The Meaning of The Speed of Light in The FLRW Universe

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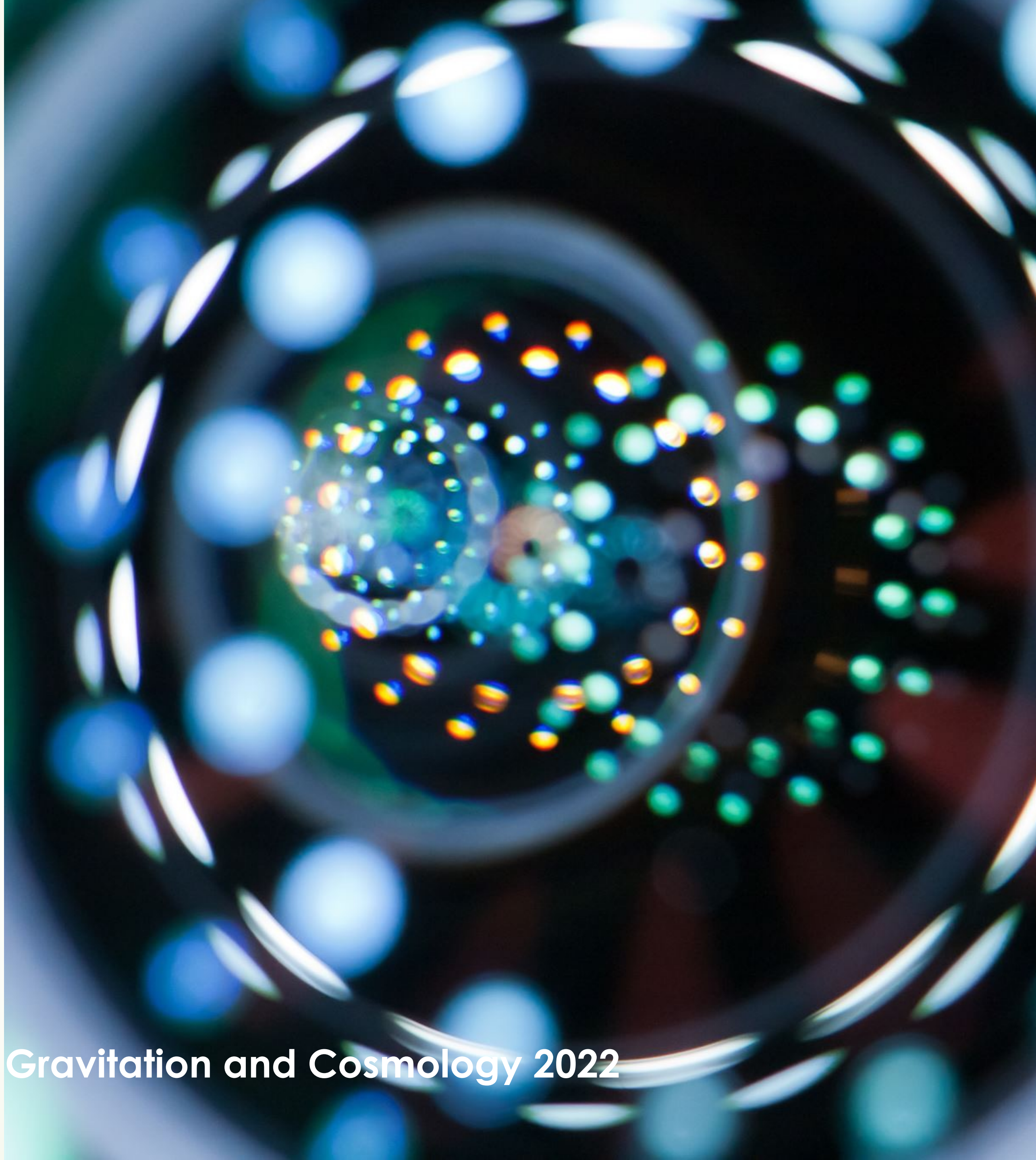


Asia-Pacific School and Workshop on Gravitation and Cosmology 2022

OUTLINE

- Lorentz invariant (Reviews)
- RW metric (Reviews)
- FLRW Universe (Reviews)
- Speed of light (correct Interpretation)
- A possible Solution : meVSL
- Summary

