Constraining Phase Transitions in Grand Unified Theories with Gravitational Waves

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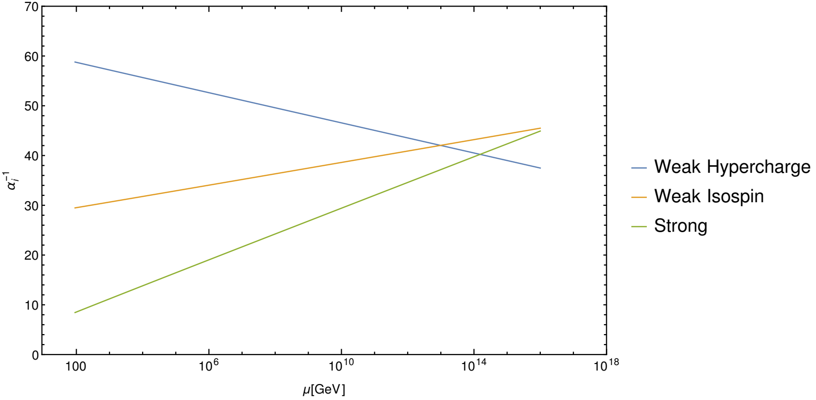
Motivation

- LISA is established as an indispensable tool for studying the nature of the electro-weak phase transition in the Standard Model (SM) and its extensions
- Some extensions of the SM, such as Grand Unified Theories (GUTs) can predict phase transitions at scales much higher than that of the electro-weak transition
- We want to assess to what extent such signals could be constrained by current and future generation ground based gravitational wave detectors which operate in a frequency range much higher than LISA

Gauge Coupling Running in the Standard Model



We used `PyR@TE 3` to compute the β function at two-loops

From the GUT to the Standard Model

- An $SO(10)$ GUT breaks into the Pati-Salam group

$$SU(4)_c \times SU(2)_L \times SU(2)_R$$

- Breaks into the left-right symmetric group

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

- Breaks into the standard model

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Pati-Salam

- Gauge group

$$SU(4)_c \times SU(2)_L \times SU(2)_R$$

- Is a left-right symmetric theory, restoring parity at high scales
- Provides an explanation for the hypercharge quantum numbers of the Standard Model

$$Y_W = T_{3R} + (B - L)/2$$

Pati-Salam - Fermions

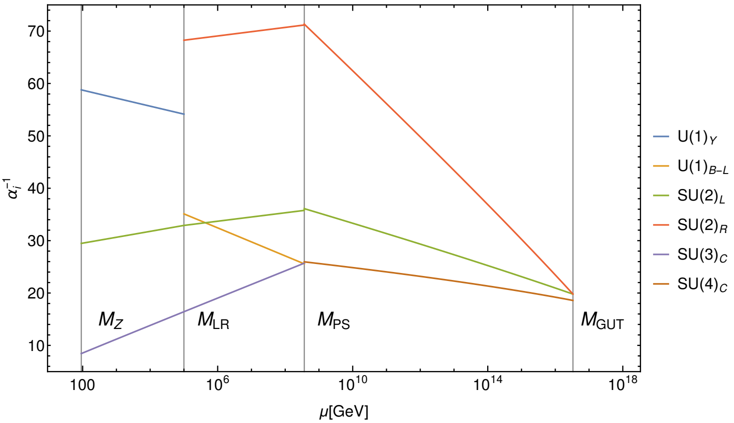
- Unifies quarks and leptons under the same representation of $SU(4)$
- Matter falls into a doublet representation of $SU(2)_{L/R}$ depending on its chirality

$$\begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix} ; \begin{pmatrix} -d_1^c & -d_2^c & -d_3^c & -e^c \\ u_1^c & u_2^c & u_3^c & \nu^c \end{pmatrix}$$

Pati-Salam - Scalars

Fields	$SU(4)_c$	$SU(2)_L$	$SU(2)_R$	Purpose
ϕ	1	2	2	Breaks SM
Δ_R	$\overline{10}$	1	3	Breaks LR
Δ_L	$\overline{10}$	3	1	Seesaw
Ξ	15	1	1	Breaks PS
Ω_R	15	1	3	Unification

