

Seeking the origin of neutrino masses: a brief review

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1. ν mass = new physics
2. ν mass = new mass scale?
3. High-scale seesaw models
4. Low-scale models
5. Radiative models
6. Closing remarks

1. ν mass = new physics

Minimal standard model:

no RH neutrinos, no $Y=2$ scalar triplet \Rightarrow massless neutrinos, perturbative $L_{e,\mu,\tau}$ conservation

old-fashioned normalisation, sorry: $Q = I_3 + Y/2$

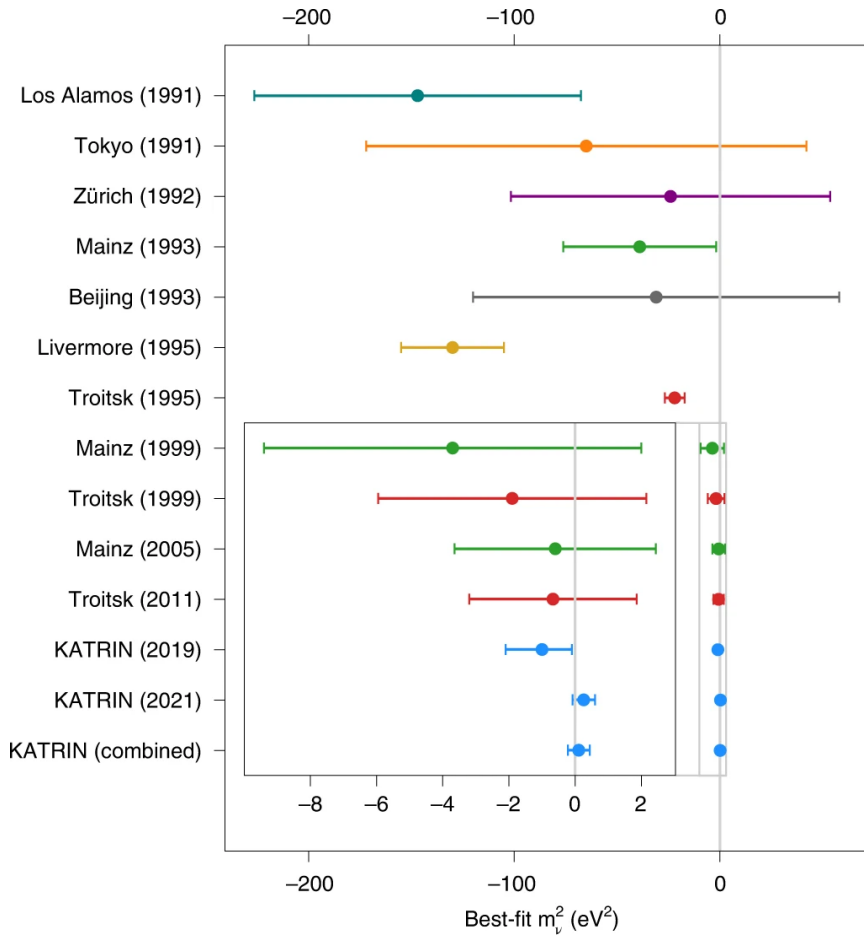
Massive neutrinos may be **Dirac** or **Majorana**.

If Majorana: first such states discovered \Rightarrow new physics. Plethora of mechanisms, always new dofs.

If Dirac: RH neutrinos needed \Rightarrow new dofs. (For singlet ν_R , no Majorana masses \Rightarrow impose L , new principle.)

2. ν mass = new mass scale?

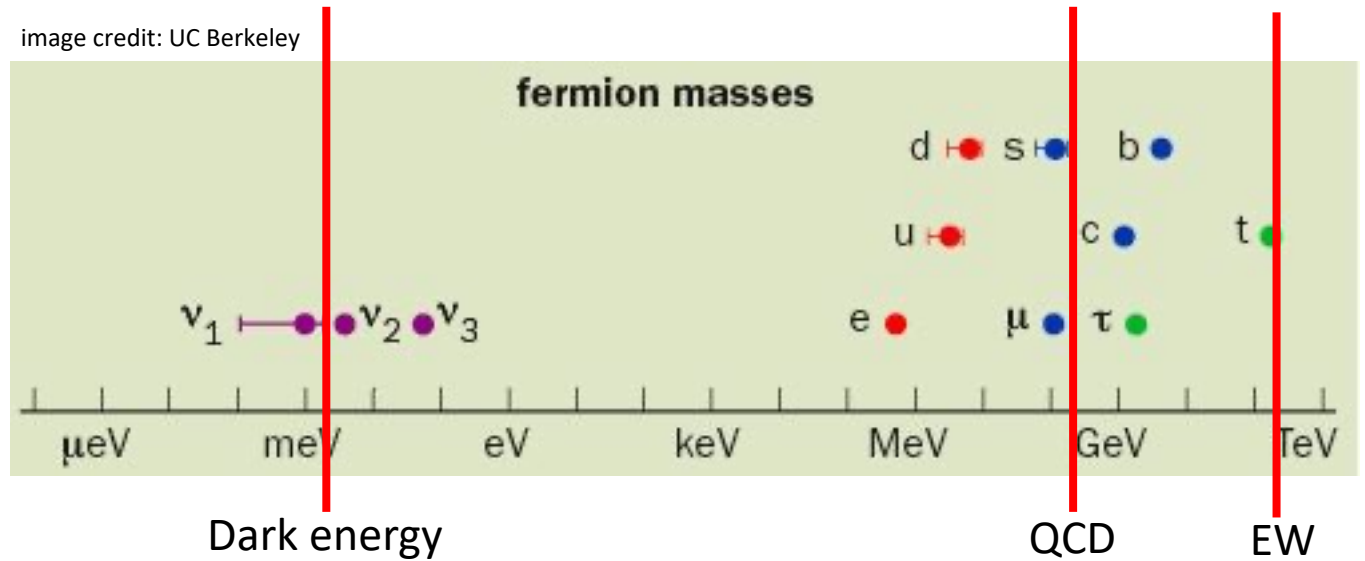
image credit: Nature Physics, KATRIN



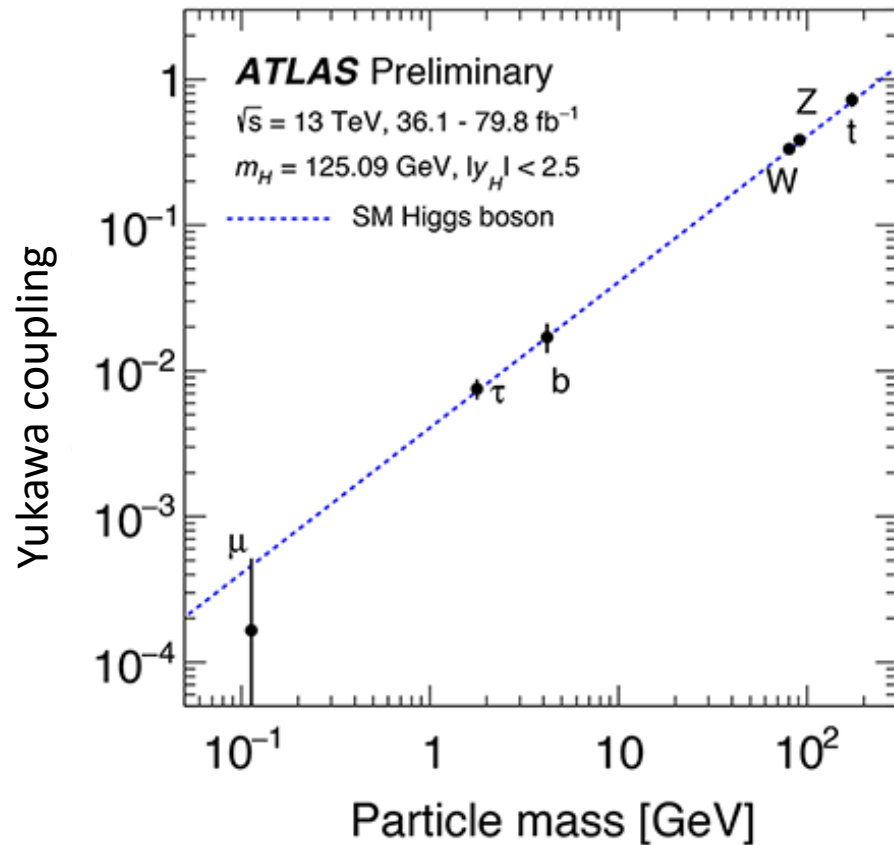
KATRIN β -decay endpoint: $m_\nu \lesssim 0.8$ eV

Cosmology: $m_\nu \lesssim 0.12 - 0.26$ eV

image credit: UC Berkeley



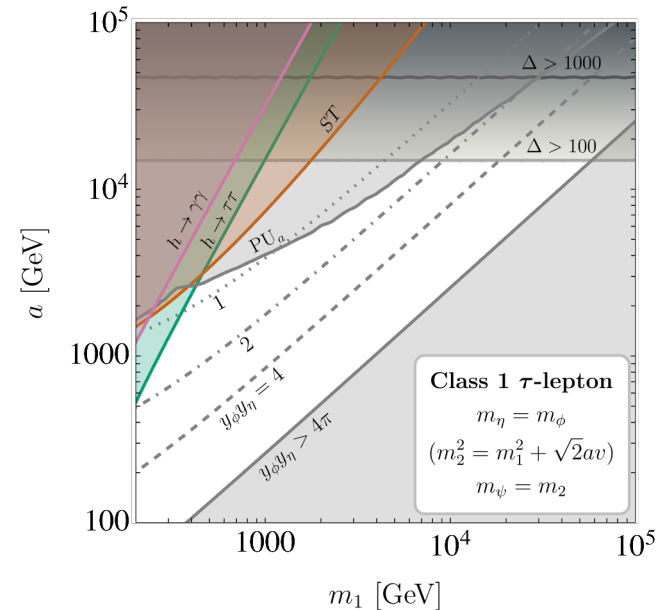
Maybe not new scale, despite $m_\nu \ll$ any other known nonzero particle mass?



