de-Sitter space as coherent state of gravitons †

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† with Lasha Berezhiani and Gia Dvali, Phys.Rev.D 105 (2022) 2, 025022 [arXiv:2111.12022]

de-Sitter state as a vacua?

A good QFT vacua

- Rigid, no time evolution
- S-matrix compatible

de-Sitter

- Gibbons-Hawking radiation
- No-S matrix formulation

de-Sitter should be state not vacua, due to incomparability with S-matrix (see e.g. Dvali 2012.02133 [hep-th])
Obvious candidate is to build as coherent state(Dvali, Gomez and Zell 1701.08776 [hep-th], the case of massive gravity)

Coherent States in QFT

Complex scalar field $\hat{\Phi}$, with canonical momenta $\hat{\Pi}$.
The coherent state $(t = 0)$has the form

$$|C\rangle = e^{-i\int d^3x \Phi_c \hat{\Pi} - \Pi_c \hat{\Phi} + h.c.} |\Omega\rangle ,$$

giving us $\langle C| \hat{\Phi} |C\rangle = \Phi_c$ and $\langle C| \hat{\Pi} |C\rangle = \Pi_c$.
For the free scalar evolution is trivial, for interacting fields there is quantum part(Berezhiani, Cintia and Zantedeschi 2108.13235 [hep-th], Berezhiani and Zantedeschi 2011.11229 [hep-th]).
What if we gauge it?

QED coherent states

Similar manner, let us build coherent state of free photon

$$|A\rangle = e^{-i\int d^3x \left(A_j^c \hat{E}_j - E_j^c \hat{A}_j\right)} |\Omega\rangle$$

but we know $\partial_j E_j = 0$!
Not all states are physical! We need physicality condition.
We need BRST quantization.

$$\mathcal{L} = -\frac{1}{4}\hat{F}_{\mu\nu}^2 - \partial_\mu \hat{B}\hat{A}_\mu + \frac{1}{2}\xi\hat{B}^2 + \partial_\mu \hat{\bar{c}}\partial_\mu \hat{c}$$

$$\delta\hat{A}_\mu = \theta\partial_\mu\hat{c}\,, \quad \delta\hat{\bar{c}} = \theta\hat{B}\,, \quad \delta\hat{c} = \delta\hat{B} = 0\,,$$

$Q_B\,|physical\rangle = 0$, giving us above condition.

Matter content

Let us add matter content

$$\mathcal{L} = |D_\mu \hat{\Phi}|^2 - m^2 |\hat{\Phi}|^2$$

Requires BRST transformation

$$\delta\Phi = ig\theta\hat{c}\hat{\Phi}\,, \quad \delta\Phi^\dagger = -ig\theta\hat{c}\hat{\Phi}^\dagger$$

Naively we could build

$$|C\rangle \times |A\rangle$$

but $Q_B\,|C\rangle \times |A\rangle \neq 0$.
The proper way is to dress operators via Dirac dressing

$$\hat{\Phi}_g = \hat{\Phi} \cdot \exp\left(-ie\frac{1}{\nabla^2}\partial_j \hat{A}_j\right)$$

Giving us $\langle C|\,\partial_j \hat{E}_j\,|C\rangle = \rho_c$

Constant source (very close to de-Sitter)

$$\mathcal{L} = -\hat{A}_\mu J^c_\mu$$

Coherent state:

$$|J\rangle = e^{-i\int d^3x \left(A^c_j \hat{E}_j - E^c_j \hat{A}_j\right)} |\Omega\rangle$$

BUT

$$\partial_j E^c_j = J_0$$

(for localized sources)
Lets take

$$J_0 = \rho = const$$

BRST compatible!

