Precise predictions and new insights for the Migdal effect

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Direct detection: current status (SI NR)

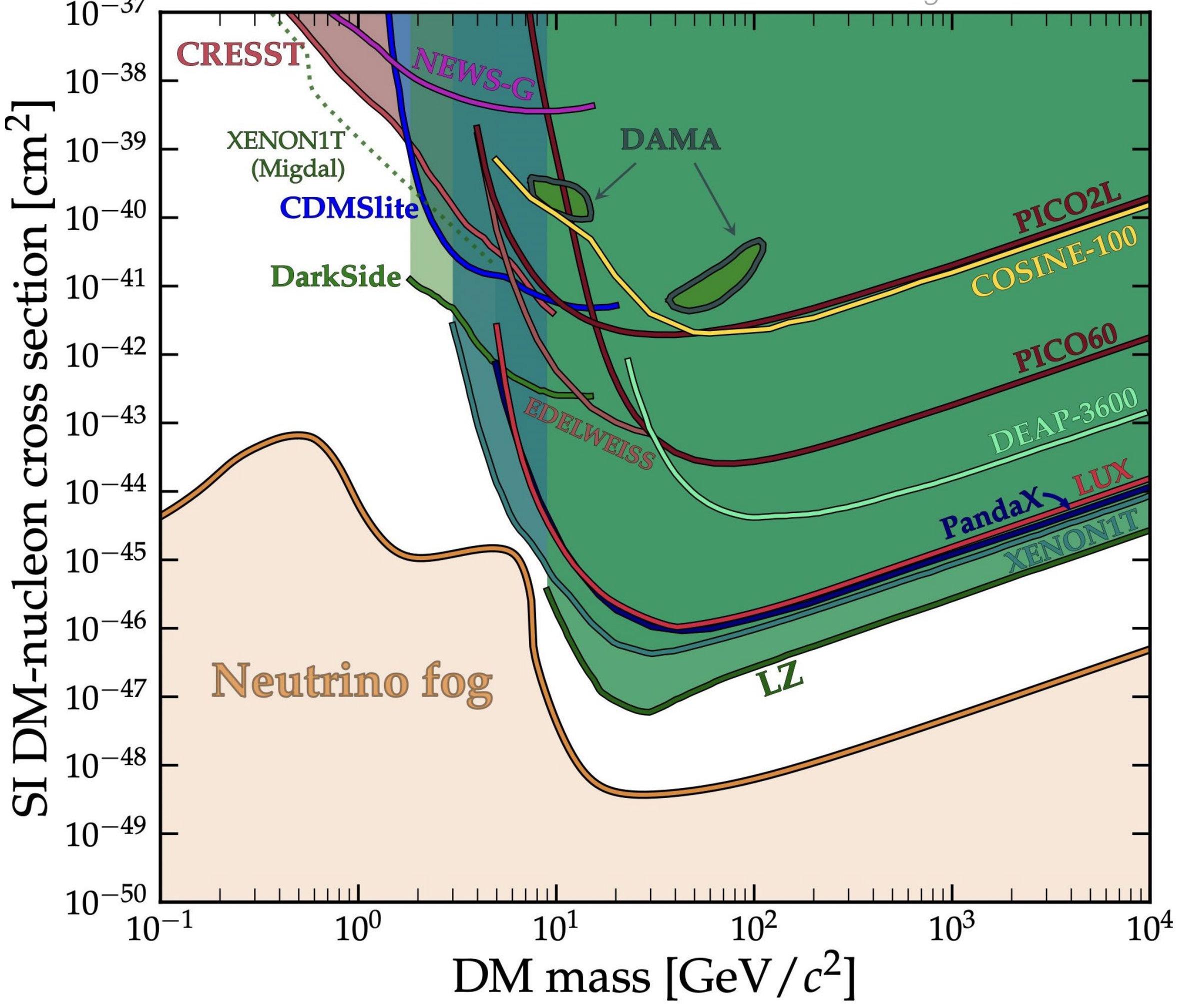
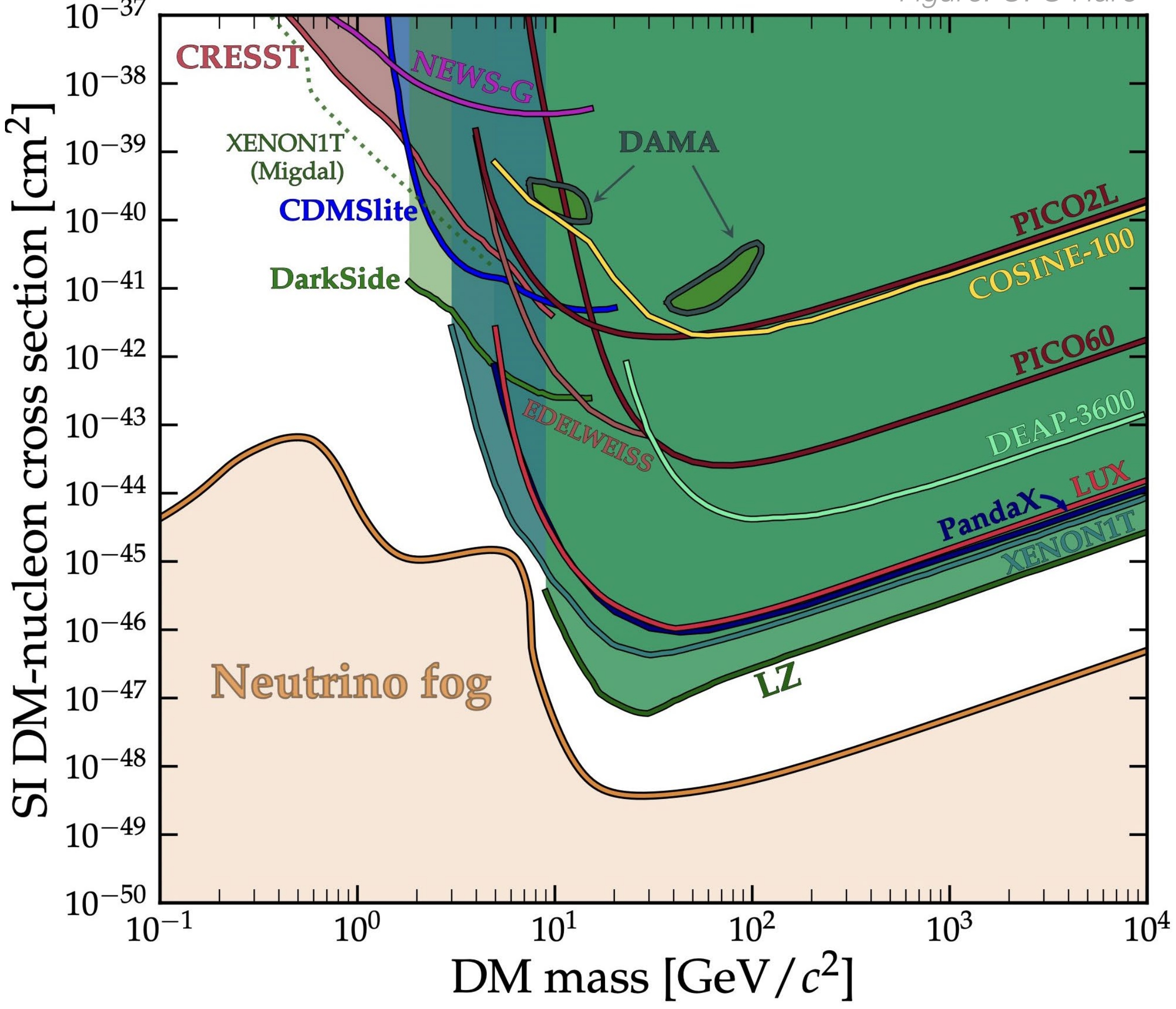


