Automating of Antenna Subtraction in Colour Space

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with X. Chen, T. Gehrmann, N. Glover, A. Huss

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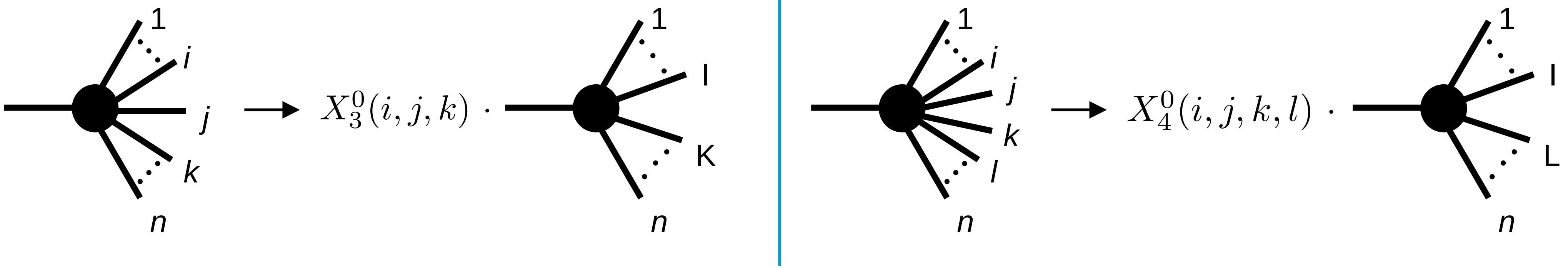
Antenna Subtraction

- NNLO subtraction scheme;
- **Antenna functions** describe the emission of unresolved partons between a pair of hard radiatiors;
- Real emission subtraction terms are constructed with antennae and reduced matrix elements;



- Analytical integration over $\mathrm{d}\Phi_{\mathrm{rad}}$ to obtain virtual subtraction terms;

Antenna Subtraction

Limitations:

- Poor scaling with the number of external partons n_p. Increasing n_p beyond previously available results requires a lot of work.
- Highly non-trivial construction of subtraction terms beyond **leading colour** for $n_p \geq 4$.

e.g. dijet production @NNLO:

	LC	FC
	[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss '17]	[Chen, Gehrmann, Glover, Huss, Mo '22]

Antenna Subtraction

Limitations:

- Poor scaling with the number of external partons n_p. Increasing n_p beyond previously available results requires a lot of work.
- Highly non-trivial construction of subtraction terms beyond **leading colour** for $n_p \geq 4$.

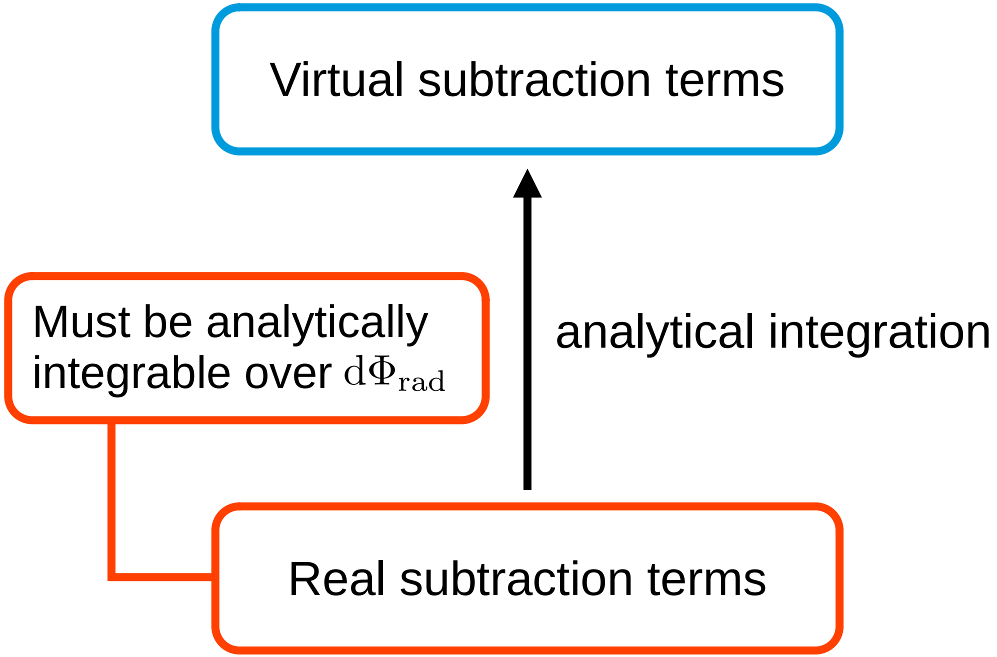
e.g. dijet production @NNLO:	LC	FC
	[Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss '17]	[Chen, Gehrmann, Glover, Huss, Mo '22]

A **new formulation** is required:

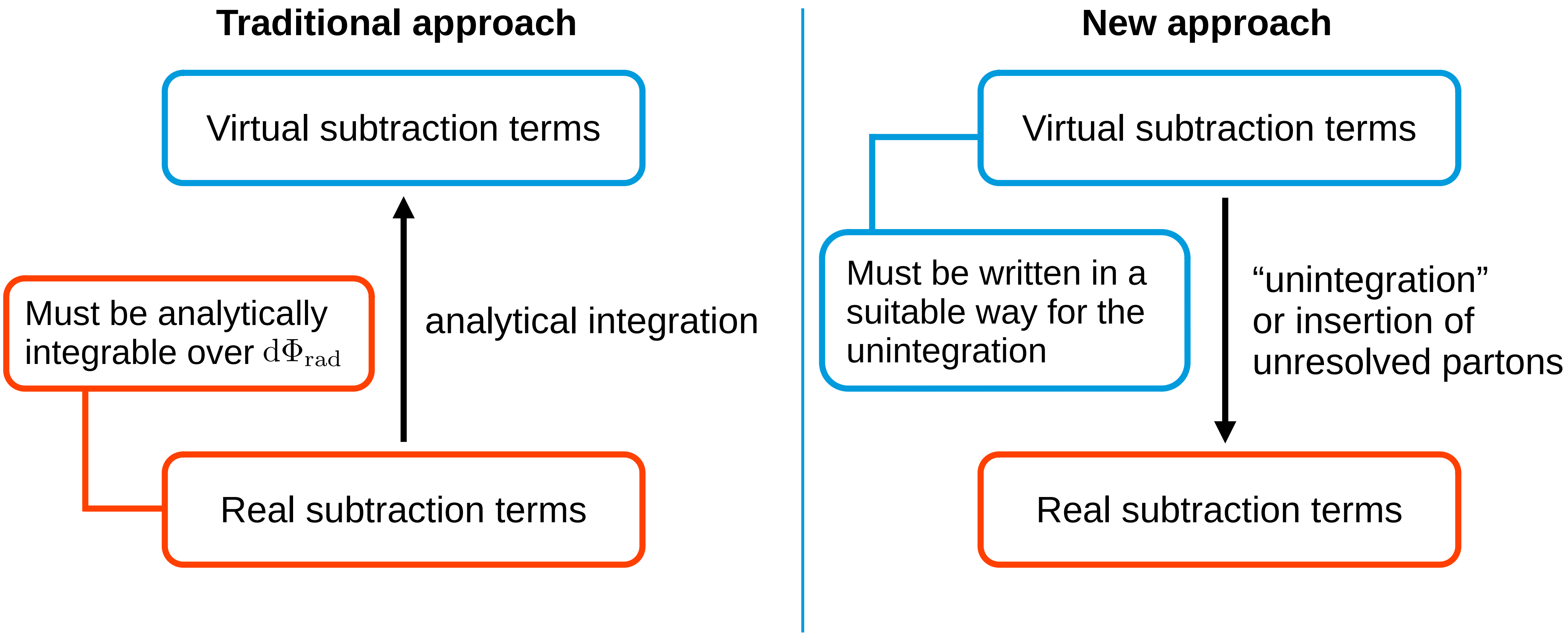
- automation and efficiency;
- improved understanding/organization of the subtraction infrastructure;
- $n_p = 5$: 3-jet production @NNLO [Czakon, Mitov, Poncelet '21]
- ($n_p \geq 5$)

