Classification of CP-violating operators in SMEFT

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Background

- Physics beyond the Standard Model (BSM)
  - Dark matter, neutrino masses, baryon asymmetry, and ...
  - The BSM at UV scales
    $\Rightarrow$ the Standard Model Effective Field Theory (SMEFT)
- CP Violation by the BSM $\gg$ CP violation by the SM
  - Measurement of CPV is very sensitive!
  - Neutron electric dipole moment, $K^0 - \bar{K}^0$ mixing, and ...

$\Rightarrow$ We need to know which operators violate CP in SMEFT.

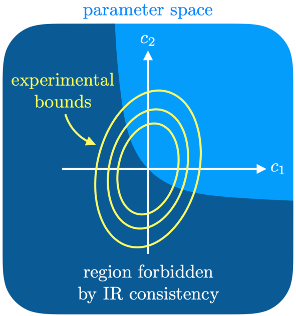
How does our research contribute to new physics?

1. Systematically list all operators violating CP. ← Our work!

   = Identify parameters that independently contribute to CPV.

2. Restrict each parameter by measuring CPV.

3. Put the above restrictions on BSM models.



[Remmen and Rodd, 2020]

Hilbert series

- Hilbert series: a series composed of singlets under given group(s)
  - Applied to construct EFT operators [Henning et al., 2017]

$$H_0(\{\phi_i\},\mathcal{D}) = \int \mathrm{d}\mu_{SO(4)}(x) \int \mathrm{d}\mu_{\text{gauge}}(y) \frac{1}{P(\mathcal{D},x)} \prod_i Z(\phi_i,\mathcal{D},x,y)$$

- The coeffecients represent the number of independent operators (modulo EOM and IBP)
  - e.g.) the Hilbert series for the SMEFT

$$\begin{aligned} H_0(\text{SMEFT}) &\supset 2HH^\dagger QQ^\dagger\mathcal{D} \\ &\Rightarrow i\left[H^\dagger(D_\mu H)-(D_\mu H^\dagger)H\right]\bar{Q}\sigma^\mu Q, \\ &\quad\; i\left[H^\dagger\sigma^a(D_\mu H)-(D_\mu H^\dagger)\sigma^a H\right]\bar{Q}\gamma^\mu\sigma^a Q \end{aligned}$$

Charge conjugation $\mathcal{C}$ for N-dimentional fundamental rep. of a compact Lie group G should be

1. linear
2. unitary
3. $\mathcal{C}^2 = e^{i\theta}\mathbb{1}$
4. $\mathcal{C}g\mathcal{C}^\dagger \in G$ for $\forall g \in G$
5. $\phi \xrightarrow{\mathcal{C}} C^*\phi^\dagger \quad \Rightarrow \quad \phi^\dagger \xrightarrow{\mathcal{C}} C\phi$

$$\Rightarrow \quad \mathcal{C}^2 = \begin{cases} +1 & (N\text{: odd}) \\ \pm 1 & (N\text{: even}) \end{cases}$$

An example of $\mathcal{C}_S^2 = +1$ & $\mathcal{C}_A^2 = -1$

$$H_i \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\mathcal{C}_S} \begin{pmatrix} \phi^{+\dagger} \\ \phi^{0\dagger} \end{pmatrix} \equiv H^{\dagger i} \xrightarrow{\mathcal{C}_S} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = +H_i$$

$$H_i \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\mathcal{C}_A} \begin{pmatrix} \phi^{0\dagger} \\ -\phi^{+\dagger} \end{pmatrix} \equiv H_i^\dagger \xrightarrow{\mathcal{C}_A} \begin{pmatrix} -\phi^+ \\ -\phi^0 \end{pmatrix} = -H_i$$

The transf. of singltes are not affected by the choice of $\mathcal{C}_S$ and $\mathcal{C}_A$.

