Seesaw Models and Higgs Physics

Tobias Felkl

In Collaboration with **Adam Lackner & Michael A. Schmidt**
Sydney Consortium of Particle Physics & Cosmology
School of Physics, University of New South Wales, Sydney, Australia

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UNSW SYDNEY

Future e^+e^- Colliders

To what precision can $\sigma(e^+e^- \to Zh)$ be measured?

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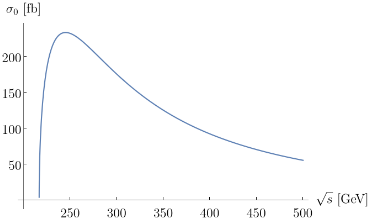
Benchmark points for ($\sqrt{s}$, precision): (240, 0.5%), (240, 1.0%), (365, 1.0%)



Collider	L_{int} [ab^{-1}]	Z-decay final states	$\sqrt{s}$ [GeV]	Precision
CEPC	20	$\ell^+\ell^-$, $q\bar{q}$, $\nu\bar{\nu}$	240	0.26%
	1	$\ell^+\ell^-$, $q\bar{q}$, $\nu\bar{\nu}$	360	1.4%
FCC-ee	5	$\ell^+\ell^-$	240	0.5%
	1.5	$\ell^+\ell^-$, $q\bar{q}$, $\nu\bar{\nu}$	365	0.9%
ILC	1.35	$\ell^+\ell^-$	250	1.1%
	0.115 (0.5)	$\ell^+\ell^-$ ($q\bar{q}$)	350	5% (1.63%)
	1.6 (0.5)	$\ell^+\ell^-$ ($q\bar{q}$)	500	2.9% (3.9%)
CLIC	0.5	$\ell^+\ell^-$, $q\bar{q}$	350	1.65%

H. Cheng et al., 2022; G. Bernardi et al., 2022; J. Yan et al., 2016; M. Thomson, 2016;

A. Miyamoto, 2013; H. Abramowicz et al., 2017

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Type-III Seesaw: Model

Like Type-I Seesaw, but with additional charged leptons!

$$\Sigma^a_{Ri} = (\Sigma^1_{Ri}, \Sigma^2_{Ri}, \Sigma^3_{Ri}) \sim (1, 3, 0) \qquad \Sigma^\pm_{Ri} \equiv \frac{\Sigma^1_{Ri} \mp i\Sigma^2_{Ri}}{\sqrt{2}}, \quad \Sigma^0_{Ri} \equiv \Sigma^3_{Ri}$$

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$$\mathcal{L}_{\Sigma_R} = \sum_{i=1}^{n_\Sigma} \overline{\Sigma^a_{Ri}} i \not{D} \Sigma^a_{Ri} - \left(\sum_{i=1}^{3} \sum_{j=1}^{n_\Sigma} Y^\Sigma_{ij} \overline{L_i} \sigma^a \Sigma^a_{Rj} \widetilde{H} + \frac{1}{2} \sum_{i,j=1}^{n_\Sigma} M^\Sigma_{ij} \overline{\Sigma^{ac}_{Ri}} \Sigma^a_{Rj} + \text{h.c.} \right)$$
$$D_\mu = \partial_\mu - i g_2 W^b_\mu T^b$$

R. Foot, H. Lew, X. G. He, G. C. Joshi, 1989.

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R. Foot, H. Lew, X. G. He, G. C. Joshi, 1989.

After EWSB:

$$\mathcal{L}_{\nu\ \text{mass}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\Sigma^{0c}_R} \end{pmatrix} \begin{pmatrix} 0 & m^\Sigma \\ m^{\Sigma T} & M^\Sigma \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \Sigma^0_R \end{pmatrix} + \text{h.c.}$$

Lepton Number Symmetry

Lepton number is (classically) conserved in SM.

Introduce two new (singlet/triplet) fermion interaction states,
with $+1$ and -1 units under lepton number.

Parametrisation of Dirac and Majorana matrices:

$$m^X_{ij} = \frac{v}{\sqrt{2}} \begin{pmatrix} Y^X & \epsilon Y^{X'} \end{pmatrix}_{ij} \qquad M^X_{ij} = \begin{pmatrix} \mu_1 M^X & M^X \\ M^X & \mu_2 M^X \end{pmatrix}_{ij}$$

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Light neutrino masses:

$$m_v = \frac{v^2}{2} \left[\mu_2 Y^X (M^X)^{-1} Y^{X^T} - \epsilon \left(Y^{X'} (M^X)^{-1} Y^{X^T} + Y^X (M^X)^{-1} Y^{X'^T} \right) \right]$$

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	ϵ	μ_1	μ_2
Inverse Seesaw	0	0	