Lorentz Transformation I (Textbook review) :**skip**

- Lorentz Trasformation
- $\begin{pmatrix} t\prime \\ x\prime \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$
- If $x' = 0$, then $x = ut$. Thus, $x' = Ct + Dx = Ct + D\,ut = \ 0 \ \rightarrow C = -u\,D$
- If $x = 0$, then $x' = -ut$. Thus, $x' = -ut' = D(-ut + x) = D(-ut + 0) \rightarrow t' = Dt$
- $t' = At + Bx \rightarrow t' = At + 0 \ \rightarrow t' = At \quad \therefore A = D$
- $t' = At + Bx = A(t + Fx)\,, F \ \equiv \frac{B}{A} \ \ , [F] = \left[\frac{\mathrm{T}}{\mathrm{L}}\right] = [\mathrm{B}]$
- $\begin{pmatrix} t\prime \\ x\prime \end{pmatrix} = A[u] \begin{pmatrix} 1 & F[u] \\ -u & 1 \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$
- A combination of two LTs also must be a LT. If a a reference fram O' moving relative to O with velocity u1 and a O'' moving relative to O' with u2 then

Lorentz Transformation II (Textbook review) : **skip**

$$\begin{pmatrix} t'' \\ x'' \end{pmatrix} = A[u_2] \begin{pmatrix} 1 & F[u_2] \\ -u_2 & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix} = A[u_2] \begin{pmatrix} 1 & F[u_2] \\ -u_2 & 1 \end{pmatrix} A[u_1] \begin{pmatrix} 1 & F[u_1] \\ -u_1 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

$$= A[u_2]A[u_1] \begin{pmatrix} 1 - F[u_2]u_1 & F[u_1] + F[u_2] \\ -u_2 - u_1 & 1 - F[u_1]u_2 \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix} \equiv A[V] \begin{pmatrix} 1 & F[V] \\ -V & 1 \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix} .$$

If one compares the coefficeints of single LT, then one obtains

$$\frac{A[V]}{A[u_2]A[u_1]} = 1 - F[u_2]u_1 = 1 - F[u_1]u_2 \Rightarrow \frac{F[u_1]}{u_1} = \frac{F[u_2]}{u_2} \equiv \frac{1}{\alpha} = \text{cons} ,$$

If one makes LT from O to O' and then from O' to O back, then one obtains

$$\begin{pmatrix} t \\ x \end{pmatrix} = A[-u]A[u] \begin{pmatrix} 1 - F[-u]u & F[u] + F[-u] \\ u - u & 1 + F[u]u \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix} = A[-u]A[u] \begin{pmatrix} 1 + \frac{u^2}{\alpha} & 0 \\ 0 & 1 + \frac{u^2}{\alpha} \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

$$\Rightarrow \quad A[-u]A[u] = 1/\left(1 + \frac{u^2}{\alpha}\right) = A[u]^2 \equiv \gamma[u]^2 \quad \text{because of space symmetry} . \quad (9)$$

Lorentz Transformation III (Textbook review)

LT can be written as

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \frac{1}{\sqrt{1+\frac{u^2}{\alpha}}} \begin{pmatrix} 1 & \frac{u}{\alpha} \\ -u & 1 \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix} .$$

Finally, if one puts $\alpha = -V^2 = \text{const}$, then Eq. (10) becomes

$$\begin{pmatrix} Vt' \\ x' \end{pmatrix} = \frac{1}{\sqrt{1-\beta^2}} \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} Vt \\ x \end{pmatrix} \quad , \quad \text{where} \quad \beta = \frac{u}{V} .$$

V is the **maximum value of u** to make $1-\beta^2 \geq 0$

V doesn't need to be specified. Its physical meaning becomes clear when one applies LT to a specific physical phenomena (**apply to EM, then V = c**)

