Neutron Star Heating: WIMP DM vs Others

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K. Hamaguchi, N. Nagata, K. Yanagi, Phys. Lett. **B795**, 484 (2019).

WIMP dark matter heating in NS

It has been discussed that the signature of WIMP DM may be detected via the neutron star (NS) temperature observations.

PHYSICAL REVIEW D **77,** 023006 (2008)

WIMP annihilation and cooling of neutron stars

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(Received 27 August 2007; published 28 January 2008)

PHYSICAL REVIEW D **81,** 123521 (2010)

Neutron stars as dark matter probes

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(Received 6 April 2010; published 18 June 2010)

PHYSICAL REVIEW D **82,** 063531 (2010)

Can neutron stars constrain dark matter?

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(Received 29 May 2010; published 28 September 2010)

PRL **119,** 131801 (2017) PHYSICAL REVIEW LETTERS week ending 29 SEPTEMBER 2017

Dark Kinetic Heating of Neutron Stars and an Infrared Window on WIMPs, SIMPs, and Pure Higgsinos

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(Received 10 April 2017; revised manuscript received 20 July 2017; published 26 September 2017)

Mechanism

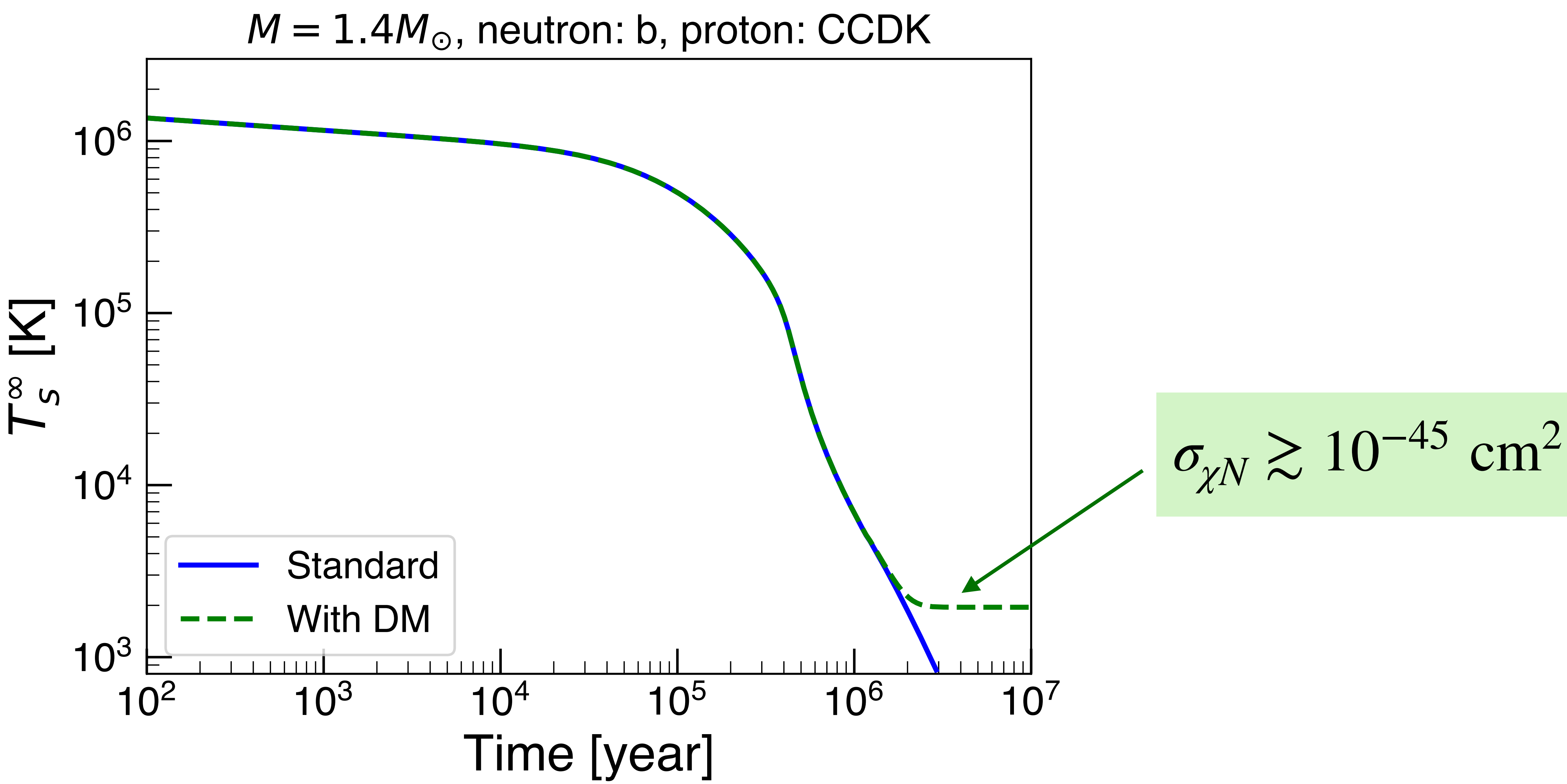
WIMP DM accretes on a neutron star.

→

Annihilation of WIMPs in the NS core causes heating effect.

WIMP dark matter heating in NS

Dark matter heating effect may be observed in old NSs.



- In the standard cooling scenario, temperature becomes very low for $t > 10^7$ years.
- With DM heating effect, $T_s^\infty \to \sim 2 \times 10^3$ K at later times.

Limits on DM models

If we find a NS with a temperature $\lesssim 10^3$ K, we can give a strong constraint on WIMP DM.

For some models, this can be stronger than direct detection limits.

DM speed is very large in NSs

- Electroweak Multiplet Dark Matter
  - Wino, Higgsino, Minimal Dark Matter, etc.
  - Inelastic scattering occurs in NSs.

M. Fujiwara, K. Hamaguchi, N. Nagata, J. Zheng, Phys. Rev. **D106**, 055031 (2022).

- Dark matter interacts only with leptons.

  → See Maura E. Ramirez-Quezada's talk on Thursday.

- WIMP-nucleon scattering is velocity-suppressed.

Other heating sources?

If there are other heating sources in NSs, DM heating effect may be concealed.

- There is no heating source in Standard NS cooling theory.

- Is it possible to have extra heating sources?

Or, even motivated?

Old warm neutron stars?

Recently, "old but warm neutron stars" have been observed.

Milli-second pulsars

- J0437-4715: $t_{\rm sd} = (6.7 \pm 0.2) \times 10^9$ years, $T_s^\infty = (1.25 - 3.5) \times 10^5$ K

O. Kargaltsev, G. G. Pavlov, and R. W. Romani, Astrophys. J. **602**, 327 (2004);
M. Durant, *et al.*, Astrophys. J. **746**, 6 (2012).

- J2124-3358: $t_{\rm sd} = 11^{+6}_{-3} \times 10^9$ years, $T_s^\infty = (0.5 - 2.1) \times 10^5$ K

B. Rangelov, *et al.*, Astrophys. J. **835**, 264 (2017).

Ordinary pulsars

- J0108-1431: $t_{\rm sd} = 2.0 \times 10^8$ years, $T_s^\infty = (1.1 - 5.3) \times 10^5$ K

R. P. Mignani, G. G. Pavlov, and O. Kargaltsev, Astron. Astrophys. **488**, 1027 (2008).

- B0950+08: $t_{\rm sd} = 1.75 \times 10^7$ years, $T_s^\infty = (1 - 3) \times 10^5$ K

G. G. Pavlov, *et al.*, Astrophys. J. **850**, 79 (2017).

These observations cannot be explained in the standard cooling.

Topics of this talk

- We need an extra heating source to explain those observations.

Proposed mechanisms

- Non-equilibrium beta processes

Today's topic

- Friction caused by vortex creep

M. Fujiwara, K. Hamaguchi, N. Nagata, M. E. Ramirez-Quezada, in preparation.

- Can we still observe the DM heating effect in the presence of this extra heating effect??

Outline of this talk

- Introduction
- Standard Cooling Theory
- Non-equilibrium β processes
- Results
- Conclusion

Standard Cooling Theory

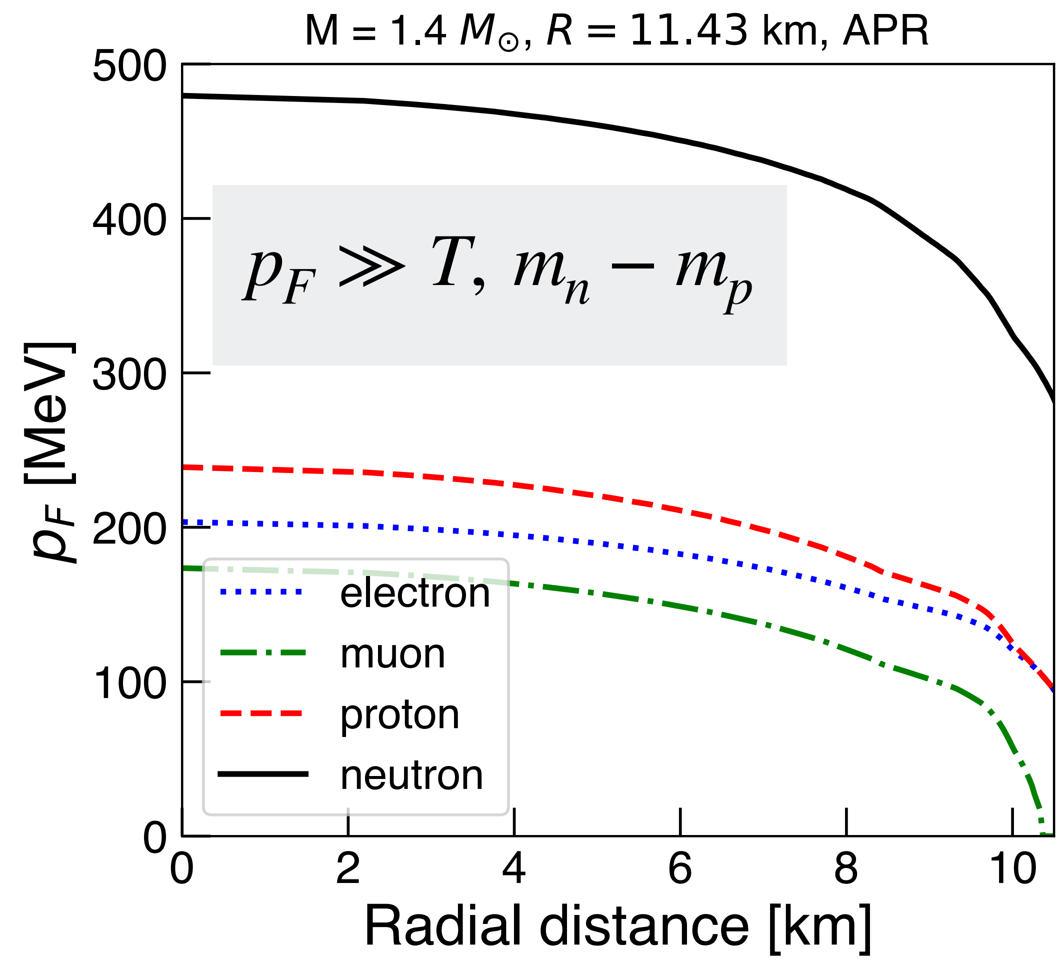
Standard Cooling of NS

D. Page, J. M. Lattimer, M. Prakash, A. W. Steiner, Astrophys. J. Suppl. **155**, 623 (2004);
M. E. Gusakov, A. D. Kaminker, D. G. Yakovlev, O. Y. Gnedin, Astron. Astrophys. **423**, 1063 (2004).

Consider a NS composed of

- Neutrons
- Protons
- Leptons (e, μ)

Form Cooper pairs



- Supposed to be in the β equilibrium.
- In Fermi degenerate states.

Equation for temperature evolution

$$C(T)\frac{dT}{dt} = -L_\nu - L_\gamma$$

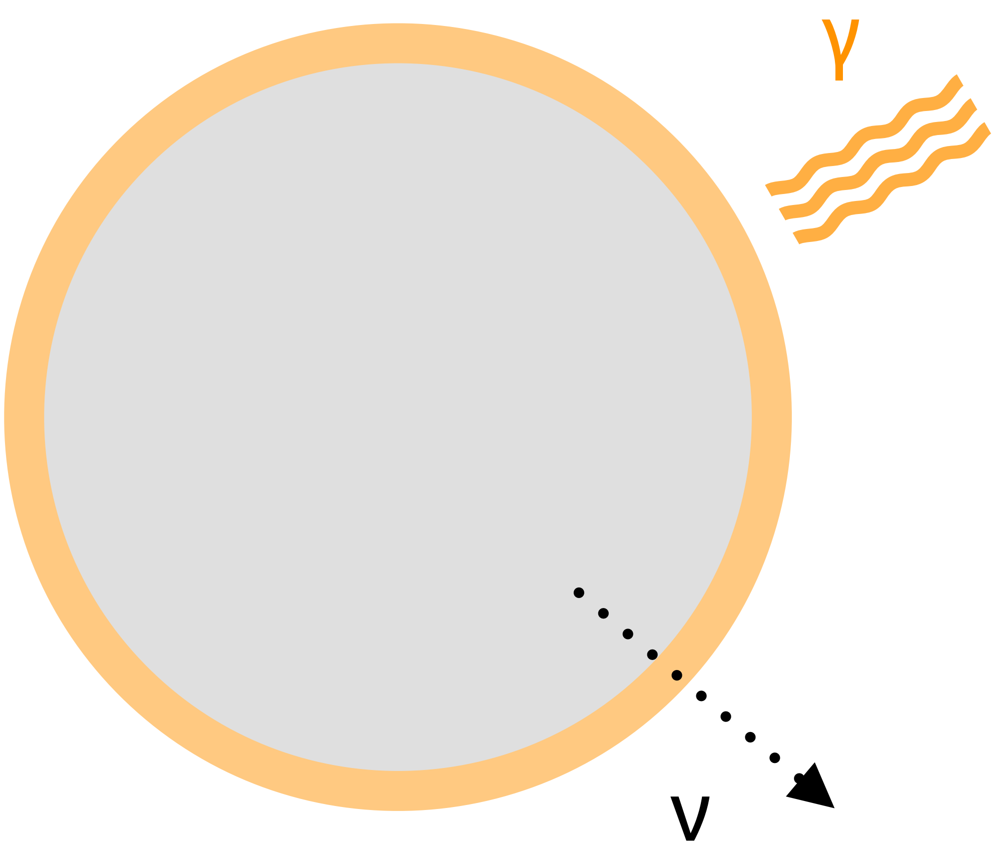
C(T): Stellar heat capacity

L_ν: Luminosity of neutrino emission

L_γ: Luminosity of photon emission

Cooling sources

Two cooling sources:



Dominant for $t \lesssim 10^5$ years

<u>Photon emission (from surface)</u>

$$L_\gamma = 4\pi R^2 \sigma_{\rm SB} T_s^4$$

Dominant for $t \gtrsim 10^5$ years

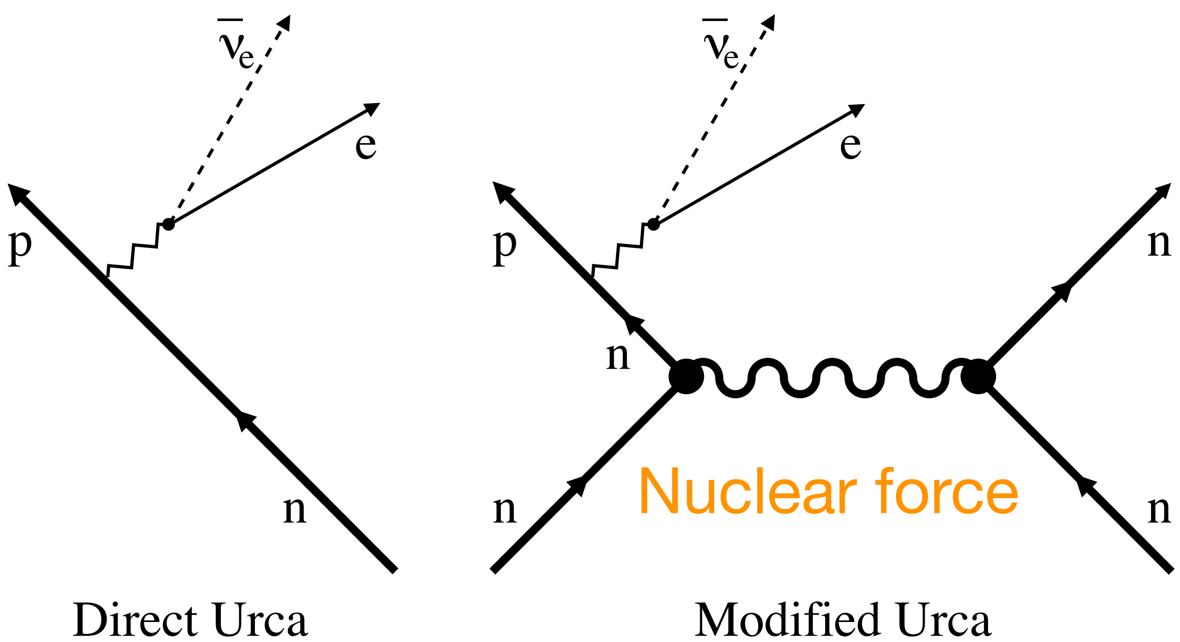
<u>Neutrino emission (from core)</u>

- Direct Urca process (DUrca)
- Modified Urca process (MUrca)
- Bremsstrahlung
- PBF process

Occurs when nucleon pairings are formed.

Urca processes

Urca processes keep NSs into β equilibrium:



<u>Chemical equilibrium</u>

$$\mu_n = \mu_p + \mu_\ell$$

$$n\ (+N) \to p + \ell^- + \bar{\nu}_\ell\ (+N),\ \ \ell^- + p\ (+N) \to n + \bar{\nu}_\ell\ (+N)$$

Rapid Direct Urca process can occur only in heavy stars.

For the APR equation of state, $M \gtrsim 1.97 M_\odot$

Effects of nucleon pairings

Nucleons in a NS form Cooper pairings.

