

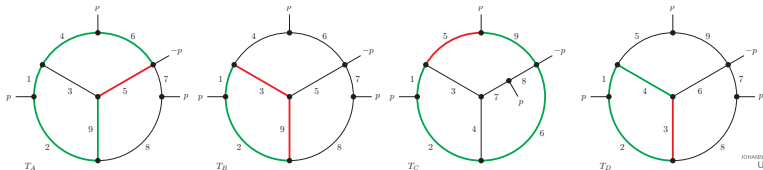
Three-loop master integrals for the mixed QCD-electroweak correction to $H \rightarrow b\bar{b}$

Ina Hönnemann¹

in collaboration with Ekta Chaubey² and Stefan Weinzierl¹

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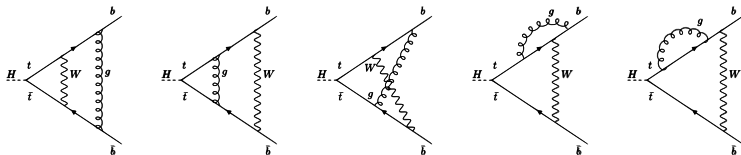
Predominant decay mode of the Higgs boson: $H \rightarrow b\bar{b}$

$$\Gamma(H \rightarrow b\bar{b}) = \Gamma^{(0)}(1 + \Delta^{(\alpha_s)} + \Delta^{(\alpha)} + \Delta^{(\alpha\alpha_s)} + \dots)$$

$$\Delta^{(\alpha\alpha_s)} = \underbrace{\Delta^{(\alpha\alpha_s)}}_{1 \text{ int. mass}} + \underbrace{\Delta_Z^{(\alpha\alpha_s)}}_{0.002} + \underbrace{\Delta_W^{(\alpha\alpha_s)}}_{0.0009} \leftarrow 3 \text{ int. masses}$$

A. Kataev, '97; L. Mihaila, B. Schmidt, M. Steinhauser, '15

Future colliders \rightarrow need for precise theoretical predictions
 \rightarrow importance of analytic computations



Relation of decay width and Higgs boson self-energy:

$$\Gamma(H \rightarrow b\bar{b}) = \frac{1}{m_H} \text{Im}(\Sigma_H(p^2 = m_H^2 + i\epsilon))$$

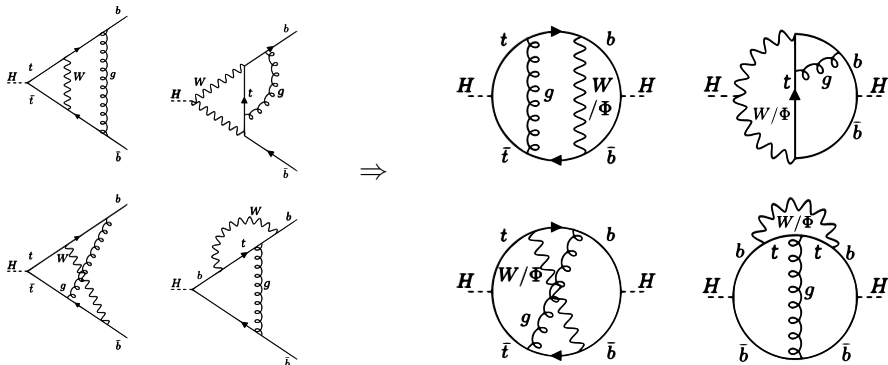
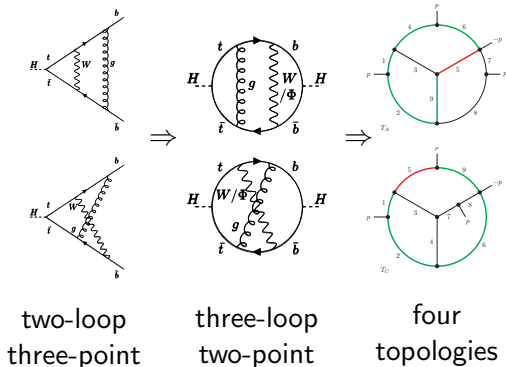


