p_T^Z Resummation Benchmarking
Status Report

Johannes Michel
MIT Center for Theoretical Physics

On behalf of the WG 1 Resummation Subgroup

LHC EW WG General Meeting
CERN and remote locations, February 17, 2022

Why we're doing this ...

- W mass measurements at the LHC use precisely measured p_T^Z (or ϕ_η^*) spectrum
- Theory input needed to relate p_T^Z and p_T^W
- Fixed-order predictions diverge for $p_T^Z \ll m_Z$ due to large Sudakov double logs
  - All-order resummation required to high (logarithmic) perturbative accuracy

Who is doing this ...

- On the theory side, p_T^Z spectrum is of key interest to many different communities

e.g. TMD/proton structure, resummation in direct QCD/SCET, fixed-order subtractions

- Many mature formalisms and codes to perform all-order resummation

Goal of the benchmarking effort

- Compare predictions & understand their differences, uncertainties, and accuracy

One theorist's implicit assumption is another theorist's uncertainty ...

Currently participating groups and codes

TMD global fit tools (Collins/Soper/Sterman formalism):

`artemide`	Scimemi, Vladimirov ’17, ’19
`NangaParbat`	Bacchetta et al. ’19
`ResBos2`	Isaacson ’17

Direct QCD (Catani/de Florian/Grazzini formalism):

`DYRes/DYTurbo`	Camarda et al. ’15, ’19, ’21
`reSolve`	Coradeschi, Cridge ’17

SCET-based tools:

`CuTe-MCFM`	Becher, Neumann ’11, ’20
`SCETlib`	Billis, Ebert, JM, Tackmann ’17, ’20

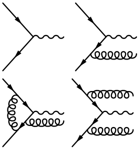
Coherent branching/momentum-space resummation:

`RadISH`	Monni, Re, Rottoli, Torrielli ’16, ’17, ’19, ’21

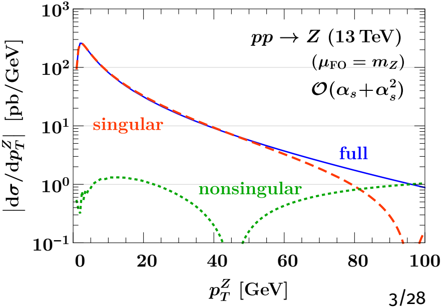
Resummation basics: Power expansion of the spectrum at small p_T

Structure of the fixed-order spectrum for $p_T \ll Q \equiv \sqrt{q^2} = m_{\ell\ell}$:

$$
\begin{aligned}
\frac{d\sigma}{dp_T} = \delta(p_T) &+ \alpha_s \Big[\frac{L}{p_T} + \frac{1}{p_T} && + \delta(p_T) + f_1^{\rm nons}(p_T) \Big] \\
&+ \alpha_s^2 \Big[\frac{L^3}{p_T} + \frac{L^2}{p_T} + \frac{L}{p_T} + \frac{1}{p_T} && + \delta(p_T) + f_2^{\rm nons}(p_T) \Big] \\
&+ \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \ddots && + \ \dots \ \Big] \\
\sim (1/p_T) \Big[& \qquad\qquad \mathcal{O}(1) && + \mathcal{O}(p_T^2/Q^2) \Big]
\end{aligned}
$$



- "Singular" or "leading power" terms
  - Large logarithms $L \equiv \ln p_T/Q$ left after real/virtual IR poles cancel
  - To be resummed to all orders
- "Nonsingular" or "subleading power"
  - Suppressed by relative p_T^2/Q^2 [for incl. Z – see talk by L. Buonocore for fiducial!]
  - Supplied by matching to full FO

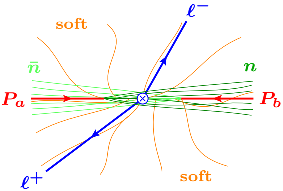


Leading-power terms factorize into hard, collinear, and soft contributions:

[Collins, Soper, Sterman '85; many different formulations] $x_{a,b} \equiv (Q/E_{\rm cm})e^{\pm Y}$

$$\frac{d\sigma_{\rm sing}}{dQ\,dY\,dp_T^2} = \sum_{a,b} H_{ab}(Q^2,\mu) \times [B_a B_b S](Q^2, x_a, x_b, \vec{p}_T, \mu)$$

$$[B_a B_b S] \equiv \int d^2\vec{k}_a\, d^2\vec{k}_b\, d^2\vec{k}_s\, \delta^{(2)}(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s) \times B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$