$$H^\dagger H = H^{\dagger i} H_i \xrightarrow{\mathcal{C}_S} H_i H^{\dagger i} = +H^\dagger H$$

$$H^\dagger H = \epsilon^{ij} H_i^\dagger H_j \xrightarrow{\mathcal{C}_A} \epsilon^{ij} (-H_i) H_j^\dagger = +H^\dagger H$$

Listing CPV operators by the Hilbert series

The Hilbert series for CP-even/odd operators:

$$H^{CP\text{-even/odd}}(\{\phi_i\}, \mathcal{D}) = \frac{1}{2}\left[H^{+}(\{\phi_i\}, \mathcal{D}) \pm H^{-}(\{\phi_i\}, \mathcal{D})\right]$$

- $H^{+}(\{\phi_i\}, \mathcal{D})$: Singlets under Lorentz and gauge transf.
  $\supset 2\mathcal{O}$ (for $\mathcal{O} \not\propto \mathcal{O}^{\dagger} \equiv (\mathcal{CP})\mathcal{O}$), $\mathcal{O}$ (for $\mathcal{O} = \pm\mathcal{O}^{\dagger}$)
- $H^{-}(\{\phi_i\}, \mathcal{D})$: Singlets under Lorentz, gauge, and CP transf.
  $\supset \pm\mathcal{O}$ (for $\mathcal{O} = \pm\mathcal{O}^{\dagger}$)

$\Rightarrow$ $H^{CP\text{-even/odd}}$ enumerates $\mathcal{O} + \mathcal{O}^{\dagger}$ and $i(\mathcal{O} - \mathcal{O}^{\dagger})$, respectively.

The Hilbert series with CP

The formulae for listing CP-even and odd operators

$$H^{CP\text{-even/odd}}(\{\phi_i\},\mathcal{D}) = \frac{1}{2}\left[H^{+}(\{\phi_i\},\mathcal{D}) \pm H^{-}(\{\phi_i\},\mathcal{D})\right],$$

where

$$H^{+}(\{\phi_i\},\mathcal{D}) = \int \mathrm{d}\mu_{\mathsf{SO(4)}}(x) \int \mathrm{d}\mu_{\text{gauge}}(y) \frac{1}{P(\mathcal{D},x)} \prod_i Z(\phi_i,\mathcal{D},x,y),$$

$$H^{-}(\{\phi_i\},\mathcal{D}) = \int \mathrm{d}\mu_{\mathsf{Sp(2)}}(\tilde{x}) \int \mathrm{d}\mu_{\widetilde{\text{gauge}}}(\tilde{y}) \frac{1}{P^{-}(\mathcal{D},\tilde{x})} \prod_i Z^{-}(\phi_i,\mathcal{D},\tilde{x},\tilde{y}).$$

Removing CP phases by rephasing

The definition of "CP-violating operators"

Add one SMEFT operator $\mathcal{O}$ to the SM Lagrangian $\mathcal{L}_{SM}$.
If the CP phase of $\mathcal{O}$ cannot be removed by rephasing while keeping the CP phases in $\mathcal{L}_{SM}$ fixed, $\mathcal{O}$ is a *CP-violating operator*.

U(1) symmetries that keep δ_{CKM}, θ_{QCD}, and $\theta_{\text{weak}}(=0)$ invariant:
$\mathrm{U}(1)_{B-L}$, $\mathrm{U}(1)_{L_1-L_2}$, and $\mathrm{U}(1)_{L_2-L_3}$.

- CP-violating operators are invariant under these U(1).
- They can be obtained just by adding the three U(1) to $H^{\pm}$.

The number of CP-odd and CP-violating operators

Mass dimension	5	6	7	8	9	10	11	12
CP-odd ops.	6	1422	771	22016	45228	1042942	1736133	37761366
CP-violating ops.	0	705	0	11777	0	437331	0	13891774
Time [s]	0.01	0.10	0.15	0.50	2.86	9.14	24.10	326.21

- Dim 6:
  705 CPV oerators $= 6$ (only bosonic) $+ 699$ (including fermions)
  - Consistent with previous results by hand [Bonnefoy et al., 2021]
- Dim 8: Classification of $H^2W^2\mathcal{D}^2$
  - Remmen and Rodd (2020) claims that there are three CP-odd operators.
  - Our Hilbert series tells that there are only two. $\leftarrow$ This is correct!