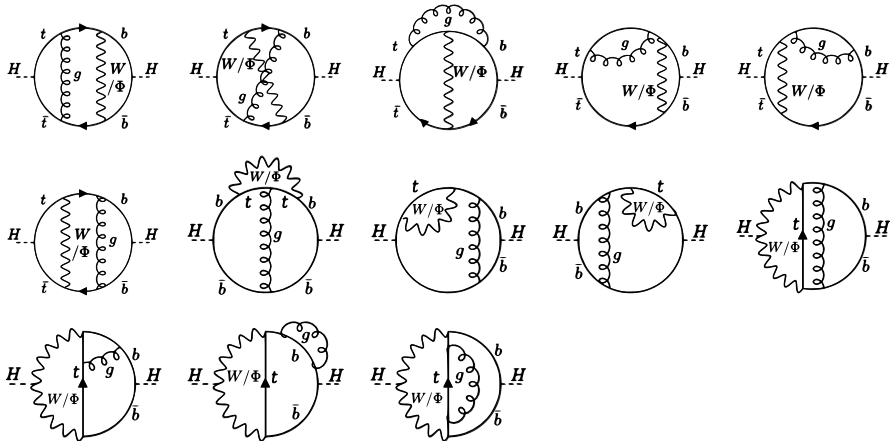
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- 2 The contributing diagrams
 - The topologies
- 3 The differential equations
 - The ϵ -form
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 - Example sector



$$d\vec{I} = A(\epsilon, x, y) \vec{I} \xrightarrow{\vec{J}=U\vec{I}} d\vec{J} = \epsilon \tilde{A}(x, y) \vec{J}$$

Contributing diagrams

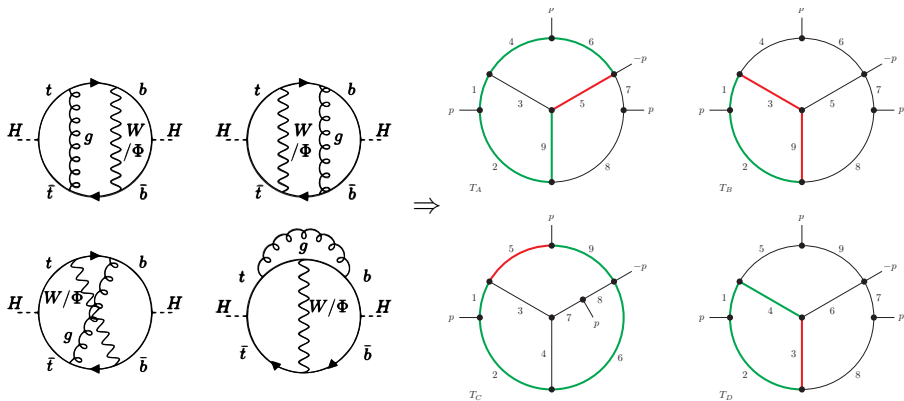


⇒ 13 pairs of three-loop two-point diagrams

2 masses m_t & m_w ($\mathcal{O}(m_b^4) = 0$) and one external momentum

⇒ 4 auxiliary topologies with $\frac{1}{2}l(l+1) + eI = 9$ propagators $p^2 = s$

The auxiliary topologies



2 masses m_t (green lines) & m_W (red lines), 1 ext. momentum $p^2 = s$

The integrals

$$I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7 \nu_8 \nu_9}^X = e^{3\epsilon\gamma_E} (\mu^2)^\nu \frac{3D}{2} \int \frac{d^D k_1}{i\pi^{\frac{D}{2}}} \frac{d^D k_2}{i\pi^{\frac{D}{2}}} \frac{d^D k_3}{i\pi^{\frac{D}{2}}} \prod_{j=1}^9 \frac{1}{(P_j^X)^{\nu_j}},$$

$$P_j = q^2 + m_j^2, \quad X \in \{A, B, C, D, G\}, \quad \nu = \sum_{j=1}^9 \nu_j, \quad D=4-2\epsilon$$

$\xRightarrow{\text{kira}}$ any integral expressible as linear combination of basis integrals
Laporta

Topology	Total number MI's	independent MI's from this topology
A	50	50
B	66	50
C	95	37
D	67	1

\Rightarrow 138 master integrals $\vec{l} = (l_1, l_2, \dots, l_{138})^T$

S. Laporta, '01; J. Klappert, F. Lange, P. Maierhöfer, J. Usovitsch, '20

The system of differential equations

The master integrals $\vec{l} = (l_1, l_2, \dots, l_{138})^T$

- depend on 2 dimensionless quantities ($\mu^2 = m_t^2$):