Dirac ν , Yukawa coupling $\sim 10^{-12}$?

SM says electron Yukawa $\sim 10^{-6}$.

Is 10^{-6} OK? If so, is another factor of 10^{-6} also OK?



Despite the far left plot, not yet proven that even the b and τ masses must have SM origin.

Radiative origin also works.

Baker, Cox, RV 2021a, 2021b

For rest of talk, hypothesise that:

- m_ν is a new scale
- ν is Majorana

Model building dominated by

- explaining small m_ν
- origin of L violation

3. High-scale seesaw models

Interesting, well-known fact: lowest non-renormalisable SM effective operator is the Weinberg operator

L = LH lepton doublet
H = Higgs doublet
 λ = dimensionless coupling
M = new $\Delta L=2$ physics scale

$$\frac{\lambda}{M} LLHH$$

\Rightarrow Majorana neutrinos

$$m_\nu \sim \lambda \frac{v^2}{M}$$

seesaw formula

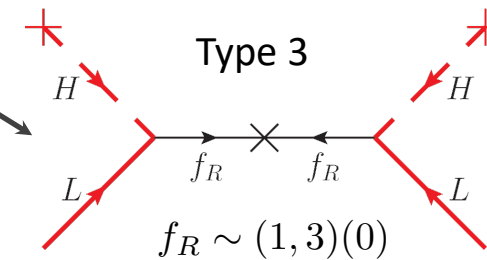
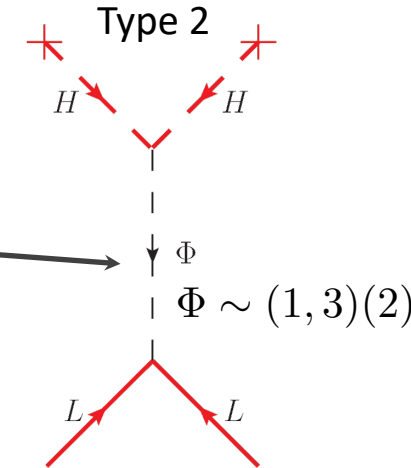
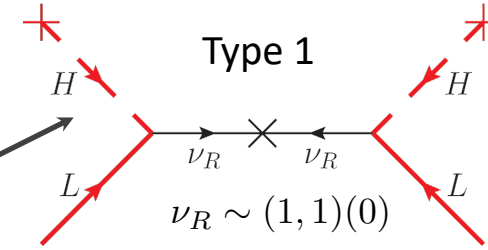
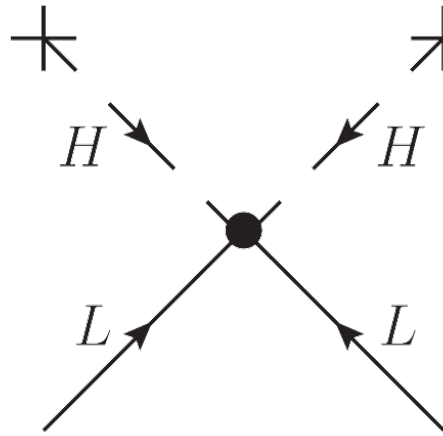
$m_\nu \ll v$ when $M \gg v$ i.e. seesaw effect when L-violation scale very high

$$m_\nu \sim 0.1 \text{ eV}, \quad v \sim 10^2 \text{ GeV} \implies M \sim 10^{14} \text{ GeV}$$

In its pure form, the seesaw scale is very high.
Testability is very low.

Type-1,2,3 seesaw models:

“Open up” LLHH in all minimal, tree-level ways.



Advantage of effective operator approach to constructing models is that you don't miss any.

Minkowski 1977
 Yanagida 1979
 Gell-Mann, Ramond, Slansky 1979
 Mohapatra, Senjanovic 1980

Magg, Wetterich 1980
 Schechter, Valle 1980
 Cheng, Li 1980
 Lazarides, Shafi, Wetterich 1981
 Wetterich 1981
 Mohapatra, Senjanovic 1981

Foot, Lew, He, Joshi 1989

Type 1: Mediator is massive Majorana ν_R gauge singlet.

If M very large, then untestable. ☹️

But has leptogenesis. 😊 Fukugita, Yanagida: 1986

(Same Lagrangian with small ν_R Majorana masses also interesting: the ν MSM.)

Asaka, Shaposhnikov 2005
Asaka, Blanchet, Shaposhnikov 2005

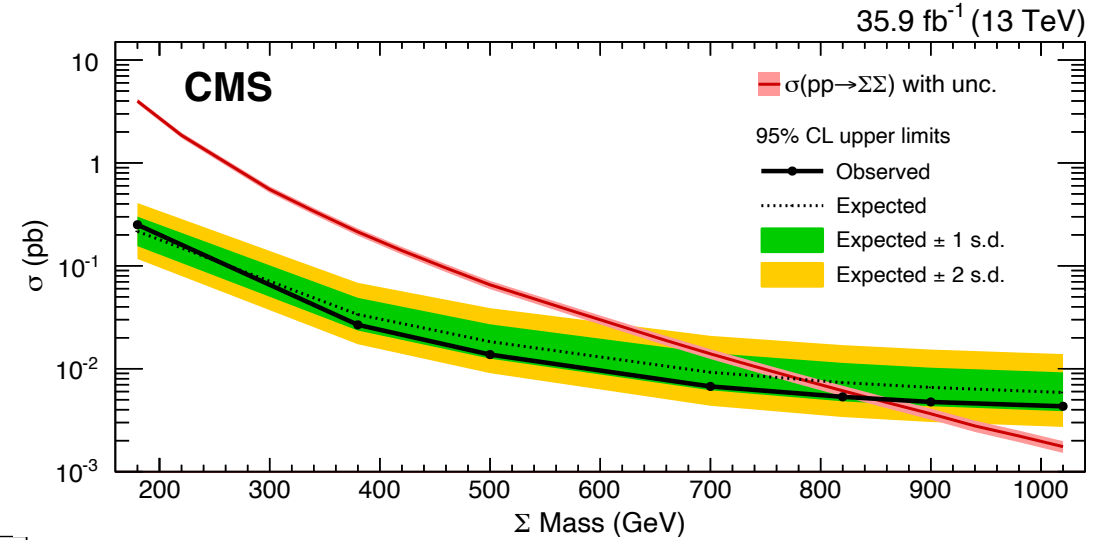
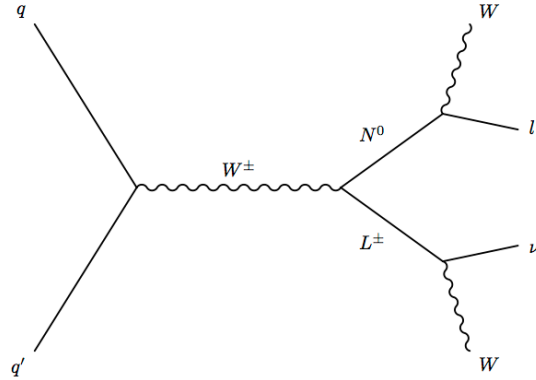
Type 2 and 3: Mediators have EW quantum numbers.

Better (but not great) prospects at colliders.

Example of search and bound: type 3 seesaw.

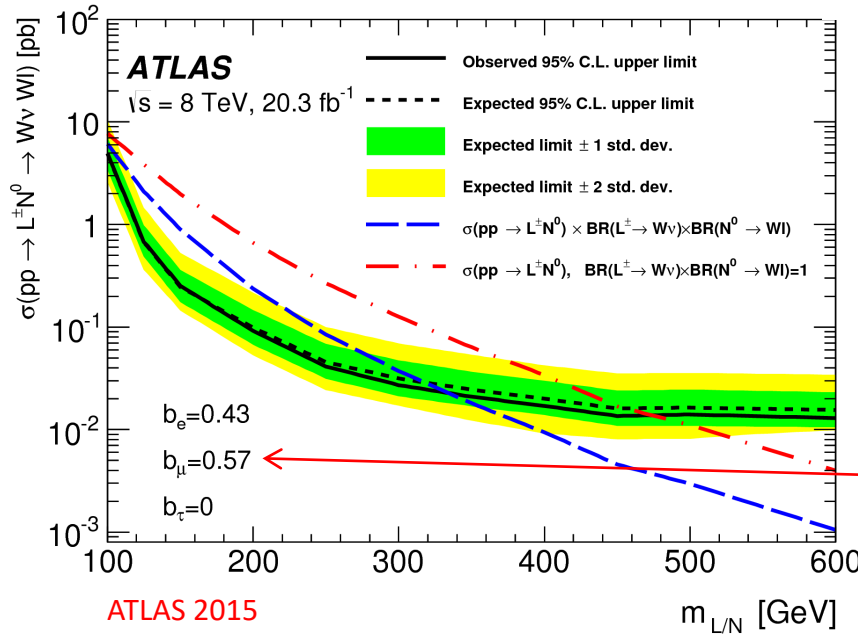
$$f_R \sim (1,3)(0) = (L^+, N^0, L^-)$$

Heavy lepton EW pair production.



