Dark energy, black holes and effective field theory

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Ref. arXiv: 2204.00228 w/ V.Yingcharoenrat
arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat
arXiv: 2301.xxxxx w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat

Also Arkani-Hamed, Cheng, Luty and Mukohyama 2004 (hep-th/0312099)
Mukohyama 2005 (hep-th/0502189)

Collaborators

V.Yingcharoenrat

K.Takahashi

K.Tomikawa

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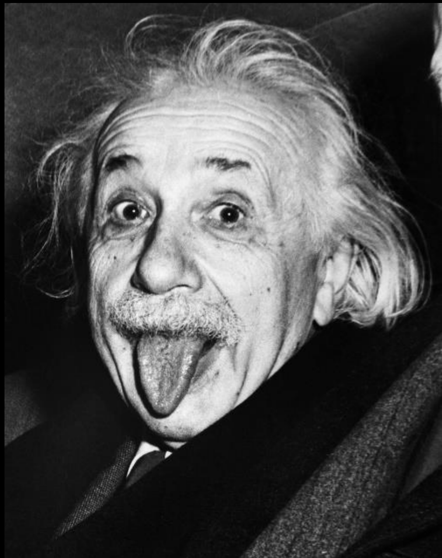
INTRODUCTION

Why gravity beyond GR?

(GR : general relativity)

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field and **tensions**
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we **understand general relativity**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.



Some examples (my personal experiences)

I. Effective field theory (EFT) approach
IR modification of gravity
motivation: dark energy/inflation, universality

II. Massive gravity
IR modification of gravity
motivation: "Can graviton have mass?" & dark energy

III. Minimally modified gravity
IR modification of gravity
motivation: tensions in cosmology, various constraints

IV. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity

V. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

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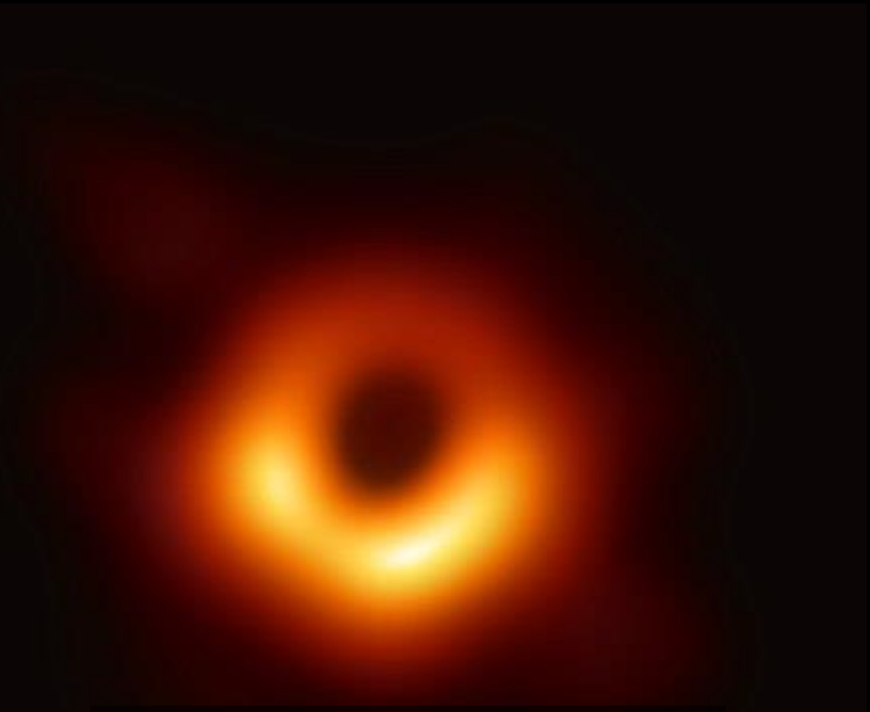
Many gravity theories

- 3 check points
  “What are the physical d.o.f. ?”
  “How do they interact ?”
  “What is the regime of validity ?”
- If two (or more) theories give the same answers to the 3 questions above then they are the same even if they look different.
  → **Effective Field Theory (EFT)**
  as universal description

Proto-type of modified gravity: scalar-tensor theory

- Metric $g_{\mu\nu}$ + scalar field ϕ
- Jordan (1955), Brans & Dicke (1961), Bergmann (1968), Wagoner (1970), …
- Most general scalar-tensor theory of gravity with 2nd order covariant EOM: Horndeski (1974)
- DHOST theories beyond Horndeski: Langlois & Noui (2016)
- U-DHOST theories beyond DHOST: DeFelice, Langlois, Mukohyama, Noui & Wang (2018)
- All of them (and more) are universally described by an effective field theory (EFT)

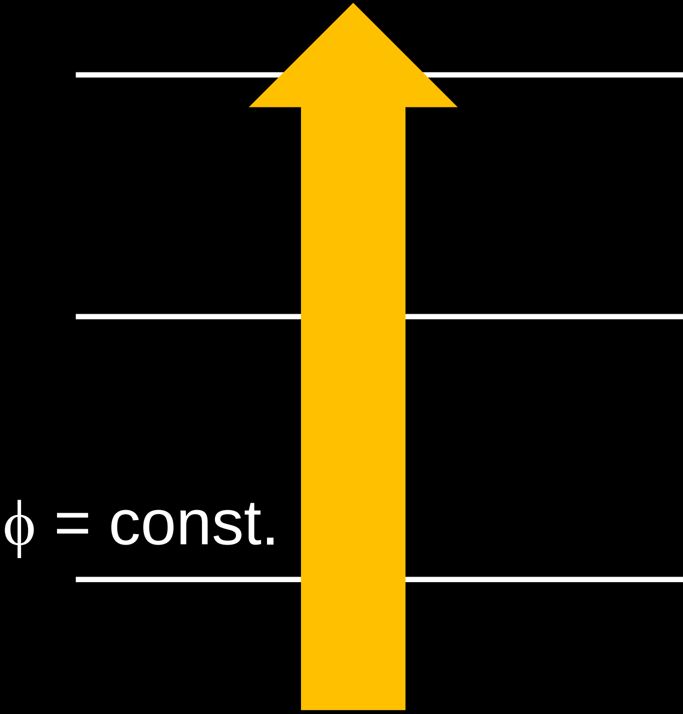
- Cosmology and black holes (BHs) play as important roles in gravitational physics as blackbody radiation and hydrogen atoms did in quantum mechanics.
- In cosmology a time-dependent scalar field can act as dark energy (DE), while BHs serve as probes of strong gravity. We then hope to probe the scalar field DE by astrophysical BHs.



Black hole

https://www.eso.org/public/images/eso1907a/

Timelike gradient



ϕ = const.