$$E^c_j = \frac{\rho}{3} x_j$$

But we have particle creation

$$R \sim \frac{m^2}{e\rho}$$

Gravity and coherent states

BRST formulation of linear gravity

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \hat{h}_{\mu\nu})^2 - \frac{1}{2}(\partial_\alpha \hat{h})^2 + \partial_\alpha \hat{h} \partial_\mu \hat{h}^{\mu\alpha} - \partial_\mu \hat{h}^{\mu\alpha} \partial_\nu \hat{h}^\nu_\alpha$$
$$-\partial_\mu \hat{B}_\nu \left(\hat{h}^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\hat{h} \right) + \frac{1}{2}\xi \hat{B}^2_\mu + \partial_\alpha \hat{\bar{C}}_\mu \partial^\alpha \hat{C}^\mu \,,$$

with

$$\delta \hat{h}_{\mu\nu} = \theta \left(\partial_\mu \hat{C}_\nu + \partial_\nu \hat{C}_\mu \right) \,,$$
$$\delta \hat{\bar{C}}_\mu = \theta \hat{B}_\mu \,,$$

Coherent states in Linear gravity

$$|h\rangle = e^{-i \int d^3x \left(h^c_{ij} \hat{\pi}_{ij} - \pi^c_{ij} \hat{h}_{ij} \right)}$$

Physicality condition

$$\left(\nabla^2 \delta_{ij} - \partial_i \partial_j \right) h^c_{ij} = 0 \,, \quad \text{and} \quad \partial_i \pi^c_{ij} = 0 \,,$$

Cosmological constant and matter

We can add cosmological constant

$$\Delta \mathcal{L} = -\lambda \hat{h}\,.$$

Physicality condition

$$\left(\nabla^2 \delta_{ij} - \partial_i \partial_j\right) h^c_{ij} - \lambda = 0\,.$$

we can dress matter

$$|C\rangle = e^{i \int d^3x \Pi^c_\phi(x) \hat{\phi}(x^\mu + \epsilon^\mu)} |\Omega\rangle\,,$$

$$\epsilon_0 = -\frac{1}{4M_{\rm pl}} \frac{1}{\nabla^2} \hat{\pi}_{kk} + \mathcal{O}({M_{\rm pl}}^{-2})$$

$$\epsilon_j = \frac{1}{M_{\rm pl}} \frac{1}{\nabla^2} \partial_i \left(\hat{h}_{ij} - \frac{1}{2} \delta_{ij} \hat{h}_{kk} \right) + \mathcal{O}({M_{\rm pl}}^{-2})$$

giving

$$\langle C | \hat{\Pi}_\phi | C \rangle (t=0) = \Pi^c_\phi + \mathcal{O}({M_{\rm pl}}^{-2})\,,$$

$$\nabla^2 \langle C | \hat{h}_{ij} | C \rangle (t=0) = \frac{\delta_{ij}}{8M_{\rm pl}} {\Pi^c_\phi}^2 + \mathcal{O}({M_{\rm pl}}^{-2})\,.$$

Double scale limit

We can only trust linear analysis. So we can make the theory exact! Lets take $M_{pl} \to \infty$, keep λ fixed and take N copies of matter fixing $\Lambda_{gr} = \frac{M_{pl}}{\sqrt{N}}$.

Results

1. $H^2 \simeq \frac{\lambda}{M_{\rm pl}}$
2. $\Gamma \sim H$
3. $\Gamma \sim H\,N$
4. corpuscular derivation $\Gamma \sim H\alpha_{\rm gr}^2 N_{\rm gr}^2\,N$, where $\alpha \sim \frac{H^2}{M_{pl}^2}$ and $N_{gr} = \frac{M_{pl}^2}{H^2}$
5. $\frac{\Gamma}{V} \sim H^4 N = \left(\frac{\lambda}{\Lambda_{\rm gr}}\right)^2$
6. $t_Q \sim \frac{{M_{\rm pl}}^2}{H^3 N} \sim \frac{\Lambda_{\rm gr}^2}{H^3}$

Conclusions

- Semi-classical objects are quantum, described very well by coherent states
- States for classical charge/cosmological constant can and should build on free/Minkowski vacua
- very strict physicality/consistency condition (BRST) is passed
- Quantum description is $\frac{1}{N}$ effect and departure from classicality