CMS 2017

Flavour democratic BR choice



ATLAS 2015

Benchmark BR

Comments on Type I seesaw

At SM gauge group level, very compelling:

- Simply add gauge-singlet RH neutrinos
- Use most general renormalisable Lagrangian (standard Yukawa, ν_R Maj. masses)
- Get leptogenesis as wonderful byproduct
- Can identify seesaw and Peccei-Quinn scales (SMASH, VISHv)

Langacker, Peccei, Yanagida 1986
Shin 1987
Salvio 2015, 2019
Ballesteros+ 2017a, 2017b, 2019
Sopov, RV 2022

But more complicated when ν_R embedded in non-trivial gauge multiplet:

- LR symmetric model: ν_R masses from Yuk with RH triplet scalar
- Pati-Salam: need $(10^*, 1, 3)$ scalar
- SO(10): need 126 scalar

Some may like connection with GUT breaking, but if you like smaller multiplets then ...

4. Low-scale models

aka inverse and linear seesaws

In LRSM = $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, add a gauge singlet fermion S_R :

$$\left(\bar{\nu}_L, \overline{(\nu_R)^c} (\overline{S_R})^c \right) \begin{bmatrix} \cancel{(3, 1)(2)} & (2, 2)(0) & (2, 1)(-1) \\ (2, 2)(0) & \cancel{(1, 3)(2)} & (1, 2)(-1) \\ (2, 1)(-1) & (1, 2)(-1) & (1, 1)(0) \end{bmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \\ S_R \end{pmatrix}$$

The triplet scalars are not needed. Doublets suffice.

SO(10) level:

$$\begin{bmatrix} \cancel{126} & 10 + \cancel{120} & 16 \\ 10 + \cancel{120} & \cancel{126} & 16 \\ 16 & 16 & 1 \end{bmatrix}$$

126 can be replaced by 16.

Putting in mass scales:

$$\begin{bmatrix} 0 & m & m_L \\ m & 0 & m_R \\ m_L & m_R & \mu \end{bmatrix}$$

Inverse seesaw: $m_L = 0$ and $\mu \ll m \ll m_R$

Wyler, Wolfenstein 1983
Mohapatra 1986
Mohapatra, Valle 1986
Ma 1987

Light neutrino mass:

$$m_\nu \sim \mu \left(\frac{m}{m_R} \right)^2$$

Double suppression: small μ and m/m_R .

Sterile admixture $\sim m/m_R$, so relatively large if $m_R \sim \text{few TeV}$.

Small μ explicitly violates L. Technically natural.

Putting $m_R \sim 10 \text{ TeV}$ gives $\frac{\mu}{\text{MeV}} \sim \frac{10}{(m/\text{GeV})^2}$ so not ridiculously small.
 $m_\nu \sim 0.1 \text{ eV}$

Linear seesaw: $\mu=0$, $m_L \leq m \ll m_R$

Akhmedov+ 1996
Malinksý, Ramão, Valle 2005

Light neutrino mass:

$$m_\nu \sim \frac{m m_L}{m_R}$$

$m_L=0$ restores L conservation, so $m_L \ll m$ technically natural.

Thus double suppression possible: small m_L and m/m_R .

Scale of new physics can again be relatively low e.g. $m_R \sim 10$ TeV get $\frac{m_L}{\text{MeV}} \sim \frac{10^{-3}}{m/\text{GeV}}$

ISS & LSS produce small m_ν

- without tiny ν Dirac masses, and
- with low scale of new physics,

but at the expense of introducing (technically natural) small L-violation scales μ and m_L .

Q: Are they improvements over regular Dirac neutrinos?

5. Radiative models

$\Delta L = 2$ effective operator \rightarrow open it up aka UV complete \rightarrow neutrino self-energy and mass

Systematic model-building and classification procedure.

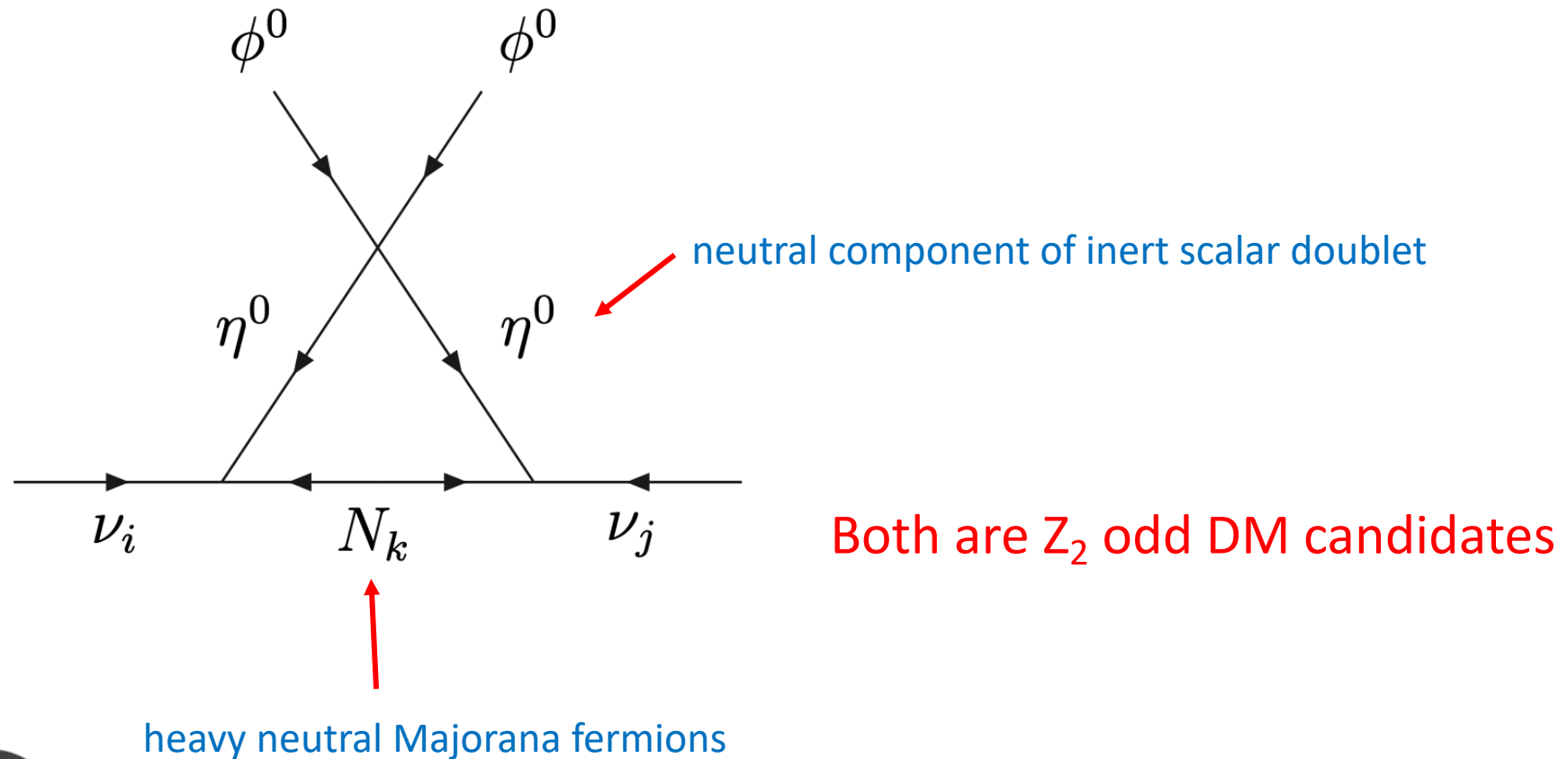
Two complementary approaches:

(Generalised) Weinberg operators $\frac{\lambda}{M^{1+2n}} LLHH (H^\dagger H)^n$ Valencia group: Hirsch, Cepedello et al (many papers)

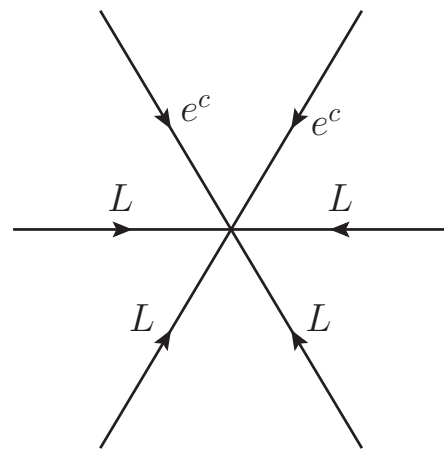
non-Weinberg operators \rightarrow ν self-energy graphs with both SM particles and exotics Babu, Leung 2001
de Gouvêa, Jenkins 2008
Melbourne group (many papers)

Possible connection with the Dark Side: scotogenesis!