D. Wyler, L. Wolfenstein, 1983; R. N. Mohapatra, 1986; R. N. Mohapatra, J. W. F. Valle, 1986.

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Light neutrino masses:

$$m_v = -\frac{v^2}{2}\epsilon \left(Y^{X'} (M^X)^{-1} Y^{X^T} + Y^X (M^X)^{-1} Y^{X'^T} \right)$$

	ϵ	μ_1	μ_2
Inverse Seesaw	0	0	
Linear Seesaw		0	0

E. K. Akhmedov, M. Lindner, E. Schnapka and J. W. F. Valle, 1996.

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Light neutrino masses:

$$m_\nu = 0$$

	ϵ	μ_1	μ_2
Inverse Seesaw	0	0	
Linear Seesaw		0	0
Exact LN limit	0	0	0

Lepton Flavour Universality

$W\ell\nu$ coupling in SM: $\mathcal{L} \supset -ig_{W,ij}\overline{\ell_i}\gamma^\mu P_L \nu_j W_\mu, \qquad g_{W,ij} = \frac{g_2}{\sqrt{2}}\delta_{ij}$

Shift in SMEFT:

$$\varepsilon_{ij} \equiv \frac{\delta g_{W,ij}}{g_2/\sqrt{2}} = -\frac{v_T^2}{c_w^2 - s_w^2}\left(c_w s_w C_{HWB} + \frac{1}{4}c_w^2 C_{HD} + \frac{1}{\sqrt{2}}c_w^2 \delta G_F\right)\delta_{ij} + v_T^2 C_{HL,ij}^{(3)}$$

$$\approx -0.04\left(\hat{C}_{HL,11}^{(3)} + \hat{C}_{HL,22}^{(3)}\right)\delta_{ij} + 0.06\,\hat{C}_{HL,ij}^{(3)}$$

$W\ell\nu$ coupling in SM: $\mathcal{L} \supset -i g_{W,ij} \overline{\ell_i} \gamma^\mu P_L \nu_j W_\mu, \qquad g_{W,ij} = \frac{g_2}{\sqrt{2}} \delta_{ij}$

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$$\approx -0.04 \left(\hat{C}_{HL,11}^{(3)} + \hat{C}_{HL,22}^{(3)} \right) \delta_{ij} + 0.06\, \hat{C}_{HL,ij}^{(3)}$$

Ratios of gauge couplings:

$$g_{\mu/e}^X \equiv \left(\frac{g_\mu}{g_e} \right)^X \simeq 1 - \varepsilon_{ee} + \varepsilon_{\mu\mu} \approx 1 + 0.06 \left(\hat{C}_{HL,22}^{(3)} - \hat{C}_{HL,11}^{(3)} \right); \qquad X = \pi, \ell, K$$
$$g_{\tau/\mu}^X \equiv \left(\frac{g_\tau}{g_\mu} \right)^X \simeq 1 - \varepsilon_{\mu\mu} + \varepsilon_{\tau\tau} \approx 1 + 0.06 \left(\hat{C}_{HL,33}^{(3)} - \hat{C}_{HL,22}^{(3)} \right); \qquad X = \pi, \ell$$

Extraction of V_{us} from different channels:

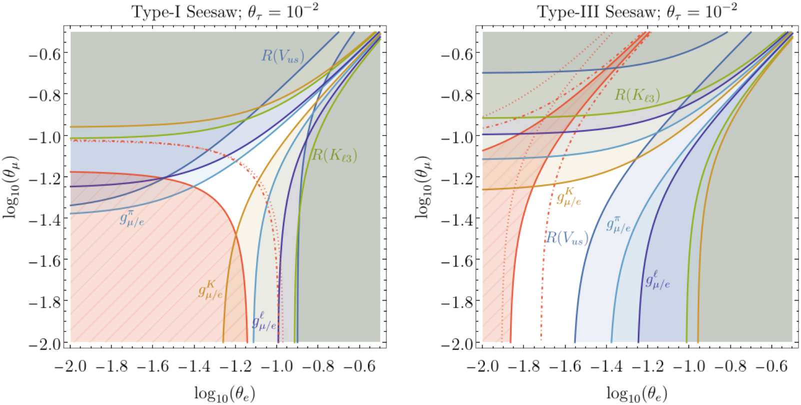
$$R(K_{\ell 3}) \simeq \frac{V_{us}^{K_{\mu 3}}}{V_{us}^{K_{e3}}} \simeq 1 - \varepsilon_{ee} + \varepsilon_{\mu\mu} \approx 1 + 0.06 \left(\hat{C}_{HL,22}^{(3)} - \hat{C}_{HL,11}^{(3)} \right)$$
$$R(V_{us}) = \frac{V_{us}^{K_{\mu 2}}}{V_{us}^{\beta}} \simeq 1 - \frac{V_{ud}}{V_{us}} \varepsilon_{\mu\mu} \approx 1 + 0.19\, \hat{C}_{HL,11}^{(3)} - 0.07\, \hat{C}_{HL,22}^{(3)}$$