CP-odd opertors at $D = 6$

$$H^{CP\text{-odd}} = 3l^2e^2 + 9d^2e^2 + 9d^2l^2 + 18d^4 + 9u^2e^2 + 9u^2l^2 + 72u^2d^2 + 18u^4 + 27qdle + 54qule + 9q^2e^2 + 18q^2l^2 + 72q^2d^2 + 162q^2ud + 72q^2u^2 + 36q^4 + G^3 + W^3 + 9hGqd + 9hGqu + 3hWle + 9hWqd + 9hWqu + 3hBle + 9hBqd + 9hBqu + 3h^2\mathcal{D}d^2 + 9h^2\mathcal{D}ud + 3h^2\mathcal{D}u^2 + 6h^2\mathcal{D}q^2 + h^2G^2 + h^2W^2 + h^2BW + h^2B^2 + 3h^3le + 9h^3qd + 9h^3qu$$

$\rightarrow$ contribute to the neutron EDM

$$H^{CP\text{-odd}} = +6\mathcal{D}^2l^2e^2 + 18\mathcal{D}^2d^2e^2 + 18\mathcal{D}^2d^2l^2 + 36\mathcal{D}^2d^4 + 18\mathcal{D}^2u^2e^2 + 18\mathcal{D}^2u^2l^2 + 144\mathcal{D}^2u^2d^2 + 36\mathcal{D}^2u^4 + 54\mathcal{D}^2qdle + 81\mathcal{D}^2qule + 18\mathcal{D}^2q^2e^2 + 36\mathcal{D}^2q^2l^2 + 144\mathcal{D}^2q^2d^2 +$$

$$243\mathcal{D}^2q^2ud + 144\mathcal{D}^2q^2u^2 + 72\mathcal{D}^2q^4 + 27Gd^2e^2 + 27Gd^2l^2 + 81Gd^4 + 27Gu^2e^2 + 27Gu^2l^2 + 324Gu^2d^2 + 81Gu^4 + 54Gqdle + 81Gqule + 27Gq^2e^2 + 54Gq^2l^2 + 324Gq^2d^2 + 486Gq^2ud +$$

$$324Gq^2u^2 + 162Gq^4 + 18G^2\mathcal{D}d^2 + 18G^2\mathcal{D}u^2 + 18G^2\mathcal{D}q^2 + 3G^4 + 15Wl^2e^2 + 12Wl^4 + 27Wd^2l^2 + 27Wu^2l^2 + 54Wqdle + 81Wqule + 27Wq^2e^2 + 81Wq^2l^2 + 162Wq^2d^2 + 243Wq^2ud +$$

$$162Wq^2u^2 + 126Wq^4 + 18WG\mathcal{D}q^2 + 3W^2\mathcal{D}l^2 + 3W^2\mathcal{D}d^2 + 3W^2\mathcal{D}u^2 + 15W^2\mathcal{D}q^2 + 3W^2G^2 + 2W^4 + 3Be^4 + 15Bl^2e^2 + 6Bl^4 + 27Bd^2e^2 + 27Bd^2l^2 + 36Bd^4 + 27Bu^2e^2 + 27Bu^2l^2 +$$

$$162Bu^2d^2 + 36Bu^4 + 54Bqdle + 81Bqule + 27Bq^2e^2 + 54Bq^2l^2 + 162Bq^2d^2 + 243Bq^2ud + 162Bq^2u^2 + 72Bq^4 + 18BG\mathcal{D}d^2 + 18BG\mathcal{D}u^2 + 18BG\mathcal{D}q^2 + 2BG^3 + 6BW\mathcal{D}l^2 +$$