- Often: Hard function $H_{ab} = \sigma^{\rm LO}_{ab\to Z} \times$ $\overline{\rm MS}$-renormalized quark (form factor)^2
- Beam and soft functions individually feature so-called rapidity divergences
- Regularize and renormalize $\Rightarrow$ 2D renormalization group in (μ, ν)

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$$\times\ B_a(x_a, \vec{k}_a, \mu, \nu/Q)\,B_b(x_b, \vec{k}_b, \mu, \nu/Q)\,S(\vec{k}_s, \mu, \nu)$$

$$= \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\,\tilde{B}_a(x_a, b_T, \mu, \nu/Q)\,\tilde{B}_b(x_b, b_T, \mu, \nu/Q)\,\tilde{S}(b_T, \mu, \nu)$$

$$= \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\,\tilde{f}_a^{\rm TMD}(x_a, b_T, \mu, \zeta_a)\,\tilde{f}_b^{\rm TMD}(x_b, b_T, \mu, \zeta_b)$$

- Often: Fourier transform to turn convolution into a product in $\vec{b}_T$ space
- Often: combine beam and $\sqrt{\text{soft}}$ function into a TMD PDF that runs as a function of (μ, ζ)
- Collins-Soper scales $\zeta_{a,b}$ = energies^2 of the scattering partons, e.g. $= Q^2$ in Z rest frame

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$$= \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{p}_T}\, \tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)\, \tilde{S}(b_T, \mu, \nu)$$
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- For $b_T \ll 1/\Lambda_{\rm QCD}$, beam functions can be calculated in terms of collinear PDFs:

$$B_i(x, b_T, \mu, \nu/Q) = \sum_j \int \frac{\mathrm{d}z}{z}\, \mathcal{I}_{ij}(z, b_T, \mu, \nu/Q)\, f_j\Big(\frac{x}{z}, \mu\Big) + \mathcal{O}(b_T^2 \Lambda_{\rm QCD}^2)$$

- Equivalent relation holds for TMD PDFs (coefficients often called C_{ij})

$$\frac{d\sigma_{\rm sing}}{p_T} = H(Q,\mu) \times B(p_T,\mu,\nu/Q)^2 \otimes S(p_T,\mu,\nu/p_T)$$

$$\ln^2\frac{p_T}{Q} = 2\ln^2\frac{Q}{\mu} + 2\ln\frac{p_T}{\mu}\ln\frac{\nu}{Q} + \ln\frac{p_T}{\mu}\ln\frac{\mu\, p_T}{\nu^2}$$

- For generic μ, ν, each function contains (potentially large) logs
- Resummation follows from solving RGEs, and evolving each function from some starting scales μ_i, ν_i to common arbitrary μ, ν

$$H(\mu) = H(\mu_H) \times U_H(\mu_H, \mu)$$
$$B(\mu,\nu) = B(\mu_B,\nu_B) \otimes U_B(\mu_B,\nu_B;\mu,\nu)$$
$$S(\mu,\nu) = S(\mu_S,\nu_S) \otimes U_S(\mu_S,\nu_S;\mu,\nu)$$

- Often: Done in b_T space, where $\mu_B, \mu_S \sim 1/b_T$ and evolution is multiplicative
  - b_T-space approaches (TMD/CSS, CFG, SCET) resum logarithms $\ln(b_T Q)$
  - Differences in choice of boundary condition and practical form of solution
  - Setups also differ in treatment of μ_B, μ_S as $1/b_T \to \Lambda_{\rm QCD}$ (Landau pole)
- RadISH instead performs the evolution in momentum space and resums logarithms $\ln(p_T^h/Q)$ of the hardest emission p_T^h

$$\frac{d\sigma_{\text{sing}}}{p_T} = H(Q,\mu) \times B(p_T,\mu,\nu/Q)^2 \otimes S(p_T,\mu,\nu/p_T)$$

$$\Rightarrow \boxed{\frac{d\sigma_{\text{sing}}^{\text{res}}}{p_T} = \frac{1}{p_T}\exp\left(-\alpha_s \ln\frac{\mu_H}{\mu_S}\right)\left[1+\alpha_s \ln\frac{p_T}{\mu_S}\right]\left[1+\alpha_s \ln\frac{\mu_H}{Q}\right]}$$

[Very schematic!]