Colourful Antenna Subtraction

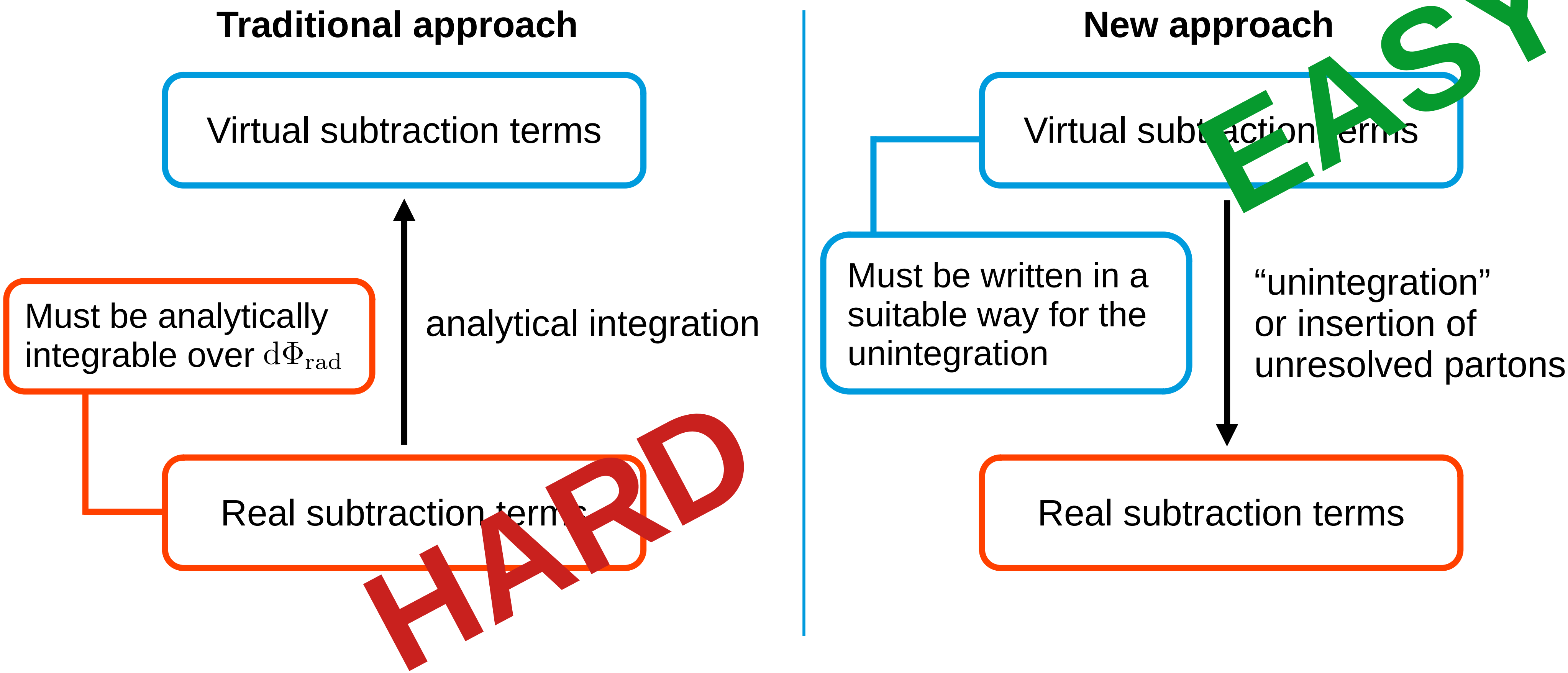
Traditional approach



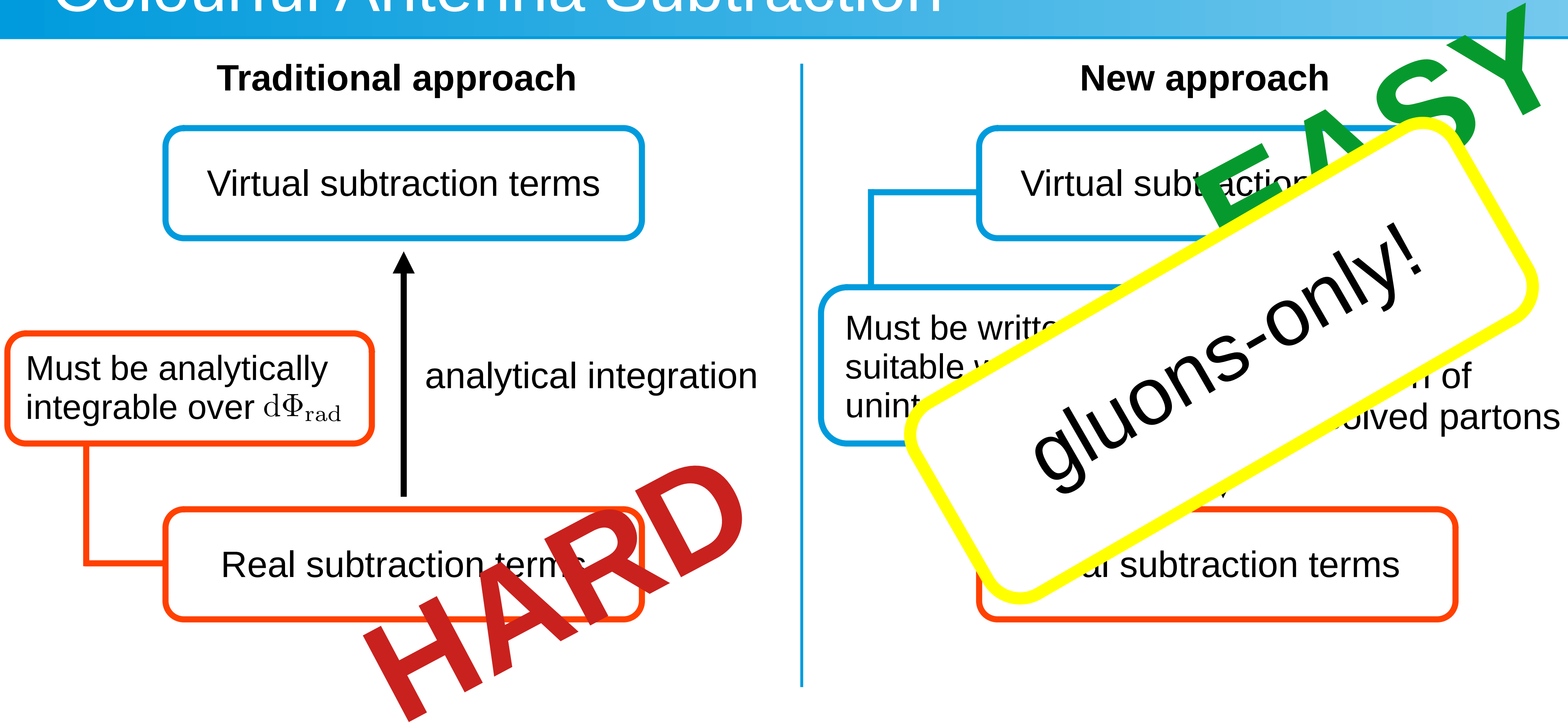
Colourful Antenna Subtraction



Colourful Antenna Subtraction



Colourful Antenna Subtraction



Colourful Antenna Subtraction @NLO

Partonic cross section @NLO:

$$\mathrm{d}\hat{\sigma}_{ab,NLO} = \int_n (\mathrm{d}\hat{\sigma}^{V}_{ab,NLO} + \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}) + \int_{n+1} \mathrm{d}\hat{\sigma}^{R}_{ab,NLO}$$

Subtraction @NLO:

$$\mathrm{d}\hat{\sigma}_{ab,NLO} = \int_n [\mathrm{d}\hat{\sigma}^{V}_{ab,NLO} - \mathrm{d}\hat{\sigma}^{T}_{ab,NLO}] + \int_{n+1} [\mathrm{d}\hat{\sigma}^{R}_{ab,NLO} - \mathrm{d}\hat{\sigma}^{S}_{ab,NLO}]$$

$$\mathrm{d}\hat{\sigma}^{T}_{ab,NLO} = -\int_1 \mathrm{d}\hat{\sigma}^{S}_{ab,NLO} - \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}$$

Colourful Antenna Subtraction @NLO

Partonic cross section @NLO:

$$\mathrm{d}\hat{\sigma}_{ab,NLO} = \int_n (\mathrm{d}\hat{\sigma}^V_{ab,NLO} + \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}) + \int_{n+1} \mathrm{d}\hat{\sigma}^R_{ab,NLO}$$

Subtraction @NLO:

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$$\mathrm{d}\hat{\sigma}^T_{ab,NLO} = -\int_1 \mathrm{d}\hat{\sigma}^S_{ab,NLO} - \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}$$

Goal:

