakon,Heymes ’14] [Caola,Melnikov,Röntsch ‘17, ...]
- Projection to Born [Cacciari et al ‘15, ...]

Why use the nested soft collinear scheme?

- Physically transparent (collinear C / soft (S) singularities made explicit)
- Modular (singularities extracted using partial fractioning & sectorization)
- Analytic (integrals obtained by Melnikov et al)
- Numerically efficient

Toy example: Color singlet decay at NLO



Notation: At NLO consider $Z/\gamma^* \to q(p_1)\bar{q}(p_2)g(p_3)$,

Real-emission cross-section:

$$d\sigma^{\mathrm{R}} = \frac{1}{2s}\int [dg_3]\mathcal{F}(1,2,3) \quad \text{and} \quad \int [dg_3] = \int \frac{d^{d-1}p_3}{(2\pi)^d 2E_3}\theta(E_{\max} - E_3)$$

$E_{\max} \leftrightarrow$ arbitrary parameter chosen large enough to accommodate all possible kinematic configurations

$$\mathcal{F}(1,2,3) = \mathrm{dLips}_V |\mathcal{M}(1,2,3)|^2 \mathcal{F}_{\mathrm{kin}}(1,2,3)$$

- dLips_V is Lorentz-invariant phase space for colorless particles including $\delta^{(d)}(p_1 + p_2 + p_3)$.
- $\mathcal{F}_{\mathrm{kin}}(1,2,3)$ is an *IR safe* observable that depends on kinematic variables of all particles in the process.

Toy example: Color singlet decay at NLO

The construction of subtraction terms:

$$\int [dg_3]\mathcal{F}(1,2,3) = \langle S_3\mathcal{F}(1,2,3)\rangle + \langle (C_{12}+C_{13})(1-S_3)\mathcal{F}(1,2,3)\rangle$$
$$+ \langle \hat{O}_{\rm NLO}\mathcal{F}(1,2,3)\rangle$$

with $\hat{O}_{\rm NLO} = (I - C_{31} - C_{32})(I - S_3)$ finite.

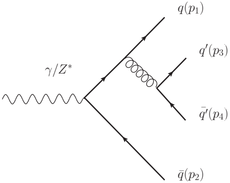
Limit operators at NLO ($2\eta_{ij} := 1 - \cos\theta_{ij}$):

- Soft: $S_i\mathcal{F} = \lim_{E_i\to 0}\mathcal{F} \quad \to \quad \propto \frac{1}{E_i^2}\sum_{j,k\neq i}\langle\mathcal{M}|\mathbf{T}_j\mathbf{T}_k|\mathcal{M}\rangle\frac{\eta_{jk}}{\eta_{ji}\eta_{ik}}$
- Collinear: $C_{ij}\mathcal{F} = \lim_{\eta_{ij}\to 0}\mathcal{F} \quad \to \quad \propto \frac{1}{s_{ij}}|\mathcal{M}|^2 \otimes P_{(ij)\to i}(z)$

N.B. Subtracted terms indeed *Analytic* e.g. soft term

$$\langle S_3\mathcal{F}(1,2,3)\rangle = -2^{1-2\varepsilon}\frac{[\alpha_s]}{\varepsilon}\left[\frac{(4\pi)^\varepsilon}{8\pi^2}\right]\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\,{}_2F_1(1,1,;,1-\varepsilon,1-\eta_{12})\eta_{12}$$

First non-trivial case: Color singlet decay at NNLO



Singularity structure of $Z/\gamma^* \to q(p_1)\bar{q}(p_2)q'(p_3)\bar{q}'(p_4)$
[Gehrmann,Gehrmann-DeRidder,Glover '03]

$$\int [\mathrm{d}g_3][\mathrm{d}g_4]|\mathcal{M}|^2 = -\frac{1}{12\varepsilon^3} - \frac{7}{18\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{11\pi^2}{72} - \frac{407}{216}\right) + \ldots$$

Singular limits: [Campbell,Glover '98], [Catani,Grazzini '99]

- Both $q'_{3,4}$ soft: $\int \mathbb{S} = \int [\mathrm{d}g_3][\mathrm{d}g_4] \sum_{i,j=1,2} \mathcal{I}_{ij}(3,4)$
- Double collinear $q'_{3,4}$: $\int C_{34} = \int [\mathrm{d}g_3][\mathrm{d}g_4] P_{q_3 q'_4}$
- Triple collinear $q'_{3,4}, q_i$: $\int \mathbb{C}_i = \int [\mathrm{d}g_3][\mathrm{d}g_4] P_{\bar{q}'_3 q'_4 q_i}$
- Iterated double collinear: $q'_{3,4}, q_i$: $\int C_{34}\mathbb{C}_i = \int [\mathrm{d}g_3][\mathrm{d}g_4] P_{qg} \otimes P_{gq'}$
- iterated soft-collinear ($\mathbb{S}C_{34}, \mathbb{S}\mathbb{C}_i, \mathbb{S}\mathbb{C}_i C_{34}$)