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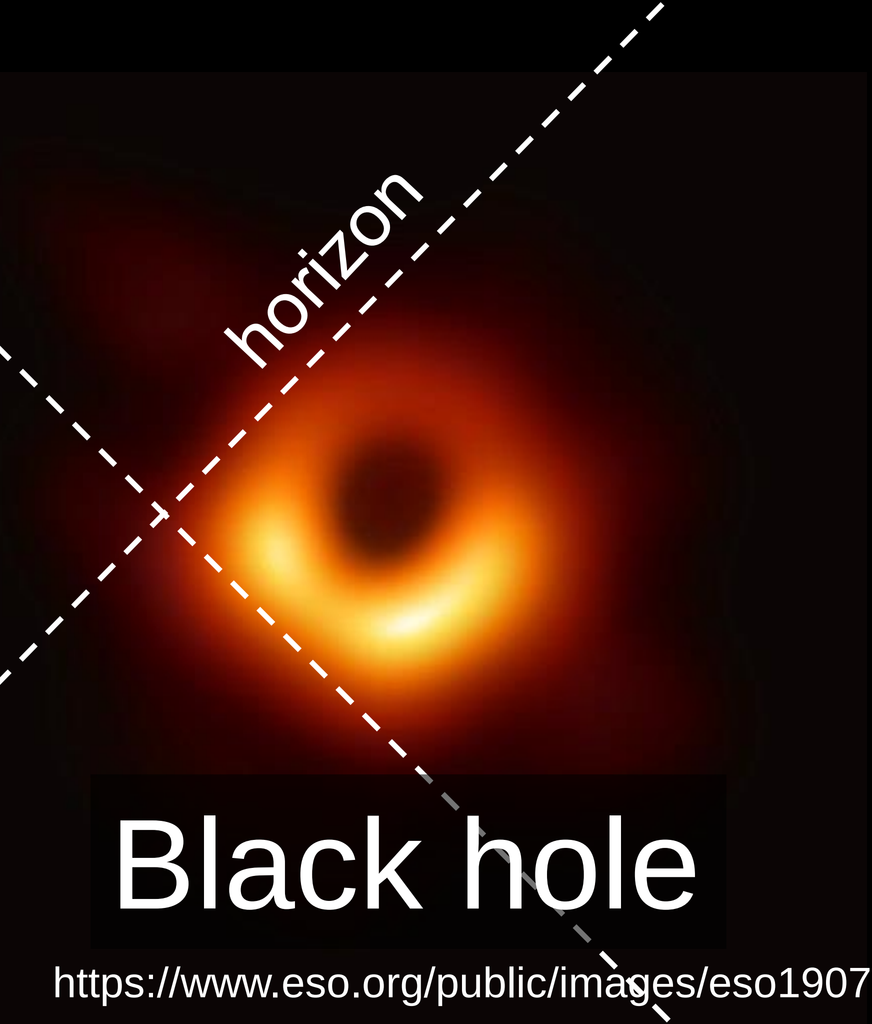

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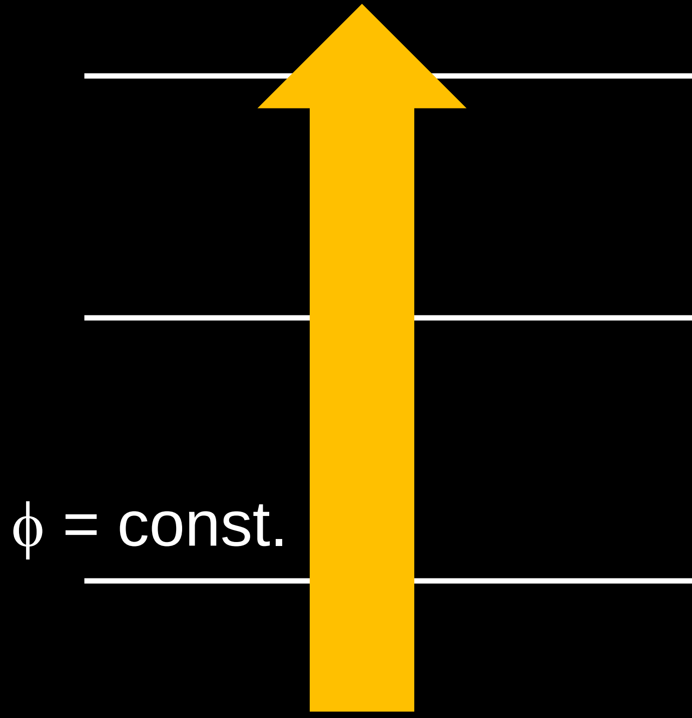
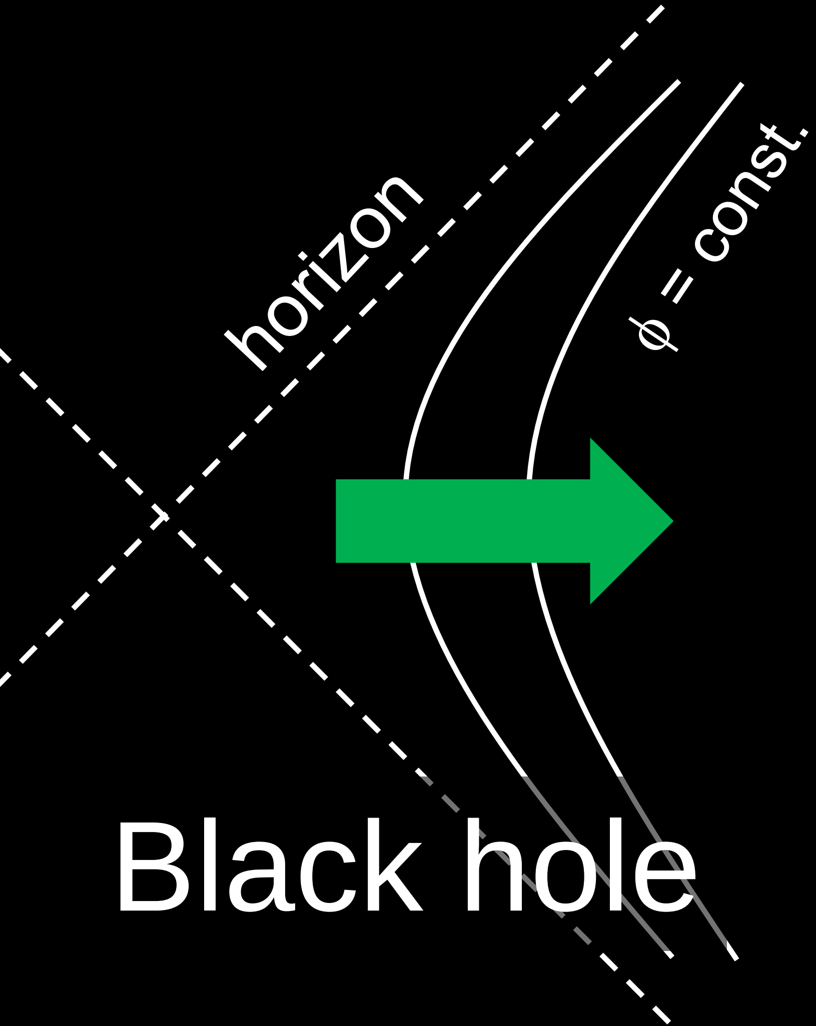


Dark energy

Unlucky case

Spacelike gradient



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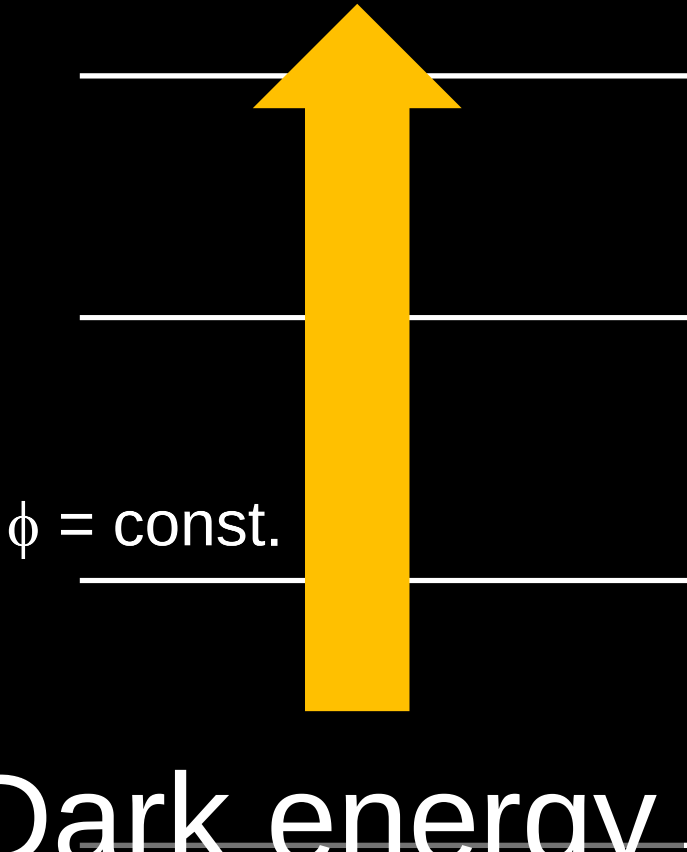
Timelike gradient