- generate $\mathrm{d}\hat{\sigma}^T$;
- systematically infer $\mathrm{d}\hat{\sigma}^S$;

Colourful Antenna Subtraction @NLO

Virtual subtraction term: $\mathrm{d}\hat{\sigma}^{T}_{gg,NLO} = \mathcal{N}_V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n\, J_n^n(\Phi_n)\cdot 2\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle$

colour space

$$\boldsymbol{\mathcal{J}}^{(1)} = \sum_{(i,j)\geq 3}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,J_2^{(1)}(i_g,j_g) + \sum_{i\neq 1,2}(\boldsymbol{T}_1\cdot\boldsymbol{T}_i)\,J_2^{(1)}(1_g,i_g) + \sum_{i\neq 1,2}(\boldsymbol{T}_2\cdot\boldsymbol{T}_i)\,J_2^{(1)}(2_g,i_g) + (\boldsymbol{T}_1\cdot\boldsymbol{T}_2)\,J_2^{(1)}(1_g,2_g)$$

One-loop colour stripped integrated dipoles:

$$J_2^{(1)}(i_g,j_g) = \frac{1}{3}\mathcal{F}_3^0(s_{ij})$$

$$J_2^{(1)}(1_g,j_g) = \frac{1}{2}\mathcal{F}^0_{3,g}(s_{1j}) - \frac{1}{2}\Gamma^{(1)}_{gg}(x_1)\delta_2$$

$$J_2^{(1)}(1_g,2_g) = \mathcal{F}^0_{3,gg}(s_{12}) - \frac{1}{2}\Gamma^{(1)}_{gg}(x_1)\delta_2 - \frac{1}{2}\Gamma^{(1)}_{gg}(x_2)\delta_1$$

Colourful Antenna Subtraction @NLO

Virtual subtraction term: $$d\hat{\sigma}^{T}_{gg,NLO} = \mathcal{N}_V \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} d\Phi_n\, J^n_n(\Phi_n)\cdot 2\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle$$

colour space

$$\boldsymbol{\mathcal{J}}^{(1)} = \sum_{(i,j)\geq 3} (\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\, J^{(1)}_2(i_g,j_g) + \sum_{i\neq 1,2} (\boldsymbol{T}_1\cdot\boldsymbol{T}_i)\, J^{(1)}_2(1_g,i_g) + \sum_{i\neq 1,2} (\boldsymbol{T}_2\cdot\boldsymbol{T}_i)\, J^{(1)}_2(2_g,i_g) + (\boldsymbol{T}_1\cdot\boldsymbol{T}_2)\, J^{(1)}_2(1_g,2_g)$$

One-loop colour stripped integrated dipoles:

$$J^{(1)}_2(i_g,j_g) = \frac{1}{3}\mathcal{F}^0_3(s_{ij})$$

$$J^{(1)}_2(1_g,j_g) = \frac{1}{2}\mathcal{F}^0_{3,g}(s_{1j}) - \frac{1}{2}\Gamma^{(1)}_{gg}(x_1)\delta_2$$

$$J^{(1)}_2(1_g,2_g) = \mathcal{F}^0_{3,gg}(s_{12}) - \frac{1}{2}\Gamma^{(1)}_{gg}(x_1)\delta_2 - \frac{1}{2}\Gamma^{(1)}_{gg}(x_2)\delta_1$$

Integrated antenna functions

Splitting kernels

$$Poles\left[J^{(1)}_2(i_g,j_g)\right] = Poles\left[\mathrm{Re}\left(\mathcal{I}^{(1)}_{i_g j_g}(\epsilon,\mu_r^2)\right)\right]$$

[Catani '98]

Colourful Antenna Subtraction @NLO

Construction of the real subtraction term:

$$\mathrm{d}\hat{\sigma}^{S}_{gg,NLO} = -\mathcal{I}ns\left[\mathrm{d}\hat{\sigma}^{T}_{gg,NLO}\right]$$

- remove splitting kernels from the integrated dipoles;

- replace integrated antennae with their unintegrated counterparts:

$$\text{FF: } \mathcal{F}_3^0(s_{ij}) \to 3\, f_3^0(i,k,j), \qquad \text{IF: } \mathcal{F}_{3,g}^0(s_{1i}) \to 2\, f_{3,g}^0(1,k,i), \qquad \text{II: } \mathcal{F}_{3,gg}^0(s_{12}) \to F_{3,gg}^0(1,k,2)$$

- adjustments: NLO momentum mapping, phase space, overall factors;

Colourful Antenna Subtraction @NNLO

Partonic cross section @NNLO:

$$\mathrm{d}\hat{\sigma}_{ab,NNLO} = \int_n (\mathrm{d}\hat{\sigma}^{VV}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{MF,2}_{ab,NNLO}) + \int_{n+1} (\mathrm{d}\hat{\sigma}^{RV}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{MF,1}_{ab,NNLO}) + \int_{n+2} \mathrm{d}\hat{\sigma}^{RR}_{ab,NNLO}$$

Subtraction @NNLO:

$$\mathrm{d}\hat{\sigma}_{ab,NNLO} = \int_n [\mathrm{d}\hat{\sigma}^{VV}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{U}_{ab,NNLO}] + \int_{n+1} [\mathrm{d}\hat{\sigma}^{RV}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{T}_{ab,NNLO}] + \int_{n+2} [\mathrm{d}\hat{\sigma}^{RR}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{S}_{ab,NNLO}]$$

$$\mathrm{d}\hat{\sigma}^{S}_{ab,NNLO} = \mathrm{d}\hat{\sigma}^{S,1}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{S,2}_{ab,NNLO}$$

$$\mathrm{d}\hat{\sigma}^{T}_{ab,NNLO} = \mathrm{d}\hat{\sigma}^{VS}_{ab,NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,1}_{ab,NNLO}$$

$$\mathrm{d}\hat{\sigma}^{U}_{ab,NNLO} = -\int_1 \mathrm{d}\hat{\sigma}^{VS}_{ab,NNLO} - \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{ab,NNLO}$$

Colourful Antenna Subtraction @NNLO

Partonic cross section @NNLO:

$$\mathrm{d}\hat{\sigma}_{ab,NNLO} = \int_n (\mathrm{d}\hat{\sigma}^{VV}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{MF,2}_{ab,NNLO}) + \int_{n+1} (\mathrm{d}\hat{\sigma}^{RV}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{MF,1}_{ab,NNLO}) + \int_{n+2} \mathrm{d}\hat{\sigma}^{RR}_{ab,NNLO}$$

Subtraction @NNLO:

$$\mathrm{d}\hat{\sigma}_{ab,NNLO} = \int_n [\mathrm{d}\hat{\sigma}^{VV}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{U}_{ab,NNLO}] + \int_{n+1} [\mathrm{d}\hat{\sigma}^{RV}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{T}_{ab,NNLO}] + \int_{n+2} [\mathrm{d}\hat{\sigma}^{RR}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{S}_{ab,NNLO}]$$

$$\mathrm{d}\hat{\sigma}^{S}_{ab,NNLO} = \mathrm{d}\hat{\sigma}^{S,1}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{S,2}_{ab,NNLO}$$

$$\mathrm{d}\hat{\sigma}^{T}_{ab,NNLO} = \mathrm{d}\hat{\sigma}^{VS}_{ab,NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,1}_{ab,NNLO}$$

$$\mathrm{d}\hat{\sigma}^{U}_{ab,NNLO} = -\int_1 \mathrm{d}\hat{\sigma}^{VS}_{ab,NNLO} - \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{ab,NNLO}$$

Goal:

- generate $\mathrm{d}\hat{\sigma}^{U}$;
- systematically infer part of $\mathrm{d}\hat{\sigma}^{T}$;
- generate missing part of $\mathrm{d}\hat{\sigma}^{T}$;
- systematically infer $\mathrm{d}\hat{\sigma}^{S}$;

Colourful Antenna Subtraction @NNLO

Double virtual subtraction term:

$$\mathrm{d}\hat{\sigma}^{U}_{gg,NNLO} = \mathcal{N}_{VV} \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n J_n^n(\Phi_n)$$

$$\times 2\Big\{\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^1_{n+2}\rangle + \langle A^1_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle - \frac{b_0 N_c}{\epsilon}\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle$$

single one-loop insertion at one-loop

$$-\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)} \otimes \boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle + \langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(2)}|A^0_{n+2}\rangle\Big\}$$

double one-loop insertion

single two-loop insertion

$$\boldsymbol{\mathcal{J}}^{(2)} = N_c \sum_{(i,j)\geq 3} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, J_2^{(2)}(i_g, j_g) + N_c \sum_{i\neq 1,2} (\boldsymbol{T}_1 \cdot \boldsymbol{T}_i)\, J_2^{(2)}(1_g, i_g) + N_c \sum_{i\neq 1,2} (\boldsymbol{T}_2 \cdot \boldsymbol{T}_i)\, J_2^{(2)}(2_g, i_g) + N_c (\boldsymbol{T}_1 \cdot \boldsymbol{T}_2)\, J_2^{(2)}(1_g, 2_g)$$

