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Graduiertenkolleg 2149
Research Training Group

Improved theoretical predictions of the dark matter relic abundance

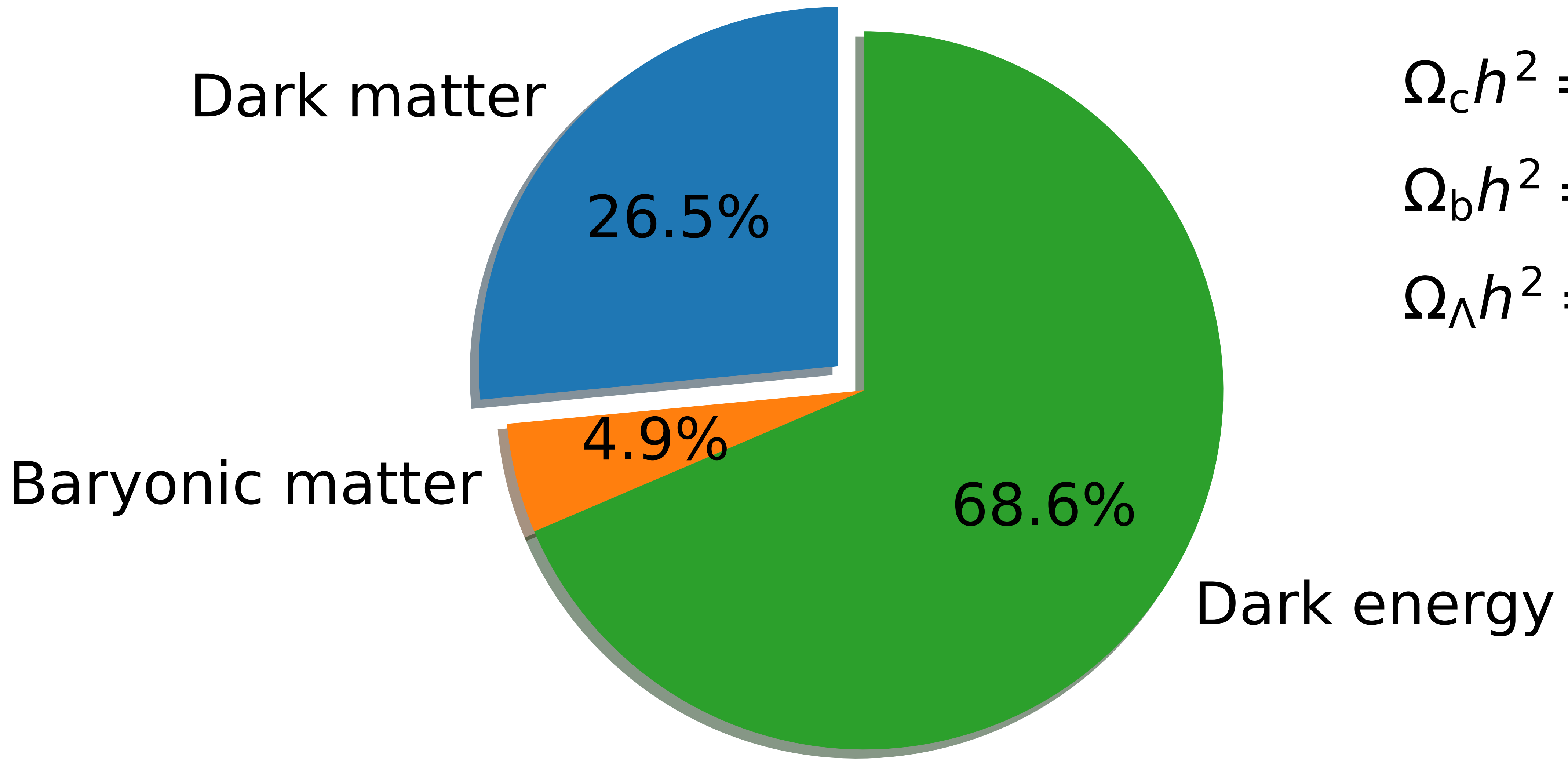
Based on arXiv: 2210.03409 in collaboration with Julia Harz, Michael Klasen and Mohamed Younes Sassi

and arXiv: 2210.05260 in collaboration with Michael Klasen and Karol Kovařík (PRD accepted)

Luca Wiggering @ DSU 2022

How much dark matter is in the universe?

- Energy composition of the universe is experimentally known at the %-level



$\Omega_c h^2 = 0.120 \pm 0.001$

$\Omega_b h^2 = 0.0224 \pm 0.0001$

$\Omega_\Lambda h^2 = 0.3107 \pm 0.0082$

[Planck 2018 - Aghanim *et al.* arXiv: 1807.06209 (astro-ph)]

➡The dark matter relic density highly constraints dark matter models

Computation of the DM freeze-out abundance

- Boltzmann equation in an expanding FLRW universe:

$$\partial_t f_X - Hp\partial_p f_X = \frac{1}{2Eg_X}\sum_{r\in\mathcal{R}_X}(\pm)_r\int\prod_{k\in(\mathcal{I}_r\cup\mathcal{F}_r)\setminus\{X\}}\delta(p_k^2-m_k^2)\mathrm{d}^4p_k\delta^{(4)}\Big(\sum_{i\in\mathcal{I}_r}p_i-\sum_{j\in\mathcal{F}_r}p_j\Big)|\mathcal{M}_{\mathcal{I}_r\to\mathcal{F}_r}|^2\prod_{i\in\mathcal{I}_r}f_i\prod_{j\in\mathcal{F}_r}(1\pm f_j)$$

- Take into account only the 0-th moment of the Boltzmann equation:

$$\dot{n}_\chi + 3Hn_\chi = \int \mathrm{d}^3p\frac{g_\chi}{E}\hat{C}[f_\chi]$$

- Neglecting quantum statistics and assuming kinetic equilibrium $f_\chi = \frac{n_\chi}{n_{\mathrm{eq}}}f_{\mathrm{eq}}$ gives in the freeze-out case:

$$\dot{n}_\chi + 3Hn_\chi = \langle\sigma_{\chi\bar{\chi}\to\mathrm{SM}}v\rangle((n_\chi^{\mathrm{eq}})^2 - n_\chi^2)$$

- Implement co-annihilation via $n_\chi \to n_\chi = \sum_{i\notin\mathrm{SM}} n_i$ and $n_i = \frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}}n_\chi$ (kinetic equilibrium and same μ) giving:

$$\langle\sigma_{\mathrm{eff}}v\rangle = \sum_{i,j}\frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}}\frac{n_j^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}}\langle\sigma_{ij\to\mathrm{SM}}v\rangle \text{ with } \frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}} \sim e^{-(m_i-m_\chi)/T}$$

[Gondolo and Gelmini, Nucl. Phys. B 360 (1991)]
[Edsjö and Gondolo, arXiv: 9704361 (hep-ph)]