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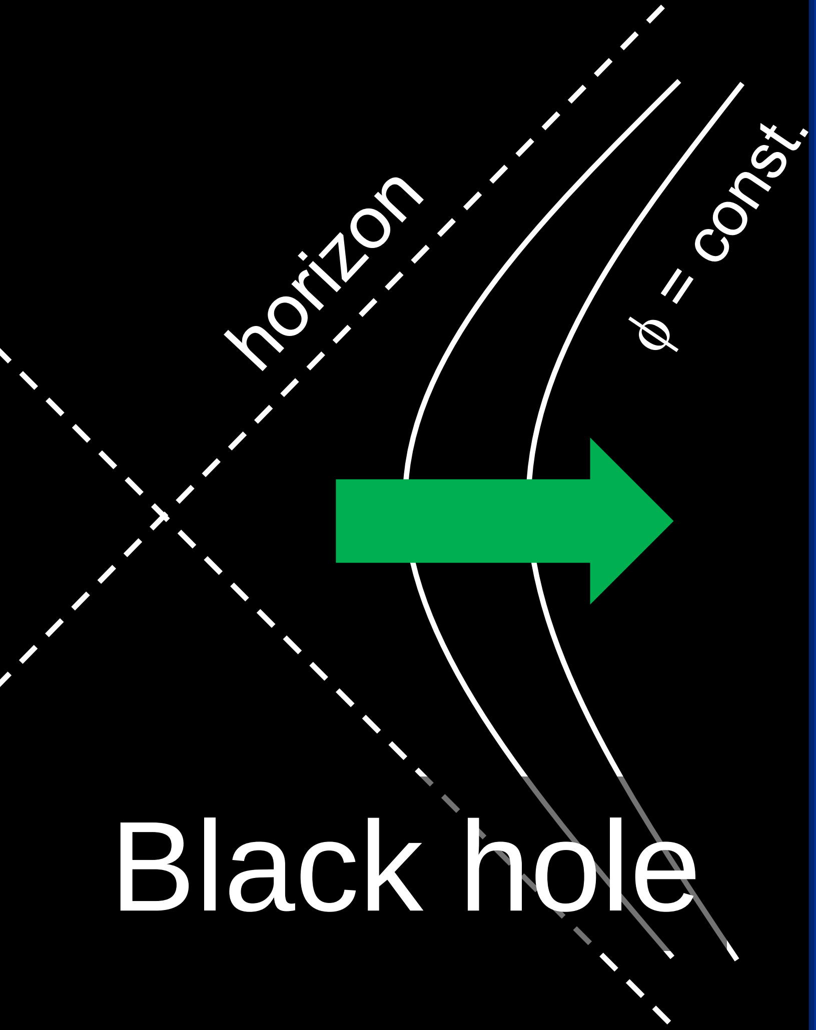
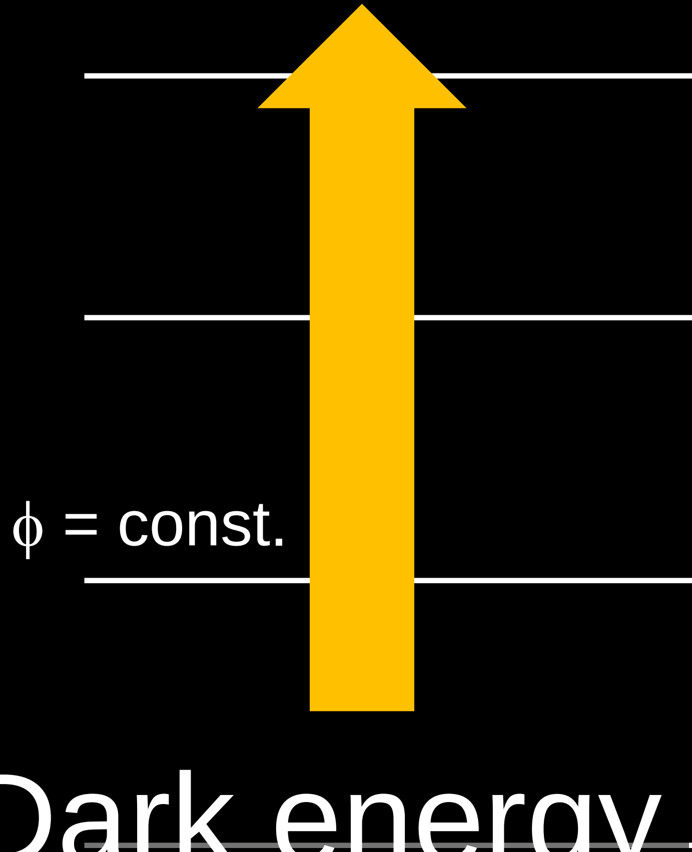


Black hole

No smooth matching

Timelike gradient



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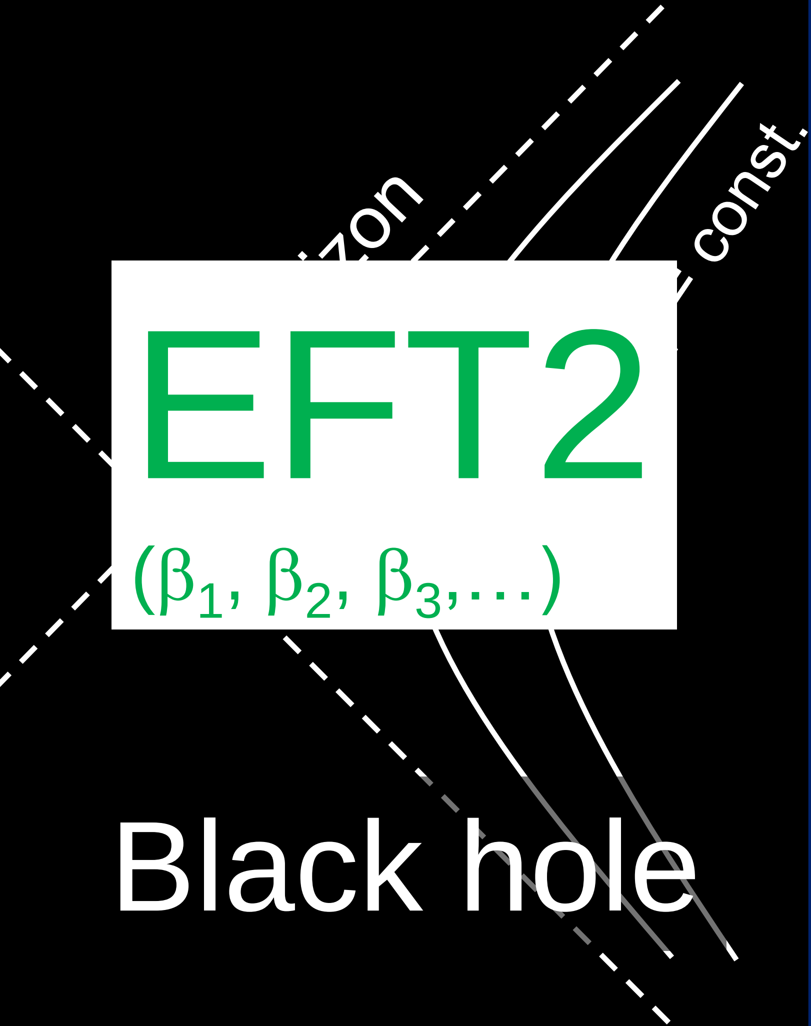


