Matter Bounce Scenario in Extended Symmetric Teleparallel Gravity

A.S. Agrawal
Department of Mathematics
BITS-Pilani, Hyderabad Campus, India
E-mail: agrawalamar61@gmail.com

Collaborators
B. Mishra, P.K. Agrawal

The 16th Workshop on the Dark Side of the Universe (DSU-2022)

Dec 08,2022

Outline of Presentation

- Introduction
- $f(Q)$ Gravity Field Equations
- Matter Bounce Scenario in $f(Q)$ Gravity
- Phase Space Analysis
- Scalar Perturbation
- Conclusion

Introduction

• Observational evidences suggest that the Universe had undergone an exponential expansion phase in the early Universe, known as the inflation phase.[1]

• During the inflationary phase, the Universe grew exponentially, expanded rapidly and in a short span of time attained an immense size.

• Geometrically, the expansion rate along the spatial directions can be obtained through the scale factor $a(t)$ and the evolution of Hubble parameter is based on the scale factor as, $H = \dot{a}(t)/a(t)$. So, there are two possibilities:

i) The scale factor attains a value zero, that leads to the big bang singularity or the space time curvature singularity.

ii) The bouncing behaviour i.e. without attain the singularity, the evolution would increase again, which is an early Universe era. Since the scale factor never zero, the space time singularity would never occur. The bounce happens when H vanishes and $\dot{H} > 0$.

[1] R. Brout, F. Englert, E. Gunzig, Ann. Phys., **115**, 78 (1978); A.A. Starobinsky, Phys. Lett. B, **91**, 99 (1980); A.H. Guth, Phys. Rev. D, **23**, 347 (1981).

Introduction

• A quantum theory of gravity may avoid such a initial cosmological singularity, but we don't know what is the correct quantum theory of gravity. However, In the absence of fully accepted quantum gravity, bounce cosmology is the most promising one that allows a non-singular Universe.

• At the time of bounce, the following conditions has to be hold

$$a \neq 0 \qquad \dot{a} = 0 \qquad \ddot{a} > 0$$

• Bouncing cosmology can be derived as a cosmological solution of loop quantum cosmology (LQC).[2]

• The extended symmetric teleparallel gravity, namely $f(Q)$ gravity is another geometrical modified theories of gravity that has been recently formulated using the non-metricity approach.[3]

• The matter bounce scenario motivated with the loop quantum cosmology in $f(Q)$ gravity would be investigated.

[2] A. Ashtekar, T. Pawlowski, P. Singh, Phys. Rev. D, **74**, 084003 (2006); M. Sami, P. Singh, S. Tsujikawa, Phys. Rev. D, **74**, 043514 (2006).

[3] J. B. Jimenez, L. Heisenberg, T. Koivisto, Phys. Rev. D, 98, 044048 (2018).

f(Q) Gravity

The metric affine connection can be expressed in three independent components as [4],

$$\Gamma^{\alpha}_{\mu\nu} = \{^{\alpha}{}_{\mu\nu}\} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu} \tag{1}$$

where the three terms on the R.H.S. denotes the Levi-Civita Connection, Contortion and the disformation tensor respectively and can be expressed as,

$$\begin{aligned} \{^{\alpha}{}_{\mu\nu}\} &\equiv \frac{1}{2}g^{\alpha\beta}\left(\partial_{\mu}g_{\beta\nu} + \partial_{\nu}g_{\beta\mu} - \partial_{\beta}g_{\mu\nu}\right) \\ K^{\alpha}{}_{\mu\nu} &\equiv \frac{1}{2}T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)}; \qquad T^{\alpha}{}_{\mu\nu} \equiv 2\Gamma^{\alpha}_{[\mu\nu]} \\ L^{\alpha}{}_{\mu\nu} &\equiv \frac{1}{2}Q^{\alpha}{}_{\mu\nu} - Q_{(\mu\nu)}{}^{\alpha}. \end{aligned} \tag{2}$$

[4] F. W. Hehl, J. D. McCrea, E. W. Mielke, Y. Neeman, Phys. Rep. 258, 1 (1995), T. Ortin, Gravity and Strings (Cambridge University Press, Cambridge, England, 2015).

f(Q) Gravity

The super potential of the model ,

$$P^{\alpha}{}_{\mu\nu} = -\frac{1}{2}L^{\alpha}{}_{\mu\nu} + \frac{1}{4}\left(Q^{\alpha} - \tilde{Q}^{\alpha}\right)g_{\mu\nu} - \frac{1}{4}\delta^{\alpha}_{(\mu}Q_{\nu)}, \tag{3}$$

where $Q_{\alpha} = g^{\mu\nu}Q_{\alpha\mu\nu}$ and $\tilde{Q}_{\alpha} = g^{\mu\nu}Q_{\mu\alpha\nu}$ with $Q_{\alpha\mu\nu}$ be the nonmetricity tensor.
The nonmetricity scalar,

$$Q = -Q_{\alpha\mu\nu}P^{\alpha\mu\nu} \tag{4}$$

The Field Equations

The action of $f(Q)$ gravity,

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}f(Q) + \mathcal{L}_M\right), \tag{5}$$

