Overview of status of QED/EW corrections at the **Z** pole

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LHC EWWG general meeting
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on behalf of the QED/EW precision subgroup WG

since last EWWG general meeting (8-9/10/2020)

- 11 November 2020 `https://indico.cern.ch/event/970391/`
- 18 December 2020 `https://indico.cern.ch/event/980547/`
- 26 February 2021 `https://indico.cern.ch/event/1006069/`
- 9 April 2021 `https://indico.cern.ch/event/1023054/`
- 14 May 2021 `https://indico.cern.ch/event/1032372/`
- 17 September 2021 `https://indico.cern.ch/event/1075230/`
- 15 October 2021 `https://indico.cern.ch/event/1085033/`
- 12 Novembre 2021 `https://indico.cern.ch/event/1093940/`
- 10 December 2021 `https://indico.cern.ch/event/1097287/`
- 28 January 2022 `https://indico.cern.ch/event/1117325/`

Studies and benchmarking on NC Drell-Yan

having in mind, as a target, the LEP precision of $1.6 \cdot 10^{-4}$ on $\sin^2 \vartheta_{eff}^{\ell}$
$$\implies \sim 5 \cdot 10^{-4} \text{ on } A_{FB}$$

Main focus since last G.M. on

- pure weak corrections
    - Gauge invariance and treatment of the Z-resonance
    - input parameter schemes
    - evaluation of residual theoretical uncertainties

- pure QED corrections on A_4
  breaking of the usual decomposition in angular coefficients

- Programs/groups involved
    - KKMC_hh
    - MCSANC
    - POWHEG_ew
    - RADY
    - WZGRAD2
    - DIZET/TauSpinner

Gauge invariance and treatment of resonances

for more details, see Denner, SD, 1912.06823 and refs. therein

Dyson summation of propagators mixes perturbative orders.

$$G(p^2) = \frac{i}{p^2 - M^2} + \frac{i}{p^2 - M^2} i\Sigma_{\mathrm{R}}(p^2)\frac{i}{p^2 - M^2} + \quad \cdots$$

$$= \frac{i}{p^2 - M^2 + \Sigma_{\mathrm{R}}(p^2)}, \qquad \Sigma_{\mathrm{R}}(M^2) = iM\Gamma$$

But:

Consistency of pert. calculations often requires complete fixed orders.

$\hookrightarrow$ Consistency jeopardized if no special care is taken!

Gauge-invariance requirements:

▶ proper cancellation of gauge-parameter dependences
(relations between self-energies, vertex corrections, boxes, etc.)

▶ validity of (internal) Ward identities
(e.g. ruling cancellations for forward scattering of $e^{\pm}$ or at high energies)

Required: schemes to introduce width Γ

▶ without breaking gauge invariance

▶ maintaining (at least) NLO accuracy everywhere in phase space

S. Dittmaier, 18/12/2020

Adopted width schemes within the WG

- complex mass scheme (CMS)
  - complex M_W and M_Z

$$\mu_V^2 = M_V^2 - i\Gamma_V M_V \implies \cos^2 \vartheta = \frac{\mu_W^2}{\mu_Z^2}$$

- factorization scheme (FS): global correction factor in the limit $\Gamma \to 0$

$$d\sigma_{\text{weak}} = \delta_{\text{weak}}^{\Gamma=0} \times d\sigma_{LO}^{\Gamma \neq 0}$$

- pole scheme (PS): amplitude organized in resonant g.i. contributions

$$
\begin{aligned}
\mathcal{M} &= \frac{R(p^2)}{p^2 - M_2} + N(p^2) = \frac{R(M^2)}{p^2 - M_2} + \frac{R(p^2)R(M^2)}{p^2 - M_2} + N(p^2) \\
&\to \frac{\bar{R}(M^2 - i\Gamma M)}{p^2 - M_2 + i\Gamma M} + \frac{R(p^2)R(M^2)}{p^2 - M_2} + \bar{N}(p^2)
\end{aligned}
$$

Adopted input parameter schemes

- (G_μ, M_W, M_Z)
  - MCSANC, POWHEG_EW, RADY, WZGRAD2

- $(\alpha(0), M_W, M_Z)$
  - MCSANC, POWHEG_EW, RADY, WZGRAD2

- $(G_\mu, \sin^2 \vartheta_{\text{eff}}^\ell, M_Z)$, $(\alpha(0), \sin^2 \vartheta_{\text{eff}}^\ell, M_Z)$
  - POWHEG_EW, RADY

- $(\alpha(0), G_\mu, M_Z)$
  - DIZET

Comparison of width schemes in RADY/Powheg (RADY results from Nov 2020)

FB asymmetry A_{FB}: (input parameters: G_μ, M_Z, M_W)

