Intrinsic Time Geometrodynamics

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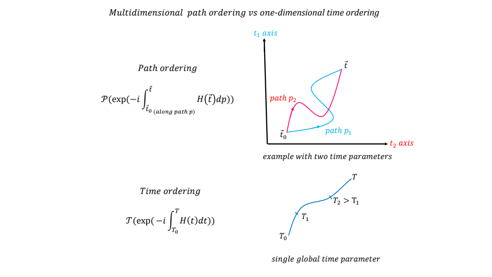
$\mathcal{I}$mportant facts and observations

- $\vec{x}$ & t are labels, $\nexists$ physical meaning, physics det'd by dynamical fields not by kinematics; in $\mathcal{Q}FT$ object of interest are $\mathcal{Q}$ fields
- $\mathcal{G}CT$, $g'_{\mu\nu}(x') = \frac{\partial x_\alpha}{\partial x'_\mu}\frac{\partial x_\beta}{\partial x'_\nu} g_{\alpha\beta}(x)$; not det'd dynamics for any theory
- Time and 4-covariance are mutually exclusive
- Cosmic expansion should be promoted to fundamental level
- Spatial dimensions can be multiple but time can only be 1



$\mathcal{QFT}$ local gauge sym. generated by 1^{st}-class field operator constraints

1. $\mathcal{G}$auge Inv. $\rightarrow$ physical states are unchanged under gauge transformations
   - $\Psi[A_{ia} + \delta_{gauge} A_{ia}] = \Psi[A_{ia}]$
   - Constraint annihilates Gauge Inv. states, $\mathcal{C}\Psi[A_{ia}] = 0$
2. $\mathcal{GR}$ w.r.t spatial $\mathcal{D}$iff. is in fact a gauge theory with metric variables
   - $\Psi[q_{ij} + \delta_{gauge} q_{ij}] = \Psi[q_{ij}] + \int (\delta_{gauge} q_{ij}(x)) \frac{\delta \Psi}{\delta q_{ij}(x)} d^3x = \Psi[q_{ij}]$
   - Since $\delta_{gauge} q_{ij} = \mathcal{L}_{\vec{N}} q_{ij}; \tilde{\pi}^{ij} = \frac{\hbar}{i} \frac{\delta}{\delta q_{ij}} \therefore \int N^i H_i \, d^3x \Psi[q_{ij}] = 0$
   - Constraints generate Gauge transformations and be 1^{st} order in canonical momenta

$\mathcal{C}$omparison of the two different local gauge structures in $\mathcal{QFT}$

	$\mathcal{D}$iffeomorphism Gauge Structures	$\mathcal{Y}$ang-$\mathcal{M}$ills Gauge Structures
$\mathcal{B}$asic Variables	Spatial metric tensor q_{ij}	Gauge connection A_{ia}
$\mathcal{S}$ymmetry Generators	$\mathcal{H}_i(\mathbf{x}) = -2q_{ik}\nabla_j\pi^{jk}(\mathbf{x})(=0)$	$\mathcal{G}^a(\mathbf{x}) = \nabla_i\pi^{ia}(\mathbf{x})(=0)$
$\mathcal{G}$auge transformation	$[q_{ij}(\mathbf{x}), \mathcal{H}_k[N^k]] = \mathcal{L}_{\vec{N}}q_{ij}(\mathbf{x});$	$[A_{ia}(\mathbf{x}), \mathcal{G}^b[\eta_b]] = -\nabla_i\eta_a(\mathbf{x})\mathcal{G}^b[\eta_b]$
	$\mathcal{H}_i[N^i] = \int N^i\mathcal{H}_i d^3\mathbf{x}$	$\mathcal{G}^b[\eta_b] = \int \eta_b\mathcal{G}^b d^3\mathbf{x}$
$\mathcal{C}$ommutation Relations	$[\mathcal{H}_i(\mathbf{x}), \mathcal{H}_j(\mathbf{y})]$ $= \mathcal{H}_j(\mathbf{x})\partial_i\delta(\mathbf{x}-\mathbf{y}) - \mathcal{H}_i(\mathbf{y})\partial_j\delta(\mathbf{x}-\mathbf{y})$	$[\mathcal{G}^a(\mathbf{x}), \mathcal{G}^b(\mathbf{y})]$ $= if^{ab}{}_c G^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})$
$\mathcal{P}$otential	$\mathcal{V} \sim [\frac{\delta\exp(CS)}{\delta q_{ij}}]^2$	$\mathcal{V} \sim [\frac{\delta\exp(CS)}{\delta A_{ia}}]^2$
$\mathcal{G}$roup $\mathcal{P}$roperties	Not product of identical group at each spatial point(i.e. $SL(3,R)$) of the base manifold, not of principle fibre structure, infinite dimensional by itself.	Infinite tensor product group $\prod_x G$, G is finite dimensional Lie group usually called 'gauge group', principle fibre structure.