Figure: C. O'Hare

Direct detection: elastic NR

Figure: C. O'Hare



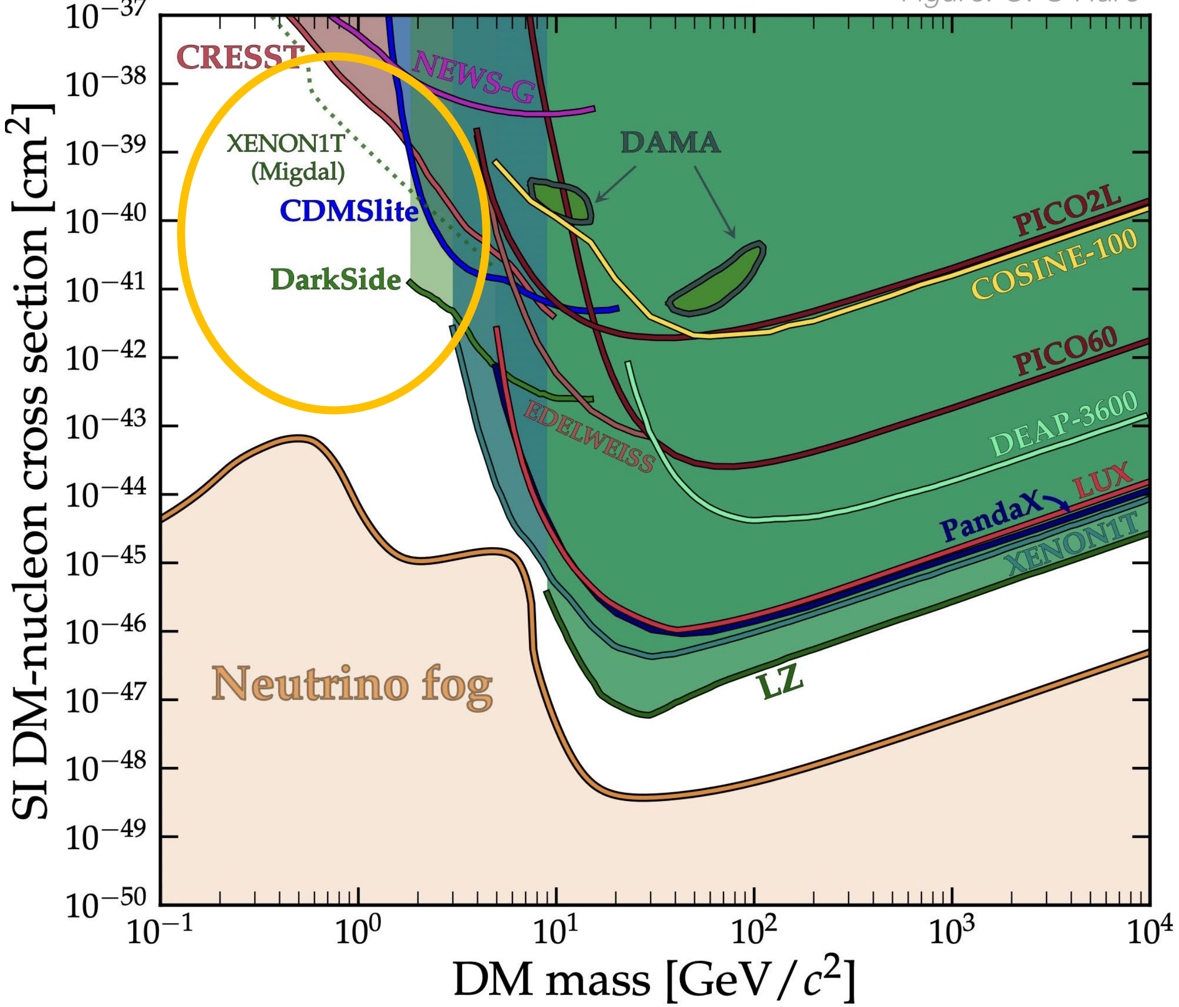
Elastic DM-nucleus scattering:

$$E_{NR} = \frac{q^2}{2m_N} \lesssim \frac{2m_\chi^2 v_\chi^2}{m_N} \quad (m_\chi < m_N)$$

$$E_{NR}^{\max} = \underline{0.1\,\mathrm{keV}} \left(\frac{131}{A}\right)\left(\frac{m_\chi}{\mathrm{GeV}}\right)^2$$

Direct detection: Migdal

Figure: C. O'Hare



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Inelastic scattering (Migdal):

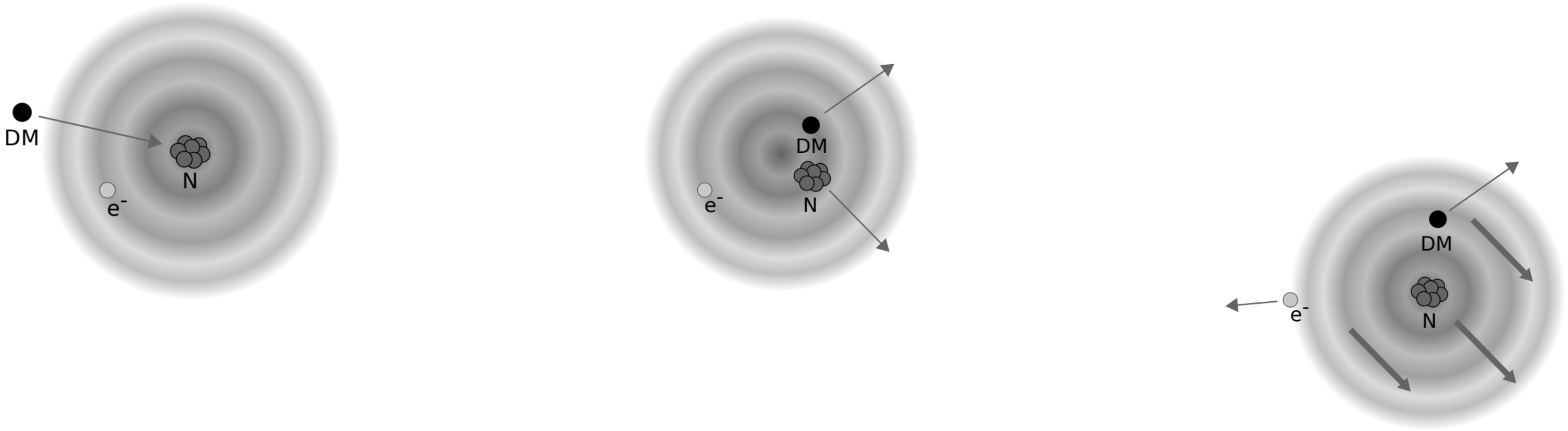
$$\omega = \boldsymbol{v}\cdot\boldsymbol{q} - \frac{q^2}{2m_\chi} \leq \frac{1}{2}m_\chi v_\chi^2$$

$$\omega_{\max} \sim \underline{3\,\mathrm{keV}} \left(\frac{m_\chi}{\mathrm{GeV}}\right)$$