The field equations of $f(Q)$ gravity,

$$\frac{2}{\sqrt{-g}}\nabla_\alpha\left(\sqrt{-g}f_Q P^\alpha{}_{\mu\nu}\right) + \frac{1}{2}g_{\mu\nu}f + f_Q\left(P_{\mu\alpha\beta}Q_\nu{}^{\alpha\beta} - 2Q_{\alpha\beta\mu}P^{\alpha\beta}{}_\nu\right) = T_{\mu\nu} \tag{6}$$

The energy momentum tensor,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_m}{\delta g^{\mu\nu}} \tag{7}$$

The homogeneous and isotropic FLRW space time,

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{8}$$

The field equations,

$$6f_Q H^2 - \frac{1}{2}f \quad = \quad \rho \tag{9}$$

$$\left(12H^2 f_{QQ} + f_Q\right)\dot{H} \quad = \quad -\frac{1}{2}(\rho + p) \tag{10}$$

Matter Bounce Scenario in $f(Q)$ Gravity

As mentioned in (LQC)[5] theory Hubble squared term take the form

$$H^2 = \frac{\rho_m(\rho_c - \rho_m)}{3\rho_c} \tag{11}$$

The matter energy density and critical energy density are represented respectively as ρ_m and ρ_c. Also, the critical energy density,

$$\rho_c = (c^2\sqrt{3})/(32\pi^2\gamma^3 G_N l_p^2), \tag{12}$$

where, $\gamma = 0.2375$ and $l_p = \sqrt{\hbar G_N/c^3}$ are respectively the Barbero-Immirzi parameter and the Planck length. We use the Planck units, $c = \hbar = G_N = 1$.

[5] A. Ashtekar, T. Pawlowski, P. Singh, Phys. Rev. D, **74**, 084003 (2006); M. Sami, P. Singh, S. Tsujikawa, Phys. Rev. D, **74**, 043514 (2006).

Matter Bounce Scenario in $f(Q)$ Gravity

The continuity equation and the energy density equations for the matter dominated case can be written as

$$\dot{\rho}_m = -3H\rho_m \qquad \text{and} \qquad \rho_m = \rho_{m0}a^{-3} \tag{13}$$

Now, the scale factor, $a(t) \propto t^{2/3}$ for the matter dominated case.

$$\rho_m = \frac{\rho_c}{\left(\frac{3}{4}\rho_c t^2 + 1\right)}, \qquad H(t) = \frac{2\rho_c t}{3\rho_c t^2 + 4}, \qquad a(t) = \left(\frac{3}{4}\rho_c t^2 + 1\right)^{\frac{1}{3}} \tag{14}$$

$$H^2 = \frac{\rho_c}{3}\left(\frac{1}{a^3} - \frac{1}{a^6}\right) \tag{15}$$

Using the relation between the e-folding parameter and the scale factor, $e^{-N} = \frac{a_0}{a}$,

$$H^2 = \frac{\rho_c}{3a_0^3}\left(e^{-3N} - \frac{e^{-6N}}{a_0^3}\right) \tag{16}$$

Matter Bounce Scenario in $f(Q)$ Gravity

We assume following quantities,

$$A = \frac{\rho_c}{3a_0^3}, \qquad b = \frac{1}{a_0^3}. \tag{17}$$

From eqn. (16), the nonmetricity scalar in the form of e-folding parameter as,

$$Q = 6H^2 = 6A\left[e^{-3N} - be^{-6N}\right] \tag{18}$$

On solving,

$$N = -\frac{1}{3}Log\left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{6Ab}\right) \tag{19}$$

In addition, we assume that the energy density (9) is of form,

$$\rho = \sum_i \rho_{i0} a_0^{-3(1+\omega_i)} e^{-3N(1+\omega_i)} \tag{20}$$

Matter Bounce Scenario in $f(Q)$ Gravity

By setting $S_i = \rho_{i0} a_0^{-3(1+\omega_i)}$, the matter energy density becomes

$$\rho_m = \left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{6Ab} \right) \tag{21}$$

We consider the Universe is filled with dust fluid only, Substituting eqn. (21) in eqn. (9), we get

$$Qf_Q - \frac{1}{2} f - \sum_i S_i \left(\frac{3A + \sqrt{9A^2 - 6AbQ}}{2Ab} \right) = 0 \tag{22}$$

On solving, we get

$$f(Q) = -\sqrt{\rho_c(\rho_c - 2Q)} - \sqrt{2\rho_c Q} \arcsin\left(\frac{\sqrt{2}\sqrt{Q}}{\sqrt{\rho_c}} \right) - \rho_c, \tag{23}$$

- The above form of $f(Q)$ produces the matter bounce evolution of the Universe.