→ x, y rationalize

$$\sqrt{s(4m_t^2 - s)}, \sqrt{\lambda(s, m_W^2, m_t^2)}; \quad \frac{s}{m_t^2} = \frac{(1-x)^2}{x}, \quad \frac{m_W^2}{m_t^2} = \frac{(1-y+2xy)(x-2y+xy)}{x(1-y^2)}$$

E. Chaubey, S. Weinzierl, '19

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E. Chaubey, S. Weinzierl, '19

- fulfill the system of differential equations:

$$d\vec{l} = (A_x(\epsilon, x, y)dx + A_y(\epsilon, x, y)dy) \vec{l} = A(\epsilon, x, y) \vec{l}$$

with

$$\frac{\partial}{\partial x/y} \vec{l} = A_{x/y} \vec{l}, \quad \frac{\partial}{\partial y} A_x - \frac{\partial}{\partial x} A_y - [A_y, A_x] = 0$$

138x138 matrices Integrability

The ϵ -form of differential equations

$$d\vec{J} = \epsilon \tilde{A} \vec{J}, \quad \tilde{A} = \sum C_j \omega_j, \quad \vec{J}(\epsilon, x) = \sum_{i=0}^1 \epsilon^i \vec{J}^{(i)}(x)$$

only ϵ

$N_{master} \times N_{master}$ matrices, only algebraic numbers

differential one-forms, only simple poles

\Rightarrow Solvable in terms of iterated integrals

J. Henn, '13

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$N_{\text{master}} \times N_{\text{master}}$ matrices, only algebraic numbers

⇒ Solvable in terms of iterated integrals

J. Henn, '13

• We have: $A(\epsilon, x, y)$

1. Basis change: $\vec{J} = U \vec{I} \Rightarrow \epsilon \tilde{A} = U A U^{-1} - U dU^{-1}$

2. Coordinate transformation:

- Two square roots already simultaneously rationalized
- Two additional square roots in topology B: $\sqrt{s(4m_w^2 \pm s)}$,
three for one sector+subsectors at maximum

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\Rightarrow MPL's: $G(z_1, \dots, z_k; y) = \int_0^y \frac{dy_1}{y_1 z_1} \int_0^{y_1} \frac{dy_2}{y_2 z_2} \dots$

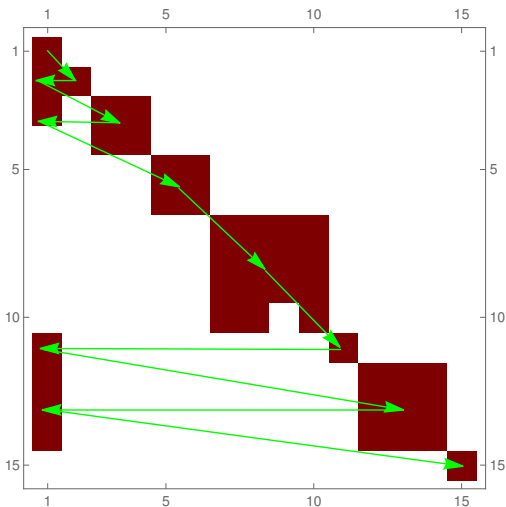
In search of a transformation: Block decomposition

- $d\vec{l} = A(\epsilon, x, y)\vec{l}$
 $\vec{J} = U\vec{l}$
 $\xrightarrow{\quad}$
 $d\vec{J} = \epsilon\tilde{A}(x, y)\vec{J}$

- sector \equiv integrals with common set $\{\nu_j > 0\}$

- Block decomposition:

→ Treat sectors one by one



schematic representation of $A_{(1 \ 15, 1 \ 15)}$

In search of a transformation for one sector

$$I \int d^D k_1 d^D k_2 d^D k_3 \prod_{j=1}^9 (P_j^X)^{-\nu_j}$$

Baikov rep.: democratic

$$I \int d^9 z B(z)^{\frac{D-l-e-1}{2}} \prod_{j=1}^9 z_j^{-\nu_j}$$

$$B(z) = \det G(k_1, k_2, k_3, p)$$

Baikov rep.: loop by loop

$$d^D k \det G(\text{mom. ext. to loop})^{\frac{-D+e+1}{2}}$$

$$\det G(k, \text{mom. ext. to loop})^{\frac{D-e-2}{2}} d^N z$$

Maximal Cut: cut all propagators with $\nu_j \neq 0$

$$\frac{1}{z_j} \rightarrow 2\pi i \delta(z_j) \Rightarrow \text{MaxCut}(I_i) = \int_{C_{\text{MaxCut}}} \varphi_i$$