$\mathcal{SR}$ does not exit as a dynamical theory & illusions of $\mathcal{SR}$

- Except at a point, Minkowski space-time is not a general limit of $\mathcal{GR}$; reason explains why usual $\mathcal{QFT}$ based on global $\mathcal{L}$orentz $\mathcal{S}$ymmetry is successful locally even $\mathcal{LS}$ is not fundamental
- Need dynamical theory for time-dilation and length contraction effects to replace global $\mathcal{LS}$ for generic curve space
- New extra term from dynamical theory will be induced and be tested

$\mathcal{H}$amiltonian constraint, quadratic in momenta $\therefore$ no temporal sym. in $\mathcal{GR}$

- $\mathcal{D}$irac algebra not even algebra of 4d $\mathcal{D}$iff; $\mathcal{QM}$ space-time not exist
- Real gauge symmetry of $\mathcal{GR}$ is 3d $\mathcal{D}$iffeomorphism invariance only
- $\mathcal{T}$ime is $\mathcal{I}$ntrinsically delivered by the cosmic expansion, $\delta T = \delta \ln q^{1/3}$ & $q_{ij} = q^{1/3}\bar{q}_{ij}$; space encodes time data
- No exotic idea is needed to build a $\mathcal{Q}$uantum $\mathcal{T}$heory of $\mathcal{G}$ravitation

$\mathcal{S}$pirit of $\mathcal{SR}$ lies in the square root dispersion relation

- $\mathcal{G}$alileo symmetry to $\mathcal{L}$orentz symmetry$\rightarrow$ $p = mv$ to $v = \frac{pc^2}{E}$
- $\mathcal{D}$ispersion relation from $E = \frac{p^2}{2m}$ $\rightarrow$ square-root-form, $E = \sqrt{\vec{p}^2 + m^2}$
- $\mathcal{S}$quare-root-form dispersion relation captures the spirit of $\mathcal{SR}$ & also particle-wave duality; $\therefore$ $\mathcal{QFT}$ built upon $\mathcal{L}$orentz sym. is successful
- $\mathcal{G}$eneralizing the sqaure-root-form dispersion relation of $\mathcal{SR}$ to $\mathcal{GR}$:
  - 3d $\mathcal{D}$iffeomorphism invariance
  - properly implement $\mathcal{E}$quivalence principle
  - existence of frame independent upper limit in speed
  - identify the cosmic expansion as intrinsic time

$\mathcal{H}$amiltonian $\mathcal{C}$onstraint of $\mathcal{G}R$ provides square root dispersion relation

$$\mathcal{H}(x) = -qR + \bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} - \beta^2\pi^2 = 0$$

$$\bar{H} = -\beta\pi = \sqrt{\bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} + \mathcal{V}}$$

$\mathcal{V}$ allows to include a dimension 3 $\mathcal{C}$otton-$\mathcal{Y}$ork tensor $\rightarrow$ a complete $\mathcal{QG}$