$$18BW\mathcal{D}q^2 + 3B^2\mathcal{D}d^2 + 3B^2\mathcal{D}u^2 + 3B^2\mathcal{D}q^2 + 3B^2G^2 + 3B^2W^2 + B^4 + 21h\mathcal{D}le^3 + 45h\mathcal{D}l^3e + 81h\mathcal{D}d^2le + 81h\mathcal{D}udle + 81h\mathcal{D}u^2le + 81h\mathcal{D}qde^2 + 162h\mathcal{D}qdl^2 + 243h\mathcal{D}qd^3 +$$

$$81h\mathcal{D}que^2 + 162h\mathcal{D}qul^2 + 486h\mathcal{D}qud^2 + 486h\mathcal{D}qu^2d + 243h\mathcal{D}qu^3 + 162h\mathcal{D}q^2le + 486h\mathcal{D}q^3d + 486h\mathcal{D}q^3u + 27hG\mathcal{D}^2qd + 27hG\mathcal{D}^2qu + 6hG^2le + 45hG^2qd + 45hG^2qu +$$

$$9hW\mathcal{D}^2le + 27hW\mathcal{D}^2qd + 27hW\mathcal{D}^2qu + 27hWGqd + 27hWGqu + 9hW^2le + 27hW^2qd + 27hW^2qu + 9hB\mathcal{D}^2le + 27hB\mathcal{D}^2qd + 27hB\mathcal{D}^2qu + 27hBGqd + 27hBGqu +$$

$$9hBWle + 27hBWqd + 27hBWqu + 6hB^2le + 18hB^2qd + 18hB^2qu + 15h^2l^2e^2 + 3h^2l^4 + 9h^2d^2e^2 + 18h^2d^2l^2 + 18h^2d^4 + 27h^2udl^2 + 9h^2u^2e^2 + 18h^2u^2l^2 + 72h^2u^2d^2 + 18h^2u^4 +$$

$$108h^2qdle + 135h^2qule + 18h^2q^2e^2 + 54h^2q^2l^2 + 234h^2q^2d^2 + 486h^2q^2ud + 234h^2q^2u^2 + 90h^2q^4 + 6h^2\mathcal{D}^3d^2 + 9h^2\mathcal{D}^3ud + 6h^2\mathcal{D}^3u^2 + 12h^2\mathcal{D}^3q^2 + 18h^2G\mathcal{D}d^2 + 18h^2G\mathcal{D}ud +$$

$$18h^2G\mathcal{D}u^2 + 36h^2G\mathcal{D}q^2 + h^2G^2\mathcal{D}^2 + h^2G^3 + 6h^2W\mathcal{D}e^2 + 18h^2W\mathcal{D}l^2 + 18h^2W\mathcal{D}d^2 + 18h^2W\mathcal{D}ud + 18h^2W\mathcal{D}u^2 + 54h^2W\mathcal{D}q^2 + 2h^2W^2\mathcal{D}^2 + h^2W^3 + 6h^2B\mathcal{D}e^2 + 12h^2B\mathcal{D}l^2 +$$

$$18h^2B\mathcal{D}d^2 + 18h^2B\mathcal{D}ud + 18h^2B\mathcal{D}u^2 + 36h^2B\mathcal{D}q^2 + 3h^2BW\mathcal{D}^2 + h^2BW^2 + h^2B^2\mathcal{D}^2 + 18h^3\mathcal{D}^2le + 54h^3\mathcal{D}^2qd + 54h^3\mathcal{D}^2qu + 9h^3Gqd + 9h^3Gqu + 6h^3Wle + 18h^3Wqd +$$

$$18h^3Wqu + 3h^3Ble + 9h^3Bqd + 9h^3Bqu + 3h^4\mathcal{D}l^2 + 3h^4\mathcal{D}d^2 + 9h^4\mathcal{D}ud + 3h^4\mathcal{D}u^2 + 15h^4\mathcal{D}q^2 + h^4G^2 + 2h^4W\mathcal{D}^2 + 2h^4W^2 + h^4B\mathcal{D}^2 + h^4BW + h^4B^2 + 3h^5le + 9h^5qd + 9h^5qu$$