Ma 2006

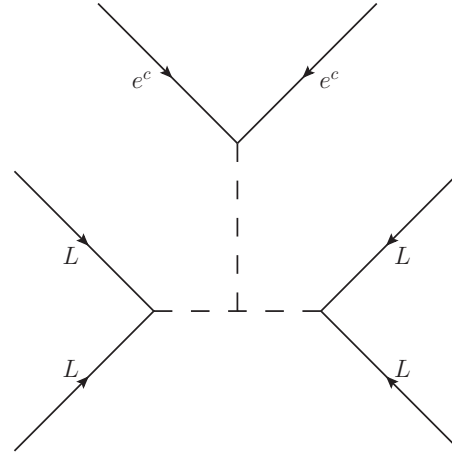


Historic example: Zee-Babu model

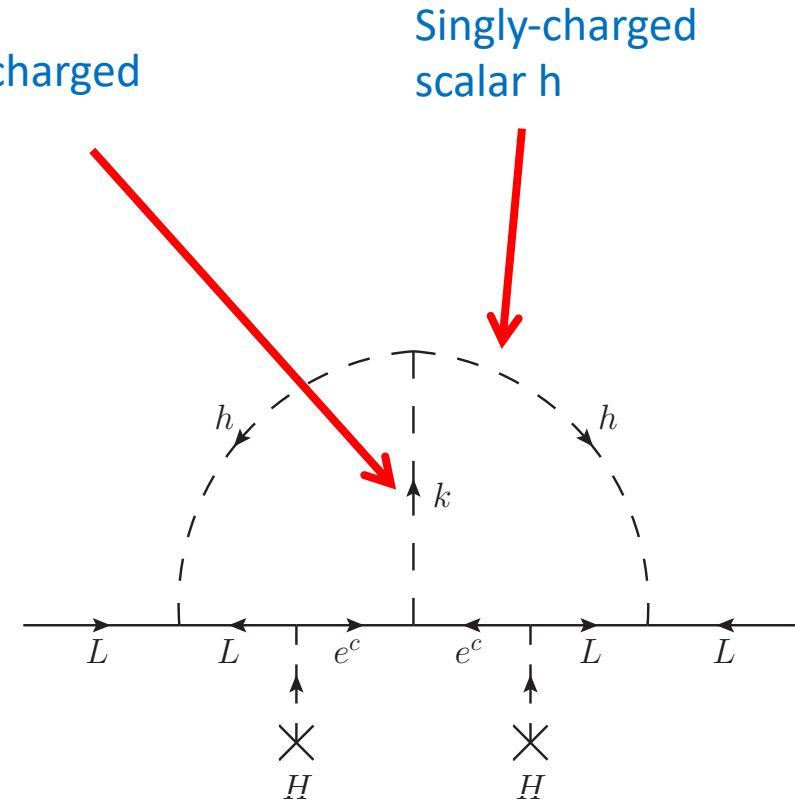


$$O_9 = LLLe^c Le^c$$

Effective op



Opening it up



2-loop ν mass diagram

The exotics k and h can be searched for at the LHC.

There are many operators
up to mass dimension 11 ...

Labels	Operator	Models	Filtered	Loops	Λ [TeV]
1	$L^i L^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
2	$L^i L^j L^k \bar{e} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	8	2	1	$4 \cdot 10^7$
3a	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ij} \epsilon_{kl}$	9	2	2	$2 \cdot 10^5$
3b	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	14	5	1	$9 \cdot 10^7$
4a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
4b	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
5a	$L^i L^j Q^k \bar{d} H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	790	36	2	$6 \cdot 10^5$
5b	$\mathcal{O}_1 \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	492	14	1,2	$6 \cdot 10^5$
5c	$\mathcal{O}_{3a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	509	0	2,3	$1 \cdot 10^3$
5d	$\mathcal{O}_{3b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	799	16	1,2	$6 \cdot 10^5$
6a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	289	14	2	$2 \cdot 10^7$
6b	$\mathcal{O}_1 \cdot \tilde{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	177	0	1,2	$2 \cdot 10^7$
6c	$\mathcal{O}_{4a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	262	0	1,2	$2 \cdot 10^7$
6d	$\mathcal{O}_{4b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	208	0	2,3	$6 \cdot 10^4$
7	$L^i \bar{e}^\dagger Q^j \tilde{Q}^k H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	240	15	2	$2 \cdot 10^5$
8	$L^i \bar{e}^\dagger \bar{u}^\dagger \bar{d} H^j \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
9	$L^i L^j L^k L^l \bar{e} \bar{e} \cdot \epsilon_{ik} \epsilon_{jl}$	14	1	2	$3 \cdot 10^3$
10	$L^i L^j L^k \bar{e} Q^l \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	50	1	2	$6 \cdot 10^3$
11a	$L^i L^j Q^k Q^l \bar{d} \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	48	0	3	$4 \cdot 10^1$
11b	$L^i L^j Q^k Q^l \bar{d} \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	72	16	2	$1 \cdot 10^4$
12a	$L^i L^j \tilde{Q}^k \bar{Q}^l \bar{u}^\dagger \bar{u}^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	19	0	2	$2 \cdot 10^7$
12b	$L^i L^j \tilde{Q}^k \bar{Q}^l \bar{u}^\dagger \bar{u}^\dagger \cdot \epsilon_{ij} \epsilon_{kl}$	17	4	3	$6 \cdot 10^4$
13	$L^i L^j L^k \bar{e} \tilde{Q}^l \bar{u}^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	12	0	2	$2 \cdot 10^5$
14a	$L^i L^j Q^k \bar{Q}^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	29	1	3	$1 \cdot 10^3$

Labels	Operator	Models	Filtered	Loops	Λ [TeV]
14b	$L^i L^j Q^k \bar{Q}^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	43	1	2	$6 \cdot 10^5$
15	$L^i L^j L^k L^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	12	1	3	$1 \cdot 10^3$
16	$L^i L^j \bar{e} \bar{e}^\dagger \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij}$	13	1	3	$1 \cdot 10^3$
17	$L^i L^j \bar{u}^\dagger \bar{d} \bar{d} \bar{d}^\dagger \cdot \epsilon_{ij}$	18	12	3	$1 \cdot 10^3$
18	$L^i L^j \bar{u} \bar{u}^\dagger \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij}$	22	8	3	$1 \cdot 10^3$
19	$L^i \bar{e}^\dagger Q^j \bar{u}^\dagger \bar{d} \bar{d} \cdot \epsilon_{ij}$	27	0	3,4	$2 \cdot 10^{-1}$
20	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij}$	27	3	3,4	$8 \cdot 10^{-1}$
21a	$L^i L^j L^k \bar{e} Q^l \bar{u} H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	3943	1	2,3	$2 \cdot 10^3$
21b	$L^i L^j L^k \bar{e} Q^l \bar{u} H^m H^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	4080	4	3	$2 \cdot 10^3$
22a	$L^i L^j L^k L^l \bar{e} \bar{e}^\dagger H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	726	0	2	$2 \cdot 10^7$
22b	$\mathcal{O}_2 \cdot \bar{L}^i \bar{e}^\dagger H^j \cdot \epsilon_{ij}$	931	0	2	$2 \cdot 10^7$
23a	$L^i L^j L^k \bar{e} \tilde{Q}^l \bar{d}^\dagger H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	780	0	2,3	$4 \cdot 10^1$
23b	$\mathcal{O}_2 \cdot \tilde{Q}^i \bar{d}^\dagger H^j \cdot \epsilon_{ij}$	969	0	2,3	$4 \cdot 10^1$
24a	$L^i L^j Q^k Q^l \bar{d} \bar{d} H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	9613	193	3	$9 \cdot 10^1$
24b	$L^i L^j Q^k Q^l \bar{d} \bar{d} H^m \tilde{H}^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	6058	110	3	$9 \cdot 10^1$
24c	$\mathcal{O}_{3a} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	6022	34	3,4	1
24d	$\mathcal{O}_{3b} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	9616	211	2,3	$9 \cdot 10^1$
24e	$\mathcal{O}_{11a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	3834	18	3,4	1
24f	$\mathcal{O}_{11b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	5915	131	2,3	$9 \cdot 10^1$
25a	$L^i L^j Q^k Q^l \bar{u} \bar{d} H^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	5960	151	2,3	$4 \cdot 10^3$
25b	$\mathcal{O}_{3a} \cdot Q^i \bar{u} H^j \cdot \epsilon_{ij}$	5913	9	3,4	10
25c	$\mathcal{O}_{3b} \cdot Q^i \bar{u} H^j \cdot \epsilon_{ij}$	14036	470	2,3	$4 \cdot 10^3$
26a	$L^i L^j \tilde{L}^k \bar{e}^\dagger Q^l \bar{d} H^m H^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	1600	0	3	$4 \cdot 10^1$
26b	$L^i L^j \tilde{L}^k \bar{e}^\dagger Q^l \bar{d} H^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	1040	0	2,3	$4 \cdot 10^1$
26c	$\mathcal{O}_{3a} \cdot \bar{L}^i \bar{e}^\dagger H^j \cdot \epsilon_{ij}$	1149	0	3	$4 \cdot 10^1$
26d	$\mathcal{O}_{3b} \cdot \bar{L}^i \bar{e}^\dagger H^j \cdot \epsilon_{ij}$	1797	0	2,3	$4 \cdot 10^1$
27a	$L^i L^j Q^k Q^l \bar{d} \bar{d}^\dagger H^m H^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	3851	164	2	$2 \cdot 10^7$
27b	$L^i L^j Q^k Q^l \bar{d} \bar{d}^\dagger H^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	2226	74	2	$2 \cdot 10^7$
27c	$\mathcal{O}_{3a} \cdot \bar{Q}^i \bar{d}^\dagger H^j \cdot \epsilon_{ij}$	2469	33	3	$6 \cdot 10^4$
27d	$\mathcal{O}_{3b} \cdot \bar{Q}^i \bar{d}^\dagger H^j \cdot \epsilon_{ij}$	3443	165	2	$2 \cdot 10^7$
28a	$L^i L^j Q^k Q^l \bar{u}^\dagger \bar{d} H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	4038	64	3	$4 \cdot 10^3$
28b	$L^i L^j Q^k Q^l \bar{u}^\dagger \bar{d} H^m \tilde{H}^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	4103	0	3,4	10
28c	$L^i L^j Q^k Q^l \bar{u}^\dagger \bar{d} H^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jn} \epsilon_{lm}$	4305	123	3	$4 \cdot 10^3$
28d	$\mathcal{O}_{3a} \cdot \bar{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	2749	7	3,4	10
28e	$\mathcal{O}_{3b} \cdot \bar{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	4304	90	2,3	$4 \cdot 10^3$
28f	$\mathcal{O}_{4a} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	4039	74	2,3	$4 \cdot 10^3$
28g	$\mathcal{O}_{4b} \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	2748	14	3,4	10