Dynamical theory for time & length contraction in generic curve space

- ADM metric, $ds^2 = -N^2 dt^2 + q_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$; $N = \frac{\sqrt{q}(\partial_t \ln q^{1/3} - \frac{2}{3}\nabla_i N^i)}{4\beta\kappa\bar{H}}$
- Lagrangian of test particle in ADM form, $\mathcal{L} = -m_0 c\sqrt{N^2c^2 - q_{ij}\left(N^i c + \dot{x}^i\right)\left(N^j c + \dot{x}^j\right)}dt$
- Conjugate momentum, $p_i == \frac{m_0 c q_{ij}\left(N^j c + \dot{x}^j\right)}{\sqrt{N^2c^2 - q_{jk}\left(N^j c + \dot{x}^j\right)\left(N^k c + \dot{x}^k\right)}} \to H = Nc\sqrt{m_0^2c^2 + p_i p^i} - N^i p_i c$
- Proper time interval, $d\tau = \frac{1}{c}\sqrt{-ds^2} = \sqrt{N^2 - q_{ij}\left(N^i + \beta^i\right)\left(N^j + \beta^j\right)}dt = N\frac{dt}{\gamma}$,

Applications

- $\mathcal{S}R$ motions: $p_i = \frac{m_0 c q_{ij}\left(N^j c + \dot{x}^j\right)}{\sqrt{N^2c^2 - q_{jk}\left(N^j c + \dot{x}^j\right)\left(N^k c + \dot{x}^k\right)}} = \frac{m_0 \dot{x}_i}{\sqrt{1-\beta^2}}$
- $\mathcal{G}$ravitational effects; $ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1}dr^2 + r^2 d\Omega^2$; $N(r) = \sqrt{1 - \frac{2GM}{rc^2}}$
  - $r_S = \frac{d\tau_s - d\tau_0}{d\tau_0} = \sqrt{1 - \frac{v_s^2}{c^2}} - 1 \approx -8.3 \times 10^{-11}$ for typical satellite spped, $v_s \approx 3.8704 \times 10^3 m/s$
  - $r_G = \frac{\sqrt{1 - \frac{2GM_E}{r_s c^2}}}{\sqrt{1 - \frac{2GM_E}{r_o c^2}}} - 1 \approx 5.3 \times 10^{-10}$ for $r_o \approx 6.3781366 \times 10^6 m$, $M_E \approx 5.9722 \times 10^{24} kg$
  - Total: $r_T = \frac{dt_s - dt_o}{dt_o} = \frac{\sqrt{\left(1 - \frac{2GM_E}{r_s c^2}\right) - \frac{v_s^2}{c^2}}}{\sqrt{\left(1 - \frac{2GM}{r_o c^2}\right) - \frac{v_o^2}{c^2}}} - 1 = r_S + r_G + r_S r_G$; $r_S \times r_G \approx -4.4 \times 10^{-20}$

$\mathcal{O}$verview on $\mathcal{I}$ntrinsic $\mathcal{T}$ime $\mathcal{G}$eometrodynamics

1. $\mathcal{D}$ecompose $(q_{ij}, \tilde{\pi}^{ij})$ irreducibly into: $(\bar{q}_{ij}, \bar{\pi}^{ij})$, $(\ln q^{\frac{1}{3}}, \pi)$; identifying $\ln q(x)$ as $\mathcal{I}ntrinsic\ \mathcal{T}ime$ & $\pi = q_{ij}\tilde{\pi}^{ij}$ the energy density($\mathcal{D}$irac's extended phase space)

2. $\mathcal{ADM}$ Hamiltonian constraint($\beta^2 = \frac{1}{6}$ for $\mathcal{GR}$) delivers dynamical Hamiltonian density
   - $H(x) = -qR + \bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} - \beta^2\pi^2 = 0$
   - Solve and obtain: $-\pi = \frac{\bar{H}(x)}{\beta} = \frac{1}{\beta}\sqrt{\bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} - qR} \rightarrow \frac{1}{\beta}\sqrt{\bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} + \mathcal{V}}$
   - Non-Relativity, $H_{NR} = K.E. + \mathcal{V}$; Relativistically, $H_{Rel} = \sqrt{momenta^2 + \mathcal{V}}$
   - $\mathcal{H}_i = -2\bar{q}_{ij}\nabla_l\bar{\pi}^{jl} = 0$ generates 3d diffeomorphism transformations in the theory
   - Overcome the no-go thm. of Hojman et al; potential can now be modified by a dimension 3 $\mathcal{C}$otton-$\mathcal{Y}$ork tensor, $\tilde{C}^{ij} = \frac{1}{2}(\tilde{\epsilon}^{imn}\nabla_m R^j{}_n + \tilde{\epsilon}^{jmn}\nabla_m R^i{}_n)$, which is unique to 3d; a full theory of ultra-violet complete $\mathcal{QG}$ theory of 3-geometry