Code/scheme:	$89 < M_{\ell\ell[GeV]} < 93$	$60 < M_{\ell\ell[GeV]} < 81$	$81 < M_{\ell\ell[GeV]} < 101$	$101 < M_{\ell\ell[GeV]} < 150$
$A_{\mathrm{FB}}(\mathrm{LO})$				
RADY/CMS	0.046551(4)	−0.202894(5)	0.044817(4)	0.226101(5)
Powheg/CMS	0.04655(2)	−0.20292(4)	0.04481(2)	0.22608(4)
RADY/PS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
Powheg/PS	0.04655(2)	−0.202975(24)	0.04481(2)	0.22608(4)
RADY/FS	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
Powheg/FS	0.04655(2)	−0.202975(24)	0.04481(2)	0.22608(4)
|X−CMS|	< 0.00001	0.00006	< 0.00001	0.00001
$A_{\mathrm{FB}}(\mathrm{NLO\ weak})$				
RADY/CMS	0.028568(4)	−0.215645(5)	0.026846(4)	0.218966(5)
Powheg/CMS	0.02855(2)	−0.21566(2)	0.02682(2)	0.21895(2)
RADY/PS	0.028574(4)	−0.215576(4)	0.026852(4)	0.218938(4)
Powheg/PS	0.02856(2)	−0.21559(2)	0.02683(2)	0.218933(25)
RADY/FS	0.028768(4)	−0.215354(4)	0.027044(4)	0.219037(4)
Powheg/FS	0.02875(2)	−0.21537(2)	0.02702(2)	0.219032(25)
|PS−CMS|	≲ 0.00001	0.0001	≲ 0.00001	≲ 0.00002
|FS−CMS|	0.0002	0.0003	0.0002	0.0001

↪ |PS-CMS| ≲ 0.00001 in resonance window

But: FS less solid than CMS and PS

New: agreement with Powheg/CMS within errors

to be finalized with comparisons on universal HO

(G_μ, M_W, M_Z) scheme: $\int d\sigma$ Factorization scheme

rescaled correction from $\alpha_{G_\mu} \to \alpha$

we could quote in the report numbers without rescaled couplings in the weak part

Code:	$89 < M_{\ell\bar{\ell}}[\text{GeV}] < 93$	$60 < M_{\ell\bar{\ell}}[\text{GeV}] < 81$	$81 < M_{\ell\bar{\ell}}[\text{GeV}] < 101$	$101 < M_{\ell\bar{\ell}}[\text{GeV}] < 150$
$\sigma(\text{LO})$ (pb)				
MCSANC	612.531(5)	46.870(2)	880.527(6)	-
POWHEG$_{ew}$ (FS)	612.529(8)	46.870(1)	880.513(9)	-
WZGRAD2	612.521(7)	46.868(4)	880.520(10)	-
RADY (FS)	612.526(1)	46.871(1)	880.520(2)	-
$\sigma(\text{NLO})/\sigma(\text{LO})$				
MCSANC	0.99167(2)	1.02865(7)	0.99206(1)	-
POWHEG$_{ew}$ (FS) $\vert$ α resc.	0.99149(3)	1.02870(4)	0.99190(2)	0.98925(3)
no α resc.	0.991193(25)	1.02971(4)	0.99162(2)	0.98887(3)
WZGRAD2	0.99198(1)	1.02913(4)	0.99239(1)	-
RADY (FS)	0.99148(1)	1.02864(4)	0.99189(1)	-
$\sigma(\text{NLO} + \text{HO})/\sigma(\text{LO})$				
MCSANC	0.99232(2)	1.02614(7)	0.99268(1)	-
POWHEG$_{ew}$ (FS) $\vert$ α resc.	0.992149(25)	1.02602(4)	0.99252(2)	0.98967(3)
no α resc.	0.991799(25)	1.02576(4)	0.99217(1)	0.98918(3)
RADY (FS)	0.99179(1)	1.02589(1)	0.99216(1)	-
TauSpinner+DIZET (estimated)	0.99211(0)	1.02321(0)	0.99264(0)	0.98884(0)

