Impact of Sommerfeld Enhancement and Bound State Effects on Simplified Dark Matter Models

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with

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2. Kirtimaan A. Mohan (MSU), DS, C.P Yuan (MSU) , Tim Tait (UC Irvine), Bin Yan (Argonne) JHEP 05 (2019) 115

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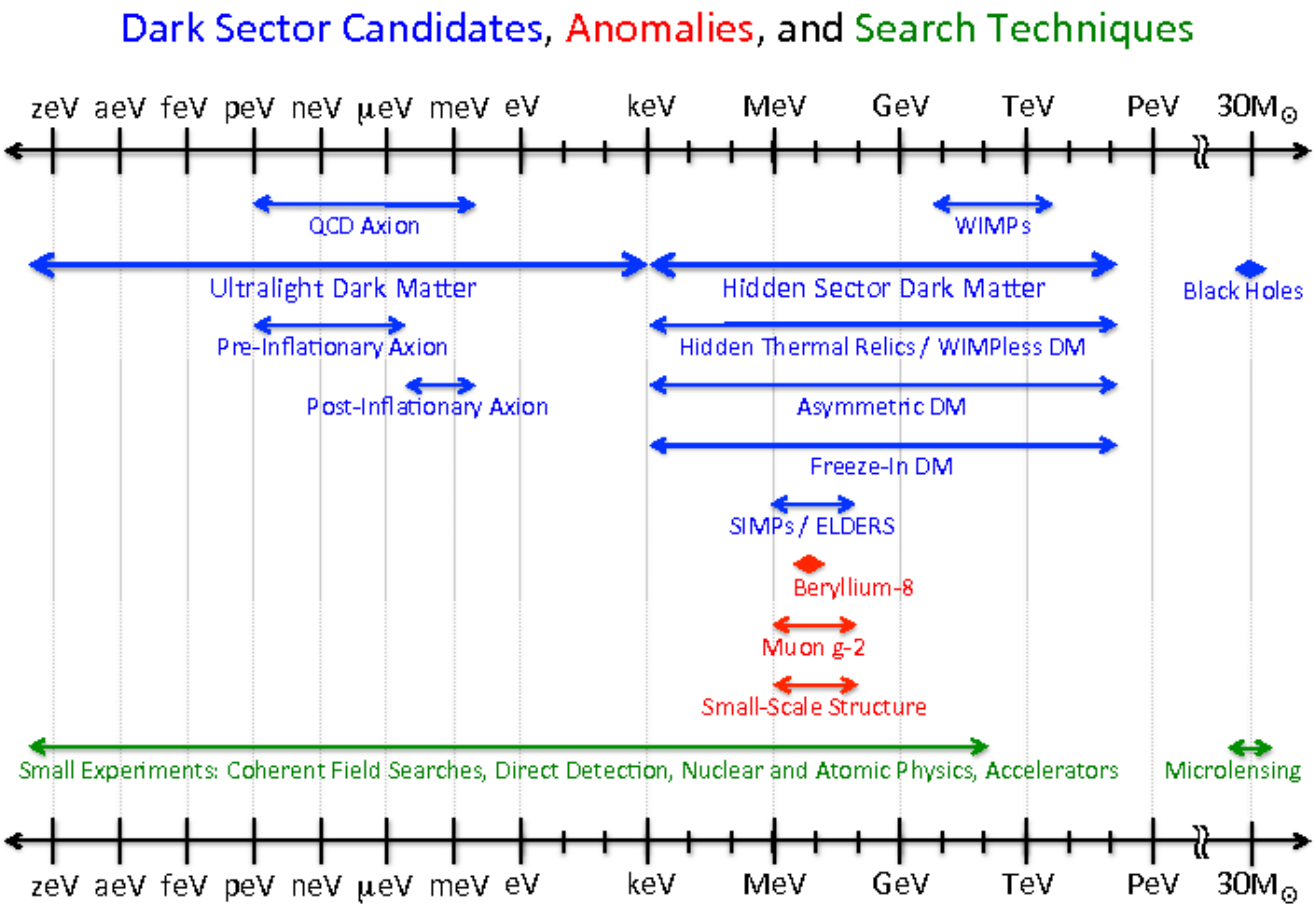
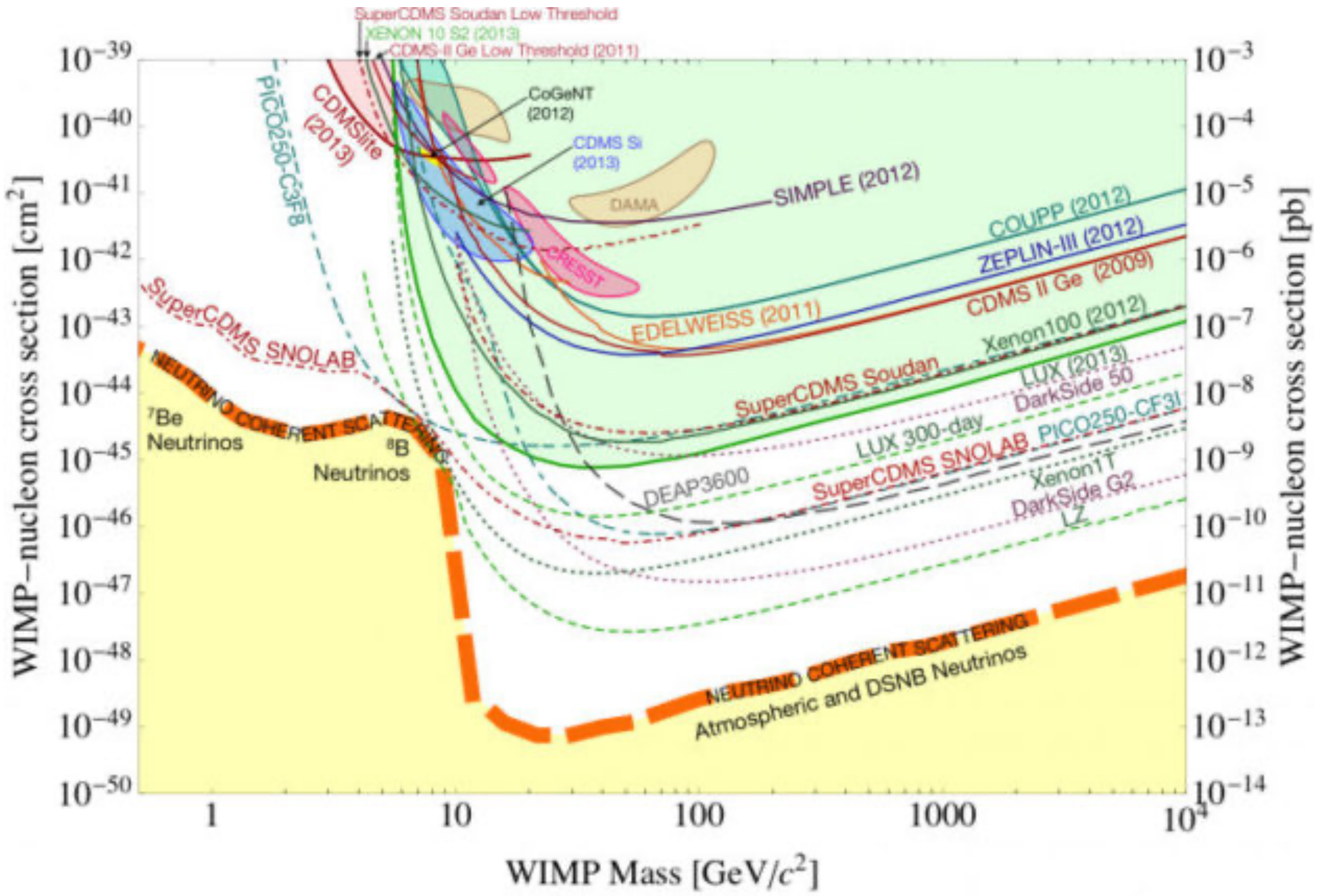
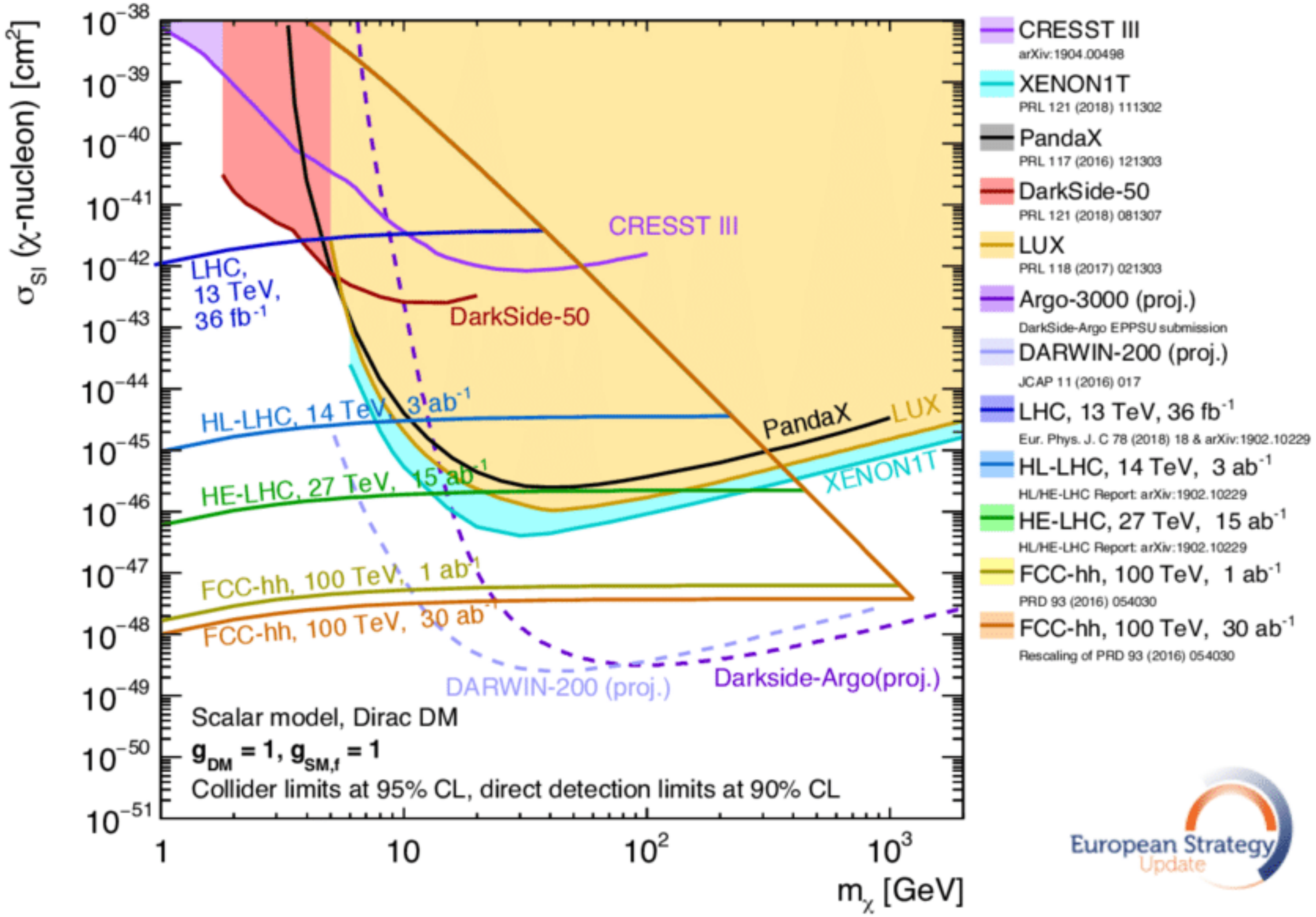
ARC CENTRE OF EXCELLENCE FOR DARK MATTER PARTICLE PHYSICS

Evidence for Dark Matter



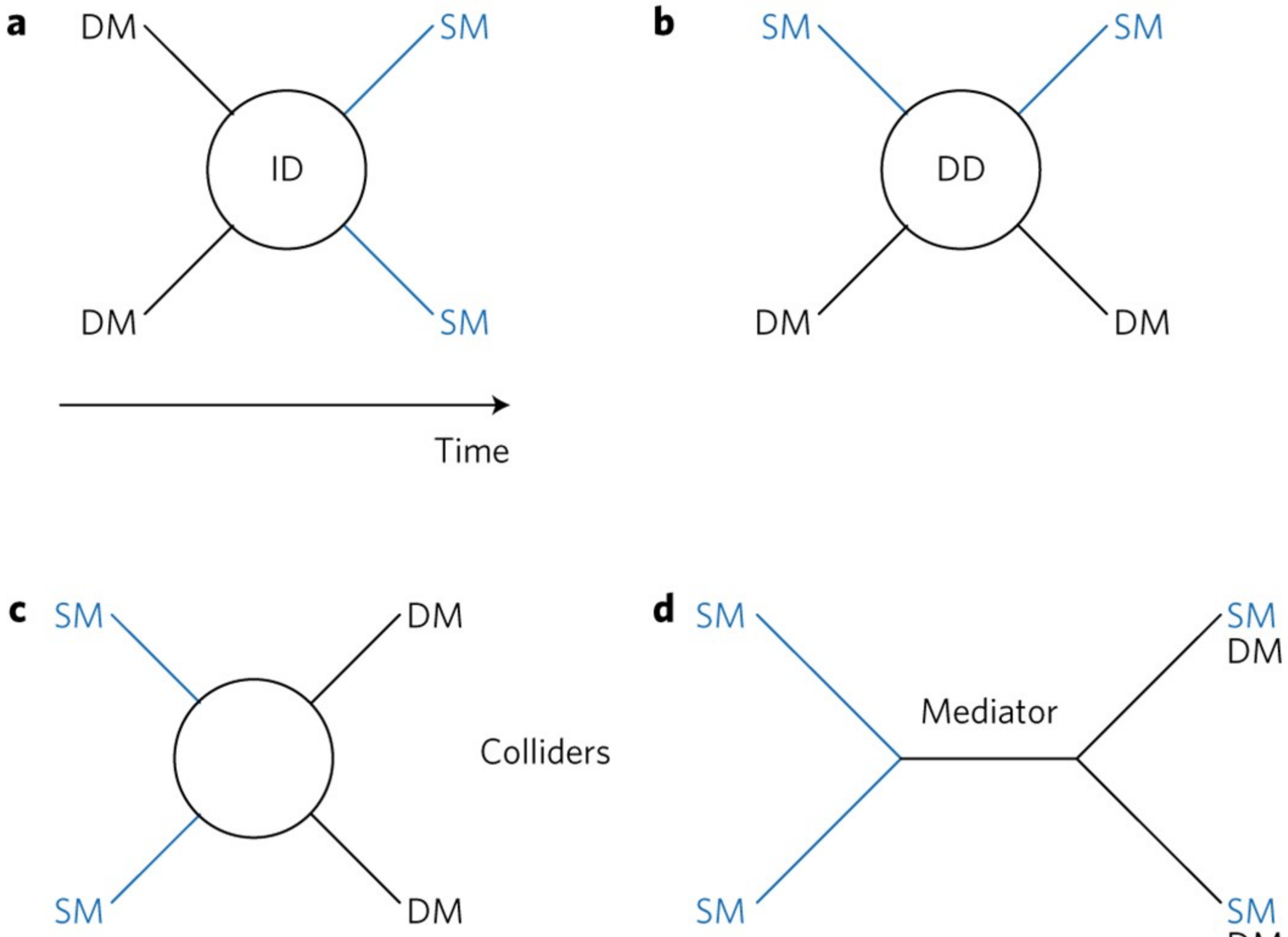
Limits on WIMP Dark Matter

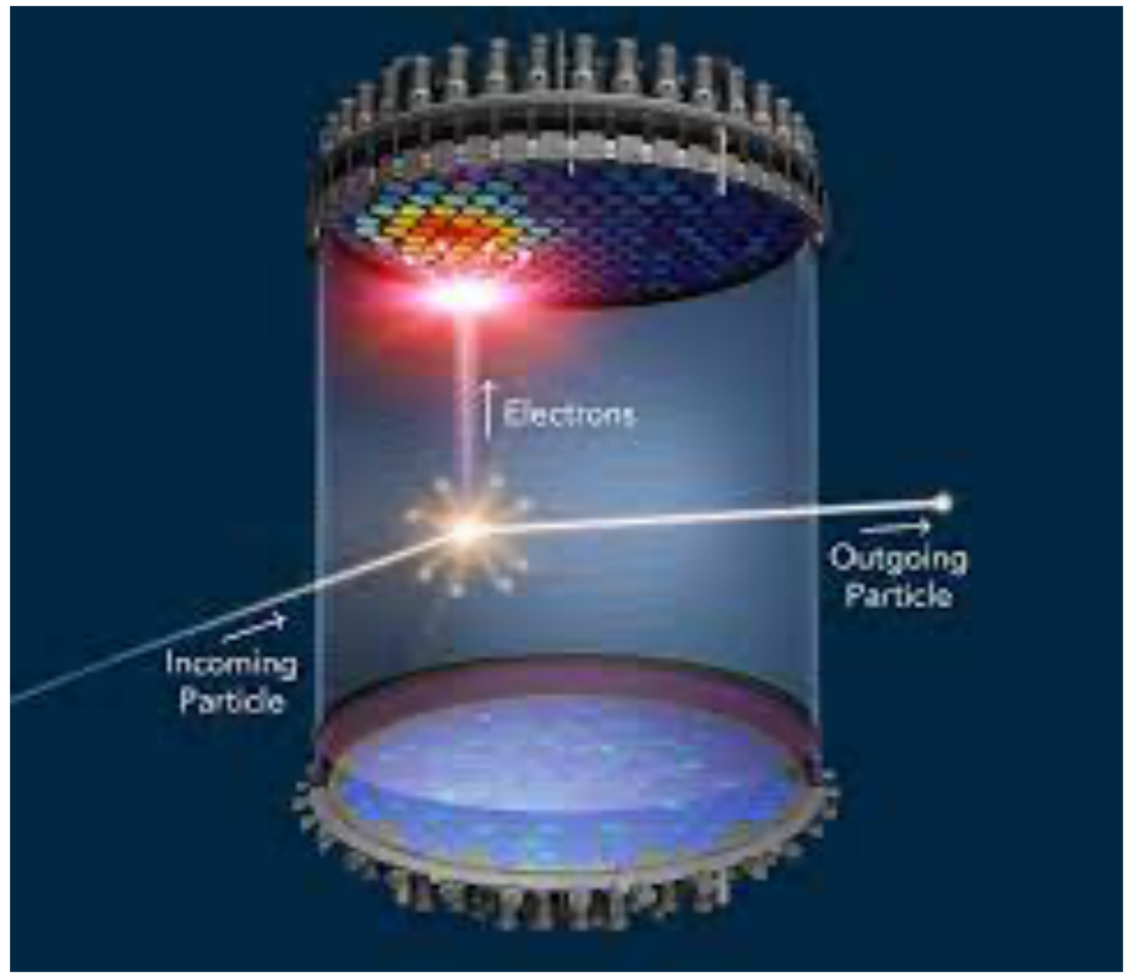


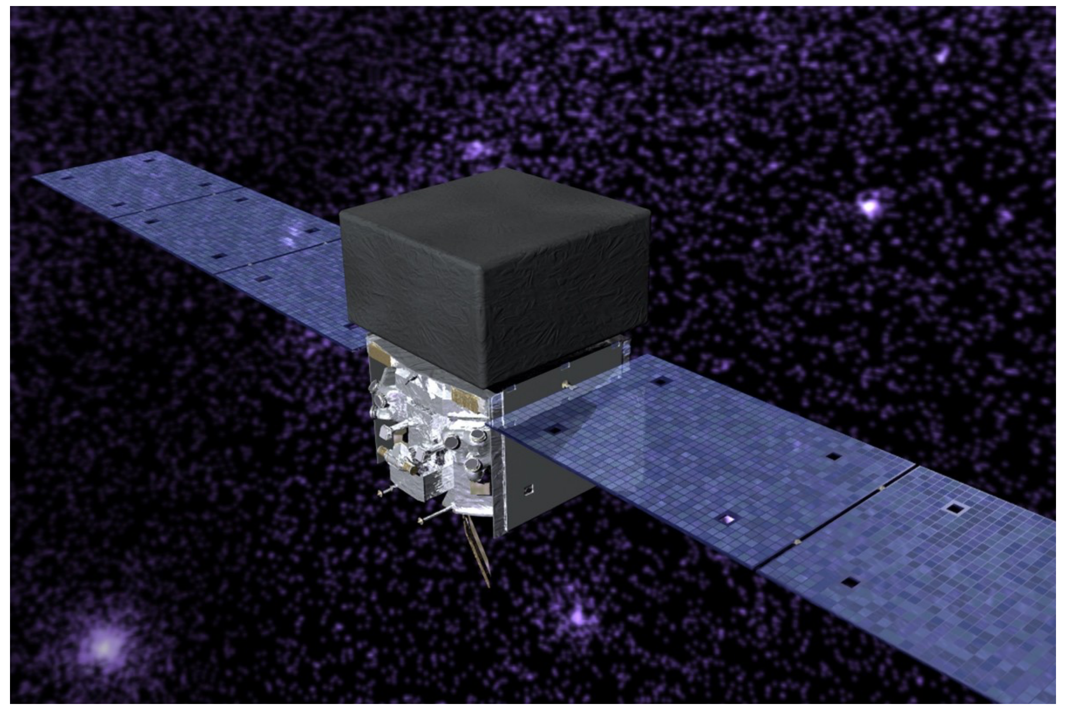


Can we improve limits, open up new parameter space for WIMPS ?