$\mathcal{Q}$uantum $\mathcal{H}$amiltonian and emergent of the $\mathcal{E}$instein potential

1. $\bar{H} = \sqrt{Q^{\dagger i}_j Q^j_i + q\mathcal{K}} = \sqrt{\bar{\pi}^{\dagger j}_i \bar{\pi}^i_j + \hbar^2(g\tilde{C}^i_j - \alpha\sqrt{q}\bar{R}^i_j)(g\tilde{C}^j_i - \alpha\sqrt{q}\bar{R}^j_i) - i\alpha\hbar\sqrt{q}[\bar{\pi}^i_j, \bar{R}^j_i] + q\mathcal{K}}$
   - $Q^{\dagger i}_j$ & Q^i_j related to $\bar{\pi}^i_j$ by $e^{\mp W_T}$, are non-Hermitian, generate 2 unitarily inequivalent rep. of the non-compact group $SL(3,R)$ at each x; commutator in $\bar{H}$ can be, $-i\alpha\hbar\sqrt{q}[\bar{\pi}^i_j, \bar{R}^j_i] = [Q^{\dagger i}_j, Q^j_i]$
   - The point split commutator term, $[Q^{\dagger i}_j(x), Q^j_i(y)] = -\frac{5}{6}\alpha\hbar^2\sqrt{q(x)}(-\nabla^2_x + R)\delta(x-y)$
2. $[Q^{\dagger i}_j(x), Q^j_i(x)] = \lim_{\epsilon\to 0} -\frac{5\alpha\hbar^2 q}{6(4\pi\epsilon)^{3/2}}\left[\frac{3}{2\epsilon} + \frac{13R}{12} + (a_1 R - \frac{a_2}{2})\epsilon + \text{terms with } \epsilon^{n\geq 2}\right]$;
   - Renormalized values are identified phenomenologically, with $\frac{1}{(2\kappa)^2} = \frac{5\alpha\hbar^2}{6(4\pi\epsilon)^{3/2}}(\frac{13}{12})$ to yield the correct Newtonian limit, the effective cosmological constant is, $\frac{2\Lambda_{\rm eff}}{(2\kappa)^2} = (\mathcal{K} - \frac{5\alpha\hbar^2}{32\pi^{3/2}\epsilon^{5/2}}) = (\mathcal{K} - \frac{2^4 3^2}{13\kappa^2\epsilon})$
   - $\alpha = \frac{72(4\pi\epsilon)^{3/2}}{65\hbar^2(2\kappa)^2}$, and finiteness of κ implies $\alpha \to 0$ as $\epsilon \to 0$. The theory actually produces all the Seeley-DeWitt coefficients, higher curvature terms with positive powers of ϵ disappear upon regulator removal

$\mathcal{F}$inally the full $\mathcal{QG}$ Hamiltonian is $\mathcal{Q}$ dynamic of 3-geometry:

$$\bar{H} = \sqrt{Q^{\dagger i}_j Q^j_i + q\mathcal{K}} = \sqrt{\bar{\pi}^{\dagger j}_i \bar{\pi}^i_j + \hbar^2 g^2 \tilde{C}^i_j \tilde{C}^j_i - \frac{q}{(2\kappa)^2}(R - 2\Lambda_{eff})}$$

Note that N doesn't show up $\to$ not a $\mathcal{Q}$ theory of space-time but 3-space

$\mathcal{B}$ackground solution and linearization for $\mathcal{GW}$

1. Robertson-Walker 3-metrics is a solution of the pair of Hamilton eqt.,

$$d\ell^2 = a^2(T)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

with $k = +1, 0, -1$ respectively for the compact S^3, $\mathbb{R}^3$ and H^3 intrinsic 3-geometries