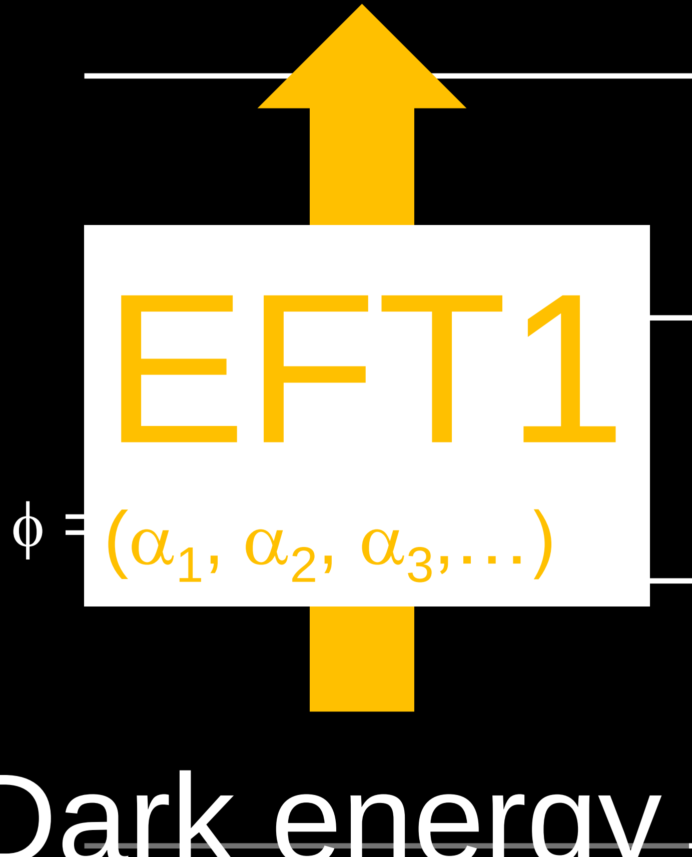
EFT2

$(\beta_1, \beta_2, \beta_3, \ldots)$

Black hole

No direct relation between EFT1 & EFT2

Timelike gradient



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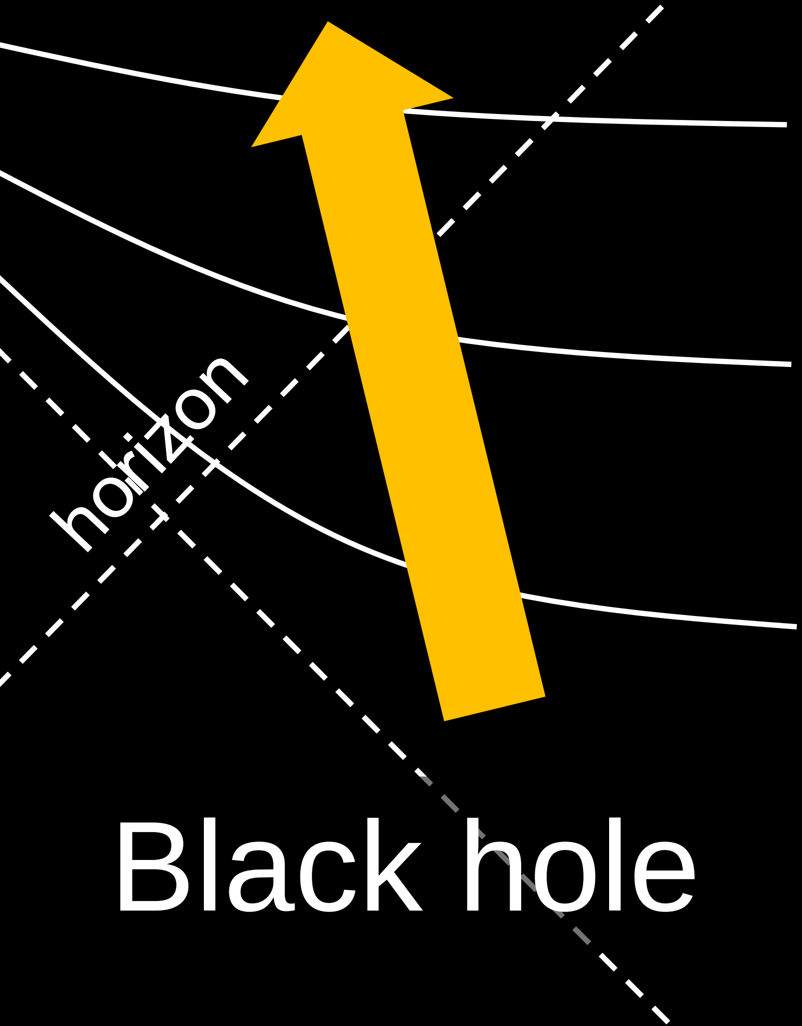
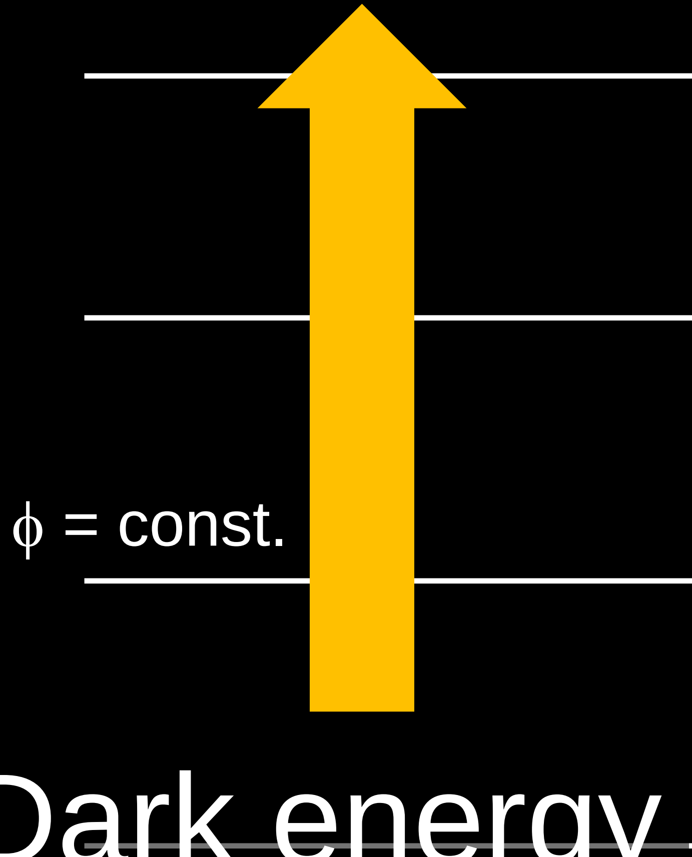
Lucky case

Timelike gradient



Black hole

Timelike gradient



Dark energy

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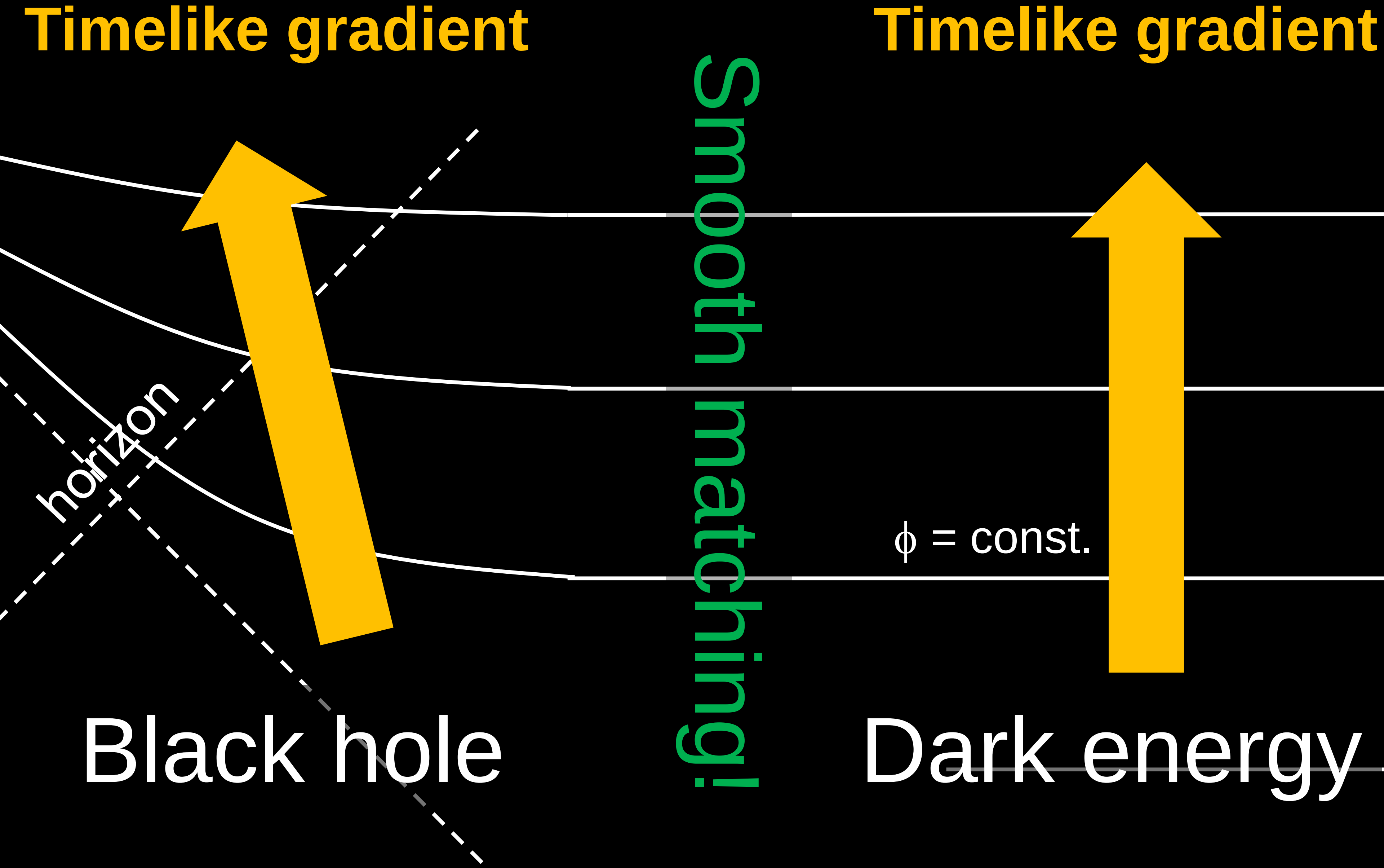