Simplified Models







Simplified Models

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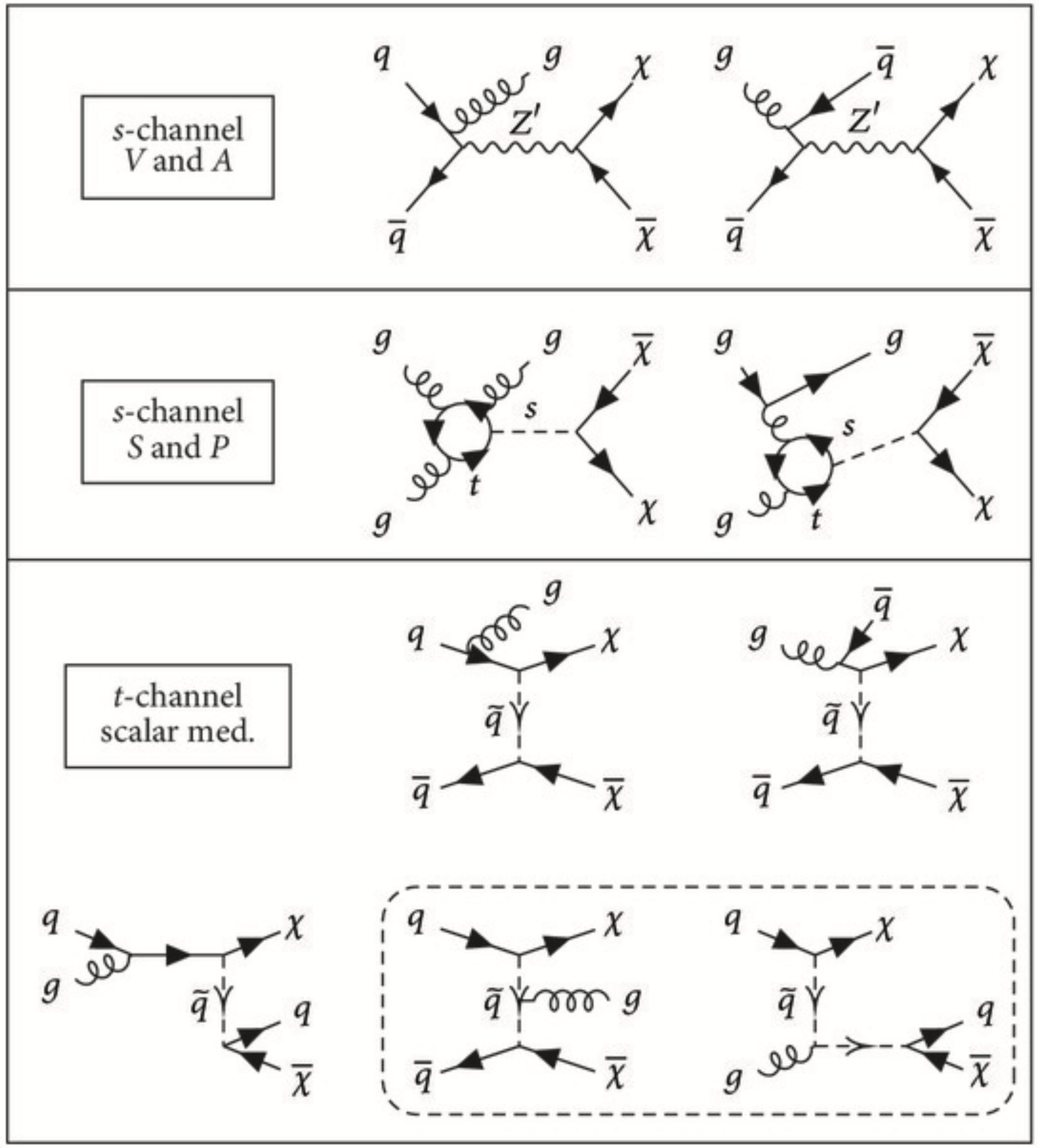
Simplified models for dark matter searches at the LHC

Jalal Abdallah [1,†], Henrique Araujo [2], Alexandre Arbey [3,4,5], Adi Ashkenazi [6],
Alexander Belyaev [7], Joshua Berger [8], Celine Boehm [9], Antonio Boveia [5],

Going beyond effective interactions

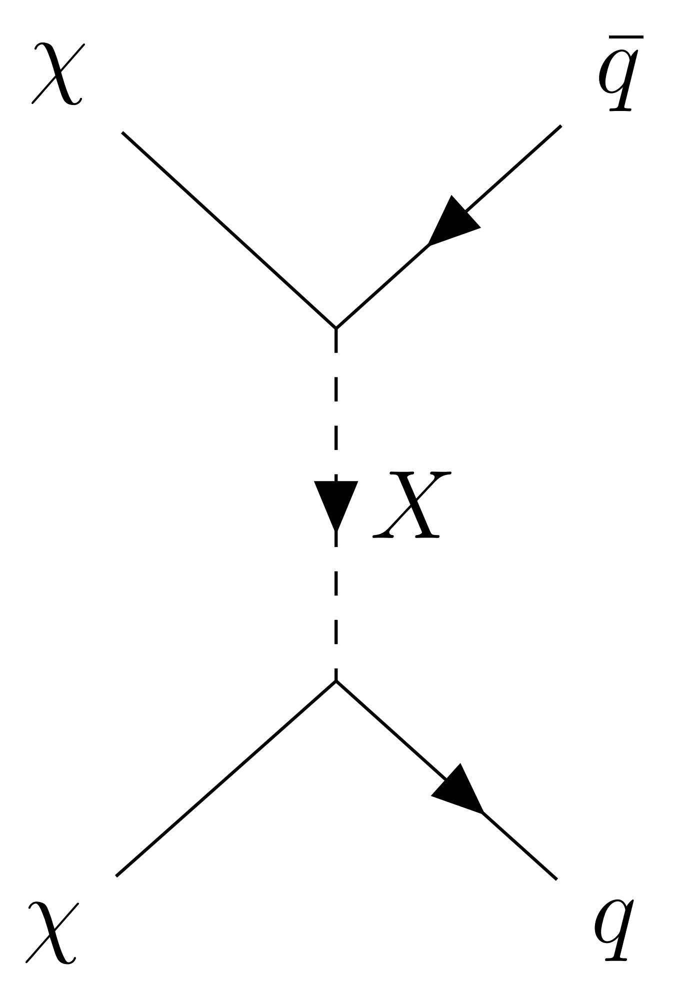


A Simplified Dark Matter Model

$$\mathcal{L} \supset \sum_i (D_\mu X_i)^\dagger (D^\mu X_i) + \sum_{i,j} \left(g_{\mathrm{DM},ij} X_i^\dagger \bar{\chi} P_R q_j + g^*_{\mathrm{DM},ij} X_i \bar{q}_j P_L \chi \right)$$

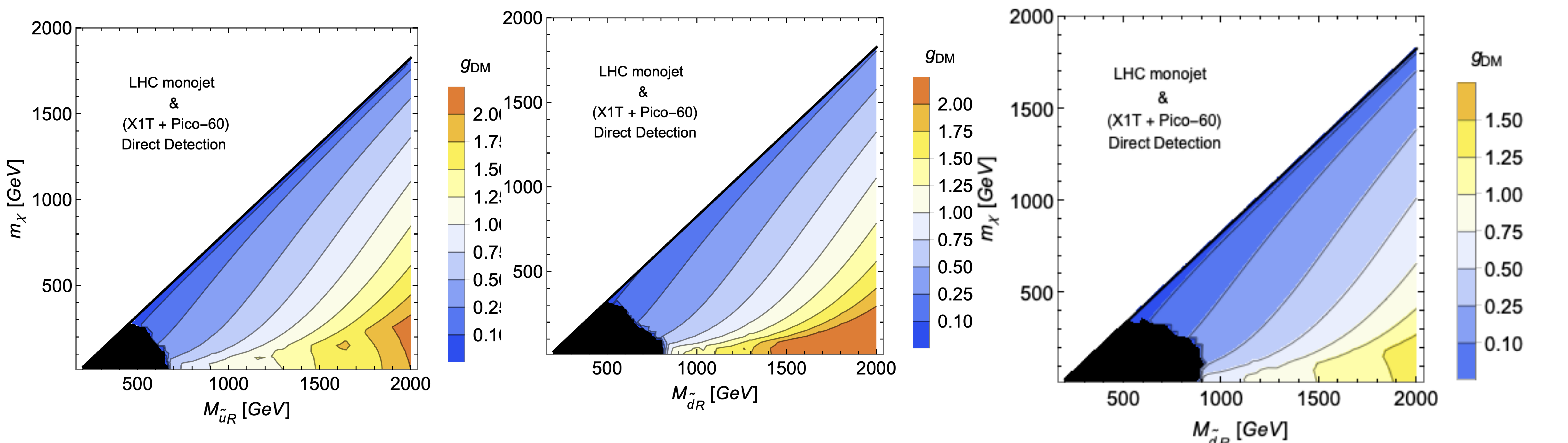
$(3,1)_{2/3}$, $(3,1)_{-1/3}$, $(3,2)_{-1/6}$

U_R D_R Q_L

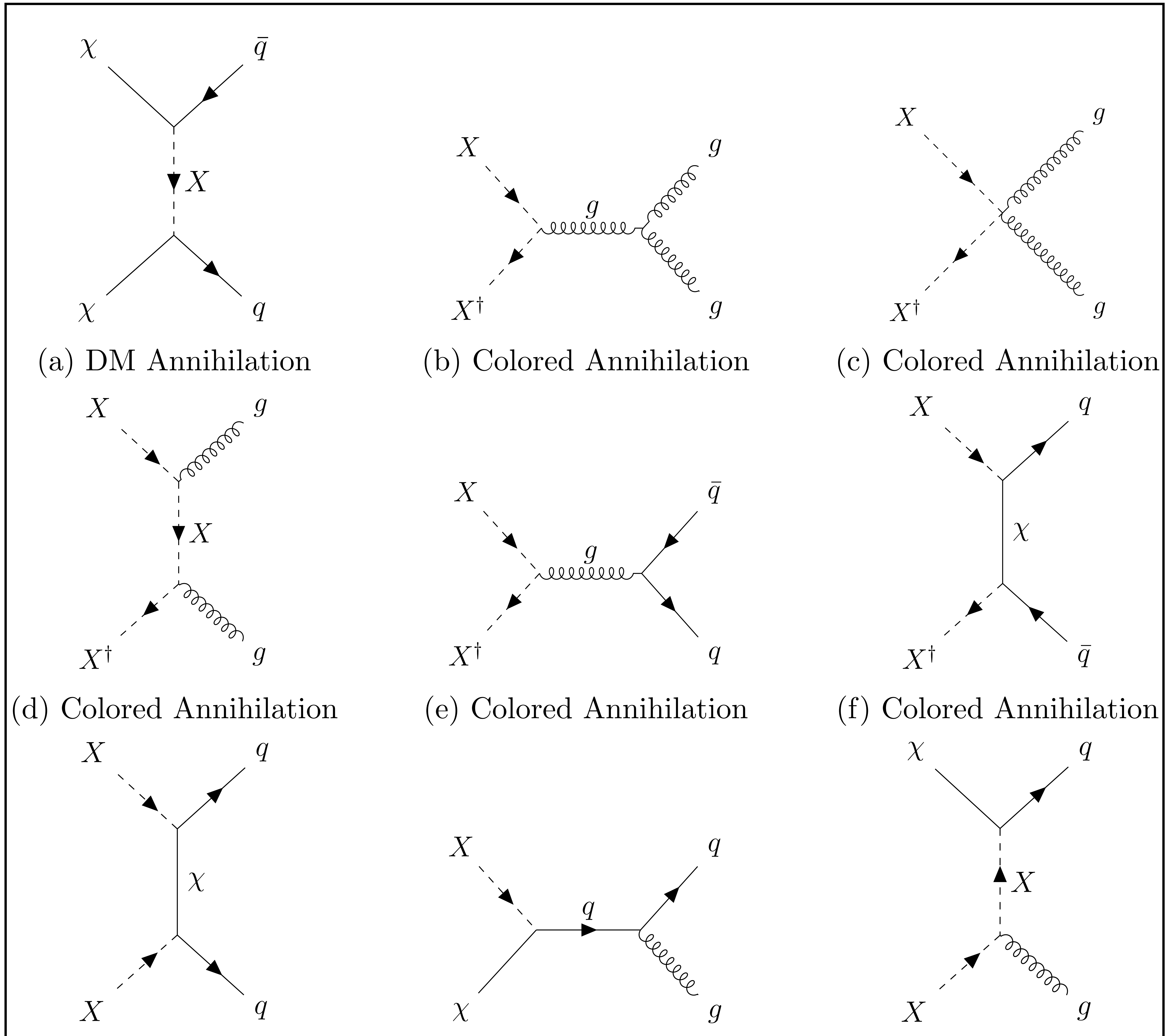


A Majorana Fermion Dark Matter interacting with Colored Scalar Mediators

$$\langle \sigma v \rangle \simeq N_c^f g_{DM}^4 \left[\frac{m_f^2 \sqrt{1 - \frac{m_f^2}{m_\chi^2}}}{64\pi (m_{\tilde{q}}^2 + m_\chi^2 - m_f^2)^2} + \beta^2 \left\{ \frac{m_\chi^2 \sqrt{m_\chi^4 + m_{\tilde{q}}^4}}{32\pi (m_\chi^2 + m_{\tilde{q}}^2)^4} + \mathcal{O}(m_f^2) \right\} \right]$$



A Simplified Dark Matter



(a) DM Annihilation (b) Colored Annihilation (c) Colored Annihilation (d) Colored Annihilation (e) Colored Annihilation (f) Colored Annihilation (g) Colored Annihilation (h) Co-annihilation (i) Co-annihilation

$$\tilde{Y} = Y_\chi + \sum_{i=u,c,t} \left(Y_{X_i} + Y_{X_i}^\dagger \right) = Y_\chi + 2 \sum_{i=u,c,t} Y_{X_i}$$

$$\frac{d\tilde{Y}}{dx} = -c\, g_{*,\mathrm{eff}}^{1/2} \frac{\langle \sigma_{\mathrm{eff}} v_{\mathrm{rel}} \rangle}{x^2} \left(\tilde{Y}^2 - \tilde{Y}_{\mathrm{eq}}^2 \right)$$

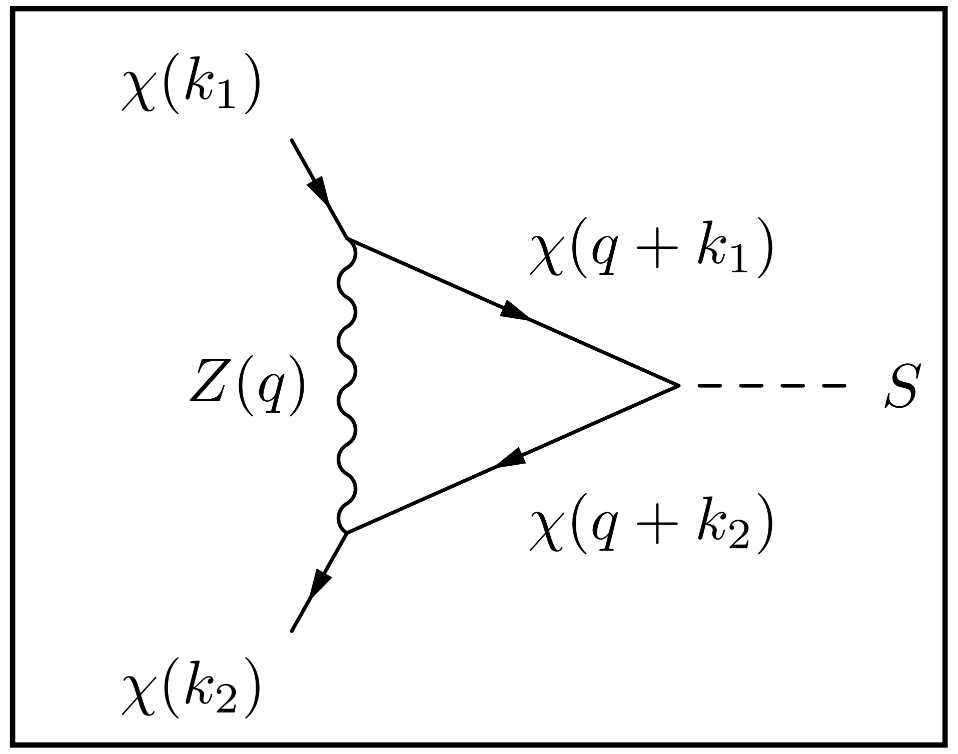
$$Y_\chi^{\mathrm{eq}} \simeq \frac{90}{(2\pi)^{7/2}} \frac{g_\chi}{g_{*S}} x^{3/2} e^{-x},$$

$$Y_X^{\mathrm{eq}} = Y_{X^\dagger}^{\mathrm{eq}} \simeq \frac{90}{(2\pi)^{7/2}} \frac{g_X}{g_{*S}} [(1+\delta)x]^{3/2} e^{-(1+\delta)x},$$

$$\delta \equiv \frac{m_X - m_\chi}{m_\chi} \equiv \frac{\Delta m}{m_\chi}, \quad \Delta m \equiv m_X - m_\chi$$

Sommerfeld Enhancement

Radiative Corrections in annihilation



In the non-relativistic limit loop scales as

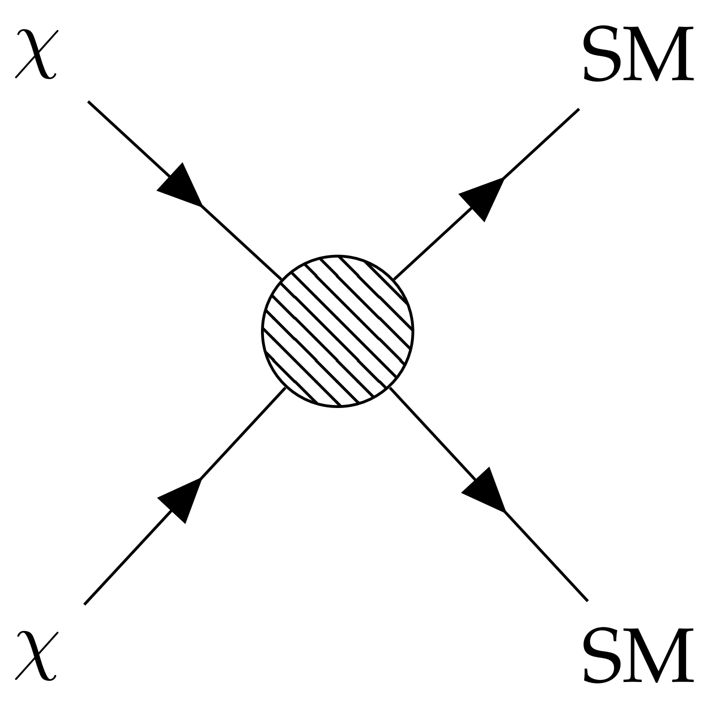
$$\frac{v}{v^2 + \frac{m_Z^2}{m_\chi^2}} \xrightarrow{m_\chi \gg m_Z} \frac{1}{v}$$

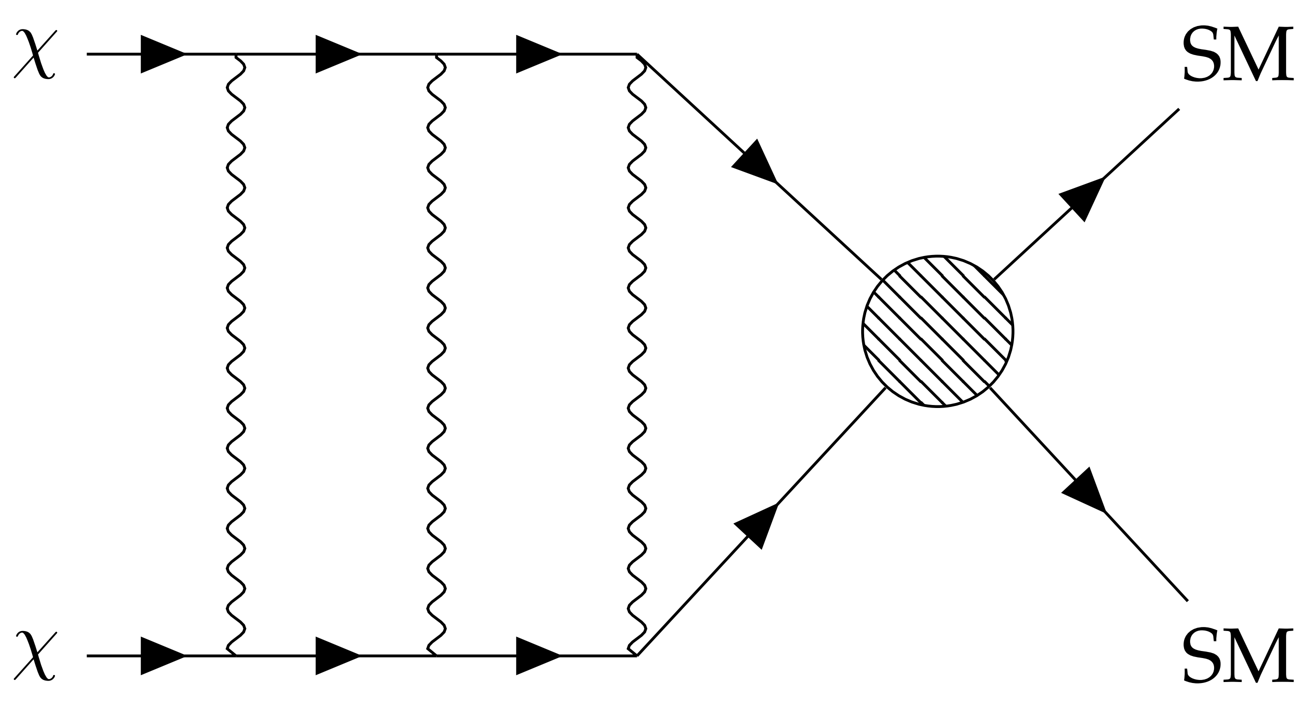
$$V(r) = \frac{g_Z^2 \delta e^{-\delta r}}{1 - e^{-\delta r}}$$

$$\delta \approx \frac{\pi^2}{6}\, m_Z$$

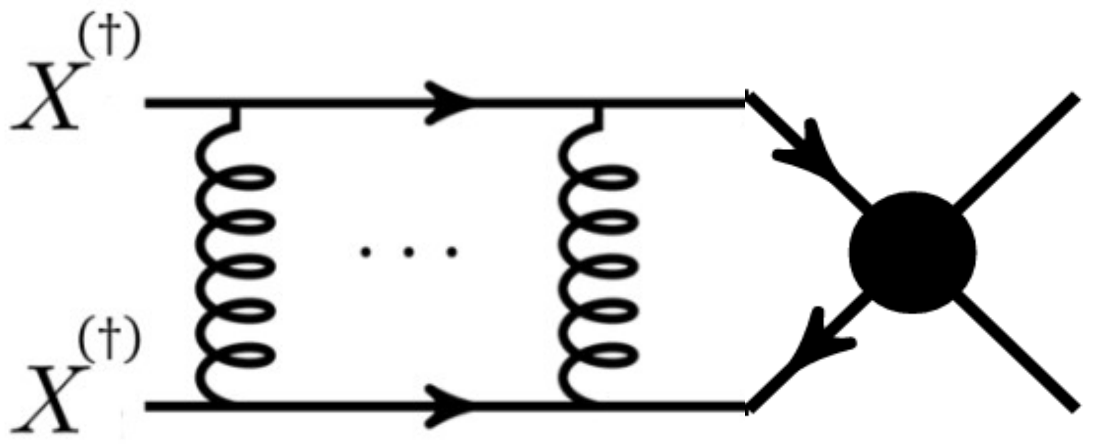
Large Enhancements for slowly moving particles for massless gauge bosons

Treat it as a non-relativistic Schrodinger equation with a long range potential





Sommerfeld Enhancement



n-gluon exchanges contribute with $\left(\frac{\alpha}{v}\right)^n$ for $\alpha \sim v$

→ Resummation required since $\alpha \sim v$

→ Reduces to Schrödinger Equation for $v \ll 1$.

Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle S(\alpha / v_{ij}) \cdot \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}} + \langle \sigma_{\text{BSF}} v \rangle_{\text{eff}} \left(\frac{n_X^{\text{eq}}}{n^{\text{eq}}} \right)^2$$

Color Decomposition and Sommerfeld Effect

A schematic color algebra of two incoming particles

$$\mathbf{R_1} \otimes \mathbf{R_2} = \bigoplus_{\hat{\mathbf{R}}} \hat{\mathbf{R}}$$

Gluonic Coulomb Potential

$$V_{[\hat{\mathbf{R}}]}(r) = -\frac{\alpha_g^{[\hat{\mathbf{R}}]}(Q)}{r}$$

$$\alpha_g^{[\hat{\mathbf{R}}]}(Q) = \alpha_s(Q) \times \frac{1}{2}[C_2(\mathbf{R_1}) + C_2(\mathbf{R_1}) - C_2(\hat{\mathbf{R}})] \equiv \alpha_s(Q) \times k_{[\hat{\mathbf{R}}]}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8} \text{ and } \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

$$V(r)_{\mathbf{3}\otimes\bar{\mathbf{3}}} = \begin{cases} -\frac{4}{3}\frac{\alpha_s}{r} & [\mathbf{1}] \\ +\frac{1}{6}\frac{\alpha_s}{r} & [\mathbf{8}] \end{cases} \quad ; \quad V(r)_{\mathbf{3}\otimes\mathbf{3}} = \begin{cases} -\frac{2}{3}\frac{\alpha_s}{r} & [\bar{\mathbf{3}}] \\ +\frac{1}{3}\frac{\alpha_s}{r} & [\mathbf{6}] \end{cases}$$

Sommerfeld Enhancement factor

$$\sigma_{\mathrm{SE},[\mathbf{R}]} v_{\mathrm{rel}} = c_{[\mathbf{R}]} S_{0,[\mathbf{R}]}\, \sigma_0$$

We will work in the most attractive singlet potential

Color Decomposition and Sommerfeld Effect

$$\sigma_{\mathrm{SE},[\mathbf{R}]} v_{\mathrm{rel}} = c_{[\mathbf{R}]} S_{0,[\mathbf{R}]}\, \sigma_0$$

$$\sigma_{\mathbf{3}\otimes\bar{\mathbf{3}}\to gg} v_{\mathrm{rel}} = \sigma_{\mathbf{3}\otimes\bar{\mathbf{3}}\to gg,0} \left(\frac{2}{7} S_{0,[\mathbf{1}]} + \frac{5}{7} S_{0,[\mathbf{8}]}\right),$$
$$\sigma_{\mathbf{3}\otimes\bar{\mathbf{3}}\to q\bar{q}} v_{\mathrm{rel}} = \sigma_{\mathbf{3}\otimes\bar{\mathbf{3}},0} \left(f_{[\mathbf{1}]}(g_s, g_{\mathrm{DM}}) S_{0,[\mathbf{1}]} + f_{[\mathbf{8}]}(g_s, g_{\mathrm{DM}}) S_{0,[\mathbf{8}]}\right)$$
$$\sigma_{\mathbf{3}\otimes\mathbf{3}\to qq} v_{\mathrm{rel}} = \sigma_{\mathbf{3}\otimes\mathbf{3}\to qq,0} S_{0,[\mathbf{6}]},$$
$$\sigma_{\mathbf{3}_i\otimes\mathbf{3}_j\to q_i q_j} = \sigma_{\mathbf{3}_i\otimes\mathbf{3}_j\to q_i q_j,0} \left(\frac{1}{3} S_{0,[\bar{\mathbf{3}}]} + \frac{2}{3} S_{0,[\mathbf{6}]}\right).$$

$$S_{0,[\mathbf{1}]} = S_0\left(\frac{4\alpha_s^S}{3v_{\mathrm{rel}}}\right),\ S_{0,[\mathbf{8}]} = S_0\left(\frac{-\alpha_s^S}{6v_{\mathrm{rel}}}\right),\ S_{0,[\bar{\mathbf{3}}]} = S_0\left(\frac{2\alpha_s^S}{3v_{\mathrm{rel}}}\right),\ S_{0,[\mathbf{6}]} = S_0\left(\frac{-\alpha_s^S}{3v_{\mathrm{rel}}}\right)$$

$$S_0(\zeta_s) = \frac{2\pi\zeta_s}{1 - e^{-2\pi\zeta_s}}$$

$$\zeta_s = \alpha_{g,[\mathbf{R}]}/v_{\mathrm{rel}} = k_{[\mathbf{R}]}\, \alpha_s/v_{\mathrm{rel}}$$

At small velocities

$$S_0 \sim \zeta_s \sim \alpha_{g,[\mathbf{R}]} v_{\mathrm{rel}}^{-1}$$

Can be positive or negative depending on the sign

For l partial waves

$$S_\ell(\zeta) = S_0(\zeta) \prod_{k=1}^{\ell} \left(1 + \frac{\zeta^2}{k^2}\right)$$

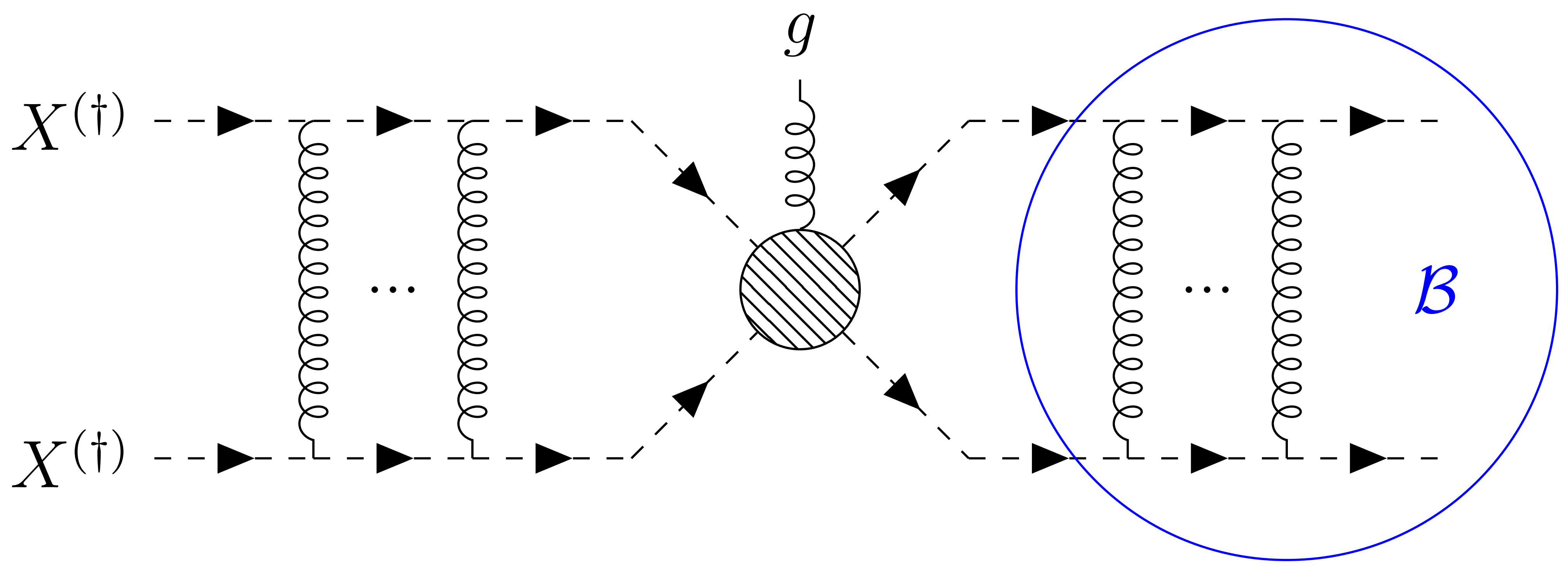
Singlet States form the most attractive potential

Does not work for light DM: Large Yukawa Exponential Suppression
Bohr radius needs to be smaller than inverse mass of force mediator

$$(\alpha\, m_X/2)^{-1} \lesssim m_A^{-1}$$

Petraki, Harz +
Becker, Copello, Harz, Mohan, DS

Color Decomposition and Bound States Effect



Colored Particles can form Unstable Bound States

$X_1 + X_2 \to \mathcal{B}(X_1 X_2) + g$

wavefunction of the bound state   Schrödinger equation

binding energies $\mathcal{E}_{n\ell m} = -\kappa^2/(2\mu\, n^2)$

Average momentum transfer between bound states

Bohr momentum $\kappa_{[\hat{\mathbf{R}}]} \equiv \mu\, \alpha^B_{g,[\hat{\mathbf{R}}]} = \mu\, k_{[\hat{\mathbf{R}}]}\, \alpha^B_{s,[\hat{\mathbf{R}}]}$

Capture into either singlet or octet states

$$(X + X^\dagger)_{[\mathbf{8}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g\}_{[\mathbf{8}]},$$
$$(X + X^\dagger)_{[\mathbf{1}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g\}_{[\mathbf{1}_S]},$$
$$(X + X^\dagger)_{[\mathbf{8}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g\}_{[\mathbf{8}_S] \text{ or } [\mathbf{8}]_A}.$$

$$\sigma^{[\mathbf{8}]\to[\mathbf{1}]}_{\{100\}} v_{\rm rel} = \frac{2^7\, 17^2}{3^5} \frac{\pi \alpha^{\rm BSF}_{s,[\mathbf{1}]} \alpha^B_{s,[\mathbf{1}]}}{m_X^2} S_{\rm BSF}(\zeta_S, \zeta_B)$$

$$S_{\rm BSF}(\zeta_S, \zeta_B) = \left(\frac{2\pi\zeta_S}{1 - e^{-2\pi\zeta_S}}\right)(1+\zeta_S^2)\frac{\zeta_B^4 e^{-4\zeta_S \,\mathrm{arccot}(\zeta_B)}}{(1+\zeta_B^2)^3}$$

$$\zeta_S \equiv \alpha_g^S / v_{\rm rel}$$

$$\zeta_B \equiv \alpha_g^B / v_{\rm rel}$$

Color Decomposition and Bound States Effect : Ionisation

Bound States can be ionized Energetic Gluons in the thermal Plasma and dissociate into constituents : High Temperature or can directly decay to constituents

$$\Gamma_{\text{dec},[\mathbf{R}]} = (\sigma^{s\text{-wave}}_{\text{ann},[\mathbf{R}]}\, v_{\text{rel}})|\psi^{[\mathbf{R}]}_{n00}(0)|^2$$

$$|\psi^{[\mathbf{1}]}_{100}(0)|^2 = 8m_X^3(\alpha^B_{s,[\mathbf{1}]})^3/27\pi$$

Formation and Subsequent annihilation of Bound States open up a new annihilation channel

Incorporated in a system of coupled Boltzmann Equations

$$\langle\sigma_{\text{BSF}} v_{\text{rel}}\rangle_{\text{eff}} \equiv \langle\sigma^{[\mathbf{8}]\to[\mathbf{1}]}_{\text{BSF}} v_{\text{rel}}\rangle \frac{\langle\Gamma_{\text{dec}[\mathbf{1}]}\rangle}{\langle\Gamma_{\text{dec}[\mathbf{1}]}\rangle + \langle\Gamma_{\text{ion},[\mathbf{1}]}\rangle}$$

At Large Temperatures : Ionisation processes dominates over decays -> Effective Contribution of Bound States in dark sector evolution is negligible.
Relic density is independent of contribution of Bound States

As Universe cools down decays dominate, efficiently depleting the dark sector, ionisation rate is exponentially suppressed

$$\langle\sigma_{XX^\dagger} v_{\text{rel}}\rangle_{\text{eff}} = \sum_i \left(\langle\sigma_{X_i X_i^\dagger} v_{\text{rel}}\rangle + \langle\sigma^{[\mathbf{8}]\to[\mathbf{1}]}_{\text{BSF}} v_{\text{rel}}\rangle \frac{\Gamma_{\text{dec}[\mathbf{1}]}}{\Gamma_{\text{dec}[\mathbf{1}]} + \Gamma_{\text{ion},[\mathbf{1}]}}\right)$$

Color Decomposition and Sommerfeld Effect

Process	Contribution to $\langle \sigma v \rangle$	v_{rel}	Color Structure	BSF
$\chi\chi \to q_i \bar{q}_i$	g_{DM}^4	$v_{\text{rel}}^2\ (m_q = 0)$ $v_{\text{rel}}^0\ (m_q \neq 0)$	none	✗
$X_i X_j^\dagger \to gg$	$g_s^4 e^{-2x\delta}$	v_{rel}^0	$\lvert\mathcal{M}\rvert^2 \sim \frac{2}{7}[\mathbf{1}] + \frac{5}{7}[\mathbf{8}]$	✓
$X_i X_j^\dagger \to q_i \bar{q}_j$	$(\alpha g_{\text{DM}}^2 + \beta g_s^2)^2 e^{-2x\delta}$	$v_{\text{rel}}^2\ (m_q = 0)$ $v_{\text{rel}}^0\ (m_q \neq 0)$	$\lvert\mathcal{M}\rvert^2 \sim f_1(g_{\text{DM}}, g_s)\,[\mathbf{1}]$ $+ f_8(g_{\text{DM}}, g_s)\,[\mathbf{8}]$	
$X_i X_j \to q_i q_j$	$g_{\text{DM}}^4 e^{-2x\delta}$	v_{rel}^0	$\lvert\mathcal{M}\rvert^2 \sim \frac{1}{3}[\bar{\mathbf{3}}] + \frac{2}{3}[\mathbf{6}]$	✓
$X_i X_i \to q_i q_i$	$g_{\text{DM}}^4 e^{-2x\delta}$	v_{rel}^0	$\lvert\mathcal{M}\rvert^2 \sim [\mathbf{6}]$	
$X_i \chi \to q_i A$	$g_{\text{DM}}^2 g_{\text{gauge}}^2 e^{-x\delta}$	v_{rel}^0	none	✗

Sommerfeld Enhancement and Bound States Effect : Implementation

$$\mathcal{L} \supset \sum_i (D_\mu X_i)^\dagger (D^\mu X_i) + \sum_{i,j} \left(g_{\mathrm{DM},ij} X_i^\dagger \bar{\chi} P_R q_j + g^*_{\mathrm{DM},ij} X_i \bar{q}_j P_L \chi \right)$$

Implemented the full effect of Sommerfeld Effect and Boltzmann Equations in micrOMEGAS 2.7

Becker, Copello, Harz, Mohan DS

Relic Abundance

Determine $g_{DM,0}$ for each data point $(m_{DM}, \Delta m)$ such that DM is *not* overproduced.

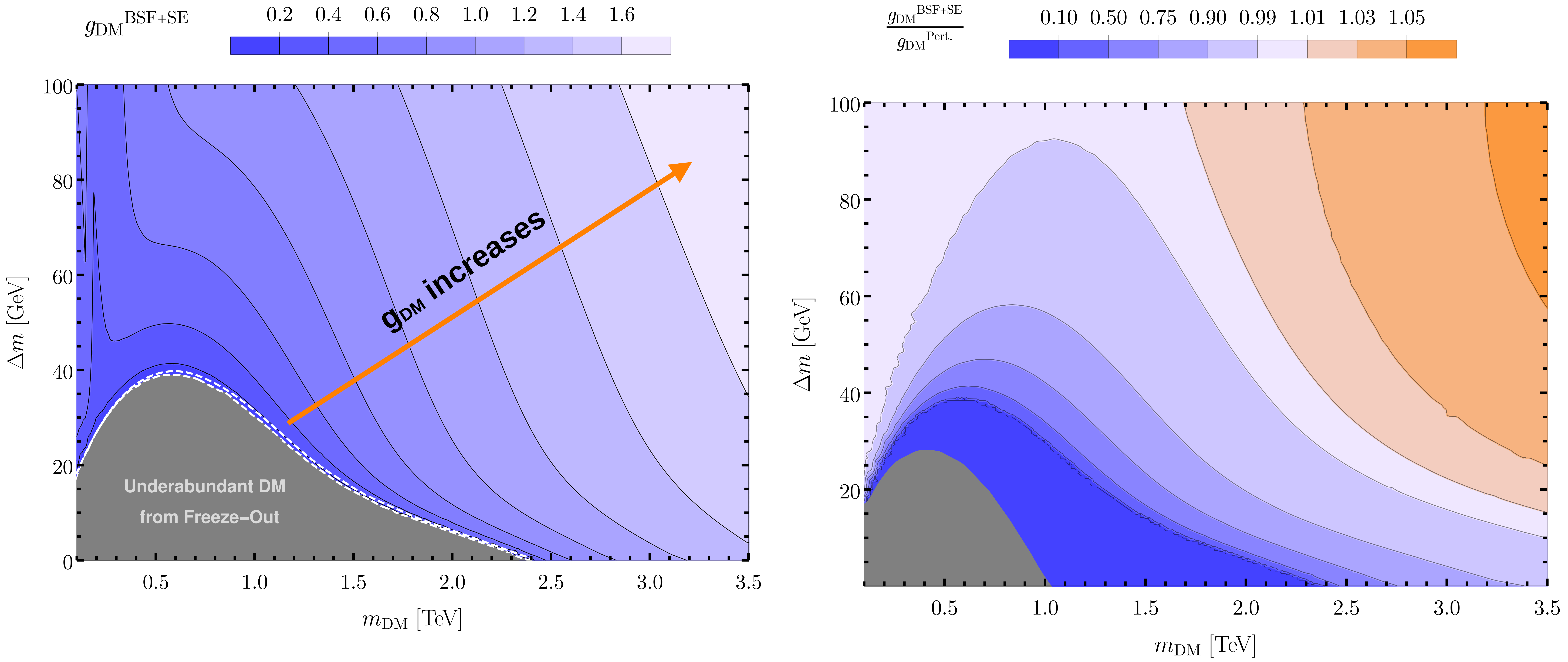
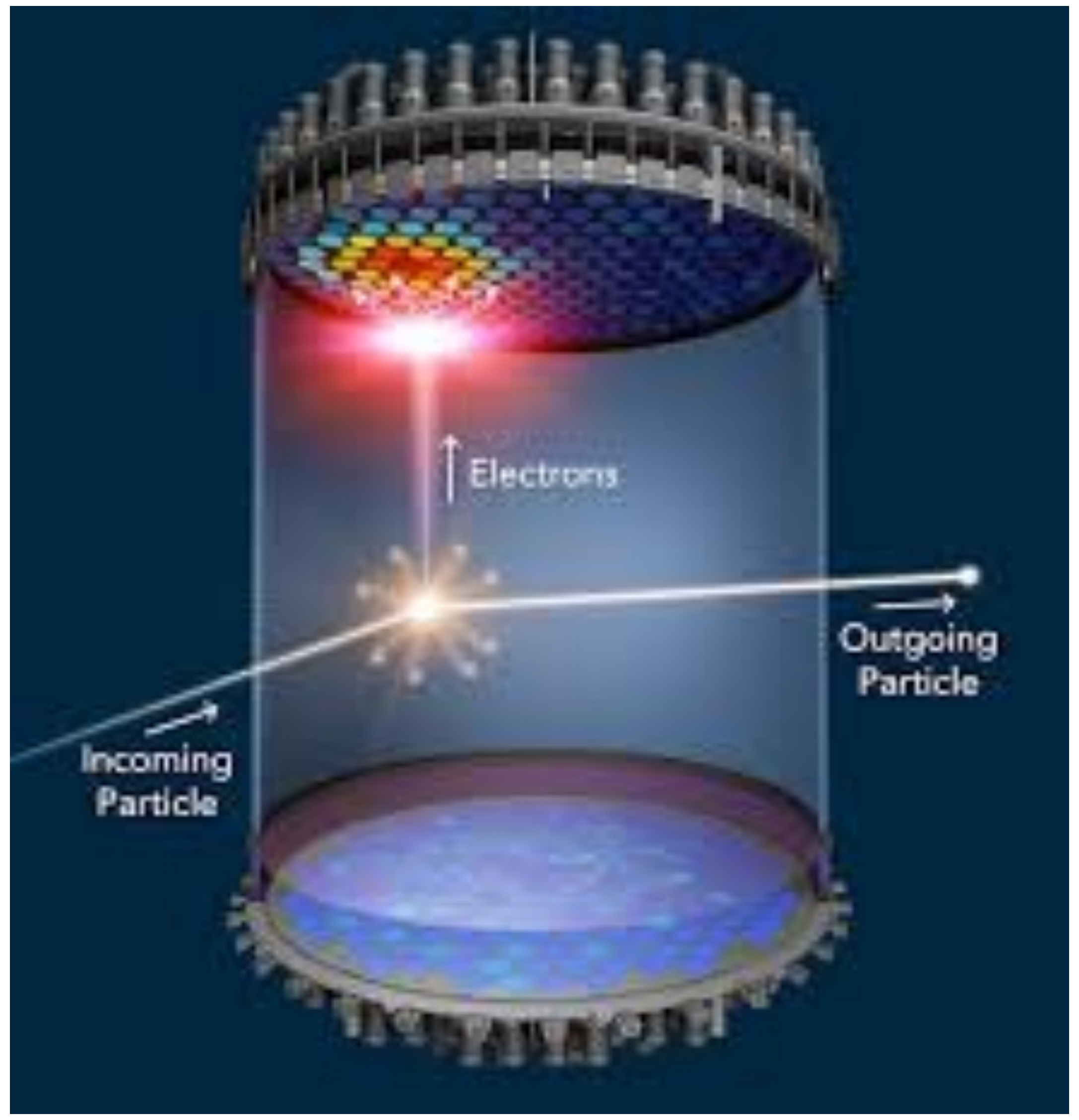