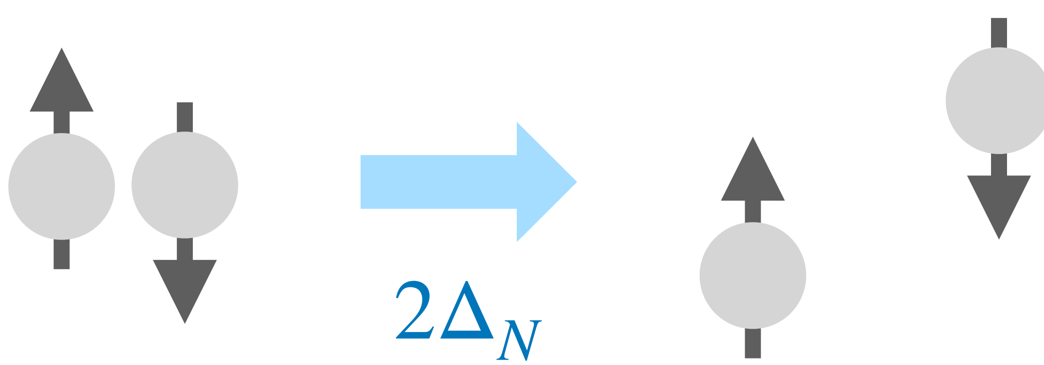
Energy spectrum

$$\epsilon_N(\boldsymbol{p}) \simeq \sqrt{\Delta_N^2 + v_{F,N}^2 (p - p_{F,N})^2}$$

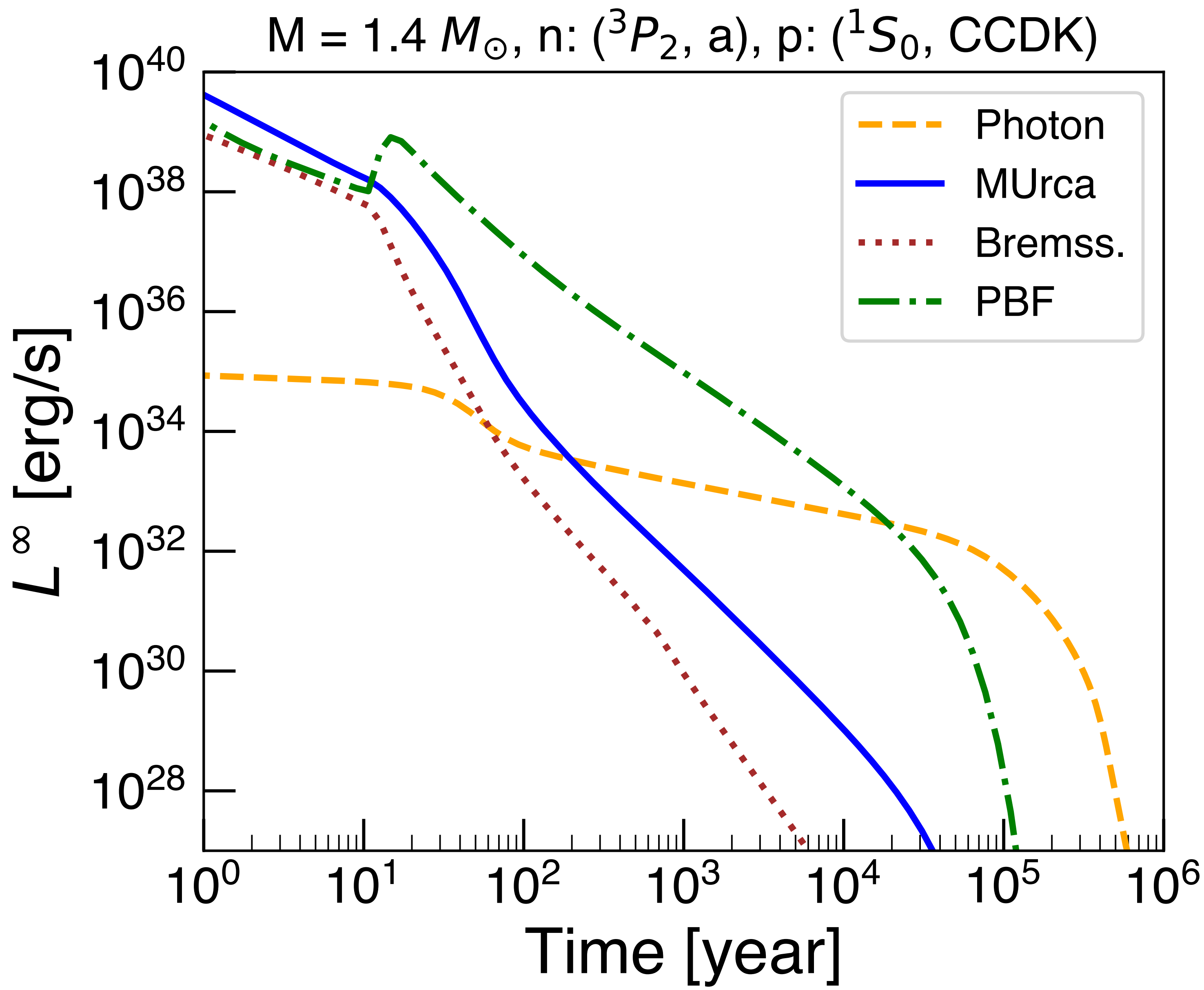
Δ_N: pairing gap

This pairing energy gap strongly suppresses the neutrino emission at low temperatures.

$$\propto e^{-\frac{\Delta_N}{T}}$$

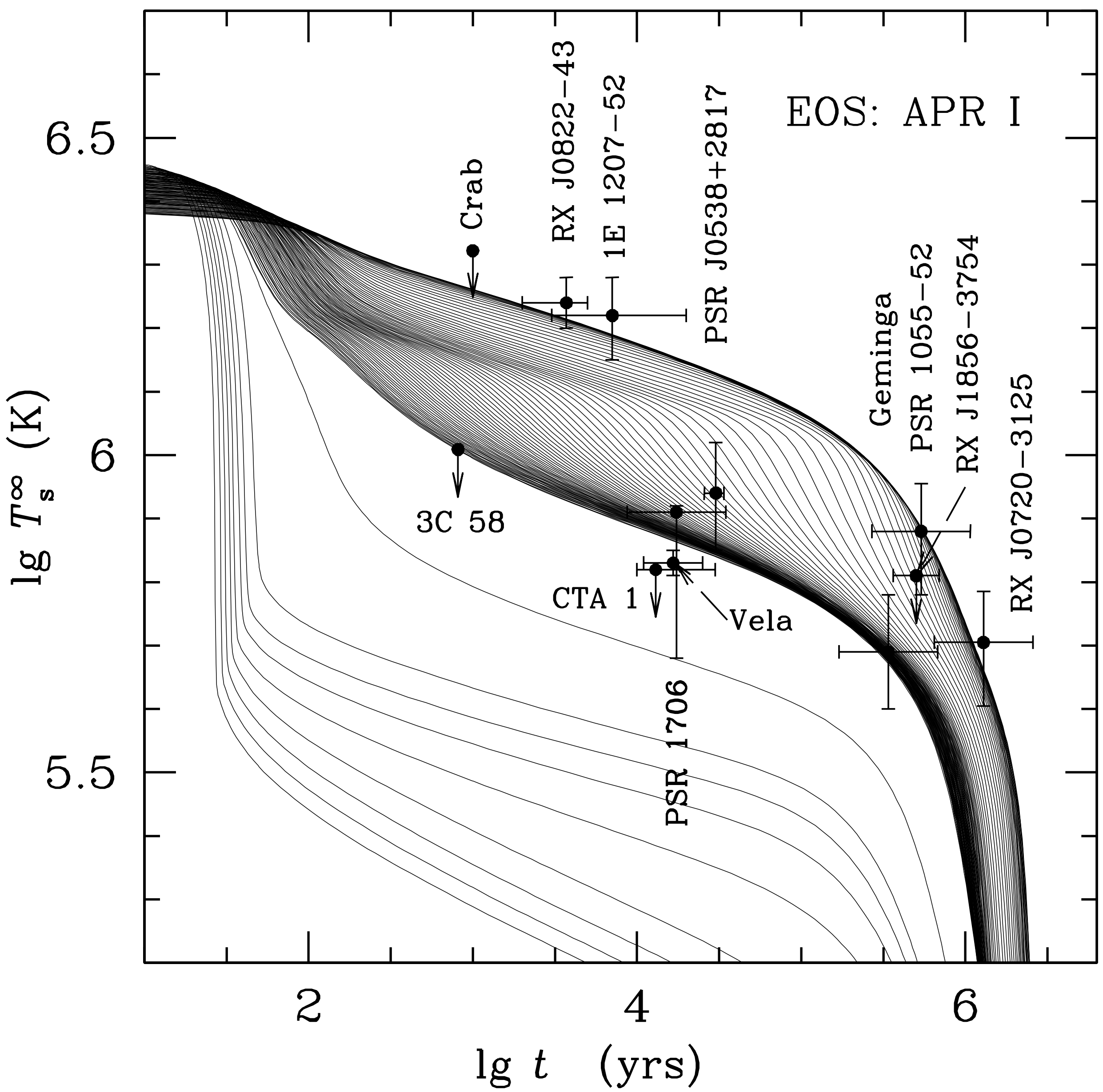


Luminosity



- Photon emission becomes dominant after ~10^5 years.
- Urca process is extremely suppressed at later times.

Success of Standard Cooling



$$M = (1.01 - 1.92)M_\odot$$

O. Y. Gnedin, M. Gusakov, A. Kaminker, D. G. Yakovlev, Mon. Not. Roy. Astron. Soc. **363,** 555 (2005).

- Temperature gets very low for $t \gtrsim 10^6$ years.
- Consistent with the observations for $t < 10^6$ years.

~ 50 NSs listed.

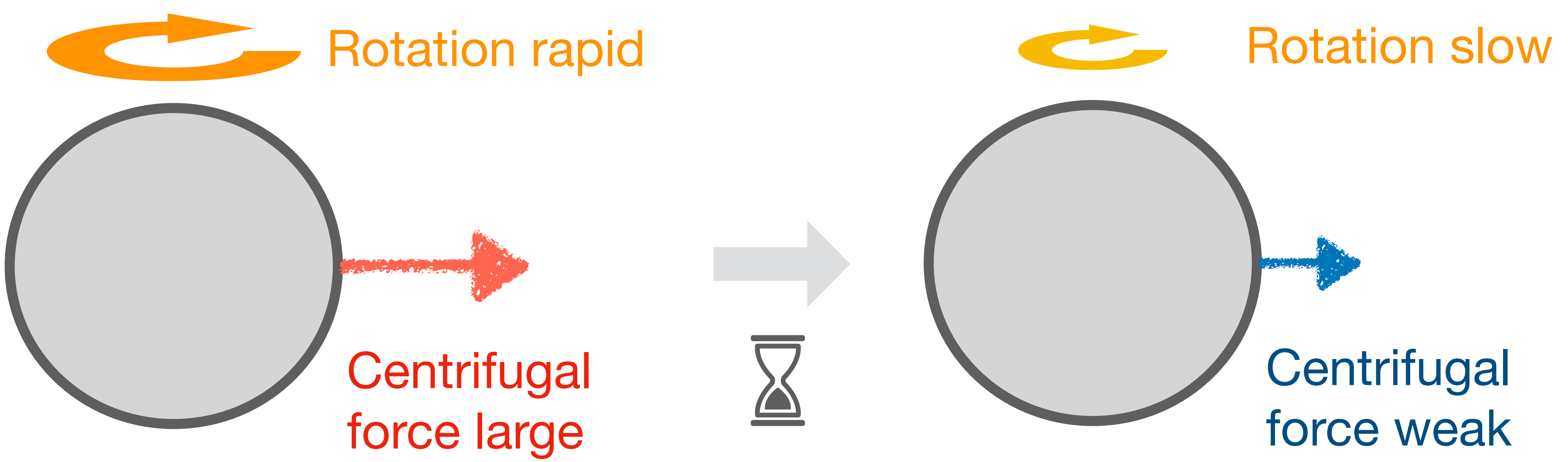
For the latest data, see http://www.ioffe.ru/astro/NSG/thermal/cooldat.html

Role of non-equilibrium β processes

Loop hole in standard cooling

In the standard cooling, β equilibrium is assumed.

In a real pulsar



Local pressure changes. Chemical equilibrium condition changes.

At low temperatures, the rate of Urca process is highly suppressed.

➡ Deviation from β equilibrium

A. Reisenegger, Astrophys. J. **442**, 749 (1995).

Out of β equilibrium

Deviation from β equilibrium is quantified by

$$\eta_\ell \equiv \mu_n - \mu_p - \mu_\ell \quad (\ell = e, \mu)$$

At early times

Urca processes are rapid.

→ NS can follow the change in the equilibrium condition.

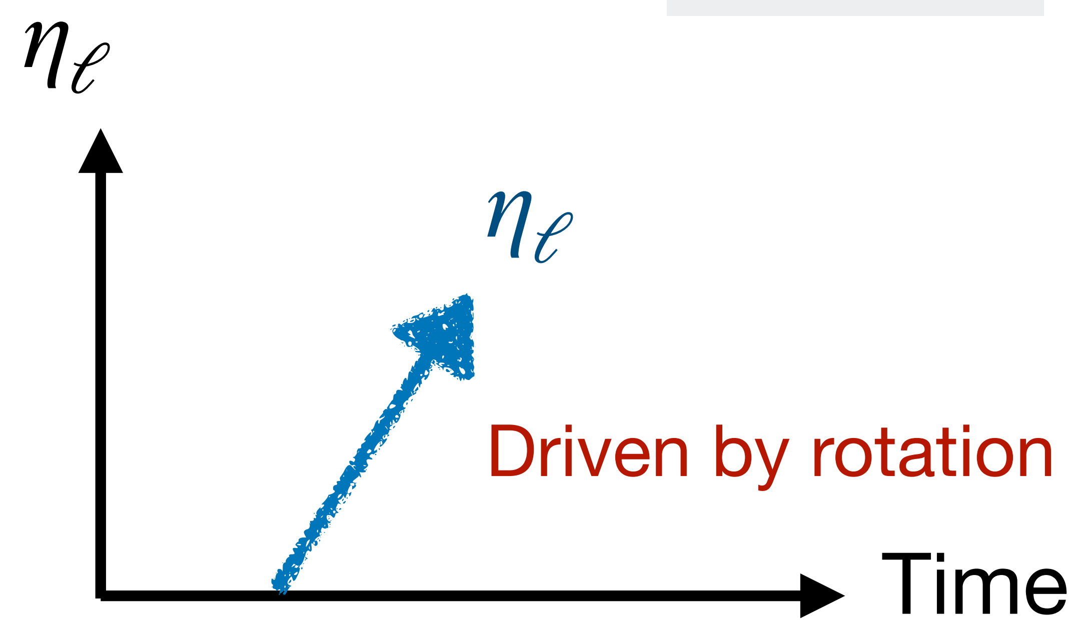
$$\eta_\ell = 0$$

At later times

Urca processes are too slow.

→ Deviation from β equilibrium

→ η_ℓ increases!

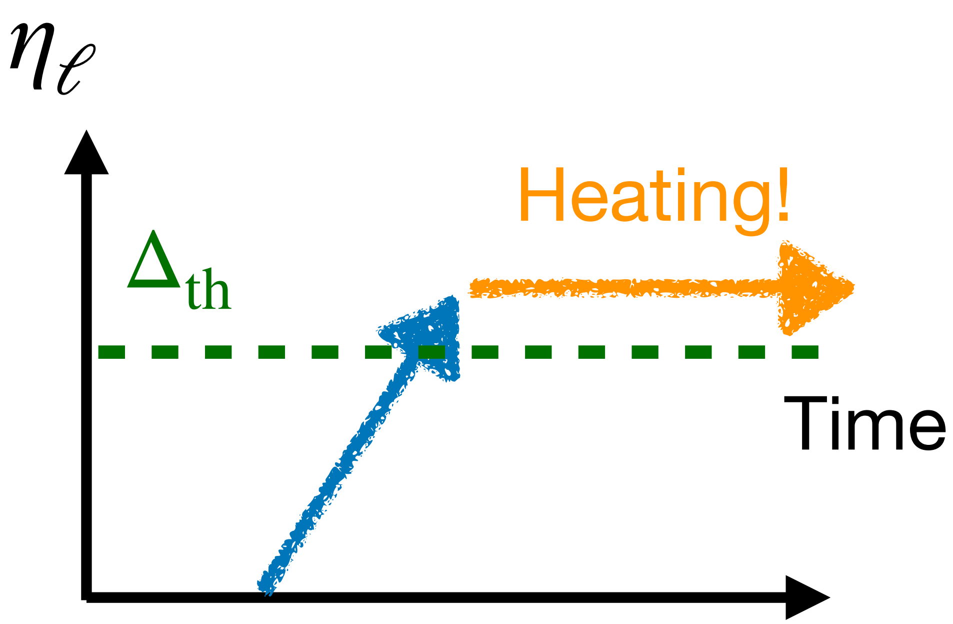


Rotochemical heating

R. Fernandez and A. Reisenegger, Astrophys. J. **625**, 291 (2005);
C. Petrovich, A. Reisenegger, Astron. Astrophys. **521**, A77 (2010).

Once η_ℓ exceeds a threshold $\Delta_{\rm th}$ determined by nucleon gaps,

$$\Delta_{\rm th} = \min\{3\Delta_n + \Delta_p, \Delta_n + 3\Delta_p\}$$



- Urca processes are enhanced.
- Generation of heat

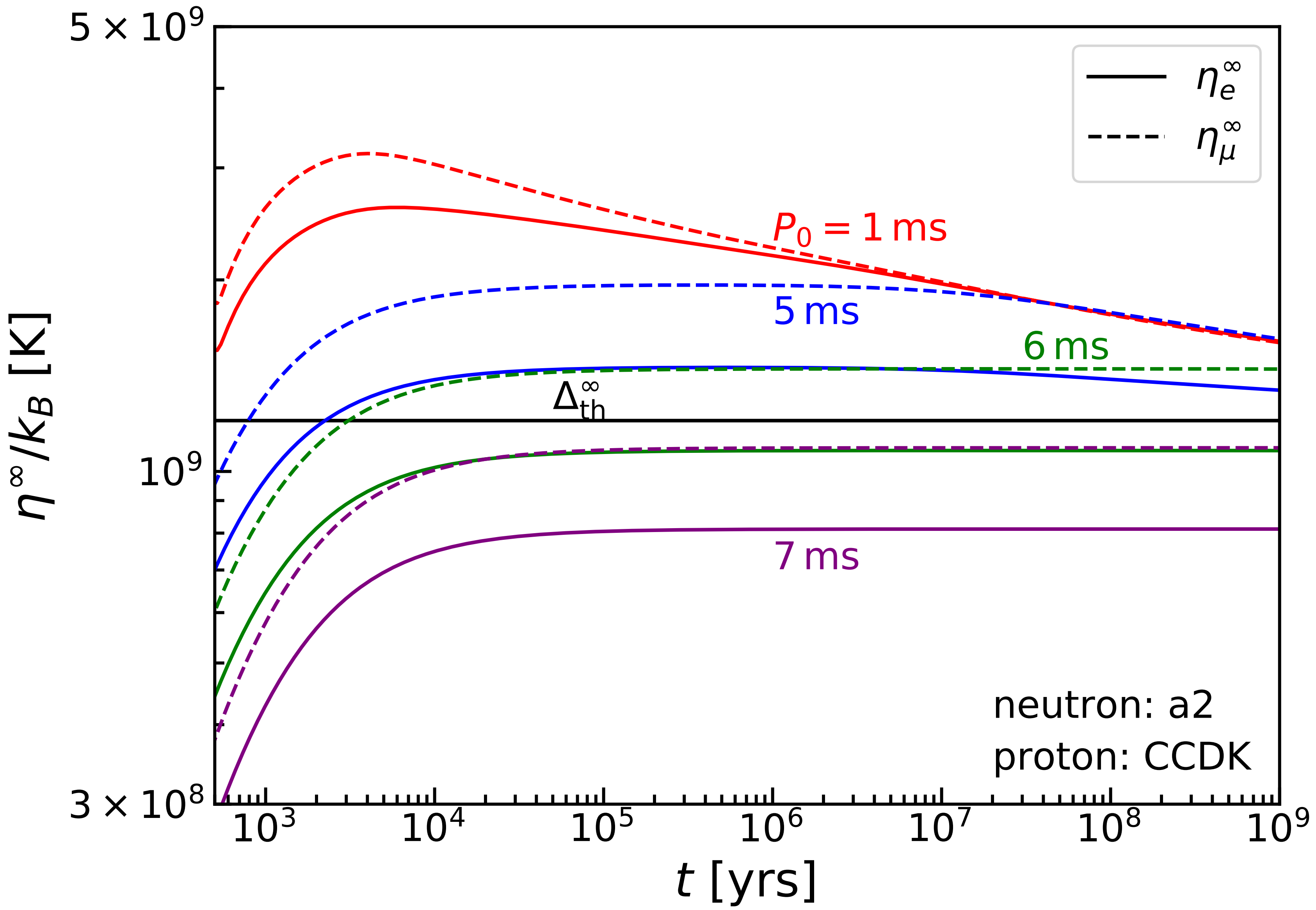
Called the rotochemical heating.

It occurs in the same setup as the standard cooling.