A. Crivellin, M. Hoferichter, 2020; D. Bryman, V. Cirigliano, A. Crivellin, G. Inguglia, 2021.

Lepton Flavour Universality

Lepton Flavour Universality		
Observable	Measurement	Model prediction
$g^{\pi}_{\mu/e}$	1.0010 ± 0.0009	$1 \pm 0.48(\lvert\theta_e\rvert^2 - \lvert\theta_\mu\rvert^2)$
$g^{\ell}_{\mu/e}$	1.0017 ± 0.0016	
$g^{K}_{\mu/e}$	0.9978 ± 0.0018	
$R(K_{\ell 3})$	1.001295 ± 0.002891	
$g^{\pi}_{\tau/\mu}$	0.9965 ± 0.0026	$1 \pm 0.48(\lvert\theta_\mu\rvert^2 - \lvert\theta_\tau\rvert^2)$
$g^{\ell}_{\tau/\mu}$	1.0011 ± 0.0014	
$R(V_{us})$	0.98898 ± 0.00606	$1 \mp 1.49\lvert\theta_e\rvert^2 \pm 0.58\lvert\theta_\mu\rvert^2 + 0.01\,\lvert\theta_\tau\rvert^2$

A. Pich, 2021; C.-Y. Seng, D. Galviz, M. Gorchtein, U.-G. Meißner, 2022; PDG 2022.



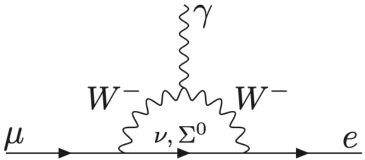
LFU constraints in the θ_e–θ_μ plane.

- red-ruled region: $|\Delta\sigma/\sigma| < 0.5\%$, $\sqrt{s} = 240$ GeV
- dot-dashed contour: $|\Delta\sigma/\sigma| < 1.0\%$, $\sqrt{s} = 240$ GeV
- dotted contour: $|\Delta\sigma/\sigma| < 1.0\%$, $\sqrt{s} = 365$ GeV

Lepton Flavour Violation

A. Abada, C. Biggio, F. Bonnet, M. B. Gavela, T. Hambye, 2008.

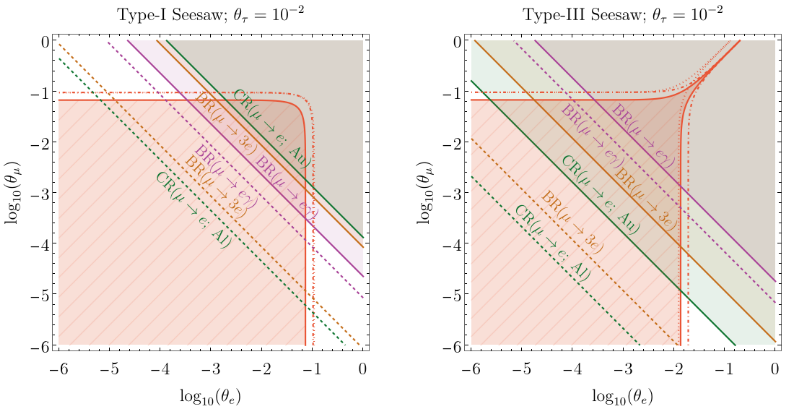
Trilepton decays and $\mu - e$ conversion induced at tree level in Type-III Seesaw, other transitions at loop level.



Lepton Flavour Violation				
Observable	Experimental bound		Model predictions	
	Current	Future	Type-I	Type-III
$\mathrm{BR}(\mu \to e\gamma)$	4.2×10^{-13}	6×10^{-14}	$0.82 \times 10^{-3} \, \lvert\theta_e\theta_\mu\rvert^2$	$1.27 \times 10^{-3} \, \lvert\theta_e\theta_\mu\rvert^2$
$\mathrm{BR}(\mu \to 3e)$	1×10^{-12}	1×10^{-16}	$0.14 \times 10^{-3} \, \lvert\theta_e\theta_\mu\rvert^2$	$0.72 \, \lvert\theta_e\theta_\mu\rvert^2$
$\mathrm{CR}(\mu - e; \mathrm{Au})$	7×10^{-13}		$0.12 \times 10^{-3} \, \lvert\theta_e\theta_\mu\rvert^2$	$27.1 \, \lvert\theta_e\theta_\mu\rvert^2$
$\mathrm{CR}(\mu - e; \mathrm{Al})$		2.6×10^{-17} 8×10^{-17}	$0.23 \times 10^{-3} \, \lvert\theta_e\theta_\mu\rvert^2$	$6.7 \, \lvert\theta_e\theta_\mu\rvert^2$
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8}	9×10^{-9}	$0.15 \times 10^{-3} \, \lvert\theta_e\theta_\tau\rvert^2$	$0.23 \times 10^{-3} \, \lvert\theta_e\theta_\tau\rvert^2$
$\mathrm{BR}(\tau \to 3e)$	2.7×10^{-8}	4.7×10^{-10}	$0.02 \times 10^{-3} \, \lvert\theta_e\theta_\tau\rvert^2$	$0.13 \, \lvert\theta_e\theta_\tau\rvert^2$