Migdal effect

Migdal 1939

- Ionisation/excitation due to displacement of nucleus after nuclear recoil



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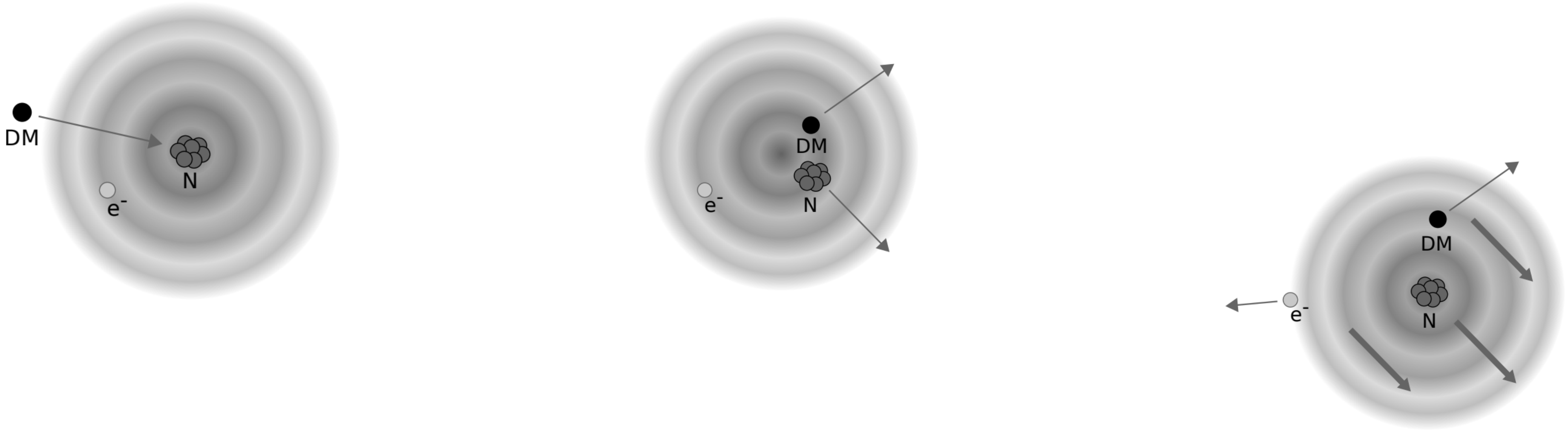
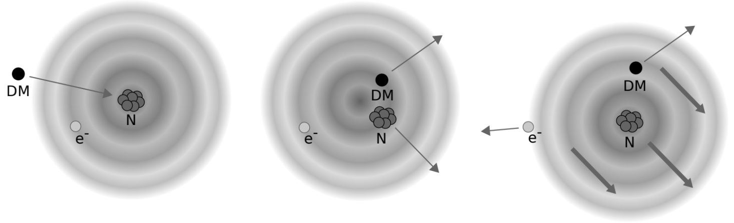


- Migdal effect observed in $\alpha, \beta^{\pm}$ decays
- Yet to be observed in neutron scattering – *important to validate theory for dark matter searches*

Calculating the Migdal effect

Migdal 1939



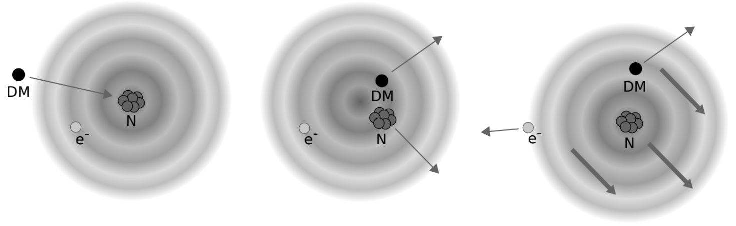
- In rest frame of nucleus, wavefunction of moving electron cloud obtained by Galilean boost:

$$\left|\Psi'\right\rangle = U(\boldsymbol{v})\left|\Psi\right\rangle = e^{-im_e\sum_k \boldsymbol{v}\cdot\boldsymbol{r}_k}\left|\Psi\right\rangle$$

sum over electrons

Calculating the Migdal effect

Migdal 1939



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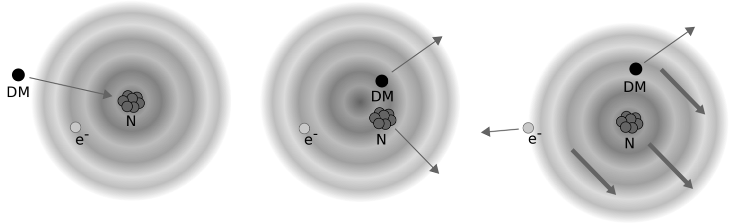
Transition probability:

$$p_v\left(\Psi_i \to \Psi_f\right) = \left|\left\langle \Psi_f \middle| \exp\left(im_e\boldsymbol{v}\cdot\sum_{k=1}^{N}\boldsymbol{r}_k\right) \middle| \Psi_i \right\rangle\right|^2$$

eigenstates of $v = 0$ Hamiltonian

Calculating the Migdal effect

Migdal 1939



- In rest frame of nucleus, wavefunction of moving electron cloud obtained by Galilean boost:

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eigenstates of $v = 0$ Hamiltonian

Migdal effect also occurs in

- Molecules (Blanco et. al. '22)
- Semiconductors (e.g. Knapen et. al. '20; Liang et. al. '22)

Dipole approximation

Need to evaluate *multi-electron* matrix element

$$\left\langle \Psi_f \middle| \exp\left(i m_e \boldsymbol{v} \cdot \underline{\sum_{k=1}^{N} \boldsymbol{r}_k} \right) \middle| \Psi_i \right\rangle$$

Standard approach: *dipole approximation*

$$\exp\left(i m_e \boldsymbol{v} \cdot \sum_{k=1}^{N} \boldsymbol{r}_k \right) \approx 1 + i m_e \boldsymbol{v} \cdot \sum_{k=1}^{N} \boldsymbol{r}_k$$

Dipole approximation

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Standard approach: *dipole approximation* $\exp\left(i m_e \boldsymbol{v} \cdot \sum_{k=1}^{N} \boldsymbol{r}_k \right) \approx 1 + i m_e \boldsymbol{v} \cdot \sum_{k=1}^{N} \boldsymbol{r}_k$

- Reduces to single electron matrix elements: $\langle \chi_j | i m_e \boldsymbol{v} \cdot \boldsymbol{r} | \psi_i \rangle$
- Migdal ionisation probability $\propto v^2$

Expected to breakdown when

$$v \gtrsim (a_o m_e)^{-1} \sim 0.007$$

Theory improvements

$$\left\langle \Psi_f \middle| \exp\left(i m_e \boldsymbol{v} \cdot \sum_{k=1}^{N} \boldsymbol{r}_k \right) \middle| \Psi_i \right\rangle$$