⋮

4 pages omitted

Babu, Leung 2001
de Gouvêa, Jenkins 2008
Angel, Rodd, RV 2013
Angel, Cai, Rodd, Schmidt, RV 2013
Cai, Clarke, Schmidt, RV 2015
Bigaran, Gargalionis, RV 2019
Gargalionis, Popa-Mateiu, RV 2020
Gargalionis, RV 2021

Labels	Operator	Models	Filtered	Loops	Λ [TeV]
D8c	$L^i L^j Q^k \bar{Q}^l (DH)^m H^n \cdot \epsilon_{ik} \epsilon_{jl} \epsilon_{mn}$	25	0	2	$10 \cdot 10^6$
D8d	$L^i L^j Q^k \bar{Q}^l (DH)^m H^n \cdot \epsilon_{im} \epsilon_{jk} \epsilon_{ln}$	53	11	1	$4 \cdot 10^9$
D8e	$L^i L^j Q^k \bar{Q}^l (DH)^m H^n \cdot \epsilon_{im} \epsilon_{jl} \epsilon_{kn}$	44	6	1	$4 \cdot 10^9$
D8f	$L^i L^j Q^k \bar{Q}^l (DH)^m H^n \cdot \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	30	5	1	$4 \cdot 10^9$
D8g	$L^i L^j Q^k \bar{Q}^l (DH)^m H^n \cdot \epsilon_{ij} \epsilon_{km} \epsilon_{ln}$	35	7	2	$10 \cdot 10^6$
D8h	$L^i L^j Q^k \bar{Q}^l (DH)^m H^n \cdot \epsilon_{ij} \epsilon_{kn} \epsilon_{lm}$	35	7	2	$10 \cdot 10^6$
D8i	$L^i L^j Q^k \bar{Q}^l (DH)^m H^n \cdot \epsilon_{ij} \epsilon_{kl} \epsilon_{mn}$	16	3	2	$10 \cdot 10^6$
D9a	$L^i L^j \bar{d} \bar{d}^\dagger (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	30	5	1	$4 \cdot 10^9$
D9b	$L^i L^j \bar{d} \bar{d}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	16	4	2	$10 \cdot 10^6$
D10a	$(DL)^i L^j \bar{u}^\dagger \bar{d} H^k \tilde{H}^l \cdot \epsilon_{il} \epsilon_{jk}$	56	13	2,3	$1 \cdot 10^3$
D10b	$(DL)^i L^j \bar{u}^\dagger \bar{d} H^k \tilde{H}^l \cdot \epsilon_{ij} \epsilon_{kl}$	36	7	2,3	$1 \cdot 10^3$
D10c	$(DL)^i L^j \bar{u}^\dagger \bar{d} H^k \tilde{H}^l \cdot \epsilon_{ik} \epsilon_{jl}$	56	13	2,3	$1 \cdot 10^3$
D11	$(DL)^i L^j (D\bar{u}^\dagger) (D\bar{d}) \cdot \epsilon_{ij}$	—	—	2,3	$1 \cdot 10^3$
D12a	$L^i L^j \bar{u} \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	30	5	1	$4 \cdot 10^9$
D12b	$L^i L^j \bar{u} \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	16	4	2	$10 \cdot 10^6$
D13a	$(DL)^i L^j \tilde{Q}^k (D\bar{u}^\dagger) H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
D13b	$(DL)^i L^j \tilde{Q}^k (D\bar{u}^\dagger) H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
D14a	$L^i \bar{e}^\dagger Q^j \bar{d} (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	53	0	2	$6 \cdot 10^3$
D14b	$L^i \bar{e}^\dagger Q^j \bar{d} (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	53	0	2	$6 \cdot 10^3$
D14c	$L^i \bar{e}^\dagger Q^j \bar{d} (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	27	0	2	$6 \cdot 10^3$
D15	$(DL)^i \bar{e}^\dagger (D\bar{u}^\dagger) \bar{d} H^j \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
D16a	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	58	8	2	$2 \cdot 10^5$
D16b	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	58	8	2	$2 \cdot 10^5$
D16c	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	27	4	2	$2 \cdot 10^5$
D17	$\bar{e}^\dagger \bar{e}^\dagger \bar{u}^\dagger \bar{d} (DH)^j H^k \cdot \epsilon_{ij}$	16	7	3,4	$2 \cdot 10^{-1}$
D18a	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	53	1	0,1	$4 \cdot 10^9$
D18b	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jl} \epsilon_{mn}$	53	1	0,1	$4 \cdot 10^9$
D18c	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{im} \epsilon_{jl} \epsilon_{kn}$	53	1	0,1	$4 \cdot 10^9$
D18d	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ij} \epsilon_{km} \epsilon_{ln}$	24	1	1,2	$10 \cdot 10^6$
D18e	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{in} \epsilon_{jl} \epsilon_{km}$	34	0	1	$4 \cdot 10^9$
D18f	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	34	0	1	$4 \cdot 10^9$
D19a	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	1	0	1	$2 \cdot 10^9$
D19b	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{il} \epsilon_{jk}$	3	3	0	$6 \cdot 10^{11}$
D19c	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
D20	$L^i \bar{e}^\dagger H^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	129	0	1,2	$2 \cdot 10^5$
D21	$(DL)^i (D\bar{e}^\dagger) H^j H^k (DH)^l \cdot \epsilon_{ik} \epsilon_{jl}$	2	0	1	$4 \cdot 10^7$
D22	$\bar{e}^\dagger \bar{e}^\dagger (DH)^i (DH)^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	9	0	2	$3 \cdot 10^3$

The following discussion is based on J. Gargalionis and RV, JHEP 01, 074 (2021)
“Exploding operators for Majorana neutrino masses and beyond”

- All gauge-invariant $\Delta L=2$ operators containing SM fields are listed, including EW index contractions and derivatives, completing the Babu+Leung list.
- An algorithm for tree-level openings of those operators is developed, validated e.g. against dim=6 SMEFT.
- The subsequent theories are filtered to produce genuine neutrino mass models.
- Statistics for the properties of the theories are derived.
- New illustrative models are constructed.
- The code and a searchable database of these models made available online for community use.
- The code can be used for all sets of effective operators, not limited to Majorana neutrino application.