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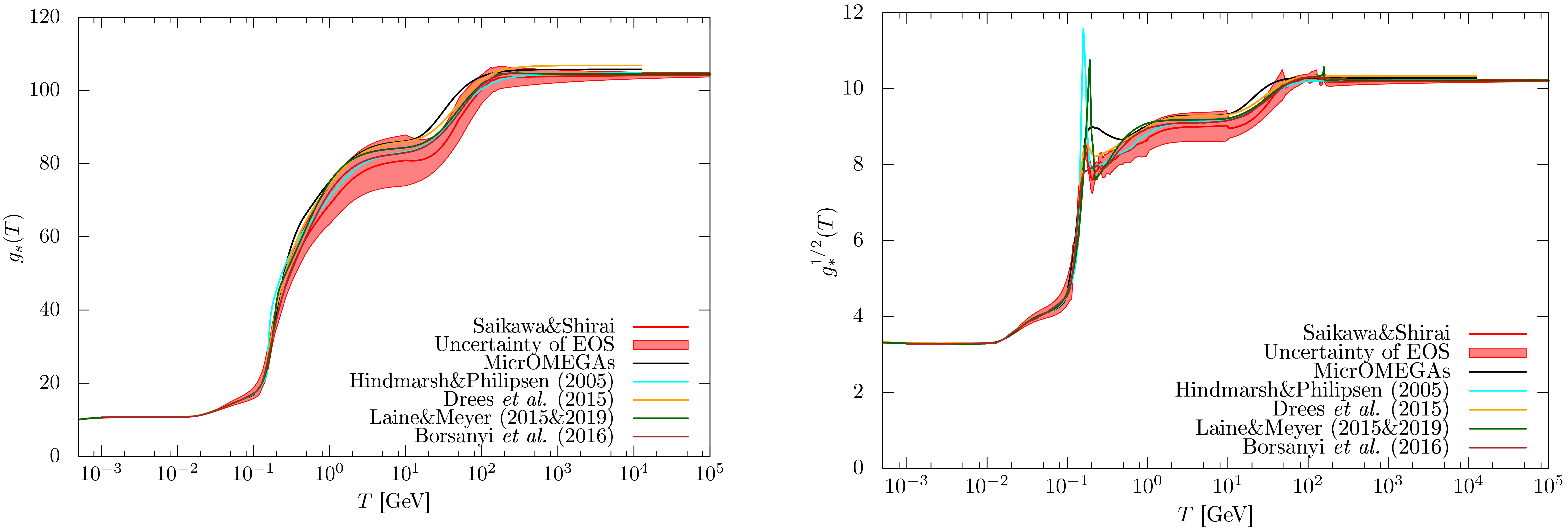
Theoretical uncertainties - h_{eff} and g_{eff}



➡ 10% difference in the final dark matter abundance

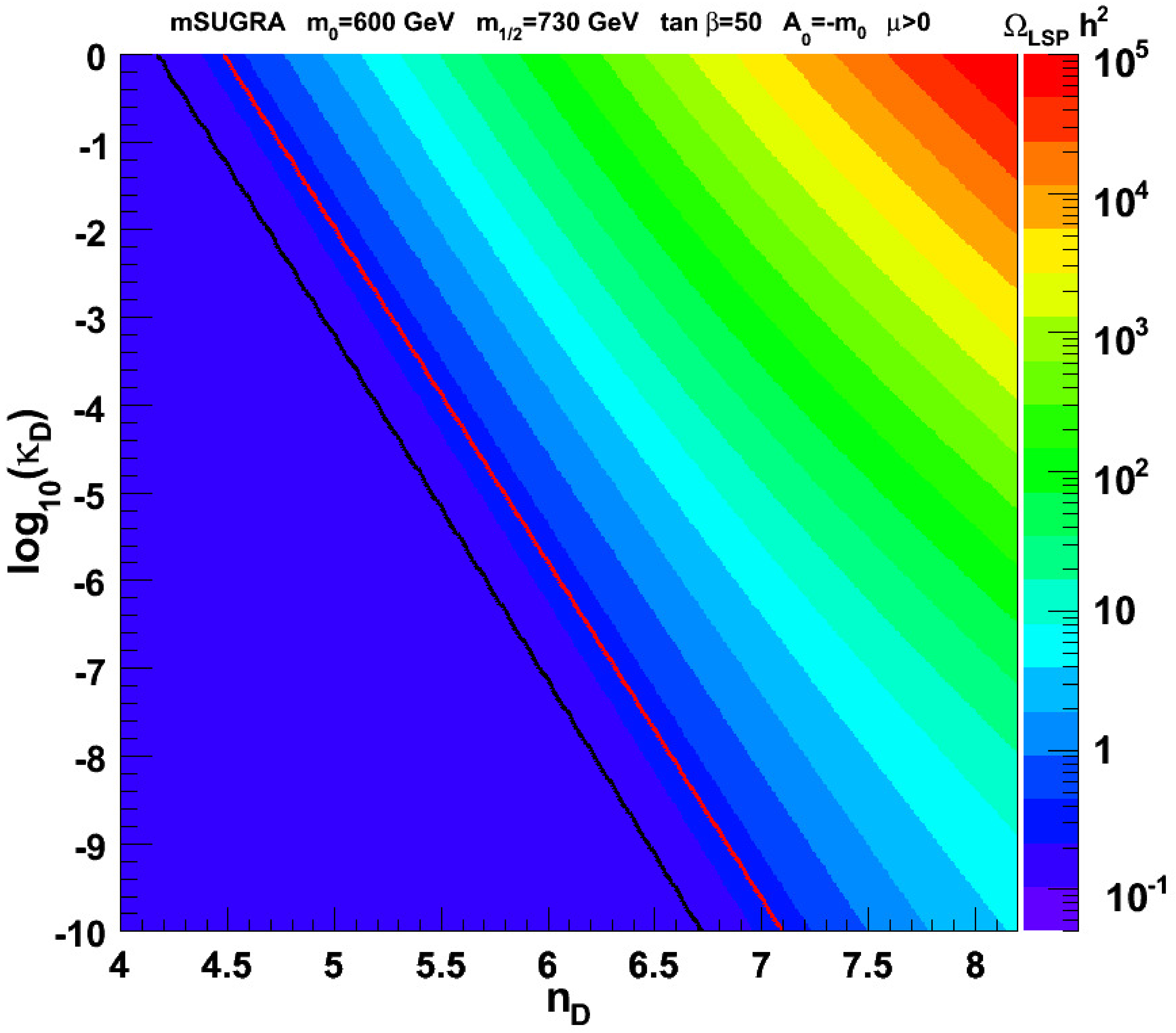
[Saikawa and Shirai, arXiv: 2005.03544 (hep-ph)]

Theoretical uncertainties - H before BBN

- Additional dark radiation:

$$\rho_D(T) = \kappa_D \rho_{\mathrm{SM}}(T_0) \left(\frac{T}{T_0} \right)^{n_D}$$

- Modification of $\Omega_{\mathrm{CDM}} h^2$ up to a factor 10^5



[Arbey and Mahmoudi, arXiv: 0803.0741 (hep-ph)]