Beyond the dipole approximation

- Wavefunction is Slater determinant of single electron orbitals:

$$\Psi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N\right) = \mathcal{A}\left(\psi_1(\boldsymbol{r}_1)\psi_2(\boldsymbol{r}_2)\ldots\psi_N(\boldsymbol{r}_N)\right)$$

- Full matrix element can be written as determinant of single electron matrix elements:

$$\left\langle \Psi_f \middle| e^{i m_e \boldsymbol{v} \cdot \sum_k \boldsymbol{r}_k} \middle| \Psi_i \right\rangle = \det(M)$$

$$M_{\beta\alpha} = \left\langle \chi_{b_\beta} \middle| e^{i m_e \boldsymbol{v} \cdot \boldsymbol{r}} \middle| \psi_{a_\alpha} \right\rangle$$

(Talman & Frolov '06)

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(Talman & Frolov '06)

Atomic wavefunctions

Dirac-Hatree-Fock method

- Relativistic effects important for large atoms
- Include full non-local exchange potential
  (c.f. local effective potential in previous calculations)

Use two complementary approaches:

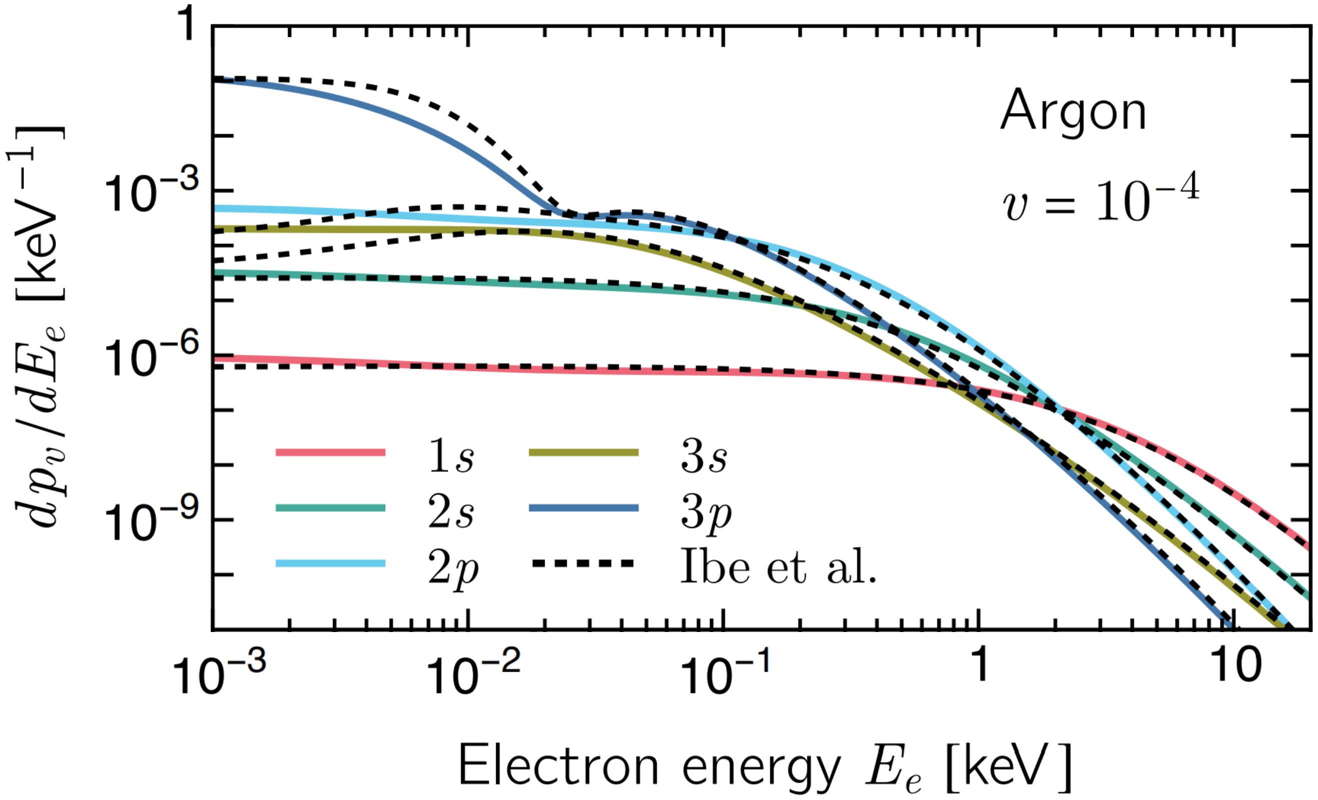
- Gaussian basis set method (BERTHA)
- Finite difference self-consistent field (GRASP/RATIP)

Ionisation probabilities

- Valence electrons dominate rate at very low e^- energies

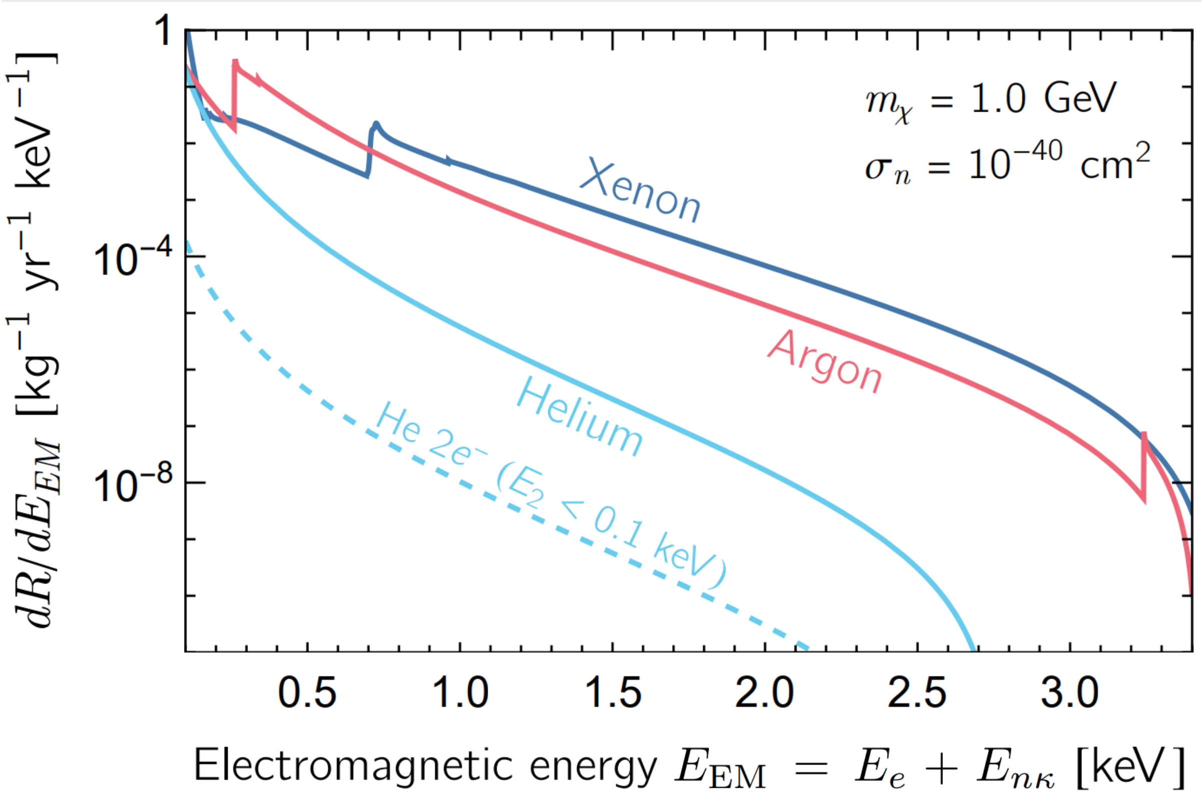
- Inner shells dominate at high energies