Timelike gradient

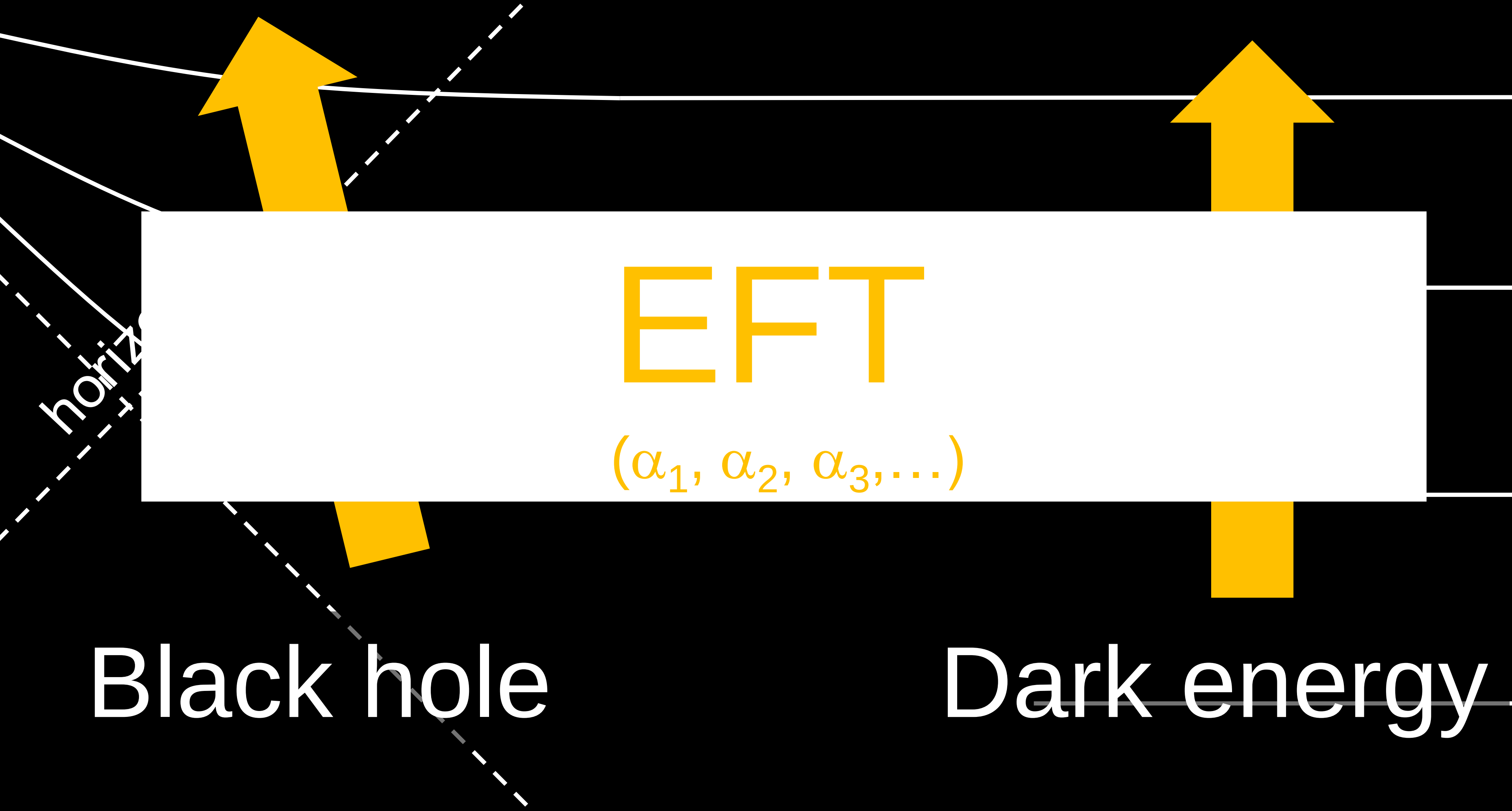
Timelike gradient

Smooth matching!

Black hole

Dark energy

Lucky case



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EFT

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EFT of scalar-tensor gravity with timelike scalar profile

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- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

EFT of scalar-tensor gravity
on Minkowski background

= ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

EFT of ghost condensation =

EFT of scalar-tensor gravity with timelike scalar profile on Minkowski background

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

Backgrounds characterized by

✧ $\left\langle \partial_\mu \phi \right\rangle \neq 0$ and timelike

✧Background metric is Minkowski.

⇒

$$\boxed{\begin{aligned} L_{eff} = L_{EH} + M^4 \Big\{ & \left(h_{00} - 2\dot{\pi}\right)^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 \\ & - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \Big\} \end{aligned}}$$

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EFT of scalar-tensor gravity
on cosmological background

= EFT of inflation/dark energy

Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

Application: non-Gaussinity of inflationary perturbation $\zeta = -H\pi$

$$I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \left\{ -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) - \dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\boxed{\dot{\pi}^3} - \boxed{\dot{\pi}\frac{(\partial_i\pi)^2}{a^2}}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta}K, \tilde{\delta}R} \right\}$$

2 types of 3-point interactions

→ power spectrum

$$P_\zeta(\vec{k}) = \frac{\Delta}{k^3}, \quad \Delta = \left.\frac{H^4}{-4M_{Pl}^2 \dot{H} c_s}\right|_{c_s k \simeq aH}$$

→ non-Gaussianity

$$\langle \zeta_{\vec{k}_1}(t)\, \zeta_{\vec{k}_2}(t)\, \zeta_{\vec{k}_3}(t) \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta$$

c_s^2 → size of non-Gaussianity

$$k^6 B_\zeta|_{k_1=k_2=k_3=k} = \frac{18}{5}\Delta^2 \left(f_{NL}^{\dot{\pi}(\partial_i\pi)^2} + f_{NL}^{\dot{\pi}^3}\right)$$