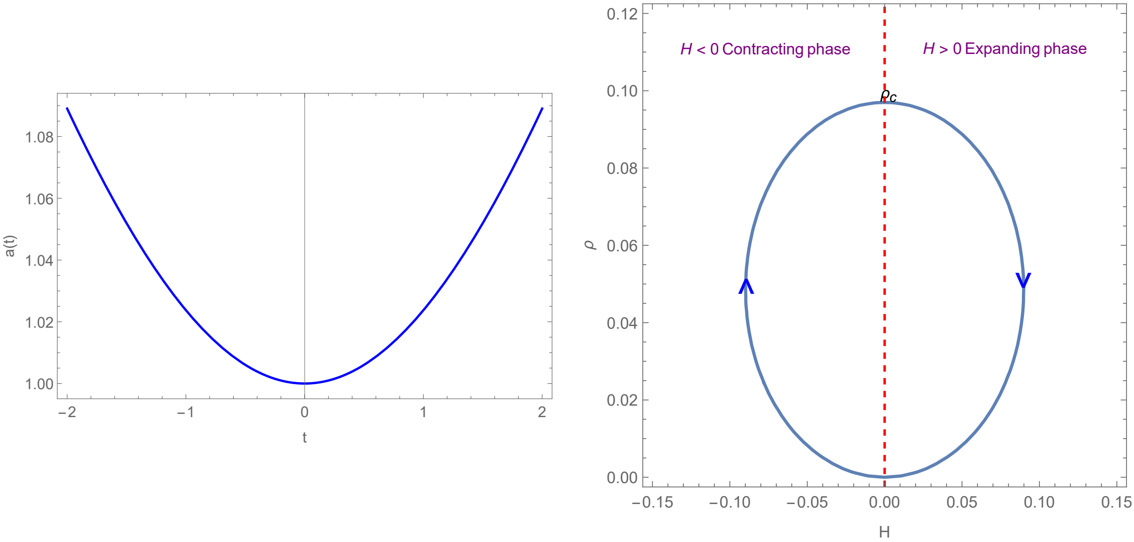


Figure: Scale factor vs cosmic time (left panel) and Energy density in Hubble parameter (right panel).

Phase Space Analysis

The general form of $f(Q)$ as $Q+\psi(Q)$ and accordingly,

$$3H^2 = \rho + \frac{\psi}{2} - Q\psi_Q \quad (24)$$

$$2\dot{H} + 3H^2 = -p - 2\dot{H}(2Q\psi_{QQ} + \psi_Q) + \left(\frac{\psi}{2} - Q\psi_Q\right) \quad (25)$$

The density parameters $\Omega_m = \frac{\rho_m}{3H^2}$, $\Omega_r = \frac{\rho_r}{3H^2}$ and $\Omega_{de} = \frac{\rho_{de}}{3H^2}$ with $\Omega_m + \Omega_r + \Omega_{de} = 1$. We consider the dimensionless variables,

$$x = \frac{\psi - 2Q\psi_Q}{6H^2} \qquad y = \frac{\rho_r}{3H^2}. \quad (26)$$

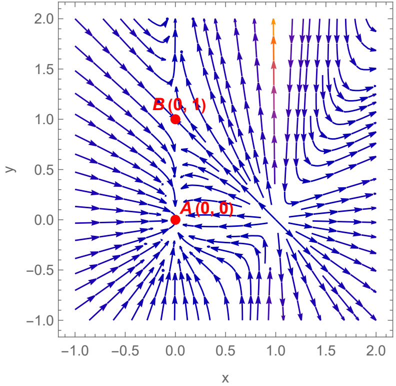
Now the dimensionless variables can be represented as,

$$x' = x\,(3(x-1) - y) \quad (27)$$

$$y' = -\frac{y(x(-3x+y+4)+y-1)}{x-1} \quad (28)$$

Note: prime denotes the derivative with respect to lna

Phase Space Analysis



(x,y)	Ω_m	Ω_r	Ω_{de}	ω_{eff}	(q)	Eigenvalues	Stability
$A(0,0)$	1	0	0	0	1/2	{-3,-1}	Stable Node
$B(0,1)$	0	1	0	1/3	1	{-4,1}	Unstable

Scalar Perturbation

The first order perturbation in the FLRW background with the perturbation geometry functions $\delta(t)$ and matter functions $\delta_m(t)$ can be expressed as,

$$H(t) \rightarrow H_b(t)(1+\delta(t)), \qquad \rho(t) \rightarrow \rho_b(t)(1+\delta_m(t)) \tag{29}$$