- No exotic physics needed.
- This effect should have been included from the beginning…

Evolution of chemical imbalance

Since the deviation from equilibrium is driven by rotation, it strongly depends on the value of initial period.



$$M = 1.4 M_\odot$$
$$P = 1\ \mathrm{s}$$
$$\dot{P} = 10^{-15}$$

Magnetic dipole radiation

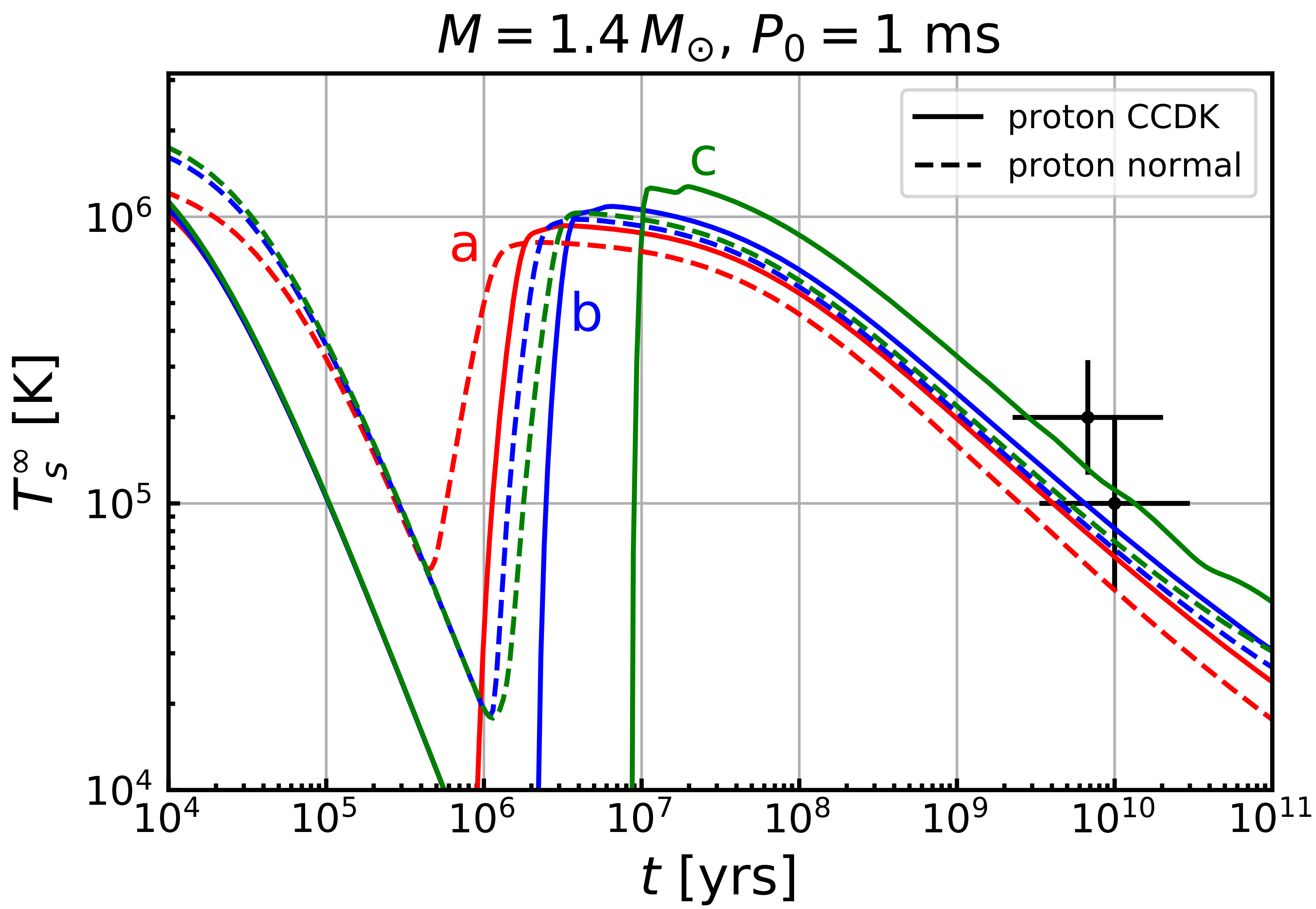
$$\dot{\Omega} = -k\Omega^3$$

Rotochemical heating occurs if the initial period P_0 is small enough.

K. Hamaguchi, N. Nagata, K. Yanagi, Phys. Lett. **B795**, 484 (2019).

Millisecond pulsars

We take account of the effect of non-equilibrium β processes.

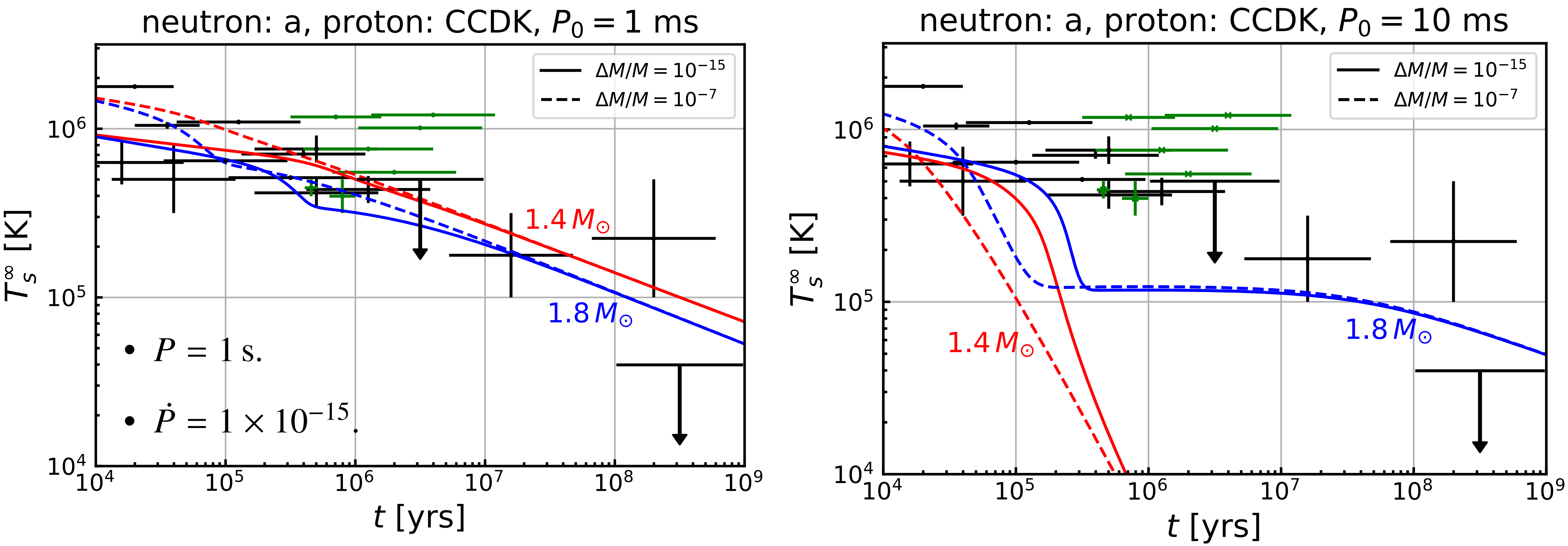


- $M = 1.4\,M_\odot$.
- $P = 5.8\,\mathrm{ms}$.
- $\dot{P} = 5.7 \times 10^{-20}$.

- Rotochemical heating always occurs in MSPs.
- We can explain the observations.

K. Hamaguchi, N. Nagata, K. Yanagi, MNRS **492**, 5508 (2020).

Ordinary pulsars



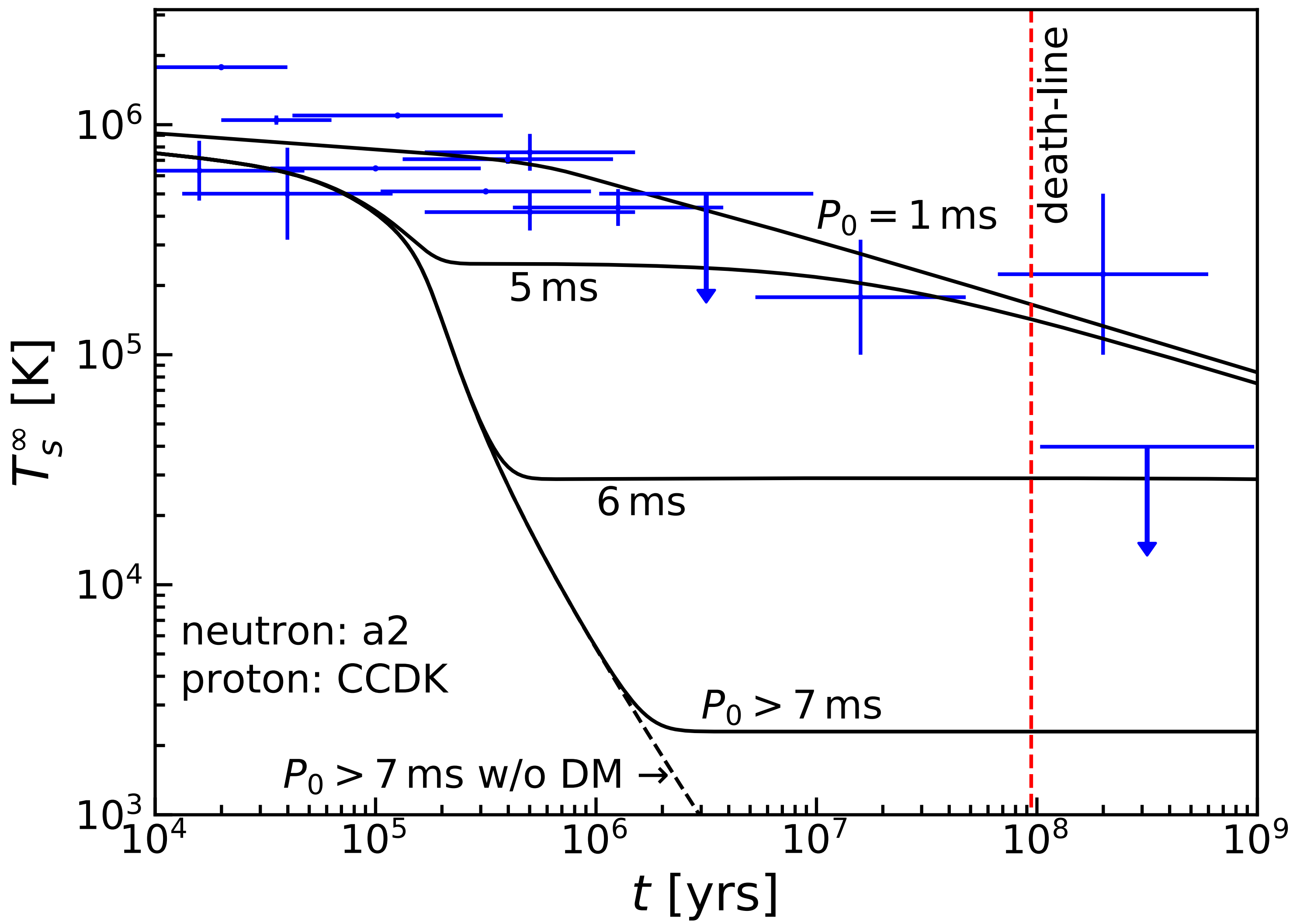
- The temperature evolution highly depends on the initial period P_0 of pulsars.
- We can explain all of the observations.
  - Cool star: large initial period → no rotochemical heating.
  - Warm star: small initial period → rotochemical heating effective.

K. Hamaguchi, N. Nagata, K. Yanagi, MNRS **492**, 5508 (2020).

Rotochemical heating vs DM heating

Rotochemical heating vs DM heating

Now we include both the DM and rotochemical heating effects.



Simulations show that P_0 can be as large as O(100) ms.

See, *e.g.*, 1811.05483.

- If P_0 is large enough, DM heating effect can be observed.
- It is always concealed in millisecond pulsars.

K. Hamaguchi, N. Nagata, K. Yanagi, Phys. Lett. **B795**, 484 (2019).

Conclusion

Conclusion

- We studied the NS temperature evolution including both the rotochemical and DM heating effects.

- For ordinary pulsars, DM heating effect can be observed if their initial period is relatively large.

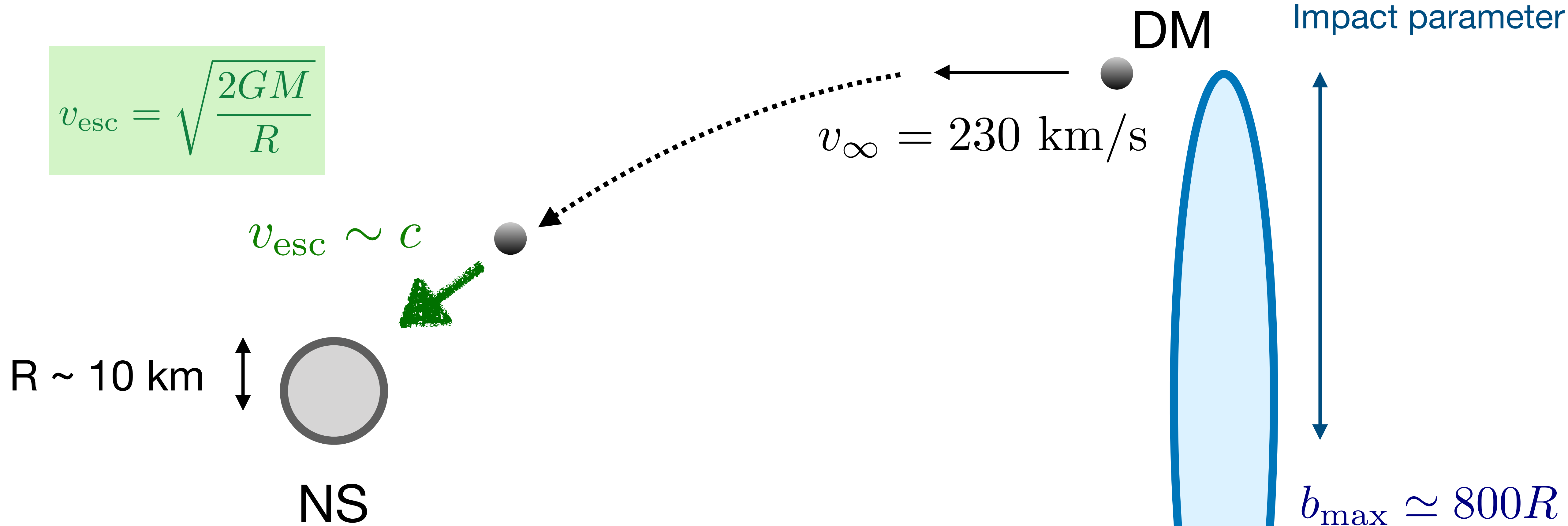
- For millisecond pulsars, DM heating effect is always hidden by the rotochemical heating.

Take-home message

To search for DM heating effect, we should look at ordinary pulsars, not millisecond pulsars.

Backup

Dark matter accretion in NS



$$b_{max} \simeq R\frac{v_{esc}}{v_\infty}$$

- DM velocity is very large on the NS surface.
- Effective geometrical cross section is very large.

DM accretion rate is

$$\dot{N} \simeq \pi b_{max}^2 v_\infty \cdot \frac{\rho_{DM}}{m_{DM}}$$

DM number density

Dark matter accretion in NS

It is found that

- One scattering is enough for WIMPs to be captured.

  Energy transfer ~ 100 MeV — 1 GeV.

- At least one scattering occurs if $\sigma_N \gtrsim 10^{-45}\ \mathrm{cm}^2$.

For old NSs, we have

Accretion rate = Annihilation rate

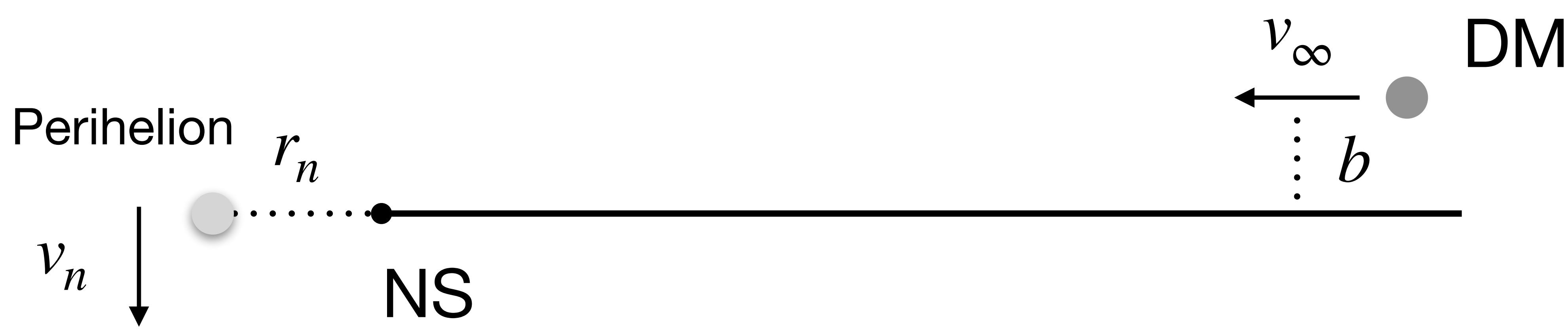
equilibrium

$$L_H \simeq m_{\mathrm{DM}} \dot{N} \simeq 2\pi G M R \rho_{\mathrm{DM}} / v_\infty$$

Independent of DM mass.

Dark matter accretion in NS

Consider a WIMP with mass m_{DM}, incoming from infinity with speed v_∞ and impact parameter b.



Energy

$$\frac{m_{\rm DM} v_\infty^2}{2} = \frac{m_{\rm DM} v_n^2}{2} - \frac{G m_{\rm DM} M}{r_n}$$

Angular momentum

$$m_{\rm DM} v_\infty b = m_{\rm DM} v_n r_n$$

$$r_n = \frac{GM}{v_\infty^2}\left[\sqrt{1 + \frac{v_\infty^4 b^2}{G^2 M^2}} - 1\right]$$

Dark matter accretion in NS

For a WIMP to be captured by a NS, $r_n \leq R$ is required.

$$\Rightarrow \quad b \leq R\left[1+\frac{v_{\rm esc}^2}{v_\infty^2}\right]^{\frac{1}{2}}$$

$v_\infty \simeq 230$ km/s

Escape velocity

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}} \simeq 2\times 10^8 \times \left(\frac{M}{1.4M_\odot}\right)^{1/2}\left(\frac{R}{10\ {\rm km}}\right)^{-1/2} {\rm m/s}$$

Close to the speed of light!

Maximum impact parameter

$$b_{\rm max} \simeq R\frac{v_{\rm esc}}{v_\infty} \simeq 0.8\times 10^7 \times \left(\frac{M}{1.4M_\odot}\right)^{1/2}\left(\frac{R}{10\ {\rm km}}\right)^{1/2} {\rm m}$$

Much larger than the NS radius.

Recoil energy

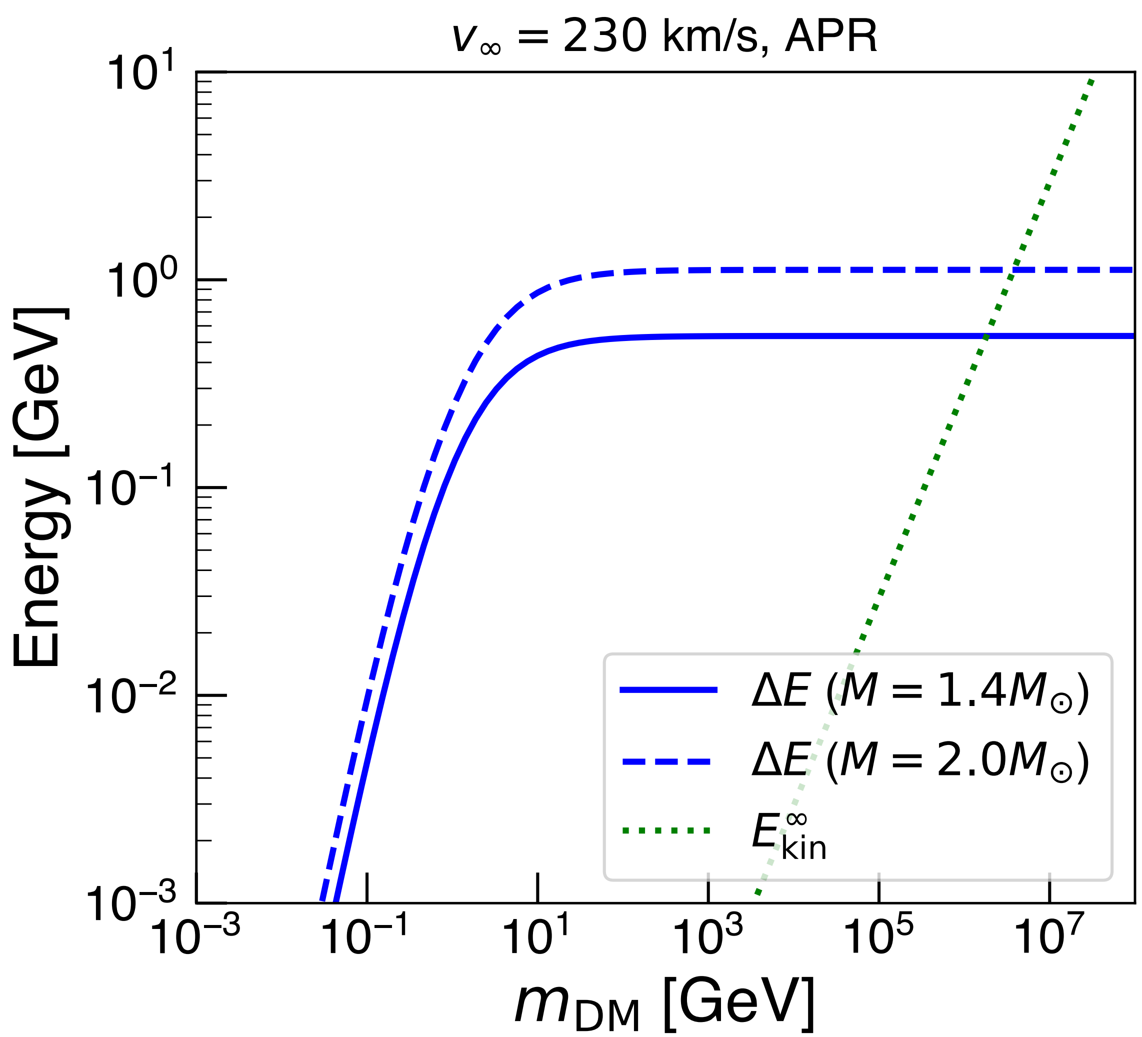
For each DM-nucleon scattering, WIMPs lose energy by

$$\Delta E = \frac{m_N m_{\rm DM}^2 \gamma_{\rm esc}^2 v_{\rm esc}^2}{m_N^2 + m_{\rm DM}^2 + 2\gamma_{\rm esc} m_{\rm DM} m_N}(1-\cos\theta_c)$$

θ_c : scattering angle in the CM frame.