Theoretical uncertainties - early kinetic decoupling

- Solve Boltzmann equation on the level of the phase space distribution function

$$\partial_t f_\chi - Hp\partial_p f_\chi = C_{\text{ann}} + C_{\text{FP}}$$

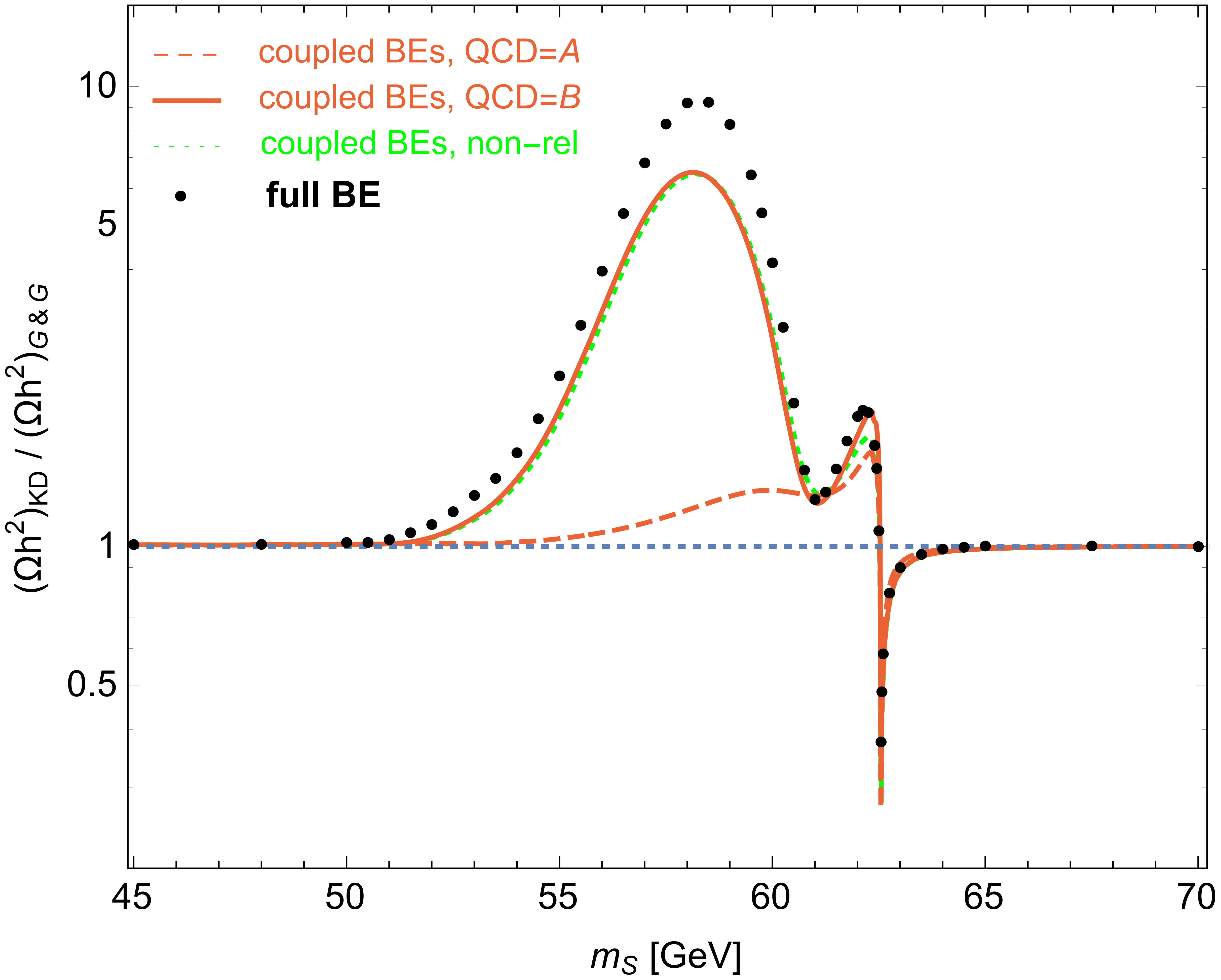
$$C_{\text{ann}} = g_\chi E \int \frac{d^3\tilde{p}}{(2\pi)^3} v\sigma_{\bar{\chi}\chi\to\bar{f}f} \left[f_{\chi,\text{eq}}(E) f_{\chi,\text{eq}}(\tilde{E}) - f_\chi(E) f_\chi(\tilde{E}) \right]$$

$$C_{\text{FP}} \simeq \frac{E}{2}\gamma(T) \left[TE\partial_p^2 + \left(p + 2T\frac{E}{p} + T\frac{p}{E} \right) \partial_p + 3 \right] f_\chi$$