nice agreement between tuned codes within few 0.01% on cross sections

rescaled correction from $\alpha_{G_\mu} \to \alpha$

we could quote in the report numbers without rescaled couplings in the weak part

Code:	$89 < M_{\ell\ell}[\text{GeV}] < 93$	$60 < M_{\ell\ell}[\text{GeV}] < 81$	$81 < M_{\ell\ell}[\text{GeV}] < 101$	$101 < M_{\ell\ell}[\text{GeV}] < 150$
$A_{\text{FB}}(\text{LO})$				
MCSANC	0.04654(1)	−0.20299(4)	0.04481(1)	-
POWHEG$_{ew}$ (FS)	0.04655(2)	−0.202975(24)	0.04481(2)	0.22608(4)
WZGRAD2	0.04654(1))	−0.20299(8)	0.04482(1)	-
RADY (FS)	0.046547(4)	−0.202955(4)	0.044812(3)	0.226090(4)
$A_{\text{FB}}(\text{NLO}) - A_{\text{FB}}(\text{LO})$				
MCSANC (FS)	−0.01717(2)	−0.01183(8)	−0.01715(2)	−0.00688(7)
POWHEG$_{ew}$ (FS) α resc.	−0.01718(3)	−0.01198(3)	−0.01718(3)	−0.00680(3)
no α resc.	−0.017796(3)	−0.01239(3)	−0.01778(3)	−0.00705(5)
WZGRAD2	−0.01716(2)	−0.01186(11)	−0.01715(2)	−0.00686(14)
RADY (FS)	−0.01717(2)	−0.01199(1)	−0.01716(1)	−0.00681(1)
$A_{\text{FB}}(\text{NLO} + \text{HO}) - A_{\text{FB}}(\text{NLO})$				
MCSANC	0.00137(2)	0.00111(8)	0.00137(2)	-
POWHEG$_{ew}$ (FS) α resc.	0.00136(3)	0.00113(3)	0.00137(3)	0.0004(2)
no α resc. NLOHO no resc.	0.00183(3)	0.00147(3)	0.00183(3)	0.00057(35)
- NLO resc.	0.00122(3)	0.00105(3)	0.001223(25)	0.00032(35)
RADY (FS)	0.00122(1)	0.00103(1)	0.00122(1)	-
$A_{\text{FB}}(\text{NLO} + \text{HO}) - A_{\text{FB}}(\text{LO})$				
MCSANC	−0.01551(2)	−0.01059(8)	−0.01551(1)	-
POWHEG$_{ew}$ (FS) α resc.	−0.01582(3)	−0.01085(3)	−0.01581(3)	−0.0064(2)
no α resc.	−0.01597(3)	−0.01092(3)	−0.015957(25)	−0.0065(5)
RADY (FS)	−0.01595(1)	−0.01096(1)	−0.01594(1)	-
TauSpinner+DIZET (estimated)	−0.01507(0)	−0.01104(0)	−0.01514(0)	0.00684(0)

small difference of about 0.01% on higher orders (maybe due to α rescaling, to be clarified)

(α, M_W, M_Z) scheme: $\int d\sigma$ Factorization scheme

Code:	$89 < M_{\ell\ell}[\text{GeV}] < 93$	$60 < M_{\ell\ell}[\text{GeV}] < 81$	$81 < M_{\ell\ell}[\text{GeV}] < 101$	$101 < M_{\ell\ell}[\text{GeV}] < 150$
$\sigma(\text{LO})$ (pb)				
MCSANC	571.412(5)	43.724(2)	821.414(6)	-
WZGRAD2	571.409(7)	43.722(4)	821.419(9)	-
RADY (FS)	571.414(1)	43.725(1)	821.420(2)	-
$\sigma(\text{NLO})/\sigma(\text{LO})$				
MCSANC	1.05117(1)	1.08830(4)	1.05157(1)	-
WZGRAD2	1.05151(1)	1.08854(9)	1.05191(1)	-
RADY (FS)	1.05101(1)	1.08816(1)	1.05142(1)	-
$\sigma(\text{NLO} + \text{HO})/\sigma(\text{LO})$				
MCSANC	1.06452(1)	1.10004(4)	1.06491(1)	-
RADY (FS)	1.06387(1)	1.09979(1)	1.06426(1)	-
TauSpinner+DIZET (estimated)	1.06558(0)	1.09892(0)	1.06613(0)	1.06202(0)

overall nice agreement

(α, M_W, M_Z) scheme: $\int d\sigma_{F-B} / \int d\sigma_{F+B}$ Factorization scheme

Code:	$89 < M_{\ell\bar{\ell}}[\text{GeV}] < 93$	$60 < M_{\ell\bar{\ell}}[\text{GeV}] < 81$	$81 < M_{\ell\bar{\ell}}[\text{GeV}] < 101$	$101 < M_{\ell\bar{\ell}}[\text{GeV}] < 150$
$A_{FB}(\text{LO})$				
MCSANC	0.04655(1)	−0.20304(4)	0.04482(1)	-
WZGRAD2	0.04654(1))	−0.20299(8)	0.04482(1)	-
RADY (FS)	0.04655(1)	−0.20295(1)	0.04481(1)	-
$A_{FB}(\text{NLO}) - A_{FB}(\text{LO})$				
MCSANC (FS)	−0.01618(1)	−0.01118(7)	−0.01618(1)	-0.00647(7)
WZGRAD2	−0.01619(2)	−0.01121(12)	−0.01617(2)	-0.00650(14)
RADY (FS)	−0.01620(1)	−0.01133(1)	−0.01619(1)	-0.00642(1)
$A_{FB}(\text{NLO} + \text{HO}) - A_{FB}(\text{NLO})$				
MCSANC	0.00077(1)	0.00068(6)	0.00078(1)	-
RADY (FS)	0.00078(2)	0.00066(1)	0.00078(1)	-
$A_{FB}(\text{NLO} + \text{HO}) - A_{FB}(\text{LO})$				
MCSANC	−0.01519(1)	−0.01035(6)	−0.01517(1)	-
RADY (FS)	−0.01542(1)	−0.01074(2)	−0.01562(2)	-
TauSpinner+DIZET (estimated)	−0.01508(0)	−0.01104(0)	−0.01515(0)	0.00684(0)