Figure from [MB,Copello,Harz,Mohan,Sengupta(2022)]

Look for elastic scattering of WIMPS with nuclei.

$$\frac{d\sigma}{dE} = \frac{m_A}{2\mu_A^2 v^2} \cdot \left(\sigma_0^{\mathrm{SI}} \cdot F_{\mathrm{SI}}^2(E) + \sigma_0^{\mathrm{SD}} \cdot F_{\mathrm{SD}}^2(E)\right)$$

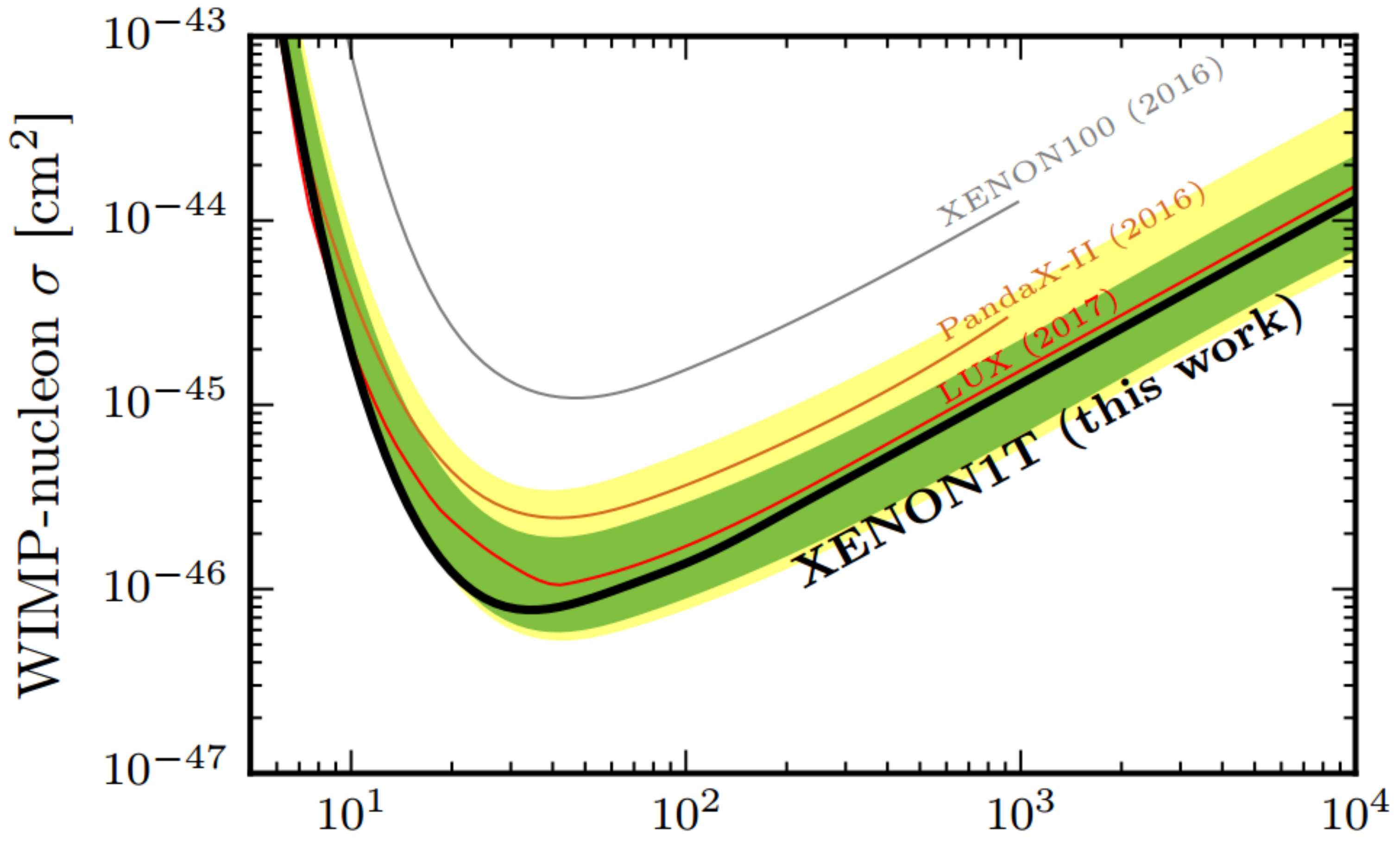
$$\sigma_0^{\mathrm{SI}} = \sigma_p \cdot \frac{\mu_A^2}{\mu^2} \cdot [Z \cdot f^p + (A - Z) \cdot f^n]^2$$



Source: KIPAC

- LO calculation tells us that model has only a spin dependent cross-section.

- Limits from direct detection are weak—large values of g_{DM} allowed.



Direct Detection Constraints

Majorana Fermion : Tree level Spin-Independent DD cross section vanishes

$$\mathcal{M} = (-ig_{DM})^2(\bar{\chi}P_R u)\frac{i}{p^2 - M_{\tilde{u}}^2}(\bar{u}P_L\chi)$$

$$\mathcal{L}_{SI}^{\text{eff}} = \sum_{q=u,d,s} \mathcal{L}_q^{\text{eff}} + \mathcal{L}_g^{\text{eff}}$$

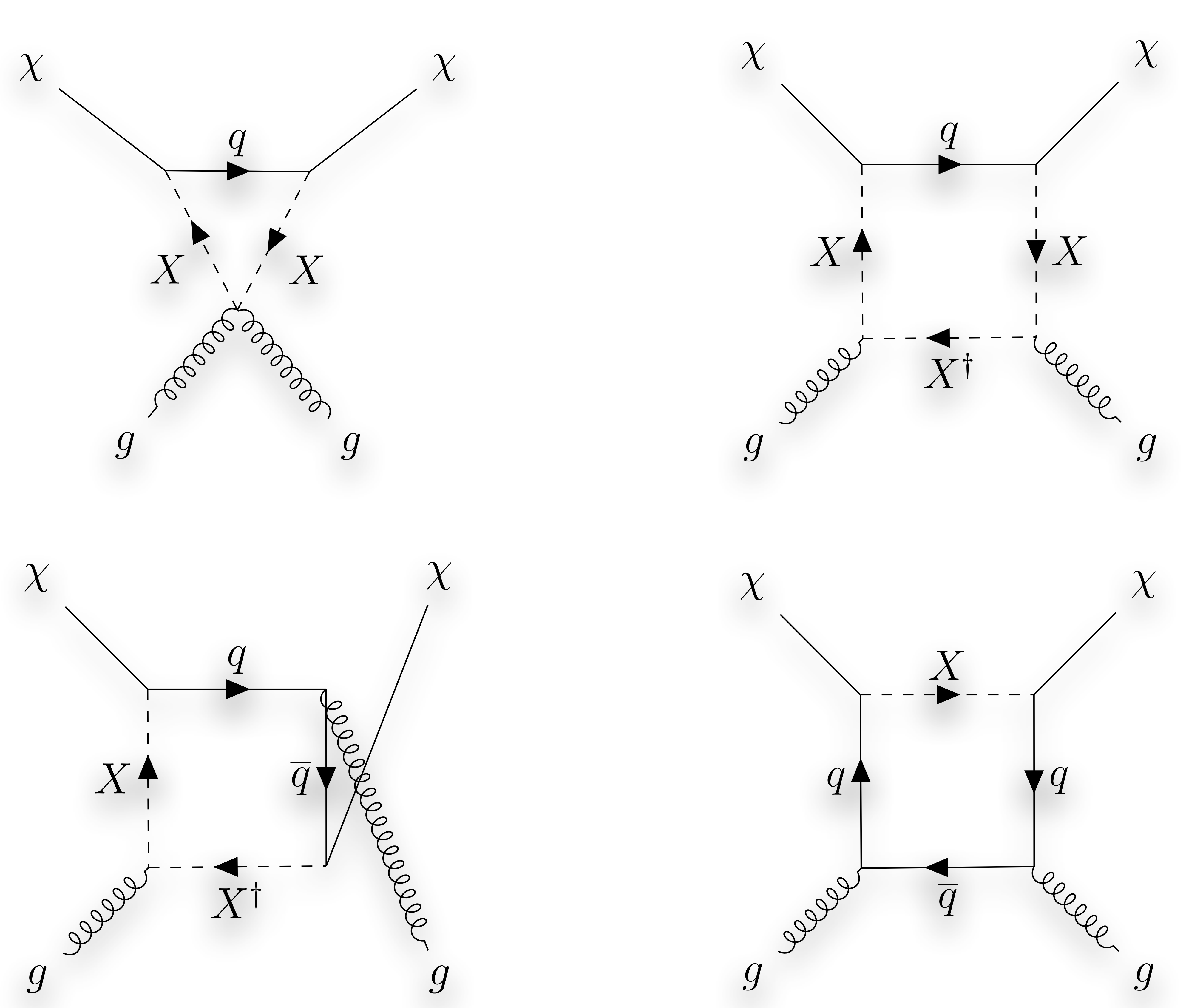
$$\mathcal{M}_{DD} \approx \frac{ig_{DM}^2}{M_{\tilde{q}_L}^2 - M_\chi^2}\frac{1}{8}[(\bar{\chi}\gamma^\mu\chi)(\bar{u}\gamma_\mu u) - (\bar{\chi}\gamma^\mu\gamma^5\chi)(\bar{u}\gamma_\mu\gamma_5 u)]$$

SI

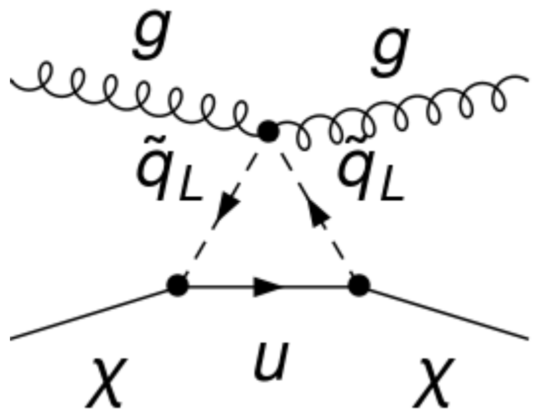
0 for Majorana

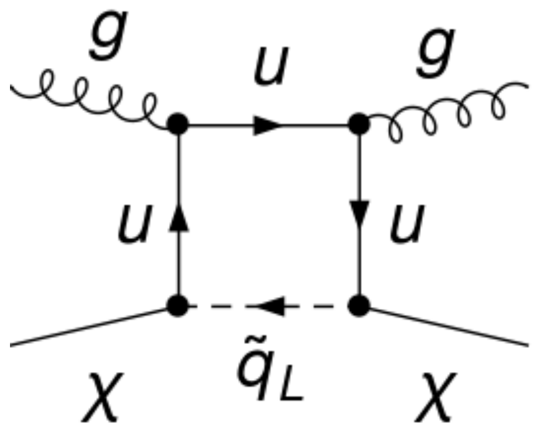
SD

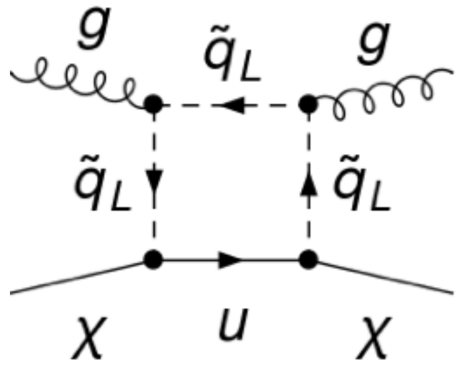
$$\sigma_p = \frac{4}{\pi}\left(\frac{M_\chi m_p}{M_\chi + m_p}\right)^2 |\langle\mathcal{M}_{DD}\rangle_{NR}|^2 \ .$$



DD @ 1-Loop







$$\mathcal{O}^q_{\mu\nu} \equiv \frac{1}{2}\bar{q}i\left(D_\mu\gamma_\nu + D_\nu\gamma_\mu - \frac{1}{2}g_{\mu\nu}\not{D}\right)q$$

$$\mathcal{O}^g_{\mu\nu} \equiv \left(G^{a\rho}_{\mu}G^a_{\rho\nu} + \frac{1}{4}g_{\mu\nu}G^a_{\alpha\beta}G^{a\alpha\beta}\right) .$$

Spin-2 Operators

Determine Wilson Coefficients for effective operators

$$\mathcal{L}^{\text{eff}}_q = f_q m_q\, \bar{\tilde{\chi}}\tilde{\chi}\, \bar{q}q + \frac{g^{(1)}_q}{m_\chi}\, \bar{\tilde{\chi}}i\partial^\mu\gamma^\nu\tilde{\chi}\, \mathcal{O}^q_{\mu\nu} + \frac{g^{(2)}_q}{m^2_\chi}\, \bar{\tilde{\chi}}(i\partial^\mu)(i\partial^\nu)\tilde{\chi}\, \mathcal{O}^q_{\mu\nu} ,$$

$$\mathcal{L}^{\text{eff}}_g = f_G\, \bar{\tilde{\chi}}\tilde{\chi}G^a_{\mu\nu}G^{a\mu\nu} + \frac{g^{(1)}_G}{m_\chi}\, \bar{\tilde{\chi}}i\partial^\mu\gamma^\nu\tilde{\chi}\, \mathcal{O}^g_{\mu\nu} + \frac{g^{(2)}_G}{m^2_\chi}\, \bar{\tilde{\chi}}(i\partial^\mu)(i\partial^\nu)\tilde{\chi}\, \mathcal{O}^g_{\mu\nu} .$$

Spin 0

Evaluate matrix element for the elastic scattering process in the non-relativistic limit.

$$f_N/m_N = \sum_{q=u,d,s} f_q f_{Tq} + \sum_{q=u,d,s,c,b} \frac{3}{4}\left(q(2)+\bar{q}(2)\right)\left(g^{(1)}_q + g^{(2)}_q\right) - \frac{8\pi}{9\alpha_s} f_{TG} f_G + \frac{3}{4}G(2)\left(g^{(1)}_G + g^{(2)}_G\right) .$$

Tools: FeynArts, FORM, PackageX

RGE

- Nucleon DM cross-sections at Non-Relativistic velocities.

- At what scale do we define coupling and masses? If at scale μ~0, then to compare with LHC we should run up. If at μ~LHC energy, then to compare we should run down.

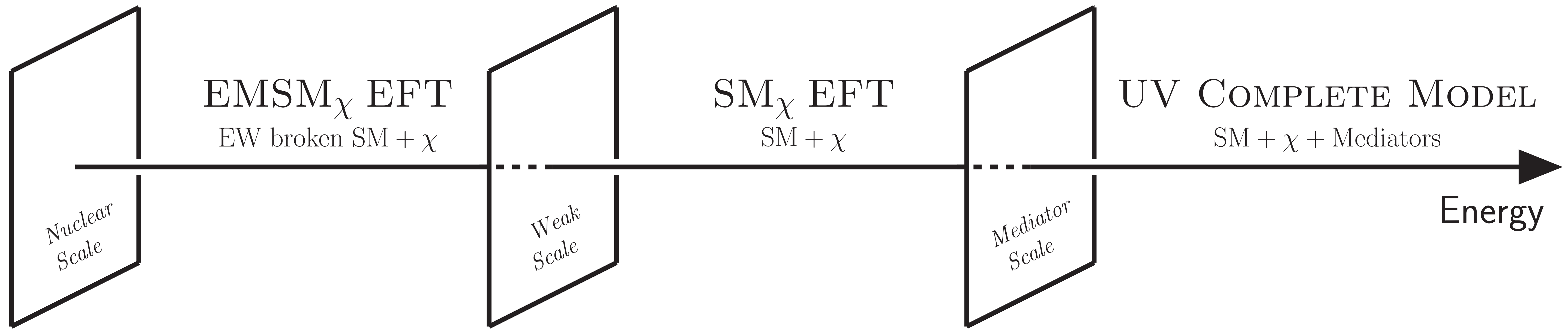
- RGE not necessary if no comparisons being made at different energy scales.

How we run

Wilson coefficients

Strategy

- Calculate RGE in full Theory.
- Apply matching conditions at each threshold of the theory.
- We will have to recalculate for every different model.
- Alternate approach available— RGE with EFT.