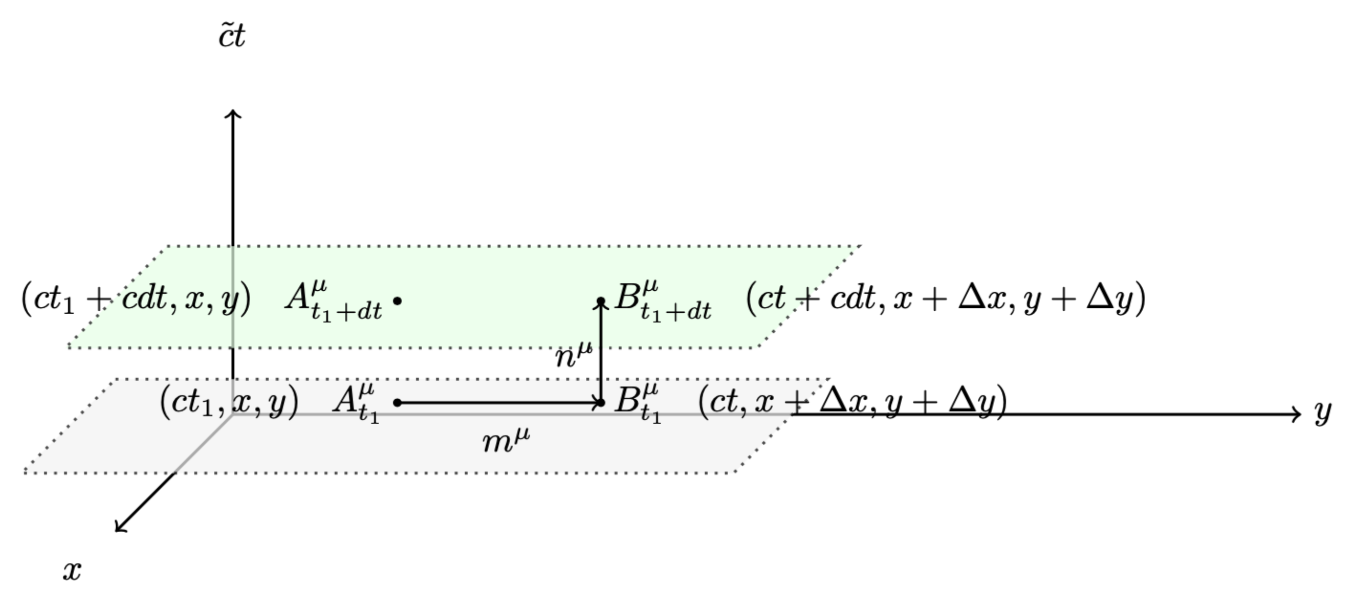
What LTs tell us (Interpretation)

- There exists an universal (*i.e*. the same for all inertial reference frames, IRFs) finite maximum speed, **V** to satisfy LTs
- V can be any value and it depends on the theory where LTs are applied
- This can be applied to each IRF

Finally, if one puts $\alpha = -V^2 = \text{const}$, then Eq. (10) becomes

$$\begin{pmatrix} Vt' \\ x' \end{pmatrix} = \frac{1}{\sqrt{1-\beta_i^2}} \begin{pmatrix} 1 & -\beta_i \\ -\beta_i & 1 \end{pmatrix} \begin{pmatrix} Vt \\ x \end{pmatrix} \quad , \quad \text{where} \quad \beta_i = \frac{u_i}{V}.$$

Minkowski space (SR) : IRF apply LT to Lorentz metric



IG. 1: Two events A and B are separated by $(cdt\,, \Delta x\,, \Delta y\,, \Delta z)$ with Lorentz metric to obatin the invariant interval $(\Delta s)^2 = -(\Delta x^0)^2 + (\Delta L)^2 = -(cdt)^2 + \Delta L^2$.

Physical quantities : interval
Invariant : independent of frame

Lorentz invariant

- SR is confirmed by QFT &EM at present
- Lorentz invarinace : locally (= at present) exact
- Spacetime interval is independent of

frames $\Delta s^2 = -c^2 dt^2 + \Delta L^2$ with $\Delta L = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

- Null interval ($\Delta s^2 = 0$): events in Minkowski spacetime are connected by signals moving at light velocity $v \equiv \frac{\Delta L}{dt} = c$ (division instead of derivative)
- c is same in any IRF : (how about c in non-IRF)

RW metric (Hom + Iso) (textbook : Islam, p.37-59)

- Universe appears to be **homogeneous & isotropic** around us on scales of more than a 100 Mlyrs
- On this scale, density of galaxies is same and all directions from us appear to be equivalent = **cosmological principle**
- The universe **looks the same** from **all positions in sapce at a particular time**
- The given $\mathbf{t_k}$: **well defined in Newtonian physics & SR**, but **not in GR**
- To define **a moment of time in GR** which is valid globally, **a particular set of circumstances are necessary**, which are satisfied by a **hom + iso Universe = RW metric**
- This makes cosmology as a science. One can study the entry history of the Universe.

Textbooks (**Skip**)

3.1 A simple derivation of the Robertson–Walker metric

As we saw in the first chapter, the universe appears to be homogeneous and isotropic around us on scales of more than a 100 million light years or so, so that on this scale the density of galaxies is approximately the same and all directions from us appear to be equivalent. From these observations one is led to the Cosmological Principle which states that the universe looks the same from all positions in space at a particular time, and that all directions in space at any point are equivalent. This is an intuitive statement of the Cosmological Principle which needs to be made more precise. For example, what does one mean by 'a particular time'? In Newtonian physics this concept is unambiguous. In special relativity the concept becomes well-defined if one chooses a particular inertial frame. In general relativity, however, there are no global inertial frames. To define 'a moment of time' in general relativity which is valid globally, a particular set of circumstances are necessary, which, in fact, are satisfied by a homogeneous and isotropic universe.

