Three-loop master integrals for the mixed QCD-electroweak correction to $H \to b\bar{b}$

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in collaboration with Ekta Chaubey[2] and Stefan Weinzierl[1]

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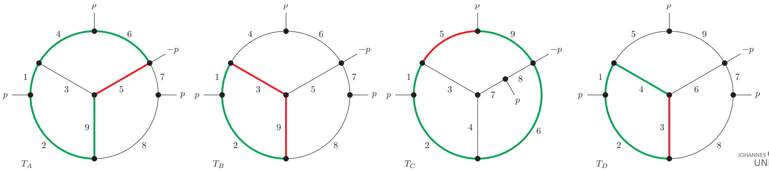
13.05.2022



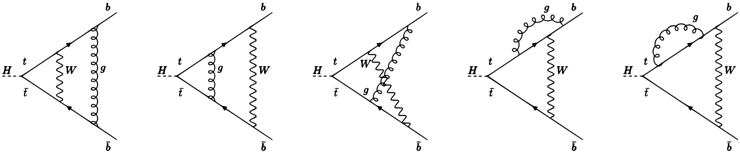
Predominant decay mode of the Higgs boson: $H \to b\bar{b}$

$$\Gamma(H \to b\bar{b}) = \Gamma^{(0)}\left(1 + \Delta^{(\alpha_s)} + \Delta^{(\alpha)} + \Delta^{(\alpha\alpha_s)} + \cdots\right)$$

$$\Delta^{(\alpha\alpha_s)} = \underbrace{\Delta_{\gamma}^{(\alpha\alpha_s)}}_{\text{1 int. mass}} + \underbrace{\Delta_{Z}^{(\alpha\alpha_s)}}_{\approx -0.002} + \underbrace{\boldsymbol{\Delta_{W}^{(\alpha\alpha_s)}}}_{\approx -0.0009} \longleftarrow \text{3 int. masses}$$

A. Kataev, '97; L. Mihaila, B. Schmidt, M. Steinhauser, '15

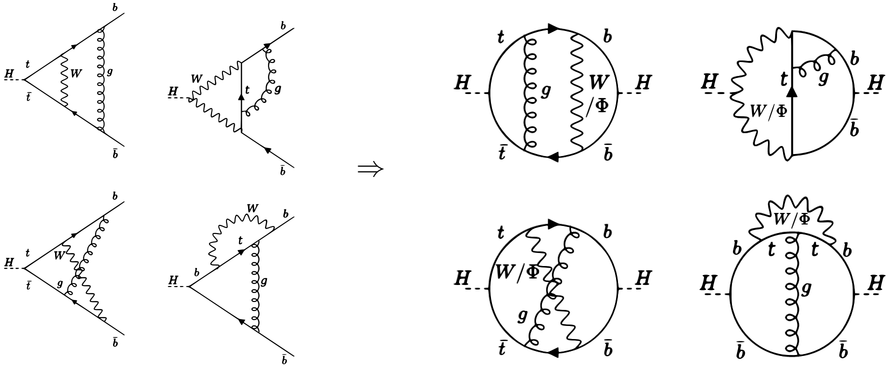
Future colliders → need for precise theoretical predictions → importance of analytic computations



The optical theorem

Relation of decay width and Higgs boson self-energy:

$$\Gamma(H \to b\bar{b}) = \frac{1}{m_H}\text{Im}(\Sigma_H(p^2 = m_H^2 + i\epsilon))$$