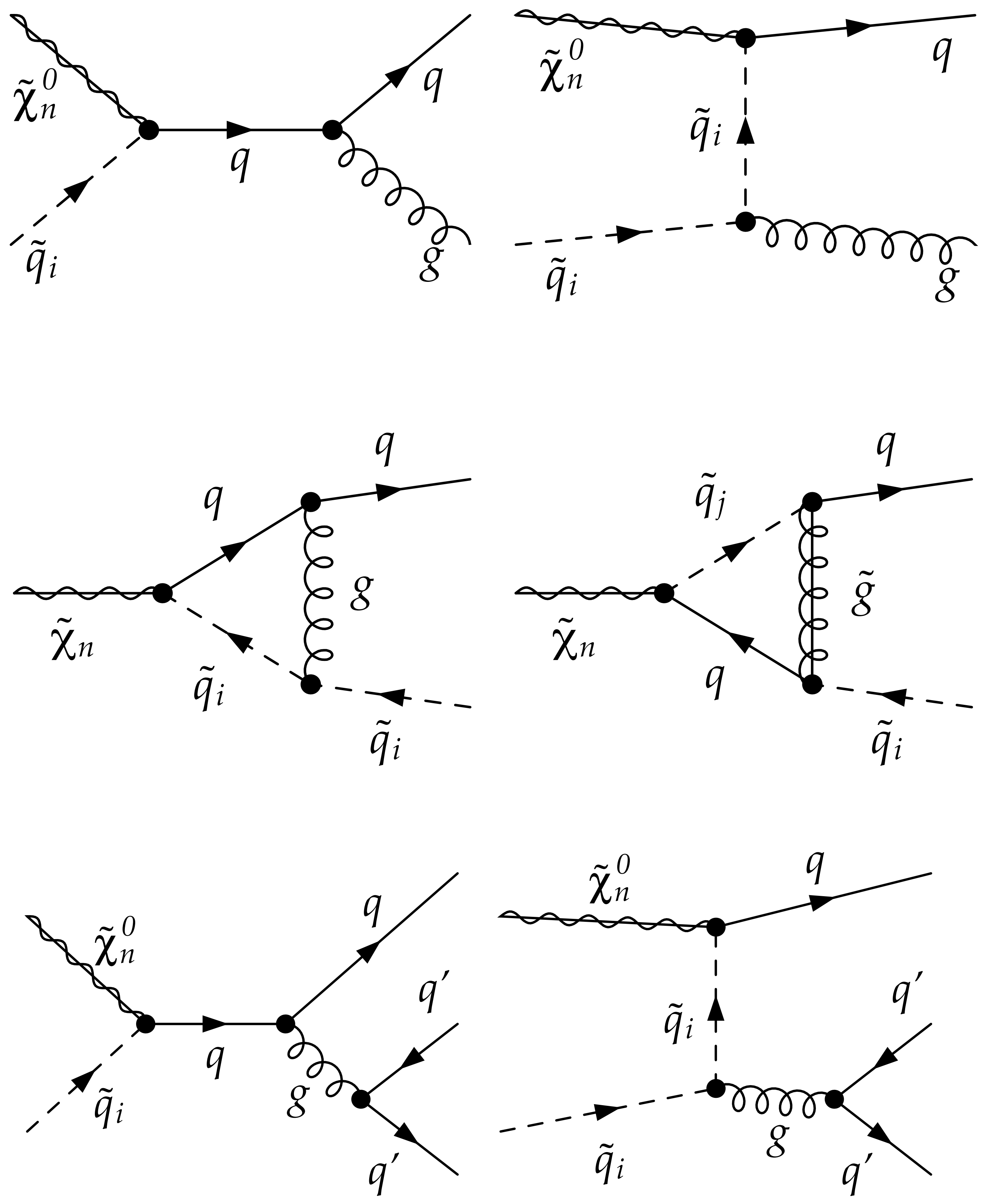
DRAKE

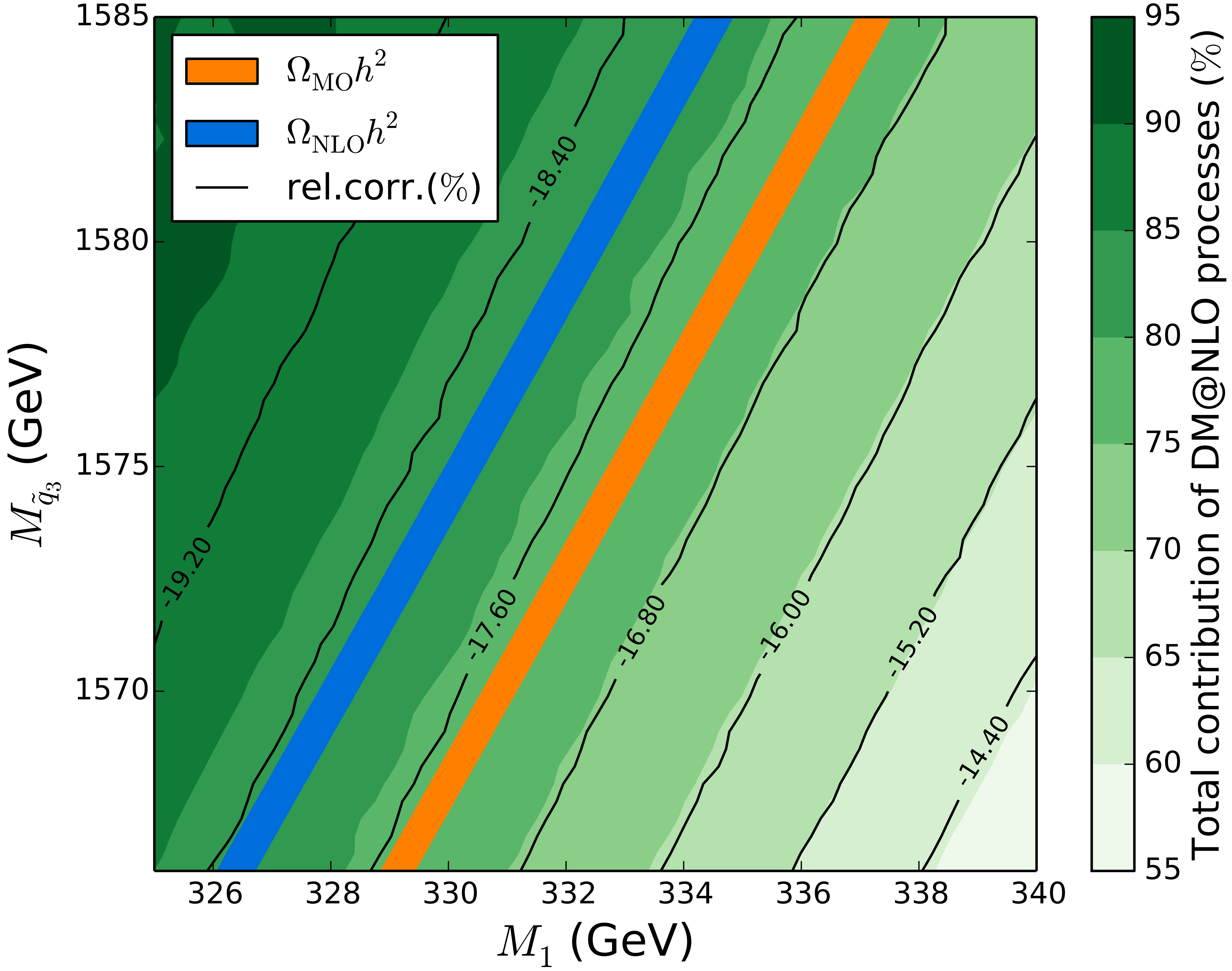
[Binder et al. arXiv: 1706.07433 (hep-ph)]

[Binder et al. arXiv: 2103.01944 (hep-ph)]



Theoretical uncertainties - higher-order corrections





[Harz et al. arXiv: 1409.2898 (hep-ph)]

The DM@NLO project

- The goal of the DM@NLO is to provide a consistent set of NLO corrections in SUSY-QCD (+resummation) for important (co)annihilation processes in the MSSM

- Ultimate goal: use of DM@NLO within GAMBIT studies, make code public ...

- Processes that have been included so far:

  ✓ $\tilde{\chi}\tilde{\chi}' \to qq'$ [Herrmann and Klasen, arXiv:0709.0043 (hep-ph); Herrmann et al. arXiv:0901.0481 (hep-ph), Herrmann et al. arXiv:0907.0030 (hep-ph); Herrmann et al. arXiv:1404.2931 (hep-ph)]

  ✓ $\tilde{q}\tilde{\chi} \to qV/qg/HV$ [Harz et al. arXiv:1212.5241 (hep-ph), Harz et al. arXiv:1409.2898 (hep-ph)]

  ✓ $\tilde{q}\tilde{q}^* \to VV/HH/VH$ [Harz et al. arXiv:1410.8063 (hep-ph)]

  ✓ $\tilde{q}\tilde{q}' \to qq'$ [Schmiemann et al. arXiv:1903.10998 (hep-ph)]

  ✓ $\tilde{\tau}\tilde{\tau}^* \to q\bar{q}$ [Branahl et al. arXiv:1909.09527 (hep-ph)]

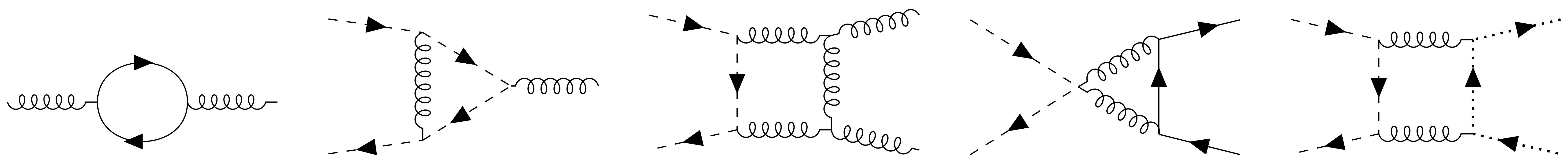
  ✓ $\tilde{t}\tilde{t}^* \to gg, q\bar{q}$ [Klasen et al. arXiv: 2210.05260 (hep-ph)]

Stop-antistop annihilation into gluons and light quarks @ NLO

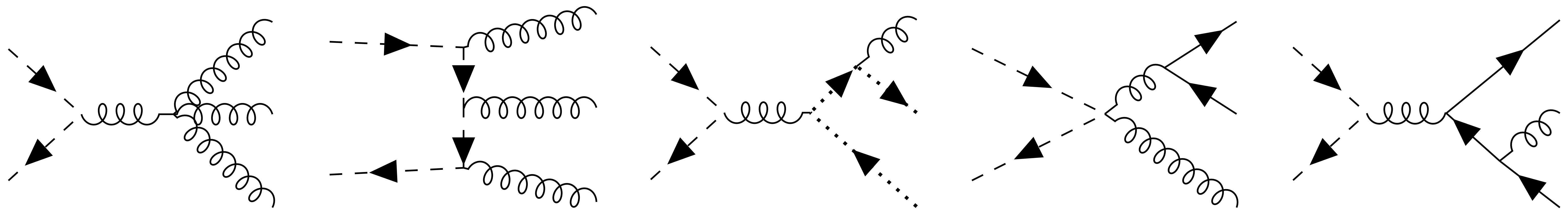
- General structure of a NLO cross section:

$$\sigma^{\mathrm{NLO}} = \sigma^{\mathrm{Tree}} + \int \mathrm{d}\sigma^{\mathrm{V}} + \int \mathrm{d}\sigma^{\mathrm{R}}$$