Colourful Antenna Subtraction @NNLO

Two-loop colour stripped integrated dipoles:

$$J_2^{(2)}(i_g, j_g) = \frac{1}{4}\mathcal{F}_4^0 + \frac{1}{3}\mathcal{F}_3^1 + \frac{1}{3}\frac{b_0}{\epsilon}\left(\frac{|s_{ij}|}{\mu_r^2}\right)^{-\epsilon}\mathcal{F}_3^0 - \frac{1}{9}[\mathcal{F}_3^0 \otimes \mathcal{F}_3^0]$$

$$J_2^{(2)}(1_g, j_g) = \frac{1}{2}\mathcal{F}_{4,g}^0 + \frac{1}{2}\mathcal{F}_{3,g}^1 + \frac{1}{2}\frac{b_0}{\epsilon}\left(\frac{|s_{1j}|}{\mu_r^2}\right)^{-\epsilon}\mathcal{F}_{3,g}^0 - \frac{1}{4}[\mathcal{F}_{3,g}^0 \otimes \mathcal{F}_{3,g}^0] - \frac{1}{2}\overline{\Gamma}_{gg}^{(2)}(x_1)\delta_2$$

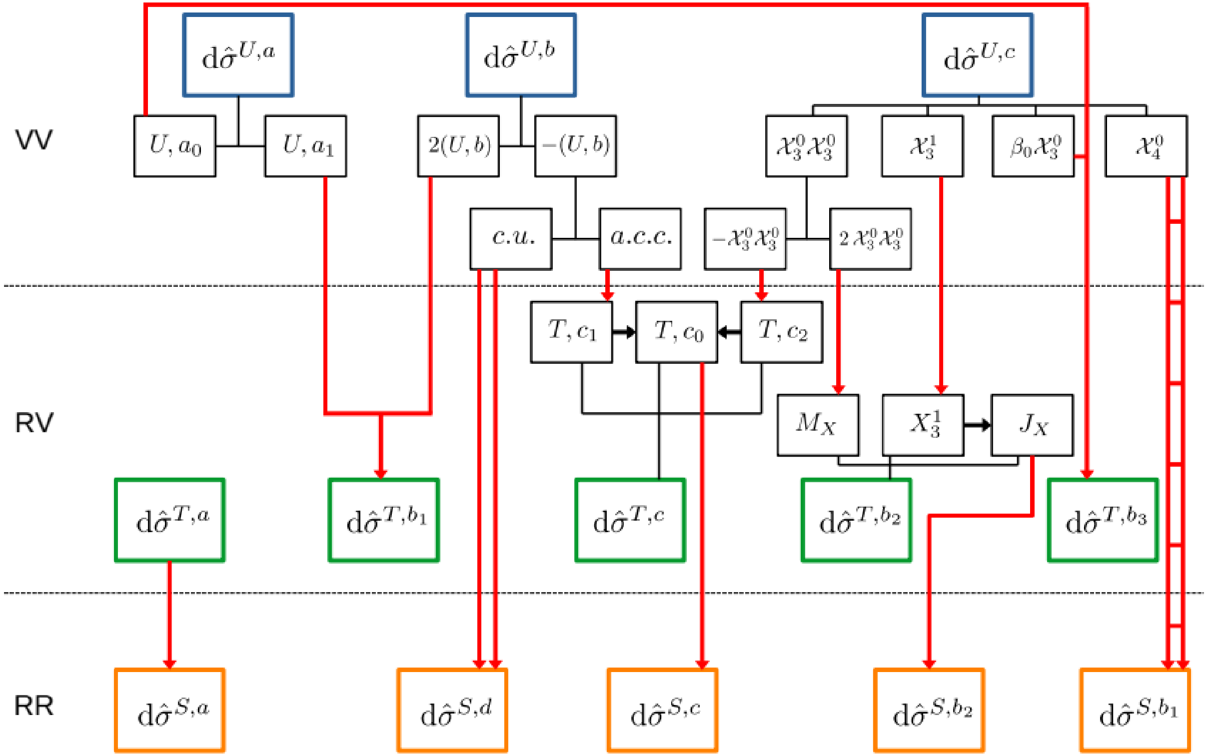
$$J_2^{(2)}(1_g, 2_g) = \mathcal{F}_{4,gg}^{0,adj} + \frac{1}{2}\mathcal{F}_{4,gg}^{0,n.adj} + \mathcal{F}_{3,gg}^1 + \frac{b_0}{\epsilon}\left(\frac{|s_{12}|}{\mu_r^2}\right)^{-\epsilon}\mathcal{F}_{3,gg}^0 - [\mathcal{F}_{3,gg}^0 \otimes \mathcal{F}_{3,gg}^0] - \frac{1}{2}\overline{\Gamma}_{gg}^{(2)}(x_1)\delta_2 - \frac{1}{2}\overline{\Gamma}_{gg}^{(2)}(x_2)\delta_1$$

$$Poles\left[J_2^{(2)}(i_g, j_g) - \frac{b_0 N_c}{\epsilon}J_2^{(1)}(i_g, j_g)\right] = Poles\left[\mathrm{Re}\left(\mathcal{I}_{i_g j_g}^{(2)}(\epsilon, \mu_r^2) - \frac{b_0 N_c}{\epsilon}\mathcal{I}_{i_g j_g}^{(1)}(\epsilon, \mu_r^2)\right)\right]$$

[Catani ‘98]
[Bern, De Freitas, Dixon ‘03]
[Becher, Neubert ‘09]

Colourful Antenna Subtraction @NNLO

- Single insertion from VV to RV;
- New terms at RV level:
  - ε-finiteness;
  - Oversubtraction;
  - Large angle soft radiation;
- Single insertion from RV to RR;
- Double insertion from VV to RR (iterated or simultaneous);



Colourful Antenna Subtraction @NNLO

Simultaneous insertion of two unresolved gluons:

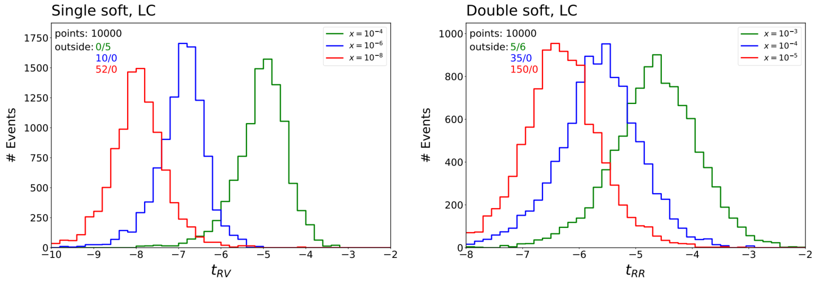
$$\mathrm{d}\hat{\sigma}^{S,b_1} = -\mathcal{I}ns_2 \left[\mathrm{d}\hat{\sigma}^{U,c,\mathcal{X}_4^0}\right]$$

Practically analogous to a single insertion with:

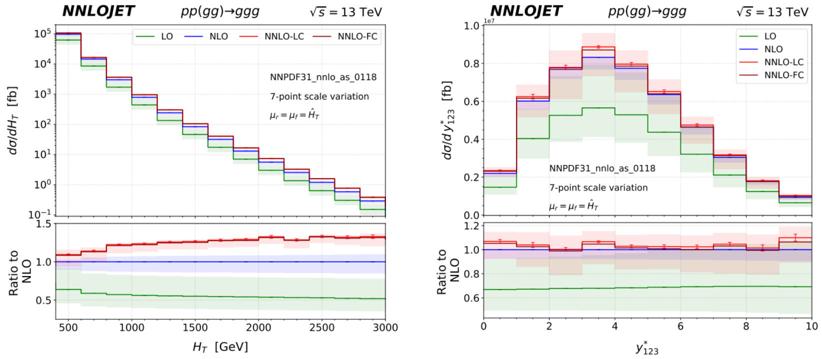
$$\begin{aligned}
\text{FF:} \quad & \mathcal{F}_4^0(s_{ij}) \to 4\left[F_{4,a}^0(i,k,l,j) + F_{4,b}^0(i,k,l,j)\right]; \\
\text{IF:} \quad & \mathcal{F}_{4,g}^0(s_{1i}) \to F_4^0(1,k,l,i); \\
\text{II:} \quad & \mathcal{F}_{4,gg}^{0,adj.}(s_{12}) \to F_4^0(1,k,l,2); \\
& \mathcal{F}_{4,gg}^{0,n.adj.}(s_{12}) \to F_4^0(1,k,2,l);
\end{aligned}$$

