Linear Power corrections for two-body kinematics

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Outline

- Prelude: introduction and motivations
- Linear fiducial power correction in the q_T subtraction method
- Numerical results
- Conclusions

PRELUDE

Introduction

- **Perturbative power corrections** induced by **fiducial cuts** on the two-body decay products in color singlet production have recently attracted great interest
  [Ebert, Tackmann, 2019], [Ebert, Michel, Stewart, Tackmann, 2020], [Alekhin, Kardos, Moch, Trocsanyi, 2021], [Salam, Slade, 2021], [Buonocore, Kallweit, Rottoli,Wiesemann, 2021],[Camarda,Cieri,Ferrera, 2021]

- This is mainly motivated by the availability of **high precision experimental data** from the LHC and of **high-order perturbative predictions** (both resummed and/or fixed-order)

- The cases of lepton pair production through the **Drell-Yan** mechanism and of **Higgs** production in the di-photon channel are of primary importance. Fiducial phase spaces are usually defined by imposing experimental cuts that **limit the transverse momenta and rapidities** of the final state leptons (photons).

- N^3LL' resummation for the **transverse momentum spectrum** of the color singlet has made possible the computation of fixed order **fiducial cross sections** at N^3LO using the q_T subtraction formalism
  [Re, Rottoli, Torrielli, 2021], [Ju, Schonherr, 2021], [Neumann, 2021], [Billis, Dehnadi, Ebert, Michel, Tackmann, 2021], [Camarda,Cieri,Ferrera, 2021]

Introduction: convergence of the perturbative expansion

Fiducial cuts may challenge the convergence of the perturbative fixed-order series

- **Symmetric cuts** on the transverse momentum of the two-body decay products lead to an **enhanced sensitivity to soft radiation** when the two particles are back-to-back in the transverse plane
  [Klaser, Kramer, 1996], [Harris, Owens, 1997], [Frixione, Ridolfi, 1997]

- They lead to **linear power corrections** in the transverse momentum spectrum of the color singlet q_T

- The linear dependence in q_T is related to a **factorial growth** of the coefficients in the perturbative series (with **alternating-sign** coefficient, hence Borel-summable) *[Salam, Slade, 2021]*

- The effect is larger for the case of the Higgs due to its **Casimir scaling**

- **Asymmetric cuts** on the transverse momentum of the hardest and the of the softest particle do not improve the situation.

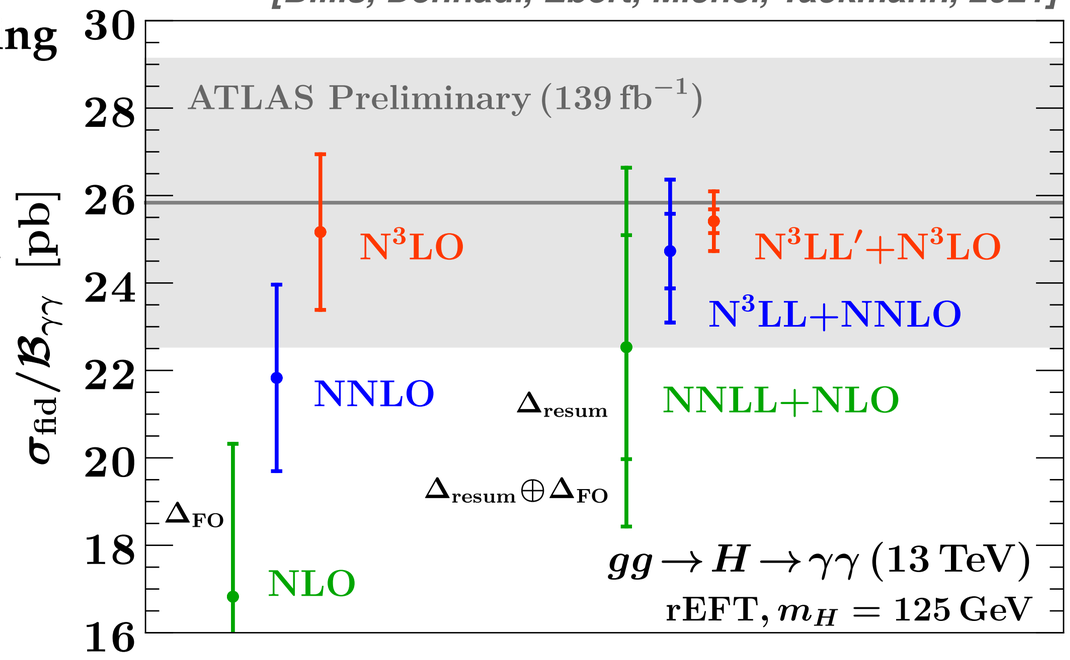
- Symmetric cuts and asymmetric cuts are commonly used for Drell-Yan and Higgs analysis, respectively.

Viable resolution strategies

- improve the convergence by **resumming** the linear power corrections

- **alternative** choices of cuts *[Salam, Slade, 2021]*

[Billis, Dehnadi, Ebert, Michel, Tackmann, 2021]



Introduction: resummation of the linear power correction

The transverse momentum spectrum can be decomposed into a **singular** and a **regular** component

$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\text{sing}}}{dq_T} + \frac{d\sigma^{\text{reg}}}{dq_T}$$

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contains contributions that diverges as $\frac{1}{q_T}$,
associated to the emission of soft and/or collinear radiation;

can be **predicted** from factorisation and **resummed** to all orders
(**Leading Power** (LP) resummation)

Introduction: resummation of the linear power correction

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$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\text{sing}}}{dq_T} + \frac{d\sigma^{\text{reg}}}{dq_T}$$

contains contributions finite in the limit $q_T \to 0$
associated to exact matrix elements and phase space

$\mathcal{O}(1)$: linear power correction
$\mathcal{O}(q_T)$: quadratic power correction
and so on

Introduction: resummation of the linear power correction

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Main results

- The lowest power correction is **quadratic** for the case of inclusive color singlet production
  [Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann, 2019], [Ebert, Moult, Stewart, Tackmann, Vita, Zhu 2019], [Cieri, Oleari, Rocco, 2019], [Buonocore, Grazzini, Tramontano, 2019]

- Symmetric/asymmetric cuts on the two-body decay lead to a **linear power corrections**
  [Ebert, Tackmann, 2019]

- For the Higgs and Drell-Yan cases, it has been formally shown that **linear power corrections can be resummed** to all orders at the same accuracy for which the LP resummation is available.
  [Ebert, Michel, Stewart, Tackmann, 2020]

- The linear power correction has a **purely kinematical origin** and can **be predicted by factorisation.**

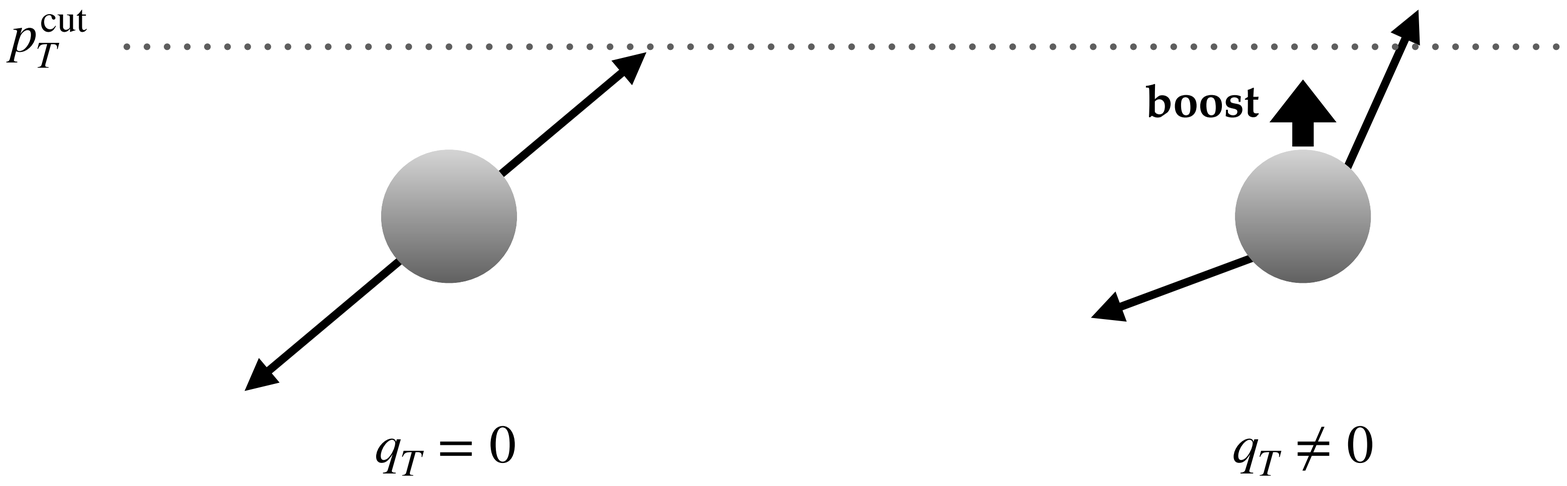
Introduction: resummation of the linear power correction

The transverse momentum spectrum can be decomposed into a **singular** and a **regular** component

$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\text{sing}}}{dq_T} + \frac{d\sigma^{\text{reg}}}{dq_T}$$

Main results

- The resummation of the linear power corrections is equivalent to a **recoil prescription**
  [Ebert, Michel, Stewart, Tackmann, 2020]
- The two-body final state system absorbs the recoil of the soft and/or collinear radiation through a boost transformation *[Catani, de Florian, Ferrera, Grazzini, 2015]*



Introduction: resummation of the linear power correction

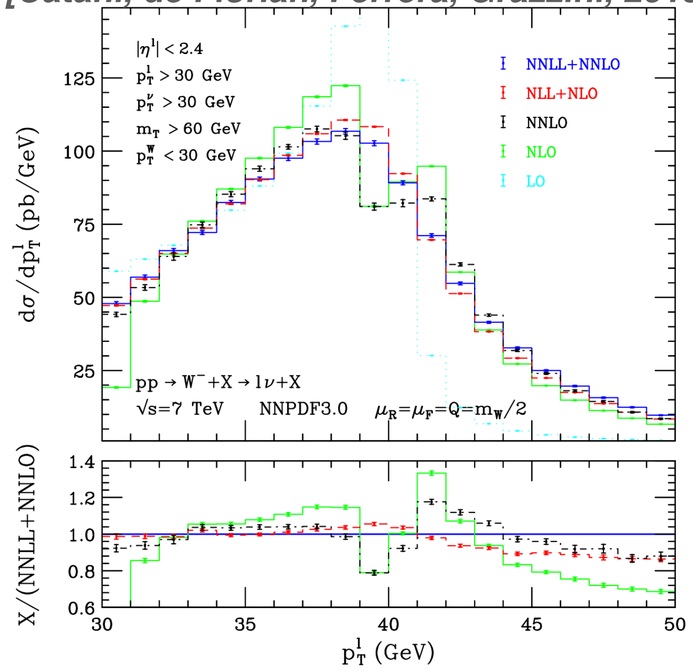
The transverse momentum spectrum can be decomposed into a **singular** and a **regular** component

$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\text{sing}}}{dq_T} + \frac{d\sigma^{\text{reg}}}{dq_T}$$

