(The) Wiggles going non-linear

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Inflation

- A number of predictions in the inflationary model have been confirmed by observations
- Seeded initial conditions for structure formation
- Specifically slow roll inflation gives us a good mechanism for explaining the inflationary dynamics at work in the early Universe.

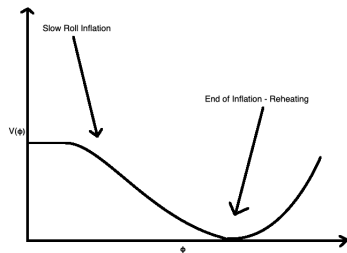- Slow roll inflation predicts a smooth power law PPS

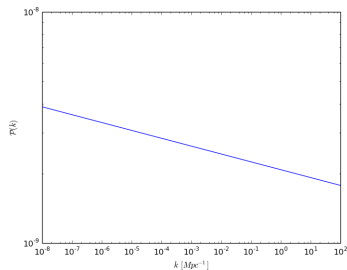Figure: Slow Roll Inflationary
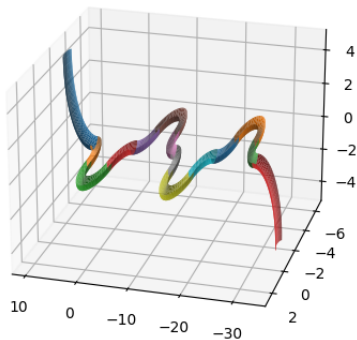Potential

Figure: Smooth Power Law PPS

Departures

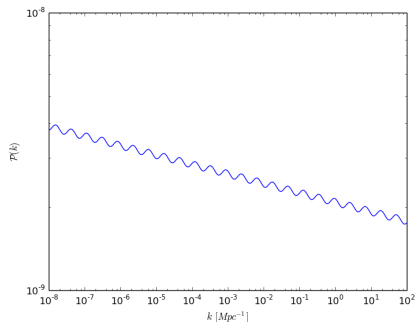Figure: Periodically oscillating 2d Inflaton Potential

Figure: PPS with global oscillatory feature

- Non-linear dynamics give rise to wave-mode coupling at low redshifts damping features in the MPS
- However, an appropriate model for this dampening can provide us insight into what features may have existed in the PPS

Definition

Primordial Power Spectrum

Primordial Amplitude

$$\mathcal{P}(k) = A_{\mathrm{s}} \left(\frac{k}{k_*} \right)^{(n_{\mathrm{s}} - 1)}$$

Spectral Index

Pivot Scale

Primordial Power Spectrum with Features

$$\tilde{\mathcal{P}}(k, \mathcal{A}, \omega, \phi) = \mathcal{P}(k) \left(1 + \mathcal{A} \cos \left[\omega \log \left(\frac{k}{k_*} \right) + \phi_k \right] \right)$$

Definition

Relative Matter Power Spectrum

$$\delta(k, \mathcal{A}, \omega, \phi) \equiv \frac{\tilde{\mathcal{P}}(k, \mathcal{A}, \omega, \phi) - \mathcal{P}(k)}{\mathcal{P}(k)}$$

$$\equiv \mathcal{A} \cos\left[\omega \log\left(\frac{k}{k_*}\right) + \phi_k\right]$$

[S. Hannestad and Y. Y. Wong 2020]

Definition

Linear Relative Matter Power Spectrum

$$\delta(k, \mathcal{A}, \omega, \phi) \equiv \mathcal{A} \cos\left[\omega \log\left(\frac{k}{k_*}\right) + \phi_k\right].$$

Non-Linear Relative Matter Power Spectrum

$$\delta_{\text{fit}}(k, \mathcal{A}, \omega, \phi, z) \equiv \mathcal{A} \cos\left[\omega \log\left(\frac{k}{k_*}\right) + \phi_k\right] \mathcal{D}(k, z, \Sigma).$$

Model

Dampening Model

$$\mathcal{D}(k, z, \Sigma) = e^{-k^2 \Sigma^2(z)/2}$$

Where Σ is the redshift dependent parameter calibrated by simulations

[M. Ballardini Et Al. 2020 & 2016]

Simulation

- PPS Generation
- Linear Evolution (CLASS)
- N-Body Initialisation
  (NGen-IC)
- N-Body Simulation
  (Gadget-2)
- Power Spectrum Analysis
  (GenPK)

Simulation

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Fitting Model to Data

Least chi-squared fit

$$\chi^2(\Sigma) = \sum_{k=k_{\mathsf{min}}}^{k_{\mathsf{max}}} \left[\frac{\delta_{\mathsf{fit}}(k,z,\Sigma) - \delta_{\mathsf{sim}}(k,z)}{\sigma(k,z)} \right]^2$$