Status

- Systematic construction of the **real virtual** and **double real** subtraction terms;
- **Full N_c dependence** retained working in colour space;
- Numerical validation of the subtraction terms against matrix elements for $gg \to ggg$;



- Successful computation of $gg \to ggg$ @NNLO in the gluons-only assumption (see 2203.13531);



Outlook

- Inclusion of the fermionic degrees of freedom:
  - quark-gluon and quark-antiquark integrated dipoles: **Done**;
  - Insertion of an unresolved quark-antiquark pair: **Done**;
  - Improvements in the $\mathrm{d}\hat{\sigma}^{T,c}$ sector: **Work in progress**;
  - Identity chaging contributions: **Work in progress**;

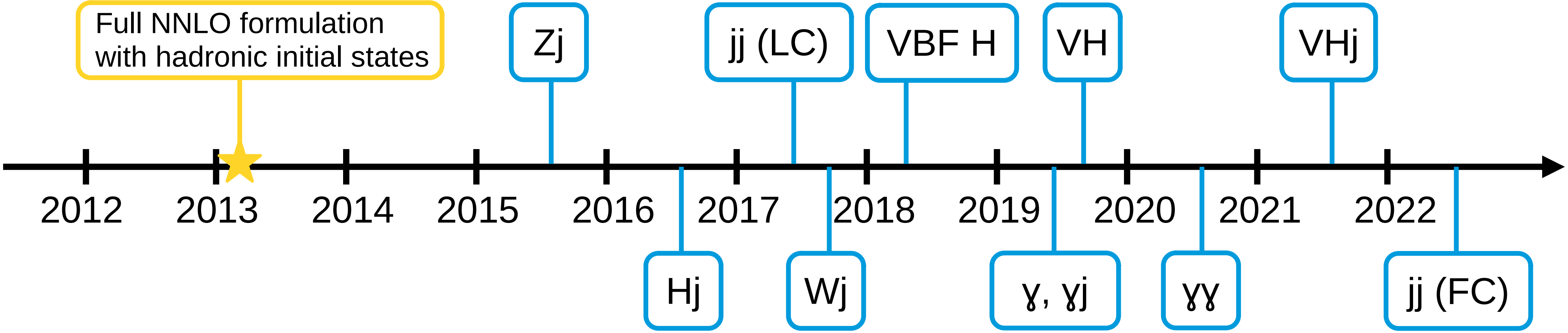
- Calculation of pp$\rightarrow$jjj @NNLO in full colour;

Thanks for your attention!

Antenna Subtraction

- Flexibility;
- Fully analytical integration of antenna functions;
- Locality (almost);

Succesfully applied to a variety of LHC processes in the past decade with ***NNLOJET***:



Backup: colour space

$$|\mathcal{A}_n^0\rangle = \sum_{\sigma\in S_n/Z_n} \mathrm{Tr}(\boldsymbol{T}^{a_{\sigma(1)}}\dots\boldsymbol{T}^{a_{\sigma(n)}})A_n^0(\sigma(p_1),\dots,\sigma(p_n))$$

$$|\mathcal{A}_n^1\rangle = \sum_{c=1}^{\lfloor n/2\rfloor+1}\sum_{\sigma\in S_n/S_{n,c}} \boldsymbol{\mathcal{C}}^1_{n,c,\sigma}A_n^1(\sigma(p_1),\dots,\sigma(p_n))$$

$$\boldsymbol{\mathcal{C}}^1_{n,1,\sigma} = N_c\mathrm{Tr}(\boldsymbol{T}^{a_{\sigma(1)}}\dots\boldsymbol{T}^{a_{\sigma(n)}})$$

$$\boldsymbol{\mathcal{C}}^1_{n,c,\sigma} = \mathrm{Tr}(\boldsymbol{T}^{a_{\sigma(1)}}\dots\boldsymbol{T}^{a_{\sigma(c-1)}})\mathrm{Tr}(\boldsymbol{T}^{a_{\sigma(c)}}\dots\boldsymbol{T}^{a_{\sigma(n)}})$$

$$f_\ell\,(p_n) = \textstyle\sum_{\sigma,\sigma'} c_n^\ell(\sigma,\sigma')\,a_n^\ell(\sigma,\sigma';\{p\})$$

$$a_n^0(\sigma,\sigma';\{p\}) = A_n^0(\sigma(\{p\}))^\dagger A_n^0(\sigma'(\{p\}))$$

$$a_{n,c}^1(\sigma,\sigma';\{p\}) = 2\mathrm{Re}\left[A_{n,c}^1(\sigma(\{p\}))^\dagger A_n^0(\sigma'(\{p\}))\right]$$

Backup: MF @NLO

Exploiting **colour conservation** any operator proportional to the identity in colour space can be written in terms of colour charge dipoles:

$$\sum_i \boldsymbol{T}_i = 0, \quad \sum_{i\neq j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j = -\boldsymbol{T}_i \cdot \boldsymbol{T}_i = -C_i\, \boldsymbol{Id}$$

The mass factorization counterterm can be written as:

$$\hat{\sigma}^{MF}_{gg,NLO} = -\mathcal{N}_V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n\, J^n_n(\Phi_n)\langle A^0_{n+2}|\boldsymbol{\Gamma}^{(1)}_{gg;gg}|A^0_{n+2}\rangle$$

$$\boldsymbol{\Gamma}^{(1)}_{gg;gg}(x_1,x_2) = -[\Gamma^{(1)}_{gg}(x_1)\delta(1-x_2)+\Gamma^{(1)}_{gg}(x_2)\delta(1-x_1)]\frac{1}{C_A}\sum_{i\neq j}\boldsymbol{T}_i\cdot\boldsymbol{T}_j, \qquad \Gamma^{(1)}_{gg}(x) = -\frac{1}{\epsilon}N_c p^0_{gg}(x)$$

Colourful Antenna Subtraction @NLO

IR singularity structure in **colour space** at one-loop: [Catani '98]

$$|A^1_{n+2}\rangle = \boldsymbol{I}^{(1)}(\epsilon,\mu_r^2)|A^0_{n+2}\rangle + \text{ finite terms}$$

$$\boldsymbol{I}^{(1)}(\epsilon,\mu_r^2) = \sum_{(i,j)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\,\mathcal{I}^{(1)}_{ij}(\epsilon,\mu_r^2), \qquad \mathcal{I}^{(1)}_{i_g j_g}(\epsilon,\mu_r^2) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon^2}+\frac{b_0}{\epsilon}\right]\left(\frac{-s_{ij}}{\mu_r^2}\right)^{-\epsilon}, \qquad b_0 = \frac{11}{6}$$

The virtual correction IR poles can be extracted in a general way:

$$Poles\left(\mathrm{d}\hat{\sigma}^V_{gg}\right) = \mathcal{N}_V \int \mathrm{d}\Phi_n\, J^n_n(\Phi_n)\, Poles\left[\sum_{(i_g,j_g)} \left\langle A^0_{n+2}\,|\boldsymbol{T}_i\cdot\boldsymbol{T}_j|\,A^0_{n+2}\right\rangle\, 2\,\mathrm{Re}\left(\mathcal{I}^{(1)}_{i_g j_g}(\epsilon,\mu_r^2)\right)\right]$$

Colourful Antenna Subtraction @NLO

Construction of the real subtraction term:

$$\mathrm{d}\hat{\sigma}^{S}_{gg,NLO} = -\mathcal{I}ns\left[\mathrm{d}\hat{\sigma}^{T}_{gg,NLO}\right]$$

- remove splitting kernels from the integrated dipoles;
- replace integrated antennae with their unintegrated counterparts according to:

$$\text{FF: } \mathcal{F}_3^0(s_{ij}) \to 3\, f_3^0(i,k,j), \qquad \text{IF: } \mathcal{F}_{3,g}^0(s_{1i}) \to 2\, f_{3,g}^0(1,k,i), \qquad \text{II: } \mathcal{F}_{3,gg}^0(s_{12}) \to F_{3,gg}^0(1,k,2)$$