Details of RGE

Operators for Spin Independent Interactions

Quark	Gluon	
$O_q^{(0)} = m_q\bar{q}q$	$O_g^{(0)} = G^A_{\mu\nu}G^{A\mu\nu}$	**Spin 0**
$O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_-^{\nu\}} - \frac{g^{\mu\nu}}{4}i\not{D}_-\right)q$	$O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_\lambda + \frac{g^{\mu\nu}}{4}(G^A_{\alpha\beta})^2$	**Spin 2**

Sum Rules
Relate operators

Spin Dependent Operators

$$A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$$

Determine Anomalous dimensions

$$\frac{d}{d\log\mu}O_i = -\gamma_{ij}O_j\,, \qquad \frac{d}{d\log\mu}c_i = \gamma_{ji}c_j$$

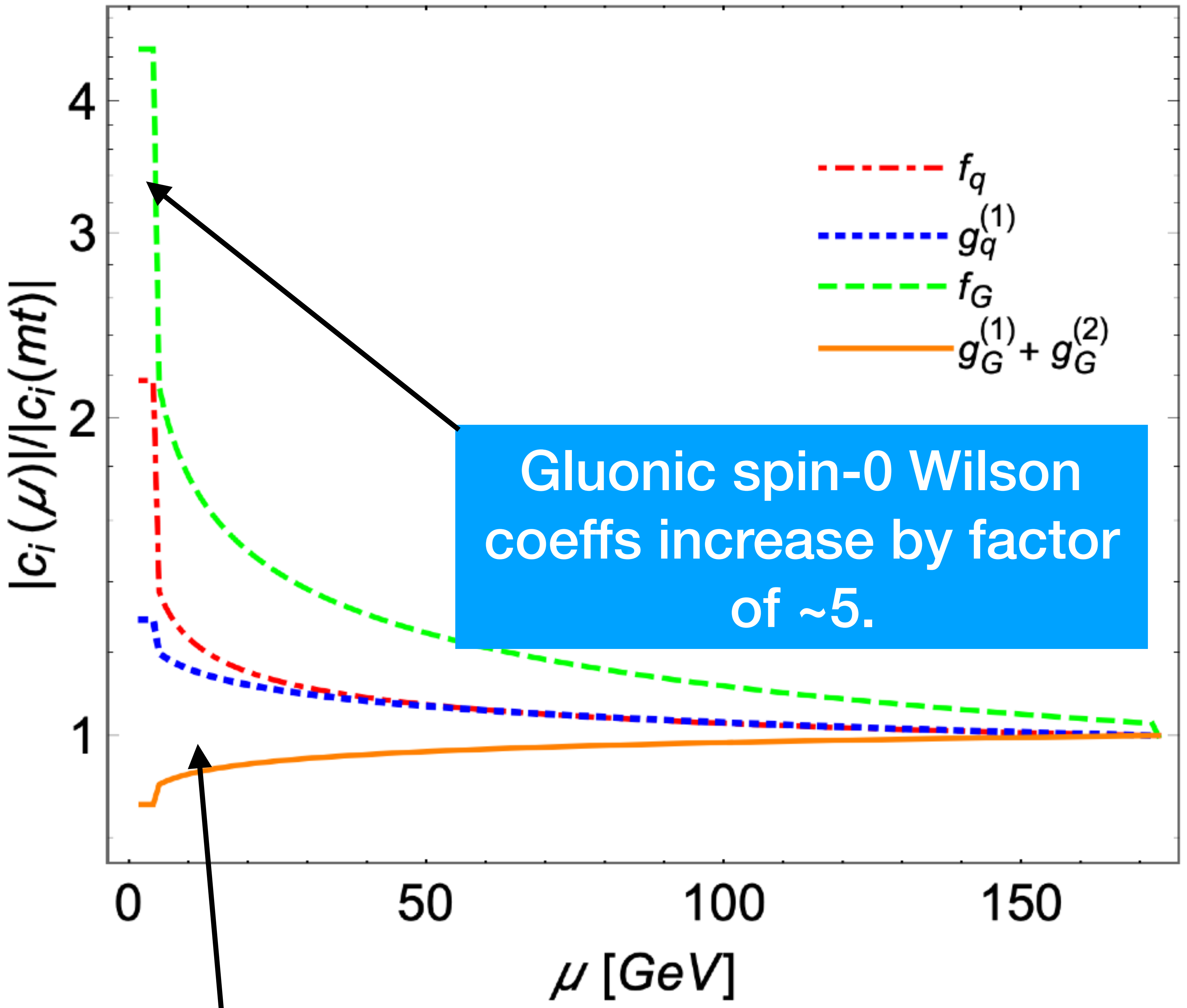
Evolve and Match at each threshold

$$c_i(\mu_l) = R_{ij}(\mu_l,\mu_h)c_j(\mu_h)\ .$$

$$c_i(\mu_Q) = M_{ij}(\mu_Q)c_j'(\mu_Q)$$

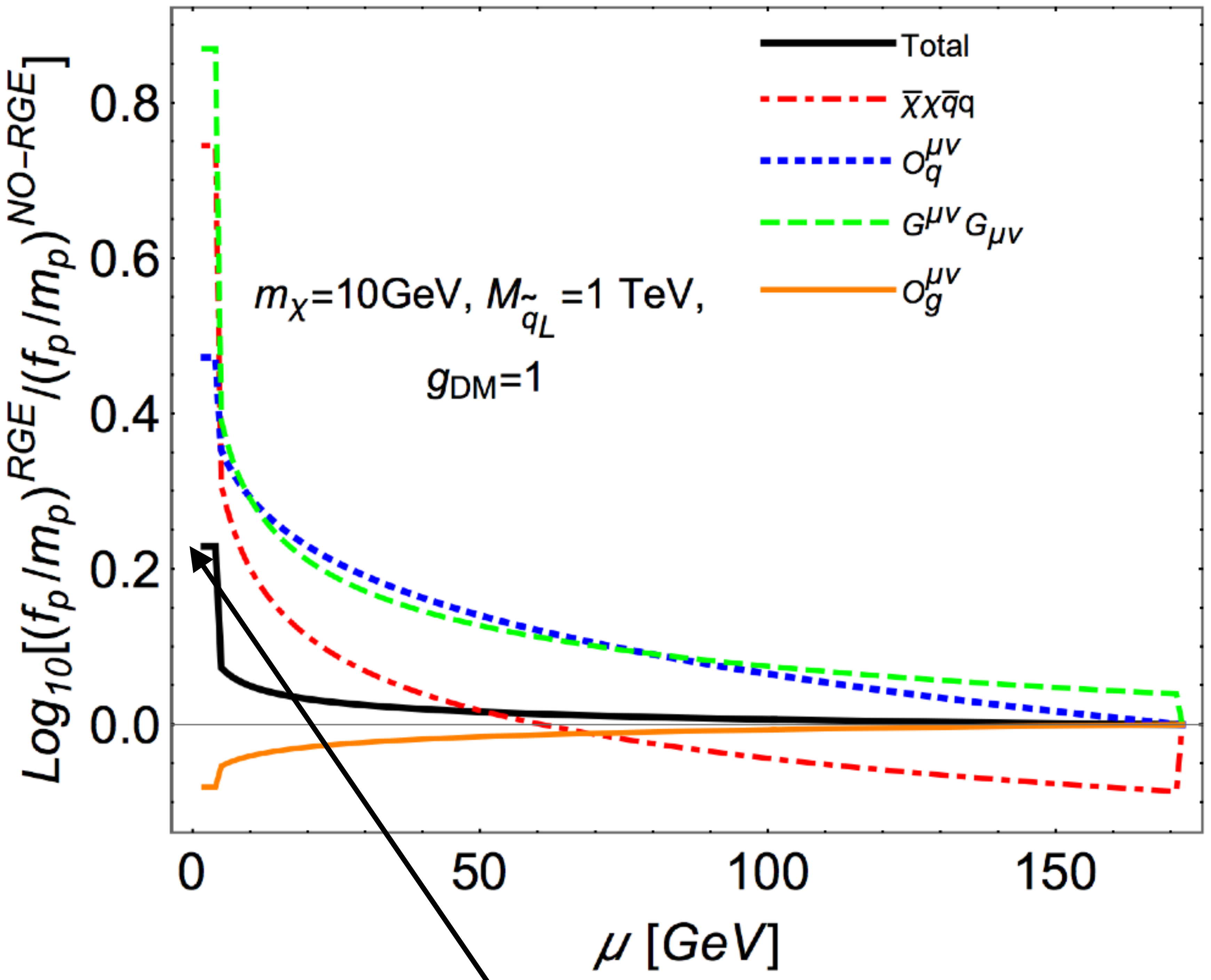
R. Hill, M. Solon 2014

How important is RGE?



Gluonic spin-0 Wilson coeffs increase by factor of ~5.

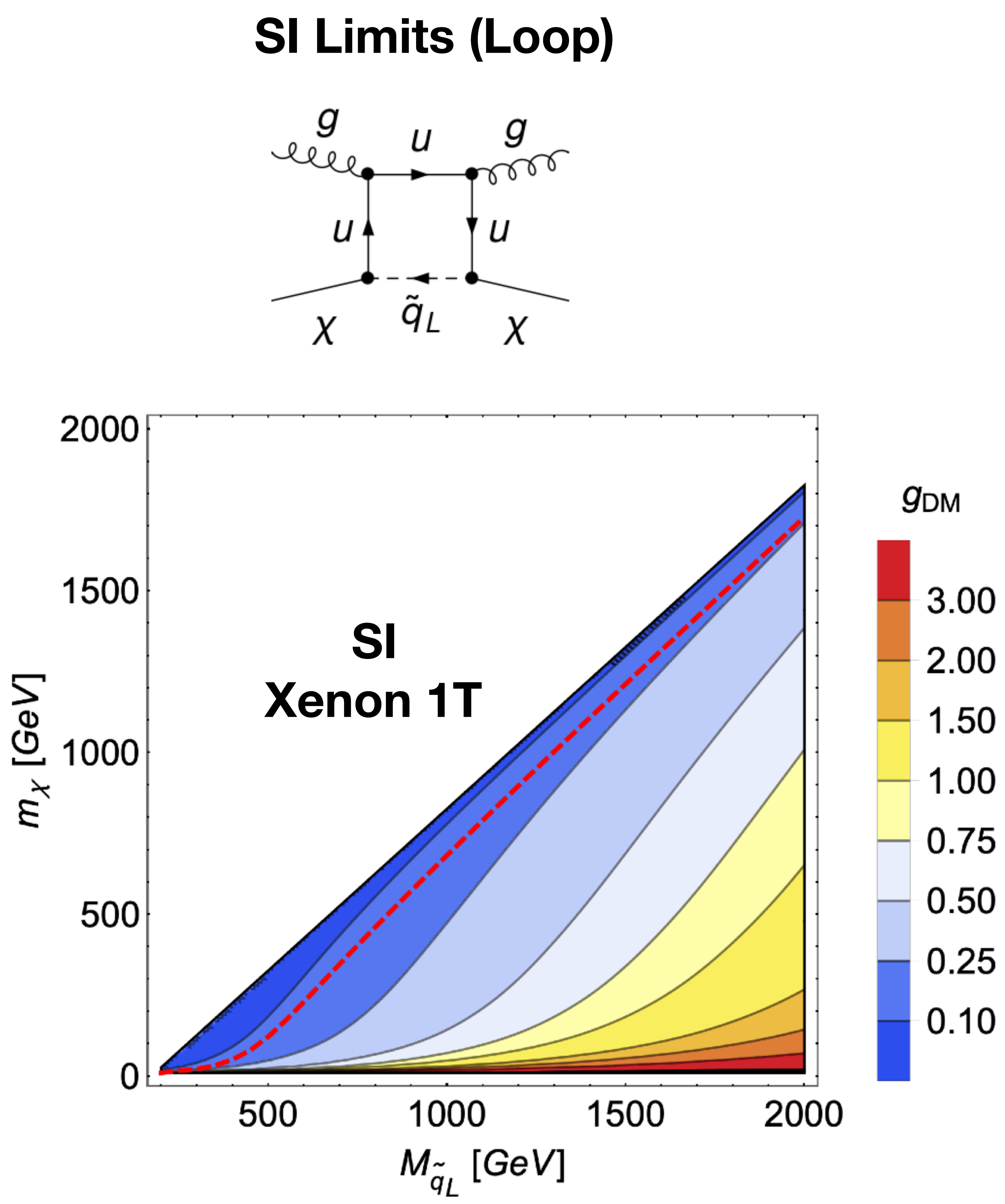
Spin-2 Wilson coefficients do not run as strongly.

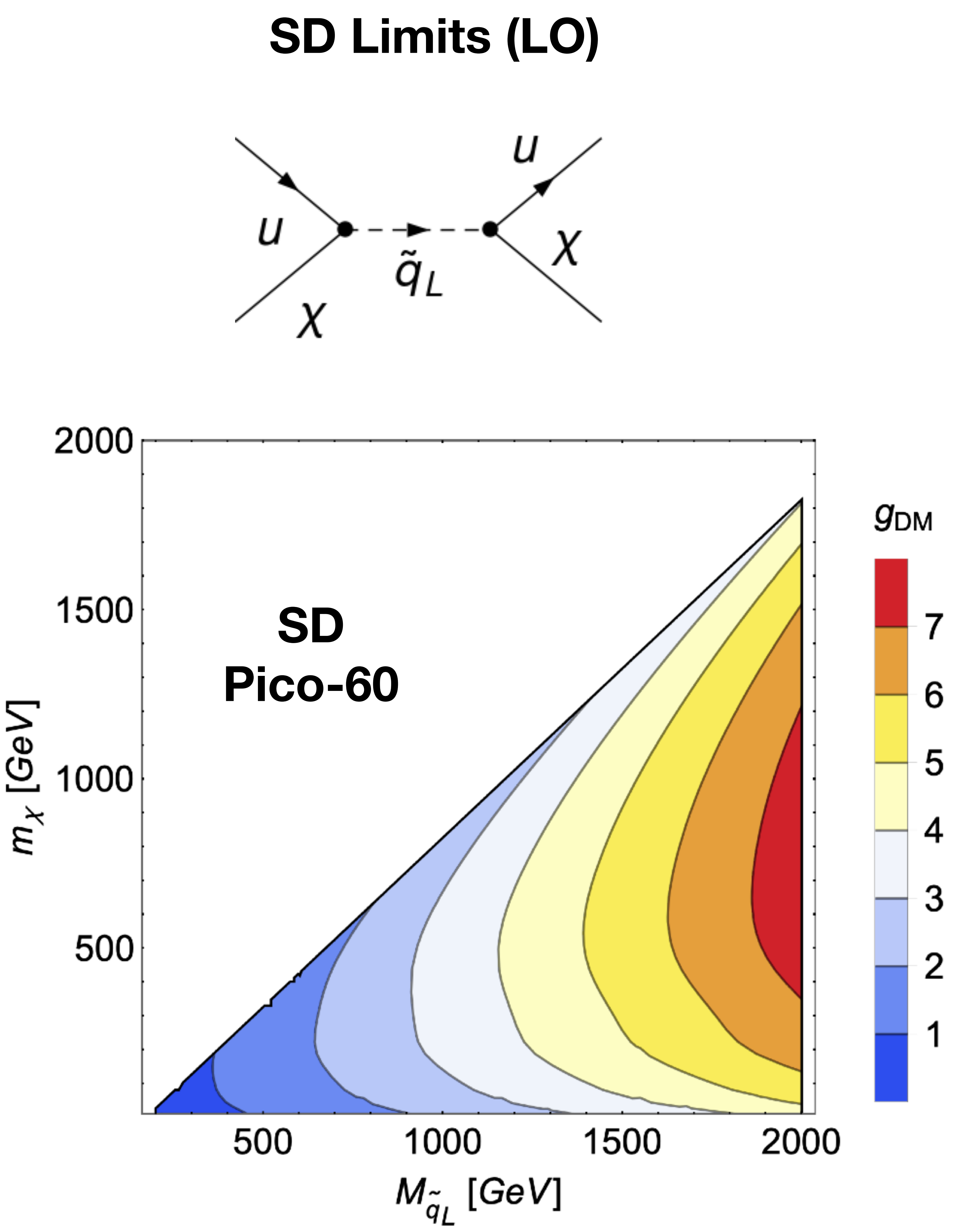


Factor ~2 increase in matrix element when performing RGE.

Factor ~4 enhancement in cross-section

Constraints from DD



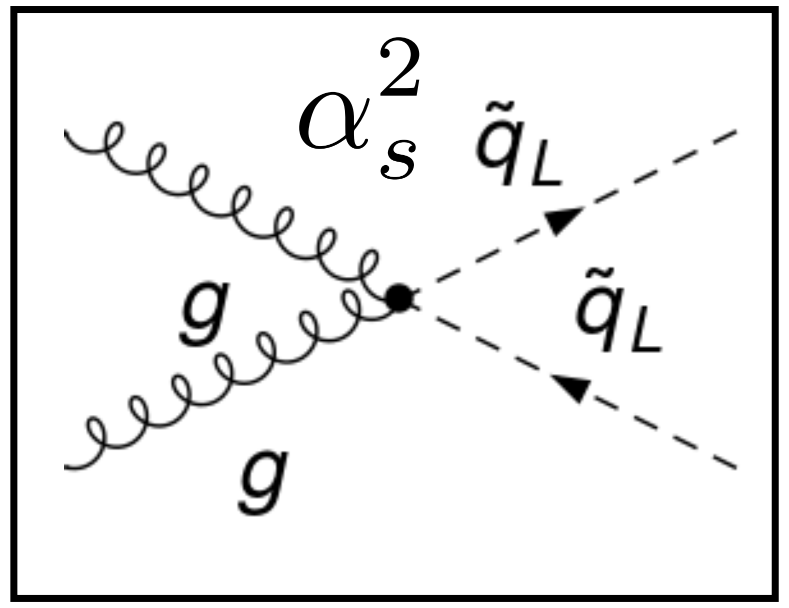


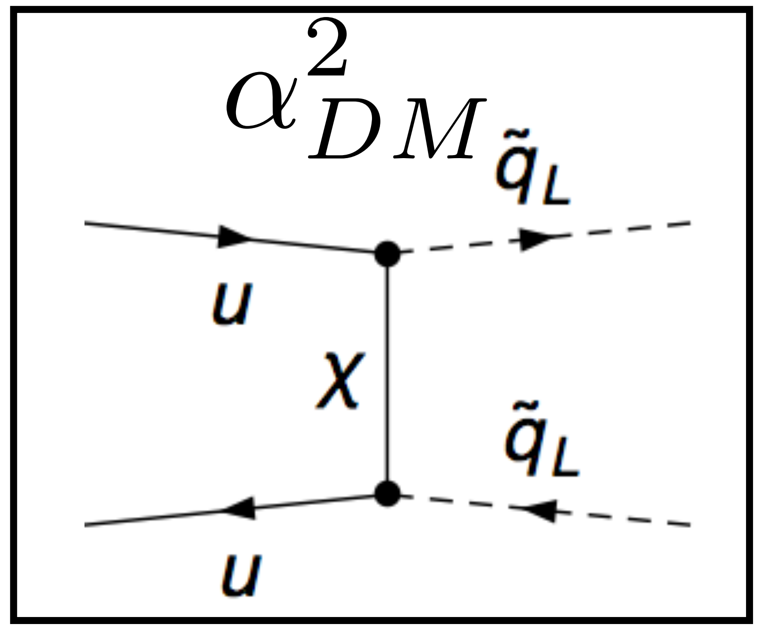
Constraints improve by an order of magnitude.

LHC constraints

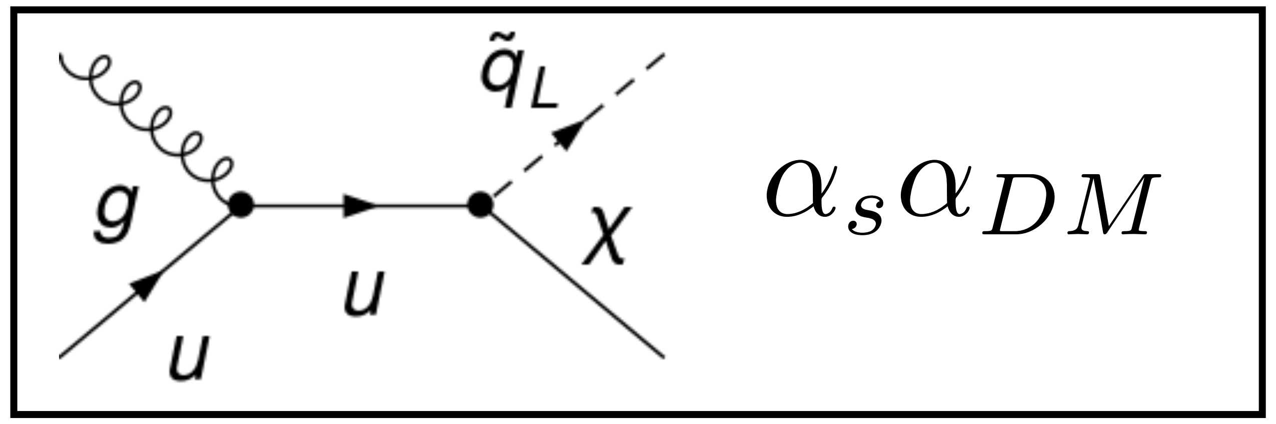
- Colored scalar mediator pair production— production cross-section (mostly QCD) depends on mass of mediator alone.

- Acceptance depends on mass of dark matter candidate also.