J. Gargalionis (2020), “neutrinomass” at <https://github.com/johngarg/neutrinomass>
Full database at <https://doi.org/10.5281/zenodo.4054618>

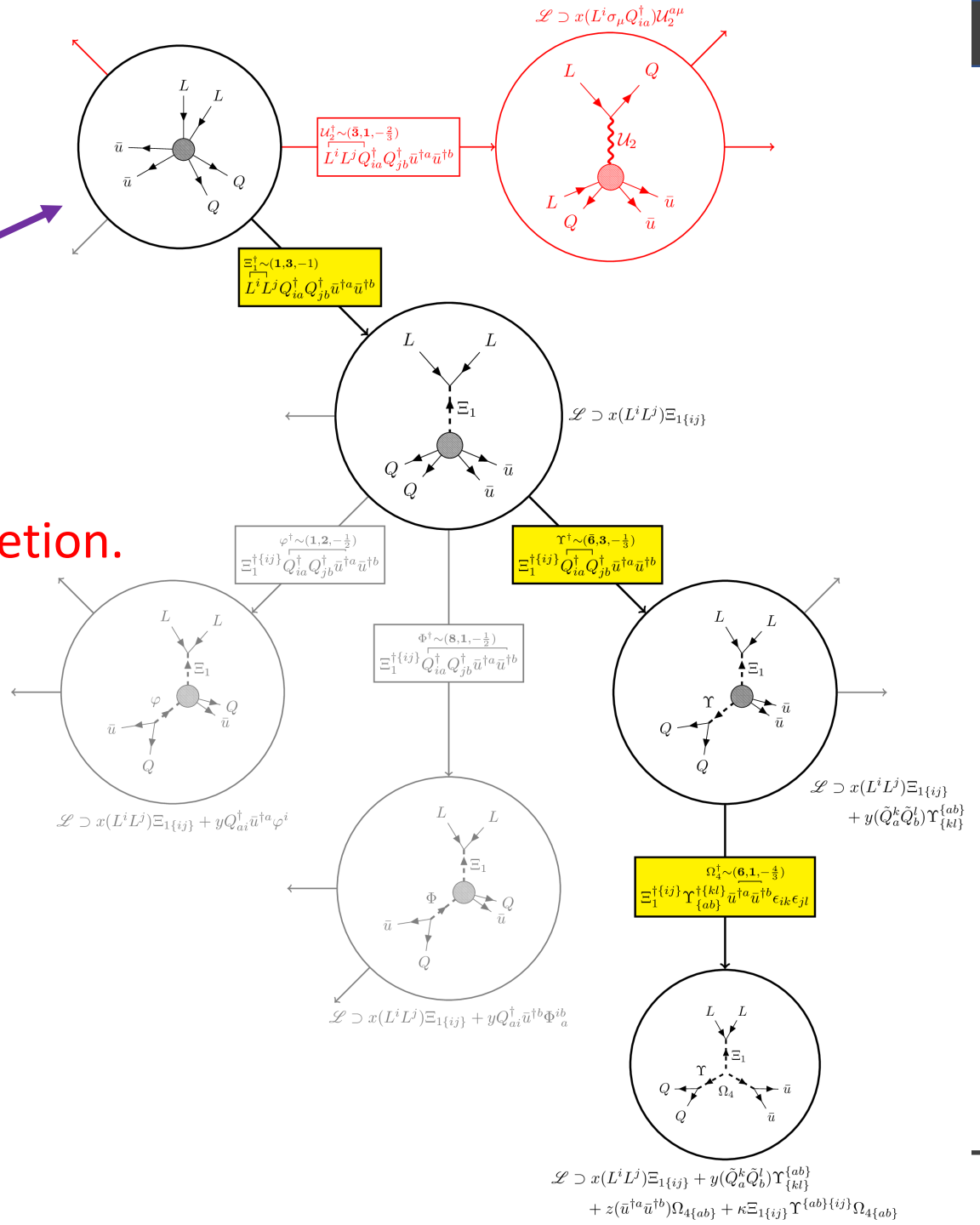
Example: opening of $\mathcal{O}_{12a} = L^i L^j \tilde{Q}^k \tilde{Q}^\ell \bar{u}^\dagger \bar{u}^\dagger \epsilon_{ik} \epsilon_{jl}$

2-component notation: $\tilde{Q}^i \equiv \epsilon^{ij} Q_j^\dagger$ $\tilde{Q} \bar{u}^\dagger = \text{colour singlet}$

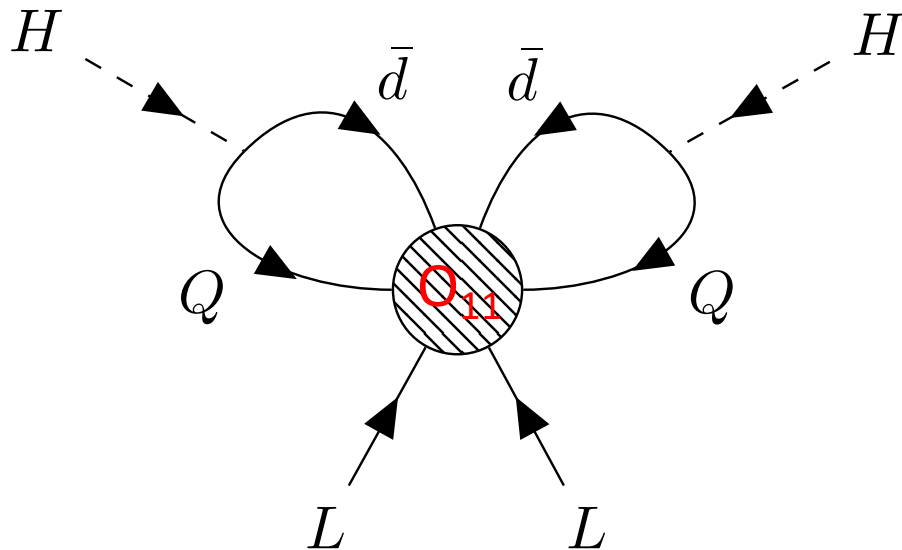
Effective operator

The yellow boxes track one pathway to opening \mathcal{O}_{12a} at tree level, producing a scalar-only completion.

The starts of other pathways are indicated in light grey.



Example: closing $\mathcal{O}_{11} = LLQQ\bar{d}\bar{d}$ into a Majorana neutrino mass diagram.



$\mathcal{O}_{11} \rightarrow LLHH$ at 2-loop order

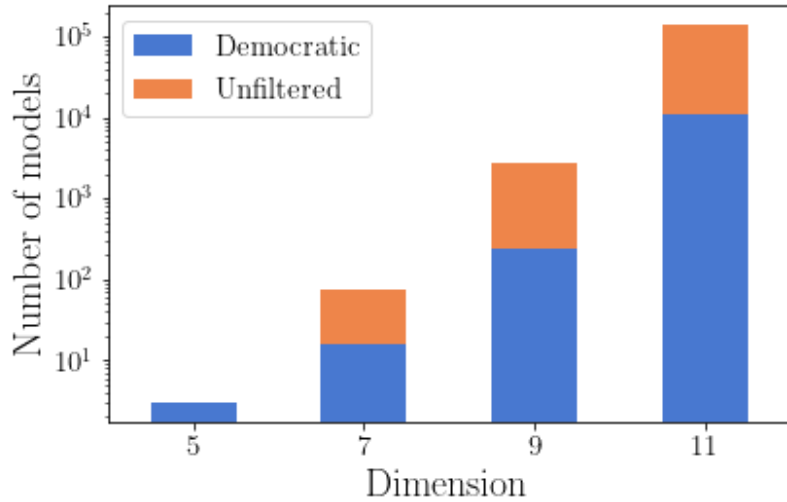
Opening up \mathcal{O}_{11} at **tree level** produces a **2-loop contribution to neutrino mass**. The virtual states consist of both exotics and SM fields.

This enables the systematic construction of radiative neutrino mass models.

Tiny ν mass due to (i) large exotic masses, (ii) $1/16\pi^2$ loop factors, and (iii) products of $\text{mag.} < 1$ coupling constants.

How many models?

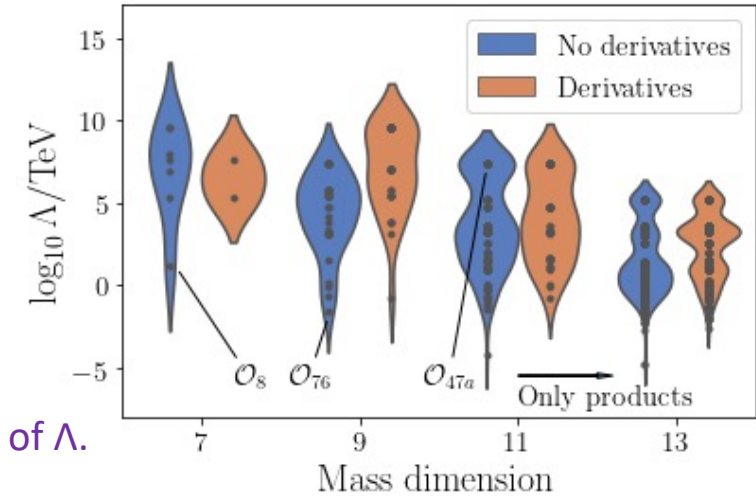
141,989 unfiltered models
11,216 filtered models



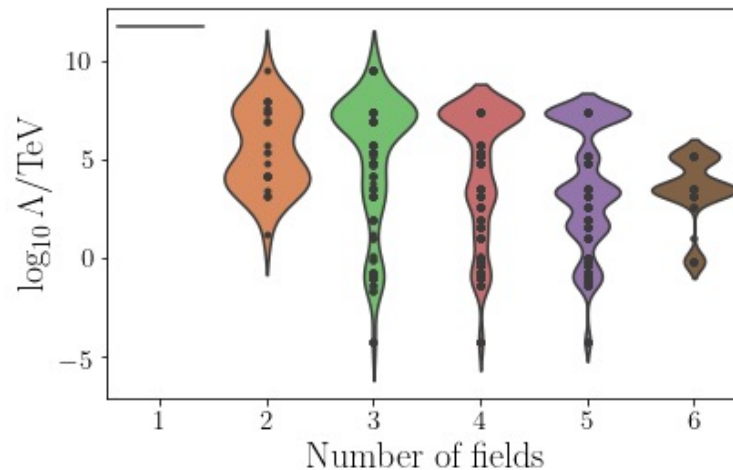
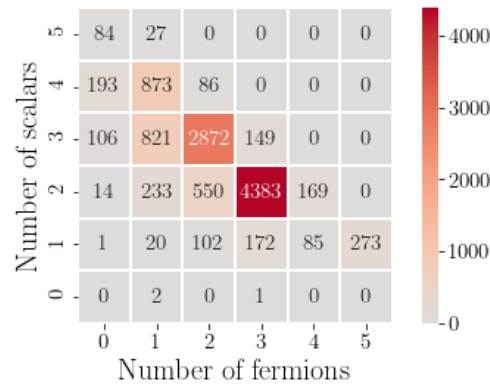
← # of models vs mass-dim

of operators for given Λ vs mass dim for O(1) couplings →

Why stop at dim=11?
At dim=11, ave. # of Yukawas = 6,
so if each ~ 0.1 , then 10^{-6} suppression of Λ .