- Virtual corrections (a few examples):



- Real corrections (a few examples):



The dipole subtraction method à la Catani-Seymour

- Auxiliary cross section: $\Delta\sigma^{\text{NLO}} = \int_{m+1}\left[\mathrm{d}\sigma^{\text{R}}_{\varepsilon=0} - \mathrm{d}\sigma^{\text{A}}_{\varepsilon=0}\right] + \int_{m}\left[\mathrm{d}\sigma^{\text{V}} + \int_{1}\mathrm{d}\sigma^{\text{A}}\right]_{\varepsilon=0}$

- Singular behaviour of $2 \to m+1$ processes in the soft and collinear limit
  - Soft limit: ${}_{m+1,a\ldots}\langle \ldots, i, \ldots, j\ldots; a, \ldots | \ldots, i, \ldots, j, \ldots; a, \ldots\rangle_{m+1,a\ldots} \xrightarrow{p_i \to 0} -4\pi\mu^{2\varepsilon}\alpha_s\, {}_{m,a\ldots}\langle \ldots, j, \ldots; a, \ldots | \mathbf{J}^{\dagger}_{\mu}\mathbf{J}^{\mu} | \ldots, j, \ldots; a, \ldots\rangle_{m,a\ldots}$

    with the eikonal current $\mathbf{J}^{\mu} = \sum_{I} \frac{p_I^{\mu}}{p_I \cdot p_i}$

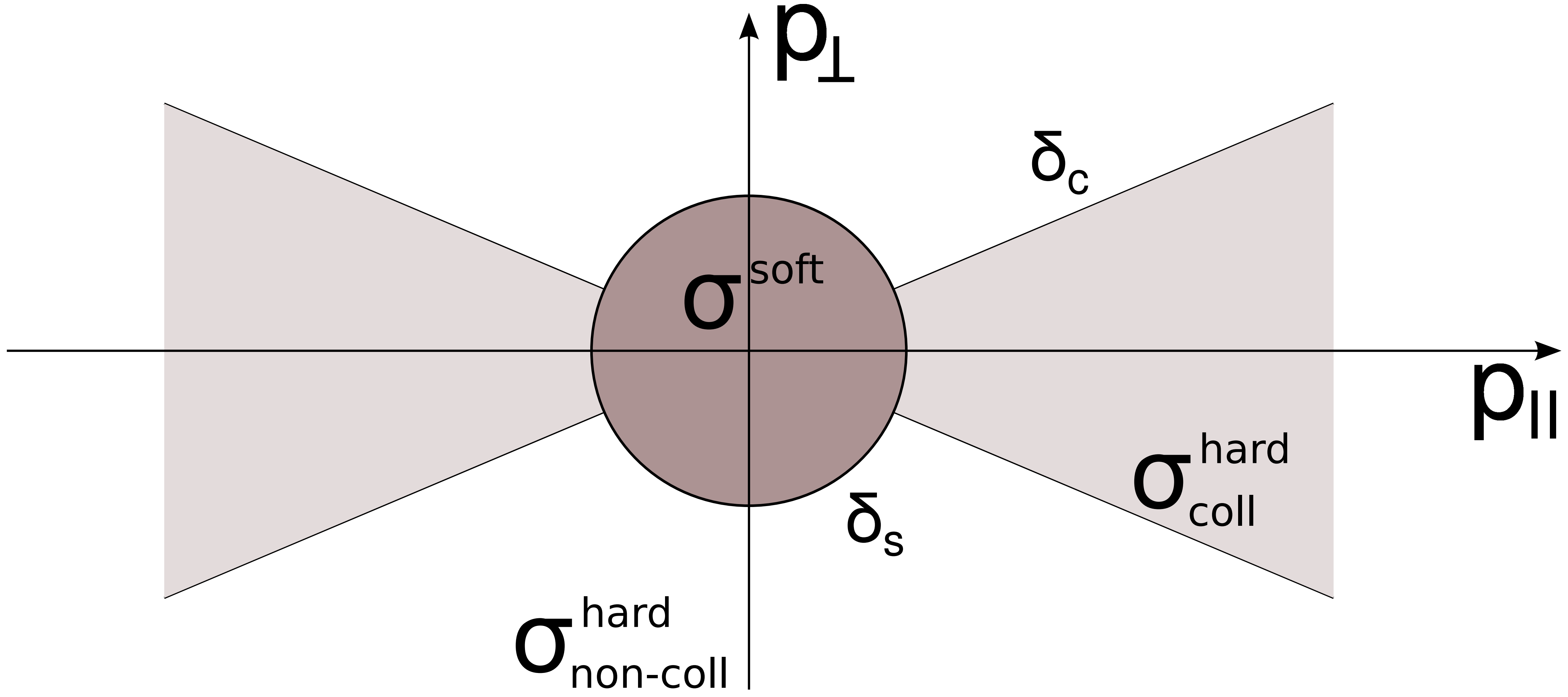
  - Collinear limit: ${}_{m+1,a\ldots}\langle \ldots, i, j, \ldots; a, \ldots | \ldots, i, j, \ldots; a, \ldots\rangle_{m+1,a\ldots} \xrightarrow{p_i \| p_j} \frac{4\pi\mu^{2\varepsilon}}{p_i \cdot p_j}\, {}_{m,a\ldots}\langle \ldots, \widetilde{ij}, \ldots; a, \ldots | \hat{P}_{\widetilde{ij},i}(z, k_\perp; \varepsilon) | \ldots, \widetilde{ij}, \ldots; a, \ldots\rangle_{m,a\ldots}$