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$$H(\mu) = H(\mu_H) \times U_H(\mu_H, \mu)$$
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Order	Boundary cond. (FO singular)	Anomalous dimensions γ_i (noncusp)	Γ_{cusp}, β	FO matching (nonsingular)
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL′ (+NLO$_0$)	α_s	1-loop	2-loop	α_s
NNLL (+NLO$_0$)	α_s	2-loop	3-loop	α_s
NNLL′ (+NNLO$_0$)	α_s^2	2-loop	3-loop	α_s^2
N^{3}LL (+NNLO$_0$)	α_s^2	3-loop	4-loop	α_s^2
N^{3}LL′ (+N^{3}LO$_0$)	α_s^3	3-loop	4-loop	α_s^3
N^{4}LL (+N^{3}LO$_0$)	α_s^3	4-loop	5-loop	α_s^3

- Resummation order is uniquely specified by perturbative order of boundary coefficients and anomalous dimensions (each is convergent on its own)
- Can show that with these ingredients, all (next-to)^{n}leading logarithmic terms $\alpha_s^k L^{k+1+n}$ in $\ln(\mathrm{d}\sigma/\mathrm{d}q_T)$ are captured for all $k \geq 1$
- At "primed" orders, boundary conditions are included to α_s^n higher in addition
  - Improves residual dependence on boundary scales
  - Ensures integral of (reexpanded) matched spectrum is N^{n}LO cross section

Definition of benchmark levels

Strategy: Increase complexity, switch on possible sources of differences step by step.

Level 1:

- Triple-differential $\mathrm{d}\sigma/\mathrm{d}Q\mathrm{d}Y\,\mathrm{d}p_T$ for Z/γ^* at 13 TeV (mostly $Q = m_Z, Y = 0$)
- Resum only "canonical" logs, i.e. $\ln(b_T Q)$ or $\ln(p_T^h/Q)$
- Resummed singular piece piece only, LL through N^{3}LL
- No nonperturbative model, Landau pole regulated in similar way
- Same PDF, same $\alpha_s(m_Z) = 0.118$, same EW settings, no cuts

Level 2:

- Still only the resummed piece
- Groups use their own default settings for Landau pole, resummation turn-off, ...
- Do not include matching piece yet

Level 3:

- Include matching to get physical spectrum everywhere

$$\frac{\mathrm{d}\sigma_{\text{matched}}}{\mathrm{d}p_T} = \frac{\mathrm{d}\sigma^{\text{res}}_{\text{sing}}}{\mathrm{d}p_T} + \left[\frac{\mathrm{d}\sigma^{\text{FO}}_{\text{full}}}{\mathrm{d}p_T} - \frac{\mathrm{d}\sigma^{\text{FO}}_{\text{sing}}}{\mathrm{d}p_T}\right]$$

Important physics understood at Level 1 & 2:

[See talk by T. Cridge at October 2020 General Meeting for details.]

- $\mathrm{N^3LL}$ with canonical logs agrees to few % at $p_T = 10 - 40\,\mathrm{GeV}$
- Understood differences in PDF evolution and quark mass thresholds
- Understood impact of different Landau pole prescriptions at $p_T \le 5\,\mathrm{GeV}$
- Understood impact of resummation scale choice $\rightarrow$ absorb by matching

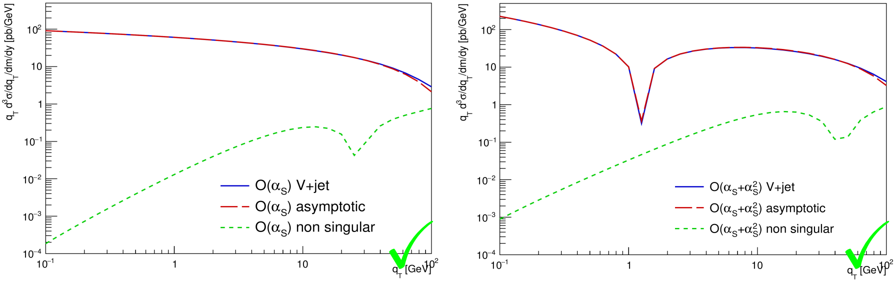
Important progress over the last year:

- Level 2/3: Converged on how to relate and interpret different uncertainties
- Level 3: Verified consistency with $\mathcal{O}(\alpha_s^2)$ fixed-order code in the singular limit

$\rightarrow$ Focus of the rest of this talk!

$$\frac{\mathrm{d}\sigma_{\mathrm{matched}}}{\mathrm{d}p_T} = \frac{\mathrm{d}\sigma^{\mathrm{res}}_{\mathrm{sing}}}{\mathrm{d}p_T} + \left[\frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}}}{\mathrm{d}p_T} - \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{sing}}}{\mathrm{d}p_T}\right]$$