Finite remainder: Subtracted $Z/\gamma^* \to q(p_1)\bar{q}(p_2)q'(p_3)\bar{q}'(p_4)$ contribution

$$\mathrm{d}\Gamma^{\mathrm{NNLO}}_{Z/\gamma^* \to q\bar{q}q'\bar{q}'} = \sum_{i=1,2} \Big\langle [I - C_{34}]\,[I - \mathbb{C}_i]\,[I - \mathbb{S}]\,[\mathrm{d}g_3][\mathrm{d}g_4] \mathcal{F}(1_q, 2_{\bar{q}}, 3_{q'}, 4_{\bar{q}'}) \Big\rangle$$

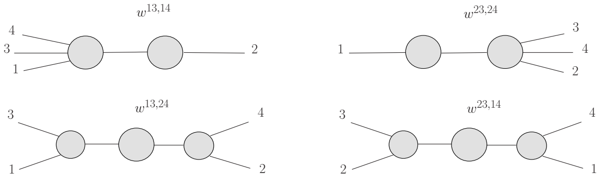
Color singlet decay (NNLO)

What remains?

- Triple collinear limits are separated using phase-space partition functions:

$$1 = w^{13,14} + w^{23,24} + w^{13,24} + w^{14,23}$$

- $w^{13,24}, w^{14,23}$ and $w^{13,14}, w^{23,24}$ project onto the relevant double and triple collinear splittings.

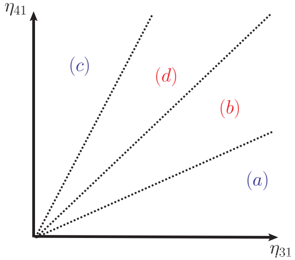


- The partitions needed to cancel all singularities which do not belong in desired sector.

Color singlet decay (NNLO)

Sectors: angular decomposition to separate singular regions:

$$1 = \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}$$
$$= \theta(\eta_{41} < \frac{\eta_{31}}{2}) + \theta(\frac{\eta_{31}}{2} < \eta_{41} < \eta_{31})$$
$$+ \theta(\eta_{31} < \frac{\eta_{41}}{2}) + \theta(\frac{\eta_{41}}{2} < \eta_{31} < \eta_{41})$$



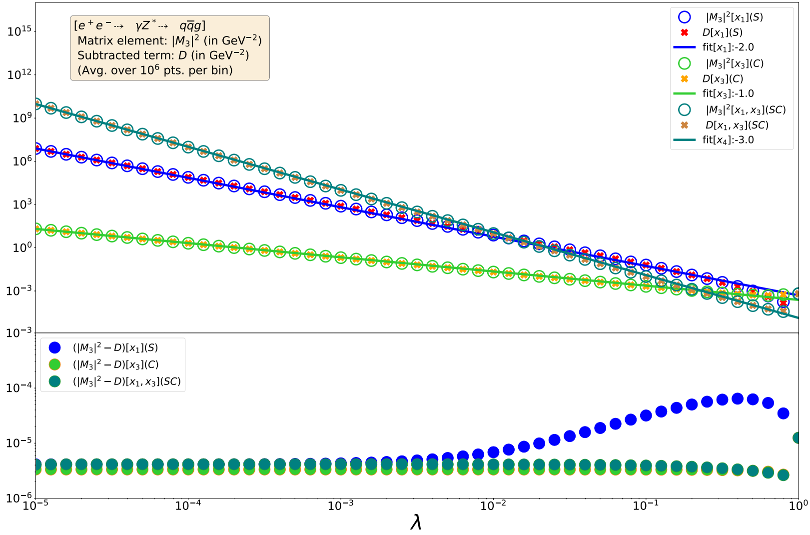
Sector-specific phase-space mapping [Czakon '10]

Corresponding limits: $\theta^{(a)} \leftrightarrow C_{41}$, $\theta^{(b)} \leftrightarrow C_{34}$, $\theta^{(c)} \leftrightarrow C_{31}$, $\theta^{(d)} \leftrightarrow C_{34}$