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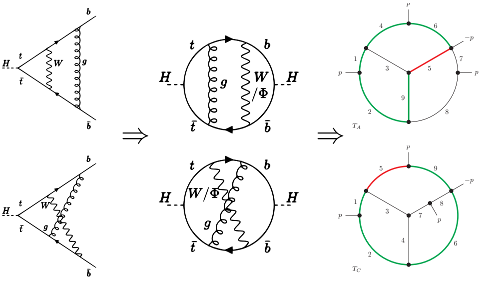
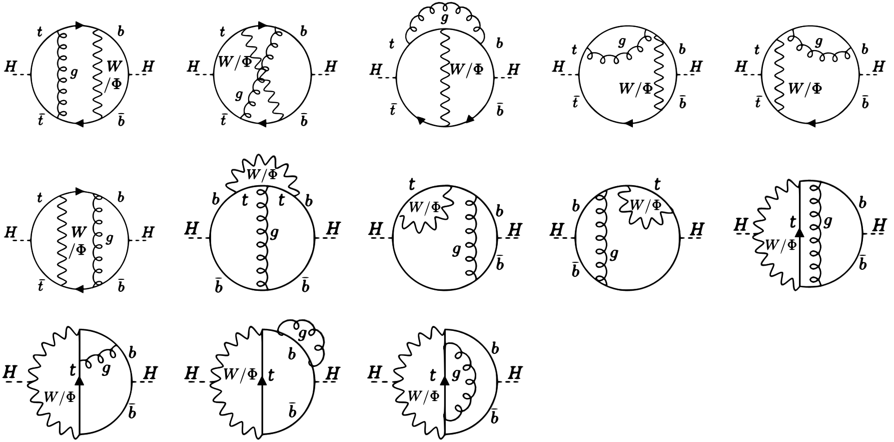


two-loop three-point | three-loop two-point | four topologies

$$d\vec{I} = A(\epsilon, x, y)\vec{I} \xrightarrow{\vec{J}=U\vec{I}} d\vec{J} = \epsilon\tilde{A}(x, y)\vec{J}$$

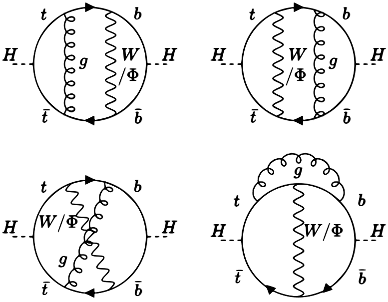


⇒ 13 pairs of three-loop two-point diagrams

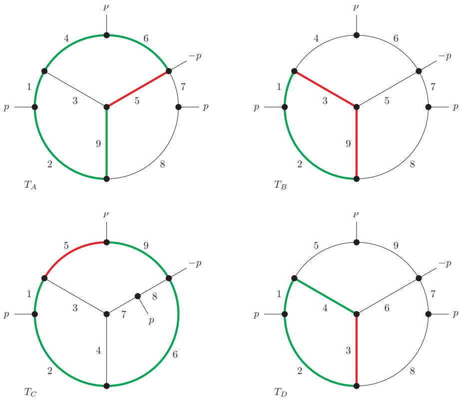
2 masses $m_t \& m_w$ ($\mathcal{O}\left(m_b^4\right) = 0$) and one external momentum

⇒ 4 auxiliary topologies with $\frac{1}{2}l(l+1) + el = 9$ propagators $p^2 = s$

The auxiliary topologies



⇒



2 masses m_t (green lines) & m_w (red lines), 1 ext. momentum $p^2 = s$

The integrals

$$I^X_{\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6\nu_7\nu_8\nu_9} = e^{3\epsilon\gamma_E}\left(\mu^2\right)^{\nu-\frac{3D}{2}} \int \frac{d^Dk_1}{i\pi^{\frac{D}{2}}}\frac{d^Dk_2}{i\pi^{\frac{D}{2}}}\frac{d^Dk_3}{i\pi^{\frac{D}{2}}} \prod_{j=1}^{9} \frac{1}{\left(P_j^X\right)^{\nu_j}},$$

$$P_j = -q_j^2 + m_j^2, \quad X \in \{A, B, C, D\}, \quad \nu = \sum_{j=1}^{9} \nu_j, \quad D = 4 - 2\epsilon$$

$\overset{kira}{\underset{Laporta}{\Rightarrow}}$ any integral expressible as linear combination of basis integrals

Topology	Total number MI's	independent MI's from this topology
A	50	50
B	66	50
C	95	37
D	67	1

$\Rightarrow$ 138 master integrals $\vec{I} = (I_1, I_2, \ldots, I_{138})^T$

S. Laporta, '01; J. Klappert, F. Lange, P. Maierhöfer, J. Usovitsch, '20

The system of differential equations

The master integrals $\vec{I} = (I_1, I_2, \ldots, I_{138})^T$

- depend on 2 dimensionless quantities $(\mu^2 = m_t^2)$:

$\rightarrow$ x, y rationalize
$\sqrt{-s(4m_t^2 - s)}$, $\sqrt{\lambda(s, m_W^2, m_t^2)}$; $\frac{s}{m_t^2} = -\frac{(1-x)^2}{x}$, $\frac{m_W^2}{m_t^2} = \frac{(1-y+2xy)(x-2y+xy)}{x(1-y^2)}$

E. Chaubey, S. Weinzierl, '19

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- fulfill the system of differential equations:

$$d\vec{I} = (A_x(\epsilon, x, y)dx + A_y(\epsilon, x, y)dy)\,\vec{I} = A(\epsilon, x, y)\vec{I}$$

with

$$\frac{\partial}{\partial x/y}\vec{I} = A_{x/y}\vec{I}, \qquad \frac{\partial}{\partial y}A_x - \frac{\partial}{\partial x}A_y - [A_y, A_x] = 0$$

$A_{x/y}$: 138x138 matrices; $[A_y, A_x] = 0$: Integrability

The ϵ-form of differential equations

$$d\vec{J} = \epsilon \tilde{A} \vec{J}, \qquad \tilde{A} = \sum C_j \omega_j, \qquad \vec{J}(\epsilon, x) = \sum_{i=0}^{\infty} \epsilon^i \vec{J}^{(i)}(x)$$

only ϵ

differential one-forms, only simple poles

$N_{master} \times N_{master}$ matrices, only algebraic numbers

$\Rightarrow$ Solvable in terms of iterated integrals

J. Henn, '13

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- We have: $A(\epsilon, x, y)$

1. Basis change: $\vec{J} = U\vec{I} \Rightarrow \epsilon\tilde{A} = UAU^{-1} - UdU^{-1}$
2. Coordinate transformation:
   - Two square roots already simultaneously rationalized
   - Two additional square roots in topology B: $\sqrt{s\,(4m_w^2 \pm s)}$, three for one sector+subsectors at maximum

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$\Rightarrow$ MPL's: $G(z_1, \ldots, z_k; y) = \int_0^y \frac{dy_1}{y_1 - z_1} \int_0^{y_1} \frac{dy_2}{y_2 - z_2} \ldots$

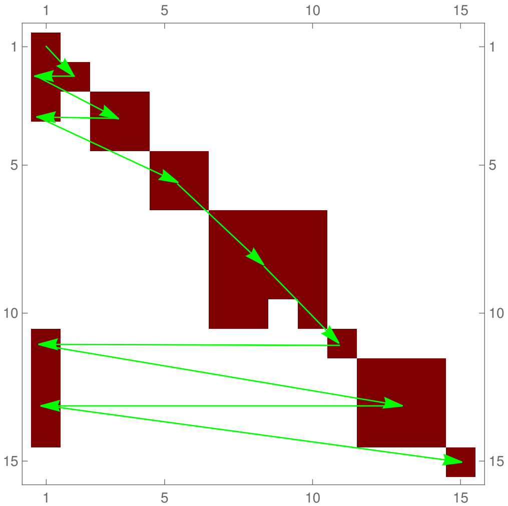
In search of a transformation: Block decomposition

- $d\vec{I} = A(\epsilon, x, y)\vec{I}$
  $\xrightarrow{\vec{J}=U\vec{I}}$
  $d\vec{J} = \epsilon\tilde{A}(x, y)\vec{J}$

- sector $\equiv$ integrals with common set $\{\nu_j > 0\}$

- Block decomposition:

$\rightarrow$ Treat sectors one by one



schematic representation of $A_{(1-15,1-15)}$

In search of a transformation for one sector

$$I \sim \int d^D k_1 d^D k_2 d^D k_3 \prod_{j=1}^{9} \left(P_j^X\right)^{-\nu_j}$$

↙ ↘

Baikov rep.: democratic

$I \sim \int d^9 z \mathcal{B}(z)^{\frac{D-l-e-1}{2}} \prod_{j=1}^{9} z_j^{-\nu_j}$