- Necessary consistency condition in all approaches:
  Total matching correction must vanish as p_T^2/Q^2 relative to leading $1/p_T$
- $\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}}$ from analytic implementation of NLO inclusive $Z+$ jet in DYTurbo
  [Gonsalves, Pawlowski, Wai ’89]



[DYTurbo, S. Camarda, April ’21]

Consistency checks for fixed-order matching

$$\frac{\mathrm{d}\sigma_{\mathrm{matched}}}{\mathrm{d}p_T} = \frac{\mathrm{d}\sigma^{\mathrm{res}}_{\mathrm{sing}}}{\mathrm{d}p_T} + \left[\frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}}}{\mathrm{d}p_T} - \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{sing}}}{\mathrm{d}p_T}\right]$$

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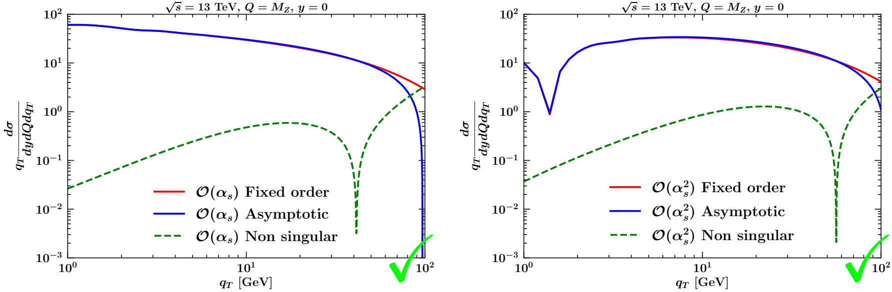

[NangaParbat, V. Bertone]

$$\frac{d\sigma_{\text{matched}}}{dp_T} = \frac{d\sigma^{\text{res}}_{\text{sing}}}{dp_T} + \left[\frac{d\sigma^{\text{FO}}_{\text{full}}}{dp_T} - \frac{d\sigma^{\text{FO}}_{\text{sing}}}{dp_T}\right]$$

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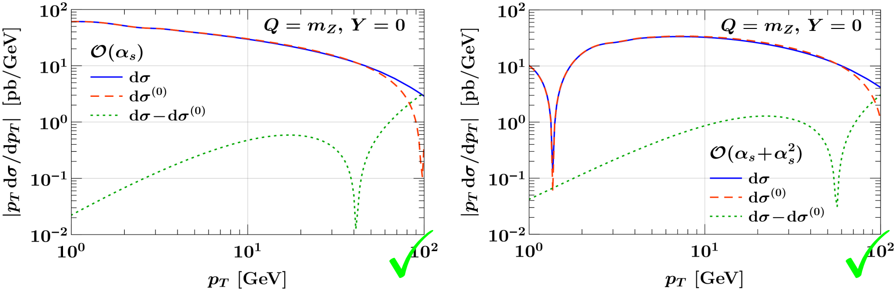

[SCETlib, F. Tackmann, November ’20]

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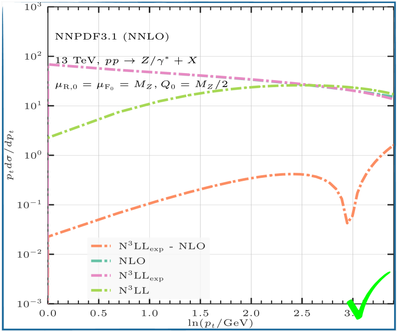

[RadISH, April '21]

- Powerful check of (reexpanded) resummed cross section
- Checks consistency of settings across the board

Many (different) scales in resummation – which ones to vary & how to interpret?

	Sudakov/ Resummation	Non-Sudakov	Matching
arTeMiDe	$\mu_f\ (\mu, \zeta_\mu)$	μ_{OPE}	No level 3
Cute-MCFM	$\mu,\ \mu_h,\ r$	$\mu_R,\ \mu_F$	Parameters of damping func.
DYTURBO	Q	$\mu_R,\ \mu_F$	Parameters of Damping func.
NangaParbat	$Q,\ \mu_b$	$\mu_R,\ \mu_F$	Still none (damping func.)
RadISH	Q	$\mu_R,\ \mu_F$	Parameters of Damping func.
ResBos	$C_1,\ C_2,\ C_3$	$\mu_R,\ \mu_F$	Parameters of damping func.
Resolve	μ_S	$\mu_R,\ \mu_F$	No level 3
SCETlib	Δ_{resum}	Δ_{FO}	Profile scales Δ_{match}