overall nice agreement

Comparison of width schemes in RADY/Powheg

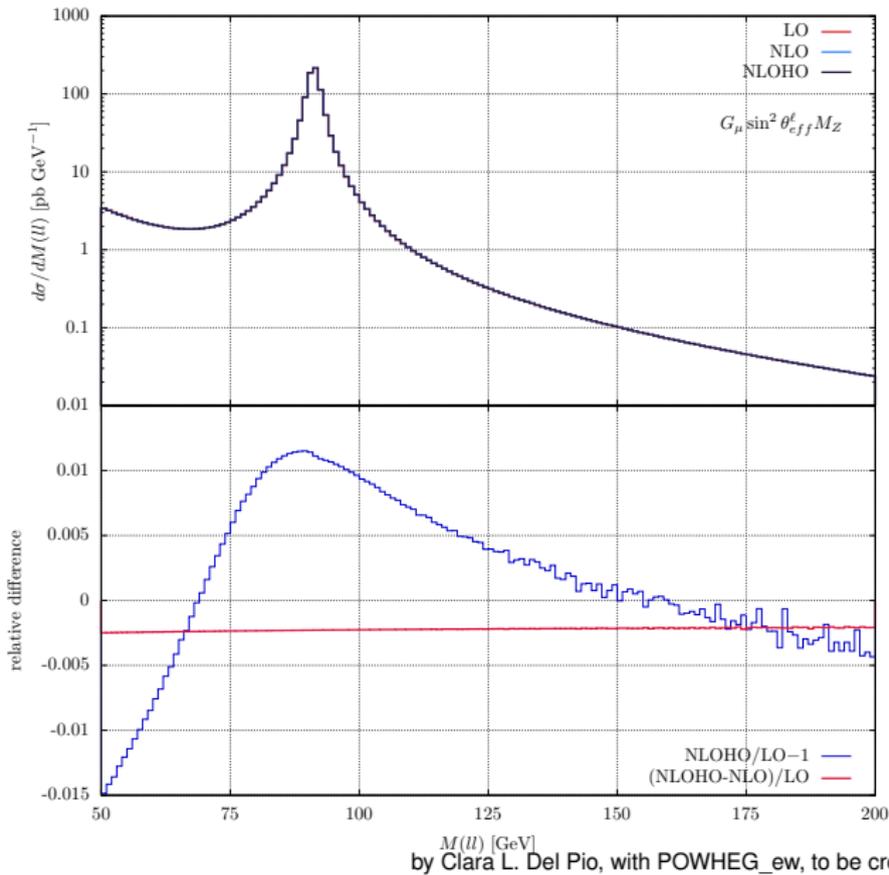**FB asymmetry A_{FB}:** (input parameters: G_μ, M_Z, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$)

(see Chiesa, Piccinini, Vicini, arXiv:1906.11569)

Code/scheme:	$89<M_{\ell\ell}[GeV]<93$	$60<M_{\ell\ell}[GeV]<81$	$81<M_{\ell\ell}[GeV]<101$	$101<M_{\ell\ell}[GeV]<150$
A_{FB}(LO)				
RADY/CMS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/CMS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
RADY/PS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/PS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
RADY/FS	0.030552(3)	−0.214572(4)	0.028815(4)	0.220793(5)
Powheg/FS	0.03056(2)	−0.21459(2)	0.02881(2)	0.22077(35)
|X−CMS|	0	0	0	0
A_{FB}(NLO weak)				
RADY/CMS	0.030459(3)	−0.214082(4)	0.028738(4)	0.219509(5)
Powheg/CMS	0.03046(2)	−0.21408(2)	0.02873(2)	0.219506(25)
RADY/PS	0.030376(3)	−0.214136(4)	0.028658(4)	0.219475(5)
Powheg/PS	0.03038(2)	−0.21413(2)	0.02865(2)	0.219472(25)
RADY/FS	0.030589(3)	−0.213854(4)	0.028871(4)	0.219573(5)
Powheg/FS	0.03059(2)	−0.21385(2)	0.02886(2)	0.219571(25)
|PS−CMS|	0.00008	0.00005	0.00008	0.00003
|FS−CMS|	0.0001	0.0002	0.0001	0.00006

$\hookrightarrow$ \|PS-CMS\| $\sim$ 0.00008 in resonance window

Agreement with Powheg/CMS within errors

S.Dittmaier   NLO weak and NLO QED results from RADY   LHC EWWG Meeting, May 14, 2020   6
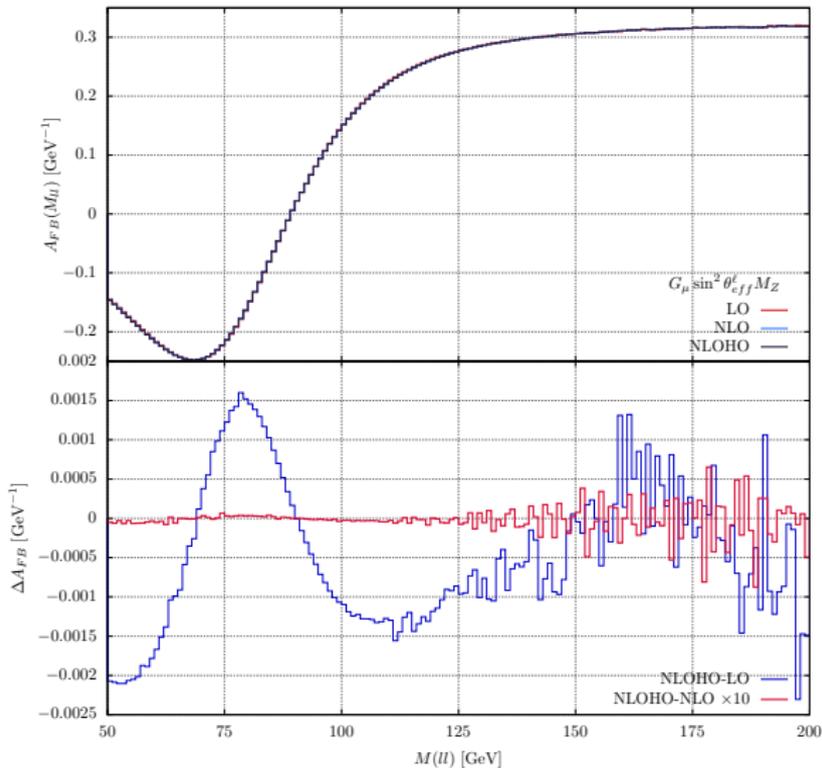
to be completed with inclusion of universal higher orders