A. M. Baldini et al., 2016 & 2021; U. Bellgardt et al., 1988; A. Blondel et al., 2013; W. H. Bertl et al., 2006; R. Abramishvili et al., 2020; M. T. Hedges, 2023; B. Aubert et al., 2010; S. Banerjee et al., 2022; K. Hayasaka et al., 2010.

Lepton Flavour Violation



LFV constraints for both Seesaw models in the θ_e–θ_μ plane.

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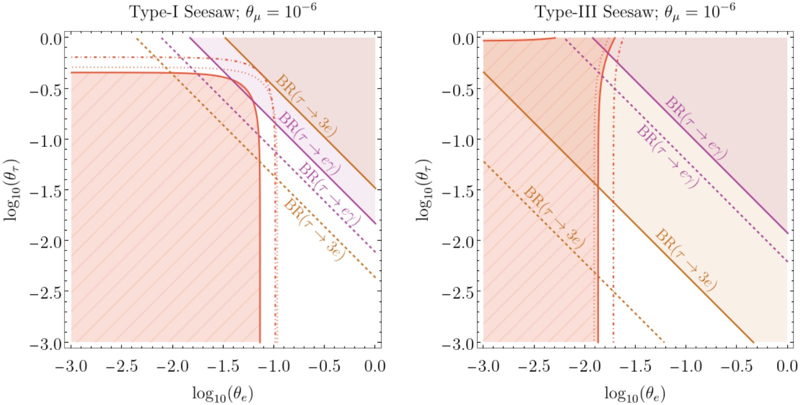

LFV constraints for both Seesaw models in the θ_e–θ_τ plane.

- red-ruled region: $|\Delta\sigma/\sigma| < 0.5\%$, $\sqrt{s} = 240\,\text{GeV}$
- dot-dashed contour: $|\Delta\sigma/\sigma| < 1.0\%$, $\sqrt{s} = 240\,\text{GeV}$
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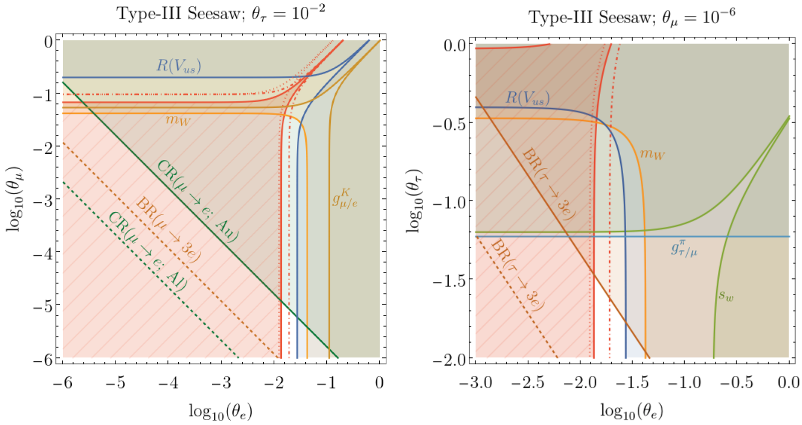
Type-III Results



Summary plots for the Type-III Seesaw model.