[Figure credit: V. Bertone, November ’21]

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arTeMiDe	$\mu_f\ (\mu, \zeta_\mu)$	μ_{OPE}	No level 3
Cu[illegible]MCFM	[illegible]	[illegible]	Parameters of [illegible]nc.
D[illegible]	[illegible]	[illegible]	[illegible] of [illegible]nc.
Na[illegible]	[illegible]	[illegible]	[illegible]nc.)
[illegible]	[illegible]	[illegible]	[illegible] of Damping func.
ResBos	C_1, C_2, C_3	μ_R, μ_F	Parameters of damping func.
Resolve	μ_S	μ_R, μ_F	No level 3
SCETlib	Δ_{resum}	Δ_{FO}	Profile scales Δ_{match}

Sudakov/resummation variation, e.g. $\mu_S \to v\mu_S$ with $v = 1/2, 2$:

$$\frac{d\sigma^{\mathrm{res}}_{\mathrm{sing}}}{p_T} = \frac{1}{p_T} \exp\left(-\alpha_s \ln\frac{\mu_H}{v\mu_S}\right)\left[1 + \alpha_s \ln\frac{p_T}{v\mu_S}\right]\left[1 + \alpha_s \ln\frac{\mu_H}{Q}\right]$$

[Figure credit: V. Bertone, November '21]

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D			… of … nc.
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Non-Sudakov variation, $\mu_S \to v\mu_S$ and $\mu_H \to v\mu_H$:

$$\frac{d\sigma^{\mathrm{res}}_{\mathrm{sing}}}{p_T} = \frac{1}{p_T}\exp\left(-\alpha_s \ln \frac{\cancel{v}\mu_H}{\cancel{v}\mu_S}\right)\left[1+\alpha_s \ln \frac{p_T}{v\mu_S}\right]\left[1+\alpha_s \ln \frac{v\mu_H}{Q}\right]$$

[Figure credit: V. Bertone, November ’21]

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Resolve	μ_S	μ_R, μ_F	No level 3
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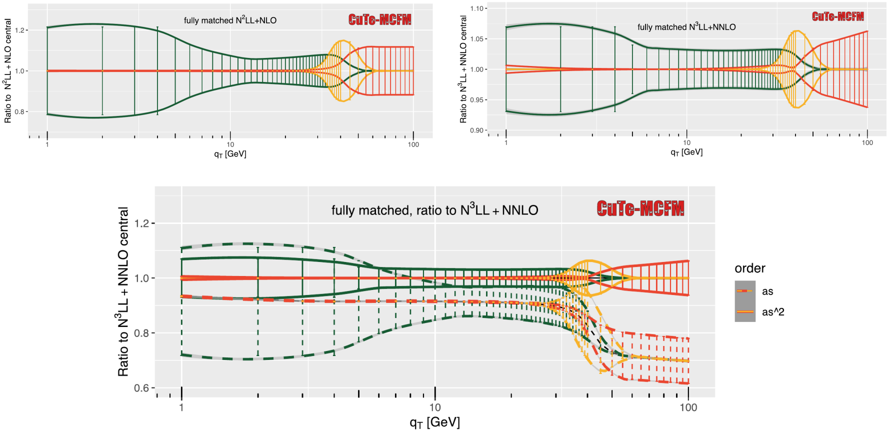
Matching variation (here: profile scales – depends on implementation!):

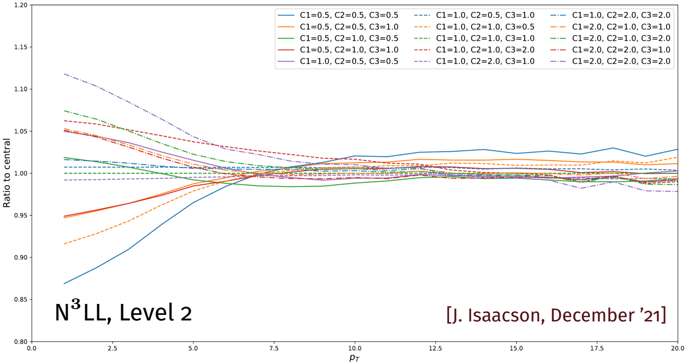
- Let $\mu_S(p_T) = f_{\text{profile}}(p_T)\,\mu_H$ where $f_{\text{profile}}(p_T \ll Q) = p_T/Q$ and $f_{\text{profile}}(p_T \to Q) \to 1$ turns resummation off
- Take f_{profile} to be a piecewise polynomial in between
- Vary transition points of polynomial
- Matching uncertainty should vanish in both deep resummation and fixed-order regimes