- The metric is Einstein, $R_{ij} = \frac{1}{3}q_{ij}R$ & $R = \frac{6k}{a^2}$; hence $\bar{R}_{ij} = 0$ & $\tilde{C}_{ij} = 0$
- $\bar{q}_{ij} = q^{-1/3}q_{ij}$ is a independent (thus of T) & $\bar{\pi}^i_j = 0$ for the RW background
- Lead to spatially covariantly constant Hamiltonian density & each term vanishes

$\Rightarrow \dot{\bar{\pi}}^i_j = 0$; the pair of Hamilton eqts satisfied identically & initial data preserved

2. Simple theorem: *any spatially constant curvature sol. of GR is also a sol. of ITQG*.
3. Note that flat Minkowski space is not even a solution of the EOM
4. Linearize the Hamilton equations about the de Sitter background $({}^*\bar{q}_{ij}, {}^*\bar{\pi}^i_j)$ with S^3 slicings, expand the variables as $\bar{q}_{ij} = {}^*\bar{q}_{ij} + \bar{h}_{ij}$ and $\bar{\pi}^i_j = {}^*\bar{\pi}^i_j + \Delta\bar{\pi}^i_j$
5. The physical fluctuations are transverse i.e. ${}^*\nabla^i\bar{h}_{ij} = 0$; and $\bar{h}_{ij}$, being perturbations of the unimodular $\bar{q}_{ij}$, are traceless (${}^*q^{ij}\bar{h}_{ij} = 0$) as well

$\mathcal{R}$ecast the 3d spatial RW into 4d RW space-time

1. In $\mathcal{ADM}$ decomposition of any 4-d classical solution with coord. time variable t, the lapse function takes the form $N = \frac{\sqrt{q}\partial_t \ln q^{1/3}}{4\beta\kappa\bar{H}}$ modulo spatial diffeomorphisms
2. Recast the background solution into the usual 4-d RW form by reparametrizing the cosmic time interval as $dt' := Ndt = \frac{(\partial_t \ln a^2)dt}{\sqrt{6}\beta\sqrt{\frac{\Lambda}{3} - \frac{k}{a^2}}}$

$$ds^2 = -dt'^2 + a^2(t')[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)],$$

3. Above relation between a and t' reduces to $\frac{da}{dt'} = \sqrt{6}\beta\sqrt{\frac{\Lambda}{3}a^2 - k}$
4. This yields, for the $\mathcal{GR}$ value of $\beta = \sqrt{\frac{1}{6}}$, the de Sitter solution with

$$a(t') = \sqrt{\frac{3}{\Lambda}}\cosh[\sqrt{\frac{\Lambda}{3}}(t'-t'_0)], Ae^{\sqrt{\frac{\Lambda}{3}}(t'-t'_0)}, \sqrt{\frac{3}{\Lambda}}\sinh[\sqrt{\frac{\Lambda}{3}}(t'-t'_0)]$$

respectively for $k = +1, 0, -1$ spatial slicing

Deriving the linearised wave equation

1. Differentiating with respect to T, the Euler-Lagrange equation for the metric fluctuation,

$$\frac{\partial^2 \bar{q}_{ij}}{\partial T^2} = \frac{\partial}{\partial T}\Big(\frac{1}{\beta \bar{H}}\bar{E}^l_{kij}\Big)\bar{\pi}^k_l + \frac{1}{\beta \bar{H}}\bar{E}^l_{kij}\frac{\partial \bar{\pi}^k_l}{\partial T},$$

yields the linearized identity

$$\frac{\partial^2 \bar{h}_{ij}}{\partial T^2} =^* [\frac{\partial}{\partial T}\Big(\frac{1}{\beta \bar{H}}\bar{E}^l_{k(ij)}\Big)]\Delta\bar{\pi}^k_l + \Delta\Big(\frac{\partial}{\partial T}\Big(\frac{1}{\beta \bar{H}}\bar{E}^l_{kij}\Big)\Big)^* \bar{\pi}^k_l$$
$$+^* \Delta\Big(\frac{1}{\beta \bar{H}}\bar{E}^l_{kij}\Big)^*\Big(\frac{\partial \bar{\pi}^k_l}{\partial T}\Big) +^* \Big(\frac{1}{\beta \bar{H}}\bar{E}^l_{kij}\Big)\frac{\partial(\Delta\bar{\pi}^k_l)}{\partial T}$$