Where $\sigma(k,z)$ is the variance in simulations, δ_{sim} is the simulated relative non-linear matter power spectrum and δ_{fit} is as above.
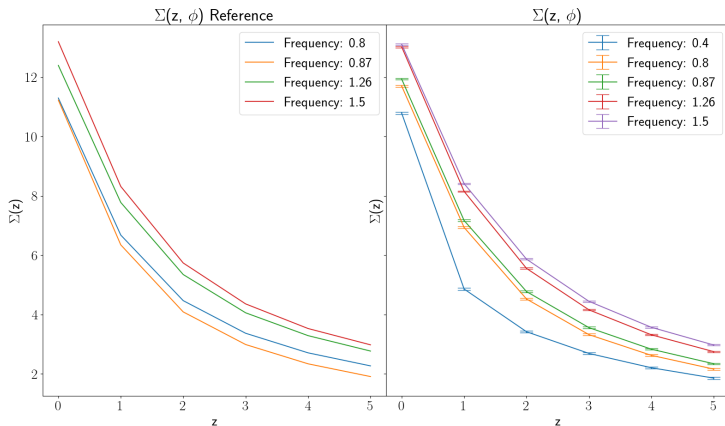
Figure: Comparison between the fit from Ballardini et al 2020 and fit obtained in this work
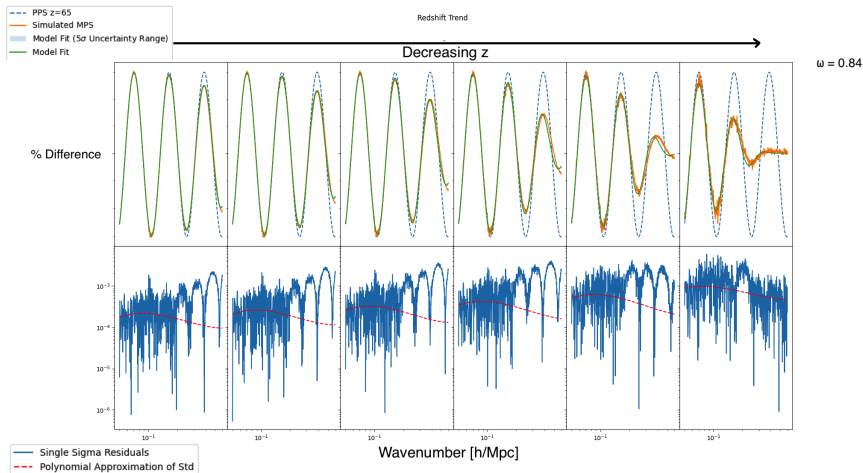
Figure: Model fit to MPS with residuals compared to simulation standard deviation
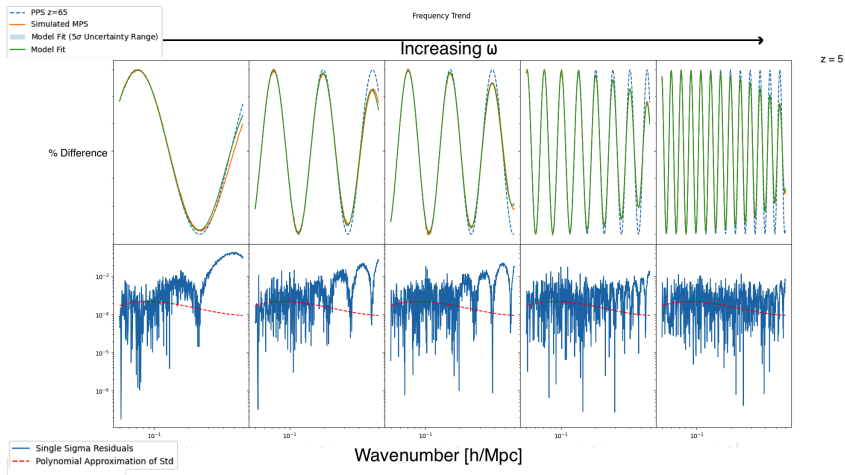
Fitted Data