[Figure credit: V. Bertone, November ’21]

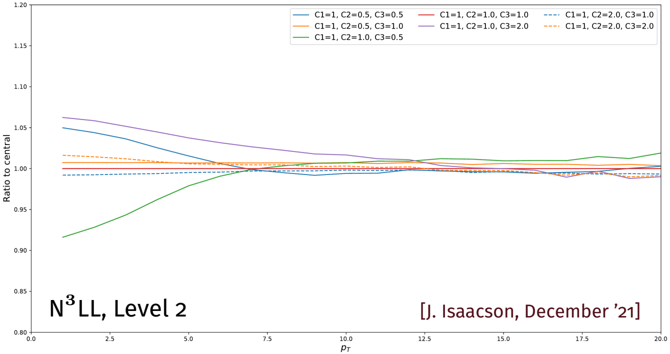


[T. Neumann, December ’21]

- Uncertainties are Sudakov, Non-Sudakov, Matching at Level 3
- Uncertainties reduced by more than factor 2 from NNLL to N^{3}LL
- Good coverage in the peak (15% to 5% at $q_T = 5\,\mathrm{GeV}$)



- Showing variations of coefficients C_1, C_2, C_3 ($\rightarrow$ Sudakov uncertainty)
- C_1 is in principle tied to nonperturbative model
  - Holding model fixed while varying C_1 may be an overestimate
- Issue: C_3 coefficient varies scale of PDF (possibly into extrapolation region)
- Uncertainty interpretation is in progress



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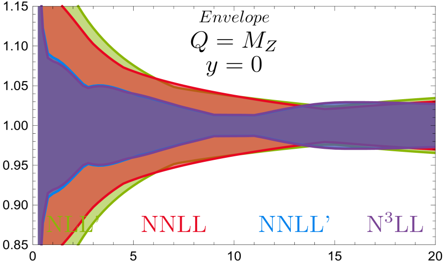
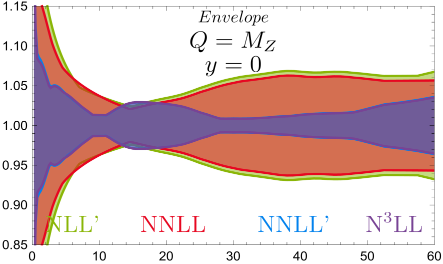
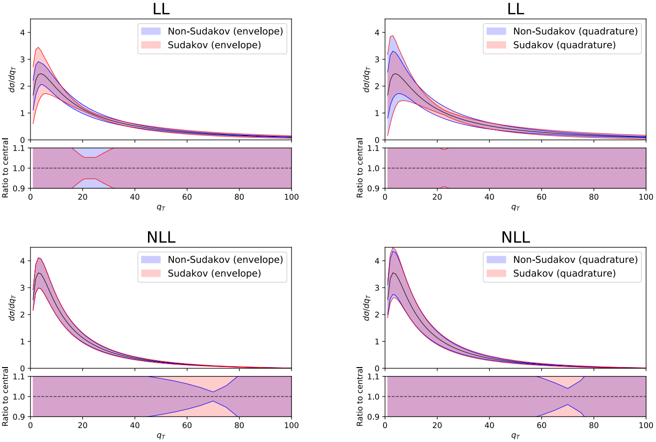




[A. Vladimirov, Januar '22]

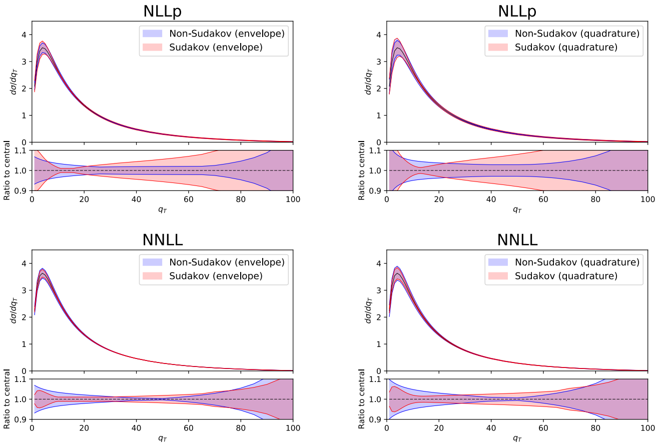
- Showing symmetrized envelope of 8 variations at different orders, Level 2
- Uncertainty improved substantially at N^{3}LL
- Dominated by a non-Sudakov variation (μ_{OPE}), also affecting the scale at which PDFs are evaluated
- Disentangling individual contributions according to table is in progress