2. On account of vanishing ${}^*\bar{\pi}^i_j$, only the first and last terms remain. Linearization yields

$$(\beta^* \bar{H})\frac{\partial \bar{h}_{ij}(x)}{\partial T} =^* \bar{E}^l_{kij}\Delta\bar{\pi}^k_l,$$

and substituting for ${}^*\bar{E}^l_{kij}\Delta\bar{\pi}^k_l$ in the first term, leads to the EOM for $\bar{h}_{ij}$ which is

$$\frac{\partial^2 \bar{h}_{ij}}{\partial T^2} = -\frac{\partial \ln {}^*\bar{H}}{\partial T}\big(\frac{\partial \bar{h}_{ij}}{\partial T}\big) +^* \Big(\frac{1}{\beta \bar{H}}\bar{E}^l_{kij}\Big)\frac{\partial(\Delta\bar{\pi}^k_l)}{\partial T}$$

$\mathcal{GW}$ equation in Intrinsic Time and speed of $\mathcal{GW}$

1. Explicit calculations lead to $\frac{\partial \ln {}^*\bar{H}}{\partial T} = \frac{(R-3\Lambda)}{(R-2\Lambda)}$, and

$$\frac{\partial(\Delta\bar{\pi}^k_l)}{\partial T} = -\frac{\alpha q}{4\beta^* \bar{H}} q^{km}(\nabla^2 - \frac{1}{3}R)h_{ml} + \ldots$$

   The background ${}^*\bar{H}$ with zero momentric, is covariantly constant, and $\frac{\alpha q}{{}^*\bar{H}^2} = \frac{1}{R-2\Lambda}$

2. The resultant $\mathcal{GW}$ eqt. on de Sitter background, expressed w.r.t. intrinsic time T,

$$\frac{\partial^2 \bar{h}_{ij}}{\partial T^2} + \frac{(R-3\Lambda)}{(R-2\Lambda)}\frac{\partial \bar{h}_{ij}}{\partial T} + \frac{1}{4\beta^2(R-2\Lambda)}(\nabla^2 - \frac{R}{3})\bar{h}_{ij} + \cdots = 0 \qquad (1)$$

3. $\frac{(R-3\Lambda)}{(R-2\Lambda)}$ is positive definite, $\frac{\partial \bar{h}_{ij}}{\partial T}$ is the frictional force, due to expansion of the universe

4. $\frac{da}{dt'} = \sqrt{6}\beta\sqrt{\frac{\Lambda}{3}a^2 - k}$, $R = \frac{6k}{a^2}$; $dT = 2d\ln a(t')$, convert Eq.(1) into coord. time t'

   yields $\left(\frac{1}{24\beta^2(\frac{\Lambda}{3}-\frac{k}{a^2})}\frac{\partial^2}{\partial t'^2} - \frac{1}{24\beta^2(\frac{\Lambda}{3}-\frac{k}{a^2})}\nabla^2\right)\bar{h}_{ij} + \ldots = 0 \Rightarrow \text{GW speed} = 1$

$\mathcal{S}^3$ Tensor density Harmonics

1. $\bar{h}_{ab} = \sum_{I=4,5} C^{(I)}_{Klm} \bar{Y}^{Klm}_{(I)ab}$; S^3 tensor harmonics carry discrete eigen values $\{Klm\}$

2. $\bar{Y}^{Klm}_{(4,5)ij}$, with $K \geq 2$ are the 2 orthogonal transverse traceless eigenfunctions of Laplacian operator, ∇^2, with (negative) eigenvalues $E'_K = \frac{2-K(K+2)}{a^2} = \frac{R[2-K(K+2)]}{6}$