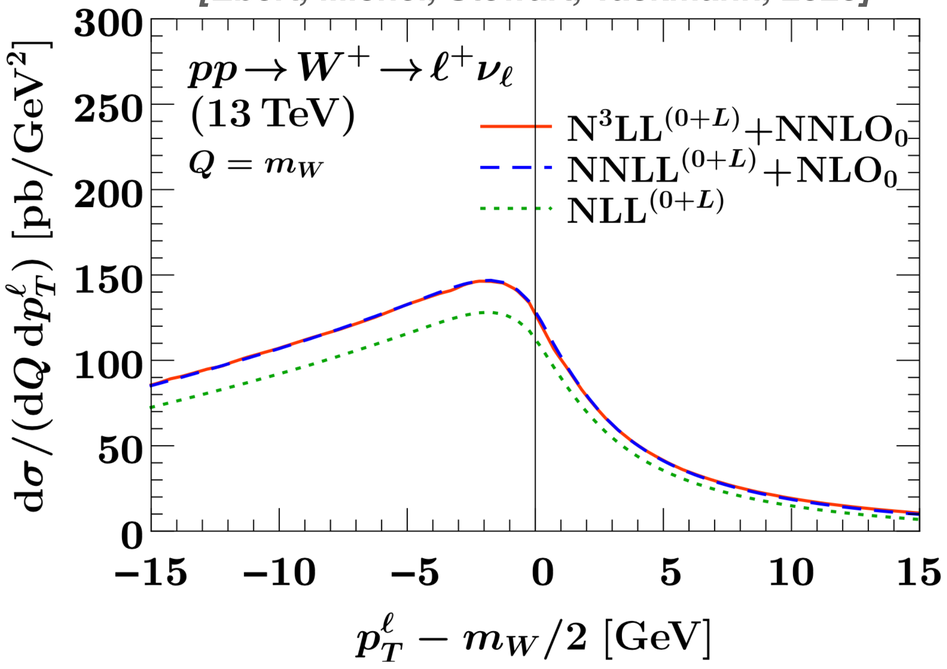
Main results

- Resummation of leptonic observables sensitive to soft gluon emission as lepton transverse momentum in Drell-Yan

[Catani, de Florian, Ferrera, Grazzini, 2015]



[Ebert, Michel, Stewart, Tackmann, 2020]



Implications for the q_T subtraction method

The resummation of the transverse momentum of the color singlet can be used to build a non-local subtraction scheme by introducing a small resolution parameter q_T^{cut}

neglecting the regular component below the cut

$$\sigma = \int^{q_T^{\text{cut}}} dq_T \frac{d\sigma}{dq_T} + \int_{q_T^{\text{cut}}} \frac{d\sigma}{dq_T} = \int^{q_T^{\text{cut}}} dq_T \frac{d\sigma^{\text{sing,LP}}}{dq_T} + \int_{q_T^{\text{cut}}} \frac{d\sigma}{dq_T} + \mathcal{O}\left((q_T^{\text{cut}})^k\right)$$

$$= \int^{\infty} dq_T \frac{d\sigma^{\text{sing,LP}}}{dq_T} + \int_{q_T^{\text{cut}}} \left[\frac{d\sigma}{dq_T} - \frac{d\sigma^{\text{sing,LP}}}{dq_T}\right] + \mathcal{O}\left((q_T^{\text{cut}})^k\right)$$

The formula is **affected by power corrections** and is formally exact only in the limit $q_T^{\text{cut}} \to 0$.

The computation can be carried out only for a finite value of q_T^{cut}.

Trade off between

- choosing q_T^{cut} sufficiently small to reduce the power corrections
- choosing q_T^{cut} sufficiently large to reduce numerical instabilities due to cancellation of very large quantities

Implications for the q_T subtraction method

The resummation of the transverse momentum of the color singlet can be used to build a non-local subtraction scheme by introducing a small resolution parameter $q_T^{\rm cut}$

neglecting the regular component below the cut

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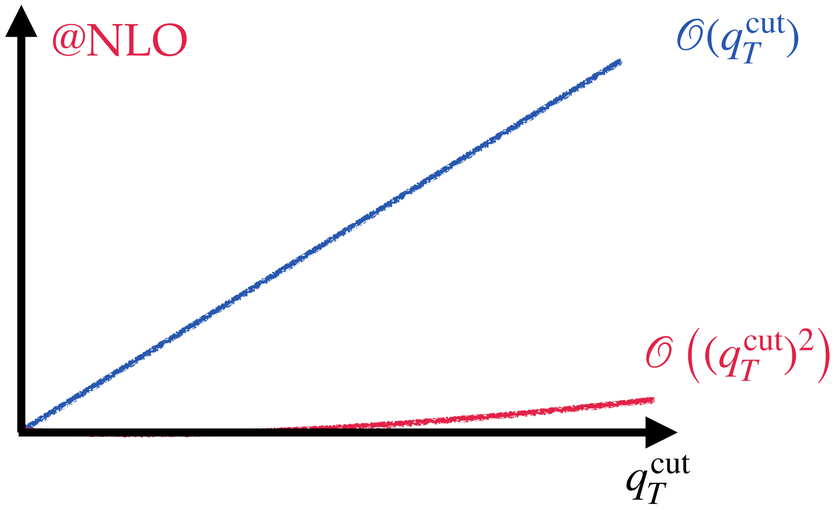

Relative size of power corrections affects the **stability** and the **performance** of the method

The larger the power corrections, the lower are the values of the slicing parameters needed for extrapolation of correct result (CPU consuming, numerically unstable)

Restoring the quadratic dependence on q_T : motivations and objective

The presence of linear power corrections may affect the **effectiveness** of the q_T subtraction method for

- high precision predictions required for processes as the Drell-Yan
  [Alekhin, Kardos, Moch, Trocsanyi, 2021]
- N^3LO calculations

Despite the issues related to the convergence of the perturbative expansion, fixed-order calculations have great relevance in precision physics at colliders and it is very desirable to have at disposal predictions at high numerical accuracy.

Objective: removing the linear power corrections associated to fiducial cuts to the two-body decay in color singlet production

FIDUCIAL POWER CORRECTION in q_T subtraction

Restoring the quadratic dependence on q_T

In the standard q_T subtraction master formula *[Catani, Grazzini, 2007]*

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathscr{H} + \int \left[d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \mathcal{O}\left(r^k_{\mathrm{cut}}\right)$$

the counterterm is given by a **pure LP expansion** of the q_T spectrum

- above r_{cut} : all power corrections are exactly provided by the real matrix element (avoiding any double counting)
- below r_{cut} : all power corrections are missing

Formally, the residual dependence on the slicing parameter r_{cut} is given by the integral of the non-singular component of the real spectrum below the cut.

$$\int d\sigma^{\mathrm{F+jet,reg}}_{\mathrm{(N)LO}} \theta(r_{\mathrm{cut}} - q_T/Q)$$

For the case of **fiducial cuts**, the leading power correction is linear ($k = 1$). It can be **predicted by factorisation** and is **equivalent** to the q_T recoil prescription

$$\boxed{\int d\Phi_{\mathrm{F+jet}} \frac{d\sigma^{\mathrm{F+jet,reg}}_{\mathrm{(N)LO}}}{d\Phi_{\mathrm{F+jet}}} \theta(r_{\mathrm{cut}} - q_T/Q)\Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F+jet}}) = \int d\Phi_{\mathrm{F}} \int_0^{r_{\mathrm{cut}}} dr' \left[\frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}\left(\Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')\right) - \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F}}) \right] + \mathcal{O}(r^2_{\mathrm{cut}})}$$

where Θ_{cuts} implements the fiducial cuts and $\Phi^{\mathrm{rec}}_{\mathrm{F}} = \Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')$ is the recoiled kinematics

Restoring the quadratic dependence on q_T

Improved q_T subtraction master formula

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathscr{H} + \int \left[d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) + \mathscr{O}\left(r_{\mathrm{cut}}^{k'}\right)$$

with the **linPC** term, $\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}})$, given by

see also *[Camarda,Cieri,Ferrera, 2021]*

$$\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) = \int d\Phi_{\mathrm{F}} \int_0^{r_{\mathrm{cut}}} dr' \left[\frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}\left(\Phi_{\mathrm{F}}^{\mathrm{rec}}(\Phi_{\mathrm{F}}, r')\right) - \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F}}) \right]$$

Remarks on the linPC term

- it affects the q_T subtraction formula at the **power corrections** level only
- its formulation is **fully differential** with respect to the Born phase space
- it is **integrable** in 4 dimensions (local cancellation of infrared singularities)
- it is completely determined by the **knowledge of the counterterm** (can be easily **implemented** in any code implementing the q_T subtraction method)

Restoring the quadratic dependence on q_T

Improved q_T subtraction master formula

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathscr{H} + \int \left[d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) + \mathscr{O}\left(r_{\mathrm{cut}}^{k'}\right)$$

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Remarks on the linPC term

- for the case of fiducial cuts, we expect that its inclusion will **change the power correction scaling from linear to quadratic** ($k = 1$ to $k' = 2$)
- for other cases, we expect that its inclusion will not make the power correction scaling worse
- in principle, given its formulation, it can be applied to any process

Restoring the quadratic dependence on q_T: implementation in MATRIX

MATRIX *[Grazzini, Kallweit, Wiesemann, 2018]*

MUNICH (by S. Kallweit)

- efficient multichannel phase space generation
- bookkeeping all subprocesses
- automatic implementation of dipole subtraction

AMPLITUDES

- 1-loop amplitudes: OpenLoops, Recola (Collier, CutTOols,…)
- 2-loop: dedicated 2-loop codes (VVamp, GiNac, TDHPL,…)

SUBTRACTION SCHEME

- @NLO: dipole and q_T subtraction
- @NNLO: q_T subtraction

MATRIX v2 https://matrix.hepforge.org

- NNLO QCD differential predictions for many color singlet processes: H, V, $\gamma\gamma$, $V\gamma$, VV for all leptonic decays
- combination with NLO EW for all leptonic V and VV processes
- loop-induced ggchannel at NLO QCD for neutral VV processes

Still not included in the public version

- NNLO QCD for heavy quarks: $t\bar{t}$ and $b\bar{b}$
- mixed QCD-EW corrections to leptonic V processes

Restoring the quadratic dependence on q_T: implementation in MATRIX

Implementation of $\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}})$

- **recoil prescription:** boost from the Collins–Soper rest frame of the colour singlet system to the laboratory frame where it has transverse momentum equal to q_T *[Catani, de Florian, Ferrera, Grazzini, 2015]*
- optimisation of integration and histogramming below r_{cut}

We checked numerically that the different recoil prescriptions lead to **sub-leading effects**.
This is consistent with the formal proof in *[Ebert, Michel, Stewart, Tackmann, 2020]*

In general, an **extrapolation procedure** can be used to deal with power corrections in slicing methods.