HO corrections to $d\sigma/dm_{ll}$

by Clara L. Del Pio, with POWHEG_ew, to be cross-checked with RADY

HO corrections to A_{FB}

by Clara L. Del Pio, with POWHEG_ew, to be cross-checked with RADY

Towards uncertainty estimates in the pure weak sector

On Z resonance (leading pole term):

$$A_4 = \frac{\sum_q X_q \, 8 \frac{v_\ell}{a_\ell} \frac{v_q}{a_q}}{\sum_q X_q \left(1 + \frac{v_\ell^2}{a_\ell^2}\right)\left(1 + \frac{v_q^2}{a_q^2}\right)} \qquad X_q = f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4 s_\ell^2, \qquad\qquad s_\ell^2 \equiv \sin^2 \theta_{\text{eff}}^\ell$$

$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q) \qquad \Delta_q = \underbrace{\Delta_{q(1)}}_{\text{implemented}} + \underbrace{\Delta_{q(2)}}_{\text{missing}}$$

$$\frac{\delta A_4}{A_4} \approx \frac{\sum_q X_q \, (-4|e_q|\Delta_{q(2)})}{\sum_q X_q \, (1 - 4|e_q| s_\ell^2)} + \frac{\sum_q X_q \, 8|e_q| \, (1 - 4|e_q| s_\ell^2) \Delta_{q(2)}}{\sum_q X_q \, [1 + (1 - 4|e_q| s_\ell^2)^2]}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

Off Z pole: need to include non-res. terms, *estimate* their missing 2-loop terms

Including photon exchange and photon form factor estimate:
(neglecting boxes and s-dependence of Z form factors)

$$A_4 = \frac{\sum_q X_q \, 4\left(\frac{v_\ell}{a_\ell}\frac{v_q}{a_q} + \frac{v_{\ell q}(s)}{a_\ell a_q}\right)}{\sum_q X_q \left(1 + \frac{v_\ell^2}{a_\ell^2} + \frac{v_q^2}{a_q^2} + \frac{v_{\ell q}^2(s)}{a_\ell^2 a_q^2}\right)} \qquad X_q = f_q(x_1)f_{\bar q}(x_2) + f_{\bar q}(x_1)f_q(x_2)$$

$$v_{\ell q}(s) = v_\ell v_q + \frac{s - M_Z^2 - iM_Z\Gamma_Z}{s}\, e^2\, e_q\, (1 + \overline{\Delta}_q)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4s_\ell^2, \qquad\qquad s_\ell^2 \equiv \sin^2\theta_{\text{eff}}^\ell$$

$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q) \qquad \Delta_q = \Delta_{q(1)} + \Delta_{q(2)}$$
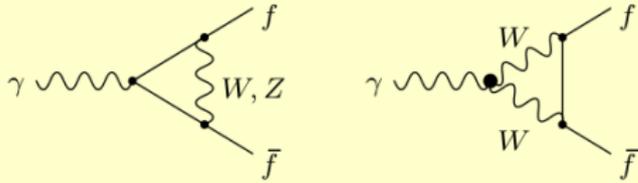
$$\Delta_q = \underbrace{\overline{\Delta}_{q(1)}}_{\text{known}} + \underbrace{\overline{\Delta}_{q(2)}}_{\text{unknown}}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

$$\overline{\Delta}_{q(2)} = \pm\overline{\Delta}_{q(1)} \times \frac{g^2}{16\pi^2}n_f, \qquad n_f = 6 + 6N_c \qquad \text{(maybe underestimate?)}$$
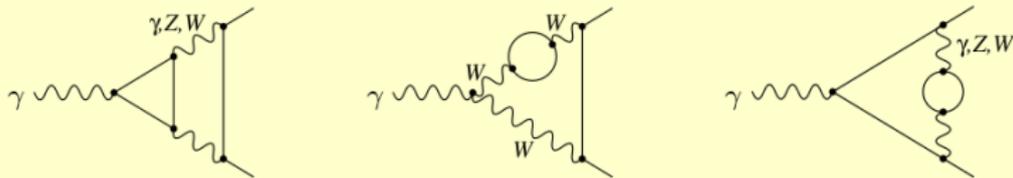
A. Freitas, 10/12/2021

Example contributions to $\Delta_{q(1)}$, $\overline{\Delta}_{q(1)}$:

Note: 1-loop boxes and s-dependence of Z vertex form factors also contribute at same order (1-loop without Z pole)

Example contributions to $\Delta_{q(1)}$, $\overline{\Delta}_{q(2)}$:

Z-pole 2-loop flavor dependence:

Assume: no EW 2-loop corrections included in analysis (i.e. they are theory unc.)