[V. Bertone/G. Bozzi, December '21, Level 2]



- Similar results with envelope of variations (left) or adding in quad (right)
- Good coverage of higher-order results in the peak
- Sudakov uncertainty dominates non-Sudakov in the peak, as expected ✓
- Level 3 results to be finalized (will make uncertainty in tail physical)

[V. Bertone/G. Bozzi, December '21, Level 2]



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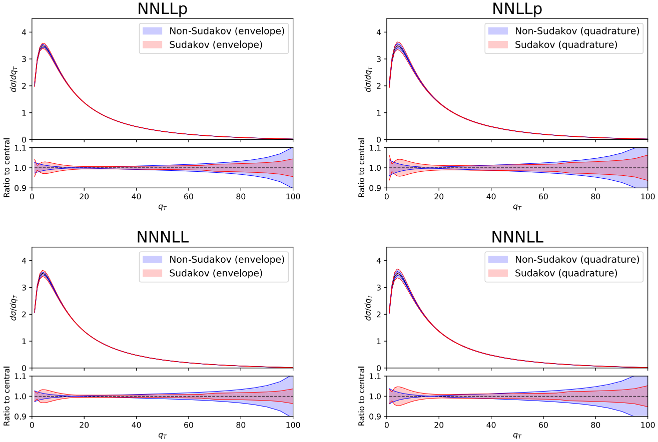
Uncertainties: NangaParbat

[V. Bertone/G. Bozzi, December '21, Level 2]



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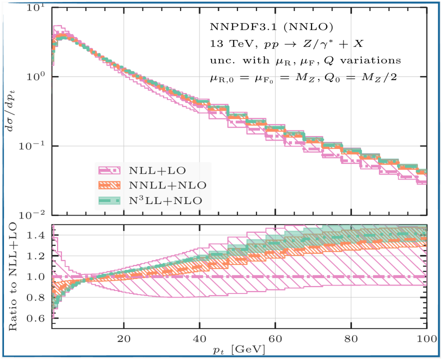
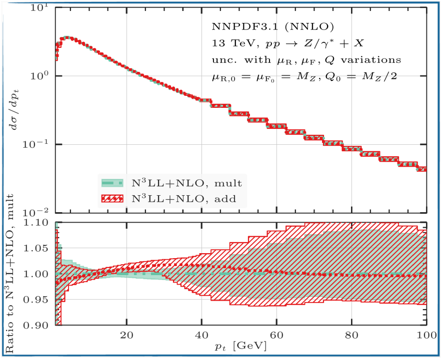
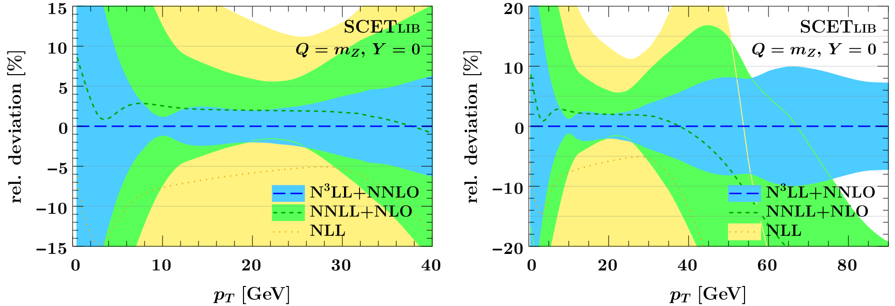




[L. Rottoli, April '21]

- Total Sudakov + non-Sudakov uncertainty (left, at different Level 3 orders) currently estimated from 9-point envelope of μ_R, μ_F, resummation scale
- Good perturbative coverage and convergence across the spectrum
- Disentangling individual sources according to table is in progress
- Matching uncertainty estimated from difference of matching schemes (right)



[F. Tackmann/JM, January '22]

- Good perturbative coverage/convergence except in far tail
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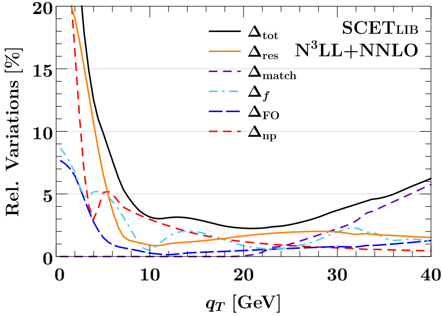