- red-ruled region: $|\Delta\sigma/\sigma| < 0.5\%$, $\sqrt{s} = 240$ GeV
- dot-dashed contour: $|\Delta\sigma/\sigma| < 1.0\%$, $\sqrt{s} = 240$ GeV
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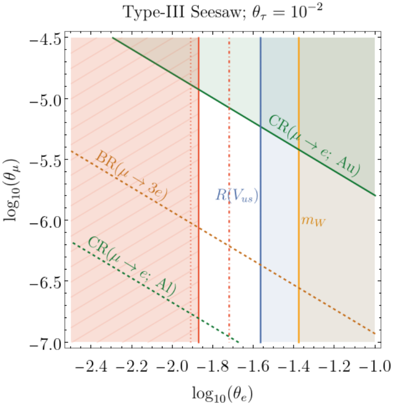
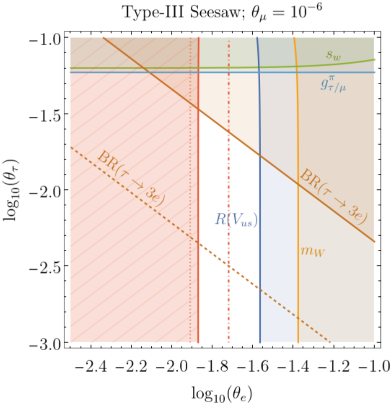
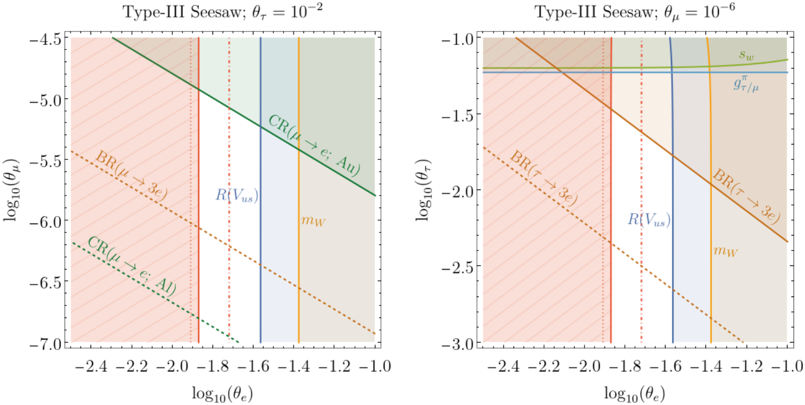
Type-III Results







Riding the Seesaw:
What Higgsstrahlung May Reveal about Massive Neutrinos.
T. Felkl, A. Lackner & M. A. Schmidt; arXiv: 2211.15954.

Thank you for your attention!

Back-Up

Future e^+e^- Colliders

Collider	Type	$\sqrt{s}$	$\mathcal{P}[\%]$ e^-/e^+	$\mathcal{L}_{\text{int}}$ ab^{-1}/IP
HL-LHC	pp	14 TeV		3
ILC and C^3	ee	250 GeV	$\pm80/\pm30$	2
c.o.m almost		350 GeV	$\pm80/\pm30$	0.2
similar		500* GeV	$\pm80/\pm30$	4
		1 TeV	$\pm80/\pm20$	8
CLIC	ee	380 GeV	$\pm80/0$	1
CEPC	ee	M_Z		60
		$2M_W$		3.6
		240 GeV		20
		360 GeV		1
FCC-ee	ee	M_Z		150
		$2M_W$		10
		240 GeV		5
		$2\ M_{top}$		1.5
muon-collider (higgs)	$\mu\mu$	125 GeV		0.02

S. Dawson, P. Meade, I. Ojalvo, C. Vernieri, 2022.

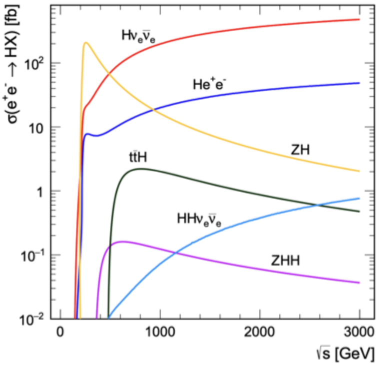
- precision measurements of Higgs-boson properties, and more
- control over initial-state momenta $\rightarrow$ smaller uncertainties
- no backgrounds from QCD processes