$\gamma_{\rm esc} \equiv (1 - v_{\rm esc}^2)^{-1/2}$

Let us compare this with the initial kinetic energy: $E_{\rm kin}^\infty = m_{\rm DM} v_\infty^2/2$



- One scattering is sufficient for WIMPs to lose the initial kinetic energy.
- Energy transfer can be as large as O(100) MeV.

One scattering in NS

WIMP-nucleon scattering occurs at least once if

$$\text{Mean Free Path} \sim (\sigma_N n)^{-1} \sim \frac{m_N R^3}{M \sigma_N} \lesssim R$$

$\Rightarrow$ $\sigma_N \gtrsim 10^{-45}\ \text{cm}^2$

σ_N : DM-nucleon scattering cross section

If this is satisfied, then all of the accreted WIMPs are captured.

If not, capture rate is suppressed by $\sigma_N / \sigma_{\text{th}}$.

Captured WIMPs eventually annihilate inside the NS core.

For old NSs, we have

Accretion rate = Annihilation rate

equilibrium

NS temperature with DM heating

At later times, the DM heating balances with the cooling by photon emission.

$$L_H = L_\gamma$$

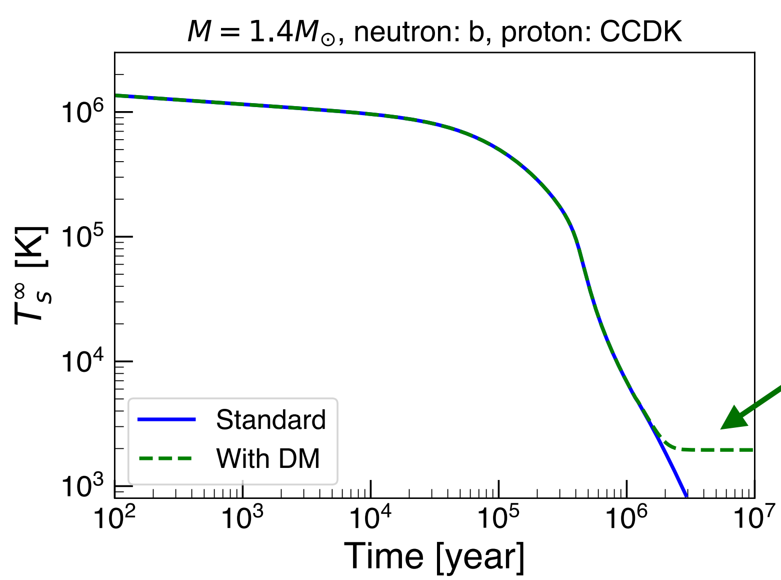
$$L_H \simeq m_{\rm DM}\dot{N} \simeq 2\pi GMR\rho_{\rm DM}/v_\infty$$

Independent of DM mass.

$$2\pi GMR\rho_{\rm DM}/v_\infty \simeq 4\pi R^2 \sigma_{\rm SB} T_s^4$$

(for $\sigma > \sigma_{\rm th}$)

$$T_s \simeq 2500 \text{ K}$$



Robust, smoking-gun prediction of DM heating.

Can we observe this??

NASA unveils first images from James Webb Space Telescope

By Joel Achenbach

Updated July 12, 2022 at 4:41 p.m. EDT | Published July 11, 2022 at 4:28 p.m. EDT

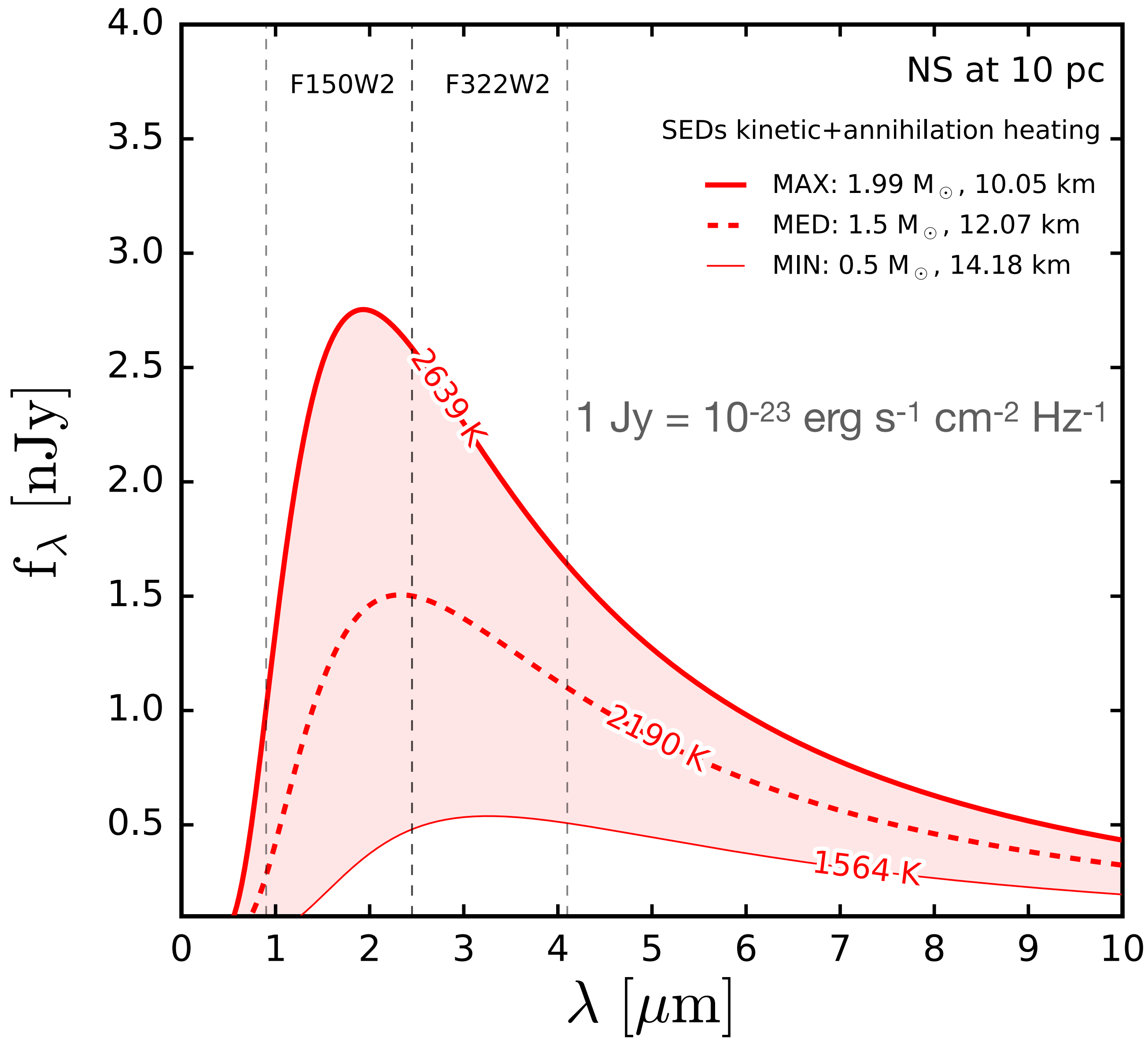
https://www.washingtonpost.com/science/2022/07/11/nasa-james-webb-space-telescope-images/

- Infrared space telescope.
- Succeed the Hubble Space Telescope.

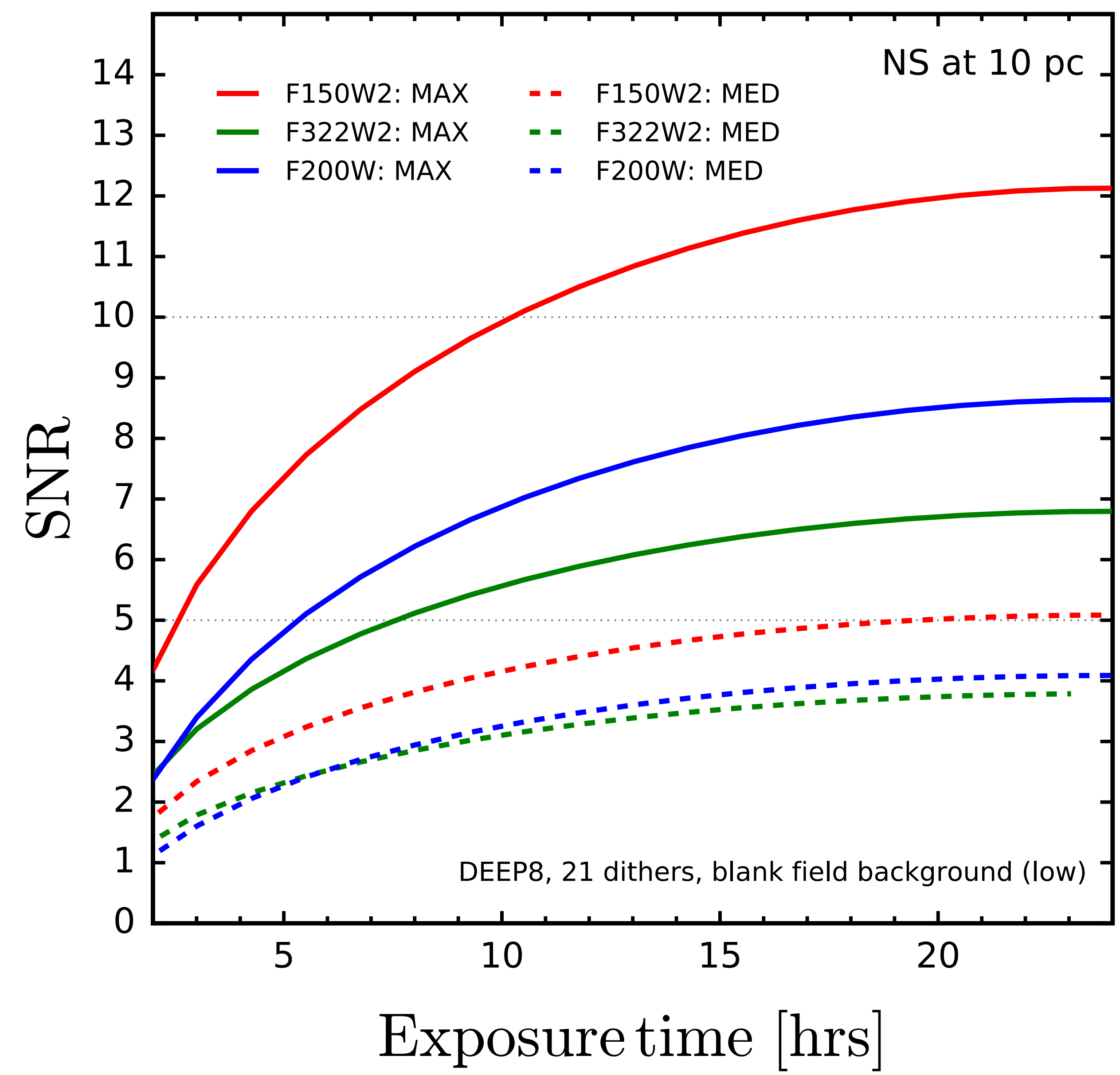
Spectrum, flux, and SNR

arXiv:2205.05048

Spectral distributions



Signal-to-noise ratio



- $\lambda \sim 2\ \mu m$ ➡ Near-Infrared Camera (NIRCam) on JWST
- With the F150W2 filter, SNR $\gtrsim 5$ is obtained for 24 hours.

FAST

Many pulsars are expected to be discovered by

Five-hundred-meter Aperture Spherical radio Telescope (FAST)

（五百米口径球面射电望远镜）

in China in the near future.

- About 5000 (4000 new) pulsars are expected to be discovered.

arXiv: 1105.3794

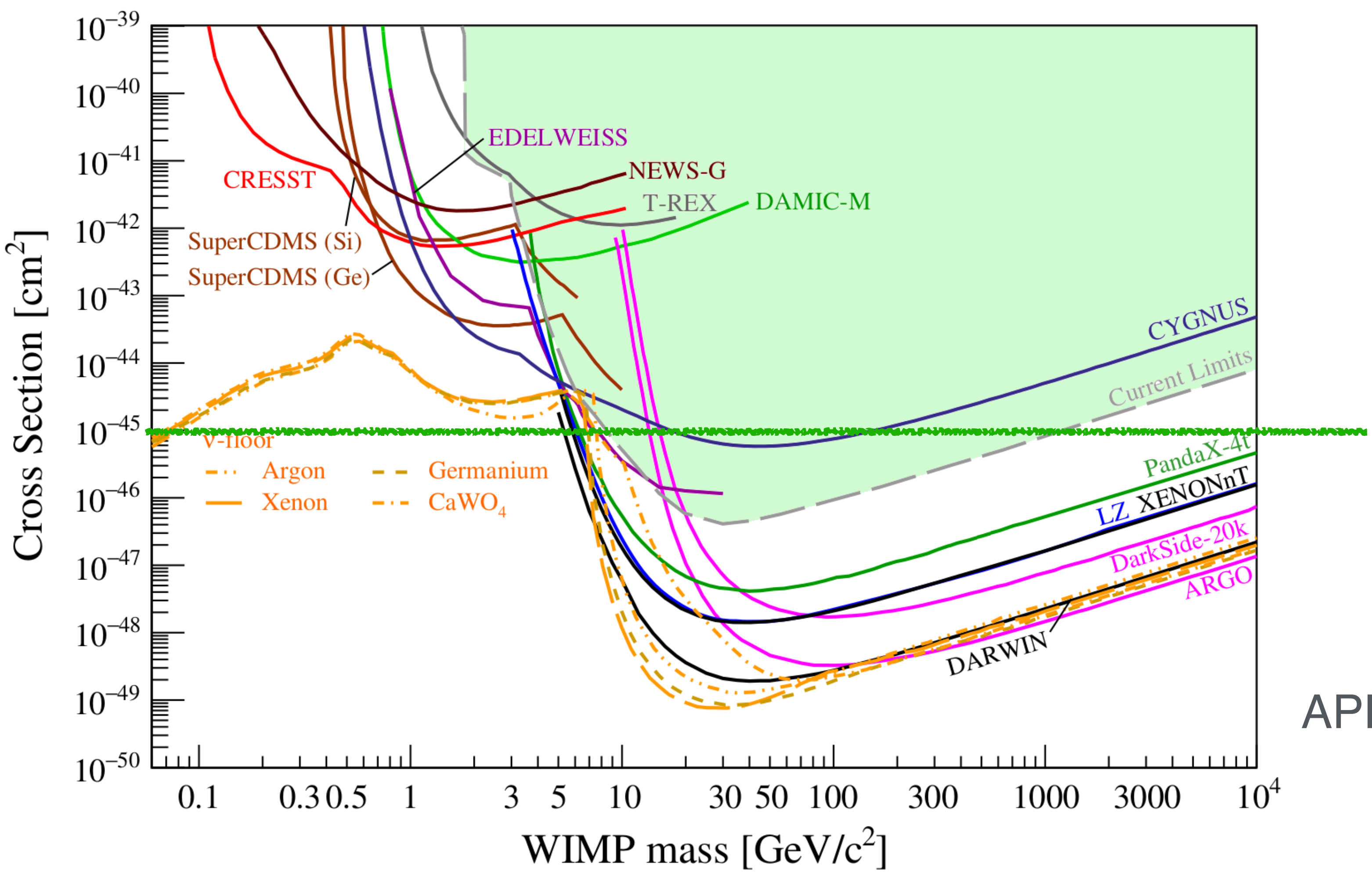
- A lot of pulsars have already been discovered.

See J. L. Han *et al.*, arXiv:2105.08460

DM heating vs direct detection

In any case, an observation of a NS with $T_s \lesssim 2 \times 10^3$ K disfavors WIMPs which have $\sigma_N \gtrsim 10^{-45}$ cm^2.

Prospects for direct detection experiments



$\sigma_N = 10^{-45}$ cm^2

APPEC Committee Report, arXiv:2104.07634.

Such a large scattering cross section can be probed in direct detection experiments. Why we should care about DM heating??

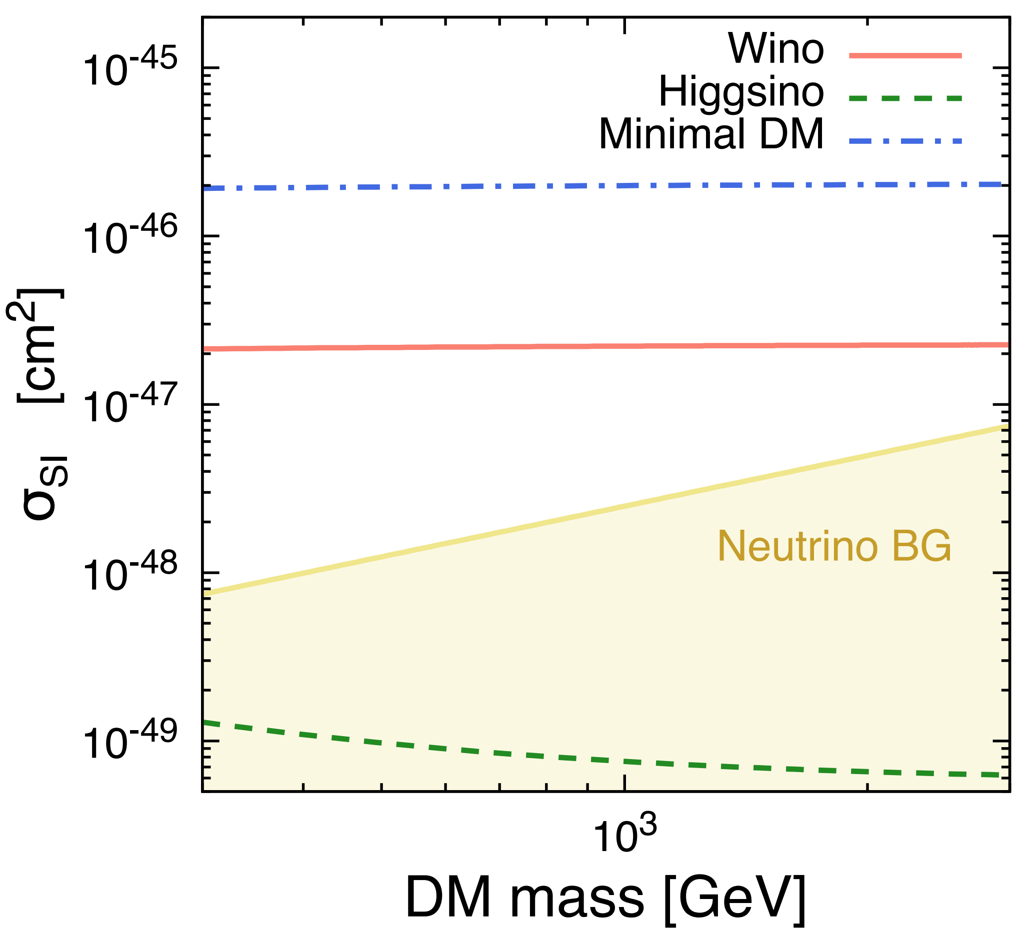
Electroweak multiplet DM

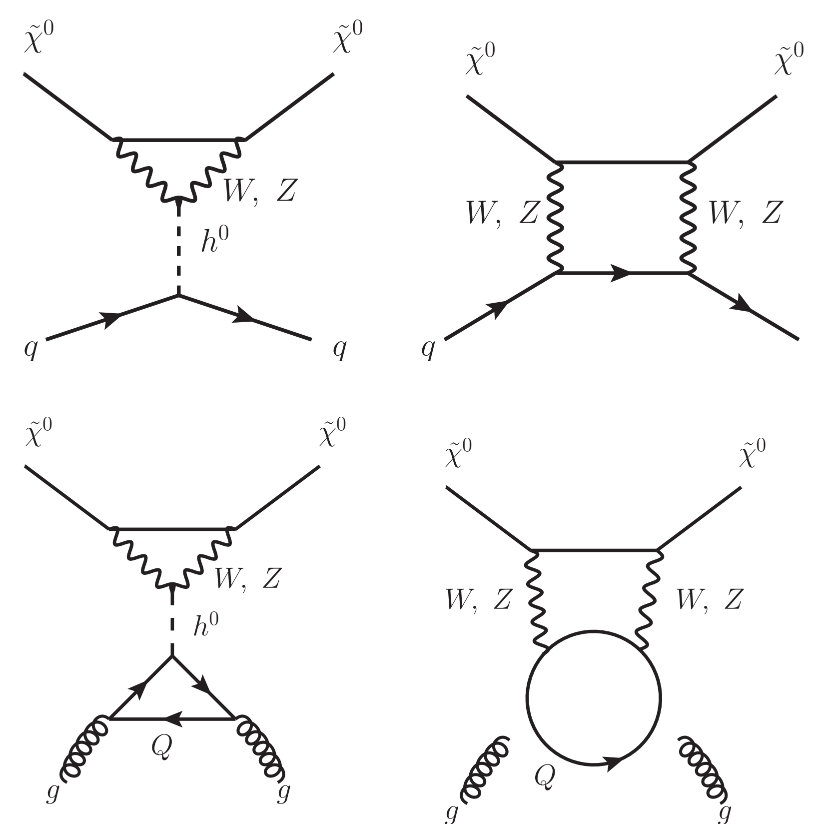
DM is electrically neutral. But, this does not fully determine its electroweak charges.

$$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y : \; (\mathbf{1},0), (\mathbf{2},\pm 1/2), (\mathbf{3},0), (\mathbf{3},\pm 1), \ldots$$

Electroweak multiplet DM

This class of DM has small DM-nucleon scattering cross section.