- perform a momenta relabelling in any function accompanying the antennae (matrix elements, jet function, …):

$$f(\ldots,p_i,\ldots,p_j,\ldots) \to f(\ldots,p_{\widetilde{ik}},\ldots,p_{\widetilde{kj}},\ldots)$$

- adjust phase space and overall factors and sum over permutations of external momenta;

Colourful Antenna Subtraction @NNLO

IR singularity structure in colour space at two-loop:

[Catani '98]
[Bern, De Freitas, Dixon '03]
[Becher, Neubert '09]

$$|A^2_{n+2}\rangle = \boldsymbol{I}^{(1)}(\epsilon,\mu_r^2)|A^1_{n+2}\rangle + \boldsymbol{I}^{(2)}(\epsilon,\mu_r^2)|A^0_{n+2}\rangle + \text{finite terms}$$

$$\boldsymbol{I}^{(2)}(\epsilon,\mu_r^2) = -\frac{1}{2}\sum_{(i,j)}\sum_{(k,l)}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)(\boldsymbol{T}_k\cdot\boldsymbol{T}_l)\,\mathcal{I}^{(1)}_{ij}(\epsilon,\mu_r^2)\mathcal{I}^{(1)}_{kl}(\epsilon,\mu_r^2)$$
$$-\frac{b_0 N_c}{\epsilon}\sum_{(i,j)}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{I}^{(1)}_{ij}(\epsilon,\mu_r^2) + \sum_{(i,j)}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{I}^{(2)}_{ij}(\epsilon,\mu_r^2)$$

$$\mathcal{I}^{(2)}_{i_g j_g}(\epsilon,\mu_r^2) = e^{-\epsilon\gamma_E}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{b_0 N_c}{\epsilon} + k_0 N_c\right)\mathcal{I}^{(1)}_{i_g j_g}(2\epsilon,\mu_r^2) - \mathcal{H}^{(2)}_{i_g j_g}(\epsilon), \qquad k_0 = \frac{67}{18} - \frac{\pi^2}{6},$$

$$\mathcal{H}^{(2)}_{i_g j_g}(\epsilon) = \frac{e^{\epsilon\gamma_E}}{2\Gamma(1-\epsilon)}\frac{N_c}{\epsilon}\left[\frac{5}{12} + \frac{11}{144}\pi^2 + \frac{\zeta_3}{2}\right]$$

Backup: VV poles

The double virtual correction IR poles can be extracted in a general way:

$$Poles\left(\hat{\sigma}_{gg}^{VV}\right) = \mathcal{N}_{VV} \int \mathrm{d}\Phi_n \, J_n^n(\Phi_n)$$

one-loop dipole insertion within one-loop amplitude

$$\times Poles\Big\{ \sum_{(i_g,j_g)} 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{i_g j_g}(\epsilon, \mu_r^2)\right] \left[\langle A^1_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|A^0_{n+2}\rangle + \langle A^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|A^1_{n+2}\rangle\right]$$

$$-\frac{1}{2}\sum_{(i_g,j_g)}\sum_{(k_g,l_g)} 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{i_g j_g}(\epsilon, \mu_r^2)\right] 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{l_g k_g}(\epsilon, \mu_r^2)\right] \langle A^0_{n+2}|(\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g})(\boldsymbol{T}_{k_g}\cdot\boldsymbol{T}_{l_g})|A^0_{n+2}\rangle$$

double one-loop dipole insertion

$$-\frac{b_0 N_c}{\epsilon}\sum_{(i,j)} 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{i_g j_g}(\epsilon, \mu_r^2)\right] \langle A^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|A^0_{n+2}\rangle$$

two-loop dipole insertion

$$+\sum_{(i,j)} 2\mathrm{Re}\left[\mathcal{I}^{(2)}_{i_g j_g}(\epsilon, \mu_r^2)\right] \langle A^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|A^0_{n+2}\rangle\Big\}$$

Backup: MF @NNLO

Analogously to the NLO case, the double virtual mass factorization counterterm is expressed in colour space:

$$\mathrm{d}\hat{\sigma}_{gg,NNLO}^{MF,2} = -\mathcal{N}_{VV}\int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n J_n^n \Phi_n \Big\{ \langle A_{n+2}^0|\boldsymbol{\Gamma}_{gg;gg}^{(1)}(x_1,x_2)|A_{n+2}^1\rangle + \langle A_{n+2}^1|\boldsymbol{\Gamma}_{gg;gg}^{(1)}(x_1,x_2)|A_{n+2}^0\rangle$$

$$-2\langle A_{n+2}^0|\left[\boldsymbol{\Gamma}_{gg;gg}^{(1)}\otimes\boldsymbol{\mathcal{J}}^{(1)}\right](x_1,x_2)|A_{n+2}^0\rangle + \frac{1}{2}\langle A_{n+2}^0|\left[\boldsymbol{\Gamma}_{gg;gg}^{(1)}\otimes\boldsymbol{\Gamma}_{gg;gg}^{(1)}\right](x_1,x_2)|A_{n+2}^0\rangle$$

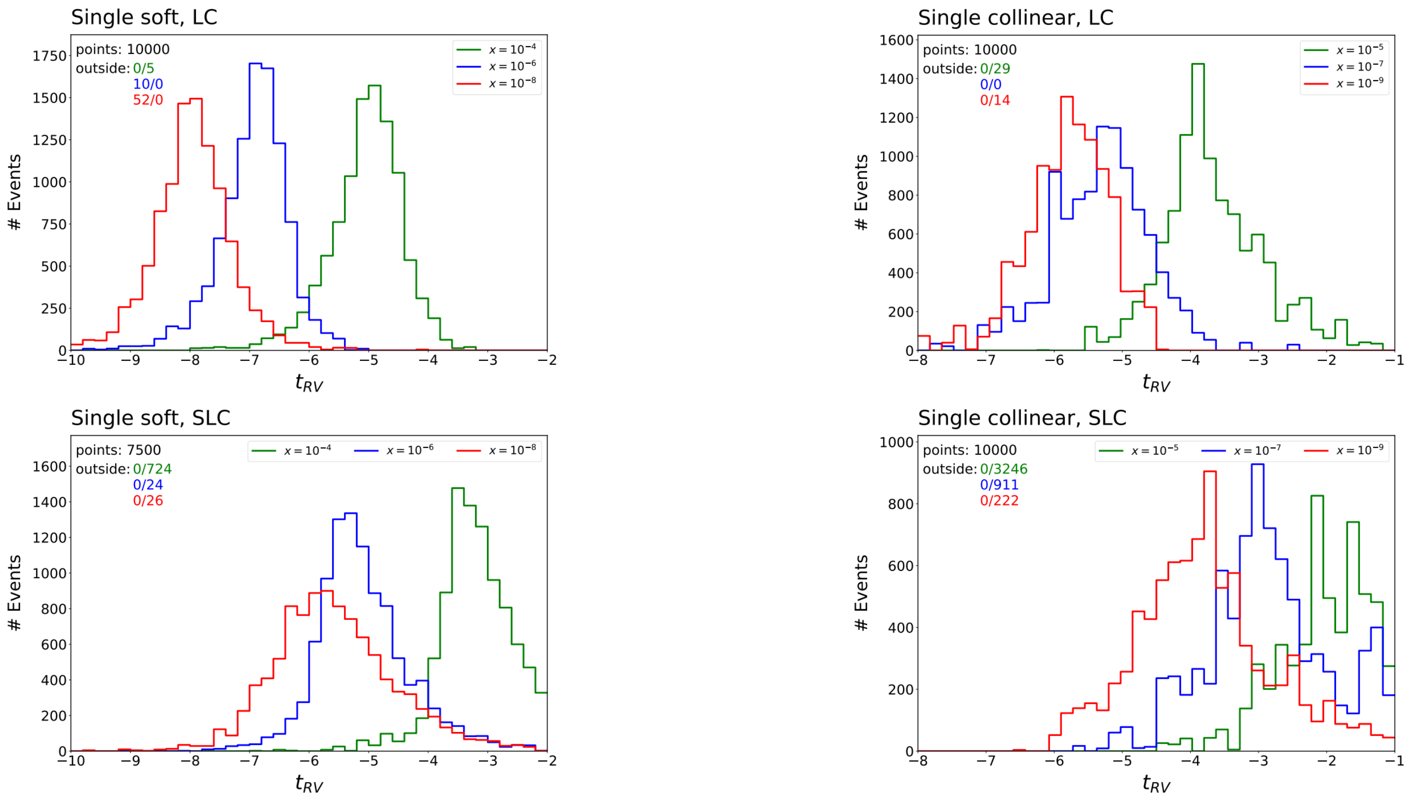
$$-\frac{b_0 N_c}{\epsilon}\langle A_{n+2}^0|\boldsymbol{\Gamma}_{gg;gg}^{(1)}(x_1,x_2)|A_{n+2}^0\rangle + \langle A_{n+2}^0|\overline{\boldsymbol{\Gamma}}_{gg;gg}^{(2)}(x_1,x_2)|A_{n+2}^0\rangle\Big\}$$