3. Eqt. encodes full-fledged information of all time dependence of the physical modes arising from gravitational pert. during different epochs of the de Sitter universe expansion

$$\ddot{C}_{\{K\}} + \frac{(R-3\Lambda)}{(R-2\Lambda)}\dot{C}_{\{K\}} + \frac{E_K}{4\beta^2(R-2\Lambda)}C_{\{K\}} + \frac{b^2 - g^2(E_K - \frac{R}{6})}{4\beta^2\alpha(R-2\Lambda)}E_K^2 C_{\{K\}} = 0;$$

wherein, $E_K := E'_K - \frac{R}{3} = -\frac{K(K+2)R}{6}$ which is the eigenvalue of $\nabla^2 - \frac{R}{3}$

4. Note the discreetness in energy levels

$\mathcal{E}$nergy for primordial $\mathcal{GW}$ excitation

1. Higher curvature/contribution $\mathcal{G}R$: $\sim -\frac{[g^2(E_K - \frac{R}{6})]E_K}{\alpha} \sim [g^2(K+1)^2]K(K+2)(\frac{l_P}{a})^4$
2. Physics is dominated by the $\mathcal{C}Y$ term at $a \to 0$, de Sitter sol. is saddle point of vacuum
3. Having both *physical local energy density* and *spatial* diff. inv. is perfect. The total Hamiltonian density is not vanishing without the paradigm of 4-cov.
4. 2 physical d.o.f. remain at each spatial point, even after spatial diff. are taken into account, precisely the transverse traceless $(\bar{h}_{ij}, \Delta\bar{\pi}^i_j)$ modes
5. Energy for the $\mathcal{GW}$ excitation is,

$$H_{\text{Phys}}[\bar{\pi}^i_j, q_{ij}] - H_{\text{Phys}}[{}^*\bar{\pi}^i_j, {}^*q_{ij}] \approx \int \frac{1}{2\beta^* \bar{H}} \Big[\Delta\bar{\pi}^m_n \Delta\bar{\pi}^n_m + \frac{\alpha q}{4} \bar{q}^{ik}\bar{q}^{jl}\bar{h}_{ij}(\nabla^2 - \frac{R}{3})\bar{h}_{kl} + (\beta\bar{H})^2 (q^{\frac{1}{3}}\mathcal{O}_{mn}{}^{kl}\bar{h}_{kl})(q^{\frac{1}{3}}\mathcal{O}^{mnij}\bar{h}_{ij})\Big] d^3x$$

6. Excitations is positive-definite for flat and compact spatial topologies since $\alpha < 0$, but is negative for open spatial topologies, $R = -6/a^2$
7. At late times, one of the mode $\to$ $constant$; may imply a memory effect
8. Matter $\rho(t)$, is easy to append to the sq. root Hamiltonian will change time dep. of $C(K)$

$\mathcal{C}$onclusions and out looks

1. 4-covariance and time are mutually exclusive
2. $\mathcal{D}$irac-algebra for $\mathcal{G}R$ is not even algebra of 4d $\mathcal{D}$iff., $\mathcal{Q}$uantum space-time doesn't exist
3. $\mathcal{T}$heories of gravitation need only 3d diffeomorphism invariance
4. $\mathcal{T}$wo types of gauge structures exist
   - $\mathcal{F}$ibre bundle type; infinite tensor product of the gauge group at each point
   - $\mathcal{D}$iffeomorphism group does not exhibit fibre bundle structure
   - $\mathcal{T}$ransversality is fundamental physics at work $\Rightarrow$may lead to new paradigm
5. $\mathcal{I}TG$ explains time dilation & length contraction effects dynamically in $\mathcal{S}R$
6. $\mathcal{Q}G$ with $\mathcal{C}$otton-$\mathcal{Y}$ork tensor is ultra-violet complete
7. $\mathcal{E}$instein's $\mathcal{R}$icci scalar potential emerges from the full $\mathcal{Q}G$ theory
8. $\mathcal{T}$he square-root-form in $\bar{H}$ may look peculiar, it produces the correct physics
9. $\mathcal{I}TG$ predicts discrete $\mathcal{G}W$ excitation energy and scale invariant primordial $\mathcal{G}W$ spectrum

$\mathcal{Q}\mathcal{G}$ needs only right idea about time;

only a dimension 3 $\mathcal{C}$otton-$\mathcal{Y}$ork tensor with a dimensionless coupling constant, g, which is *special* to 3d will success a full $\mathcal{Q}G$ theory