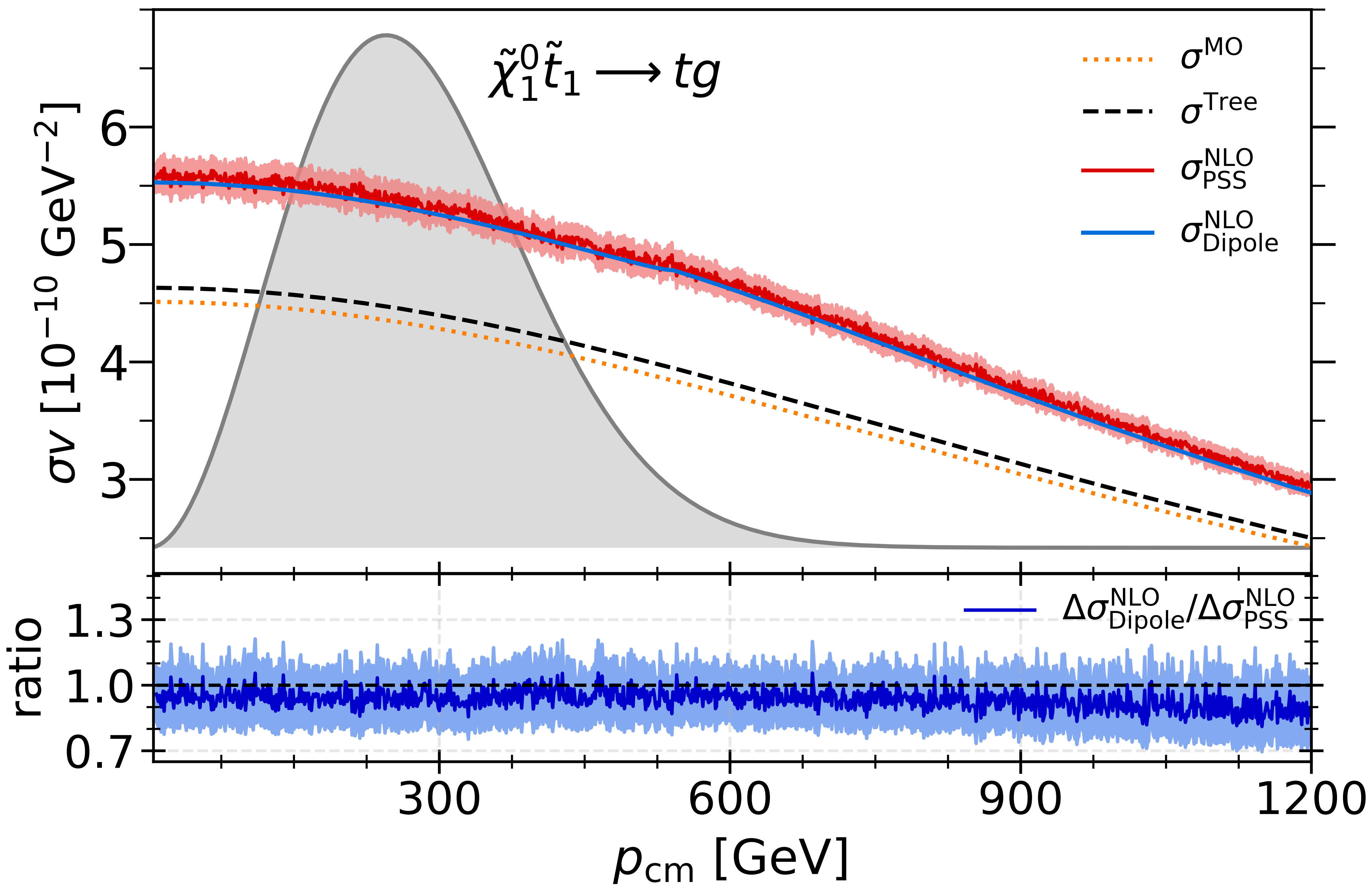
- Dipole functions: $\mathscr{D}^{a}_{ij} = \frac{1}{-2p_i \cdot p_j}\frac{1}{x_{ij,a}}\, {}_{m,a\ldots}\langle \ldots, \widetilde{ij}, \ldots; \tilde{a}, \ldots | \frac{\mathbf{T}_a \cdot \mathbf{T}_{ij}}{\mathbf{T}^2_{ij}} \mathbf{V}^{a}_{ij} | \ldots, \widetilde{ij}, \ldots; \tilde{a}, \ldots\rangle_{m,a}$

- Phase space factorisation: $\int \mathrm{d}\phi_{m+1}\left(p_i, p_j, p_k; p_a + p_b\right)\theta(x_{ij,a} - x_0) = \int_{x_0}^{1}\mathrm{d}x \int \mathrm{d}\phi_m\left(P(x), p_k; p_a + p_b\right)\int\left[\mathrm{d}p_i\left(Q^2, x, z_i\right)\right]$

Comparison with the phase space slicing method

- Phase space slicing method: $\sigma^{\mathrm{R}} = \sigma^{\text{hard}}_{\text{non-coll}}(\delta_s, \delta_c) + \sigma^{\text{hard}}_{\text{coll}}(\delta_s, \delta_c) + \sigma^{\text{soft}}(\delta_s)$ [Harris and Owens, arXiv:0102128 (hep-ph)]
- Advantages of the dipole subtraction method: no cutoff dependence, no separation of squared diagrams into collinear, soft and soft-collinear divergent contributions, easy to automatise (see e.g. `AutoDipole`, `MadDipole`), ...

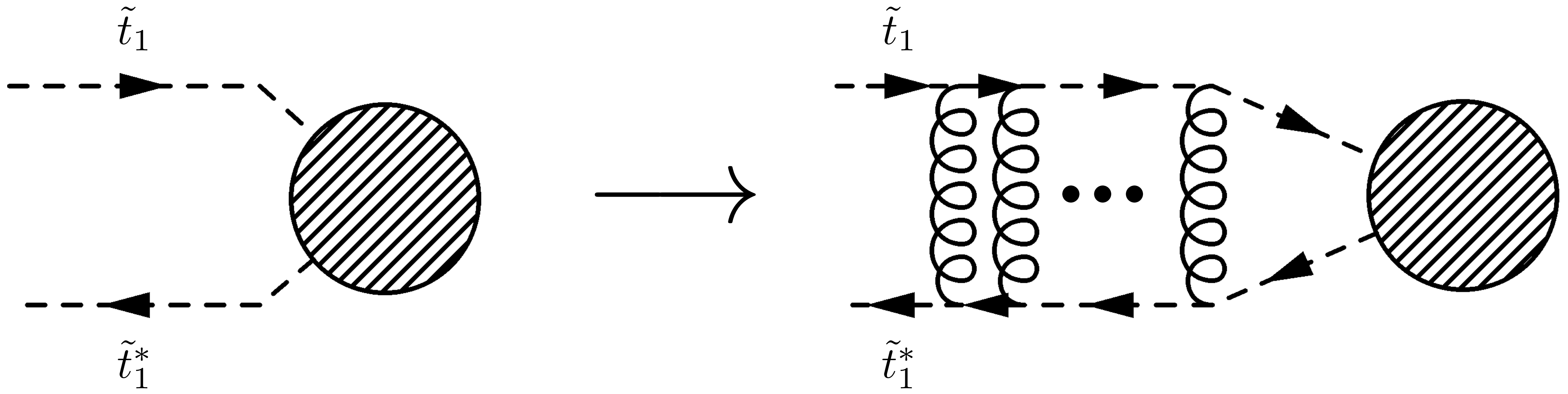




Sommerfeld enhancement

- For small relative velocities v between the incoming stop-antistop pair the annihilation cross section grows as $(\alpha_s/v)^n$ for the exchange of n potential gluons [Sommerfeld, A. Annalen Phys. 403, 257–330 (1931)]

  ➡ All order resummation within the framework of non-relativistic QCD



- Solve Schrödinger equation for the one-loop QCD Coulomb potential

$$\tilde{V}^{[\mathbf{R}]}(\vec{q}) = -\,C^{[\mathbf{R}]}\,\frac{4\pi\alpha_s(\mu_C)}{\vec{q}^{\,2}}\left\{1+\frac{\alpha_s(\mu_C)}{4\pi}\left[\beta_0\ln\left(\frac{\mu_C^2}{\vec{q}^{\,2}}\right)+a_1\right]\right\}$$

with the colour decomposition

$$\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{1}\oplus\mathbf{8}$$
$$\mathbf{8}\otimes\mathbf{8} = \mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\overline{\mathbf{10}}\oplus\mathbf{10}\oplus\mathbf{27}$$

[Kiyo et al. arXiv:0812.0919 (hep-ph)]