SMEFT matching conditions

	Coefficient	Type-I	Type-III
Tree	$C_{5,ij}$	$\frac{1}{2}\left(Y^{\nu *}(M^{\nu\dagger})^{-1}Y^{\nu\dagger}\right)_{ij}$	$\frac{1}{2}(Y^{\Sigma *}(M^{\Sigma\dagger})^{-1}Y^{\Sigma\dagger})_{ij}$
	$C^{(1)}_{HL,ij}$	$\frac{1}{4}\left(Y^{\nu}(M^{\nu\dagger}M^{\nu})^{-1}Y^{\nu\dagger}\right)_{ij}$	$\frac{3}{4}\left(Y^{\Sigma}(M^{\Sigma\dagger}M^{\Sigma})^{-1}Y^{\Sigma\dagger}\right)_{ij}$
	$C^{(3)}_{HL,ij}$	$-\frac{1}{4}\left(Y^{\nu}(M^{\nu\dagger}M^{\nu})^{-1}Y^{\nu\dagger}\right)_{ij}$	$\frac{1}{4}\left(Y^{\Sigma}(M^{\Sigma\dagger}M^{\Sigma})^{-1}Y^{\Sigma\dagger}\right)_{ij}$
	$C_{eH,ij}$	0	$\left(Y^{\Sigma}(M^{\Sigma\dagger}M^{\Sigma})^{-1}Y^{\Sigma\dagger}Y^{e}\right)_{ij}$
Loop	$C_{eB,ij}$	$\frac{1}{16\pi^2}\frac{g_1}{24}\left(Y^{\nu}(M^{\nu\dagger}M^{\nu})^{-1}Y^{\nu\dagger}Y^{e}\right)_{ij}$	$\frac{1}{16\pi^2}\frac{g_1}{8}\left(Y^{\Sigma}(M^{\Sigma\dagger}M^{\Sigma})^{-1}Y^{\Sigma\dagger}Y^{e}\right)_{ij}$
	$C_{eW,ij}$	$\frac{1}{16\pi^2}\frac{5g_2}{24}\left(Y^{\nu}(M^{\nu\dagger}M^{\nu})^{-1}Y^{\nu\dagger}Y^{e}\right)_{ij}$	$\frac{1}{16\pi^2}\frac{3g_2}{8}\left(Y^{\Sigma}(M^{\Sigma\dagger}M^{\Sigma})^{-1}Y^{\Sigma\dagger}Y^{e}\right)_{ij}$

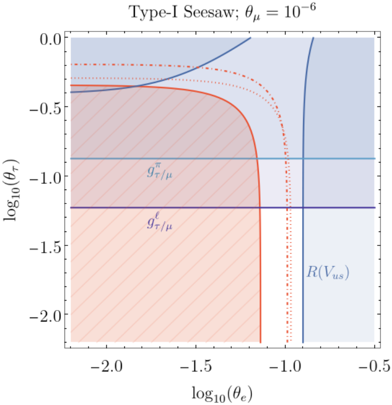
Y. Du, X.-X. Li, J.-H. Lu, 2022; D. Zhang, S. Zhou, 2021; R. Coy, M. Frigerio, 2022.

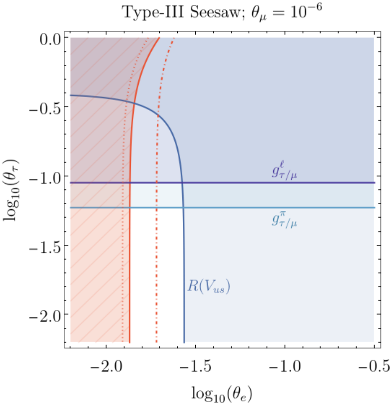
Higgs Production Channels



S. Dawson, P. Meade, I. Ojalvo, C. Vernieri, 2022.

Lepton Flavour Universality





LFU constraints in the θ_e–θ_τ plane.

Low-Energy Effective Theory (LEFT)

Physics at low energies: Remove t, h, Z, $W^{\pm}$ as dynamical d.o.f.
Use JMS operator basis.

E. E. Jenkins, A. V. Manohar, P. Stoffer, 2017.

$$\begin{aligned}\mathcal{L} \supset\ & C^{VLL}_{ee,ijkl}(\overline{\ell_i}\gamma^\mu P_L \ell_j)(\overline{\ell_k}\gamma_\mu P_L \ell_l) + C^{VRR}_{ee,ijkl}(\overline{\ell_i}\gamma^\mu P_R \ell_j)(\overline{\ell_k}\gamma_\mu P_R \ell_l) \\ & + C^{VLR}_{ee,ijkl}(\overline{\ell_i}\gamma^\mu P_L \ell_j)(\overline{\ell_k}\gamma_\mu P_R \ell_l) + \left[C^{SRR}_{ee,ijkl}(\overline{\ell_i} P_R \ell_j)(\overline{\ell_k} P_R \ell_l) + \mathrm{h.c.}\right] \\ & + \left[C_{e\gamma,ij}(\overline{\ell_i}\sigma^{\mu\nu} P_R \ell_j) F_{\mu\nu} + \mathrm{h.c.}\right].\end{aligned}$$

$$\begin{aligned}\mathcal{L} \supset\ & C^{VLL}_{eq}(\bar{\ell}\gamma^\mu P_L \ell)(\overline{q}\gamma_\mu P_L q) + C^{VRR}_{eq}(\bar{\ell}\gamma^\mu P_R \ell)(\overline{q}\gamma_\mu P_R q) \\ & + C^{VLR}_{eq}(\bar{\ell}\gamma^\mu P_L \ell)(\overline{q}\gamma_\mu P_R q) + C^{VLR}_{qe}(\overline{q}\gamma^\mu P_L q)(\bar{\ell}\gamma_\mu P_R \ell) \\ & + \Big[C^{SRR}_{eq}(\bar{\ell} P_R \ell)(\overline{q} P_R q) + C^{SRL}_{eq}(\bar{\ell} P_R \ell)(\overline{q} P_L q) \\ & + C^{TRR}_{eq}(\bar{\ell}\sigma^{\mu\nu} P_R \ell)(\overline{q}\sigma_{\mu\nu} P_R q) + \mathrm{h.c.}\Big].\end{aligned}$$

Match SMEFT onto LEFT, perform RG running down to relevant scales.