MATRIX features

- an **automatic and robust extrapolation** procedure for the total cross section (available in the public version)
- an **automatic bin-wise extrapolation** for distributions (under development)

Warnings

- the extrapolation is non trivial for sizeable power corrections; reaching the asympotic behaviour is required
- the bin-wise extrapolation requires a larger amount of storing resources

NUMERICAL RESULTS

Numerical results: setups

Neutral current Drell-Yan

$\sqrt{s} = 13\,\text{TeV}$ $\mu_R = \mu_F = M_Z$

pdfset: NNPDF31_nnlo_as_0118

$66\,\text{GeV} < m_{\ell\ell} < 116\,\text{GeV}$

$|y_{\ell^\pm}| < 2.5$

1) Symmetric cuts
$p_{\ell^\pm} > 27\,\text{GeV}$

2) Asymmetric cuts
$p_{\ell,1} > 27\,\text{GeV}, p_{\ell,2} > 25\,\text{GeV}$

3) Staggered cuts
$p_{\ell^+} > 27\,\text{GeV}, p_{\ell^-} > 25\,\text{GeV}$

On-shell ZZ production

$\sqrt{s} = 13\,\text{TeV}$ $\mu_R = \mu_F = M_Z$

pdfset: NNPDF31_nnlo_as_0118

$|y_Z| < 2.5$

Symmetric cuts
$p_Z > 27\,\text{GeV}$

Matrix parameter

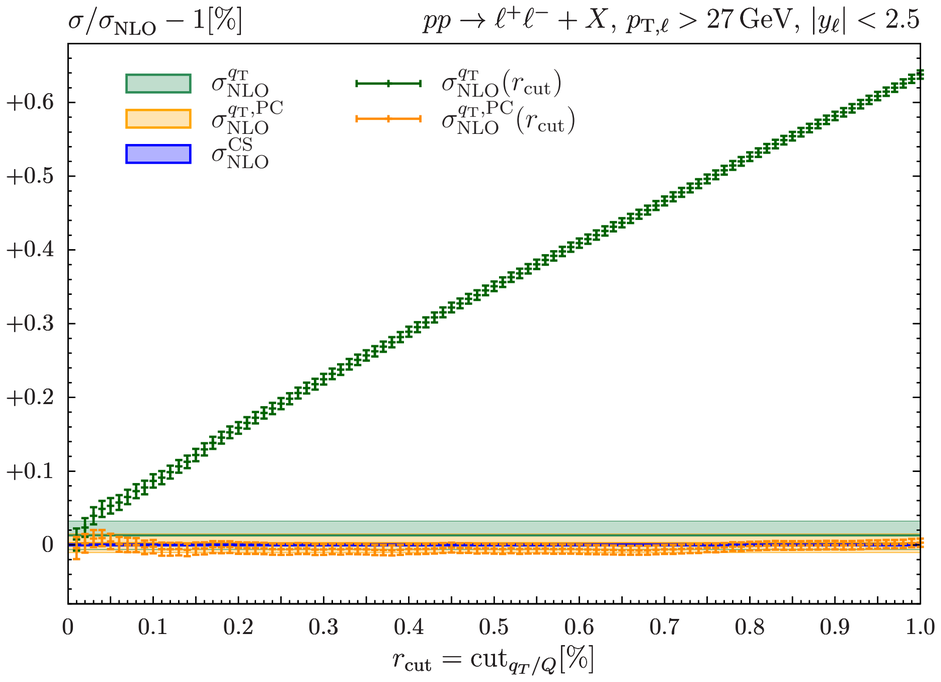
- reference slicing parameter r_{cut}=0.01%
- range of the extrapolation procedure: $r_{\text{cut}} \in [0.01,1]\%$ (about 10 MeV - 1 GeV)

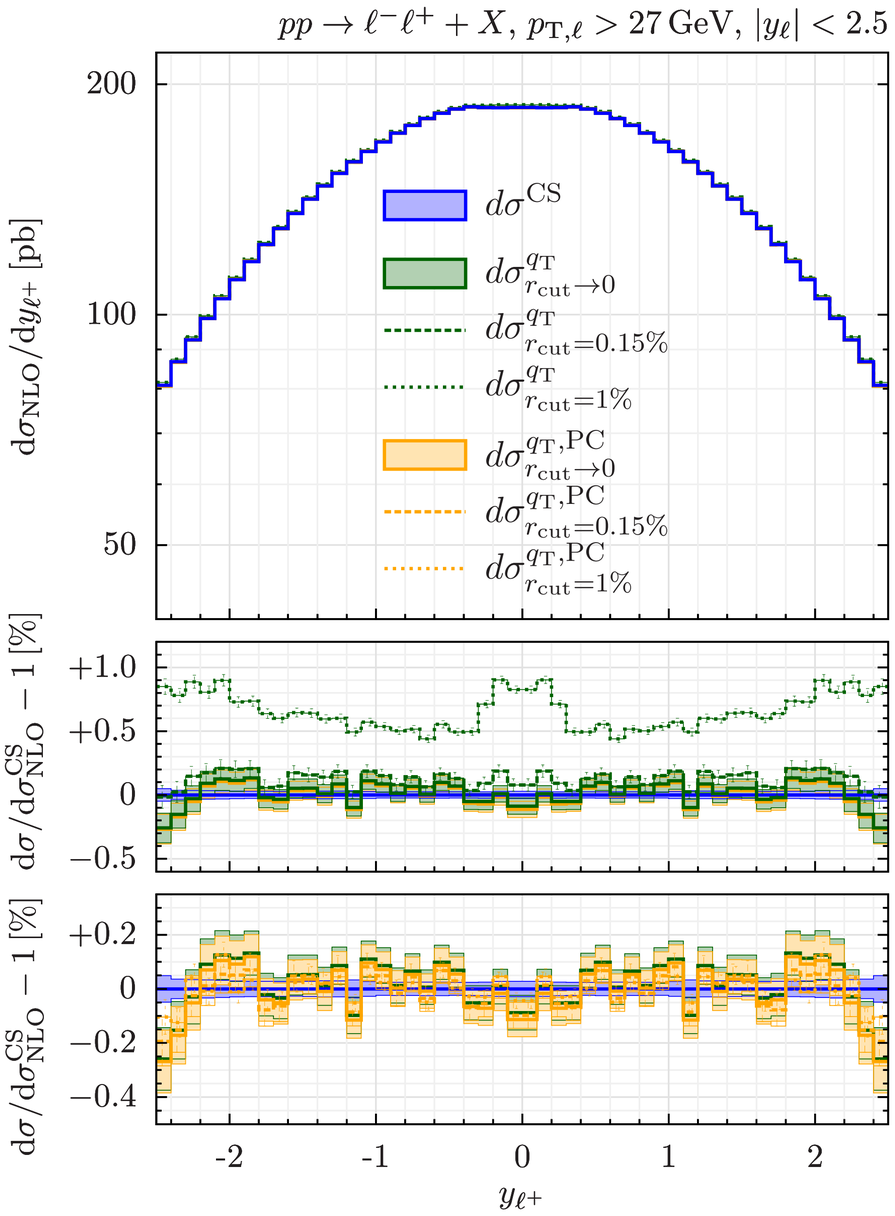
Color code

- standard q_T subtraction
- q_T subtraction with linPCs
- NLO with dipole subtraction

Drell-Yan symmetric cuts @NLO: fiducial cross section

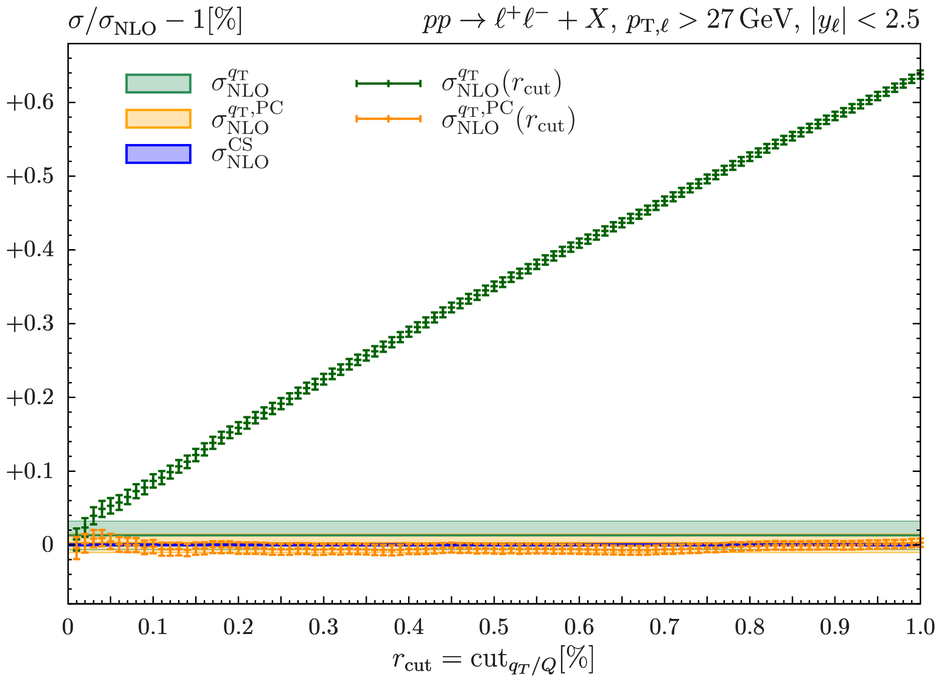
- Standard q_T subtraction calculation (green) displays the **distinctive linear behaviour** in the nominal range $r_{\rm cut} \in [0.01,1]$ %
- The inclusion of $\Delta\sigma^{\rm linPCs}$ renders the **dependence practically flat**, consistently with a residual quadratic power correction.
- Permyriad agreement with the $r_{\rm cut}$ independent computation, performed with dipole subtraction (CS), already at $r_{\rm cut} = 1$ %
- Reduced extrapolation uncertainty