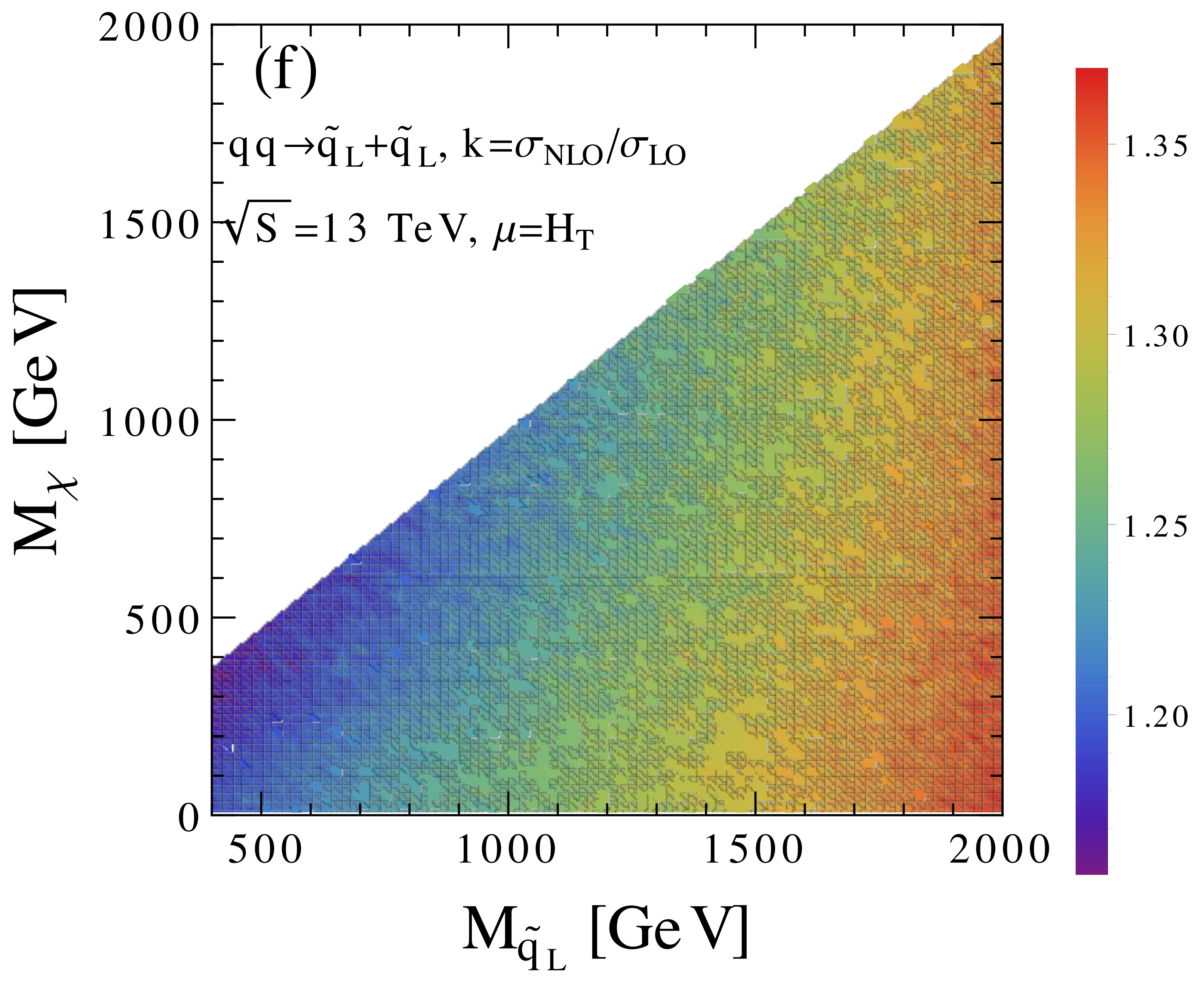


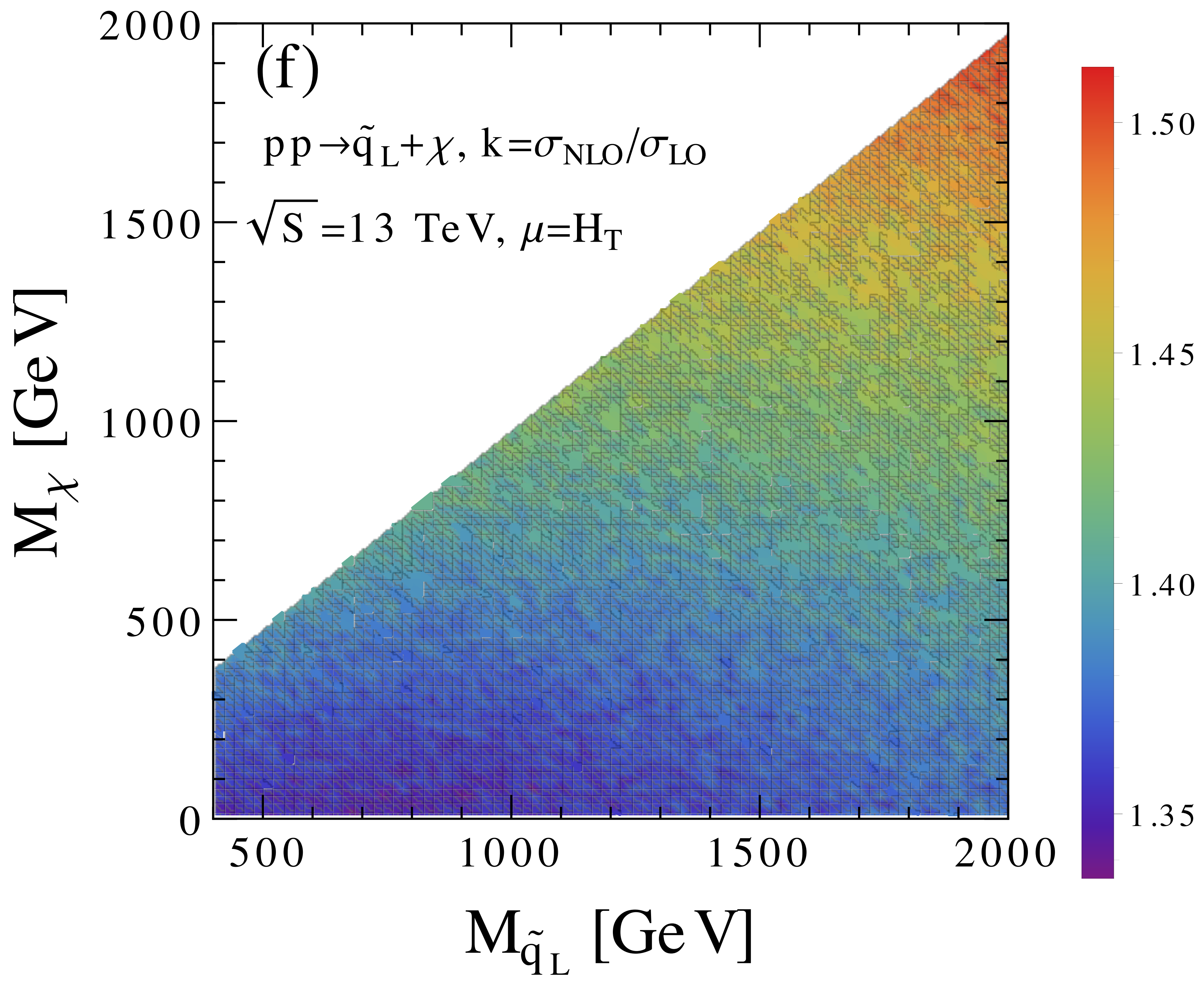


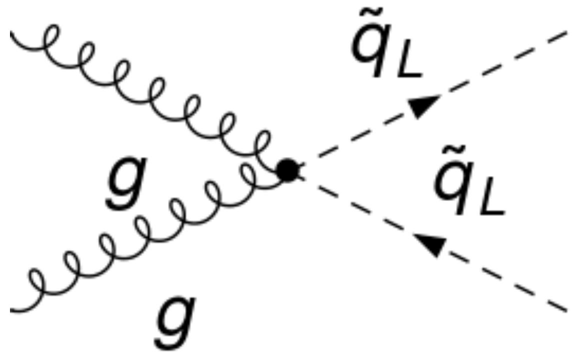
- Associated production of colored mediator and dark matter candidate— depends on all three model parameters.

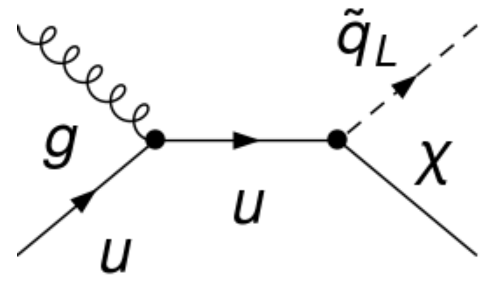


K factors









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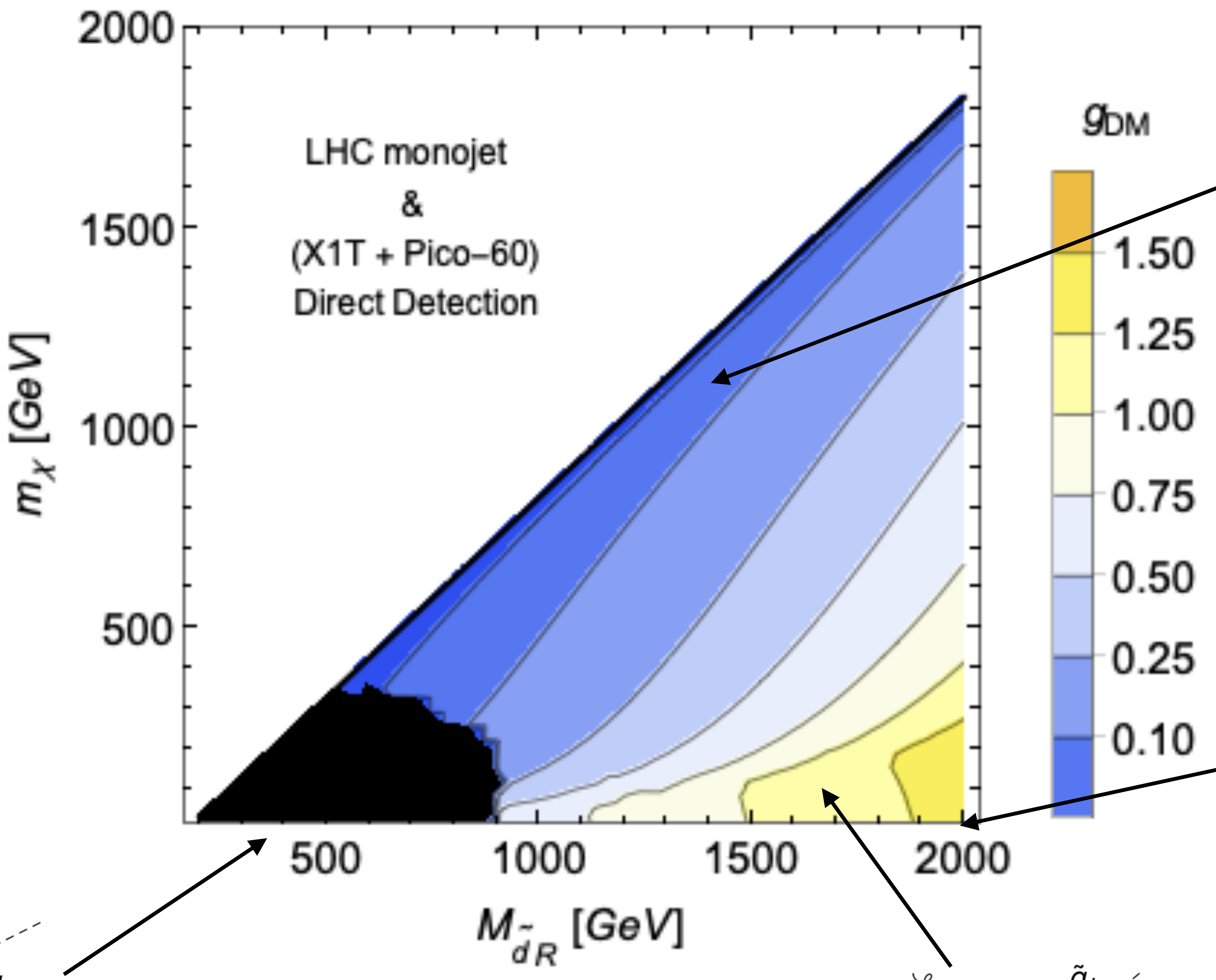
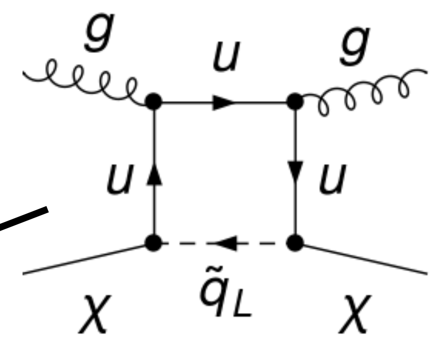
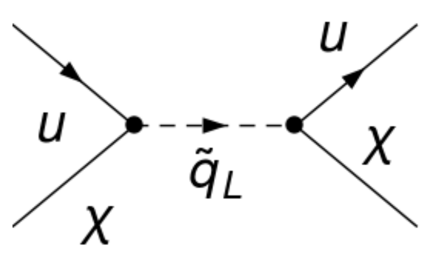
Complementarity of DD & LHC experiments



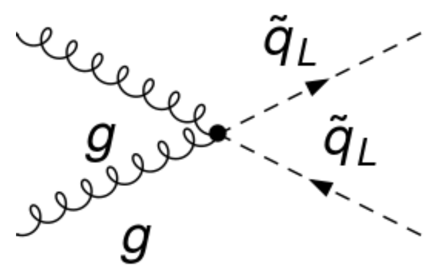


SI Limits

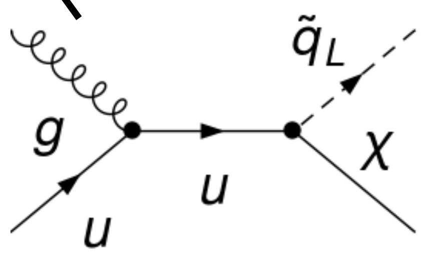
Tools : FeynRules, NLOCT, MadGraph5_aMC@NLO, Mad



SD Limits



LHC Pair production



LHC Associated production

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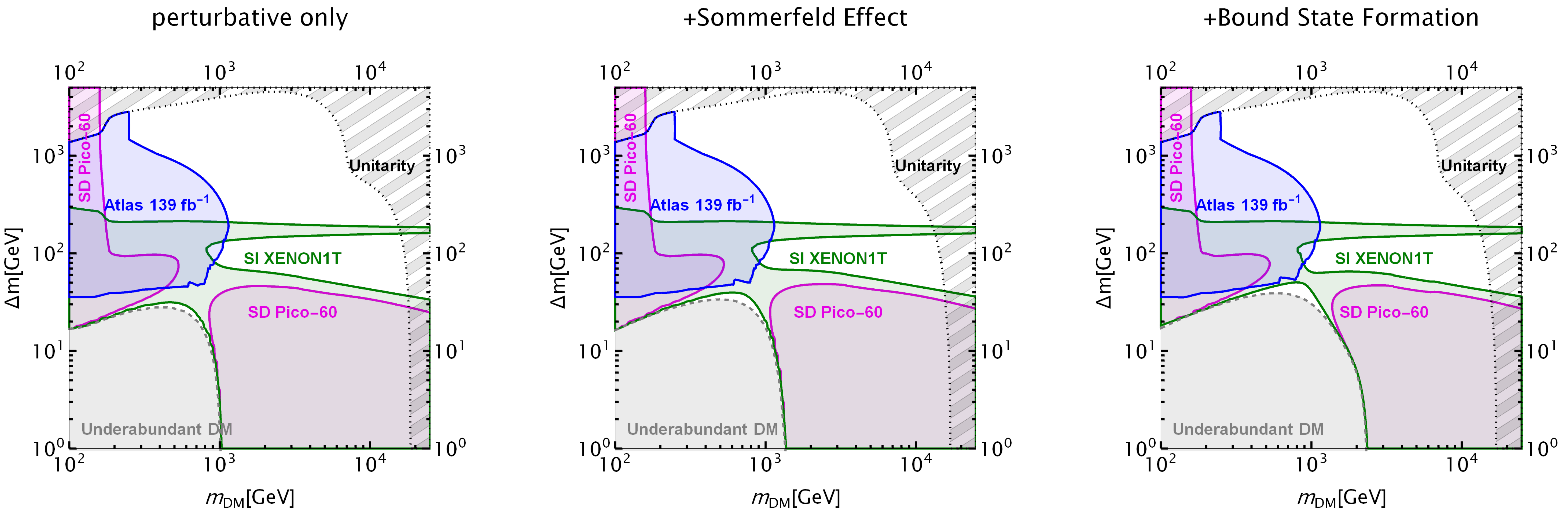
Full Impact in this seemingly trivial model

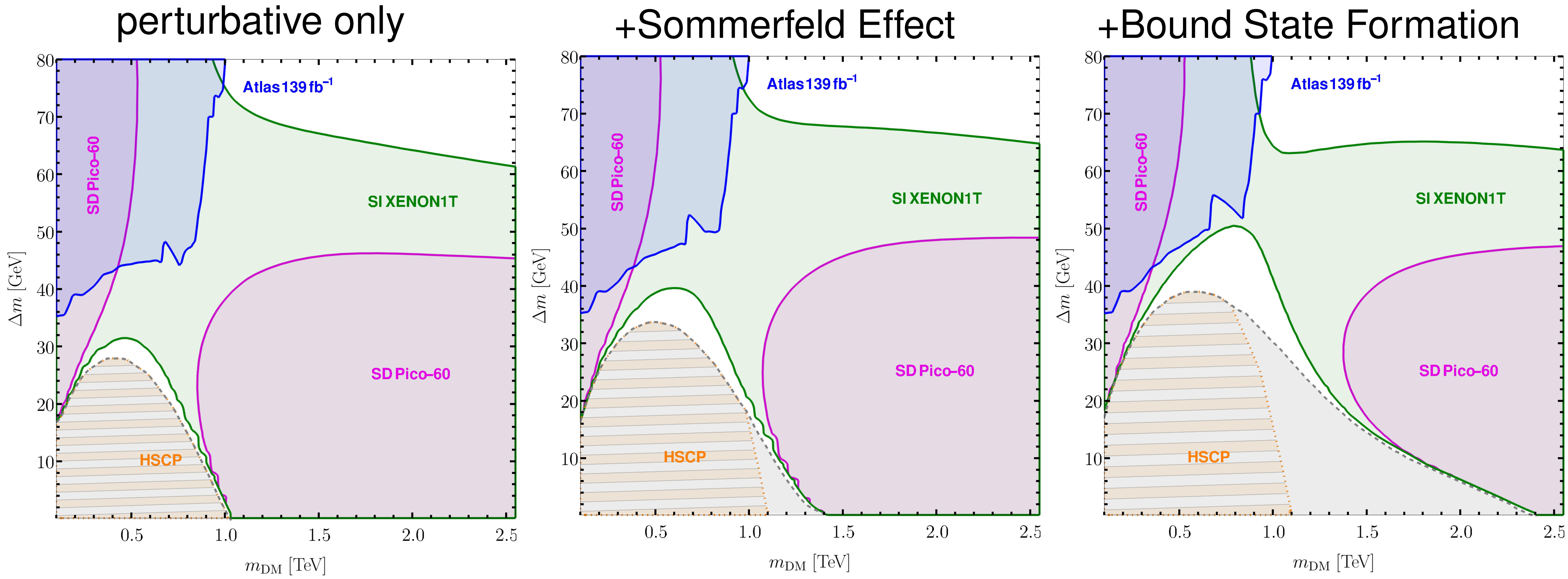
RGE improved Direct Detection [Mohan et. al (2019)]

mono-jet + ETmiss search by ATLAS [arXiv:1711.03301]

multi-jets + ETmiss search by CMS [arXiv:1704.07781]



Full Impact in this seemingly trivial model



$(m_{\rm DM}, \Delta m) < (1\,TeV, 30\,GeV)$ to $(1.4\,TeV, 40\,GeV)$ (Sommerfeld Effect) and $(2.4\,TeV, 50\,GeV)$ (Bound State Formation)

Figure from [MB,Copello,Harz,Mohan,Sengupta(2022)]

Bound State Formation at the LHC

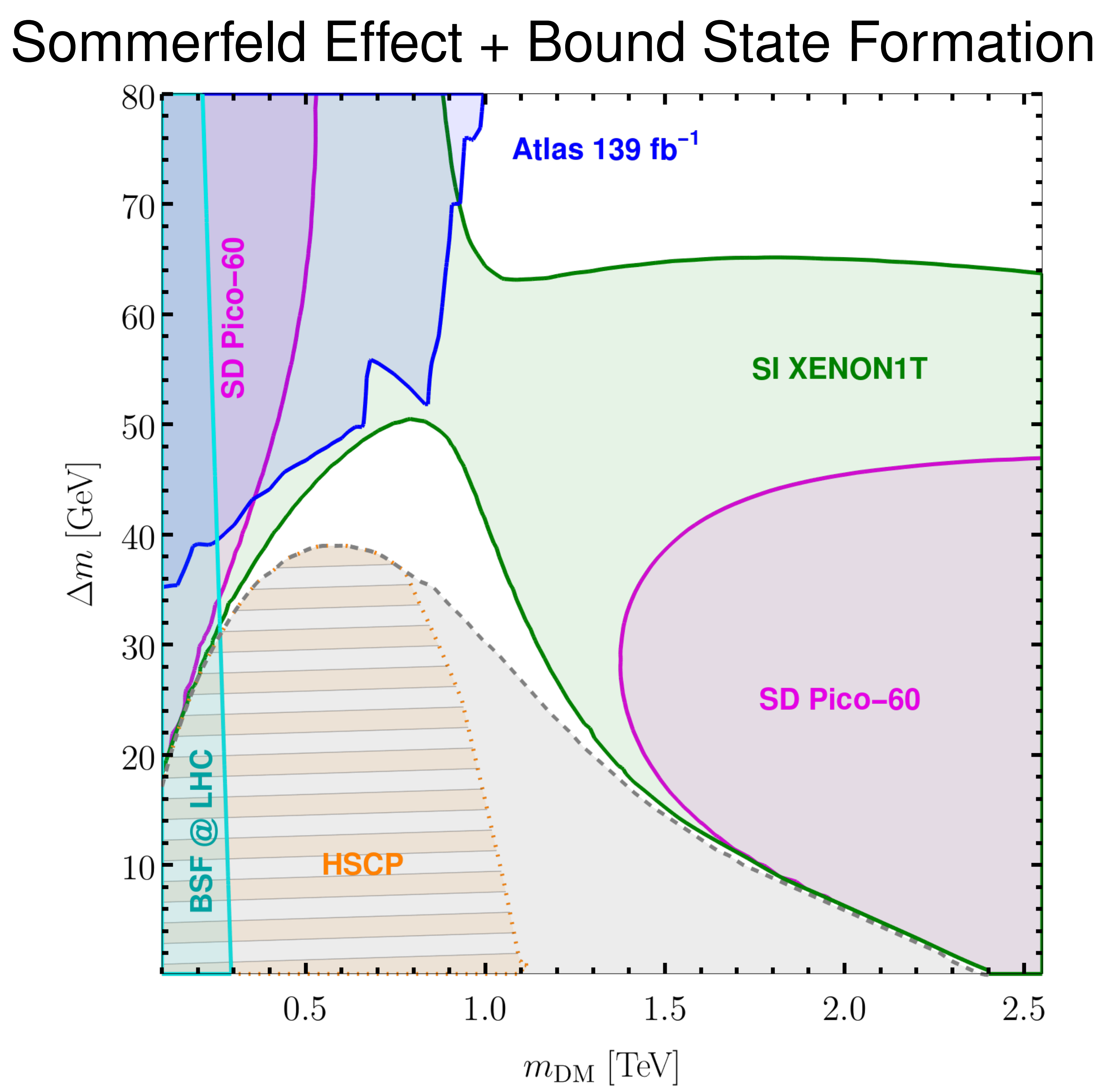
Production Cross Section

$$\sigma\left(pp \to \mathcal{B}(XX^\dagger)\right) = \frac{\pi^2}{8 m_{\mathcal{B}}^3} \Gamma\left(\mathcal{B}(XX^\dagger) \to gg\right) \mathcal{P}_{gg}\left(\frac{m_{\mathcal{B}}}{13\ \text{TeV}}\right)$$

$\rightarrow$ try to observe the bound state resonance in $\gamma\gamma$ final state. ATLAS (2017)

Efficient for all $g_{\rm DM}$ small enough such that $\Gamma_X < E_B$, roughly speaking $g_{\rm DM} \lesssim g_s$.