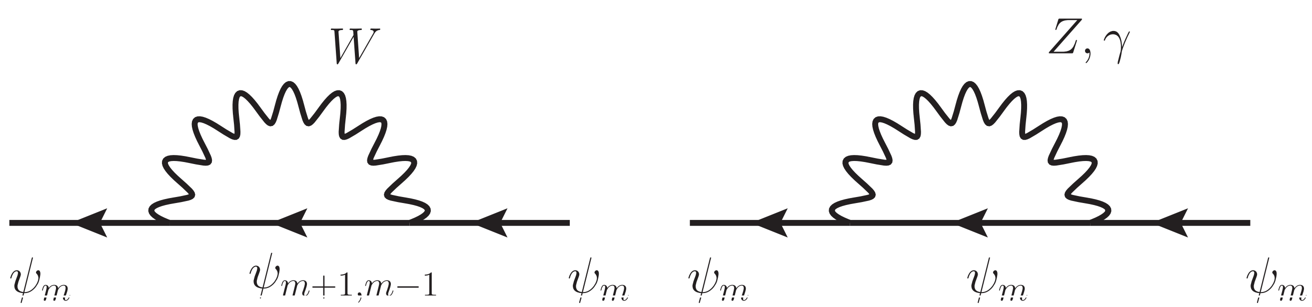


J. Hisano, K. Ishiwata, N. Nagata, JHEP **1506**, 097 (2015).

Electroweak multiplet DM

Electroweak multiplet DM is accompanied by charged particles, which are degenerate in mass.

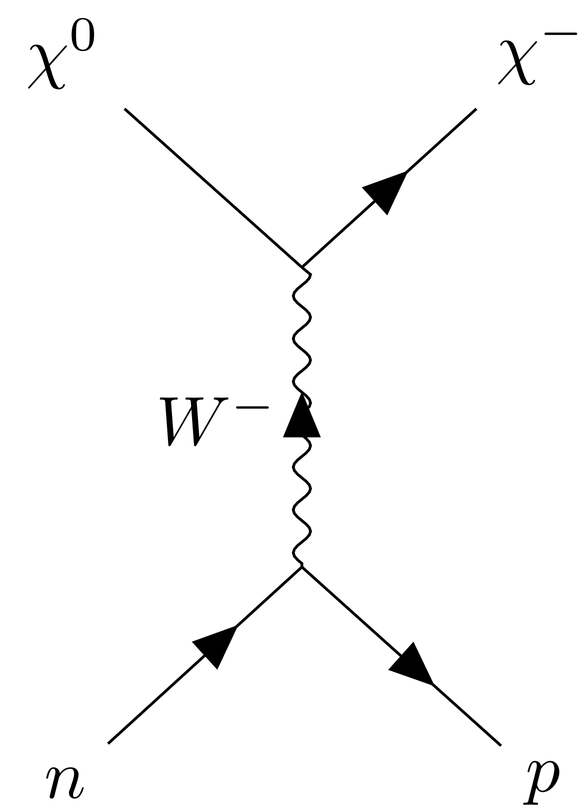
Mass splitting



$$\Delta M \simeq \alpha_2 m_W \sin^2 \frac{\theta_W}{2} + \alpha_2 Y m_W \left(\frac{1}{\cos\theta_W} - 1 \right)$$

O(100) MeV

- Inelastic scattering can occur.
- Cross section is large enough for such a DM to be captured in NS.
- NS can be a promising probe for this class of DM candidates.

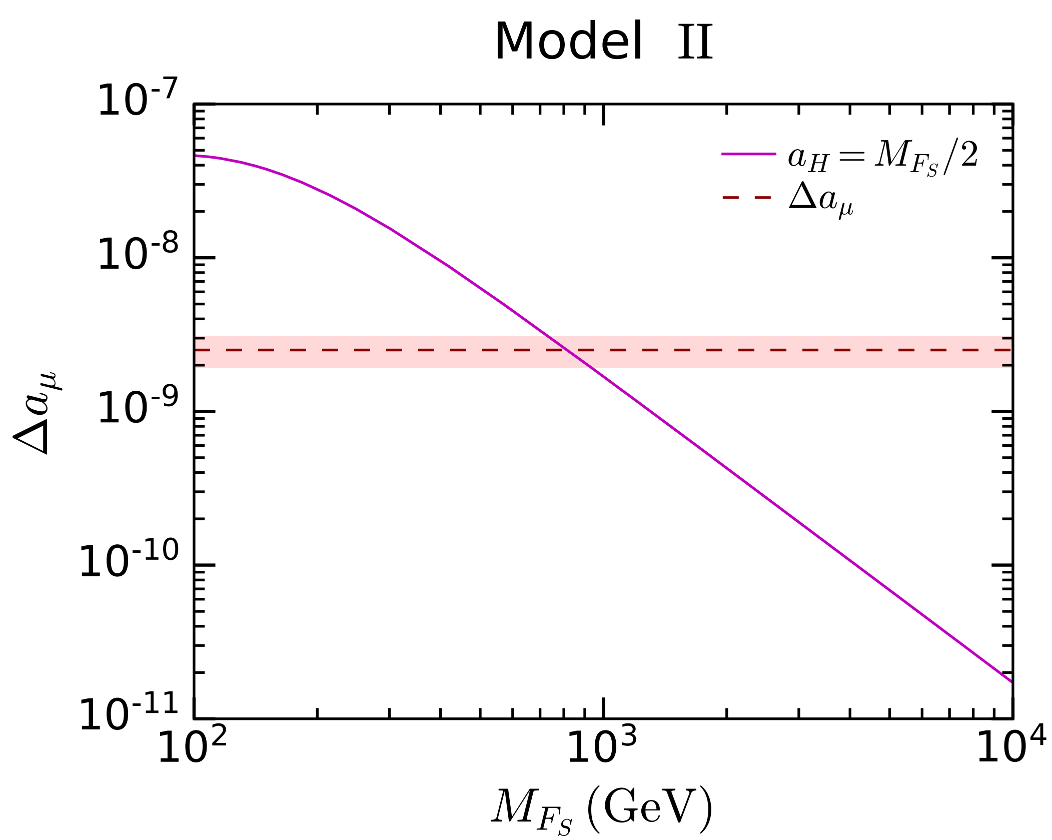


M. Fujiwara, K. Hamaguchi, N. Nagata, J. Zheng, arXiv:2204.02238.

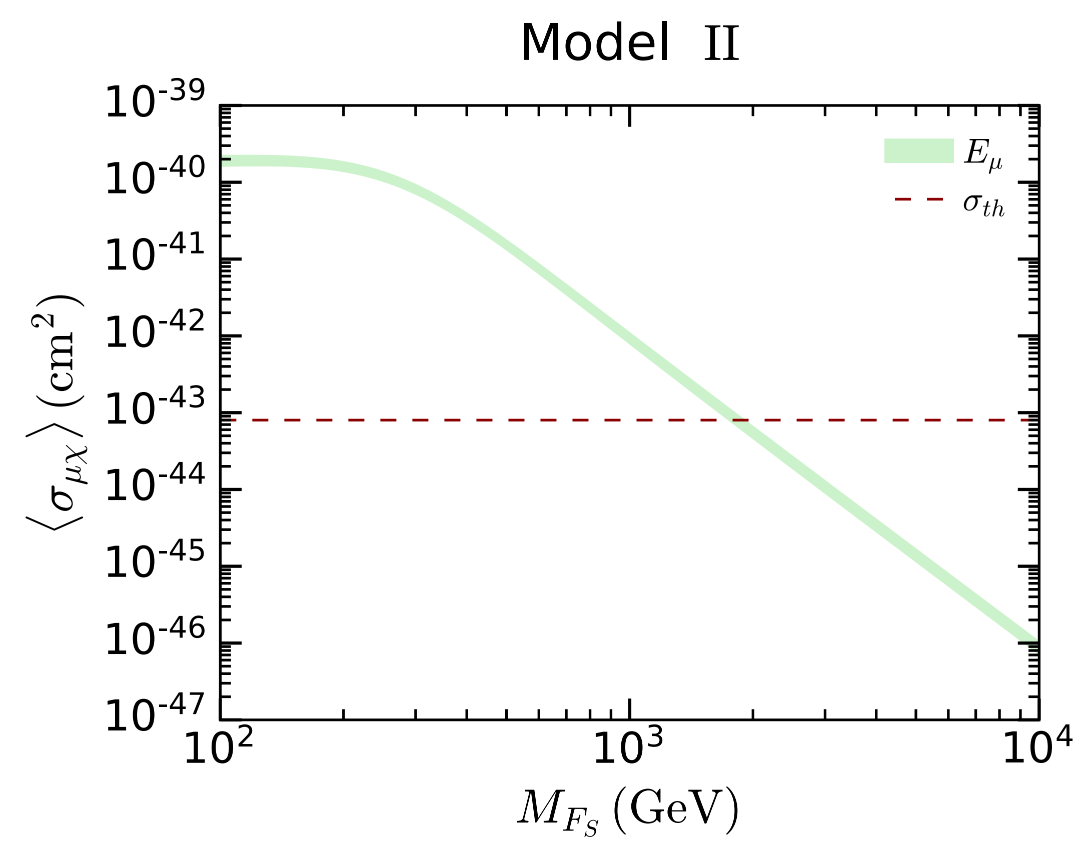
Muon g-2 and DM

NS heating can occur for DM models that couple only to leptons.

Muon g-2



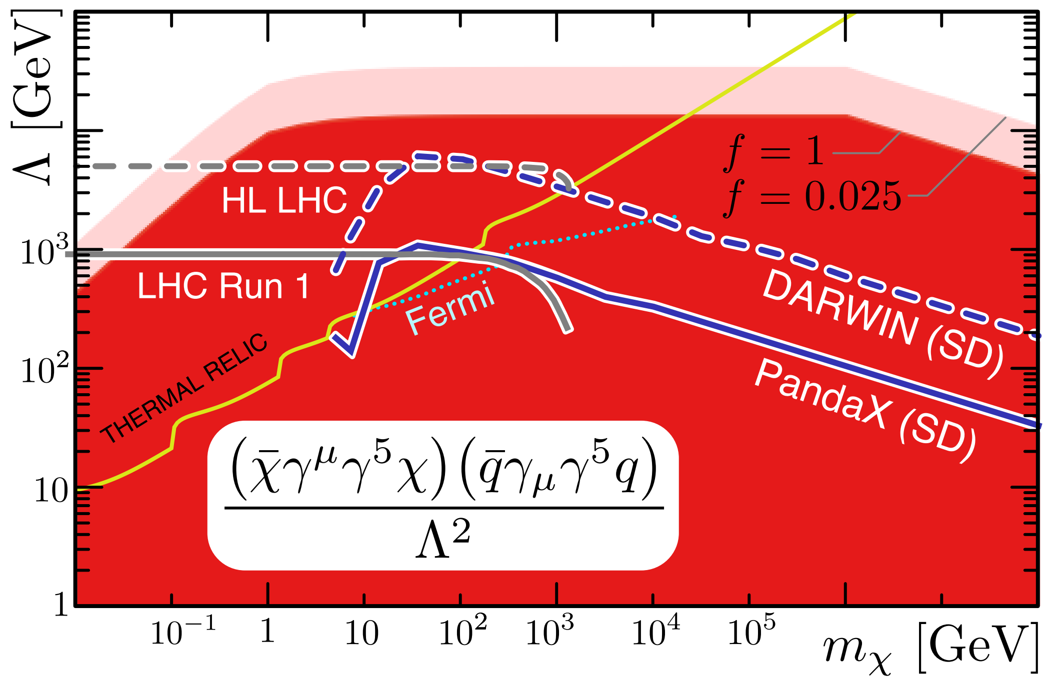
DM-muon scattering cross section

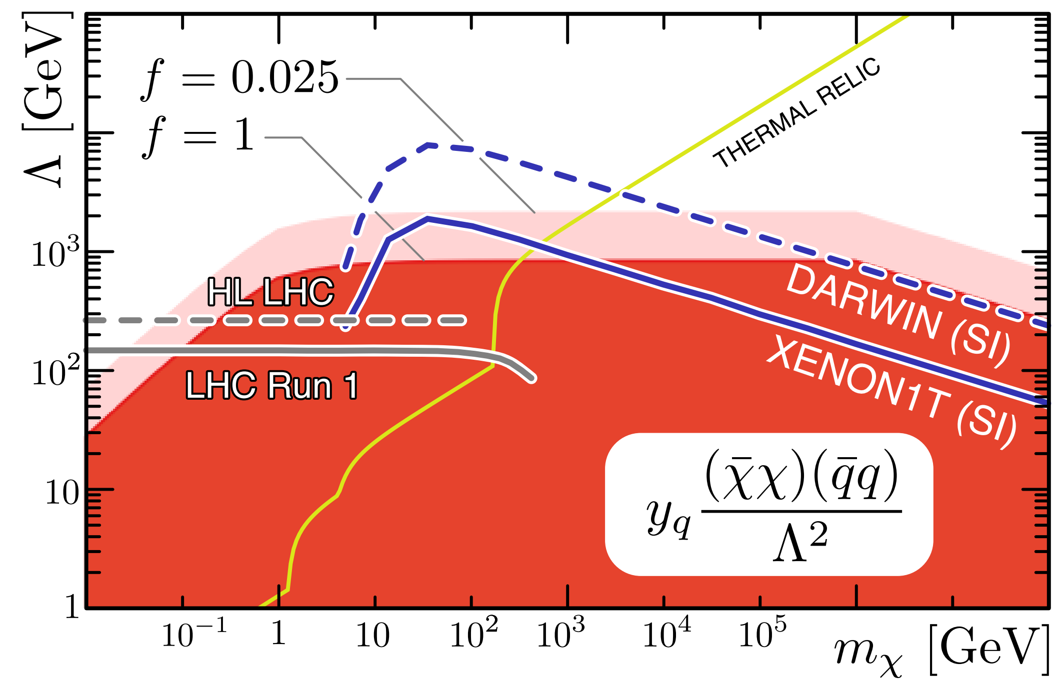


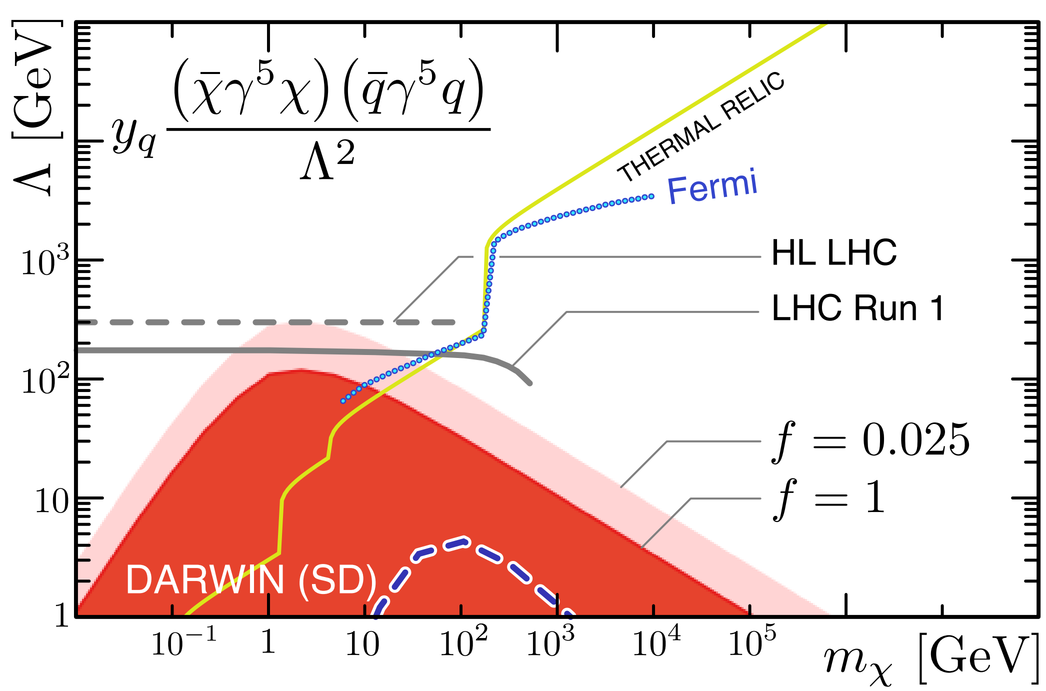
In the parameter regions where the muon g-2 anomaly is explained, DM-muon scattering is sufficiently large.

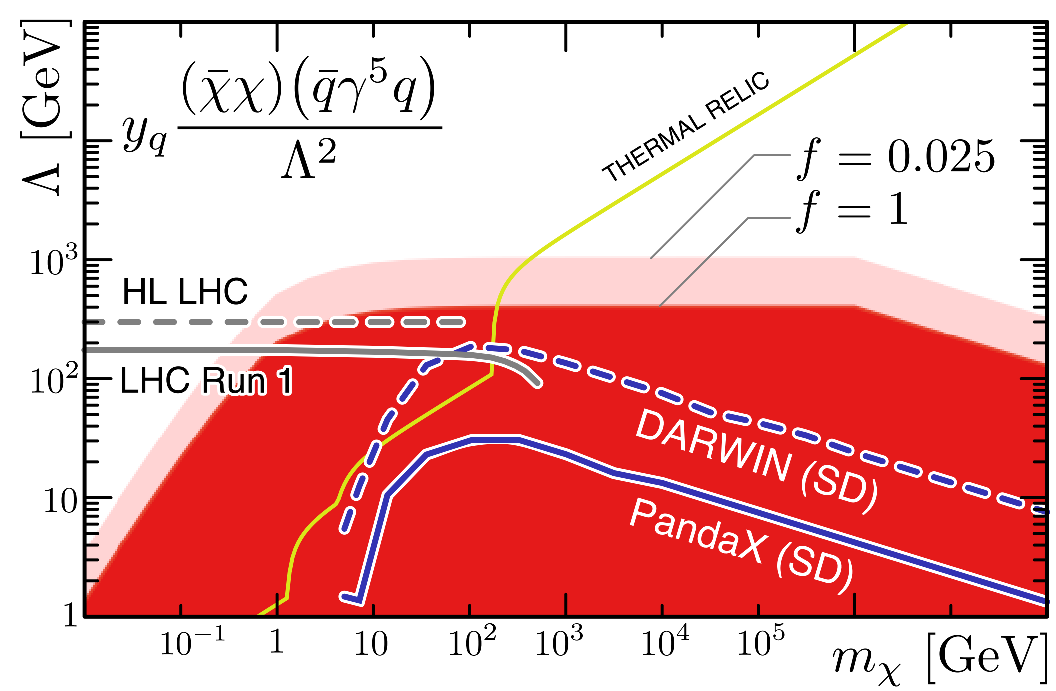
K. Hamaguchi, N. Nagata, M. E. Ramirez-Quezada, arXiv:2204.02413.