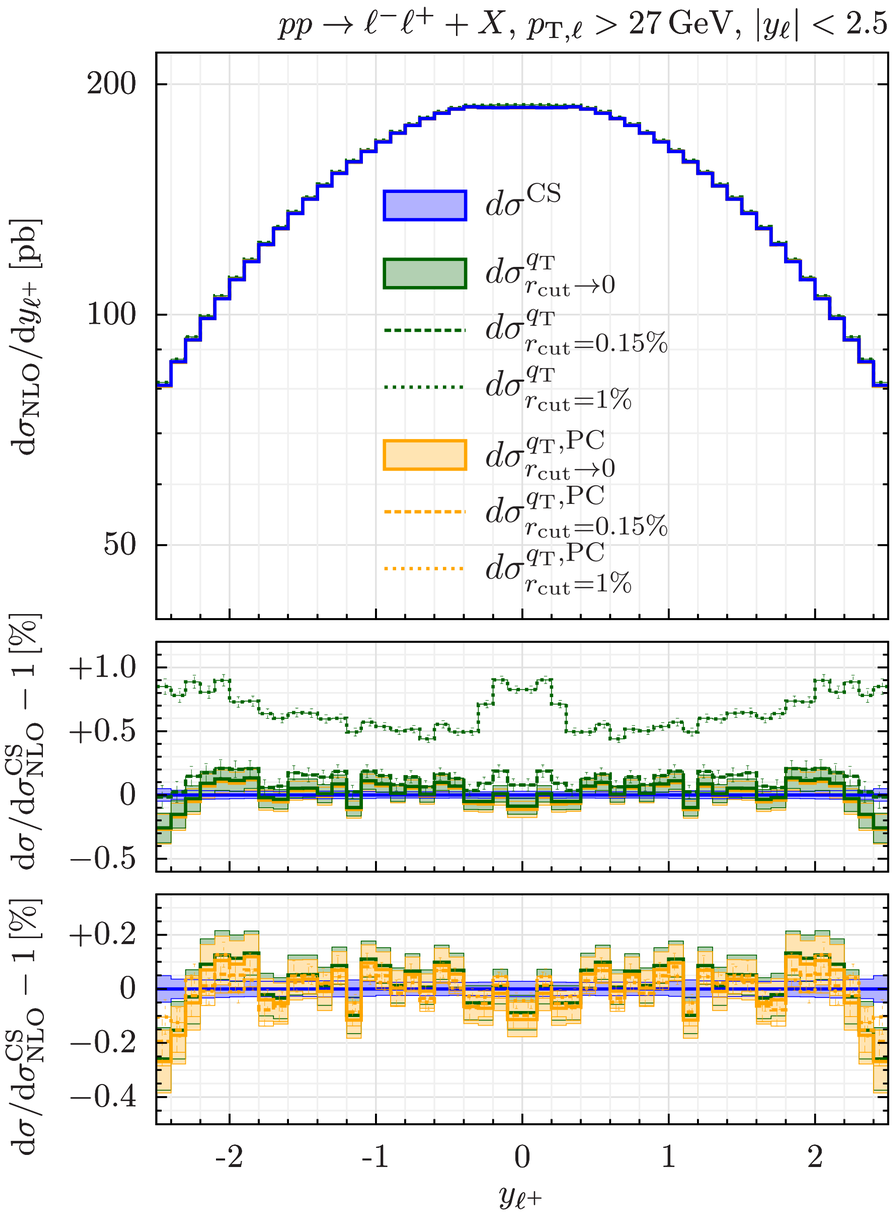




Drell-Yan symmetric cuts @NLO: differential distributions

- The lepton rapidity features the largest effects due to power corrections among the ones we studied
- The inclusion of $\Delta\sigma^{\mathrm{linPCs}}$ removes the 1% difference at r_{cut}=1%
- In all cases, truly remarkable agreement with the CS calculation for the extrapolated results for every bin of the distribution





Drell-Yan symmetric cuts @NNLO: fiducial cross section

NNLO fiducial corrections

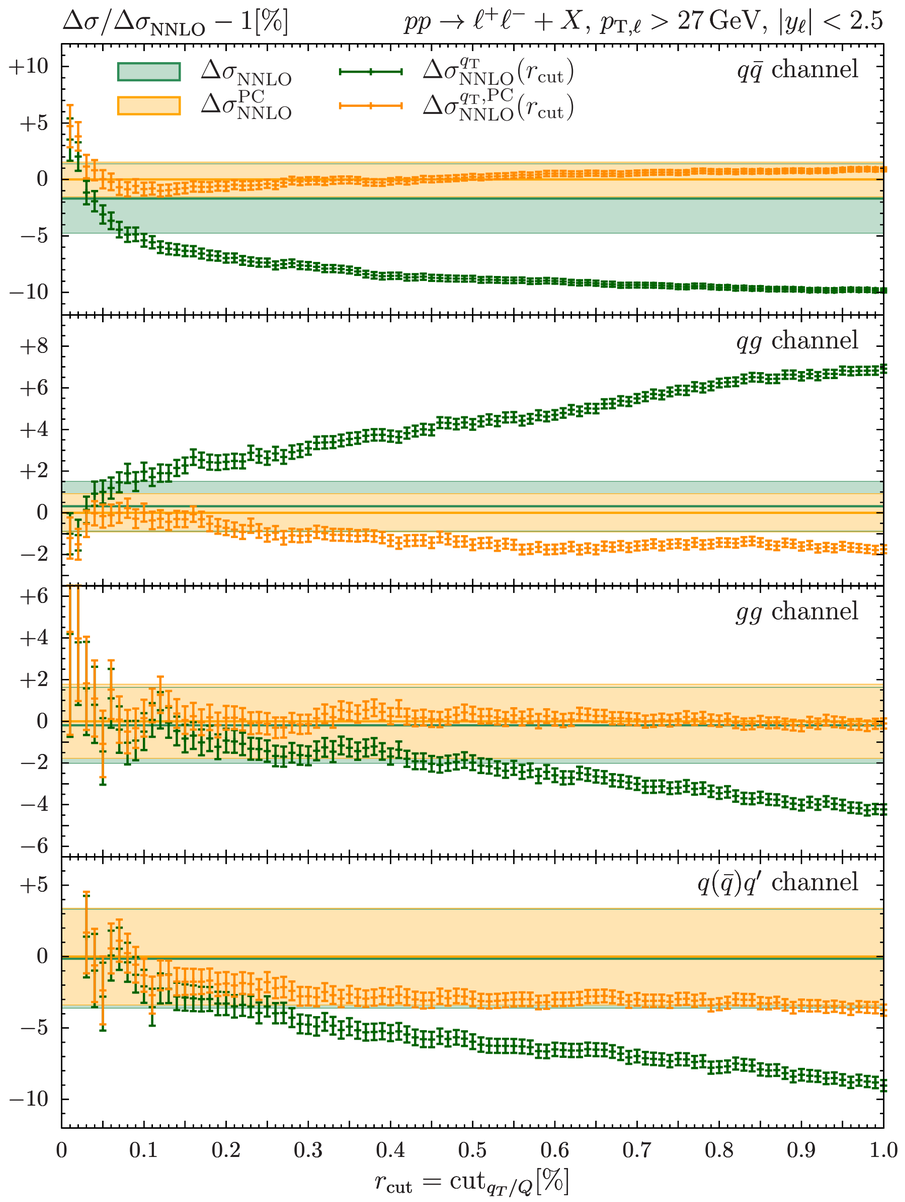
- going from NLO to NNLO, the linear power correction

$$r_{\rm cut} \to r_{\rm cut} \ln^2 r_{\rm cut}$$

- **green**: **remarkable control** of the corrections for each partonic channel (at the few percent level) thanks to the extrapolation procedure
- **yellow**: very nice **stabilisation** of the dependence on $r_{\rm cut}$ when including linPCs and in general reduction of extrapolation uncertainties
- agreement between the two extrapolations within uncertainties

Comments

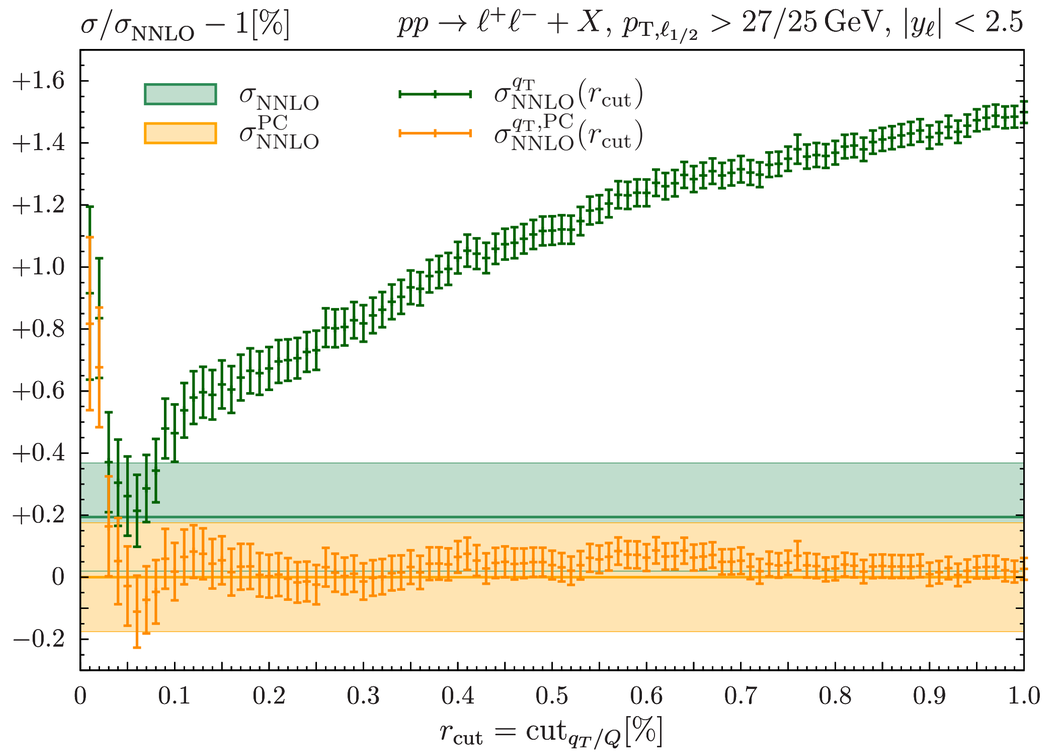
- extrapolation procedure **not yet optimised** for the calculation including linPCs
- the results in the region of small $r_{\rm cut}$ values **are dominated by uncertainties due to real integration**
- when including linPCs, the reference $r_{\rm cut}$ can be raised!

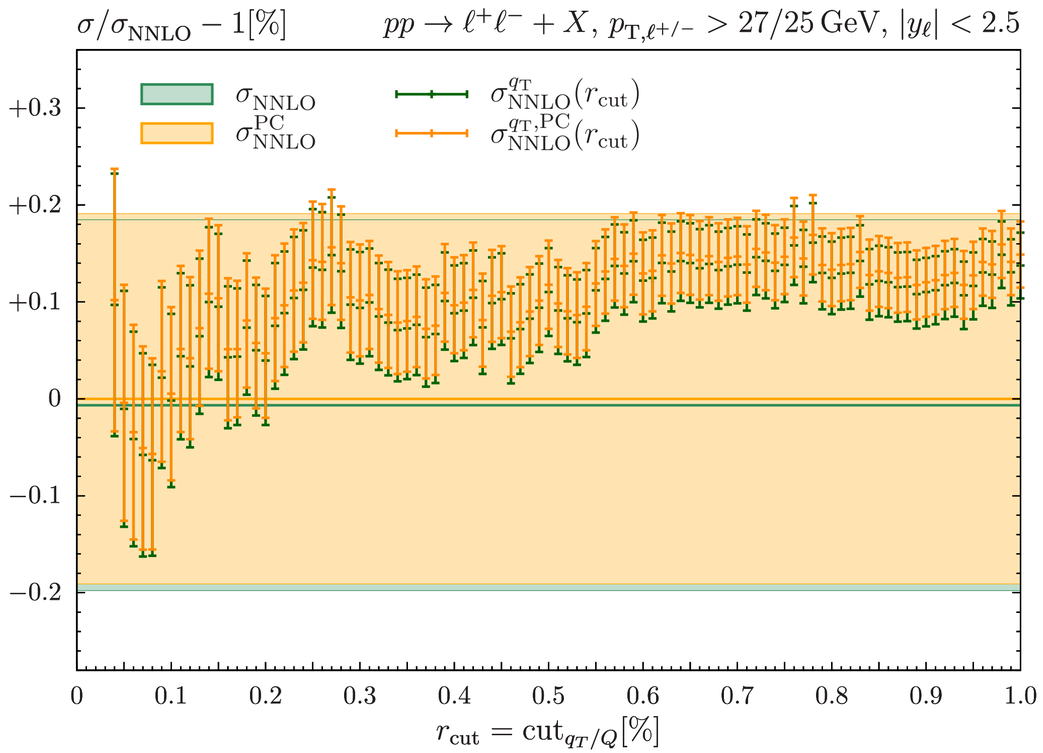


Drell-Yan asymmetric cuts and staggered cuts

Asymmetric cuts

- The standard q_T subtraction computation features a **residual linear dependence** on the slicing parameter
- The inclusion of $\Delta\sigma^{\mathrm{linPCs}}$ renders the dependence flat, compatible with a **residual quadratic power correction**
- The two extrapolated results are consistent within their uncertainties