→ Additional x-ray / auger electrons from de-excitation



Dark matter Migdal rates

$$\frac{d^2R}{dE_R\,dE_e} = \underline{\frac{A^2\sigma_n|F_{\mathrm{N}}|^2}{2m_\chi\mu_{\chi n}^2}} \sum_{n\kappa} \underbrace{\frac{dp_v(n\kappa \to E_e)}{dE_e}}_{\text{Migdal ionisation probability}} \underline{\frac{\rho_\chi}{m_\chi} g_\chi(v_{\mathrm{min}})}$$



Dark matter Migdal rates

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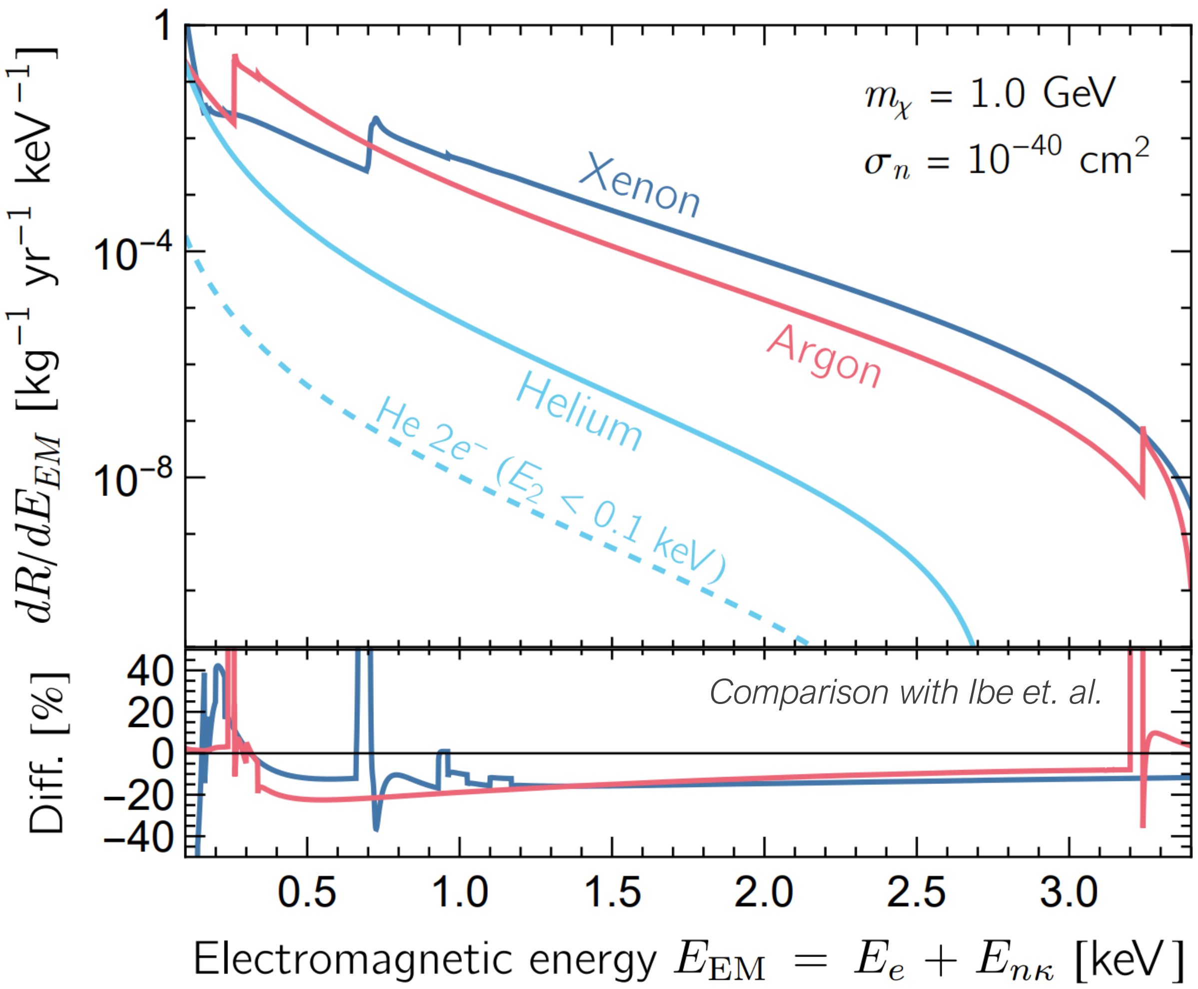
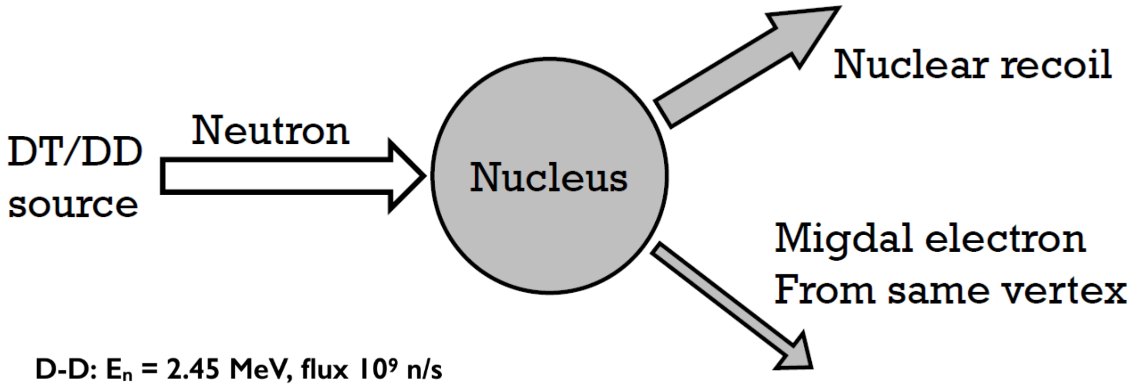
- Good agreement with dipole approximation calculation *(Ibe et. al. '17)*
- Differences due to orbital energies & atomic potential, particularly at low E_{EM}