Lepton flavour violation

$$\mathrm{BR}(\ell_i \to \ell_j \gamma) = \frac{m_{\ell_i}^3}{4\pi \Gamma_{\ell_i}} \left(|C_{e\gamma,ij}|^2 + |C_{e\gamma,ji}|^2 \right)$$

Y. Kuno, Y. Okada, 2001.

$$\mathrm{BR}(\mu \to e\gamma) \approx 7.117 \times 10^6 \left| \hat{C}_{eB,12} - 0.55\, \hat{C}_{eW,12} + \left(1.77\, \hat{C}^{(3)}_{HL,12} - 0.48\, \hat{C}^{(1)}_{HL,12}\right) 10^{-6} \right|^2$$

$$\mathrm{BR}(\tau \to e\gamma) \approx 0.004 \times 10^6 \left| \hat{C}_{eB,13} - 0.55\, \hat{C}_{eW,13} + \left(29.69\, \hat{C}^{(3)}_{HL,13} - 8.12\, \hat{C}^{(1)}_{HL,13}\right) 10^{-6} \right|^2$$

$$\begin{aligned}\mathrm{BR}(\ell_i \to \ell_j \ell_j \overline{\ell_j}) = \frac{m_{\ell_i}^5}{3(16\pi)^3 \Gamma_{\ell_i}} \Bigg[& 64 \left| C^{VLL}_{ee,jijj} \right|^2 + 64 \left| C^{VRR}_{ee,jijj} \right|^2 + 8 \left| C^{VLR}_{ee,jijj} \right|^2 + 8 \left| C^{VLR}_{ee,jjji} \right|^2 \\ & + \frac{256 e^2}{m_{\ell_i}^2} \left(\ln \frac{m_{\ell_i}^2}{m_{\ell_j}^2} - \frac{11}{4} \right) \left(\left| C^{ij}_{e\gamma} \right|^2 + \left| C^{ji}_{e\gamma} \right|^2 \right) \\ & - \frac{64 e}{m_{\ell_i}} \mathrm{Re} \left[\left(4\, C^{VLL}_{ee,jijj} + C^{VLR}_{ee,jijj} \right) C^{ji*}_{e\gamma} + \left(4\, C^{VRR}_{ee,jijj} + C^{VLR}_{ee,jjji} \right) C^{ij}_{e\gamma} \right] \Bigg]\end{aligned}$$

L. Calibbi, X. Marcano, J. Roy, 2021.

$$\mathrm{BR}(\mu \to 3e) \approx 1.2 \times 10^{-4} \left[64 \left| 0.27 \left(\hat{C}^{(1)}_{HL,12} + \hat{C}^{(3)}_{HL,12} \right) \right|^2 + 8 \left| 0.49 \left(\hat{C}^{(1)}_{HL,12} + \hat{C}^{(3)}_{HL,12} \right) \right|^2 \right]$$

$$\mathrm{BR}(\tau \to 3e) \approx 0.2 \times 10^{-4} \left[64 \left| 0.27 \left(\hat{C}^{(1)}_{HL,13} + \hat{C}^{(3)}_{HL,13} \right) \right|^2 + 8 \left| 0.49 \left(\hat{C}^{(1)}_{HL,13} + \hat{C}^{(3)}_{HL,13} \right) \right|^2 \right]$$

Lepton flavour violation

$$\omega_{\rm conv} = \left| -\frac{C_{e\gamma,12}}{2m_\mu} D + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)} \right|^2 + \left| -\frac{C^*_{e\gamma,21}}{2m_\mu} D + \tilde{g}_{RV}^{(p)} V^{(p)} + \tilde{g}_{RV}^{(n)} V^{(n)} \right|^2$$