$\mathcal{B}(z) = \det G(k_1, k_2, k_3, p)$

Baikov rep.: loop by loop

$d^D k \sim \det G(\text{mom. ext. to loop})^{\frac{-D+e+1}{2}} *$

$\det G(k, \text{mom. ext. to loop})^{\frac{D-e-2}{2}} d^N z$

↘ ↙

Maximal Cut: cut all propagators with $\nu_j \neq 0$

$\frac{1}{z_j} \to 2\pi i \delta(z_j) \Rightarrow \text{MaxCut}\left(I_i\right) = \int_{\mathcal{C}_{MaxCut}} \varphi_i$

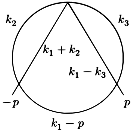
↓

Search $\mathcal{C}_i$ & modify φ_i → **Leading term of Laurent expansion** ($\epsilon = 0$) of $\int_{\mathcal{C}_i} \varphi_i'$ is **constant of weight zero**; π:weight 1, ϵ:weight -1

P. Baikov, '96, '97; H. Frellesvig, C. Papadopoulos, '17

Sector 334: first MI

- 5 propagators: $\nu_2,\ \nu_3,\ \nu_4,\ \nu_7,\ \nu_9 > 0$
- 3 master integrals
- Baikov rep. loop by loop $\rightarrow$ int. over 6 variables



$$\mathrm{MaxCut}\, I^{B}_{011100101}(2) = \int_{\mathcal{C}} \frac{32 i \pi^2 m_t^4\ dz_1}{\left(-m_t^2 + m_w^2 + z_1\right)^2 \sqrt{s\left(4m_t^2 - s\right) - 2 s z_1 - z_1^2}}$$

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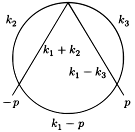

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- closed, anticlockwise contour around slit $[a, b]$: $\int_{\circlearrowleft} \frac{dz}{\sqrt{(z-a)(z-b)}} = 2\pi i$

$$\Rightarrow \int_{\circlearrowleft} dz_1 \frac{\left(-m_t^2+m_w^2+z_1\right)^2}{m_t^4}\, \varphi = \int_{\circlearrowleft} \frac{32\pi^2 dz_1}{\sqrt{\left(z_1-(-2m_t\sqrt{s}-s)\right)\left(z_1-(2m_t\sqrt{s}-s)\right)}} = 64\pi^3 i$$

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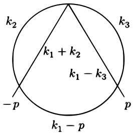

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$$J_1 = \epsilon^3 \left((1-w)^2 \mathbf{D}^- I^{B}_{011100101} - 2(1-w)\mathbf{D}^- I^{B}_{-111100101} + \mathbf{D}^- I^{B}_{-211100101} \right)$$

dimensional shift operator lowers dimension by 2

Sector 334: second MI

$$\text{MaxCut}\, I^{B}_{011100101}(2) = \int_{\mathcal{C}} \frac{32 i \pi^2 m_t^4 \; dz_1}{\left(-m_t^2 + m_w^2 + z_1\right)^2 \sqrt{s\left(4m_t^2 - s\right) - 2sz_1 - z_1^2}}$$

- small circle around z_0: $\int_{\circlearrowleft} \frac{f(z)dz}{z - z_0} = 2\pi i\, f(z_0)$

$$\Rightarrow \int_{\circlearrowleft} dz_1 \frac{\left(-m_t^2 + m_w^2 + z_1\right)}{m_t^4}\, \varphi = -\frac{64\pi^3 m_t^4}{\sqrt{s(4m_t^2 - s) - 2s(m_t^2 - m_w^2) - (m_t^2 - m_w^2)^2}}$$

Sector 334: second MI

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$$J_2 = \epsilon^3 \frac{(-1+x)(1+x-2y+2xy+y^2+xy^2)}{x(-1+y)(1+y)} \left((-1+w)\mathbf{D}^- I^B_{011100101} + \mathbf{D}^- I^B_{-111100101}\right)$$

Sector 334: third MI

- $\left(I^B_{011100101}, I^B_{-111100101}, I^B_{0111-10101}\right)^T \Rightarrow \vec{J'} = \left(J_1, J_2, \epsilon^3 \mathbf{D}^- I^B_{0111-10101}\right)^T$

$$\Rightarrow \frac{\partial}{\partial y}\vec{J'} = \epsilon \tilde{A}_y(x,y)\vec{J'}, \text{ but } \frac{\partial}{\partial x}\vec{J'} = \begin{pmatrix} \epsilon a_{11} & \epsilon a_{12} & \epsilon a_{13} \\ \epsilon a_{21} & \epsilon a_{22} & \epsilon a_{23} \\ \epsilon a_{31} & \epsilon a_{32} & \epsilon a_{33} + \frac{1-x^2}{x^3-x^2+x} \end{pmatrix} \vec{J'}$$