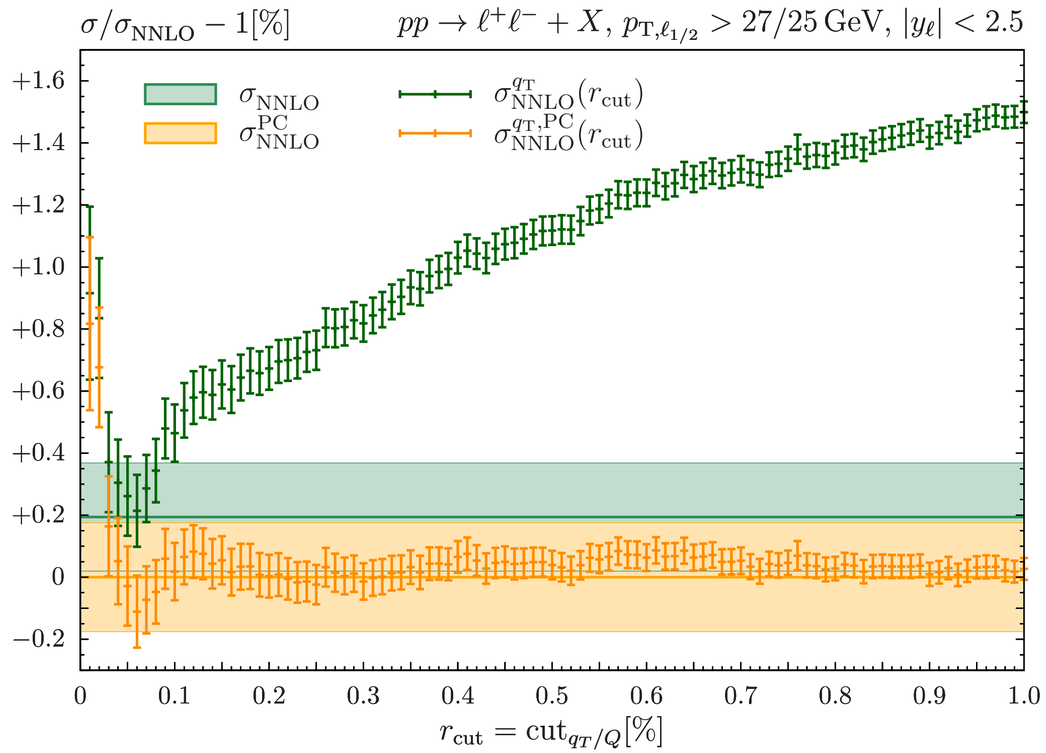


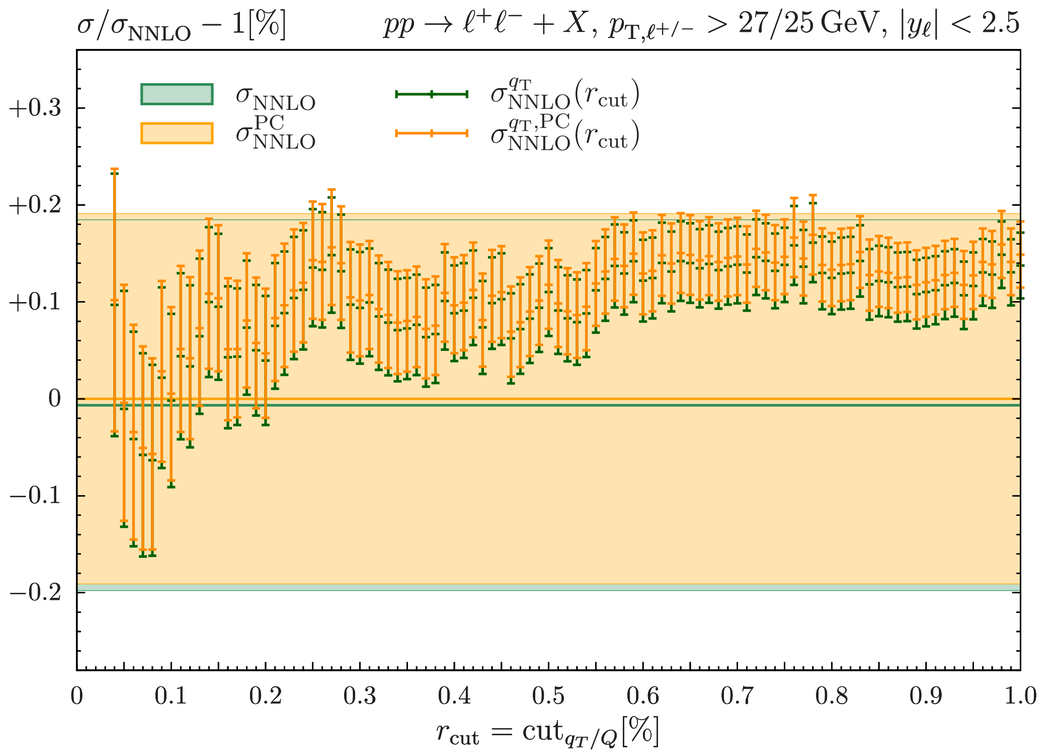


Drell-Yan asymmetric cuts and staggered cuts

Staggered cuts ($\delta p_T = 2$ GeV)

- The behaviour is compatible with a quadratic power correction already for the standard q_T computation.
- The inclusion of $\Delta\sigma^{\text{linPCs}}$ does not spoil the standard q_T subtraction result.
- Given their reduced sensitivity to very small values of q_T, staggered cuts represent a simple and more theoretically sounded alternative choice to standard Drell-Yan cuts whilst being still experimentally viable.

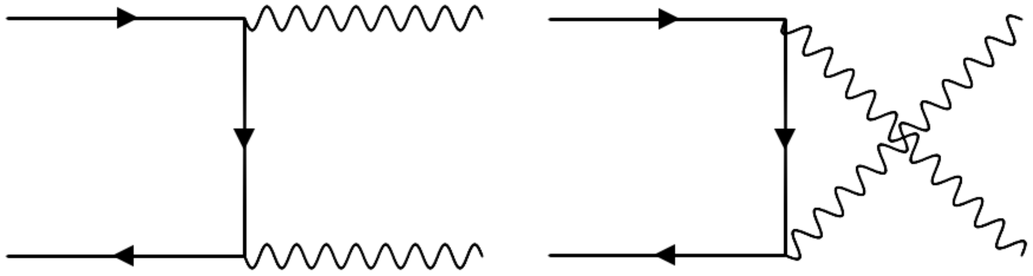




On-shell ZZ production with symmetric cuts

Generic cuts on the full 4-lepton final state in the hadro-production of EW heavy-boson pairs **do not usually lead to a departure from the quadratic behaviour**

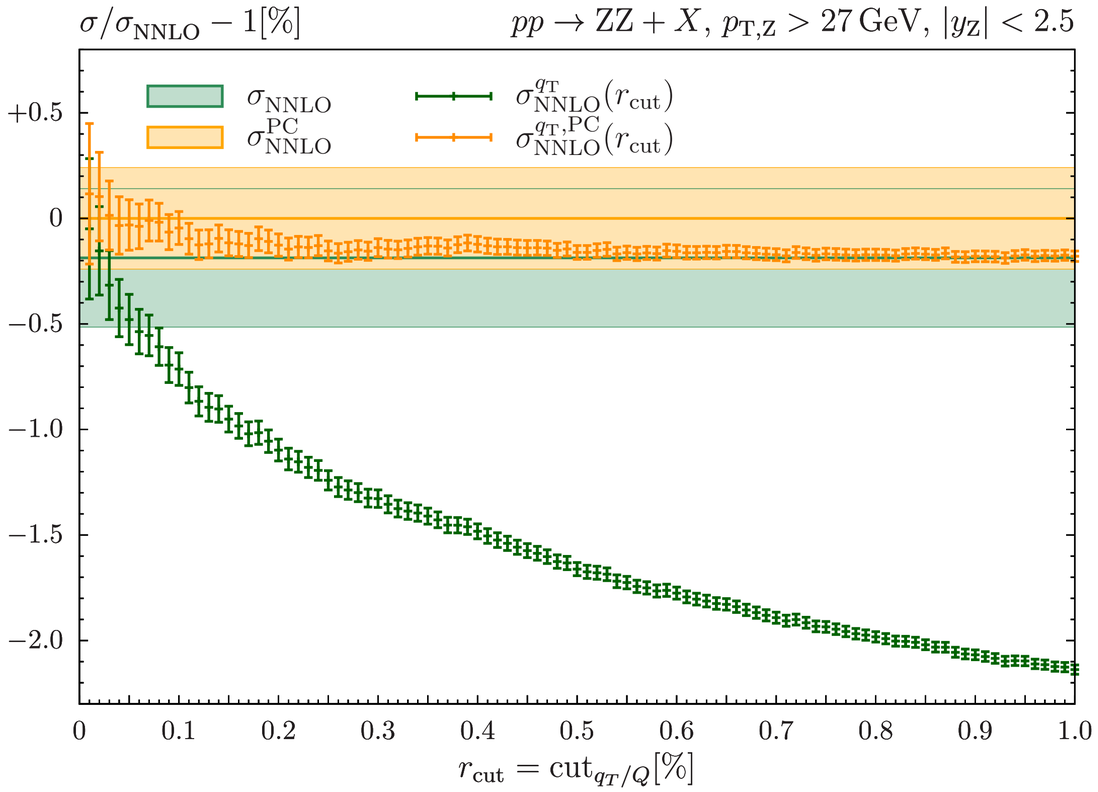
At the lowest order, on-shell *ZZ* pairs production proceeds via the exchange of a t- or u-channel quark



The **dynamics** is rather different from the s-channel exchange, as in Drell-Yan

The argument of *[Ebert, Michel, Stewart, Tackmann, 2020]*, based on the **factorisation of production and decay using the hadronic tensor decomposition**, cannot be straightforwardly repeated to determine the linear power corrections