Schemes:
- α' : Use α, M_W, M_Z as inputs, perturb. exp. in α
- α : Use α, G_μ, M_Z as inputs, perturb. exp. in α
- G_μ : Use G_μ, M_W, M_Z as inputs, perturb. exp. in G_μ

Scheme:	α'	α	G_μ
$\Delta_{u(\alpha^2)}$ [10^{-5}]	-1.74	-1.82	-1.45
$\Delta_{d(\alpha^2)}$ [10^{-5}]	-1.49	-1.67	-0.88
including non-factorizable EW$\times$QCD corrections:			
$\Delta_{u(\alpha^2+\alpha\alpha_s)}$ [10^{-5}]	$+1.47$	$+1.38$	$+1.74$
$\Delta_{d(\alpha^2+\alpha\alpha_s)}$ [10^{-5}]	$+2.34$	$+2.15$	$+2.95$

Czarnecki, Kühn '96

Harlander, Seidensticker, Steinhauser '97

Inputs: $M_Z = 91.1876$ GeV, $M_W = 80.385$ GeV, $M_H = 125.7$ GeV

$m_t = 173.5$ GeV, $\Delta\alpha = 0.059$, $\alpha_s = 0.1184$, $G_\mu = 1.16638 \times 10^{-5}$ GeV^{-2}

A. Freitas, 10/12/2021

Including photon exchange and photon form factor estimate:

Impact of EW 2-loop contributions (without EW×QCD):

$\delta A_4/A_4$: $[10^{-4}]$

$m_{\ell\ell}$ [GeV] Scheme:	α'	α	G_μ
60	0.37	0.35	15.50
70	0.52	0.60	8.99
80	1.53	1.61	37.37
M_Z-2	17.54	10.27	208.5
M_Z-1	2.14	1.97	27.6
M_Z	0.58	0.59	0.57
M_Z+2	0.45	0.46	10.61
M_Z+1	0.55	0.55	16.15
100	0.84	0.83	24.85
110	0.80	0.81	21.71
130	0.53	0.56	12.34
150	0.34	0.38	6.04

- dominated by photon form factor unc. $\overline{\Delta}_q$

- artifically large corrections for G_μ scheme

  [same for (G_μ, s_ℓ, M_Z) scheme?]

A. Freitas, 10/12/2021

- enhancement in G_μ scheme seems to be due to residual $\Delta\rho$ effects affecting the γ-exchange

- however for off-peak points also box contributions and form factor dependence on s become important

- a careful analysis with the results from generators with different input parameter schemes could be useful

- inclusion of $\mathcal{O}(\alpha\alpha_s)$ contributions

QED corrections (at QCD LO)

- important to disentangle on A_4
  - ISR
  - FSR
  - IFI

- A_4 defined as $8/3 A_{FB}$ or $4\langle\cos\vartheta\rangle$ (equivalent options at LO)

- IFI contributions defined as

$$A_{FB}^{\text{IFI}} = \frac{(\sigma_F - \sigma_B)^{\text{NLO}} - (\sigma_F - \sigma_B)^{\text{ISR}} - (\sigma_F - \sigma_B)^{\text{FSR}} + 2(\sigma_F - \sigma_B)^{\text{LO}}}{(\sigma_F + \sigma_B)^{\text{NLO}} - (\sigma_F + \sigma_B)^{\text{ISR}} - (\sigma_F + \sigma_B)^{\text{FSR}} + 2(\sigma_F + \sigma_B)^{\text{LO}}}$$

$$\langle\cos\vartheta\rangle^{\text{IFI}} = \frac{\int\cos\vartheta\, d\sigma_{\text{NLO}} - \int\cos\vartheta\, d\sigma_{\text{ISR}} - \int\cos\vartheta\, d\sigma_{\text{FSR}} + -\int\cos\vartheta\, d\sigma_{\text{LO}}}{\int d\sigma_{\text{NLO}} - \int d\sigma_{\text{ISR}} - \int d\sigma_{\text{FSR}} + 2\int d\sigma_{\text{LO}}}$$

- tuned comparison at fixed order (NLO) level for all codes except for KKMC-HH which produces only exponentiated results for both ISR and FSR