Effective operator analysis



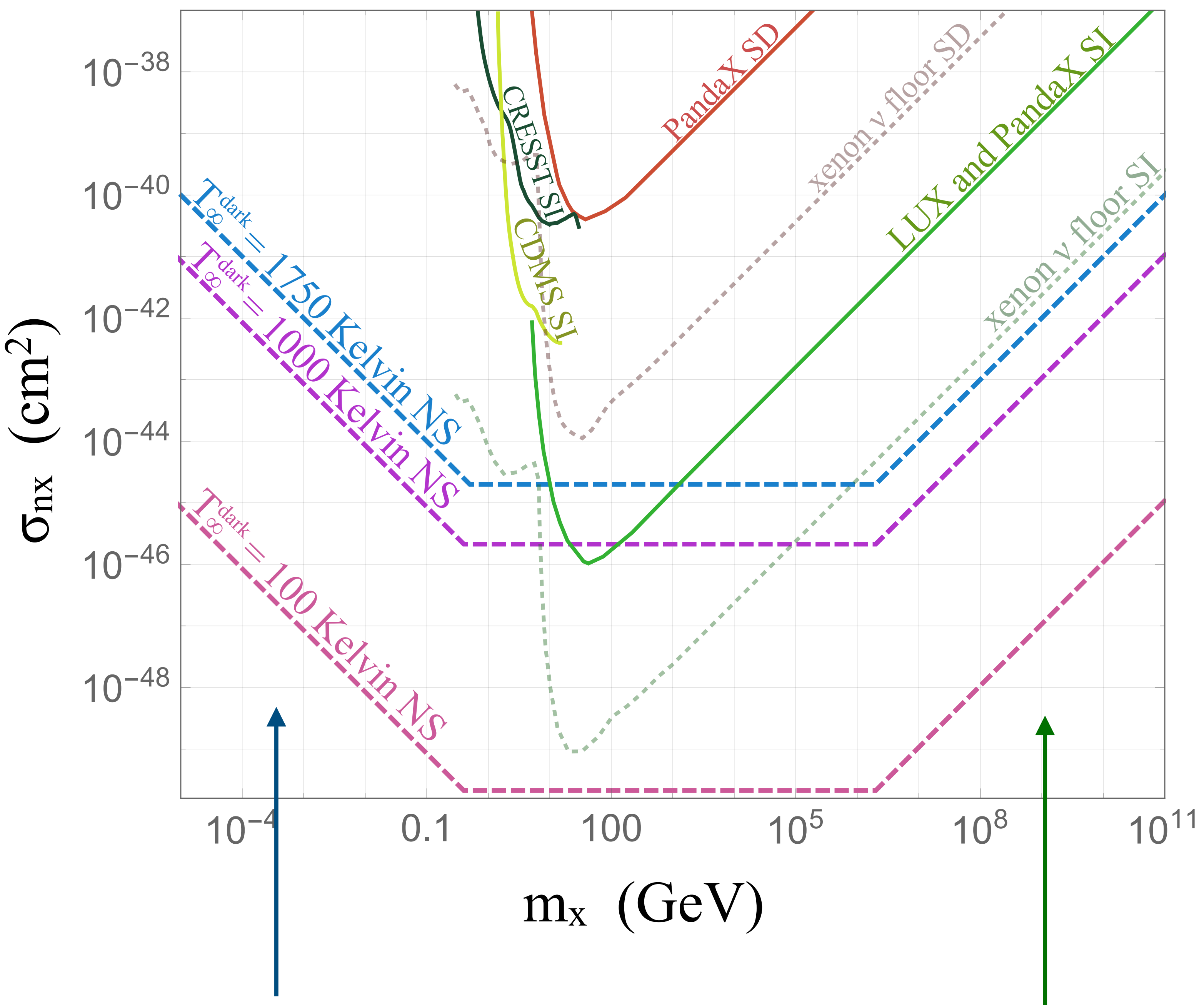






N. Raj, P. Tanedo, H. Yu, Phys. Rev. **D97**, 043006 (2018).

Dark kinetic heating

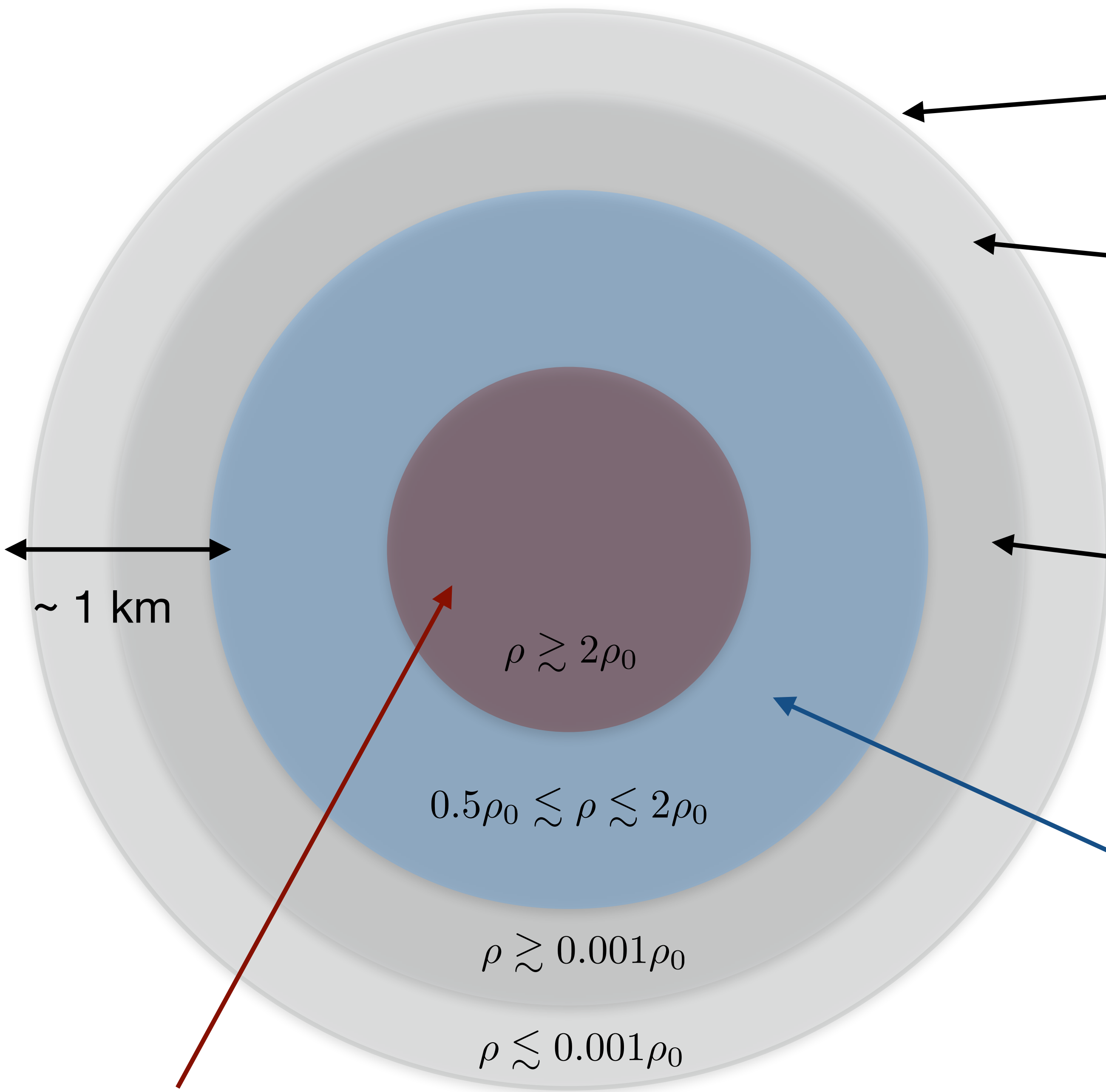


Effect of Pauli blocking

Multiple scattering required

M. Baryakhtar, J. Bramante, S. W. Li, T. Linden, N. Raj, Phys. Rev. Lett. **119**, 131801 (2017).

Neutron star structure



Envelope, atmosphere

Outer crust

Neutron-rich nucleus (crystal)

Electron

Inner crust

Neutron-rich nucleus, electron

Neutron superfluid: ${}^1S_0$

Outer core

Electron, muon

Neutron superfluid: ${}^3P_2$

Proton superconductor: ${}^1S_0$

Inner core

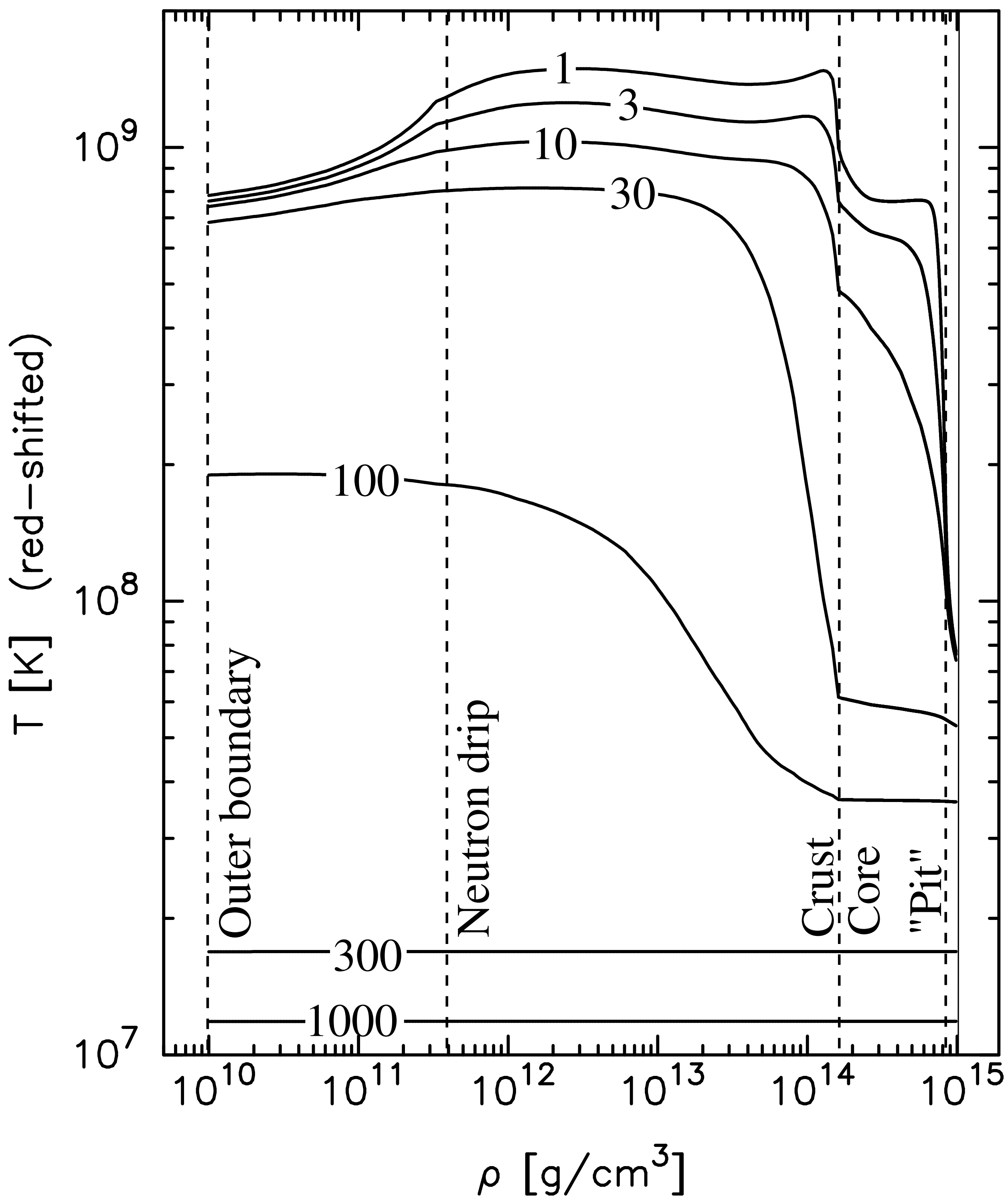
$$\rho_0 = 2.8 \times 10^{14}\ \mathrm{g \cdot cm^{-3}}$$

Neutron superfluid: ${}^3P_2$

~~Hyperons, π/K condensation, quarks (?)~~

We do not consider them in this talk.

Temperature distribution



Relaxation in the Core done in ~ 100 years.

D. Page, J. M. Lattimer, M. Prakash, A. W. Steiner [arXiv: 1302.6626].

Name: Urca

APRIL 1, 1941 PHYSICAL REVIEW VOLUME 59

Neutrino Theory of Stellar Collapse

G. Gamow, *George Washington University, Washington, D. C.*
M. Schoenberg,* *University of São Paulo, São Paulo, Brazil*

(Received February 6, 1941)

of β-particles. In fact, when the temperature and density in the interior of a contracting star reach certain values depending on the kind of nuclei involved, we should expect processes of the type

$$\begin{cases} {}_{Z}N^{A}+e^{-}\rightarrow {}_{Z-1}N^{A}+\text{antineutrino} \\ {}_{Z-1}N^{A}\rightarrow {}_{Z}N^{A}+e^{-}+\text{neutrino}, \end{cases} \quad (3)$$

which we shall call, for brevity, "urca-processes."

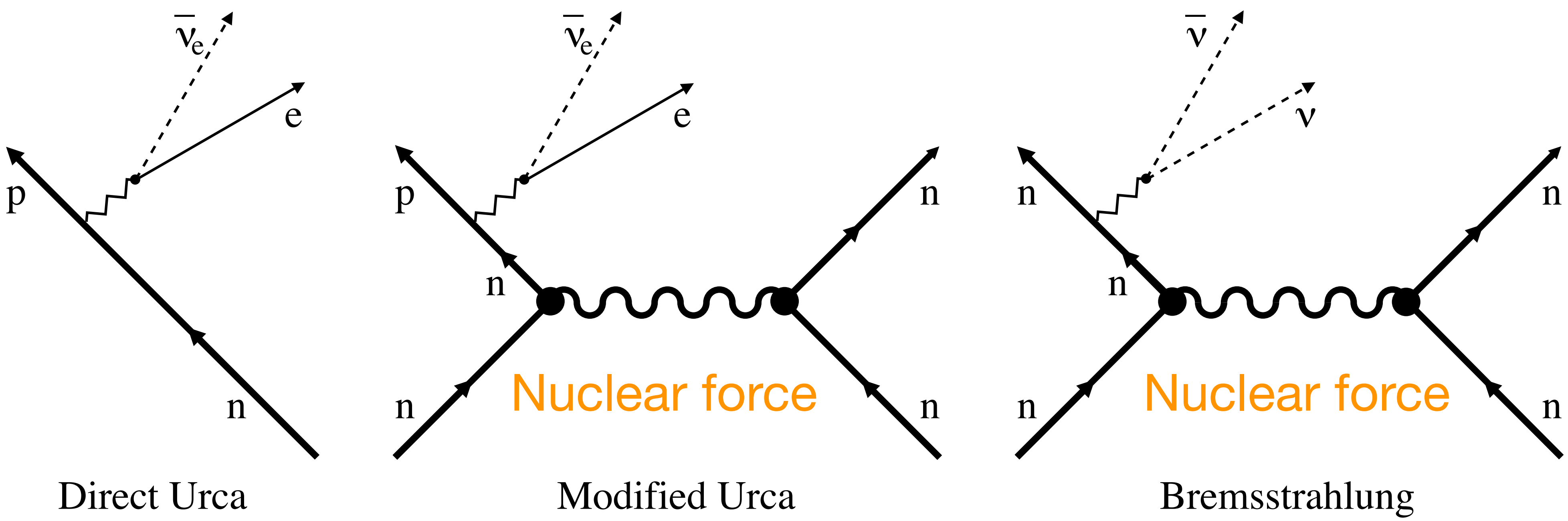
Named after a casino in Rio de Janeiro:

Cassino da Urca

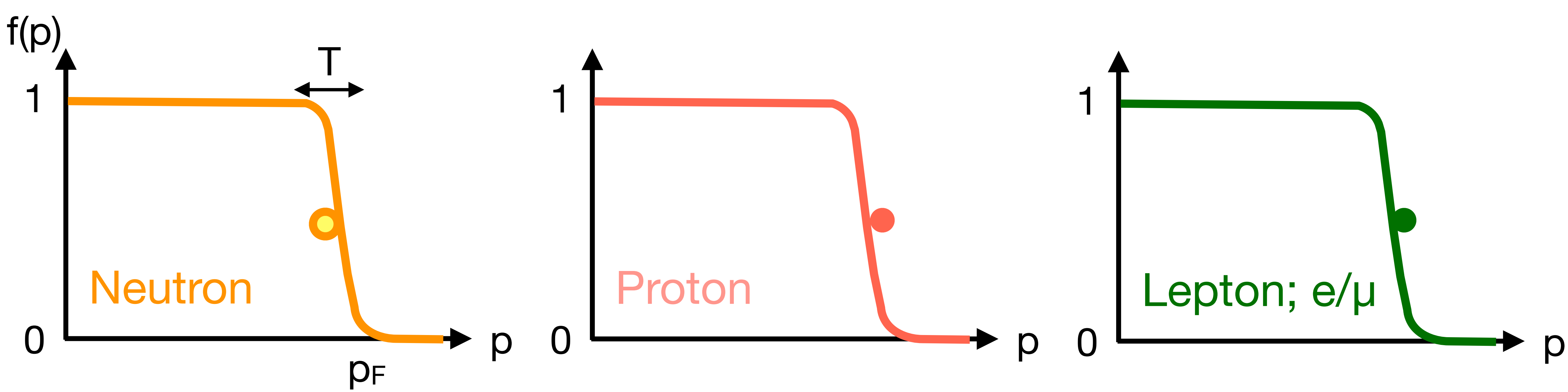
- To commemorate the casino where they first met.
- Rapid disappearance of energy (money) of a star (gambler).
- **U**n**R**ecordable **C**ooling **A**gent.
- "Urca" means "thief" in Russian.

Neutrino emission

First we consider the processes that occur without superfluidity.



These processes occur only near the Fermi surface.



β equilibrium

Inside neutron stars, β equilibrium is achieved via the direct/modified Urca reactions

Chemical equilibrium

$$\mu_e + \mu_p = \mu_n$$

Chemical potential of neutrino is zero since it can escape from neutron star.

Charge neutrality

$$n_p = n_e$$

Muons also appear in the region where $\mu_e > m_\mu$.