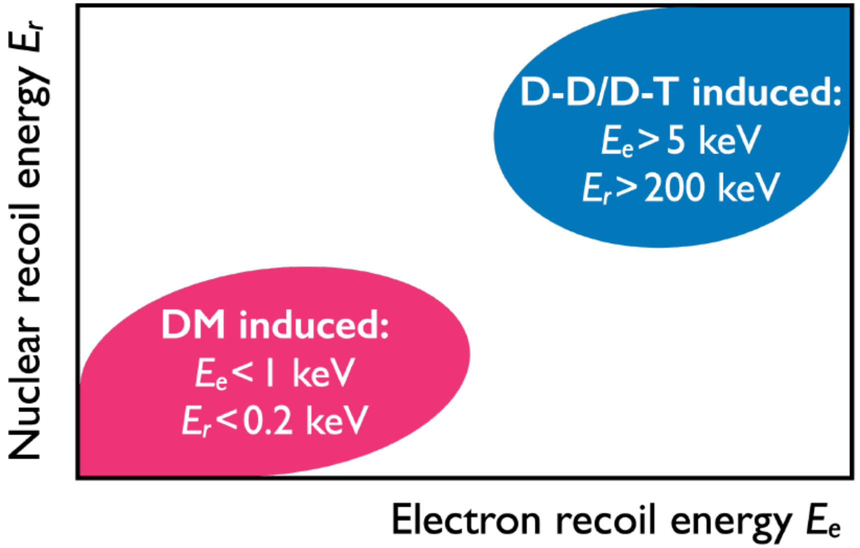
MIGDAL

Migdal In Galactic Dark mAtter expLoration

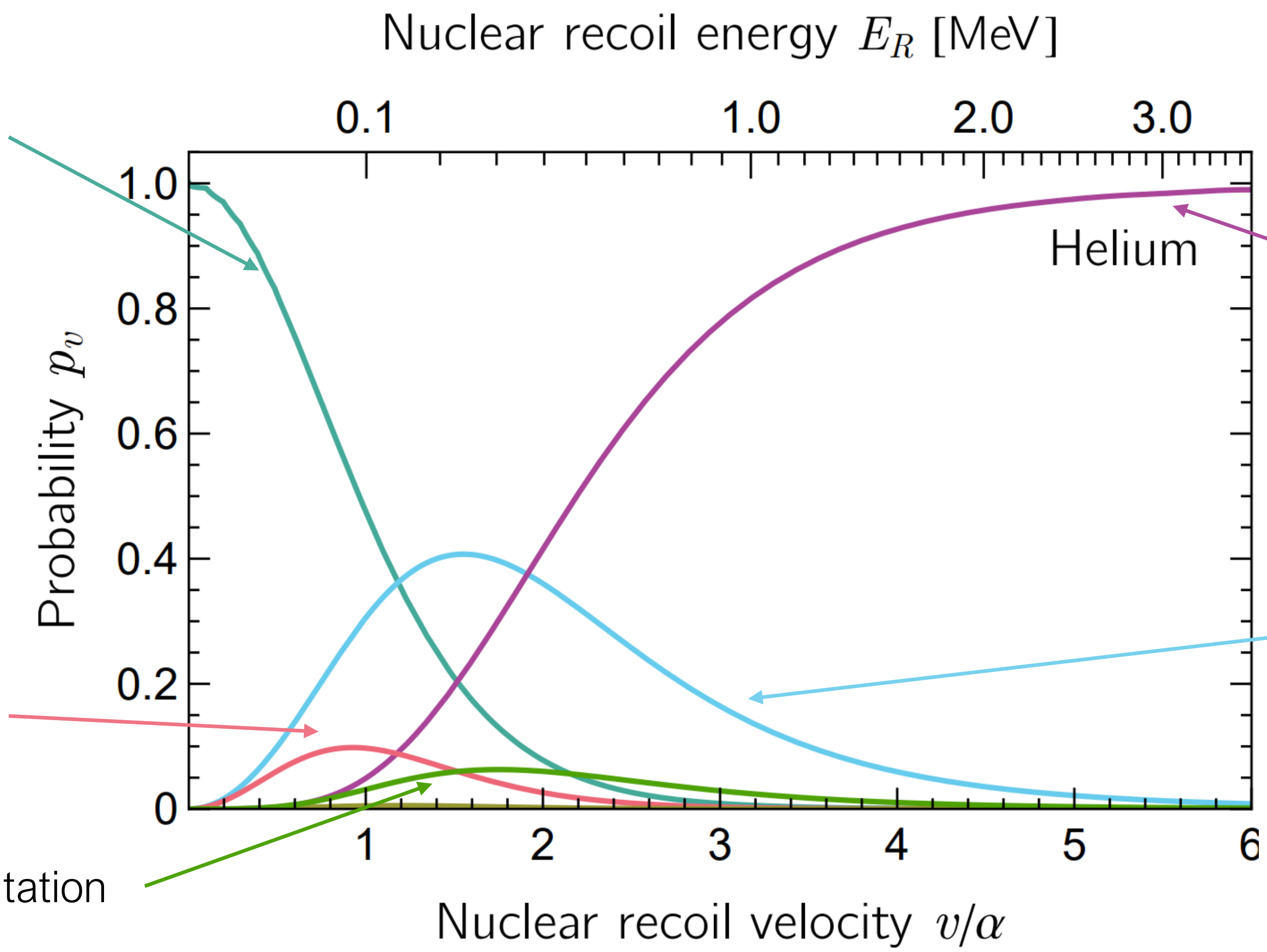
- Observe Migdal effect in neutron scattering using optical TPC
- Phase 1: $\mathrm{CF_4}$
  Phase 2: $\mathrm{CF_4}$ + noble gases