Even if not realistic, this case is **theoretically interesting**



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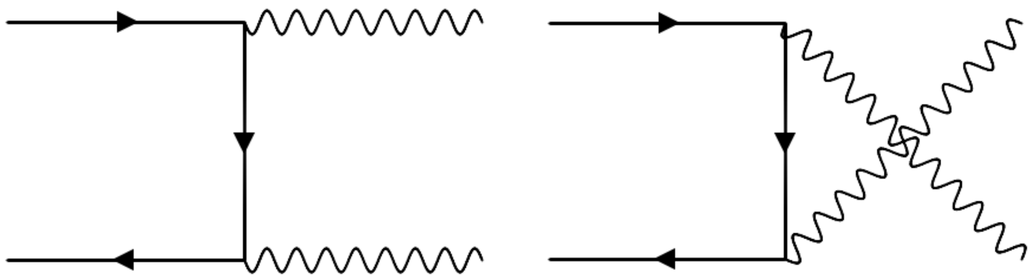
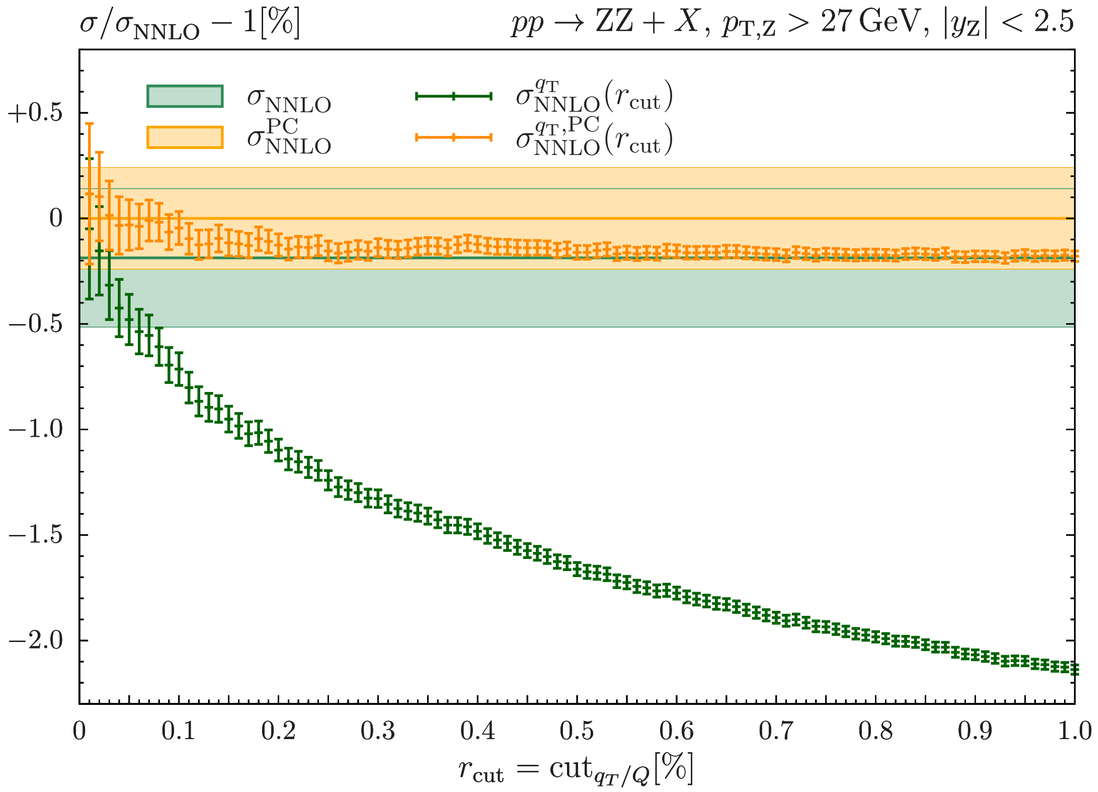




Analogously to the previous cases, we observe that

- the standard q_T subtraction computation features a **linear power correction**
- the inclusion of $\Delta\sigma^{\rm linPCs}$ renders the dependence flat, improving the stability of the calculation

This represents a strong numerical indication of the pure kinematic origin of the linear power correction

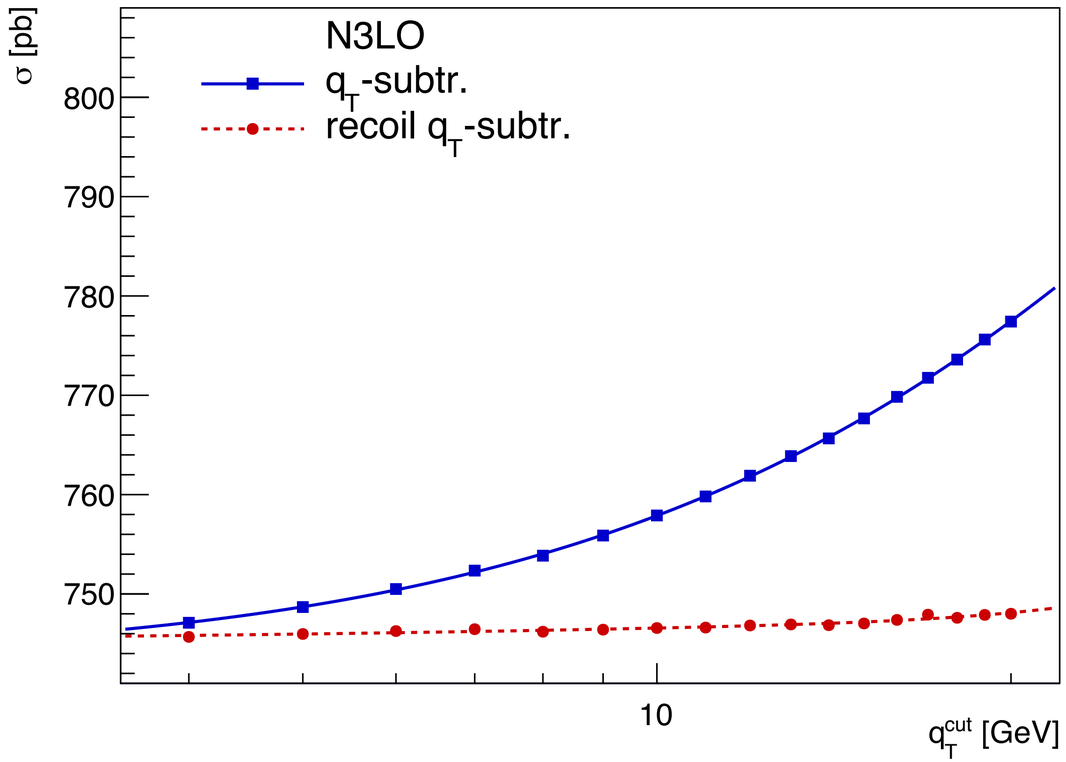
Relevance for N3LO calculations

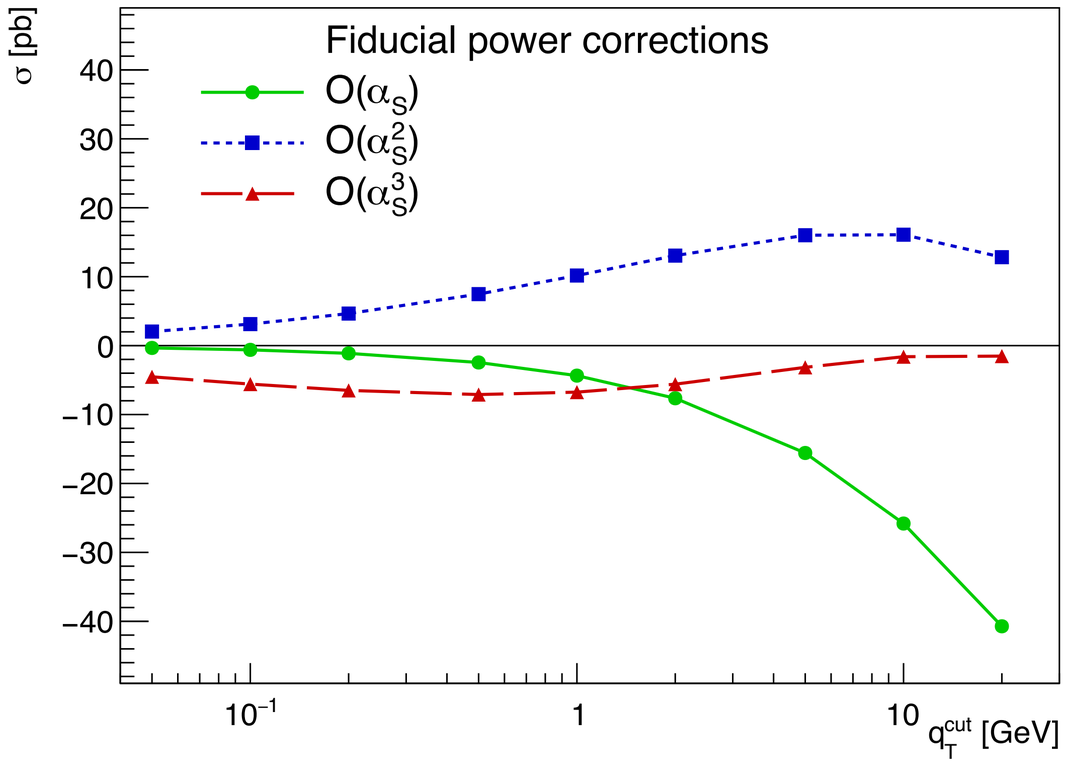
[Camarda, Cieri, Ferrera 2021]

Another implementation of the same formula for linPCs, **independently obtained** by another group, is available in DYTurbo up to N3LO for the Drell-Yan process

Linear power corrections are sizeable and their inclusion is mandatory to control the N3LO correction

The pattern of the fiducial power corrections at different perturbative order is compatible with an alternating-sign factorial growth





Conclusions

- The availability of **high precision experimental data** and of **high-order perturbative predictions** has raised the attention to **linear power corrections** to the Higgs and the Drell-Yan fiducial cross sections

- Fiducial cuts which induce a linear sensitivity to small q_T **eventually spoils the convergence** of the fixed-order coefficients of the perturbative series.

  Re-thinking of fiducial cuts and/or relying on resummed results required

  For Drell-Yan, **staggered cuts** can be a viable option

- Reliable fixed-order predictions for Drell-Yan are anyway relevant and lots of data available with symmetric cuts. Based on recent results in resummation, we **show a method to remove the linear power corrections in q_T subtraction,** applicable in principle to any perturbative order.

- We implement the procedure in MATRIX and compare the results with those obtained with the extrapolation procedure, both at the level of fiducial cross sections and differential distributions.

  All the new features **will be available in a future version** of the code

- Crucial for N3LO fiducial cross section computed with q_T **subtraction**

BACKUP

Origin of linear power corrections

Kinematics of the two-body decay

[Ebert, Michel, Stewart, Tackmann, 2020], [Alekin, Kardos, Moch, Trocsanyi, 2021], [Salam, Slade, 2021]

$$q^\mu = (m_T \cosh Y, q_T, 0, m_T \sinh Y)$$

$$p_1^\mu = p_{T,1}\left(\cosh(Y+\Delta y), \cos\phi, \sin\phi, m_T \sinh(Y+\Delta y)\right)$$

$$p_1^\mu = q^\mu - p_1^\mu$$

in the small q_T limit

$$p_{T,1} = \frac{Q}{2\cosh\Delta y}\left[1 + \frac{q_T}{Q}\frac{\cos\phi}{\cosh\Delta Y} + \mathcal{O}\left(q_T^2/Q^2\right)\right]$$

$$p_{T,2} = p_{T,1} - q_T\cos\phi + \mathcal{O}\left(q_T^2/Q^2\right)$$

$$\eta_1 = Y + \Delta y$$

$$\eta_2 = Y - \Delta y - 2\frac{q_T}{Q}\cos\phi\sinh\Delta y + \mathcal{O}\left(q_T^2/Q^2\right)$$

The two-body decay **phase space** with cuts is given by

$$\Phi_{q\to p_1+p_2}(q_T) = \frac{1}{8\pi^2}\int_0^{2\pi} d\phi \int d\Delta y \frac{p_{T,1}^2}{Q^2}\Theta_{\text{cuts}}(q_T, \phi, \Delta y; \text{cuts})$$

The integrand has a dependence on q_T through the combinations q_T^2 and $q_T\cos\phi$. It follows that

presence of **linear fiducial power corrections** ⟷ use of cuts **breaking the azimuthal symmetry**

Origin of linear power corrections: simplified cuts (no rapidity cuts)

Symmetric cuts: $p_{T,i} > p_T^{\rm cut}, \quad i = 1,2 \implies \min(p_{T,1}, p_{T,2}) > p_T^{\rm cut}$

$$\min(p_{T,1}, p_{T,2}) = \begin{cases} p_{T,1} & \cos\phi < 0 \\ p_{T,1} - q_T \cos\phi & \cos\phi > 0 \end{cases}$$

two different integrands: **breaking of azimuthal symmetry**

$$\Phi(q_T) - \Phi(0) = -\frac{1}{2\pi^2}\frac{q_T}{Q}\frac{p_T^{\rm cut}/Q}{\sqrt{1-(2p_T^{\rm cut})^2/Q^2}}\int_0^{\pi/2} d\phi \cos\phi$$

Asymmetric cuts: $p_T^{\rm hard} > p_T^{\rm cut,h}$ and $p_T^{\rm soft} > p_T^{\rm cut,s} \implies \min(p_{T,1}, p_{T,2}) > p_T^{\rm cut,s}$

Equivalent to the symmetric cuts case: **it does not solve the issue of the appearance of linear power corrections**!