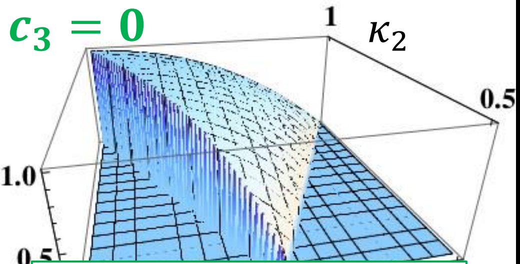
$$f_{NL}^{\dot{\pi}(\partial_i\pi)^2} = \frac{85}{324}\left(1 - \boxed{\frac{1}{c_s^2}}\right) \qquad f_{NL}^{\dot{\pi}^3} = \frac{5\boxed{c_3}}{81}\left(1 - \boxed{\frac{1}{c_s^2}}\right) \qquad \propto \frac{1}{c_s^2}$$

for small c_s^2

c_3 → shape of non-Gaussianity

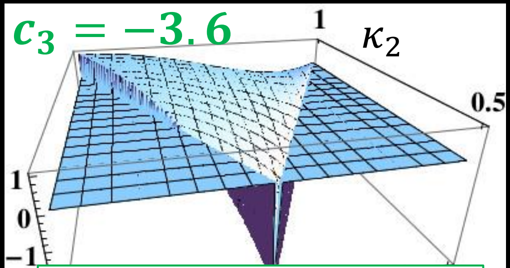
plots of $B_\zeta(k, \kappa_2 k, \kappa_3 k)/B_\zeta(k, k, k)$

$c_3 = 0$



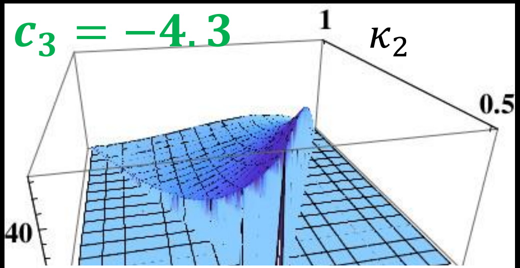
Prototype of the equilateral shape

$c_3 = -3.6$



Prototype of the orthogonal shape

$c_3 = -4.3$



Linear combination of the two shapes

Parametrization suitable for DE → EFT of DE

Gubitosi, Piazza, Vernizzi 2012
Gleyzes, Langlois, Piazza, Vernizzi 2013

- Matter (in addition to DE) needs to be added → Jordan frame description is convenient
- In Jordan frame the coefficient of the 4d Ricci scalar is not constant.

$$
\begin{aligned}
S = \frac{1}{2}\int d^4x\sqrt{-g}\,\Big[\boxed{M_*^2 f R} &- \rho_D + p_D - M_*^2(5H\dot{f} + \ddot{f}) - \left(\rho_D + p_D + M_*^2(H\dot{f} - \ddot{f})\right) g^{00} \\
&+ M_2^4(\delta g^{00})^2 - \bar{m}_1^3\,\delta g^{00}\delta K - \bar{M}_2^2\,\delta K^2 - \bar{M}_3^2\,\delta K_\mu^{\ \nu}\delta K^\mu_{\ \nu} + m_2^2 h^{\mu\nu}\partial_\mu g^{00}\partial_\nu g^{00} \\
&+ \lambda_1\delta R^2 + \lambda_2\delta R_{\mu\nu}\delta R^{\mu\nu} + \mu_1^2\delta g^{00}\delta R + \gamma_1 C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + \gamma_2\epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}^{\ \ \ \kappa\lambda}C_{\rho\sigma\kappa\lambda} \\
&+ \frac{M_3^4}{3}(\delta g^{00})^3 - \bar{m}_2^3\,(\delta g^{00})^2\delta K + \ldots\Big] ,
\end{aligned}
$$

EFT of scalar-tensor gravity with timelike scalar profile

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EFT of scalar-tensor gravity on Minkowski background = ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

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EFT of scalar-tensor gravity on arbitrary background = EFT of BH perturbations

arXiv: 2204.00228 w/ Vicharit Yingcharoenrat

Taylor expansion of the general action

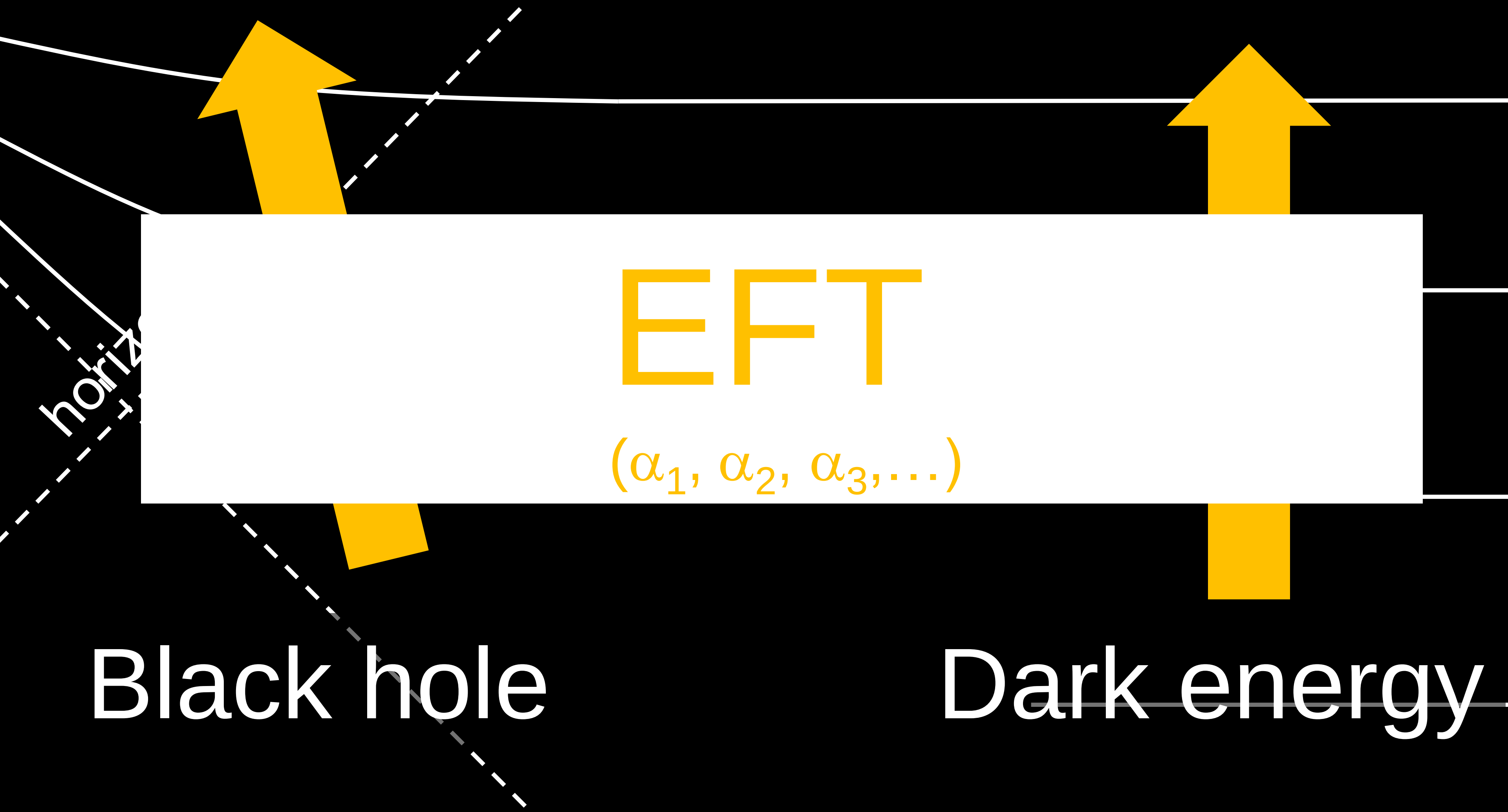
$$S = \int d^4x\sqrt{-g}\ F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$$

$$S = \int d^4x\sqrt{-g}\left[\bar{F} + \bar{F}_{g^{\tau\tau}}\delta g^{\tau\tau} + \bar{F}_K\delta K + \quad \dots\right]$$

Consistency relations ← S is invariant under spatial diffeo but the background breaks it.