$$\overline{\boldsymbol{\Gamma}}_{gg;gg}^{(2)}(x_1,x_2) = -[\overline{\Gamma}_{gg}^{(2)}(x_1)\delta(1-x_2) + \overline{\Gamma}_{gg}^{(2)}(x_2)\delta(1-x_1)]\frac{1}{C_A}\sum_{i\neq j}\boldsymbol{T}_i\cdot\boldsymbol{T}_j$$

$$\overline{\Gamma}_{gg}^{(2)}(x) = -\frac{1}{2\epsilon}\left(N_c^2 p_{gg}^1(x) + \frac{b_0 N_c^2}{\epsilon}p_{gg}^0(x)\right)$$

Backup: validation RV

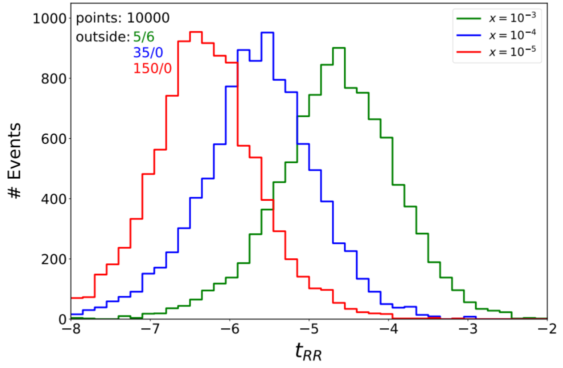
$t = \log_{10}\left(|1 - \mathrm{ME}/\mathrm{sub}|\right)$



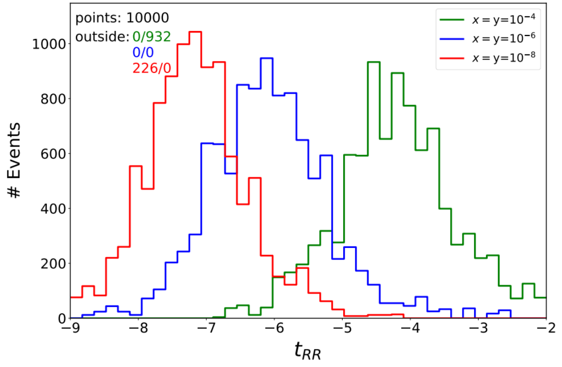
Backup: validation RR

$$t = \log_{10}\left(|1 - \mathrm{ME}/\mathrm{sub}|\right)$$

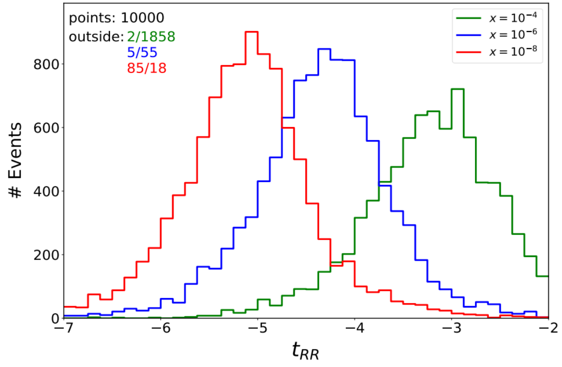
Double soft, LC



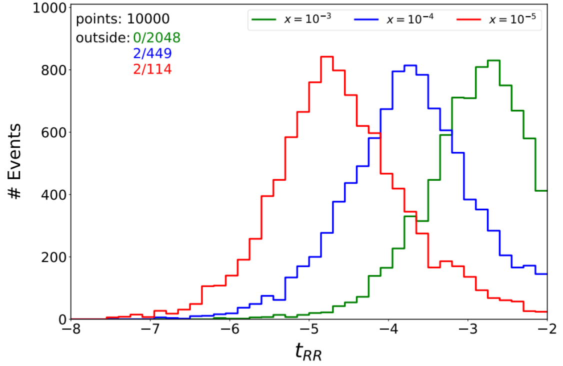
Soft and collinear, LC



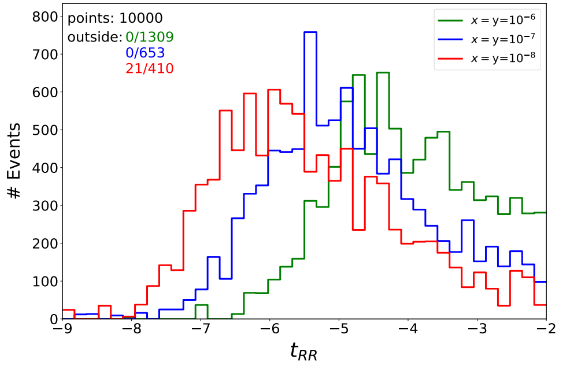
Triple collinear, LC



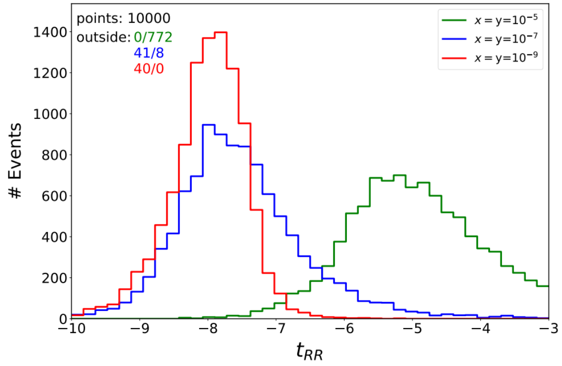
Double soft, SLC



Soft and collinear, SLC



Double collinear, LC



Backup: Computational setup

- VV ME 5-gluon two-loop: public C++ implementation;

  [Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov ‘19]
  [Abreu, Cordero, Ita, Page, Sotnikov ‘21]
  [Chicherin, Sotnikov 10] [Gehrmann, Henn, Lo Presti 18]

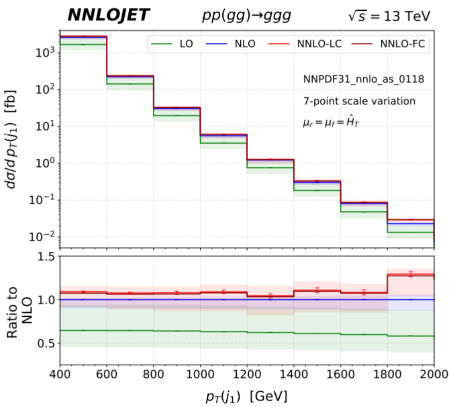
- RV ME 6-gluon one-loop: **OpenLoops, crucial IR stability;**

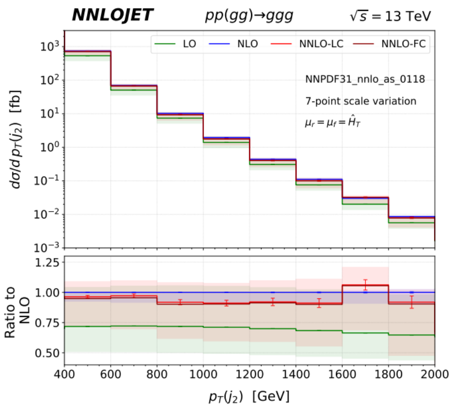
  [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller ‘19]

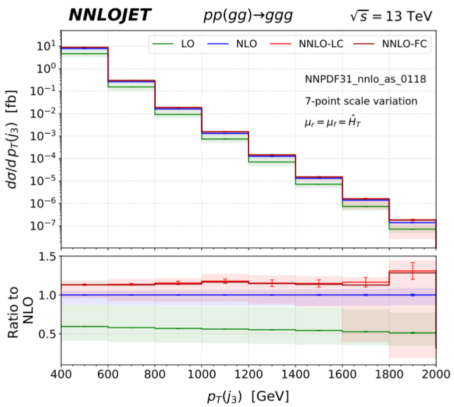
- RR ME 7-gluon tree-level: analytical;