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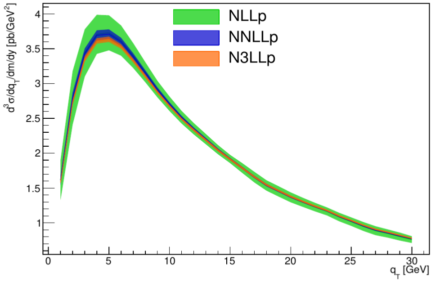
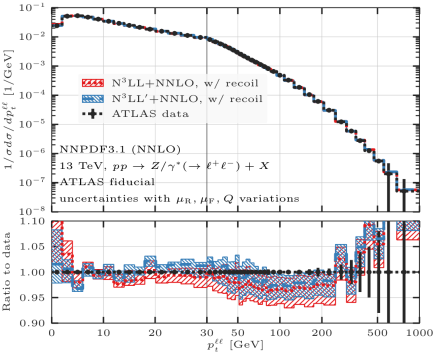
Future directions:

- Many groups already have working codes (or published results!) at N^{3}LL′
- $\mathcal{O}(\alpha_s^3)$ matching corrections to be provided by `NNLOjet` collaboration
  - Will enable level 3 benchmarking at full three-loop accuracy



[DYTurbo, April ’21]
[See also 2103.04974]



[`RadISH`, 2104.07509]

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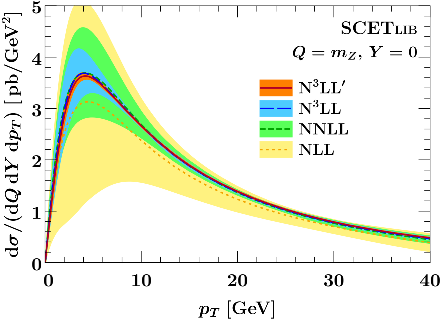
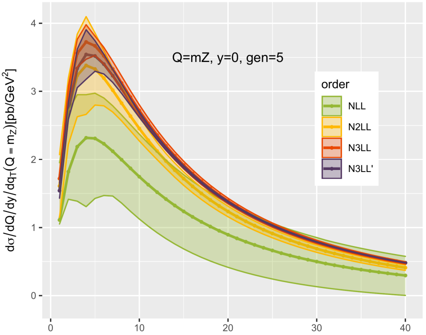


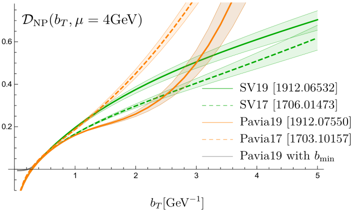
[SCETlib, January ’22]

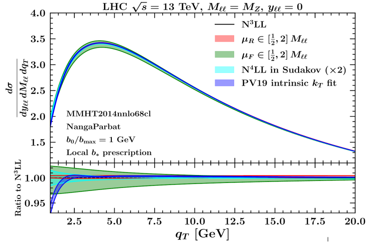


[CuTe-MCFM, December ’21]

Future directions:

- Some groups (`artemide`, `NangaParbat`) have performed dedicated global fits of the nonperturbative TMD structure at $b_T \sim 1/\Lambda_{\rm QCD}$
  [Scimemi, Vladimirov ’19; Bacchetta et al. ’19]





- Fit includes low-energy Drell-Yan and (for `artemide`) SIDIS data
- Accounts for nontrivial x dependence of the nonperturbative model
- Recently also considered flavor dependence of the model, can partially compensate differences between collinear PDF sets
  [Bury, Leal-Gomez, Scimemi, Vladimirov ’22]

▶ Level 3.5 benchmark with nonperturbative effects included?

Summary

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✓ Level 1 benchmark already completed as of October '20

- Very good agreement between codes, many new effects understood.

✓ Level 2 benchmark essentially complete

- Understand which effects will be compensated by the matching.

✓ Consistency checks for $\mathcal{O}(\alpha_s^2)$ level 3 matching complete

✓ Converged on uncertainty interpretation for level 2/3 predictions

✓ Many matched level 3 contributions already with detailed uncertainties!

- Next step: quantitative comparisons & compatibility in each category

Good progress over the last year, again many great discussions!

Many thanks to all participating groups and to Daniel and Aram
for organizing and coordinating this effort.

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