- CP violation is the key for BSM.
- I have introduced an algorithm for systematic classification of the SMEFT operators based on their CP properties.
  - By the Hilbert series techniques.
  - Also considered the rephasing redundancies.
- Results
  - We succeeded in reproducing and correcting an error of previous studies' results.
  - Higher-dimensional operators can be obtained in a few minutes.
- Our method can be applied to other EFTs such as QCD EFT or the SMEFT w/ gravity.

- Integrand Z

$$Z(\phi_i, \mathcal{D}, x, y) = \exp\left[\sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{1}{n} \left(\frac{\phi_i}{\mathcal{D}^{\Delta_i}}\right)^n \chi_i(\mathcal{D}^n, x^n, y^n)\right]$$

The coefficient of ϕ_i^n for boson (fermion)

$\rightarrow$ (Anti-)symmetric product of character χ_i

- The factor P removes the IBP redundancy

$$P(\mathcal{D}, x) = \frac{1}{(1 - \mathcal{D}x_1)(1 - \mathcal{D}x_1^{-1})(1 - \mathcal{D}x_2)(1 - \mathcal{D}x_2^{-1})}$$

- Pick up only singlets by orthogonality of characters

$$\int \mathrm{d}\mu(x)\ \chi_R^*(x) \chi_{R'}(x) = \delta_{RR'}$$

- We use $\check{\phi} = \phi \oplus \phi^\dagger$ to calculate the Hilbert series $H^\pm$.
  - e.g.) $\check{H} \ni H,\ H^\dagger,\ H + H^\dagger,\ H - H^\dagger,\ \cdots$
- The operator $\check{\mathcal{O}}$ in terms of $\check{\phi}$
  - e.g.) $\check{\psi}_1\check{\psi}_2\check{\psi}_3\check{\psi}_4 \ni \psi_1^\dagger\psi_2^\dagger\psi_3\psi_4,\ \psi_1\psi_2^\dagger\psi_3^\dagger\psi_4,\ \psi_1\psi_2\psi_3^\dagger\psi_4^\dagger,\ \cdots$
- $H^\pm$ counts independent operators in $\check{\mathcal{O}}$

$$H^{CP\text{-odd}}(SM) \supset 2\check{H}\check{H}\check{W}\check{W}\check{\mathcal{D}}\check{\mathcal{D}}$$

$$= i\epsilon^{IJK}(\mathcal{D}^\mu H^\dagger \sigma^I \mathcal{D}^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu),$$

$$(\mathcal{D}^\mu H^\dagger \mathcal{D}^\mu H)W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$$

$H^2W^2\mathcal{D}^2$

- Remmen and Rodd (2020) claims that

$$i\epsilon^{IJK}(\mathcal{D}^\mu H^\dagger\sigma^I\mathcal{D}^\nu H)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$$

is CP odd.

- In fact, this is CP even:

$$\begin{aligned}
&i\epsilon^{IJK}[\mathcal{D}^\mu H^{\dagger i}(\sigma^I)_i{}^j\mathcal{D}^\nu H_j](W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu) \\
&\xrightarrow{\mathcal{C}_A} i\epsilon^{IJK}[-\mathcal{D}^\nu H^{\dagger i}(\sigma^I)_i{}^j\mathcal{D}^\mu H_j][W^J_{\mu\rho}(-\widetilde{W}^{K\rho}_\nu) + (-\widetilde{W}^J_{\mu\rho})W^{K\rho}_\nu] \\
&= +i\epsilon^{IJK}[\mathcal{D}^\mu H^{\dagger i}(\sigma^I)_i{}^j\mathcal{D}^\nu H_j](W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)
\end{aligned}$$