Bound State formation at the LHC



Limits at 37 fb^{-1} relatively weak in mass ($\sim 300\,\mathrm{GeV}$)
But huge potential: Closes the gap between prompt and LLP searches

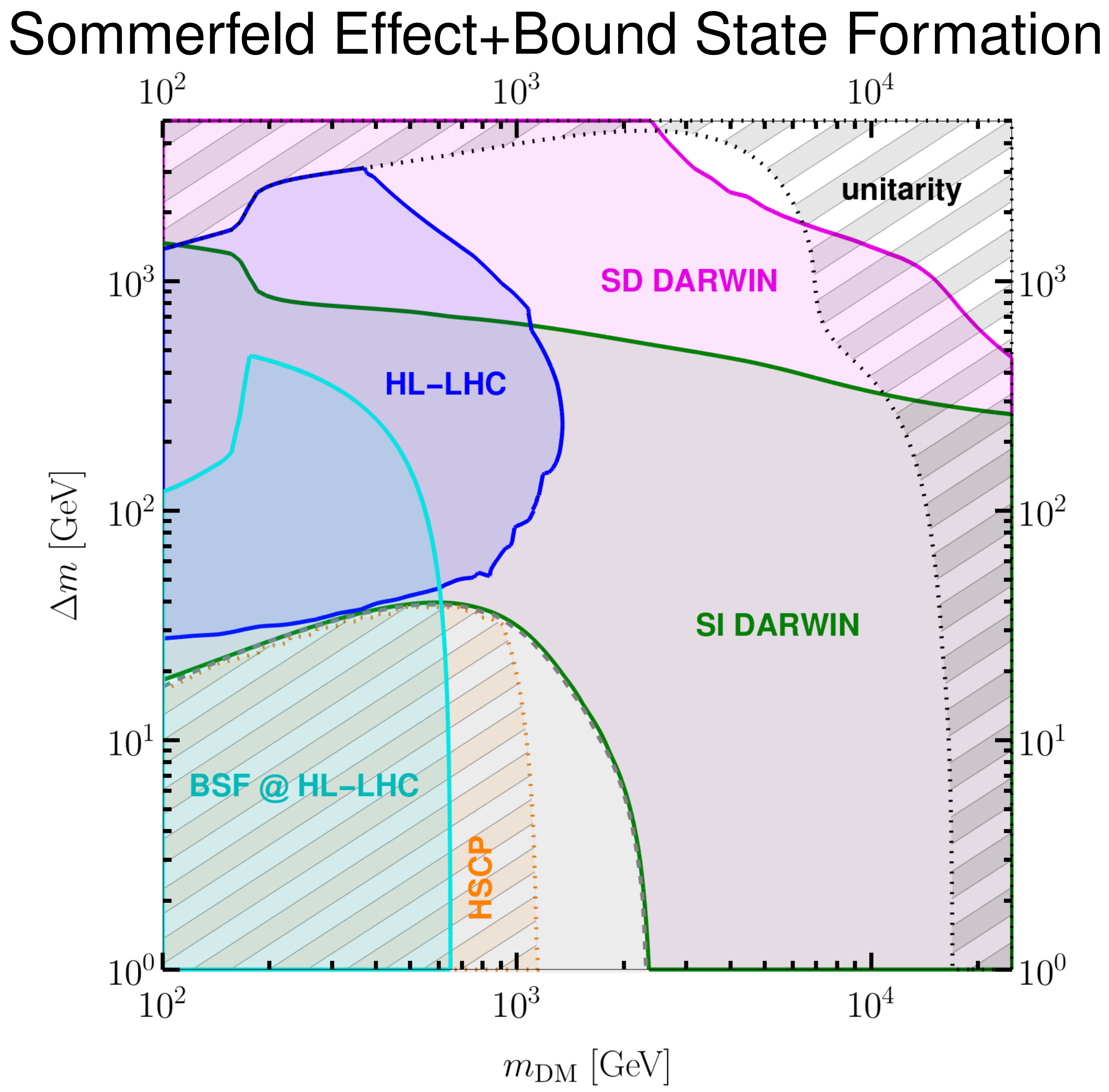
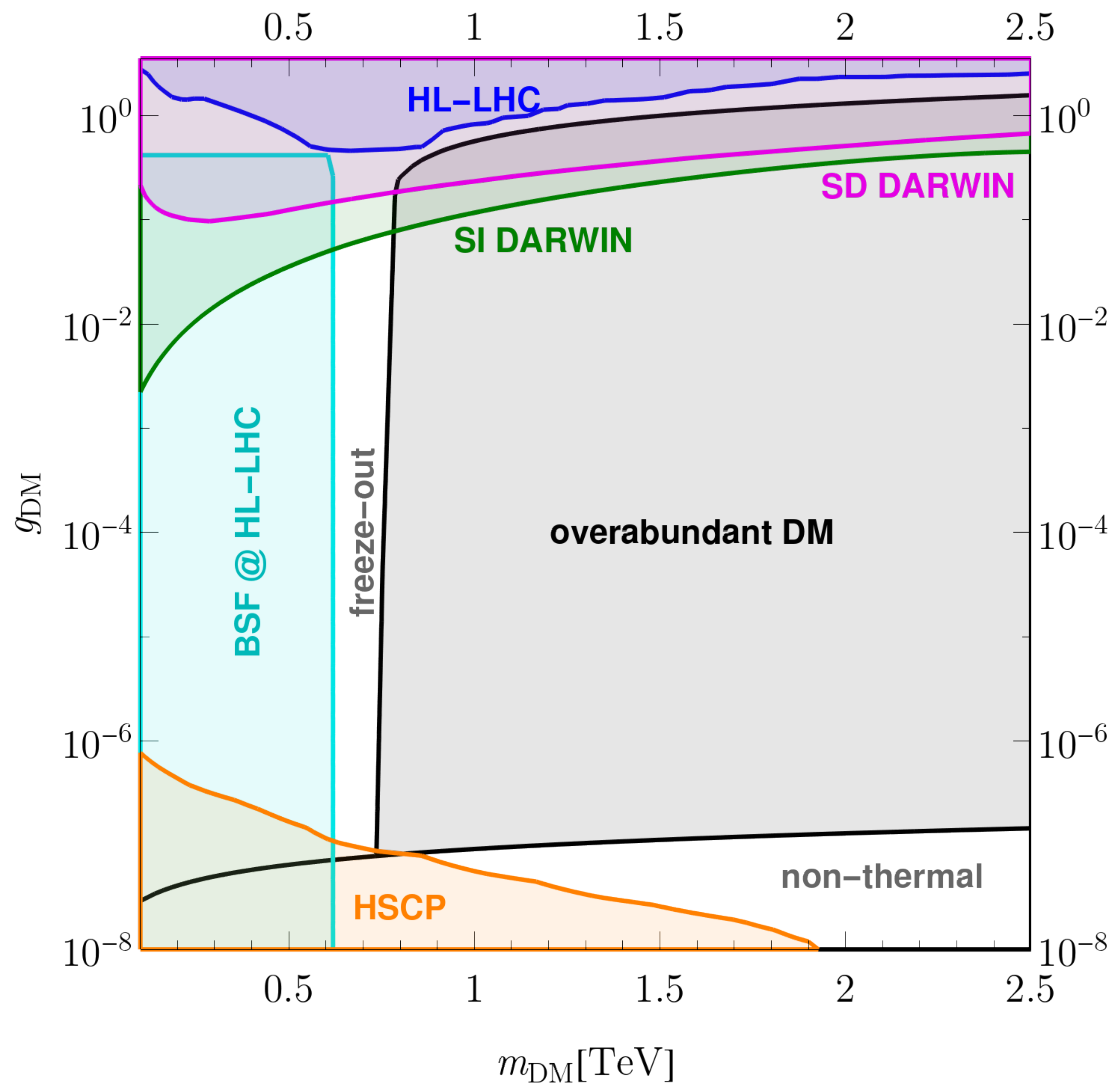


Figure from [MB,Copello,Harz,Mohan,Sengupta(2022)]

- Highly testable: Parameter space almost completely probed
- Remember: HSCP not a strict exclusion here (BSF@LHC is!)
- Bound State effects enlarge the area still necessary to test

Bound State formation at the LHC

Potential of BSF@LHC



Note: We fix $\Delta m = 0.05 m_{\rm DM}$ here!

Figure from [MB,Copello,Harz,Mohan,Sengupta(2022)]

Conclusion

- Non-perturbative Effects can increase or decrease the annihilation cross section of DM
  $\rightarrow$ Cannot be handled by a flat correction factor!

- Non-perturbative Effects are non-neglible in scenarios of colored coannihilation and open up small mass parameter space:

  Viable Parameter space shifts from $(m_{\text{DM}}, \Delta m) < (1\,TeV, 30\,GeV)$ to $(1.4\,TeV, 40\,GeV)$ (Sommerfeld Effect) and $(2.4\,TeV, 50\,GeV)$ (Bound State Formation)
  $\rightarrow$ Sommerfeld Effect alone not a good approximation!

- Bound State searches at colliders close the gap in "coupling space" between prompt and long-lived-particle searches

Direct Detection 101

matrix element for dark matter participating in SI scattering

$$f_N/m_N = \sum_{q=u,d,s} f_{Tq} f_q + \sum_{q=u,d,s,c,b} \frac{3}{4}\left[q(2)+\bar{q}(2)\right]\left(g_q^{(1)}+g_q^{(2)}\right) - \frac{8\pi}{9\alpha_s} f_{TG} f_G + \frac{3}{4} G(2)\left(g_G^{(1)}+g_G^{(2)}\right) ,$$

hadronic matrix elements:

$$\langle N|m_q \bar{q}q|N\rangle/m_N \equiv f_{Tq} ,$$
$$\langle N| -\frac{9\alpha_s}{8\pi} G^A_{\mu\nu}G^{A\mu\nu}|N\rangle/m_N \equiv f_{TG} ,$$
$$\langle N(p)|\mathcal{O}^{(2)}_{q,\mu\nu}|N(p)\rangle = \frac{1}{m_N}\left(p_\mu p_\nu - \frac{1}{4}m_N^2 g_{\mu\nu}\right)\left[q(2)+\bar{q}(2)\right]$$
$$\langle N(p)|\mathcal{O}^{(2)}_{g,\mu\nu}|N(p)\rangle = \frac{1}{m_N}\left(p_\mu p_\nu - \frac{1}{4}m_N^2 g_{\mu\nu}\right) G(2) .$$

matrix elements of the light quarks $(q = u,\ d,\ s)$ determined from lattice pion nucleon sigma term

$$\Sigma_{\pi N} = \frac{m_u + m_d}{2}\langle N|(\bar{u}u + \bar{d}d)|N\rangle ,$$
$$\Sigma_- = (m_d - m_u)\langle N|(\bar{u}u - \bar{d}d)|N\rangle .$$

matrix elements of the twist-2 operators

Related to second moments of PDF

$$\left[q(2)+\bar{q}(2)\right] = \int_0^1 dx\ x\ \left[q(x)+\bar{q}(x)\right] ,$$
$$G(2) = \int_0^1 dx\ x\ g(x) ,$$

$$m_u = 2.2 \text{ MeV}, \quad m_d = 4.7 \text{ MeV}, \quad m_s = 95 \text{ MeV},$$
$$m_c = 1.3 \text{ GeV}, \quad m_b = 4.2 \text{ GeV}, \quad m_t = 172 \text{ GeV},$$
$$m_Z = 91.188 \text{ GeV}, \quad \alpha_s(m_Z) = 0.1184,$$
$$m_n = 0.9396 \text{ GeV} \quad m_p = 0.9383 \text{ GeV} \ .$$

$$[f_{T_u}]_p = 0.018, \quad [f_{T_d}]_p = 0.030, \quad [f_{T_s}]_p = 0.043,$$
$$[f_{T_u}]_n = 0.015, \quad [f_{T_d}]_n = 0.034, \quad [f_{T_s}]_n = 0.043,$$
$$f_{T_G}|_{\text{NNNLO}} = 0.80 \ .$$

$$f_{T_G} = -\frac{9\alpha_S(\mu)}{4\pi\beta(\mu)}\left[1 - (1+\gamma_m(\mu))\sum_{u,d,s} f_{Tq}\right]$$

$$[u(2) + \bar{u}(2)]_p = 0.3481, \quad \left[d(2) + \bar{d}(2)\right]_p = 0.1902,$$
$$[s(2) + \bar{s}(2)]_p = 0.0352, \quad [c(2) + \bar{c}(2)]_p = 0.0107 \ ,$$
$$[G(2)]_p = [G(2)]_n = 0.4159 \ .$$

$$\Delta u^{(p)} = 0.84, \quad \Delta d^{(p)} = -0.43, \quad \Delta s^{(p)} = -0.09,$$
$$\Delta u^{(n)} = \Delta d^{(p)}, \quad \Delta d^{(n)} = \Delta u^{(p)}, \quad \Delta s^{(n)} = \Delta s^{(p)}.$$

Those large logs!

A closer look at the Wilson Coefficients

$$\frac{g_G^{(1)}}{m_\chi} = \alpha_s \alpha_{DM} \left[f_1(m_q, M_{\tilde{q}_L}, m_\chi) \log\left(\frac{m_q}{M_{\tilde{q}_L}}\right) + f_2(m_q, M_{\tilde{q}_L}, m_\chi) \right]$$

- For light quarks, large logs dominate the loop integral.
- Including RGE ensures large logs cancel

$$\Delta g_G^{(1)} = \frac{\alpha_s g_{DM}^2 m_\chi}{24\pi(M^2 - m_\chi^2)^2} \log\left(\frac{M}{m}\right) .$$

$$\Delta g_G^{(1)}\Big|_{\mu_l} \simeq \frac{m_\chi g_{DM}^2}{72\pi^2(M_{\tilde{q}}^2 - m_\chi^2)^2} \left[3\pi\alpha_s(\mu_h) \log\left(\frac{\mu_l}{\mu_h}\right) + \alpha_s(M_{\tilde{q}}) \log\left(\frac{M_{\tilde{q}}}{m_b}\right) \left(3\pi - 5\alpha_s(\mu_h) \log\left(\frac{\mu_l}{\mu_h}\right)\right) \right]$$

$$R = \left(\begin{array}{ccc|c} & & & R_{qg} \\ & \mathbb{I}(R_{qq} - R_{qq'}) + \mathbb{J}R_{qq'} & & \vdots \\ & & & R_{qg} \\ \hline R_{gq} & \cdots & R_{gq} & R_{gg} \end{array}\right),$$

$$R^{(0)}_{qq} = 1\,, \quad R^{(0)}_{qg} = 2[\gamma_m(\mu_h) - \gamma_m(\mu_l)]/\tilde{\beta}(\mu_h)\,,$$
$$R^{(0)}_{gq} = 0\,, \quad R^{(0)}_{gg} = \tilde{\beta}(\mu_l)/\tilde{\beta}(\mu_h)$$

$$R^{(2)}_{qq} - R^{(2)}_{qq'} = r(0) + \mathcal{O}(\alpha_s)\,, \quad R^{(2)}_{qq'} = \frac{1}{n_f}\left[\frac{16r(n_f) + 3n_f}{16 + 3n_f} - r(0)\right] + \mathcal{O}(\alpha_s$$
$$R^{(2)}_{qg} = \frac{16[1 - r(n_f)]}{16 + 3n_f} + \mathcal{O}(\alpha_s)\,,$$
$$R^{(2)}_{gq} = \frac{3[1 - r(n_f)]}{16 + 3n_f} + \mathcal{O}(\alpha_s)\,, \quad R^{(2)}_{gg} = \frac{16 + 3n_f r(n_f)}{16 + 3n_f} + \mathcal{O}(\alpha_s)$$

$$\langle O'^{(0)}_q \rangle = \langle O^{(0)}_q \rangle + \mathcal{O}(1/m_Q)$$

$$M = \left(\begin{array}{ccc|c|c} 1 & & & 0 & 0 \\ & \ddots & & \vdots & \vdots \\ & & 1 & 0 & 0 \\ \hline 0 & \cdots & 0 & M_{gQ} & M_{gg} \end{array}\right)$$

$$M^{(0)}_{gQ} = -\frac{\alpha'_s(\mu_Q)}{12\pi}\left\{1 + \frac{\alpha'_s(\mu_Q)}{4\pi}\left[11 - \frac{4}{3}\log\frac{\mu_Q}{m_Q}\right] + \mathcal{O}(\alpha_s^2)\right\}$$
$$M^{(0)}_{gg} = 1 - \frac{\alpha'_s(\mu_Q)}{3\pi}\log\frac{\mu_Q}{m_Q} + \mathcal{O}(\alpha_s^2)$$

Operators Mix

$$c_j(\mu_0) = R_{jk}(\mu_0, \mu_c) M_{kl}(\mu_c) R_{lm}(\mu_c, \mu_b) M_{mn}(\mu_b) R_{ni}(\mu_b, \mu_t) c_i(\mu_t)$$

$$A^{\mu_1\mu_2} = i8 f_G \left({k_2}^{\mu_1} {k_1}^{\mu_2} - g^{\mu_1\mu_2} \left(k_1 \cdot k_2 \right) \right)$$

Define Projection Operators

$$\begin{aligned}
C^{\mu_1\mu_2} &= i2\frac{g_G^{(1)}}{m_\chi}\Big[g^{\mu_1\mu_2}(k_1\cdot k_3)\gamma\cdot k_2 - g^{\mu_1\mu_2}(k_1\cdot k_4)\gamma\cdot k_2 + g^{\mu_1\mu_2}(k_2\cdot k_3)\gamma\cdot k_1 \\
&- g^{\mu_1\mu_2}(k_2\cdot k_4)\gamma\cdot k_1 + (k_1\cdot k_2)(g^{\mu_1\mu_2}(\gamma\cdot k_4 - \gamma\cdot k_3) + \gamma^{\mu_2}(k_3^{\mu_1} - k_4^{\mu_1}) + \gamma^{\mu_1}(k_3^{\mu_2} - k_4^{\mu_2})) \\
&- \gamma^{\mu_2} k_2^{\mu_1}(k_1\cdot k_3) + \gamma^{\mu_2} k_2^{\mu_1}(k_1\cdot k_4) - k_3^{\mu_2} k_2^{\mu_1}(\gamma\cdot k_1) + k_4^{\mu_2} k_2^{\mu_1}\gamma\cdot k_1 \\
&+ k_1^{\mu_2}(\gamma^{\mu_1}(k_2\cdot k_4 - k_2\cdot k_3) + k_2^{\mu_1}(\gamma\cdot k_3 - \gamma\cdot k_4) + (k_4^{\mu_1} - k_3^{\mu_1})\gamma\cdot k_2)\Big] .
\end{aligned}$$

$$\begin{aligned}
B^{\mu_1\mu_2} &= i\frac{g_G^{(2)}}{m_\chi^2}\Big[g^{\mu_1\mu_2}\Big(2(k_1\cdot k_3)(k_2\cdot k_3) + 2(k_1\cdot k_4)(k_2\cdot k_4) - (k_3^2 + k_4^2)(k_1\cdot k_2)\Big) \\
&+ k_1^{\mu_2}\Big((k_3^2 + k_4^2)k_2^{\mu_1} - 2(k_3^{\mu_1}(k_2\cdot k_3) + k_4^{\mu_1}(k_2\cdot k_4))\Big) \\
&+ k_3^{\mu_2}\Big(2k_3^{\mu_1}(k_1\cdot k_2) - 2k_2^{\mu_1}(k_1\cdot k_3)\Big) \\
&+ 2k_4^{\mu_2}\Big(k_4^{\mu_1}(k_1\cdot k_2) - k_2^{\mu_1}(k_1\cdot k_4)\Big)\Big] ,
\end{aligned}$$