Fully-regulated double-real emission contribution in 4 dimensions

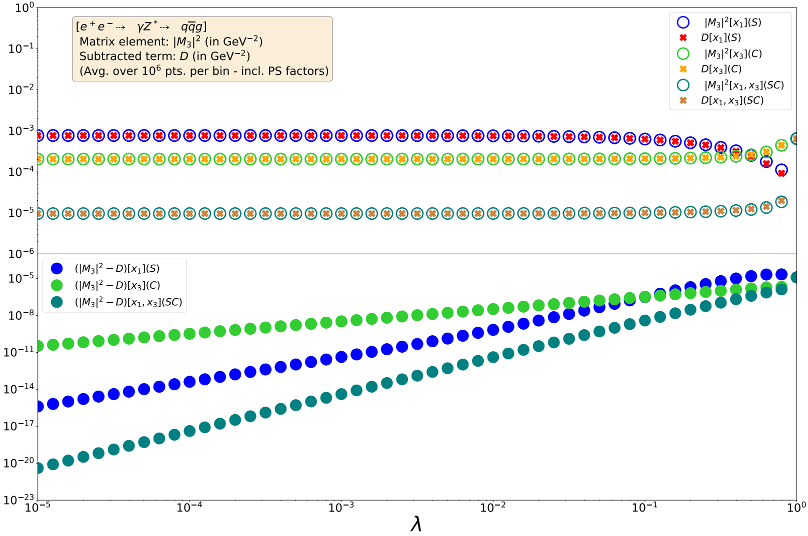
$$d\sigma^{RR} = \sum_{i,j=1,2,i\neq j} \left\langle [1 - \mathbb{S}][dg_4][dg_3]w^{i3,j4}\mathcal{F}(1,2,3,4)\right\rangle$$
$$+ \sum_{i=1,2,i\neq j} \left\langle \left[(1 - \mathbb{S})\,(1 - \mathbb{C}_i)\,\left(1 - (\theta^{(a)} + \theta^{(b)})C_{34}\right)\right]\right.$$
$$\left.[dg_4][dg_3]w^{i3,i4}\mathcal{F}(1,2,3,4)\right\rangle$$

Scaling (NLO)

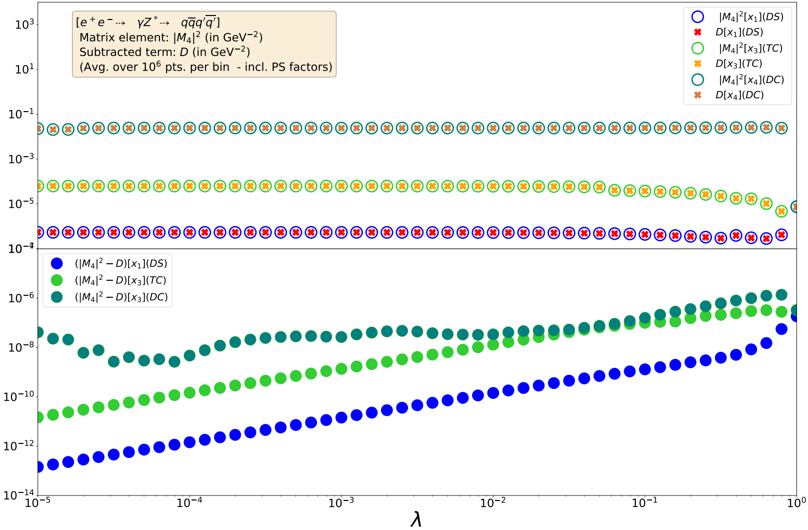


$[e^+e^- \dashrightarrow \ \gamma Z^* \dashrightarrow \ q\bar{q}g]$
Matrix element: $|M_3|^2$ (in GeV^{-2})
Subtracted term: D (in GeV^{-2})
(Avg. over 10^6 pts. per bin)
$|M_3|^2[x_1](S)$
$D[x_1](S)$
fit$[x_1]$:-2.0
$|M_3|^2[x_3](C)$
$D[x_3](C)$
fit$[x_3]$:-1.0
$|M_3|^2[x_1,x_3](SC)$
$D[x_1,x_3](SC)$
fit$[x_4]$:-3.0
$(|M_3|^2 - D)[x_1](S)$
$(|M_3|^2 - D)[x_3](C)$
$(|M_3|^2 - D)[x_1,x_3](SC)$
λ