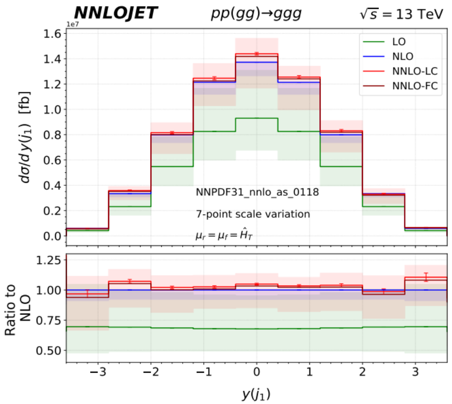
- Subtraction terms: 5- and 6-gluon tree-level, 5-gluon one-loop: analytical

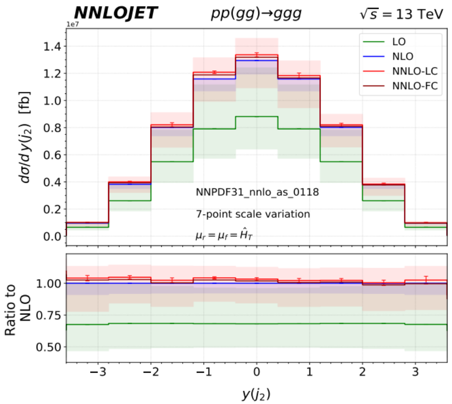
Backup: ggggg @NNLO

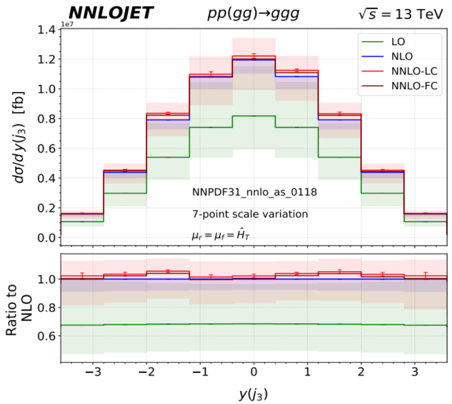




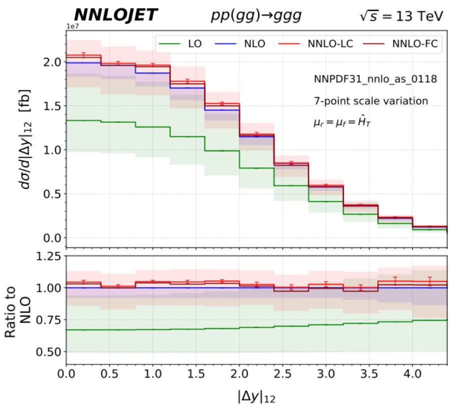


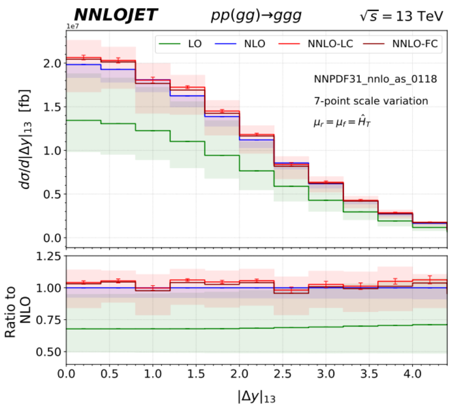


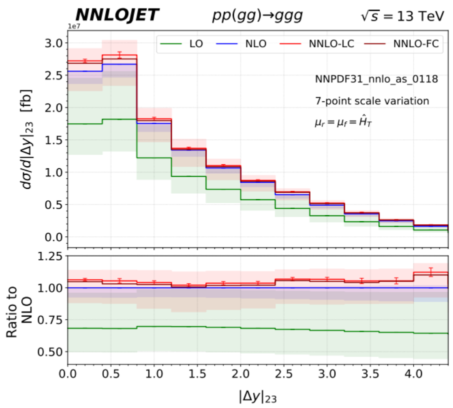


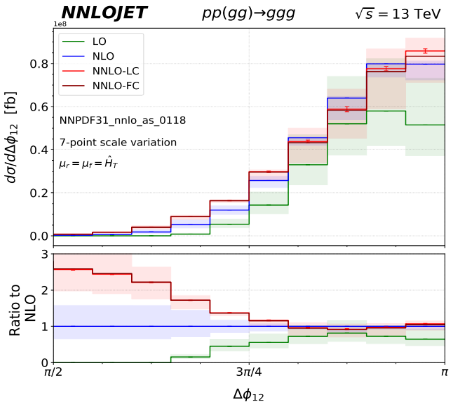


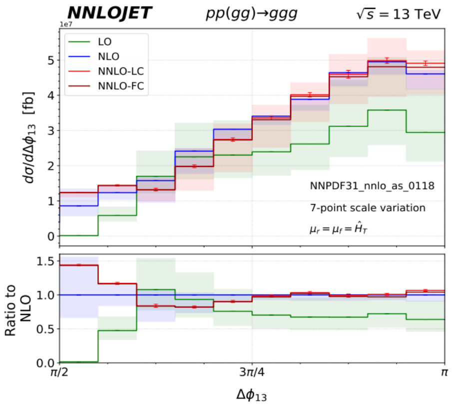
Backup: ggggg @NNLO

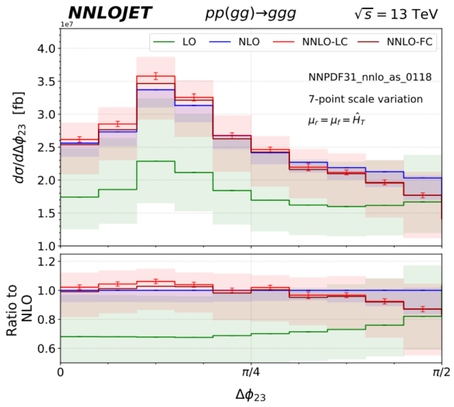




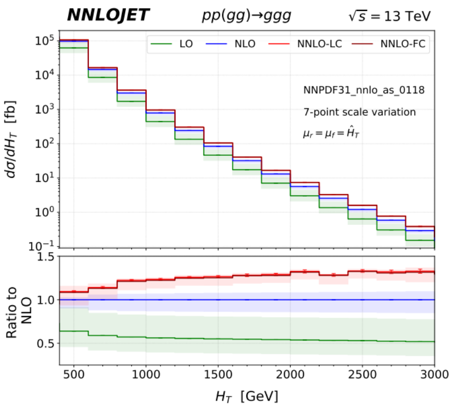


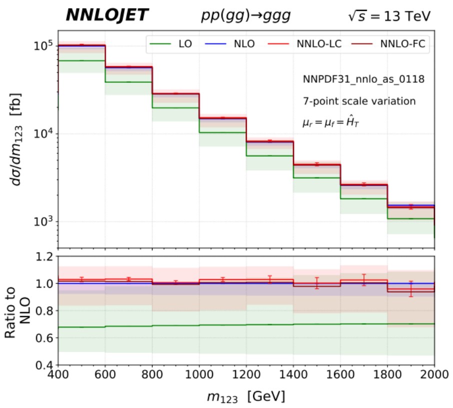


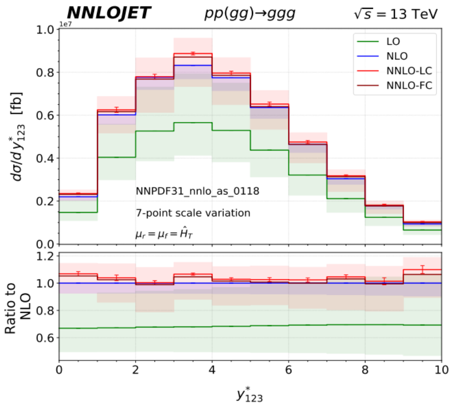


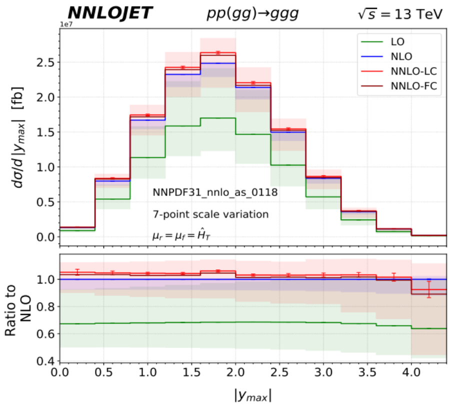


Backup: ggggg @NNLO









Backup: scale variation