Figure: Model fit to MPS with residuals compared to simulation standard deviation
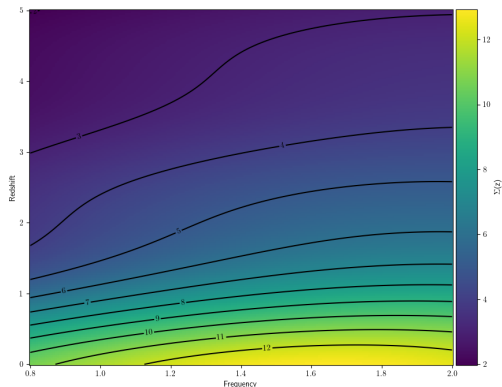
Figure: Colour plot of Frequency vs Redshift with colouring given by the value for $\Sigma(z)$ at each point. Black lines represent lines of constant $\Sigma(z)$
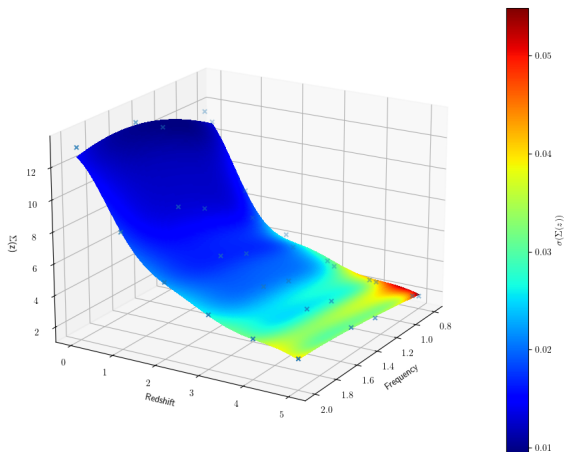
Figure: Surface plot of Frequency vs Redshift vs $\Sigma(z)$ with colouring given by the value for $\frac{\Delta\Sigma(z)}{\Sigma(z)}$ at each point
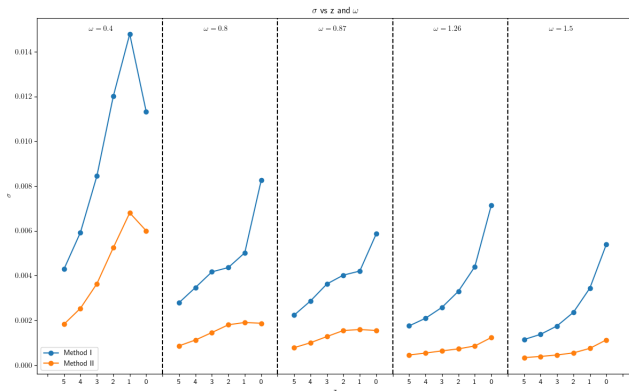
Accuracy Estimation

Figure: Plot of two methods for estimating the accuracy of our fitting function. Frequency is indicated by the labelled regions and redshift is indicated on the x-axis. Method I (blue) utilises the maximum deviation between the simulated data and model fit to obtain an estimate for the accuracy while Method II (orange) utilises optimisation of the standard deviation parameter in our χ^2 fit to obtain an estimate.

Future Outlook

- Applying this model to forecasted data from LSST, EUCLID, etc..
- Testing other similarly motivated dampening models
- Applying this model to real data from next generation LSS surveys
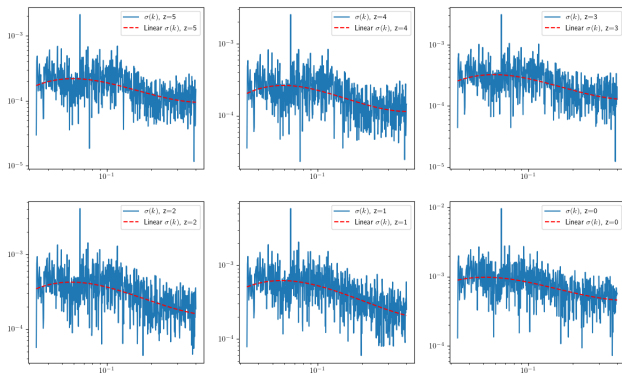
Variance Data

Figure: Polynomial approximation (red dotted line) to variance of simulated data (blue)
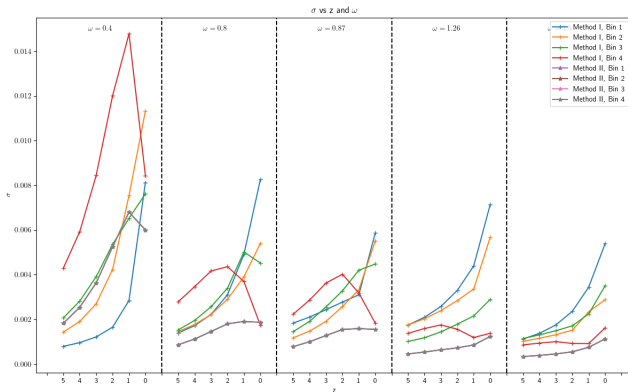
Accuracy Estimation

Figure: Plot of two methods for estimating the accuracy of our fitting function over 4 seperate bins of data. Frequency is indicated by the labelled regions and redshift is indicated on the x-axis. Method I (blue) utilises the maximum deviation between the simulated data and model fit to obtain an estimate for the accuracy while Method II (orange) utilises optimisation of the standard deviation parameter in our χ^2 fit to obtain an estimate.