The perturbation of the function $f(Q)$ and f_Q can be calculated as,

$$\delta f = f_Q \delta Q, \qquad \delta f_Q = f_{QQ} \delta Q, \tag{30}$$

Neglecting higher power of $\delta(t)$, the Hubble parameter becomes,

$$6H^2 = 6H_b^2(1+\delta(t))^2 = 6H_b^2(1+2\delta(t)) \tag{31}$$

Now from equation (10) one can easily write

$$Q(2Qf_{QQ}+f_Q)\delta = \rho\delta_m, \tag{32}$$

Now, to obtain the analytical solution to the perturbation function, we consider the perturbation continuity equation as,

$$\dot{\delta}_m + 3H(1+\omega)\delta = 0 \tag{33}$$

Scalar Perturbations

From eqns. (32)-(33), the first order differential equation can be obtained,

$$\dot{\delta}_m + \frac{3H(1+\omega)\rho}{Q(2Qf_{QQ}+f_Q)}\delta_m = 0 \tag{34}$$

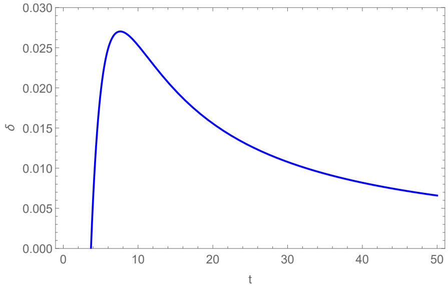
Further using the tt-component field equation and eqn. (34), the simplified relation can be obtained,

$$\dot{\delta}_m - \frac{\dot{H}}{H}\delta_m = 0, \tag{35}$$

which provides $\delta_m = C_1 H$, where C_1 is the integration constant. Subsequently from eqn. (33), we obtain

$$\delta = C_2\frac{\dot{H}}{H} \tag{36}$$

where, $C_2 = -\frac{C_1}{3(1+\omega)}$. The evolution behaviour of δ and δ_m are given in FIG.



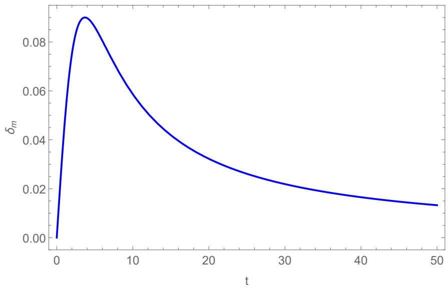


Figure: Deviation of Hubble parameter and energy density in cosmic time.

At the beginning both the deviations, $\delta(t)$ and $\delta_m(t)$, have some increment before declining through time and approaching zero at late times. As a result, we can say that though at the beginning the model shows unstable behaviour for a brief period, but in most of the time it shows stable behaviour under the scalar perturbation approach.

Conclusion

- The matter bounce scenario of the Universe has been reconstructed in an extended symmetric teleparallel gravity; a specific form of $f(Q)$ has been obtained that shows the matter bounce scenario.

- As expected, the model fails to explain the dark energy era, which has been observed from the dynamical stability analysis.

- From the critical points, the eigenvalues and the corresponding cosmology are obtained. Two critical points are obtained, one provides stable node and the other one unstable.

- To check the stability of the reconstructed model, the deviation of the Hubble parameter and the energy density in cosmic time, it has been observed that both the deviations (i.e., $\delta(t)$ and $\delta_m(t)$) approaching zero at late times.

- Further study can be carried out on the reconstructed form of $f(Q)$, which may give some more results on the bouncing scenario.

Paper in arXiv: 2206.02783

Matter Bounce Scenario in Extended Symmetric Teleparallel Gravity

A. S. Agrawal[1,*] B. Mishra[1,†] and P. K. Agrawal[2,‡]

[1]*Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad-500078, India.*

[2]*Department of Computer Sciences, Genba Sopanrao Moze, Arts Commerce and Science College, Yerwada Pune-411006, India.*

(Dated: June 8, 2022)

Abstract: In this paper, we have shown the matter bounce scenario of the Universe in an extended symmetric teleparallel gravity, the $f(Q)$ gravity. Motivated from the bouncing scenario and loop quantum cosmology (LQC), the form of the function $f(Q)$ has been obtained at the backdrop of Friedmann-Lemaître-Robertson Walker (FLRW) space time. Considering the background cosmology dominated by dust fluid, the e-folding parameter has been expressed, which contains the nonmetricity term. The dynamics of the model has been studied through the phase space analysis, where both the stable and unstable nodes are obtained. Also, the stability analysis has been performed with the first order scalar perturbation of the Hubble parameter and matter energy density to verify the stability of the model.

Keywords: Symmetric teleparallel gravity, Loop quantum cosmology, Bouncing scenario, Phase space analysis, Scalar perturbation.


Thank You
For Your
Attention