Staggered cuts: $p_{T,1} > p_T^{\rm cut} + \delta p_T$ and $p_{T,2} > p_T^{\rm cut} \implies \min(p_{T,1} - \delta p_T, p_{T,2}) > p_T^{\rm cut}$ *[Grazzini, 2020],*

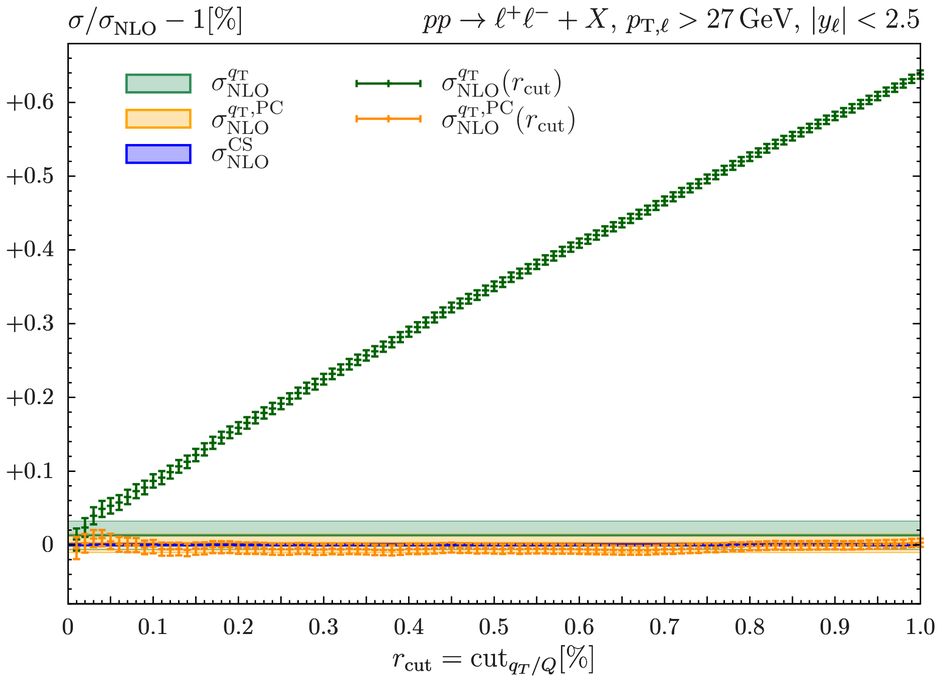
$$\min(p_{T,1} - \delta p_T, p_{T,2}) = \begin{cases} p_{T,1} - \delta p_T & \cos\phi < \delta p_T/q_T \\ p_{T,1} - q_T \cos\phi & \cos\phi > \delta p_T/q_T \end{cases}$$

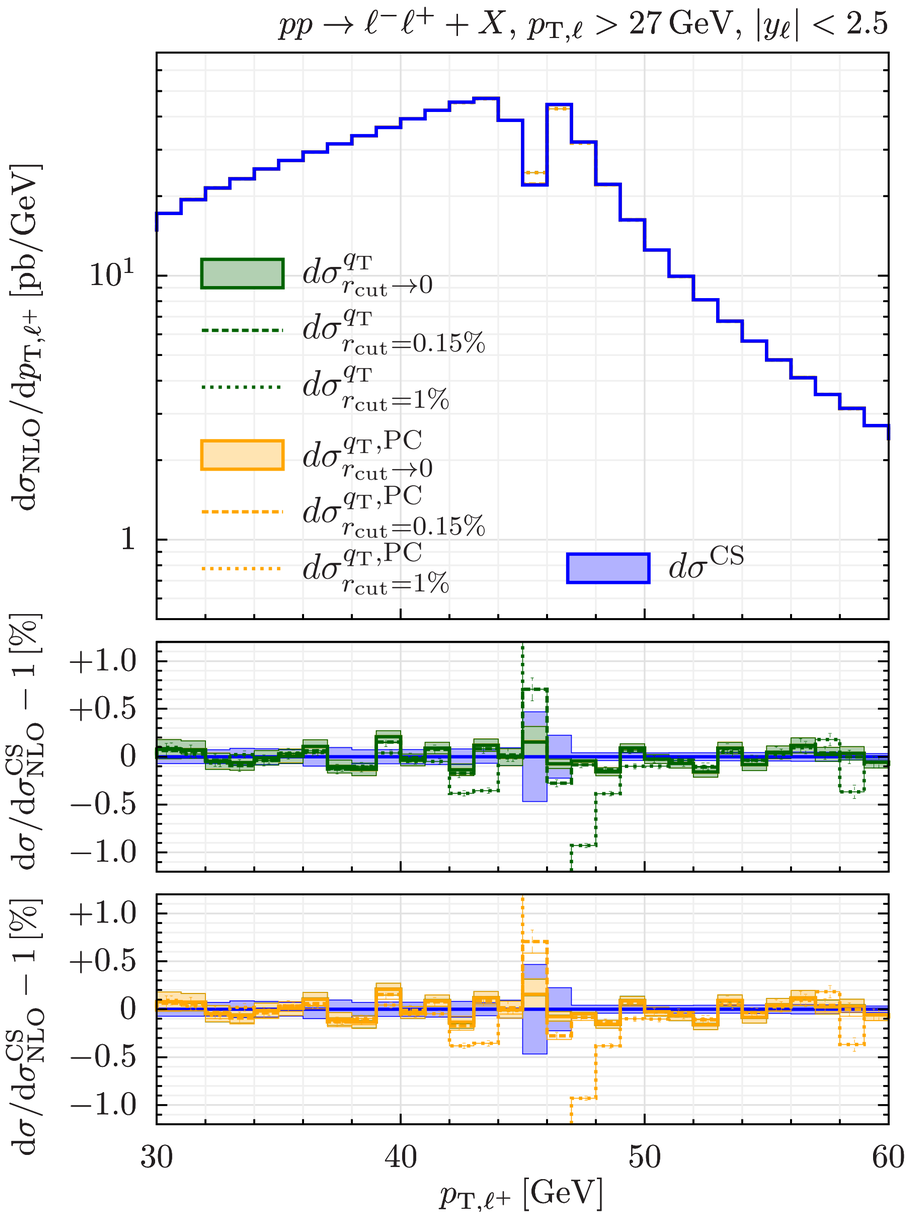
if $\delta p_T/q_T > 1$ **there are no overlapping regions** and the **azimuthal symmetry is preserved**

In the region $q_T < \delta p_T$, the **quadratic dependence on q_T is restored,** as numerically observed in *[Grazzini,Kallweit,Wiesemann, 2017]*

Drell-Yan symmetric cuts @NLO: differential distributions

- The inclusion of linPCs cannot cure problematic regions in phase space where **resummation is required**
- The kinematic edge related to the Jacobean peak of the lepton transverse momentum is an illustrative example
- There is no benefit in including the linPCs in the fixed-order calculation. Conversely, they are required for resummation





Drell-Yan symmetric cuts @NNLO with asymmetric rapidity cuts

The analytical study of rapidity cuts is more involved. Brief summary:

Symmetric rapidity cuts only

$|\eta_i| < \eta^{\text{cut}}, i = 1,2 \implies \max(|\eta_1|, |\eta_2|) < \eta^{\text{cut}}$

The condition $|\eta_1| = |\eta_2|$ defines the turning point azimuthal angle

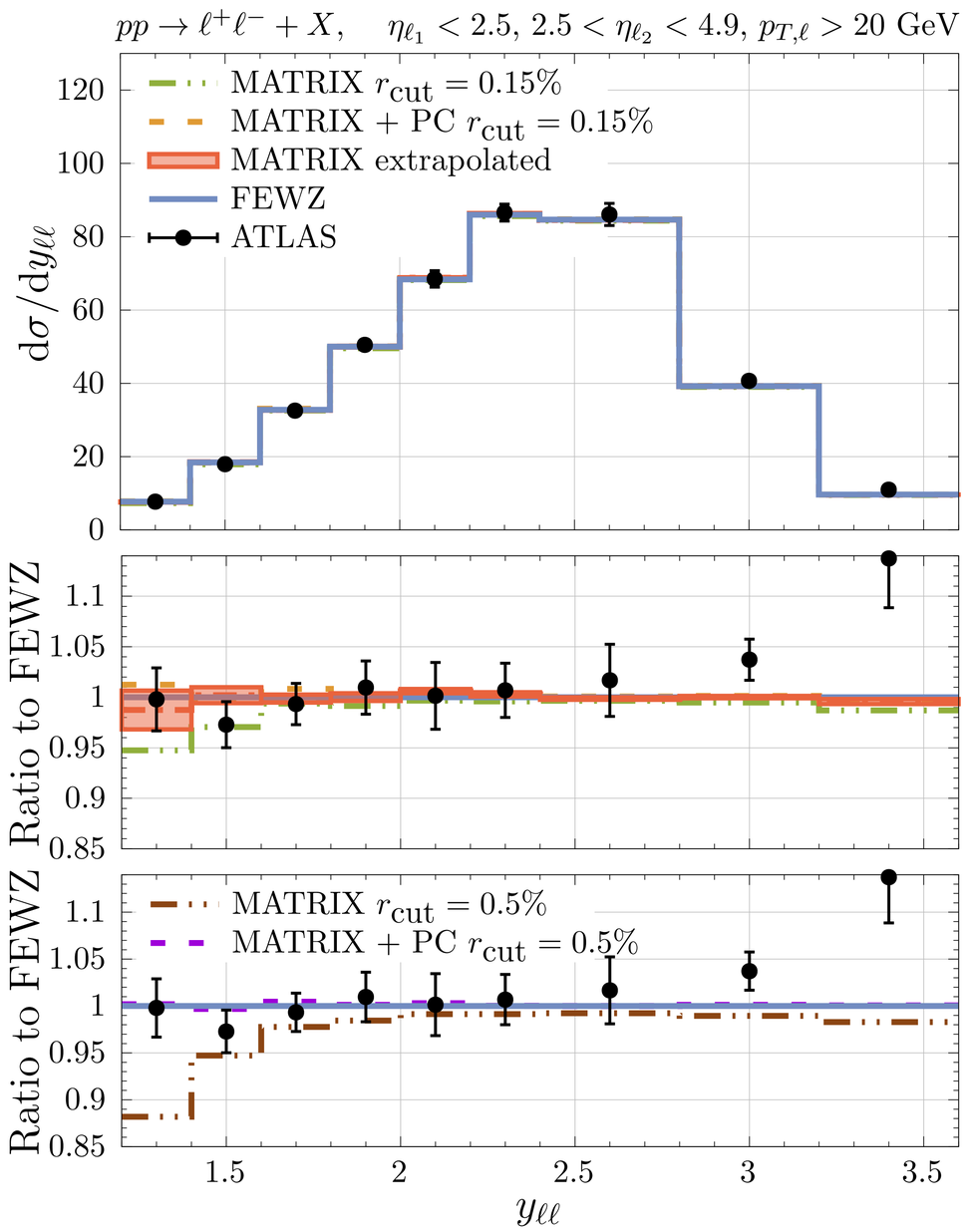
$$\cos\phi^* = \frac{Q}{2q_T}\frac{\sinh(2Y)}{\sinh(2Y+\Delta y)} + \mathcal{O}\left(q_T/Q\right)$$

if $|\cos\phi^*| > 1$, the azimuthal symmetry is not broken and the dependence is quadratic.