D-D: E_n = 2.45 MeV, flux 10^9 n/s
D-T: E_n = 14.1 MeV, flux 10^{10} n/s



Beyond dipole approx: multiple ionisation



atomic recoil

double ionisation

single ionisation

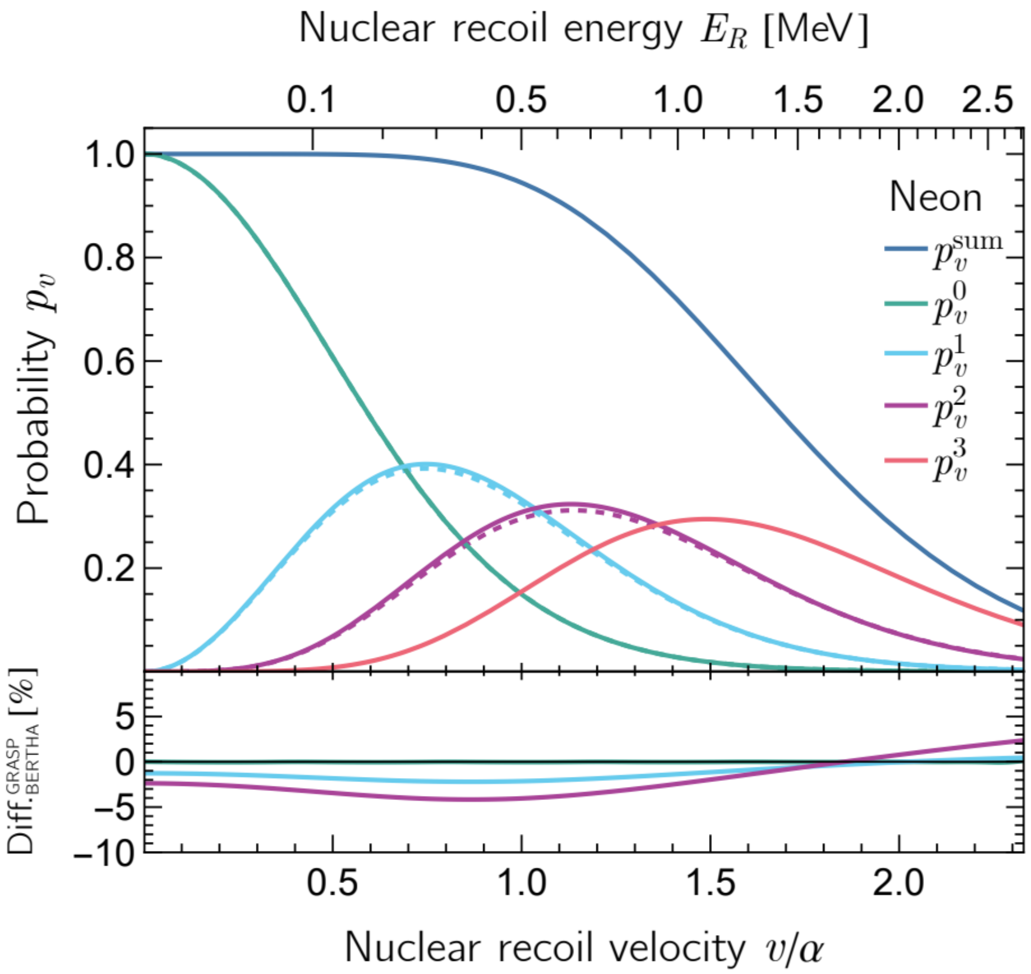
electronic excitation

ionisation + excitation

Multiple ionisation

- Triple and higher ionisation important for large recoil velocities

- *But*, impractical to individually calculate all higher order transitions for large atoms