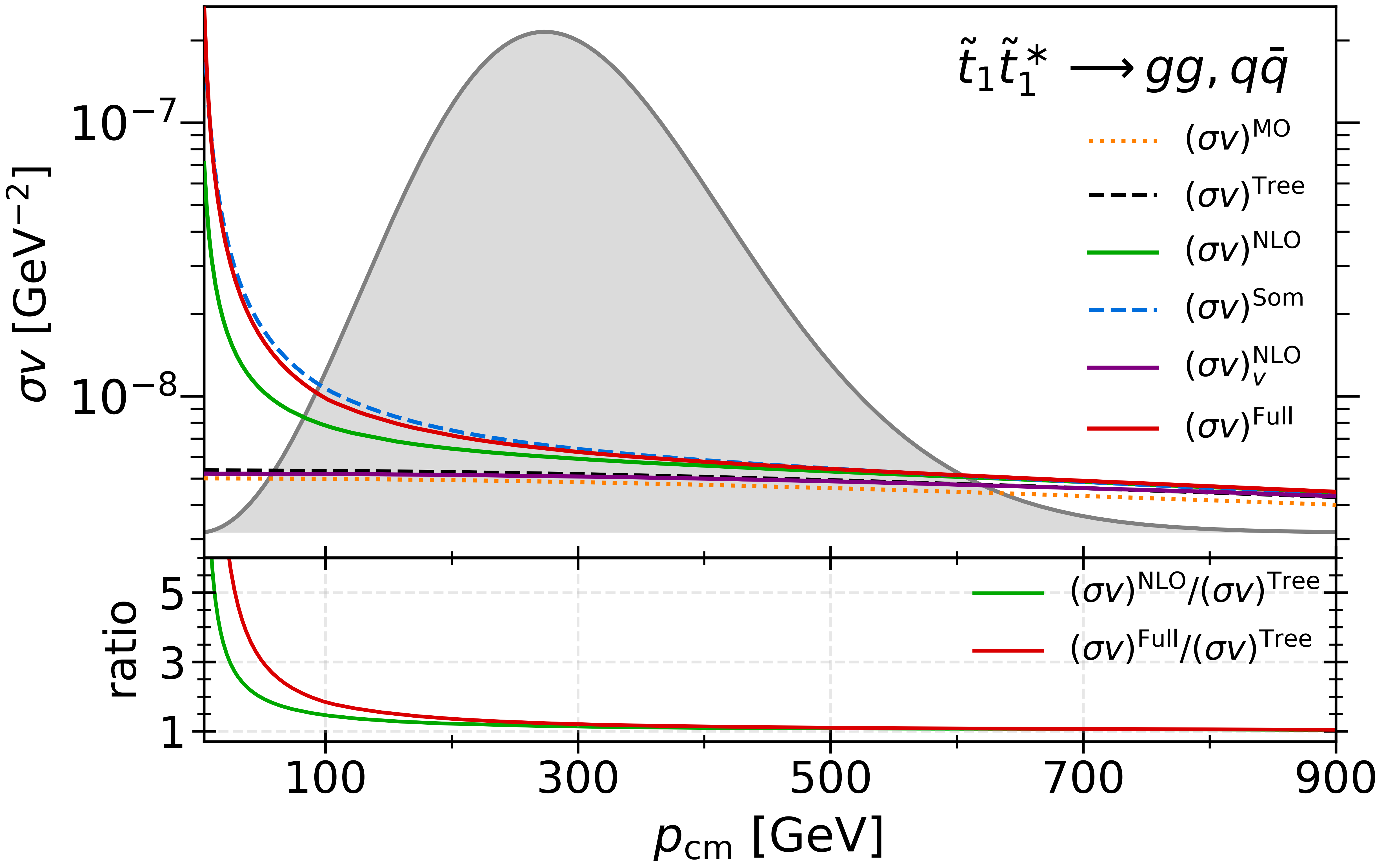
- Express Sommerfeld enhanced cross section through Sommerfeld factor

$$(\sigma v)^{\text{Som}} = S_{0,[\mathbf{8}]}\left((\sigma v)^{\text{Tree}}_{gg,[\mathbf{8_S}]} + (\sigma v)^{\text{Tree}}_{gg,[\mathbf{8_A}]} + N_f(\sigma v)^{\text{Tree}}_{q\bar{q},[\mathbf{8}]}\right) + S_{0,[\mathbf{1}]}\,(\sigma v)^{\text{Tree}}_{gg,[\mathbf{1}]}$$

Impact on the annihilation cross section

- Viable pMSSM-19 scenario:

$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{g}}$	$m_{\tilde{\tau}_1}$	m_{h^0}	m_{H^0}	Z_{11}	$\Omega_{\tilde{\chi}_1^0} h^2$
1435.7	1884.4	1882.9	1446.3	2248.0	3059.3	2613.5	124.0	3742.9	0.9976	0.1201

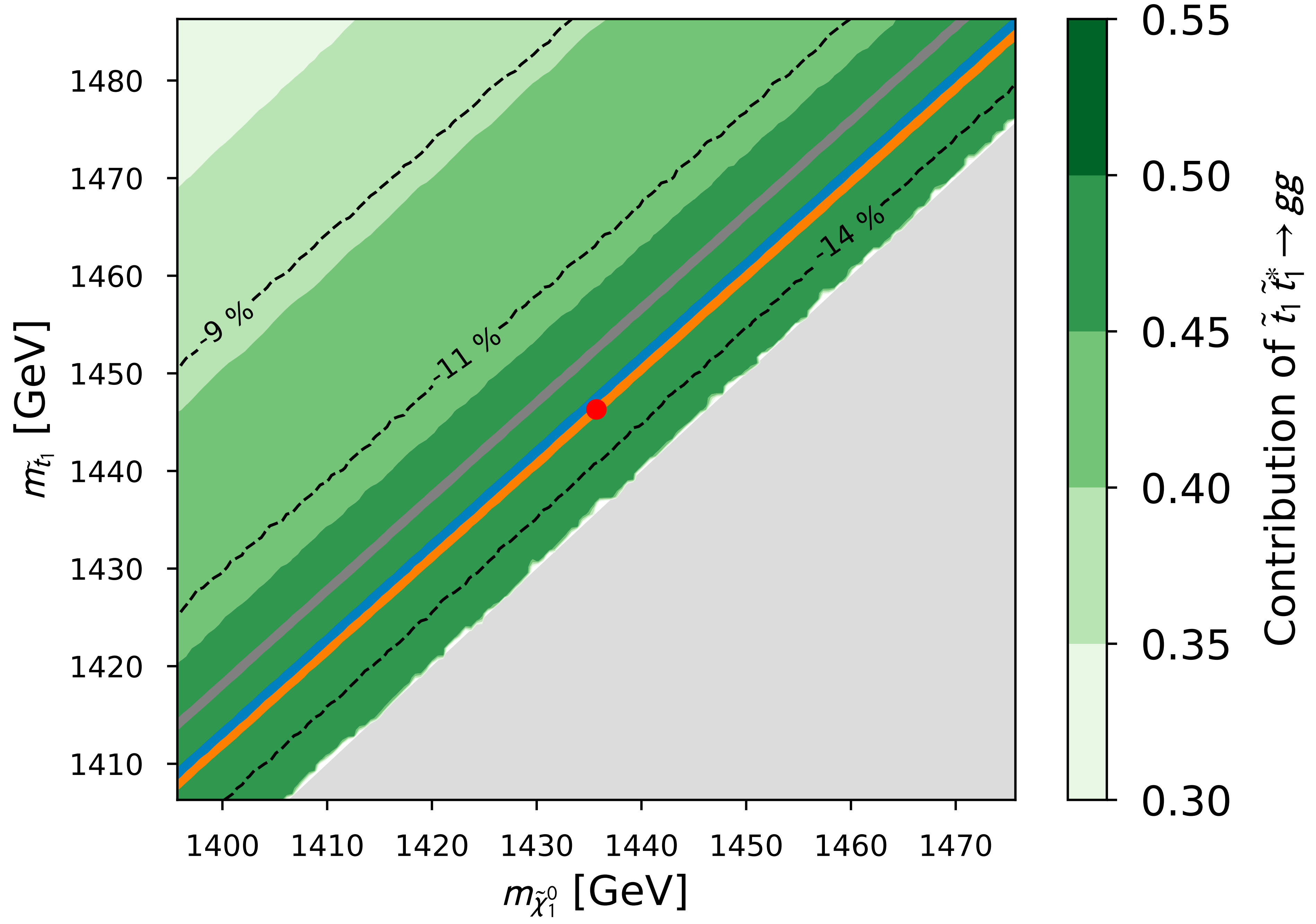


Impact on the relic density

- Viable pMSSM-19 scenario:

$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{g}}$	$m_{\tilde{\tau}_1}$	m_{h^0}	m_{H^0}	Z_{11}	$\Omega_{\tilde{\chi}_1^0} h^2$
1435.7	1884.4	1882.9	1446.3	2248.0	3059.3	2613.5	124.0	3742.9	0.9976	0.1201

Channel	Contribution
$\tilde{t}_1\ \tilde{t}_1^* \to g\ g$	47 %
$\tilde{t}_1\ \tilde{t}_1 \to t\ t$	23 %
$\tilde{\chi}_1^0\ \tilde{t}_1 \to g\ t$	7 %
$\tilde{t}_1\ \tilde{t}_1^* \to \gamma\ g$	7 %
$\tilde{t}_1\ \tilde{t}_1^* \to t\ \bar{t}$	5 %
$\tilde{t}_1\ \tilde{t}_1^* \to Z^0\ g$	2 %



Relic abundance from full Boltzmann equation

- Boltzmann equation:

$$\hat{L}[f_\chi] = \partial_t f_\chi - Hp\partial_p f_\chi = \frac{\mathrm{d}}{\mathrm{d}t} f_\chi \longrightarrow \frac{\mathrm{d}}{\mathrm{d}t} f_\chi(q/a, t) = \hat{C}[f_\chi]$$

- General parameterization for a $2 \to 2$ process:

$$C_{\text{coll}}\left[f_1\right] = \frac{1}{(2\pi)^4 E_1} \int \frac{\mathrm{d}p_2 p_2^2}{2E_2} \int \frac{\mathrm{d}p_3 p_3^2}{2E_3} \int_{-1}^{1} \mathrm{d}\cos\theta_1 \int_{x_1}^{x_2} \mathrm{d}\cos\theta_2 \frac{|\mathcal{M}(s,t)|^2}{\sqrt{a(\cos\theta_2 - x_1)(\cos\theta_2 - x_2)}} \Theta(b^2 - 4ac)\Lambda(f_1, f_2, f_3, f_4)$$

[Hannestad and Madsen, arXiv: 9506015 (astro-ph)]

➡But: $\mathcal{O}(N^5)$ scaling of the runtime $\longrightarrow$ appropriate gridding and interpolation techniques are inevitable

- Even more technical problems related to the full collision term:

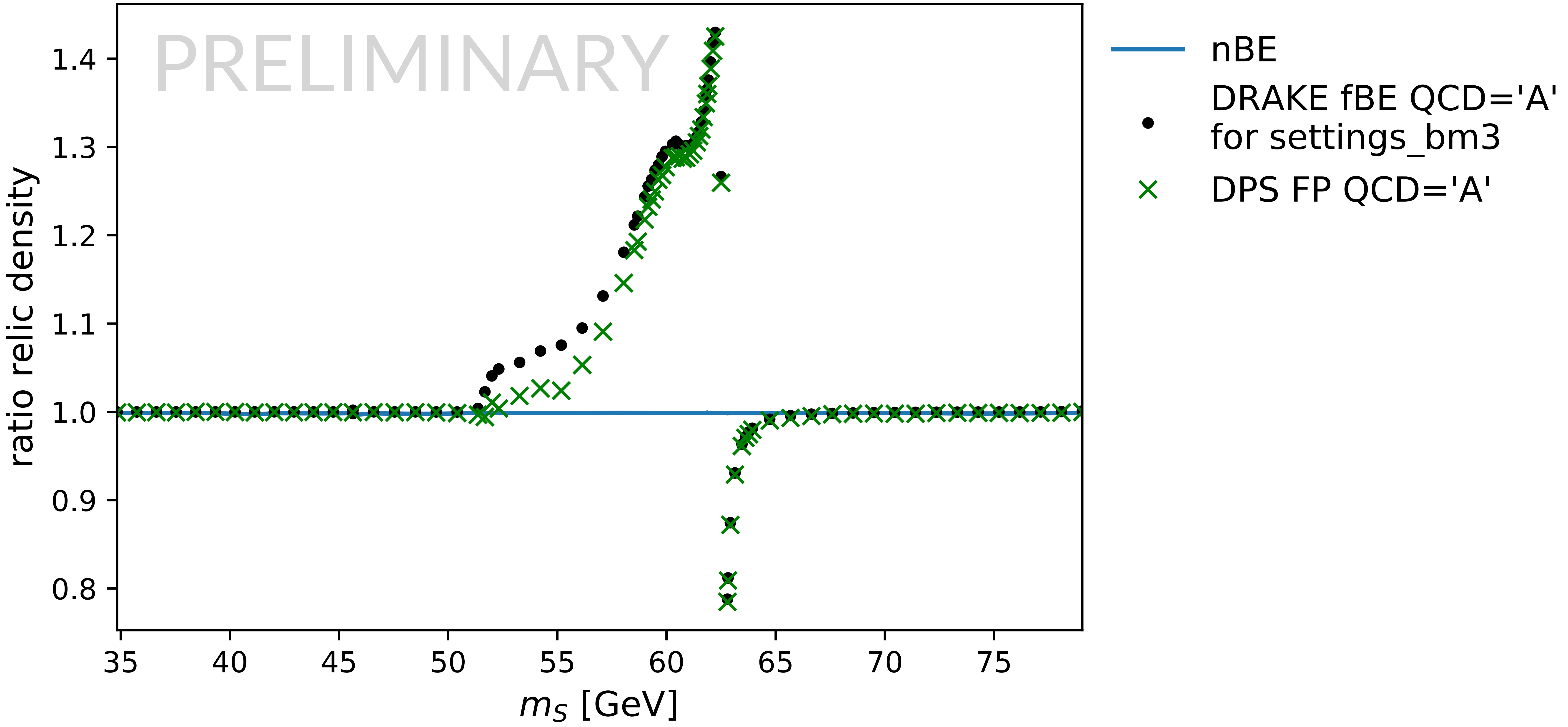
  - Inclusion of higher-order corrections when including statistical factors $\longrightarrow$ finite temperature dipole method (relevant for *relativistic freeze-out* or *freeze-in*)

  - t-channel singularities in scattering diagrams $\longrightarrow$ regularisation via thermal self-energy

[Grzadkowski et al. arXiv: 2108.01757 (hep-ph)]
[Iglicki, arXiv: 2212.00561 (hep-ph)]

DarkPhaseSpace vs DRAKE

- Real Singlet Scalar as dark matter: $\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{\lambda_S}{2} S^2 H^\dagger H - \frac{\eta}{4!} S^4$
  - DarkPhaseSpace: $\mathcal{O}(\mathrm{min})$
  - DRAKE: $\mathcal{O}(\mathrm{h})$



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Main messages

- For the standard relic density calculation (e.g. `MicrOMEGAs`): experimental uncertainty < theoretical uncertainty ⟶ %-level theoretical error is too strict

- Higher-order corrections often shift relic density beyond the experimental uncertainty
⟶ DM@NLO provides NLO corrections for large number of annihilation processes in the MSSM (reduction to simplified DM model is possible)

- Good news: full NLO corrections for dark matter models containing coloured scalars are negligible, the Sommerfeld enhancement is sufficient

- Number density approach is not sufficient for models with a strong velocity dependence
⟶necessary to develop fast tools for the evolution of the phase space distribution function

- Solution of the full Boltzmann equation is especially interesting in the freeze-in/freeze-out transition regime

Backup - Scale variation