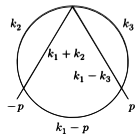


Search C_i & modify $\varphi_i \rightarrow$ **Leading term of Laurent expansion** ($\epsilon = 0$)
of $\int_{C_i} \varphi_i^0$ is **constant of weight zero**; π : weight 1, ϵ : weight -1

P. Baikov, '96, '97; H. Frellesvig, C. Papadopoulos, '17

Sector 334: first MI

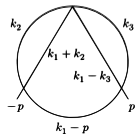
- 5 propagators: $\nu_2, \nu_3, \nu_4, \nu_7, \nu_9 > 0$
- 3 master integrals
- Baikov rep. loop by loop \rightarrow int. over 6 variables



$$\text{MaxCut} I_{0111100101}^B(2) = \int_C \frac{32i\pi^2 m_t^4 dz_1}{(-m_t^2 + m_w^2 + z_1)^2 \sqrt{s(4m_t^2 - s) - 2sz_1 - z_1^2}}$$

Sector 334: first MI

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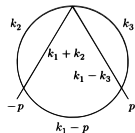
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- closed, anticlockwise contour around slit $[a, b]$: $\int \frac{dz}{\sqrt{(z-a)(z-b)}} = 2\pi i$

$$\Rightarrow \int dz_1 \frac{(m_t^2 + m_w^2 + z_1)^2}{m_t^4} \varphi = \int \frac{32\pi^2 dz_1}{\sqrt{(z_1 - 2m_t^2/s)(z_1 - (2m_t^2/s + s))}} = 64\pi^3 i$$

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- 5 propagators: $\nu_2, \nu_3, \nu_4, \nu_7, \nu_9 > 0$
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$$\text{MaxCut} I_{011100101}^B(2) = \int_C \frac{32i\pi^2 m_t^4 dz_1}{(-m_t^2 + m_w^2 + z_1)^2 \sqrt{s(4m_t^2 - s) - 2sz_1 - z_1^2}}$$

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$$J_1 = \epsilon^3 \left((1-w)^2 \mathbf{D} I_{011100101}^B - 2(1-w) \mathbf{D} I_{111100101}^B + \mathbf{D} I_{211100101}^B \right)$$

\uparrow dimensional shift operator lowers dimension by 2

$$\text{MaxCut}I_{011100101}^B(2) = \int_C \frac{32i\pi^2 m_t^4 dz_1}{(-m_t^2 + m_w^2 + z_1)^2 \sqrt{s(4m_t^2 - s) - 2sz_1 - z_1^2}}$$

• small circle around z_0 : $\int \frac{f(z)dz}{z - z_0} = 2\pi i f(z_0)$

$$\Rightarrow \int dz_1 \frac{(m_t^2 + m_w^2 + z_1)}{m_t^4} \varphi = -\frac{64\pi^3 m_t^4}{\sqrt{s(4m_t^2 - s)} 2s(m_t^2 - m_w^2) (m_t^2 - m_w^2)^2}$$

$$\text{MaxCut} I_{011100101}^B(2) = \int_C \frac{32i\pi^2 m_t^4 dz_1}{(-m_t^2 + m_w^2 + z_1)^2 \sqrt{s(4m_t^2 - s) - 2sz_1 - z_1^2}}$$

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$$J_2 = \epsilon^3 \frac{(-1+x)(1+x-2y+2xy+y^2+xy^2)}{x(-1+y)(1+y)} \\ \left((-1+w)\mathbf{D} I_{011100101}^B + \mathbf{D} I_{111100101}^B \right)$$

$$\bullet (I_{0111100101}^B, I_{-1111100101}^B, I_{01111-10101}^B)^T \Rightarrow \vec{J}^{\flat} = (J_1, J_2, \epsilon^3 \mathbf{D} \quad I_{01111 \quad 10101}^B)^T$$

$$\Rightarrow \frac{\partial}{\partial y} \vec{J}^{\flat} = \epsilon \tilde{A}_y(x, y) \vec{J}^{\flat}, \text{ but } \frac{\partial}{\partial x} \vec{J}^{\flat} = \begin{pmatrix} \epsilon a_{11} & \epsilon a_{12} & \epsilon a_{13} \\ \epsilon a_{21} & \epsilon a_{22} & \epsilon a_{23} \\ \epsilon a_{31} & \epsilon a_{32} & \epsilon a_{33} + \frac{1}{x^3} \frac{x^2}{x^2+x} \end{pmatrix} \vec{J}^{\flat}$$

$$\bullet (I_{0111100101}^B, I_{-111100101}^B, I_{0111-10101}^B)^T \Rightarrow \vec{J}^\flat = (J_1, J_2, \epsilon^3 \mathbf{D} \ I_{0111}^B \ 10101)^T$$