$$\frac{d}{dx^i}\bar{F} = \bar{F}_{g^{\tau\tau}}\frac{\partial\bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K\frac{\partial\bar{K}}{\partial x^i} + \dots$$

Lucky case



Timelike gradient

Timelike gradient

EFT

$(\alpha_1, \alpha_2, \alpha_3, \ldots)$

Black hole

Dark energy

Stealth solutions in k-essence

Mukohyama 2005

- Action in Einstein frame

$$I = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + P(X)\right] \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- EOMs

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}P'(X)g^{\mu\nu}\partial_\nu\phi\right) = 0$$

$$M_{\rm Pl}^2 G_{\mu\nu} = 2P'(X)\partial_\mu\phi\partial_\nu\phi + P(X)g_{\mu\nu}$$

- Stealth sol with $X = X_0$, where $P'(X_0)=0$

$$G_{\mu\nu} = \Lambda_{\rm eff}\, g_{\mu\nu} \qquad \Lambda_{\rm eff} = P(X_0)/M_{\rm Pl}^2$$

- $X = X_0$ ($\neq 0$)

⟺ $u^\mu = g^{\mu\nu}\partial_\nu\phi$ defines geodesic congruence

$$(u^\nu\nabla_\nu u^\mu = -\nabla^\mu X/2 = 0)$$

⟺ $\phi/\sqrt{|X_0|}$ **defines Gaussian normal coord.**

Stealth solutions with $\phi = qt + \psi(r)$

- Schwarzschild in k-essence (Mukohyama 2005)
- Schwarzschild-dS in Horndeski theory (Babichev & Charmousis 2013, Kobayashi & Tanahashi 2014) Schwarzshild-dS in DHOST (Ben Achour & Liu 2019, Motohashi & Minamitsuji 2019)
- Kerr-dS in DHOST (Charmousis & Crisotomi & Gregory & Stergioulas 2019)
- However, perturbations around most of those stealth solutions are infinitely strongly coupled (de Rham & Zhang 2019) . This means the solutions cannot be trusted.
- Fortunately, Scordatura (= detuning of degeneracy condition) solves the strong coupling problem (Motohashi & Mukohyama 2019).
- EFT of ghost condensation already includes scordatura (Arkani-Hamed & Cheng & Luty & Mukohyama 2004)
- Approximate Schwarzschild in ghost condensation (Mukohyama 2005). Also in U-DHOST (DeFelice & Mukohyama & Takahashi, to appear)

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Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
  [arXiv: 2204.00228 w/ V.Yingcharoenrat]
- Odd-parity perturbation around spherical BH
  → Generalized Regge-Wheeler equation
  [arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat]
  [see also arXiv: 2208.02823 by Khoury, Noumi, Trodden, Wong]
  → Quasi-normal mode
  [work in progress w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]
- Even-parity perturbation around spherical BH
  [work in progress w/ K.Takahashi & V.Yingcharoenrat]
- Rotating BH
- Dynamical BH
- …

SUMMARY

- Ghost condensation universally describes all scalar-tensor theories of gravity with timelike scalar profile on Minkowski background.

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- Extension of ghost condensation to FLRW backgrounds results in the EFT of inflation/DE.
- These EFTs provide universal descriptions of all scalar-tensor theories of gravity with timelike scalar profile on each background, including Horndeski theory, DHOST theory, U-DHOST theory, and more.

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- Other applications? Further extensions?

Further extension of the web of EFTs

"The Effective Field Theory of Vector-Tensor Theories"
Katsuki Aoki, Mohammad Ali Gorji, Shinji Mukohyama, Kazufumi Takahashi, , JCAP 01 (2022) 01, 059 [arXiv: 2111.08119].

Residual symmetry in the unitary gauge

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

$$t \to t - g_M \chi(x) \, , \quad A_\mu \to A_\mu + \partial_\mu \chi(x)$$

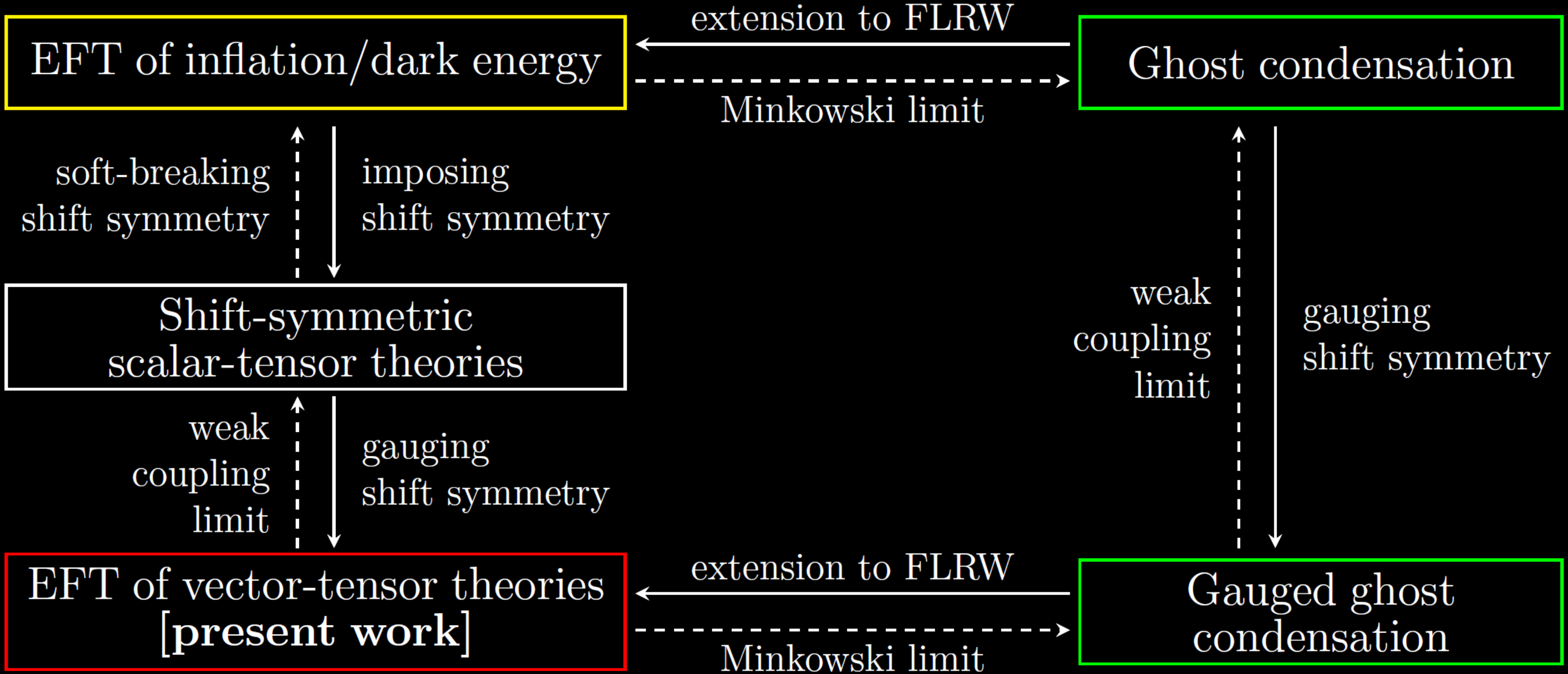
leaving $\tilde{\delta}^0{}_\mu = \delta^0{}_\mu + g_M A_\mu$ invariant

c.f. Residual symmetry in unitary gauge
for scalar-tensor theories

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

The web of EFTs

Arkani-Hamed, Cheng, Luty and Mukohyama 2004



EFT of inflation/dark energy

extension to FLRW

Minkowski limit

Ghost condensation

soft-breaking shift symmetry

imposing shift symmetry

Shift-symmetric scalar-tensor theories

weak coupling limit

gauging shift symmetry

weak coupling limit

gauging shift symmetry

EFT of vector-tensor theories **[present work]**

extension to FLRW

Minkowski limit

Gauged ghost condensation

Cheng, Luty, Mukohyama and Thaler 2004

Thank you!