R. Kitano, M. Koike, Y. Okada, 2007; V. Cirigliano, R. Kitano, Y. Okada, P. Tuzon, 2009.

$$\tilde{g}_{LV}^{(p)} = 2\left(C^{VLL}_{eu,1211} + C^{VLR}_{eu,1211}\right) + \left(C^{VLL}_{ed,1211} + C^{VLR}_{ed,1211}\right)$$
$$\tilde{g}_{RV}^{(p)} = 2\left(C^{VRR}_{eu,1211} + C^{VLR}_{ue,1112}\right) + \left(C^{VRR}_{ed,1211} + C^{VLR}_{de,1112}\right)$$
$$\tilde{g}_{LV}^{(n)} = \left(C^{VLL}_{eu,1211} + C^{VLR}_{eu,1211}\right) + 2\left(C^{VLL}_{ed,1211} + C^{VLR}_{ed,1211}\right)$$
$$\tilde{g}_{RV}^{(n)} = \left(C^{VRR}_{eu,1211} + C^{VLR}_{ue,1112}\right) + 2\left(C^{VRR}_{ed,1211} + C^{VLR}_{de,1112}\right)$$

$$\mathrm{CR}(\mu \to e) \approx \left(\frac{m_\mu}{\mathrm{GeV}}\right)^5 \left\{ \begin{array}{c} 1.16 \\ 21.54 \end{array} \right\} \times 10^5$$
$$\left| \left[2\left(\hat{C}^{VLL}_{eu,1211} + \hat{C}^{VLR}_{eu,1211}\right) + \left(\hat{C}^{VLL}_{ed,1211} + \hat{C}^{VLR}_{ed,1211}\right)\right] \left\{ \begin{array}{c} 0.0974 \\ 0.0161 \end{array} \right\} \right.$$
$$\left. + \left[\left(\hat{C}^{VLL}_{eu,1211} + \hat{C}^{VLR}_{eu,1211}\right) + 2\left(\hat{C}^{VLL}_{ed,1211} + \hat{C}^{VLR}_{ed,1211}\right)\right] \left\{ \begin{array}{c} 0.146 \\ 0.0173 \end{array} \right\} \right|^2$$

RG running in LEFT and matching onto SMEFT

$$\frac{C_{e\gamma,ij}\,(\mu = 5\,\text{GeV})}{\text{GeV}} \approx 150.732\, C_{eB,ij} - 82.394\, C_{eW,ij} + 3.204\, C^{(3)}_{LeQu,ij33} + A_{ij}\left(C^{(3)}_{HL,ij} - 0.27353\, C^{(1)}_{HL,ij}\right)$$

$$A_{ij} = 10^{-3}\begin{pmatrix} - & 0.2661 & 4.4758 \\ 0.0013 & - & - \\ 0.0013 & - & - \end{pmatrix}_{ij}$$

$$C^{VLL}_{ee,jijj} \approx -0.266\, C^{(1)}_{HL,ji} - 0.271\, C^{(3)}_{HL,ji} + 0.973\, C_{LL,jjji},$$

$$C^{VRR}_{ee,jijj} \approx 0.974\, C_{ee,jjji} + 0.235\, C_{He,ji} - 0.006\, C_{eu,ji33} + 0.003\, C_{Qe,33ji},$$

$$C^{VLR}_{ee,jijj} \approx 0.4912\, C^{(1)}_{HL,ji} + 0.4909\, C^{(3)}_{HL,ji} + 1.018\, C_{Le,jijj} - 0.012\, C_{Lu,ji33}, \quad \text{and}$$

$$C^{VLR}_{ee,jjji} \approx -0.556\, C_{He,ji} + 1.018\, C_{Le,jjji} - 0.015\, C_{Qe,33ji} + 0.011\, C_{eu,ji33}$$

$$C^{VLL}_{eu,1211} \approx 0.708\, C^{(1)}_{HL,12} + 0.734\, C^{(3)}_{HL,12} - 1.047\, C^{(3)}_{LQ,1211},$$

$$C^{VLR}_{eu,1211} \approx -0.3172 C^{(1)}_{HL,12} - 0.3170\, C^{(3)}_{HL,12} + 0.984\, C_{Lu,1211},$$

$$C^{VRR}_{eu,1211} \approx -0.321\, C_{He,12} + 0.008\, C_{eu,1233} - 0.005\, C_{Qe,3312},$$

$$C^{VLR}_{ue,1112} \approx 0.696\, C_{He,12} + 0.017\, C_{Qe,3312} - 0.014\, C_{eu,1233},$$

$$C^{VLL}_{ed,1211} \approx -0.856\, C^{(1)}_{HL,12} - 0.864\, C^{(3)}_{HL,12} + 0.987\, C^{(3)}_{LQ,1211},$$

$$C^{VLR}_{ed,1211} \approx 0.1617\, C^{(1)}_{HL,12} + 0.1615\, C^{(3)}_{HL,12} + 1.006\, C_{Ld,1211},$$

$$C^{VRR}_{ed,1211} \approx 0.158\, C_{He,12} - 0.004\, C_{eu,1233} + 0.002\, C_{Qe,3312}, \quad \text{and}$$

$$C^{VLR}_{de,1112} \approx -0.867\, C_{He,12} - 0.020\, C_{Qe,3312} + 0.018\, C_{eu,1233}$$