At small q_T, a physical solution can be obtained for sufficiently small values of the color singlet rapidity Y

$$\frac{q_T}{Q} > \left|\frac{Y}{\sinh\eta^{\text{cut}}}\right| = \frac{q_T^*}{Q} \implies \begin{cases} q_T > q_T^* & \text{linear dependence} \\ q_T < q_T^* & \text{quadratic dependence} \end{cases}$$

For non-vanishing rapidities Y, the dependence is quadratic if q_T is sufficiently small

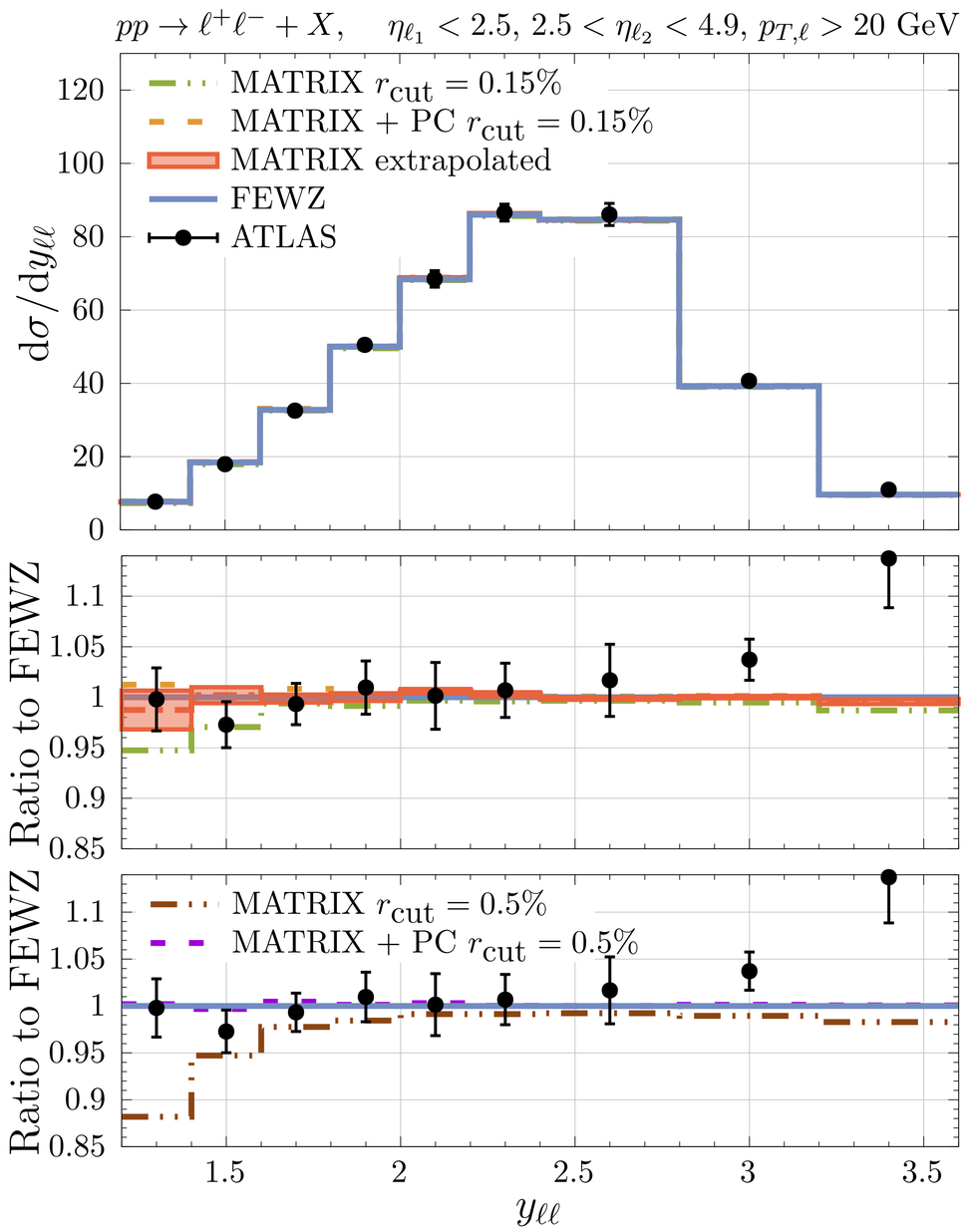


Drell-Yan symmetric cuts @NNLO with asymmetric rapidity cuts

The analytical study of rapidity cuts is more involved. Brief summary:

Symmetric transverse momentum+rapidity cuts
$\min(p_{T,1}, p_{T,2}) < p_T^{\text{cut}}$ and $\max(|\eta_1|, |\eta_2|) < \eta^{\text{cut}}$

- **for small rapidities** Y, the transverse momentum cut is usually more stringent, leading to a **linear dependence**
- **for large values of the rapidity** Y, the rapidity cuts becomes more important, leading to a **quadratic dependence**



Drell-Yan symmetric cuts @NNLO with asymmetric rapidity cuts

The analytical study of rapidity cuts is more involved. Brief summary:

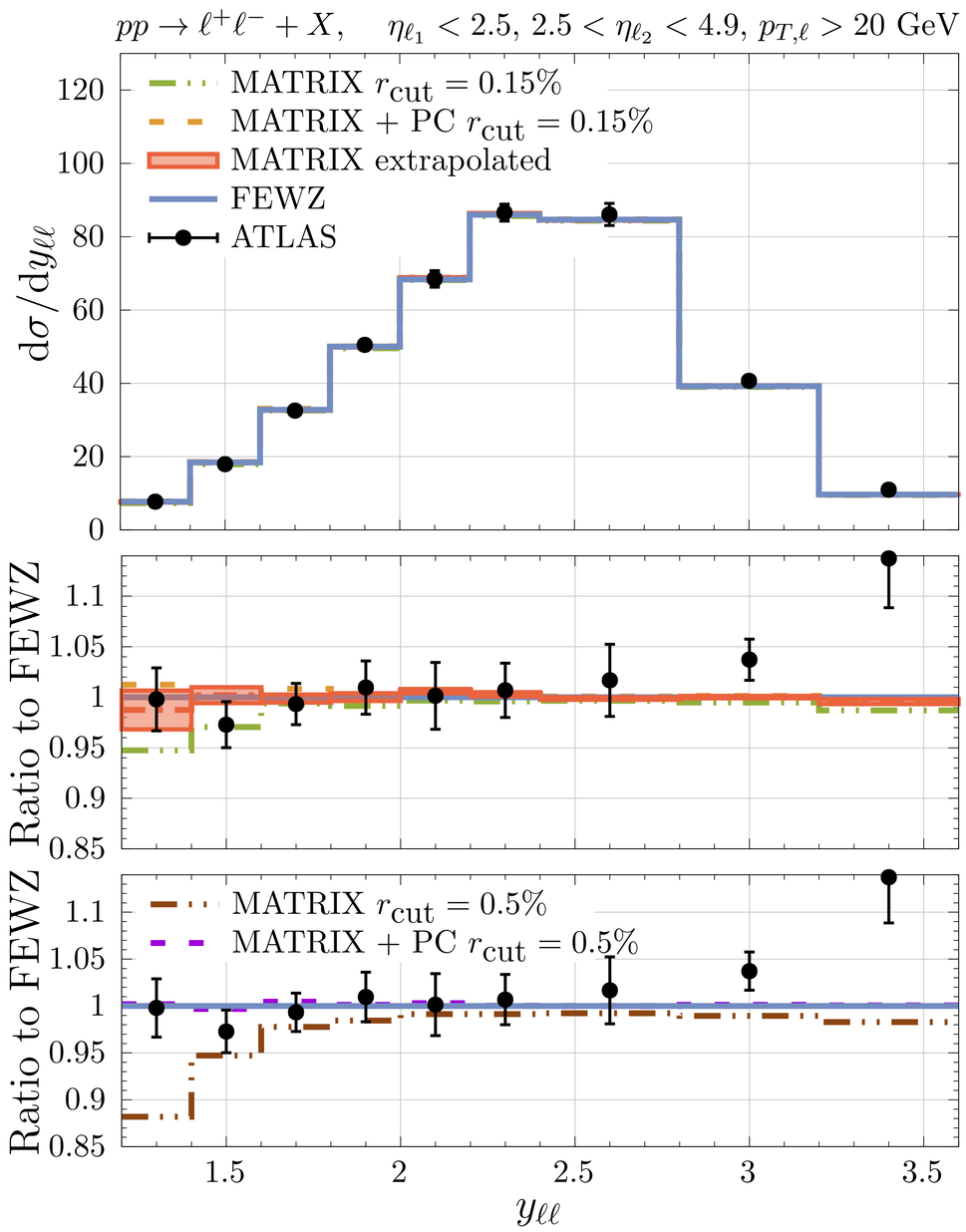
Symmetric transverse momentum + asymmetric rapidity cuts

$$\min(p_{T,1}, p_{T,2}) < p_T^{\rm cut} \text{ and } |\eta^{\rm hard}| < \eta^{\rm cut,h} < |\eta^{\rm soft}| < \eta^{\rm cut,s}$$

- in general, **linear dependence**
- the coefficient of the power correction depends on the rapidity of the color singlet: **the larger is $y_{\ell\ell}$, the smaller is the coefficient**

Comparison with ATLAS 8 TeV dataset

- setup from *[Alekhin, Kardos, Moch, Trocsanyi, 2021]*
- large corrections at small $y_{\ell\ell}$: 5%(10%) at $r_{\rm cut}$=0.15%(0.5%)
- for the standard q_T subtraction computation, **bin-wise extrapolation required** to correctly model the first two bins
- excellent agreement when including $\Delta\sigma^{\rm linPCs}$ already at $r_{\rm cut}$=0.5%



Di-photon@NNLO