Chemical equilibrium

$$\mu_e = \mu_\mu \quad (\mu_\mu + \mu_p = \mu_n)$$

Charge neutrality

$$n_p = n_e + n_\mu$$

Direct Urca

Emissivity ≡ energy loss per volume per time.

of the direct Urca process is given by

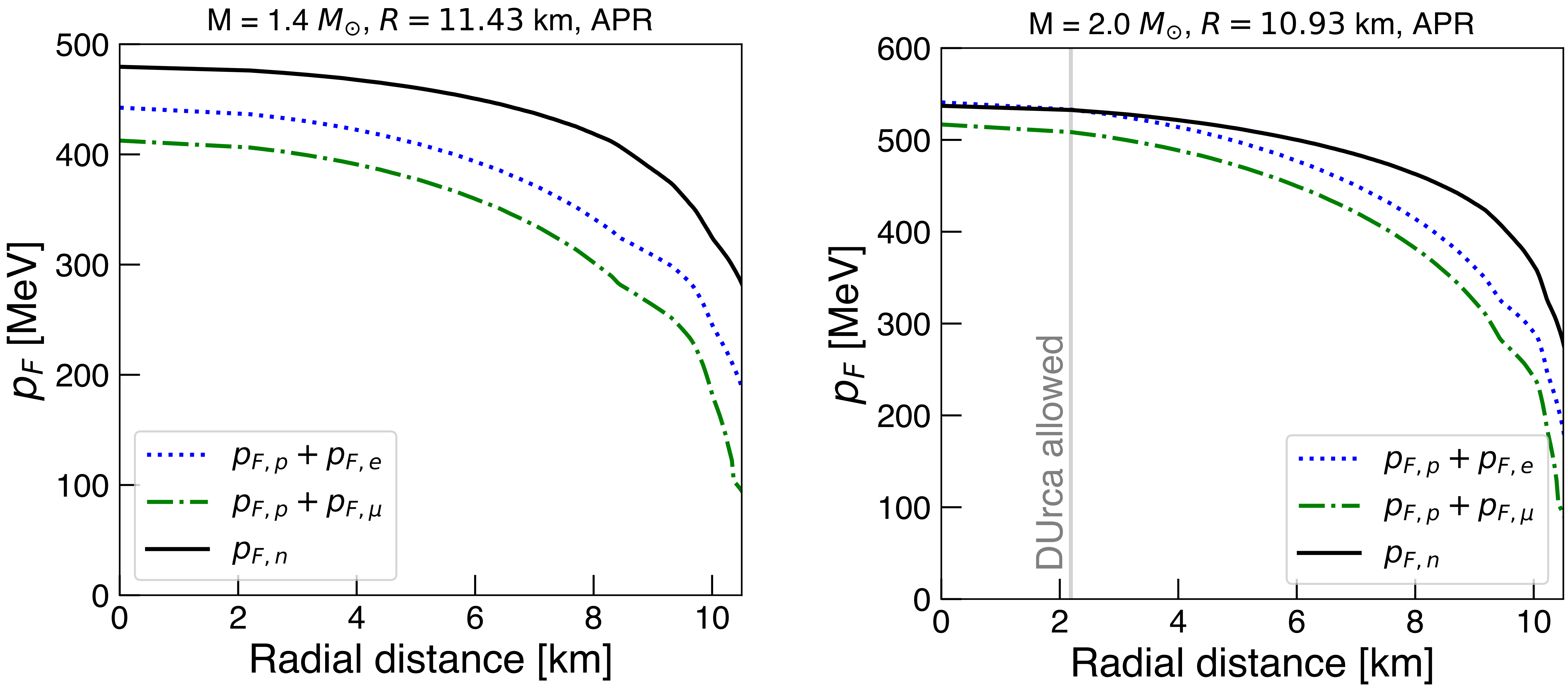
$$Q_D = \frac{457\pi}{10080} G_F^2 V_{ud}^2 (1+3g_A^2) m_{*,n} m_{*,p} m_{*,e} T^6 \Theta(p_{F,p} + p_{F,e} - p_{F,n})$$
$$\simeq 4 \times 10^{27} \times \left(\frac{T}{10^9\ \mathrm{K}}\right)^6 \Theta(p_{F,p} + p_{F,e} - p_{F,n})\ \mathrm{erg} \cdot \mathrm{cm}^{-3} \cdot \mathrm{s}^{-1}$$

- The step function comes from the momentum conservation.

$$p_{F,p} + p_{F,e} > p_{F,n}$$

- Direct Urca is the dominant process, if it occurs.

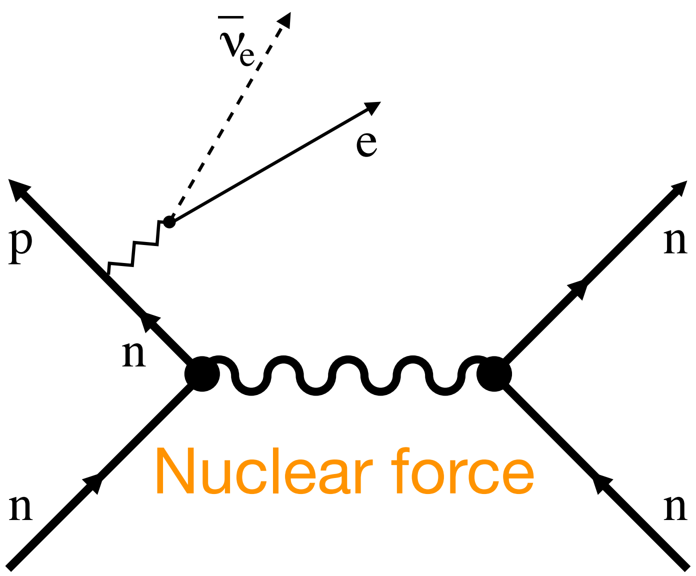
Direct Urca condition



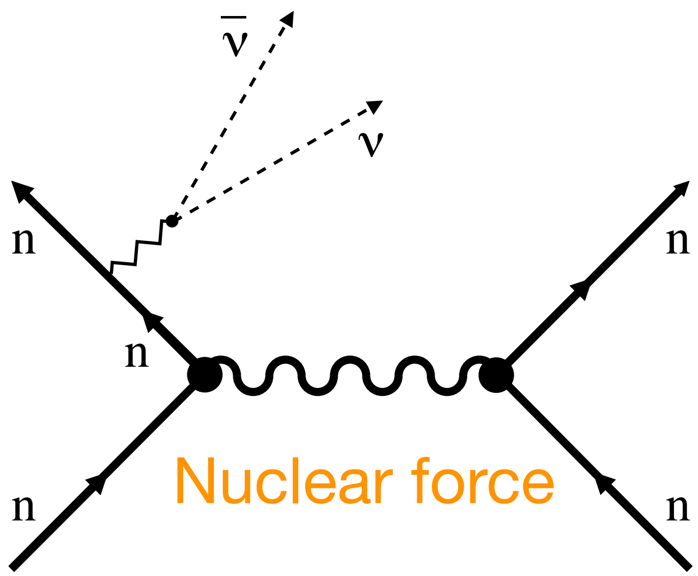
- Direct Urca can occur only in the high density region.
- It can occur only in relatively heavy stars.

For the APR equation of state, $M \gtrsim 1.97 M_\odot$

Modified Urca/bremsstrahlung



Modified Urca



Bremsstrahlung

If Direct Urca does not operate, Modified Urca/bremsstrahlung processes become dominant.

Momentum exchange with a spectator allows these processes to satisfy the momentum conservation.

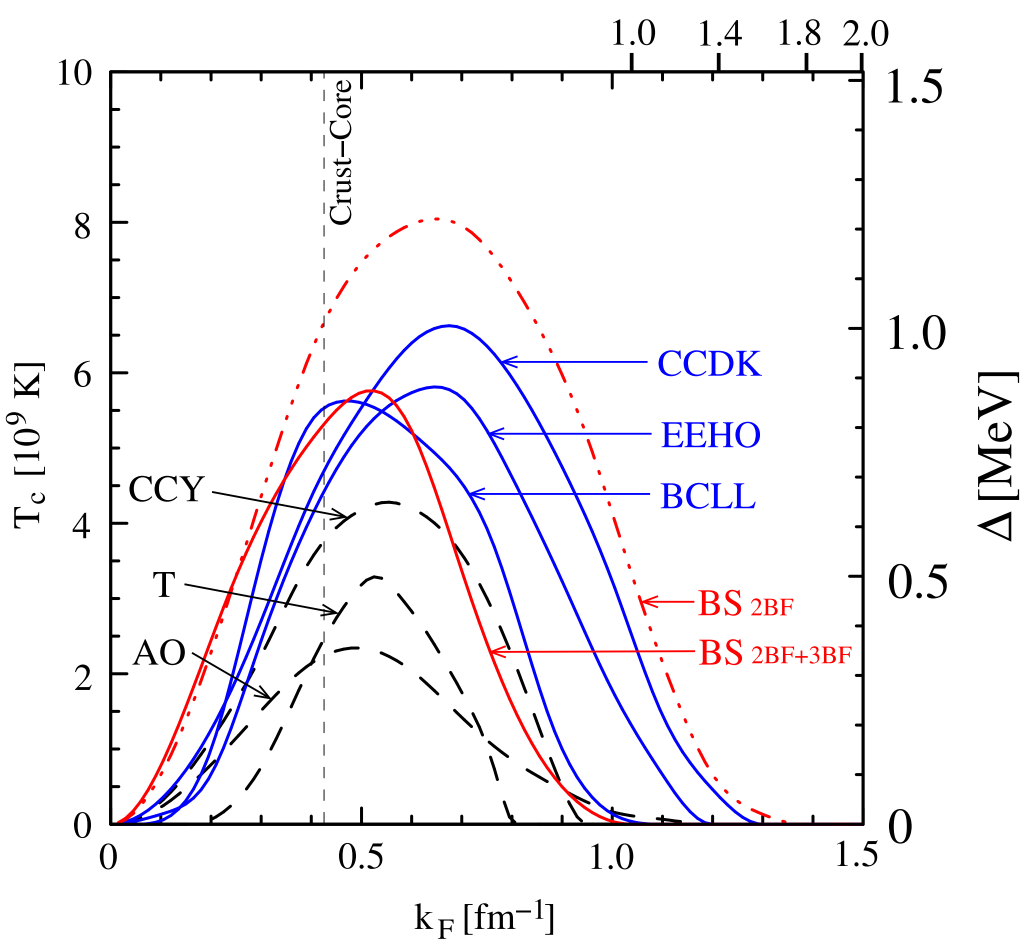
Nucleon pairing

Nucleons in a NS form pairings below their critical temperatures:

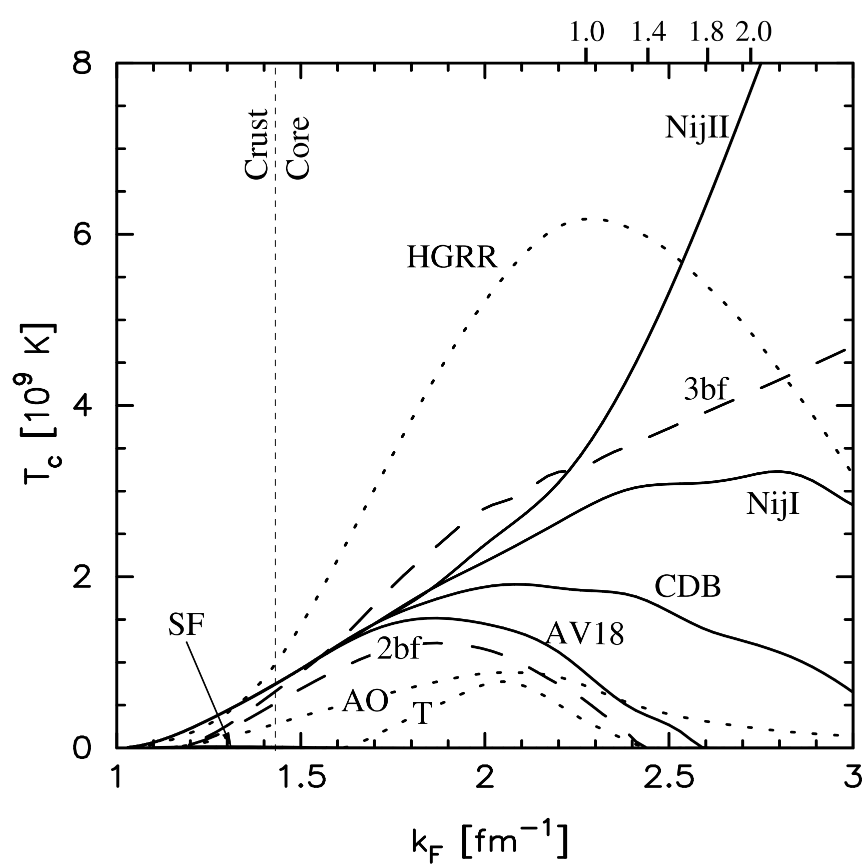
- Neutron singlet $^1S_0$ ← Only in the crust. Less important.
- Proton singlet $^1S_0$
- Neutron triplet $^3P_2$

} ← Form in the core. Important.

Proton singlet pairing gap



Neutron triplet pairing gap



D. Page, J. M. Lattimer, M. Prakash, A. W. Steiner [arXiv: 1302.6626].

Effects of nucleon parings

Nucleons in a NS form Cooper parings.

Energy spectrum

$$\epsilon_N(\boldsymbol{p}) \simeq \sqrt{\Delta_N^2 + v_{F,N}^2(p - p_{F,N})^2}$$

Δ_N: paring gap

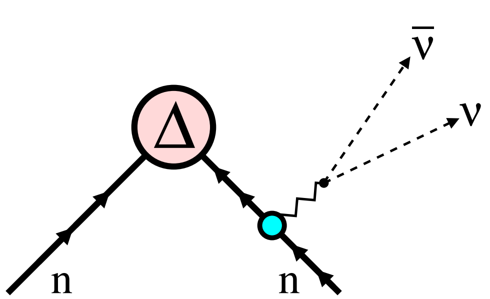
The paring gap introduces a suppression factor to

- Neutrino emission processes
- Heat capacity

$$\propto e^{-\frac{\Delta_N}{T}}$$

In addition, a new neutrino emission process is turned on:

- Pair-breaking and formation (PBF) process



PBF process

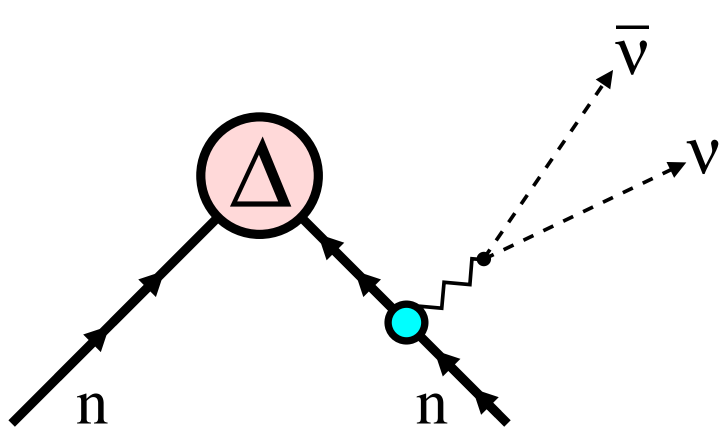
Thermal disturbance induces the breaking of nucleon pairs.

During the reformation of cooper pairs, the gap energy is released via neutrino emission.

This process significantly enhances the neutrino emission only when

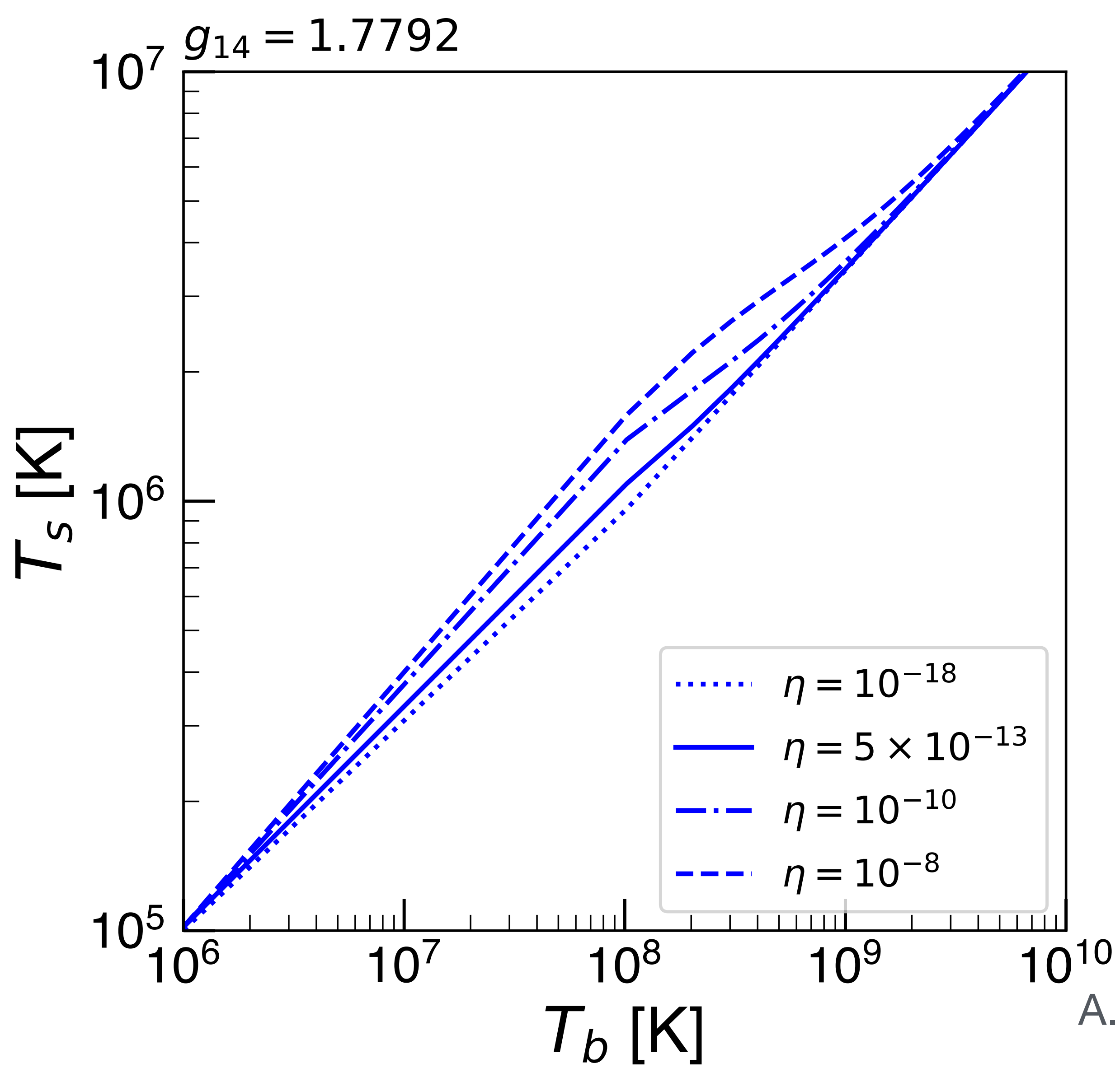
$$T \lesssim T_C$$

- If $T > T_C$, this process does not occur.
- If $T << T_C$, pair breaking rarely occurs.



Surface temperature

It is the surface temperature that we observe, so we need to relate it to the internal temperature.



This relation depends on the amount of light elements in the envelope.

$$\eta \equiv g_{14}^2 \Delta M / M$$

g_{14}: surface gravity in units of 10^{14} cm s^{-2}.

ΔM: mass of light elements.

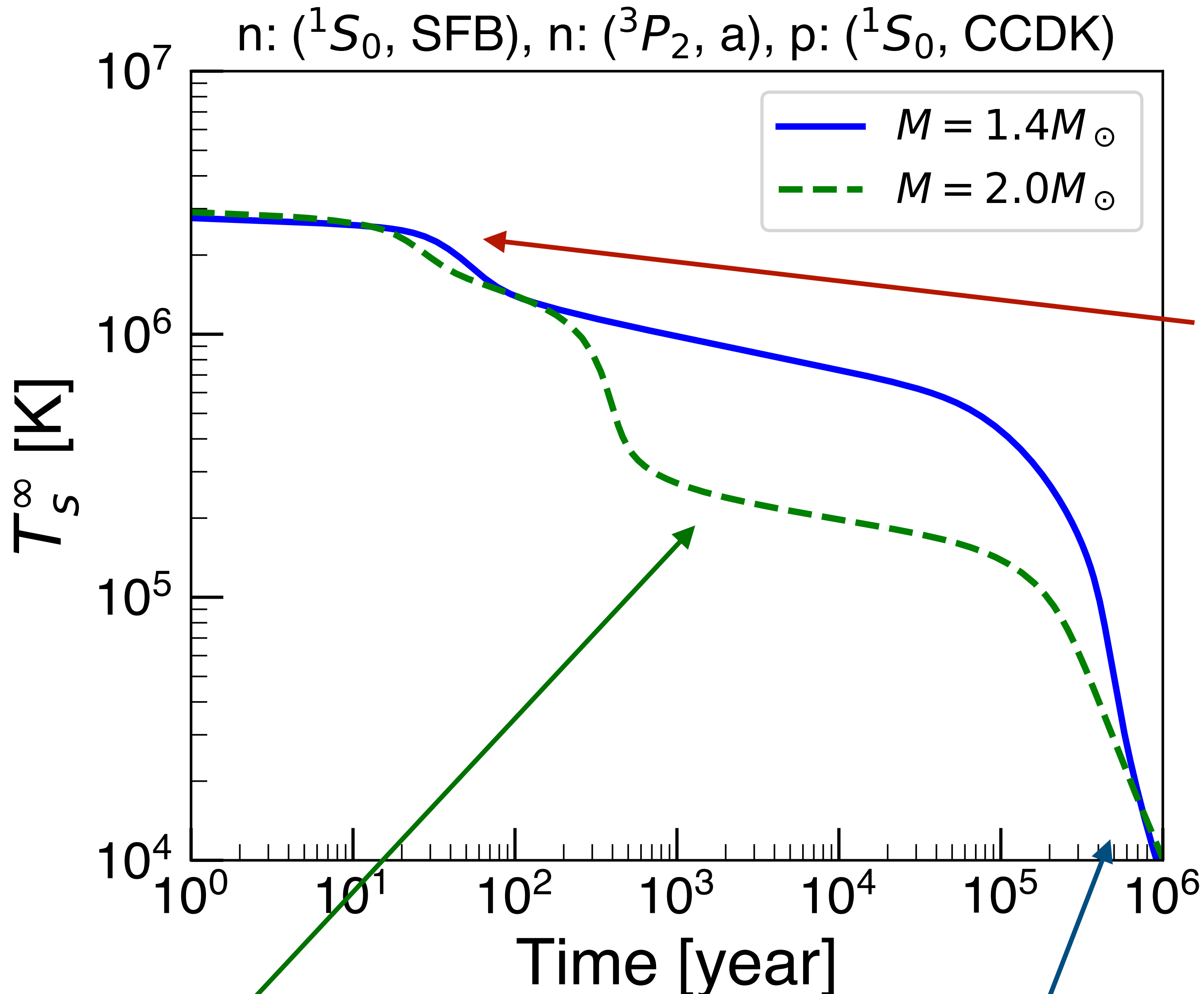
A. Y. Potekhin, G. Chabrier, and D. G. Yakovlev, A&A **323,** 415 (1997).