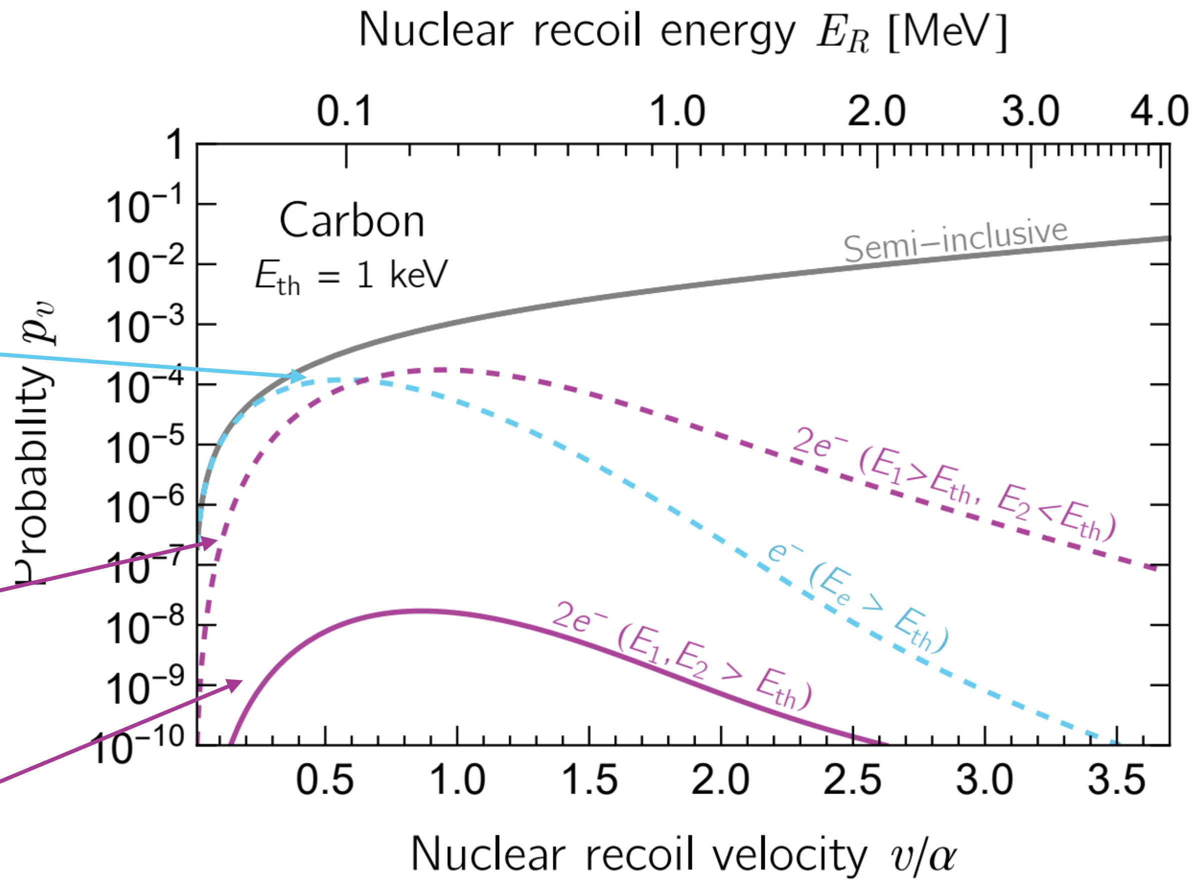


Semi-inclusive probability

- At high recoil velocities, observable rate dominated by

1 hard e^- + soft electrons

- Low probability to produce multiple hard e^-



Single e^-

1 hard e^- & 1 soft e^-

2 hard e^-

Semi-inclusive probability

- At high recoil velocities, observable rate dominated by

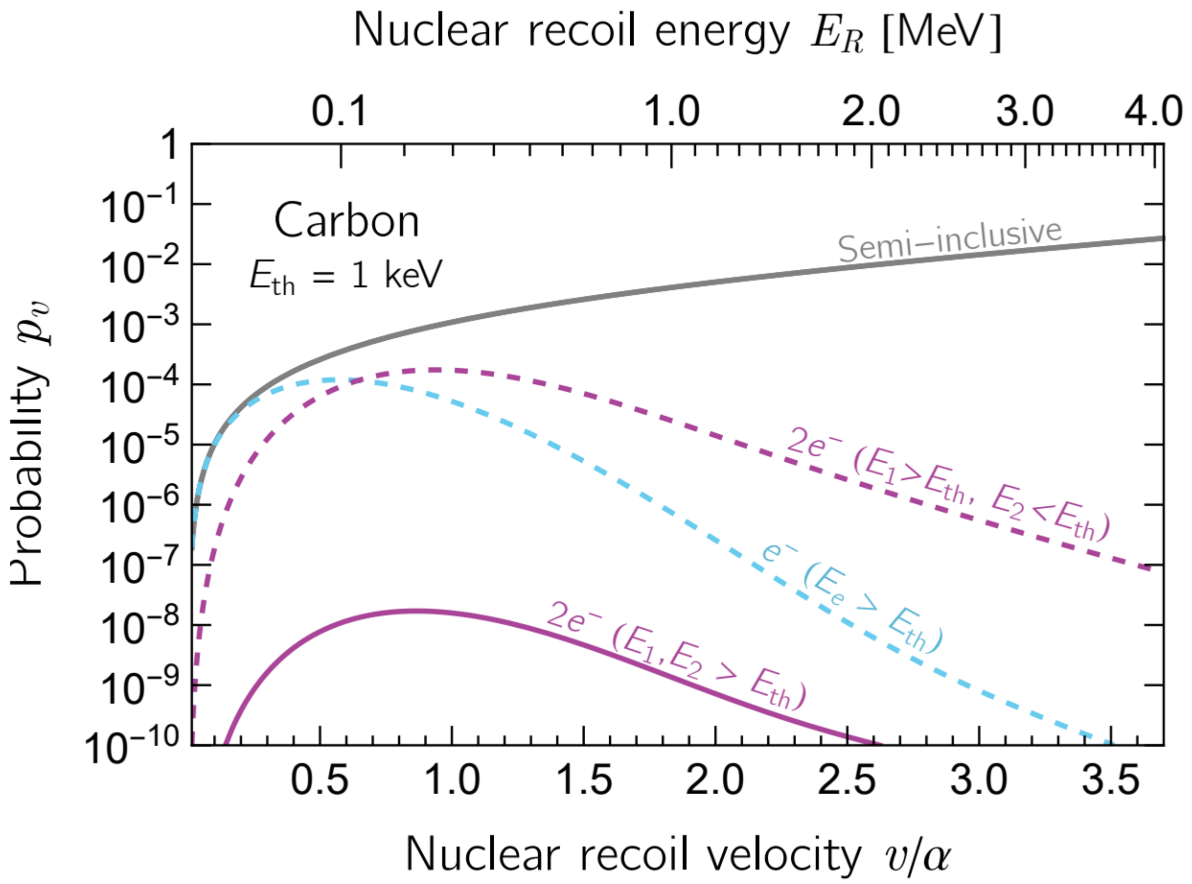
1 hard e^- + soft electrons

- Introduce *semi-inclusive* probability:

$$p_v(|\Psi_i\rangle \to |\chi_{b_1} X_{\text{soft}}\rangle) \approx \sum_{\alpha=1}^{N} \Big| \underline{\langle \chi_{b_1} | e^{i m_e \boldsymbol{v}\cdot\boldsymbol{r}} | \psi_{a_\alpha} \rangle} \Big|^2$$

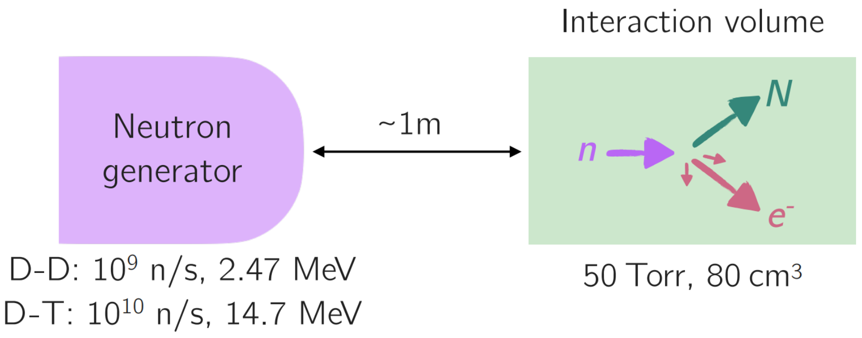
~keV ionisation e^-

Sum of single-electron matrix elements



Neutron scattering rate (MIGDAL experiment)

- *Essential* to include multiple ionisation processes to obtain accurate predictions (*semi-inclusive rate*)



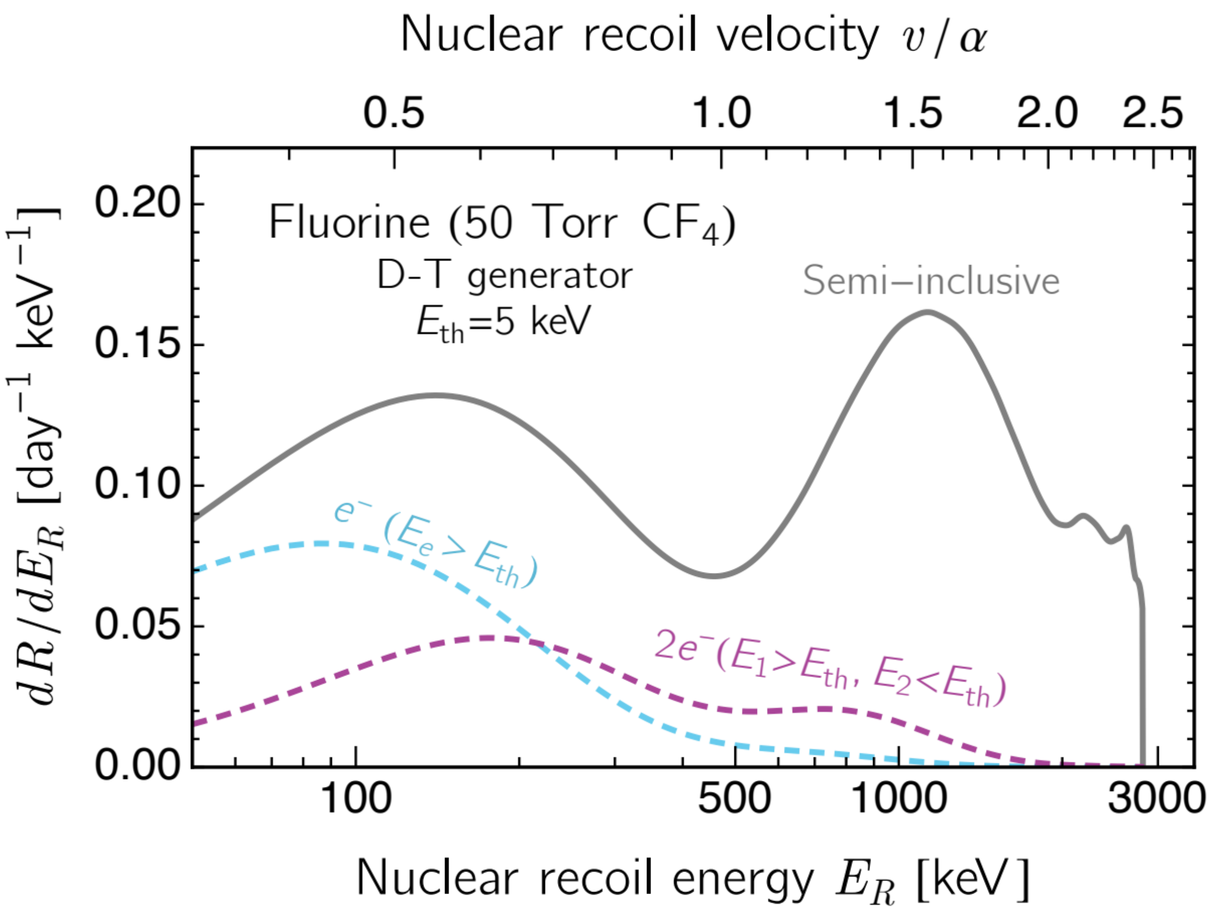
Interaction volume

Neutron generator

~1m

D-D: 10^9 n/s, 2.47 MeV
D-T: 10^{10} n/s, 14.7 MeV

50 Torr, 80 cm^3



Summary

- Migdal effect provides some of the strongest limits on sub-GeV dark matter

- Observation in neutron scattering is important to validate theory
  (MIGDAL experiment taking data soon)

- At high recoil energies, multiple ionisation dominates
  (beyond dipole approximation, semi-inclusive rate)

- Public code with Migdal probabilities for dark matter & neutron scattering
  (https://petercox.github.io/Migdal)