Multiply with loop Integrals and solve for Wilson coefficients after performing an expansion in energy

$$\begin{aligned}
A\cdot(A+B+C) &= 32 f_G^2 S^2 \\
B\cdot(A+B+C) &= -2m_\chi^3 S^2 (m_\chi \frac{g_G^{(2)}}{m_\chi^2} + 2\frac{g_G^{(1)}}{m_\chi}) + \mathcal{O}(S^3) \\
C\cdot(A+B+C) &= 2m_\chi^2 S^2 (m_\chi \frac{g_G^{(2)}}{m_\chi^2} + 4\frac{g_G^{(1)}}{m_\chi}) + \mathcal{O}(S^3)
\end{aligned}$$

$$[\mathcal{M}^{\nu}_{\mathbf{k}\to\{n\ell m\}}]^{a}_{ii',jj'} = \frac{1}{\sqrt{2\mu}} \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3}\ \tilde{\psi}^{*}_{n\ell m}(\mathbf{p})\ \tilde{\phi}_{\mathbf{k}}(\mathbf{q})\ [\mathcal{M}^{\nu}_{\text{trans}}(\mathbf{q},\mathbf{p})]^{a}_{ii',jj'}\,,$$

$$[\mathcal{M}^{\nu}_{\text{trans}}(\mathbf{q},\mathbf{p})]^{a}_{ii',jj'} = \frac{1}{\mathcal{S}_0(\mathbf{q};K)\,\mathcal{S}_0(\mathbf{p};P)} \int \frac{dq^0}{2\pi}\frac{dp^0}{2\pi}\ [\mathcal{C}^{\nu}(q,p;K,P)]^{a}_{ii',jj'}\,. \tag{2.17}$$

Here, $[\mathcal{C}^{\nu}(q,p;K,P)]^{a}_{ii',jj'}$ is the sum of all connected diagrams contributing to the process

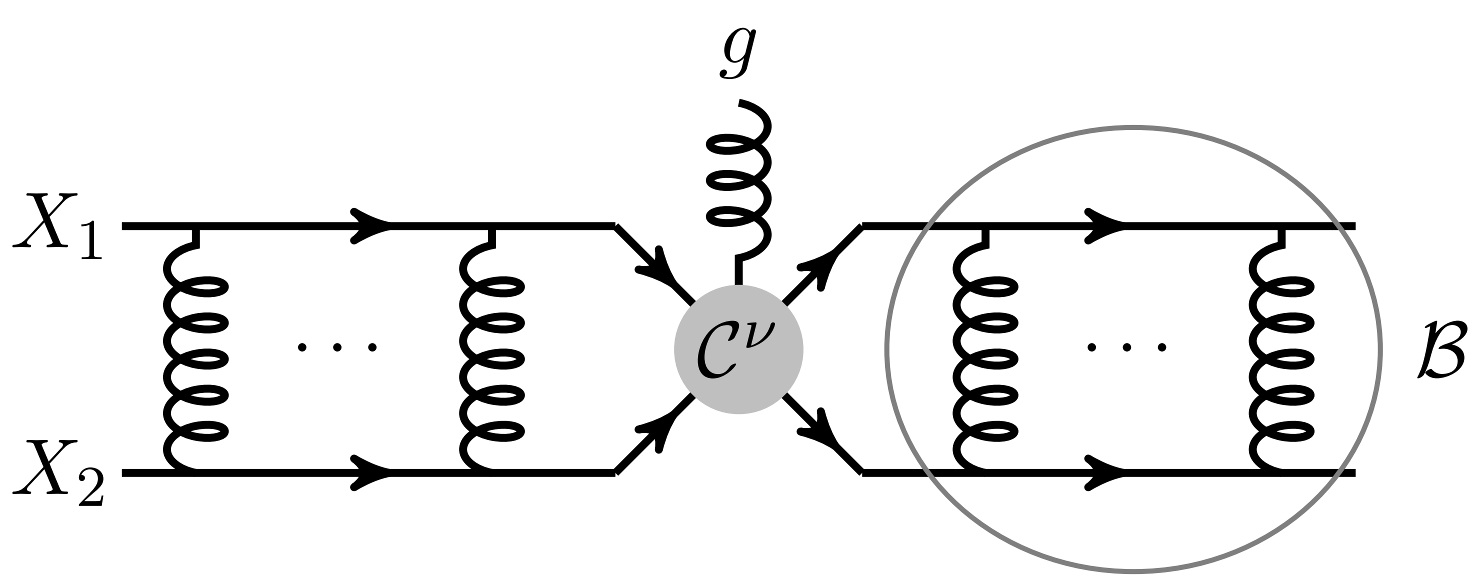
$$X_{1,i}\,(\eta_1 K + q) + X_{2,j}\,(\eta_2 K - q) \to X_{1,i'}\,(\eta_1 P + p) + X_{2,j'}\,(\eta_2 P - p) + g^a(P_g), \tag{2.18}$$

$$S(p;P) \equiv S_1(\eta_1 P + p)\ S_2(\eta_2 P - p)\,,$$

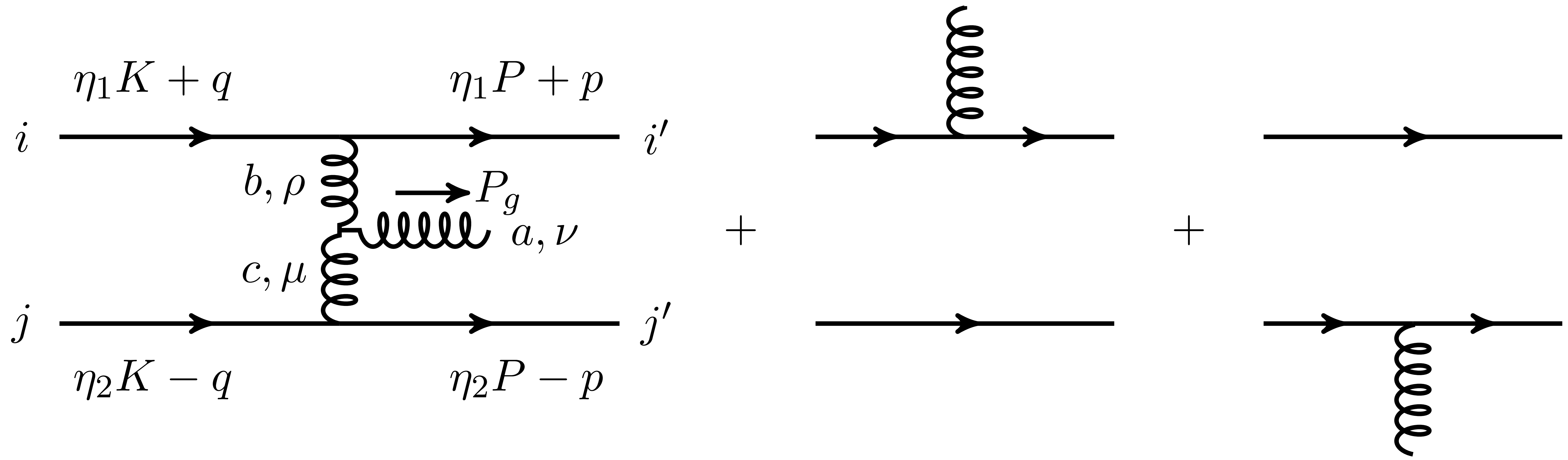
$$\mathcal{S}_0(\mathbf{p};P) \equiv \int \frac{dp^0}{2\pi}\ S(p;P)\,.$$

To leading order in the non-relativistic regime [57, appendix C],

$$\mathcal{S}_0(\mathbf{p};P) \simeq \left[-i4M\mu\left(P^0 - M - \frac{\mathbf{P}^2}{2M} - \frac{\mathbf{p}^2}{2\mu}\right)\right]^{-1}\,,$$



(a) The amplitude for the radiative capture consists of the (non-perturbative) initial and final state wavefunctions, and the perturbative 5-point function that includes the radiative vertices.



(b) The leading order diagrams contributing to $\mathcal{C}^\nu$. The external-momentum, colour-index and space-time-index assignments are the same in all three diagrams.

Figure 1. Radiative capture into bound states.

Direct Detection 101

We need the Wilson Co-efficients

determined by matching to matrix elements

Tree level quark Wilson coefficients $f_q,\ g_q^{(1)}$ and $g_q^{(2)}$

$$f_q = \frac{g_{DM}^2\, m_\chi}{16(M^2-m_\chi^2)^2}\,,$$

$$g_q^{(1)} = \frac{g_{DM}^2\, m_\chi}{8(M^2-m_\chi^2)^2}\,,$$

$$g_q^{(2)} = 0\ .$$

Suppressed compared to SD matrix elements by

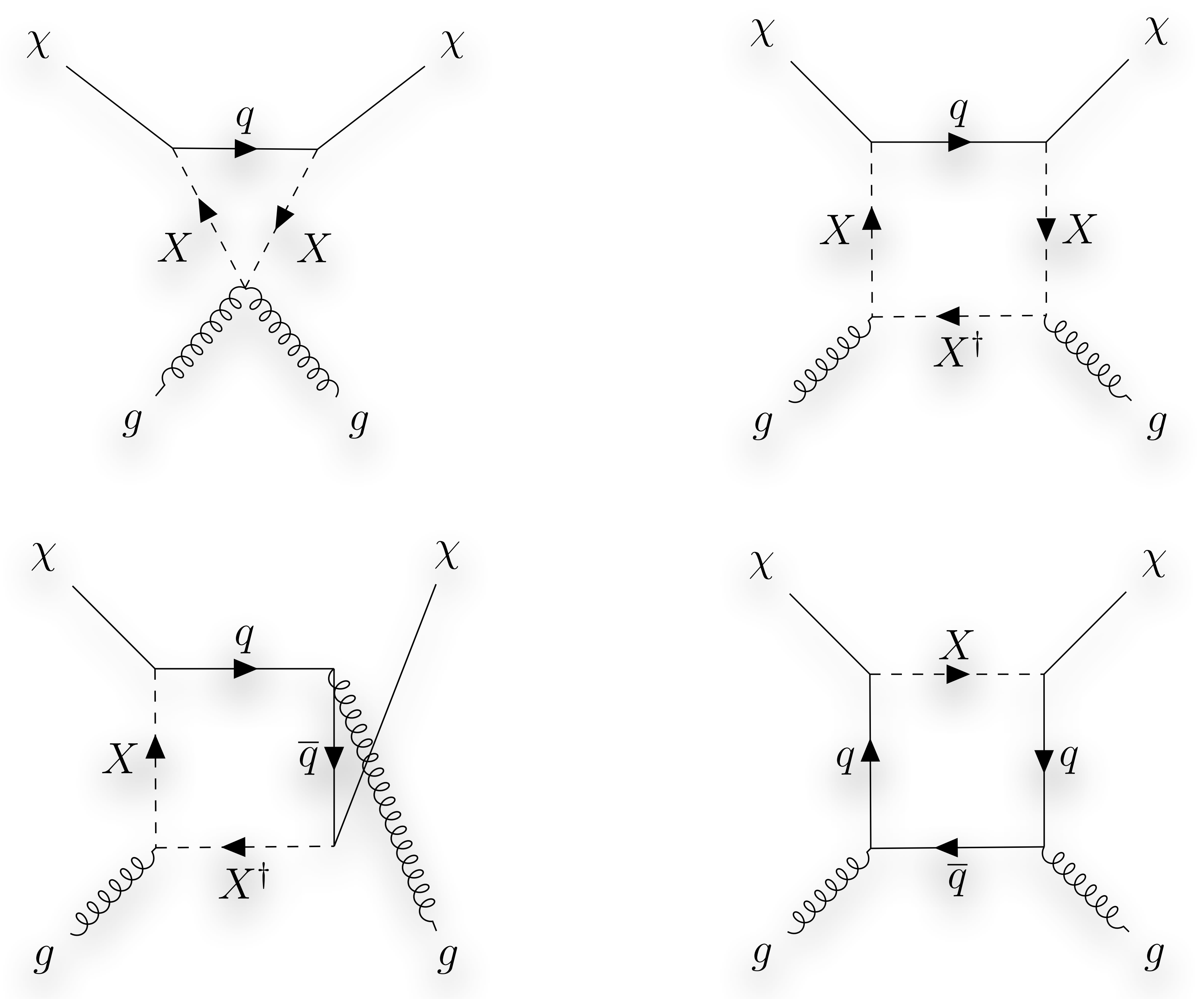
$$1/(M^2-m_\chi^2)$$

Gluon Wilson Coefficients

$$f_G \simeq \frac{\alpha_s g_{DM}^2 m_\chi}{192\pi}\frac{(m_\chi^2-2M^2)}{M^2(M^2-m_\chi^2)^2},$$

$$\frac{g_G^{(2)}}{m_\chi^2} \simeq \alpha_s g_{DM}^2\frac{-2M^2m_\chi^2+2\left(M^2-m_\chi^2\right)^2\log\left(\frac{M^2}{M^2-m_\chi^2}\right)+3m_\chi^4}{48\pi m_\chi^5\left(M^2-m_\chi^2\right)^2}$$

$$\frac{g_G^{(1)}}{m_\chi} \simeq \frac{\alpha_s g_{DM}^2}{96\pi m_\chi^4\left(M^2-m_\chi^2\right)^2}\left[-2m_\chi^4\log\left(\frac{m_q^2}{M^2}\right)-m_\chi^2\left(M^2+3m_\chi^2\right)\right.$$
$$\left.+\left(M^2-3m_\chi^2\right)\left(M^2+m_\chi^2\right)\log\left(\frac{M^2}{M^2-m_\chi^2}\right)\right]$$



Color Decomposition and Bound States Effect

For a scalar-antiscalar pair transforming in the fundamental representation degenerate masses m_X

$$\sigma^{[\mathbf{8}]\to[\mathbf{1}]}_{\{100\}} v_{\rm rel} = \frac{2^7\,17^2}{3^5}\frac{\pi\alpha^{\rm BSF}_{s,[\mathbf{1}]}\alpha^B_{s,[\mathbf{1}]}}{m_X^2} S_{\rm BSF}(\zeta_S,\zeta_B)$$

$$S_{\rm BSF}(\zeta_S,\zeta_B) = \left(\frac{2\pi\zeta_S}{1-e^{-2\pi\zeta_S}}\right)(1+\zeta_S^2)\frac{\zeta_B^4 e^{-4\zeta_S \,{\rm arccot}(\zeta_B)}}{(1+\zeta_B^2)^3}$$

S-wave Coulomb Sommerfeld P-wave correction

$$\zeta_S \equiv \alpha_g^S/v_{\rm rel}$$
$$\zeta_B \equiv \alpha_g^B/v_{\rm rel}$$

at large velocities we have $S_{\rm BSF} \sim \zeta_B^4 \sim (\alpha_g^B/v_{\rm rel})^4 \ll 1$ at low velocities $S_{\rm BSF} \sim \alpha_g^S/v_{\rm rel}$

Work for attractive singlet states $\zeta_S \gtrsim 1$ and $\zeta_B \gtrsim 1$ **BSF cross section in enhanced and compete with Sommerfeld effect**

$$\langle \sigma_{\rm BSF} v_{\rm rel}\rangle = \left(\frac{\mu}{2\pi T}\right)^{3/2}\int {\rm d}^3 v_{\rm rel}\exp\left(-\mu v_{\rm rel}^2/2T\right)\left[1+f_g(\omega)\right]\sigma_{\rm BSF} v_{\rm rel}$$

$\omega = \mu/2\left[(\alpha_g^B)^2 + v_{\rm rel}^2\right]$ is the energy emitted by the radiated gluon

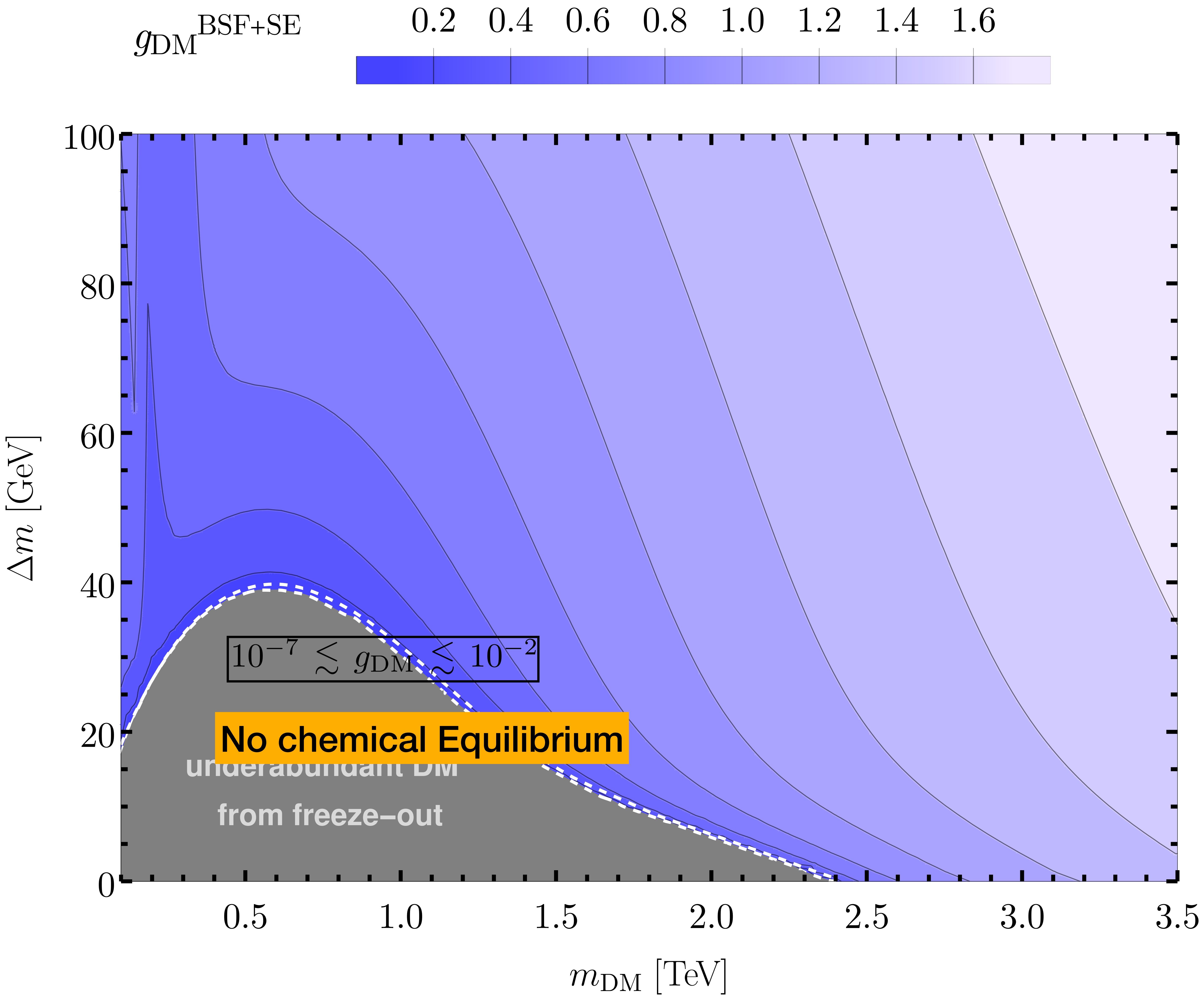
binding energy $\mathcal{E}_{100} = -\mu(\alpha_g^B)^2/2$ $f_g(\omega) = (\exp(\omega/T)-1)^{-1}$ is the gluon distribution function

Bose-enhancement factor $1+f_g(\omega)$ from the final state gluon

$$Q^{\rm BSF} \equiv \omega \simeq \mathcal{E}_{\mathbf{k}} - \mathcal{E}_{n\ell m} = \frac{\mu}{2}\left[v_{\rm rel}^2 + (\alpha^B_{g,[\hat{\mathbf{R}}]})^2\right]$$

ensure the detailed balance between bound-state formation and ionization reactions

Relic Abundance



$$\Delta m = m_X - M_{\rm DM}$$

Demand chemical equilibrium assumption holds

$$\Gamma_X \frac{Y_X^{\rm eq}}{Y_\chi^{\rm eq}} > H$$

$$\tilde{g}_{\rm DM} \gtrsim \sqrt{\frac{m_{\rm DM}}{\rm GeV}\left(1 \cdot 10^{-9} + 6.8 \cdot 10^{-11}\frac{m_{\rm DM}}{\Delta m}\right)}$$

Color Decomposition and Bound States Effect : Ionisation

At Large Temperatures : Ionisation processes dominates over decays -> Effective Contribution of Bound States in dark sector evolution is negligible.
Relic density is independent of contribution of Bound States

As Universe cools down decays dominate, efficiently depleting the dark sector, ionisation rate is exponentially suppressed

effect of BSF on the Boltzmann equation relevant at temperatures close to the bound state binding energy $(T \gtrsim \mathcal{E}_{\mathcal{B}})$

$$\langle \sigma_{XX^\dagger} v_{\rm rel} \rangle_{\rm eff} = \sum_i \left(\langle \sigma_{X_i X_i^\dagger} v_{\rm rel} \rangle + \langle \sigma_{\rm BSF}^{[\mathbf{8}]\to[\mathbf{1}]} v_{\rm rel} \rangle \, \frac{\Gamma_{{\rm dec}[\mathbf{1}]}}{\Gamma_{{\rm dec}[\mathbf{1}]} + \Gamma_{{\rm ion},[\mathbf{1}]}} \right)$$