K.Aoki


M.A.Gorji


K.Takahashi


V.Yingcharoenrat


K.Tomikawa

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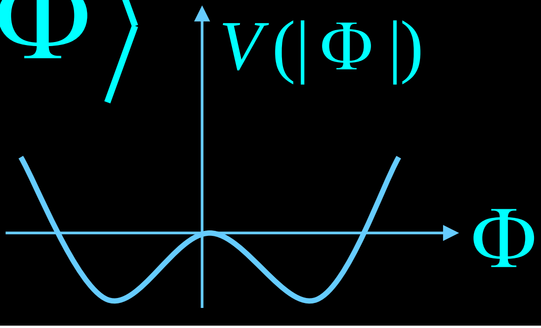
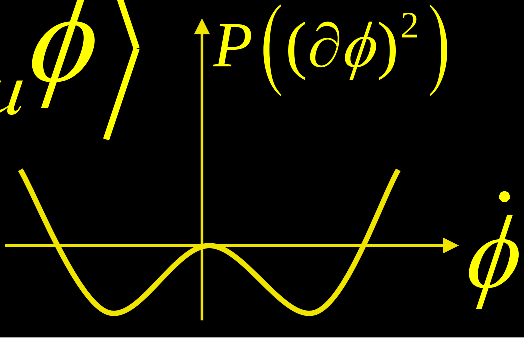
arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat
arXiv: 2301.xxxxx w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat
arXiv: 2111.08119 w/ K.Aoki, M.A.Gorji & K.Takahashi

Also Arkani-Hamed, Cheng, Luty and Mukohyama 2004 (hep-th/0312099)
Mukohyama 2005 (hep-th/0502189)

Backup slides

	Higgs mechanism	**Ghost condensate** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

EFT of ghost condensation =

EFT of scalar-tensor gravity with timelike scalar profile on Minkowski background

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

Backgrounds characterized by

✧ $\langle \partial_\mu \phi \rangle \neq 0$ and timelike

✧ Background metric is Minkowski.

➡

$$L_{eff} = L_{EH} + M^4 \left\{ \left(h_{00} - 2\dot{\pi} \right)^2 - \frac{\alpha_1}{M^2} \left(K + \vec{\nabla}^2 \pi \right)^2 - \frac{\alpha_2}{M^2} \left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi \right) \left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi \right) + \cdots \right\}$$

Gauge choice: $\phi(t,\vec{x}) = t.$ $\pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \rightarrow \vec{x}'(t,\vec{x})$

⇨ Write down most general action invariant under this residual symmetry.

(⇨ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

$$\begin{cases} \left(h_{00}\right)^2 & \textbf{OK} \\ \cancel{\left(h_{0i}\right)^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{\left(h_{00}\right)^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K^{ij}K_{ij} + \cdots\right\}$$

Action invariant under ξ^i

$$\begin{cases} (h_{00})^2 & \textbf{OK} \\ \cancel{(h_{0i})^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

⇒ $$L_{eff} = L_{EH} + M^4\left\{(h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K^{ij}K_{ij} + \cdots\right\}$$

Action for π

$$\xi^0 = \pi \qquad \begin{cases} h_{00} \to h_{00} - 2\partial_0\pi \\ K_{ij} \to K_{ij} + \partial_i\partial_j\pi \end{cases}$$

⇒ $$\boxed{\begin{aligned} L_{eff} = L_{EH} + M^4\Big\{&(h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2\pi\right)^2 \\ &- \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i\vec{\nabla}^j\pi\right)\left(K_{ij} + \vec{\nabla}_i\vec{\nabla}_j\pi\right) + \cdots\Big\} \end{aligned}}$$

$$\left.\begin{array}{l} E \to rE \\ dt \to r^{-1} dt \\ dx \to r^{-1/2} dx \\ \pi \to r^{1/4} \pi \end{array}\right\}$$

Make invariant ➡ $\int dt d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2 \pi)^2}{M^2} + \cdots \right]$

Leading nonlinear operator in infrared $\int dt d^3x \frac{\dot{\pi}(\nabla \pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

⇨ Good low-E effective theory

Robust prediction

e.g. Ghost inflation [Arkani-hamed, Creminelli, Mukohyama, Zaldarriaga 2004]

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⇒

$$\boxed{\begin{aligned} L_{eff} = L_{EH} + M^4 \Big\{ & \left(h_{00} - 2\dot{\pi}\right)^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 \\ & - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \Big\} \end{aligned}}$$

Extension to FLRW background = EFT of inflation/dark energy

Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

- Action invariant under x^i → $x^i(t,x)$
- Ingredients

  $g_{\mu\nu}, g^{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu,$ $\boxed{\text{t \& its derivatives}}$
- 1st derivative of t

$$\partial_\mu t = \delta^0_\mu \qquad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}} = \frac{\delta^0_\mu}{\sqrt{-g^{00}}}$$

$$g^{00} \qquad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

- 2nd derivative of t

$$K_{\mu\nu} \equiv h^\rho_\mu \nabla_\rho n_\nu$$

Unitary gauge action

$$\boxed{I = \int d^4x\sqrt{-g}L(t, \delta^0_\mu, K_{\mu\nu}, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma})}$$

derivative & perturbative expansions

$$\begin{aligned} I &= M_{Pl}^2 \int dx^4\sqrt{-g}\left[\frac{1}{2}R + c_1(t) + c_2(t)g^{00} \right. \\ &\quad \left. + L^{(2)}(\tilde{\delta}g^{00}, \tilde{\delta}K_{\mu\nu}, \tilde{\delta}R_{\mu\nu\rho\sigma}; t, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu)\right] \\ L^{(2)} &= \lambda_1(t)(\tilde{\delta}g^{00})^2 + \lambda_2(t)(\tilde{\delta}g^{00})^3 + \lambda_3(t)\tilde{\delta}g^{00}\tilde{\delta}K^\mu_\mu \\ &\quad + \lambda_4(t)(\tilde{\delta}K^\mu_\mu)^2 + \lambda_5(t)\tilde{\delta}K^\mu_\nu\tilde{\delta}K^\nu_\mu + \cdots \end{aligned}$$

$$\tilde{\delta}g^{00} \equiv g^{00} + 1 \qquad \tilde{\delta}K_{\mu\nu} \equiv K_{\mu\nu} - H\gamma_{\mu\nu}$$

$$\tilde{\delta}\mathrm{R}_{\mu\nu\rho\sigma} \equiv \mathrm{R}_{\mu\nu\rho\sigma} - 2(H^2 + \mathfrak{K}/a^2)\gamma_{\mu[\rho}\gamma_{\sigma]\nu} + (\dot{H} + H^2)(\gamma_{\mu\rho}\delta^0_\nu\delta^0_\sigma + (3\mathrm{perm.}))$$

NG boson

- Undo unitary gauge $t \to \tilde{t} = t - \pi(\tilde{t}, \vec{x})$

$$\begin{aligned} H(t) &\to H(t+\pi), \quad \dot{H}(t) \to \dot{H}(t+\pi), \\ \lambda_i(t) &\to \lambda_i(t+\pi), \quad a(t) \to a(t+\pi), \\ \delta^0_\mu &\to (1+\dot{\pi})\delta^0_\mu + \delta^i_\mu \partial_i \pi, \end{aligned}$$

- NG boson in decoupling (subhorizon) limit

$$\boxed{\begin{aligned} I_\pi &= M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \left\{ -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \right. \\ &\left. -\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta}K, \tilde{\delta}R} \right\} \end{aligned}}$$

$$\frac{1}{c_s^2} = 1 - \frac{4\lambda_1}{\dot{H}}, \quad c_3 = c_s^2 - \frac{8c_s^2\lambda_2}{-\dot{H}}\left(\frac{1}{c_s^2} - 1\right)^{-1}$$

- Sound speed

c_s : speed of propagation for modes with $\omega \gg H$

$\omega^2 \simeq c_s^2 \frac{k^2}{a^2}$ for $\pi \sim A(t) \exp\left(-i\int \omega dt + i\vec{k}\cdot\vec{x}\right)$