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$$\bullet \text{ Transformation } U = \text{Diag}(1, 1, u(x))$$

$$\bullet \left[U A_x(\epsilon, x, y) U^{-1} - U \frac{\partial}{\partial x} U^{-1} \right]_{(3,3)} = \epsilon a_{33} + \underbrace{\frac{1 \cdot x^2}{x^3} \frac{x^2}{x^2+x} + \frac{u'(x)}{u(x)}}_{\stackrel{!}{=} 0}$$

$$\bullet (I_{0111100101}^B, I_{-111100101}^B, I_{0111-10101}^B)^T \Rightarrow \vec{J}^0 = (J_1, J_2, \epsilon^3 \mathbf{D} \ I_{0111 \ 10101}^B)^T$$

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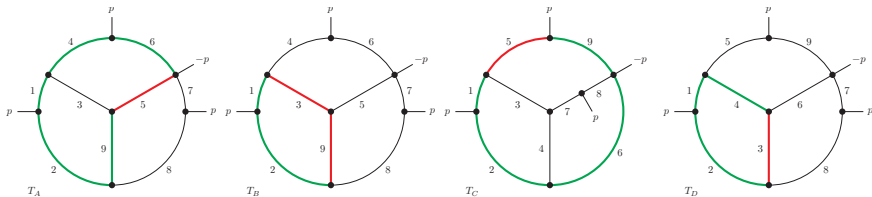
$$\Rightarrow u(x) = \exp\left(-\int \frac{1 \cdot x^2}{x^3} \frac{x^2}{x^2+x} dx\right) = \frac{(1-x-x^2)}{x}$$

$$\Rightarrow J_3 = \epsilon^3 \frac{(1-x-x^2)}{x} \mathbf{D} \ I_{0111 \ 10101}^B$$

$$\vec{J} = (J_1, J_2, J_3)^T, \quad d\vec{J} = \epsilon \tilde{A}(x, y) \vec{J}$$

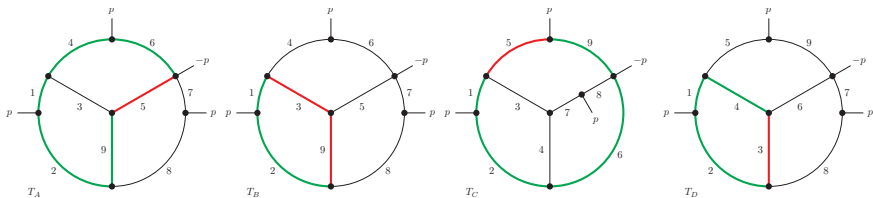
Summary

$$\Gamma(H \rightarrow b\bar{b}) = \Gamma^{(0)}(1 + \Delta(\alpha_s) + \Delta(\alpha) + \Delta_\gamma^{(\alpha\alpha_s)} + \Delta_Z^{(\alpha\alpha_s)} + \Delta_W^{(\alpha\alpha_s)} + \dots)$$



- 138 MI's kinematically depending on two dimensionless quantities
1. Basis change: $\vec{J} = U\vec{I} \Rightarrow d\vec{J} = \epsilon\tilde{A}(x, y)\vec{J}$
 - Block decomposition
 - Maximal cuts with constant leading singularities
 2. Coordinate transformation (4 square roots, max. 3 for sector+sub.)

Thank you for your attention.