As the amount of light elements gets increased, the surface temperature becomes larger.

Light elements have large thermal conductivities.

Temperature evolution

We can now solve the equation for temperature evolution:

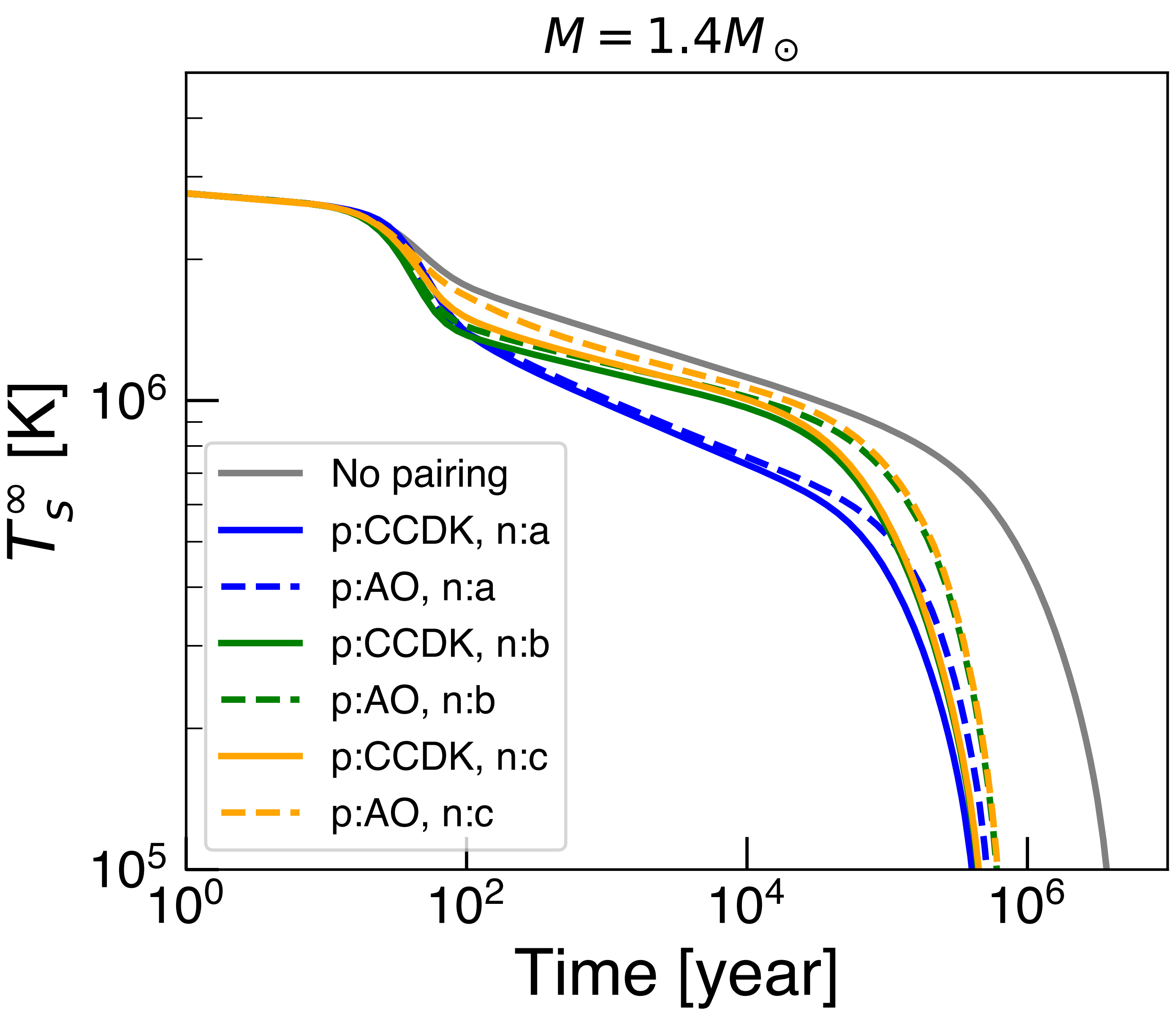


Before the thermal relaxation completed, the surface temperature does not follow the internal temperature.

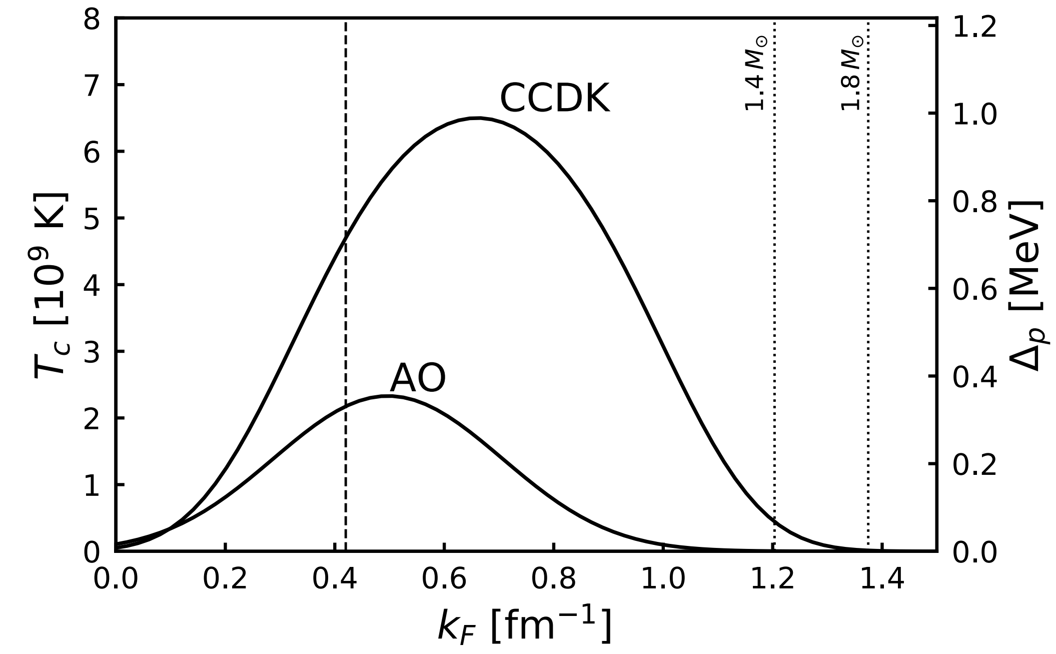
If Direct Urca occurs, the neutron star cools down rapidly.

Temperature of NSs (older than 10^6 years) is very low.

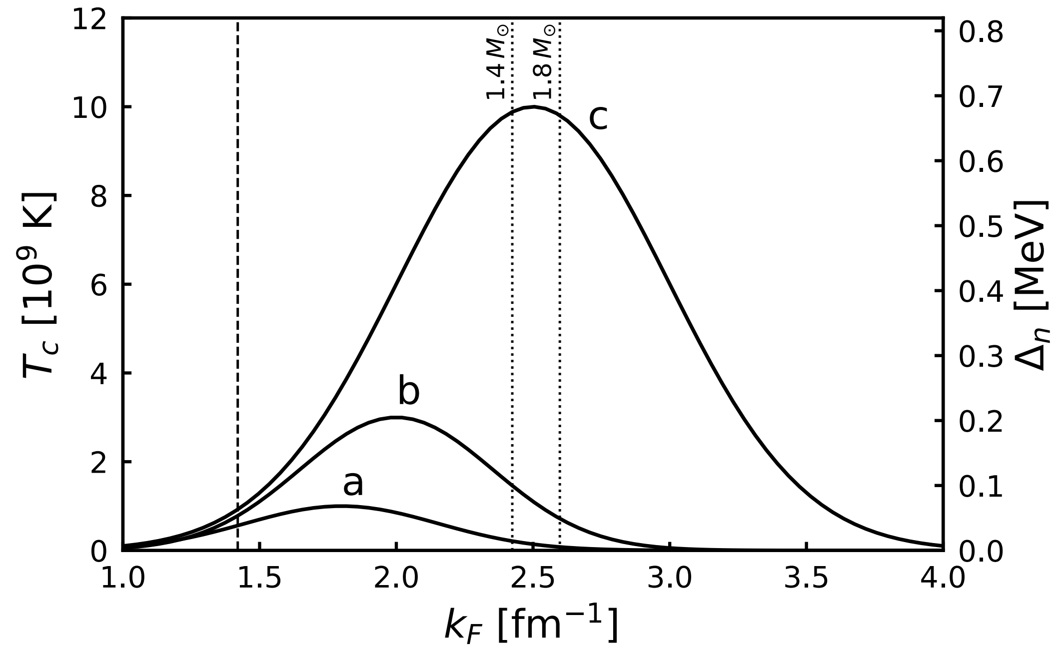
Temperature evolution (gap dependence)



Proton singlet gap



Neutron triplet gap



Uncertainty in nucleon gap models lead to the theoretical errors in the cooling calculation.

Challenge for standard cooling

On the other hand, there is an example of old cool neutron star.

- J2144-3933: $t_{\rm sd} = 3.33 \times 10^8$ years, $T_s^\infty < 4.2 \times 10^4$ K

S. Guillot, *et al.*, Astrophys. J. **874**, 175 (2019).

Is there any theory that can explain these observations on the equal footing??

Out of β equilibrium

The excess of energy is dissipated by

- Increase of neutrino emission
- Generation of heat

P. Haensel, Astron. Astrophys. **262**, 131 (1992); A. Reisenegger, Astrophys. J. **442**, 749 (1995).

Deviation from β equilibrium is quantified by

$$\eta_\ell \equiv \mu_n - \mu_p - \mu_\ell \quad (\ell = e, \mu)$$

Heating luminosity

$$L_H = \sum_{\ell=e,\mu} \sum_{N=n,p} \int dV \, \eta_\ell \cdot \Delta\Gamma_{M,N\ell}$$

where

$$\Delta\Gamma_{M,N\ell} \equiv \Gamma(n + N \to p + N + \ell + \bar{\nu}_\ell) - \Gamma(p + N + \ell \to n + N + \nu_\ell)$$

Evolution of chemical imbalance

The time evolution off η_ℓ is determined by

$$\frac{d\eta_e}{dt} = -\sum_{N=n,p} \int dV \left(Z_{npe} \Delta\Gamma_{M,Ne} + Z_{np} \Delta\Gamma_{M,N\mu} \right) + 2W_{npe} \Omega \dot{\Omega}$$

Bring the system back to equilibrium.

Drive the system out of equilibrium.

W < 0, Z > 0: coefficients which depend on NS structure.

R. Fernandez and A. Reisenegger, Astrophys. J. **625**, 291 (2005).

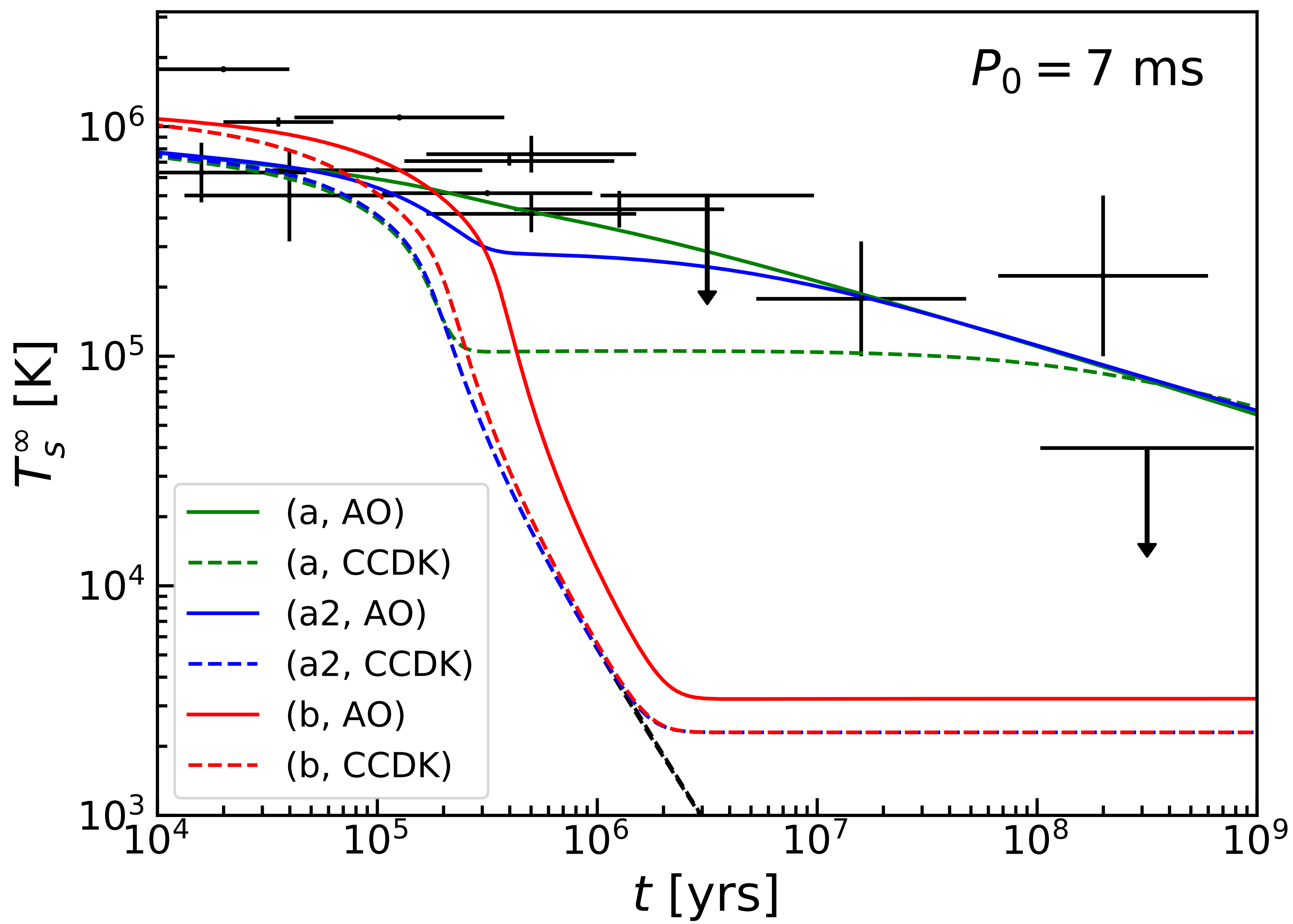
Once the second term wins, the imbalance increases.

Magnetic dipole radiation

$$\dot{\Omega} = -k\Omega^3 \quad \Rightarrow \quad \Omega\dot{\Omega} = -\frac{4\pi^2 P_{\rm now} \dot{P}_{\rm now}}{(P_0^2 + 2P_{\rm now}\dot{P}_{\rm now} t)^2}$$

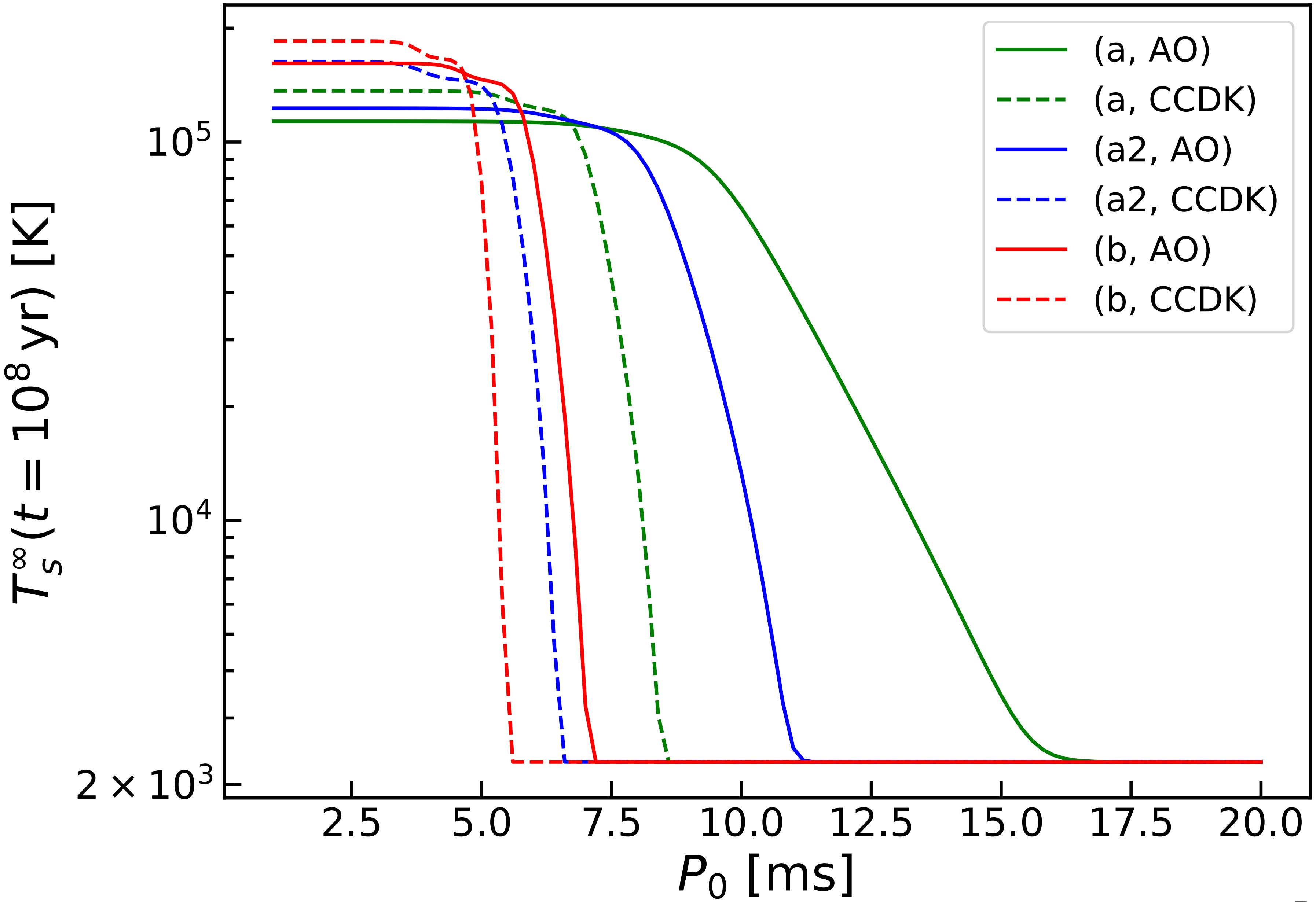
(P_0: initial period)

Gap dependence



K. Hamaguchi, N. Nagata, K. Yanagi, Phys. Lett. **B795**, 484 (2019).

Gap dependence



Courtesy of K. Yanagi.

Spin-down age

For magnetic dipole radiation,

$$\dot{\Omega} = -k\Omega^3 \qquad k = \frac{2B_s^2 \sin^2\alpha\, R^6}{3c^3 I} = -\frac{\dot{\Omega}_{\rm now}}{\Omega_{\rm now}^3} = \frac{P_{\rm now}\dot{P}_{\rm now}}{4\pi^2}$$

By solving this, we have

$$P(t) = \sqrt{P_0^2 + 2P_{\rm now}\dot{P}_{\rm now} t}$$

(P_0: initial period)

In particular, for $P_0 \ll P_{\rm now}$, we can estimate the neutron star age

$$t_{\rm sd} = \frac{P_{\rm now}}{2\dot{P}_{\rm now}}$$

t_{sd} is called spin-down age or characteristic age.

Pulsar age

Let us compare the spin-down age with the actual age in the case of the Crab pulsar.

Actual age

It was born in 1054, so its age is 967 years old.

Spin-down age

$$P = 0.033392 \text{ s}, \quad \dot{P} = 4.21 \times 10^{-13}$$

$$\Rightarrow \quad t_{\rm sd} = \frac{P}{2\dot{P}} = 1.26 \times 10^{3} \text{ yrs}$$

Agrees within ~ 30%.