- input parameter scheme: (α, M_W, M_Z)

$$A_4 = 8/3 A_{FB}$$

| Code:	$89 < M_{\ell\bar{\ell}}[\text{GeV}] < 93$	$60 < M_{\ell\bar{\ell}}[\text{GeV}] < 81$	$81 < M_{\ell\bar{\ell}}[\text{GeV}] < 101$	$101 < M_{\ell\bar{\ell}}[\text{GeV}] < 150$
$8/3 \cdot [\text{A}_{FB} \text{ (NLO QED ISR)} - \text{A}_{FB} \text{ (LO)}]/10^{-4}$				
MCSANC	$0.2(3)$	$-5(2)$	$0.2(3)$	$5(2)$
WZGRAD2	$0.2(5)$	$-5(3)$	$0.3(5)$	$6(4)$
KKMC-hh	$-1.0(6)$	$0(1)$	$-0.5(5)$	$-8(2)$
KKMC-hh (NISR)	$-1(2)$	$0(4)$	$0(1)$	$6(8)$
RADY (CMS)	$0.16(4)$	$-4.05(3)$	$0.12(3)$	$4.90(3)$
A. Huss	$0.17(1)$	$-4.07(1)$	$0.11(1)$	$4.94(4)$
POWHEG$_{ew}$	$0.1(1)$	$-4.0(4)$	$0.1(1)$	$4.5(7)$
$8/3 \cdot [\text{A}_{FB} \text{ (NLO QED IFI)} - \text{A}_{FB} \text{ (LO)}]/10^{-4}$				
MCSANC	$-2.8(5)$	$-34(2)$	$-4.0(4)$	$-60(3)$
WZGRAD2	$-1.1(5)$	$-37(3)$	$-2.3(5)$	$-51(4)$
KKMC-hh	$-3.8(6)$	$-25(1)$	$-2.1(1)$	$-53(1)$
KKMC-hh (NISR)	$-3.1(6)$	$-17(1)$	$-3.2(5)$	$-60(3)$
RADY (CMS)	$-1.5(1)$	$-33.6(4)$	$-2.49(7)$	$-59.5(1)$
A. Huss	$-1.42(6)$	$-33.9(6)$	$-2.57(7)$	$-58.7(3)$
POWHEG$_{ew}$ $\quad \mu_F = M_{\ell\bar{\ell}\gamma}$	$-1.2(3)$	$-62(1)$	$-2.5(4)$	$-59(2)$
$\quad\quad\quad\quad\quad \mu_F = M_{\ell\bar{\ell}}$	$-1.3(6)$	$-34(2)$	$-2.6(7)$	$-59(3)$

● POWHEG$_{ew}$ $\mu_F(1) \Longrightarrow M_{ll}$ for real rad calculated with underlying Born momenta

● POWHEG$_{ew}$ $\mu_F(2) \Longrightarrow M_{ll}$ for real rad calculated with radiative event momenta

● differences between $\mu_F(1)$ and (2) expected to decrease when including also QCD corrections

$$A_4 = 4\langle\cos\vartheta\rangle$$

Code:	$89 < M_{\ell\bar{\ell}}$[GeV] < 93	$60 < M_{\ell\bar{\ell}}$[GeV] < 81	$81 < M_{\ell\bar{\ell}}$[GeV] < 101	$101 < M_{\ell\bar{\ell}}$[GeV] < 150
		$[A_4\,(\text{NLO QED ISR}) - A_4\,(\text{LO})]/10^{-4}$		
RADY (CMS)	0.15(3)	$-4.05(3)$	0.10(2)	4.89(2)
A. Huss	0.16(1)	$-4.07(1)$	0.11(1)	4.87(2)
POWHEG$_{ew}$	0.07(9)	$-4.0(3)$	0.10(7)	4.8(4)
		$[A_4\,(\text{NLO QED IFI}) - A_4\,(\text{LO})]/10^{-4}$		
RADY (CMS)	$-1.7(1)$	$-42.3(4)$	$-2.97(6)$	$-71.6(2)$
A. Huss	$-1.68(6)$	$-42.4(6)$	$-3.05(8)$	$-71.2(3)$
POWHEG$_{ew}$ $\mu_F = M_{\ell\bar{\ell}\gamma}$	$-1.5(5)$	$-70(1)$	$-3.0(4)$	$-71(3)$
$\mu_F = M_{\ell\bar{\ell}}$	$-1.5(5)$	$-43(1)$	$-3.0(4)$	$-71(3)$

- POWHEG$_{ew}$ $\mu_F(1) \Longrightarrow M_{ll}$ for real rad calculated with underlying Born momenta

- POWHEG$_{ew}$ $\mu_F(2) \Longrightarrow M_{ll}$ for real rad calculated with radiative event momenta

- at low and high M_{ll} virtual QED boxes and I-F real radiation interference break factorization assumption for angular coefficients and the LO equality between the two A_4 def's

- differences between $\mu_F(1)$ and (2) expected to decrease when including also QCD corrections