Gauge Coupling Running in the Pati-Salam Model

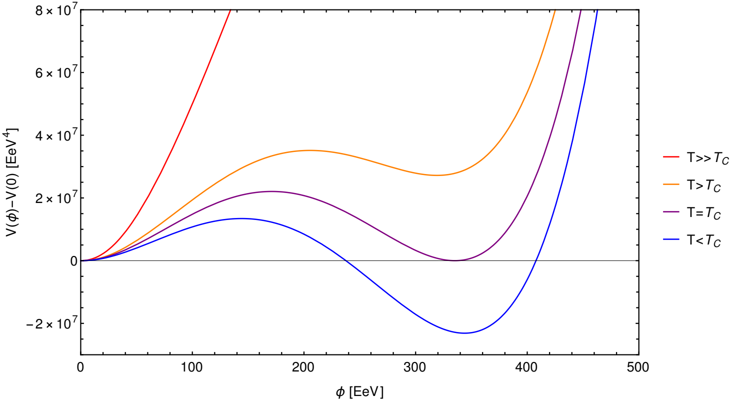


Threshold corrections $\alpha_i^{-1}(\mu) = \alpha_j^{-1}(\mu) - \lambda_{ij}(\mu)$

First Order Phase Transitions

- First order phase transitions can generate gravitational waves
- Phase transitions occur in the early universe when a scalar field ϕ acquires a non-zero vacuum expectation value (VEV) $\langle\phi\rangle$
- They proceed via the nucleation and expansion of bubbles of true vacuum (broken phase)
- In our case $SU(4)$ is broken when the fifteenth component of Ξ develops a VEV

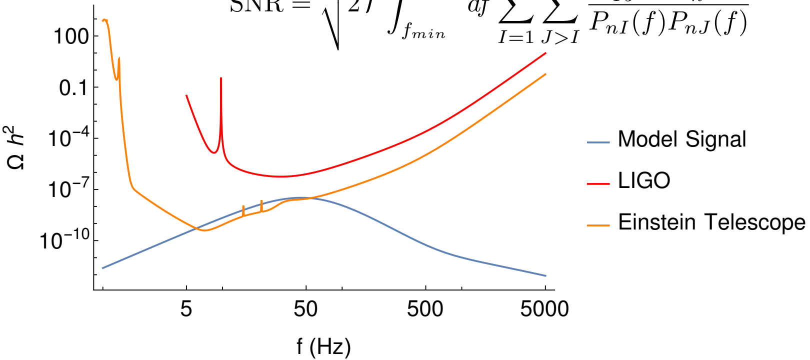
Temperature Dependence of the Potential



$$V_{eff} = V_{tree} + V_{CW} + V_T + V_{Daisy}$$

Signals

$$\mathrm{SNR} = \sqrt{2\mathcal{T} \int_{f_{min}}^{f_{max}} df \sum_{I=1}^{M} \sum_{J>I}^{M} \frac{\Gamma_{IJ}^2(f) S_h^2(f)}{P_{nI}(f) P_{nJ}(f)}}$$

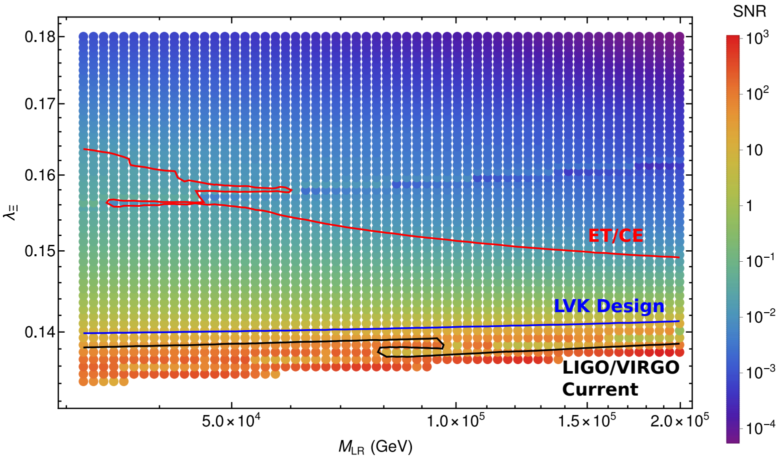


$$\Omega_{GW} = \Omega_{col} + \Omega_{sw} + \Omega_{turb}$$

(S.Hild et al., Classical and Quantum Gravity, Volume 28, Issue 9, pp. 094013, 2011)

(Abbott, B.P., et al., Prospects for observing... Living Rev Relativ 23, 3 (2020).)

Results: Signal to Noise Ratio



$$\rho_{\Omega_R} = 0.956 \;\; \rho_{\Delta_L} = 0.885 \;\; \rho_{\Delta_R} = 0.902 \;\; \alpha_{F_L} = 1.84 \times 10^{-3} \;\; \alpha_{F_L} = 5.24 \times 10^{-4}$$

Conclusion

- We constructed an $SO(10)$ grand unified theory with an intermediate Pati-Salam scale and demonstrated gauge coupling unification using renormalisation group equations
- Across the parameter space we found that the effective potential exhibits a first order phase transition
- We showed that for some sections of the parameter space the gravitational waves should have been observable in LIGO