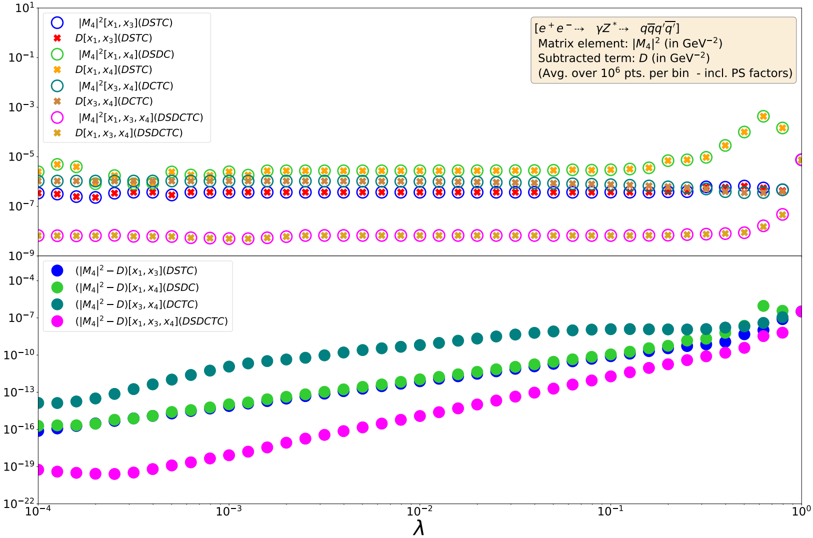
Scaling (NLO)


$[e^+e^- \dashrightarrow \gamma Z^* \dashrightarrow q\bar{q}g]$
Matrix element: $|M_3|^2$ (in GeV^{-2})
Subtracted term: D (in GeV^{-2})
(Avg. over 10^6 pts. per bin - incl. PS factors)
$|M_3|^2[x_1](S)$
$D[x_1](S)$
$|M_3|^2[x_3](C)$
$D[x_3](C)$
$|M_3|^2[x_1,x_3](SC)$
$D[x_1,x_3](SC)$
$(|M_3|^2 - D)[x_1](S)$
$(|M_3|^2 - D)[x_3](C)$
$(|M_3|^2 - D)[x_1,x_3](SC)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
10^{-20}
10^{-23}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
λ

Scaling (NNLO)



Scaling (NNLO)



Integration (NNLO)

- **LEP setup**: $e^+e^- \to \gamma^* \to 2$ jets (E_{cms}=91.2 GeV)
- Numerical safety cutoff at $k_{T,3\to4} = Q\sqrt{y_{3\to4}}$
  (Durham jet algorithm: $y_{ij} = 2(1-\cos\theta_{ij})\text{min}(E_i^2, E_j^2)/s$)
- Subtracted double-real contribution from $(d\bar{d}u\bar{u})$ final state

$k_{T,3\to4}$	σ	time
5 MeV	3.66(19) fb	96 s
5 keV	3.96(17) fb	100 s
5 eV	3.95(11) fb	100 s

- Diagrammatic calculation can lead to minor instabilities in double-soft limit:

$$|\mathcal{M}(1,2,3,4)|^2 = \frac{2s_{13}s_{14}}{s_{134}^2 s_{34}^2} - \frac{s_{13}s_{24}+s_{14}s_{23}-s_{12}s_{34}}{s_{134}s_{234}s_{34}^2} + \begin{pmatrix} 1 \leftrightarrow 2 \\ 3 \leftrightarrow 4 \end{pmatrix} + \dots \quad \text{vs}$$

$$\sum_{i,j=1,2} \mathcal{I}_{ij}(3,4) = \frac{2s_{12}}{(s_{13}+s_{14})(s_{23}+s_{24})s_{34}} \left(1 - \frac{s_{12}s_{34}(s_{13}s_{24}-s_{14}s_{23})^2}{(s_{13}+s_{14})(s_{23}+s_{24})}\right)$$

Scaling tests affected below $\lambda \approx 10^{-3}$ but integration results unchanged

Summary and outlook

So far:

- Implemented technique for simplest double-real test case
- Used all building blocks except single soft subtractions
- Encouraging results when compared to other methods

subtraction			slicing		
$k_{T,3\to4}$	$\Delta\sigma$	time	ϵ_τ	$\Delta\sigma$	time
5 MeV	0.38 fb	96 s	0.001	1.15 fb	110 s
5 keV	0.34 fb	100 s	0.0001	3.05 fb	110 s

- If pattern unchanged by single-soft subtractions, expect significant speedup for practical calculations

Next steps:

- Include integrated counterterms and assemble complete calculation
- Implement into MCFM in generic form
- Provide interfaces to other codes (e.g. for PS matching)

n_f-contribution to double-real correction (slicing method)