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- Transformation $U = Diag(1, 1, u(x))$
- $\left[U A_x(\epsilon, x, y) U^{-1} - U \frac{\partial}{\partial x} U^{-1}\right]_{(3,3)} = \epsilon a_{33} + \underbrace{\frac{1-x^2}{x^3-x^2+x} + \frac{u'(x)}{u(x)}}_{\overset{!}{=} 0}$

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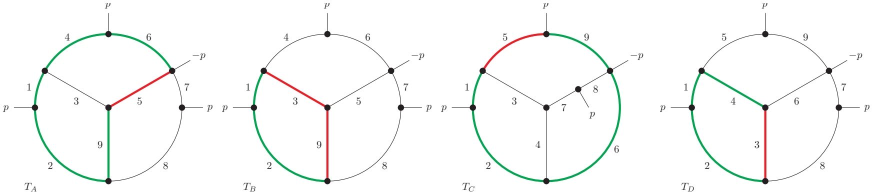
$$\Rightarrow u(x) = \exp\left(-\int \frac{1-x^2}{x^3-x^2+x}dx\right) = \frac{(1-x+x^2)}{x}$$

$$\Rightarrow J_3 = \epsilon^3 \frac{(1-x+x^2)}{x}\mathbf{D}^- I^B_{0111-10101}$$

$$\vec{J} = (J_1, J_2, J_3)^T, \quad d\vec{J} = \epsilon \tilde{A}(x,y)\vec{J}$$

Summary

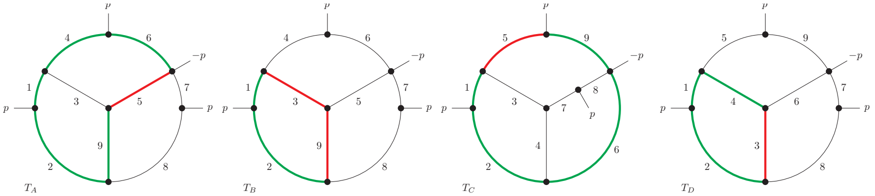
$$\Gamma(H \to b\bar{b}) = \Gamma^{(0)}(1 + \Delta^{(\alpha_s)} + \Delta^{(\alpha)} + \Delta_\gamma^{(\alpha\alpha_s)} + \Delta_Z^{(\alpha\alpha_s)} + \boldsymbol{\Delta_W^{(\alpha\alpha_s)}} + \cdots)$$



- 138 MI's kinematically depending on two dimensionless quantities

1. Basis change: $\vec{J} = U\vec{I} \Rightarrow d\vec{J} = \epsilon\tilde{A}(x,y)\vec{J}$
   - Block decomposition
   - Maximal cuts with constant leading singularities
2. Coordinate transformation (4 square roots, max. 3 for sector+sub.)

Thank you for your attention.



The decay rate $\Gamma(H \to b\bar{b})$

$$\Gamma(H \to b\bar{b}) = \Gamma^{(0)}(1 + \Delta^{(\alpha_s)} + \Delta^{(\alpha)} + \Delta^{(\alpha\alpha_s)} + \cdots),$$
$$\Delta^{(\alpha\alpha_s)} = \Delta_\gamma^{(\alpha\alpha_s)} + \Delta_Z^{(\alpha\alpha_s)} + \Delta_W^{(\alpha\alpha_s)}$$