The decay rate $\Gamma(H \rightarrow b\bar{b})$

$$\Gamma(H \rightarrow b\bar{b}) = \Gamma^{(0)}(1 + \Delta^{(\alpha_s)} + \Delta^{(\alpha)} + \Delta^{(\alpha\alpha_s)} + \dots),$$
$$\Delta^{(\alpha\alpha_s)} = \Delta_\gamma^{(\alpha\alpha_s)} + \Delta_Z^{(\alpha\alpha_s)} + \Delta_W^{(\alpha\alpha_s)}$$

Analytical:

$$\Gamma^{(0)} = \frac{N_C G_F m_H m_b^2}{4\sqrt{2}\pi}, \quad \Delta^{(\alpha_s)} = \frac{C_F \alpha_s}{\pi} \left(\frac{17}{4} - \frac{3}{2} \ln\left(\frac{m_H^2}{\mu^2}\right) \right), \quad \Delta^{(\alpha)} = \frac{Q_b^2 \alpha}{\pi} \left(\frac{17}{4} - \frac{3}{2} \ln\left(\frac{m_H^2}{\mu^2}\right) \right),$$
$$\Delta_\gamma^{(\alpha\alpha_s)} = \frac{C_F \alpha_s}{\pi} \frac{Q_b^2 \alpha}{\pi} \left(\frac{691}{32} - \frac{9}{2} \zeta_2 - \frac{9}{2} \zeta_3 - \frac{105}{8} \ln\left(\frac{m_H^2}{\mu^2}\right) + \frac{9}{4} \ln\left(\frac{m_H^2}{\mu^2}\right)^2 \right)$$

Numerical:

$$\Delta^{(\alpha_s)} = 0.2040, \quad \Delta^{(\alpha)} = 0.0011, \quad \Delta_\gamma^{(\alpha\alpha_s)} = 0.0001,$$
$$\Delta_Z^{(\alpha\alpha_s)} \approx -0.002, \quad \Delta_W^{(\alpha\alpha_s)} \approx -0.0009$$

A.L. Kataev, '97; L. Mihaila, B. Schmidt, M. Steinhauser, '15

The optical theorem

- Decay rate of one particle state:

$$\Gamma(A \rightarrow X) = \frac{(2\pi)^4}{2m_A} \int d\varphi^{(X)} \delta^4(p_A - p_X) |M(A \rightarrow X)|^2$$

- Generalized optical theorem:

$$\text{Im}(M(A \rightarrow A)) = \frac{(2\pi)^4}{2} \sum_X \int d\varphi^{(X)} \delta^4(p_A - p_X) |M(A \rightarrow X)|^2$$

⇒ **Relation of decay width and Higgs boson self-energy:**

$$\Gamma(H \rightarrow b\bar{b}) = \frac{1}{m_H} \text{Im}(\Sigma_H(p^2 = m_H^2 + i\epsilon))$$

- Diagrams entering r.h.s have to end in $Hb\bar{b}$ -vertex

$$\mu^2 \frac{\partial}{\partial m_{t/w}^2} I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7 \nu_8 \nu_9}^X = - \sum_{j \in 2J_{m_{t/w}^2}^X} \nu_j j^+ I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7 \nu_8 \nu_9}^X$$

\uparrow raises $\nu_j \rightarrow \nu_j + 1$
 \uparrow edges carrying $m_{t/w}^2$

$$\sum_{x=m_w^2, m_t^2, s} x \frac{\partial}{\partial x} I_{\nu_1 \dots \nu_9}^X = \left(\frac{3D}{2} - \nu \right) I_{\nu_1 \dots \nu_9}^X \longrightarrow \mu^2 \frac{\partial}{\partial s} I_{\nu_1 \dots \nu_9}^X$$

\Rightarrow IBP Differential equations for master integrals

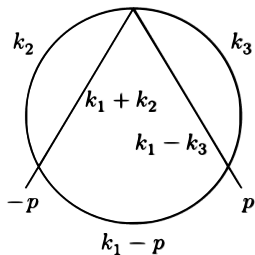
- $\mu^2 = m_t^2 \Rightarrow$ Integrals depend on 2 dimensionless quantities

$$\nu = \frac{s}{m_t^2}, \quad w = \frac{m_w^2}{m_t^2} \longrightarrow \frac{\partial}{\partial w} \vec{I} = A_w \vec{I}, \quad \frac{\partial}{\partial \nu} \vec{I} = A_\nu \vec{I}$$

Computing master integrals \longrightarrow Solving differential equations

Example from Topology B: sector 334

- Sector 334: $\nu_2, \nu_3, \nu_4, \nu_7, \nu_9 > 0$
- 3 master integrals, $\vec{I} = (I_{011100101}^B, I_{111100101}^B, I_{0111 10101}^B)$



Baikov representation loop by loop:

1. k_3 , momentum external to loop: k_1
 2. k_2 , momentum external to loop: k_1
 3. k_1 , momentum external to loop: p
- \Rightarrow integration over 6 Baikov variables

MaxCut \Rightarrow integration over one variable left



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