IFI contribution to A_4 according to different definitions

plot by Clara L. Del Pio with POWHEG_EW
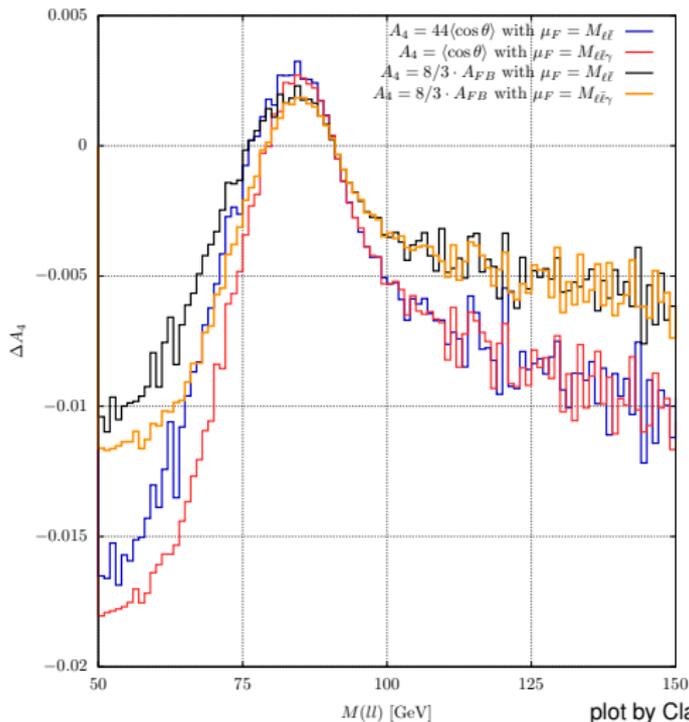
- virtual QED boxes and I-F real radiation interference give different contributions according to the definition of A_4
- results from contributing codes can be superimposed

Shape of QED IFI and ISR corrections to A4
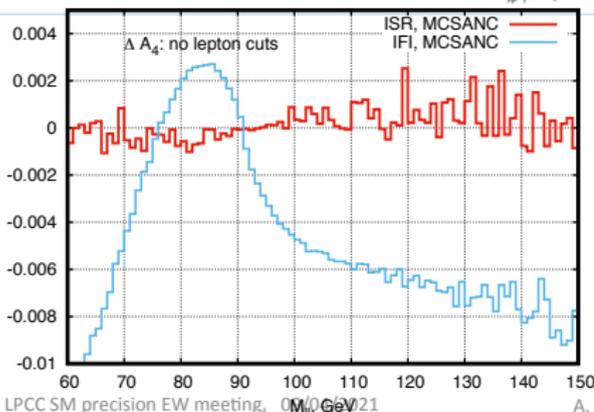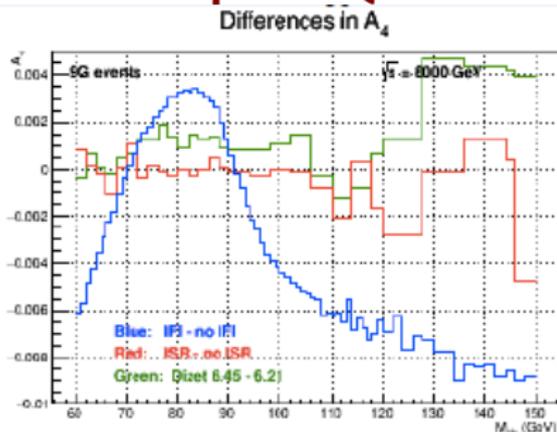
- Shown here from KKMC-hh (top) and MC-SANC (bottom) in full phase space of decay leptons

- Bare muons, A4 = 8/3 AFB, AFB computed as $(\sigma_F - \sigma_B)/(\sigma_F + \sigma_B)$

- ISR is small, < 1 in units of 10^{-4} and flat versus m_{ll}

- IFI is small around Z pole. 1-3 in units of 10^{-4}, but has strong shape vs m_{ll}, with values between -100 and +50 in units of 10^{-4}. It follows roughly shape of asymmetry.

- Note that $\Delta A4 = 1 \cdot 10^{-4}$ corresponds to ~ $1 \cdot 10^{-5}$ for $\sin^2\theta^{eff}_l$, which means that $\Delta AFB = 1 \cdot 10^{-4}$ → ~ $3 \cdot 10^{-5}$

towards higher perturbative accuracy:
new $\mathcal{O}(\alpha\alpha_s)$ calculation

Exact mixed NNLO QCD-EW corrections to the Neutral Current Drell-Yan process

Alessandro Vicini
University of Milano, INFN Milano

EW WG meeting, CERN/online, December 10th 2021

in collaboration with: R.Bonciani, F.Buccioni, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano,

based on arXiv:2106.11953, arXiv:2007.06518, arXiv:2111.12694

see also previous talk by F. Buccioni

Estimate of the residual uncertainties

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
                                                         reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

A representative example from the results for the on-shell Z production total cross section R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518
→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu - \alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

the LO + NLO-EW result would suffer of only 0.55% spread;
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence (→0.88%)
    which is reduced by the NNLO QCD-EW (→0.23%)

The availability of N3LO-QCD and NNLO QCD-EW results can bring the study of EW gauge bosons in the per mille arena !!!
Is the full NNLO-EW calculation negligible at this level ?

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- an important step would be estimating the size of the effects for A_4
- comparison with the approximation implemented in event generators
  featuring (QCD $\times$ EW)NLOPS

Summary and outlook

- different codes by different groups now converging to the required accuracy for future LHC measurements

- target of our studies is also a sound evaluation of residual theoretical uncertainties

- validation of calculations which can be used by our exp. colleagues for complete MC simulations

$\Longrightarrow$ next talk by Aleko