Analytical:

$$\Gamma^{(0)} = \frac{N_C G_F m_H m_b^2}{4\sqrt{2}\pi}, \quad \Delta^{(\alpha_s)} = \frac{C_F \alpha_s}{\pi}\left(\frac{17}{4} - \frac{3}{2}\ln\left(\frac{m_H^2}{\mu^2}\right)\right), \quad \Delta^{(\alpha)} = \frac{Q_b^2 \alpha}{\pi}\left(\frac{17}{4} - \frac{3}{2}\ln\left(\frac{m_H^2}{\mu^2}\right)\right),$$
$$\Delta_\gamma^{(\alpha\alpha_s)} = \frac{C_F \alpha_s}{\pi}\frac{Q_b^2 \alpha}{\pi}\left(\frac{691}{32} - \frac{9}{2}\zeta_2 - \frac{9}{2}\zeta_3 - \frac{105}{8}\ln\left(\frac{m_H^2}{\mu^2}\right) + \frac{9}{4}\ln\left(\frac{m_H^2}{\mu^2}\right)^2\right)$$

Numerical:

$$\Delta^{(\alpha_s)} = 0.2040,\ \Delta^{(\alpha)} = 0.0011,\ \Delta_\gamma^{(\alpha\alpha_s)} = 0.0001,$$
$$\Delta_Z^{(\alpha\alpha_s)} \approx -0.002, \quad \Delta_W^{(\alpha\alpha_s)} \approx -0.0009$$

A.L. Kataev, ‘97; L. Mihaila, B. Schmidt, M. Steinhauser, ‘15

The optical theorem

- Decay rate of one particle state:

$$\Gamma(A \to X) = \frac{(2\pi)^4}{2m_A} \int d\varphi^{(X)} \delta^4(p_A - p_X) |M(A \to X)|^2$$

- Generalized optical theorem:

$$\text{Im}(M(A \to A)) = \frac{(2\pi)^4}{2} \sum_X \int d\varphi^{(X)} \delta^4(p_A - p_X) |M(A \to X)|^2$$

$\Rightarrow$ **Relation of decay width and Higgs boson self-energy**:

$$\Gamma(H \to b\bar{b}) = \frac{1}{m_H} \text{Im}(\Sigma_H(p^2 = m_H^2 + i\epsilon))$$

- Diagrams entering r.h.s have to end in $Hb\bar{b}$-vertex

Differential equations

$$\mu^2 \frac{\partial}{\partial m^2_{t/w}} I^X_{\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6\nu_7\nu_8\nu_9} = - \sum_{j \in J^X_{m^2_{t/w}}} \nu_j \, \mathbf{j}^+ I^X_{\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6\nu_7\nu_8\nu_9}$$

$\mathbf{j}^+$: raises $\nu_j \to \nu_j + 1$

$J^X_{m^2_{t/w}}$: edges carrying $m^2_{t/w}$

$$\sum_{x = m_w^2, m_t^2, s} x \frac{\partial}{\partial x} I^X_{\nu_1 \ldots \nu_9} = \left(\frac{3D}{2} - \nu \right) I^X_{\nu_1 \ldots \nu_9} \longrightarrow \mu^2 \frac{\partial}{\partial s} I^X_{\nu_1 \ldots \nu_9}$$

$\overset{IBP}{\Rightarrow}$ Differential equations for master integrals

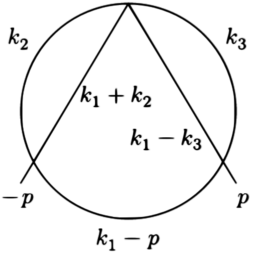
- $\mu^2 = m_t^2 \Rightarrow$ Integrals depend on 2 dimensionless quantities

$$v = \frac{s}{m_t^2}, \quad w = \frac{m_w^2}{m_t^2} \longrightarrow \frac{\partial}{\partial w} \vec{I} = A_w \vec{I}, \quad \frac{\partial}{\partial v} \vec{I} = A_v \vec{I}$$

Computing master integrals $\longrightarrow$ Solving differential equations

Example from Topology B: sector 334

- Sector 334: $\nu_2,\ \nu_3,\ \nu_4,\ \nu_7,\ \nu_9 > 0$
- 3 master integrals, $\vec{I} = (I^B_{011100101}, I^B_{-111100101}, I^B_{0111-10101})$



Baikov representation loop by loop:

1. k_3, momentum external to loop: k_1
2. k_2, momentum external to loop: k_1
3. k_1, momentum external to loop: p

$\Rightarrow$ integration over 6 Baikov variables

MaxCut $\Rightarrow$ integration over one variable left

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