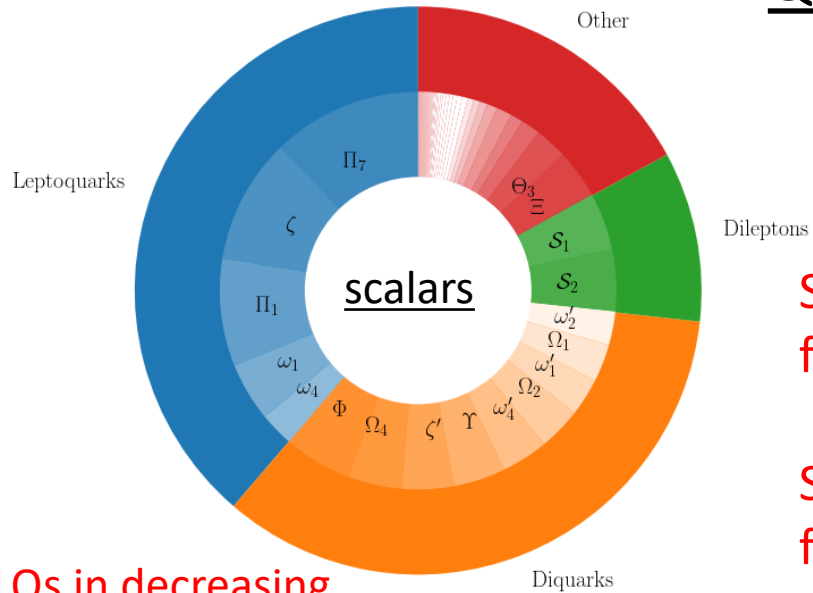
Most models have 5 exotics



New physics scale trends down (slowly) with both mass dim and # of exotics.

Many models at testable scales. Some already ruled out.

Quantum numbers of the exotics species



LQs in decreasing order of frequency

- $\Pi_7 = R_2 \sim (3, 2, \frac{7}{6})$
- $\zeta = S_3 \sim (\bar{3}, 3, \frac{1}{3})$
- $\Pi_1 = \tilde{R}_2 \sim (3, 2, \frac{1}{6})$
- $\omega_1 = S_1 \sim (\bar{3}, 1, \frac{1}{3})$
- $\omega_4 = \tilde{S}_1 \sim (\bar{3}, 1, \frac{4}{3})$

S_1, S_3, R_2 of great interest for B-meson anomalies.

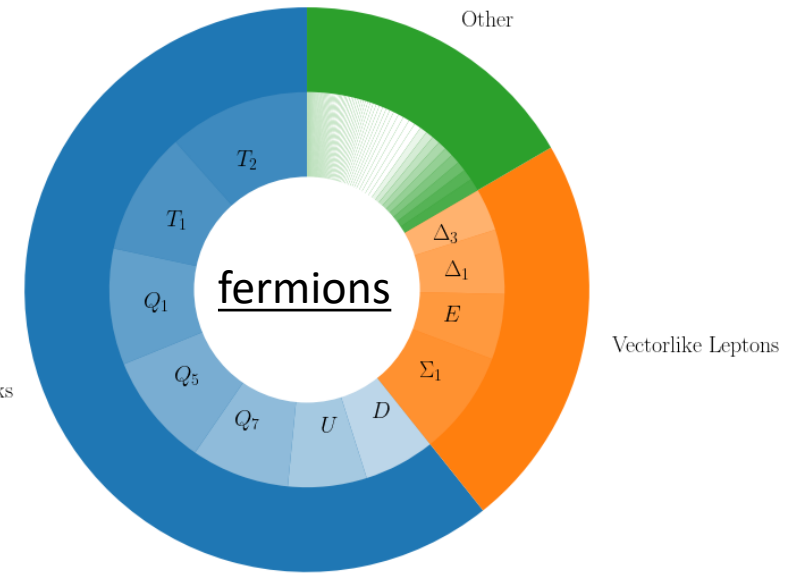
S_1, R_2 of great interest for g-2 of muon and electron.

Symbol	$(SU(3)_c, SU(2)_L, U(1)_Y)$	Interactions	$F = 3B + L$
S_3	$(\bar{3}, 3, 1/3)$	$\bar{Q}^C L$	-2
R_2	$(3, 2, 7/6)$	$\bar{u}_R L, \bar{Q} e_R$	0
\tilde{R}_2	$(3, 2, 1/6)$	$\bar{d}_R L$	0
\tilde{S}_1	$(\bar{3}, 1, 4/3)$	$\bar{d}_R^C e_R$	-2
S_1	$(\bar{3}, 1, 1/3)$	$\bar{Q}^C L, \bar{u}_R^C e_R$	-2

Table from Doršner+ 2020

Name	N	E	Δ_1	Δ_3	Σ	Σ_1
Irrep	$(1, 1, 0)$	$(1, 1, 1)$	$(1, 2, \frac{1}{2})$	$(1, 2, \frac{3}{2})$	$(1, 3, 0)$	$(1, 3, 1)$

Name	U	D	Q_1	Q_5	Q_7	T_1	T_2
Irrep	$(3, 1, \frac{2}{3})$	$(\bar{3}, 1, \frac{1}{3})$	$(3, 2, \frac{1}{6})$	$(3, 2, -\frac{5}{6})$	$(3, 2, \frac{7}{6})$	$(\bar{3}, 3, \frac{1}{3})$	$(3, 3, \frac{2}{3})$



Economical and testable models: less than 4 exotic multiplets and $\Lambda < 100$ TeV

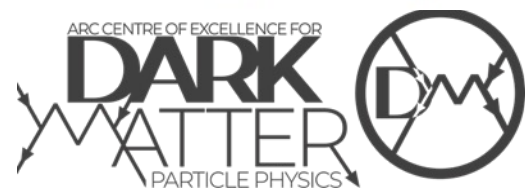
Field content	Operators	Λ [TeV]	Dominant?
$(3, 2, \frac{1}{6})_S, (3, 2, \frac{7}{6})_F$	8, $D15$	15	Y
$(1, 2, \frac{1}{2})_F, (1, 1, 1)_S, (1, 2, \frac{3}{2})_S$	$62b$	16	N
$(\bar{3}, 2, \frac{5}{6})_S, (3, 2, \frac{1}{6})_F, (3, 2, \frac{1}{6})_S$	$8'$	1	N
$(\bar{3}, 1, \frac{1}{3})_S, (\bar{6}, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_F$	$24f$	89	N
$(\bar{3}, 3, \frac{1}{3})_F, (\bar{6}, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_S$	$24d$	89	N
$(\bar{3}, 2, \frac{5}{6})_S, (1, 2, \frac{3}{2})_F, (3, 2, \frac{1}{6})_S$	$8'$	1	N
$(\bar{3}, 3, \frac{1}{3})_F, (\bar{6}, 4, \frac{1}{6})_S, (3, 2, \frac{1}{6})_S$	$24f$	89	N
$(\bar{3}, 1, \frac{1}{3})_F, (\bar{6}, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_S$	$24d$	89	N
$(\bar{6}, 2, \frac{7}{6})_F, (8, 2, \frac{1}{2})_S, (3, 2, \frac{1}{6})_S$	20	0.8	Y
$(6, 1, \frac{4}{3})_S, (6, 1, \frac{1}{3})_F, (3, 2, \frac{1}{6})_S$	20	0.8	Y
$(6, 2, \frac{5}{6})_S, (3, 2, \frac{1}{6})_F, (3, 2, \frac{1}{6})_S$	$50a, b$	10	Y
$(\bar{6}, 2, \frac{1}{6})_S, (\bar{3}, 2, \frac{5}{6})_F, (3, 2, \frac{1}{6})_S$	$50a, b$	10	Y

← The only previously known model.

Y. Cai+ 2015
Klein, Lindner, Ohmer 2019

Dominant contribution is from loop-level exotic-only completion of Weinberg operator.

The Y models have upper bounds on New Physics scale in the range 0.8-15 TeV.

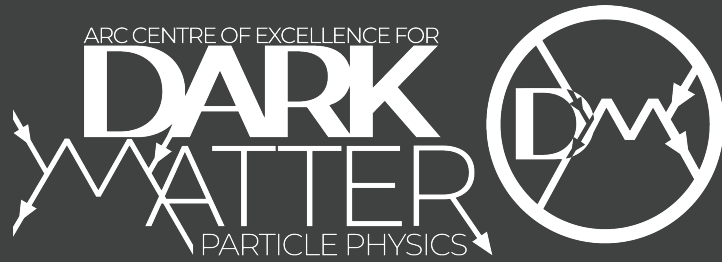


Note: Sextets can be replaced by anti-triplets.

6. Closing remarks

- We know neutrinos have tiny masses, but not what Lagrangian¹ to write in the textbooks.
- Whatever it is, it is New Physics.
- Neutrino mass scale probably a new mass scale in physics.
- High-scale seesaw models: well-motivated, leptogenesis benefit, but not very testable.
- Inverse and linear seesaws are interesting lower-energy alternatives.
- Radiative models have much new physics; connections with B-meson and g-2 anomalies.

¹ I really really want to know this.
I really really want to know what dark matter is too.



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Back up slides

Filtering

Opening up an operator produces **sets of exotic field content**, defining **unfiltered models**.

These fields then implicitly define the **most general renormalisable Lagrangian**.

If necessary, baryon-number conservation is imposed. Fields with the same SM quantum numbers but different B are considered different.

It often happens that the resulting interactions generate the **largest*** contribution to neutrino masses from an **effective operator that is different from that used to derive the field content**.

The resulting models are then re-tagged against the dominant operator, giving **filtered models**.

* We assume that there are no special parameter hierarchies when deriving the neutrino mass. This is called **democratic filtering** in the paper.

Our analysis looked at tree-level openings only. Some of our filtered models may produce exotic-only, loop-level contributions to the Weinberg operator that are larger than the closure of the dominant operator: use Valencia group's results to do more filtering.

Models with exotic scalars and fermions only.

Scale of new physics assuming $O(1)$ exotic couplings

unfiltered

The tree-level seesaw models

Some operators produce no filtered models

Labels	Operator	Models	Filtered	Loops	Λ [TeV]
1	$L^i L^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
2	$L^i L^j L^k \bar{e} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	8	2	1	$4 \cdot 10^7$
3a	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ij} \epsilon_{kl}$	9	2	2	$2 \cdot 10^5$
3b	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	14	5	1	$9 \cdot 10^7$
4a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
4b	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
5a	$L^i L^j Q^k \bar{d} H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	790	36	2	$6 \cdot 10^5$
5b	$\mathcal{O}_1 \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	492	14	1,2	$6 \cdot 10^5$
5c	$\mathcal{O}_{3a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	509	0	2,3	$1 \cdot 10^3$
5d	$\mathcal{O}_{3b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	799	16	1,2	$6 \cdot 10^5$
6a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	289	14	2	$2 \cdot 10^7$
6b	$\mathcal{O}_1 \cdot \tilde{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	177	0	1,2	$2 \cdot 10^7$
6c	$\mathcal{O}_{4a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	262	0	1,2	$2 \cdot 10^7$
6d	$\mathcal{O}_{4b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	208	0	2,3	$6 \cdot 10^4$
7	$L^i \bar{e}^\dagger Q^j \tilde{Q}^k H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	240	15	2	$2 \cdot 10^5$
8	$L^i \bar{e}^\dagger \bar{u}^\dagger \bar{d} H^j \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
9	$L^i L^j L^k L^l \bar{e} \bar{e} \cdot \epsilon_{ik} \epsilon_{jl}$	14	1	2	$3 \cdot 10^3$
10	$L^i L^j L^k \bar{e} Q^l \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	50	1	2	$6 \cdot 10^3$
11a	$L^i L^j Q^k Q^l \bar{d} \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	48	0	3	$4 \cdot 10^1$
11b	$L^i L^j Q^k Q^l \bar{d} \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	72	16	2	$1 \cdot 10^4$
12a	$L^i L^j \tilde{Q}^k \tilde{Q}^l \bar{u}^\dagger \bar{u}^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	19	0	2	$2 \cdot 10^7$
12b	$L^i L^j \tilde{Q}^k \tilde{Q}^l \bar{u}^\dagger \bar{u}^\dagger \cdot \epsilon_{ij} \epsilon_{kl}$	17	4	3	$6 \cdot 10^4$
13	$L^i L^j L^k \bar{e} \tilde{Q}^l \bar{u}^\dagger \cdot \epsilon_{ik} \epsilon_{jl}$	12	0	2	$2 \cdot 10^5$
14a	$L^i L^j Q^k \tilde{Q}^l \bar{u}^\dagger \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	29	1	3	$1 \cdot 10^3$

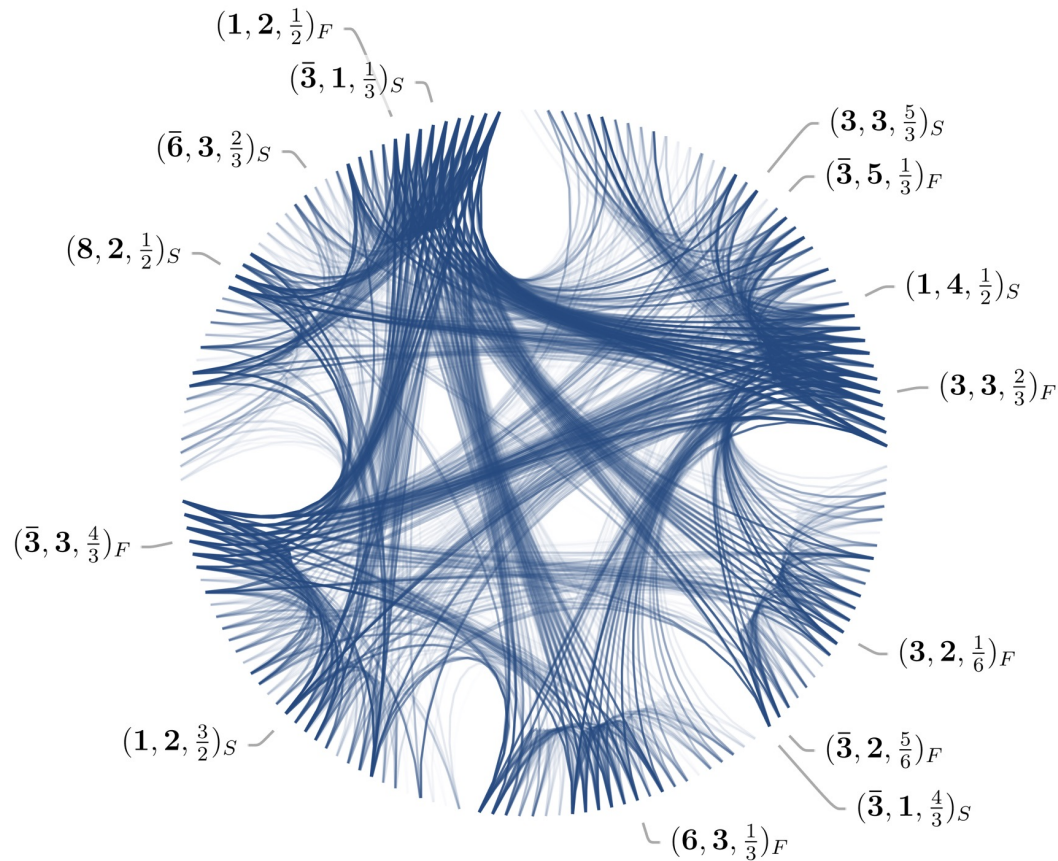
1,2
1,2
2,3

Subtlety to do with the higher-dim Weinberg ops

Zee-Babu model

⋮

Which exotics often occur together?



Rank	Edge
1	$(3, 3, \frac{2}{3})_F, (3, 4, \frac{1}{6})_S$
2	$(3, 2, \frac{1}{6})_S, (3, 2, \frac{1}{6})_F$
3	$(3, 3, \frac{2}{3})_S, (3, 2, \frac{7}{6})_S$
4	$(3, 2, \frac{7}{6})_F, (3, 2, \frac{1}{6})_S$
5	$(3, 3, \frac{2}{3})_F, (3, 4, \frac{7}{6})_F$
6	$(\bar{3}, 3, \frac{1}{3})_S, (3, 4, \frac{1}{6})_S$
7	$(3, 2, \frac{1}{6})_F, (3, 3, \frac{2}{3})_S$
8	$(\bar{3}, 3, \frac{4}{3})_F, (\bar{3}, 2, \frac{5}{6})_F$
9	$(3, 2, \frac{1}{6})_S, (3, 3, \frac{2}{3})_S$
10	$(3, 2, \frac{7}{6})_S, (\bar{3}, 2, \frac{5}{6})_F$

The ten most common pairings.

Each point on circumference is an exotic field.
 Lines between points indicate those fields occur together.
 The darker the colour, the more often a pairing occurs.