To define 'a particular time' in general relativity which is valid globally in this case, we proceed as follows. Introduce a series of non-intersecting space-like hypersurfaces, that is, surfaces any two points of which can be connected to each other by a curve lying entirely in the hypersurface which is space-like everywhere. We make the assumption that all galaxies lie on such a hypersurface in such a manner that the surface of simultaneity of the local Lorentz frame of any galaxy coincides locally with the hypersurface (see Fig. 3.1). In other words, all the local Lorentz frames of the galaxies 'mesh' together to form the hypersurface. Thus the four-velocity of a galaxy is orthogonal to the hypersurface. This series of hypersurfaces can be labelled by a parameter which may be taken as the proper time of any

the proper time along the galaxy. Then according to our assumptions $x^\mu(\tau)$ is given as follows:

$$(x^0 = c\tau,\ x^1 = \text{constant},\ x^2 = \text{constant},\ x^3 = \text{constant}). \tag{3.2}$$

From (3.1) and (3.2) we see that the proper time τ along the galaxy is, in fact, equal to the coordinate time t. This is because from (3.2) $\mathrm{d}x^i = 0$ along the worldline so that putting $\mathrm{d}x^i = 0$ in (3.1) yields $\mathrm{d}s = c\,\mathrm{d}\tau = c\,\mathrm{d}t$, so that $\tau = t$. Clearly a vector along the worldline given by $A^\mu = (c\,\mathrm{d}t, 0, 0, 0)$ and the vector $B^\mu = (0, \mathrm{d}x^1, \mathrm{d}x^2, \mathrm{d}x^3)$ lying in the hypersurface $t = \text{constant}$ are orthogonal, that is,

$$g_{\mu\nu} A^\mu B^\nu = 0, \tag{3.3}$$

since $g_{0i} = 0$ $(i = 1, 2, 3)$ in the metric given by (3.1). Further, the worldline given by (3.2) satisfies the geodesic equation

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}s} + \Gamma^\mu_{\lambda\nu} \frac{\mathrm{d}x^\lambda}{\mathrm{d}s} \frac{\mathrm{d}x^\nu}{\mathrm{d}s} = 0. \tag{3.4}$$

This can be seen from the fact that, from (3.2), we have

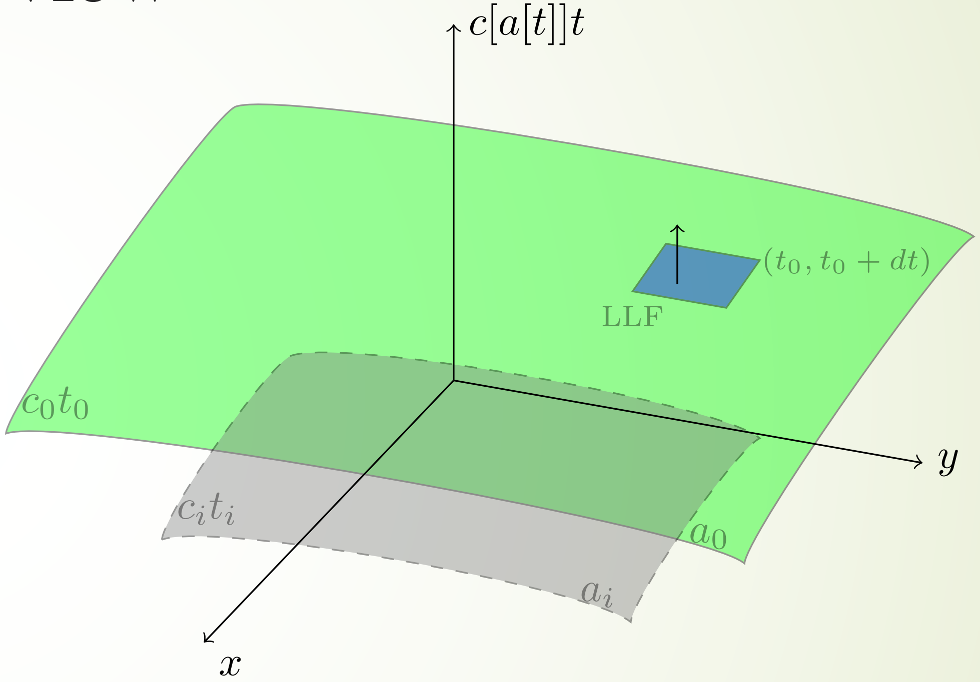
$$\mathrm{d}x^\mu/\mathrm{d}s = (1, 0, 0, 0) \tag{3.5}$$

so that (3.4) is satisfied if $\Gamma^\mu_{00} = 0$. In fact

$$\Gamma^\mu_{00} = \tfrac{1}{2} g^{\mu\nu}(2g_{\nu 0,0} - g_{00,\nu}). \tag{3.6}$$

RW metric (Hom + Iso) : Review

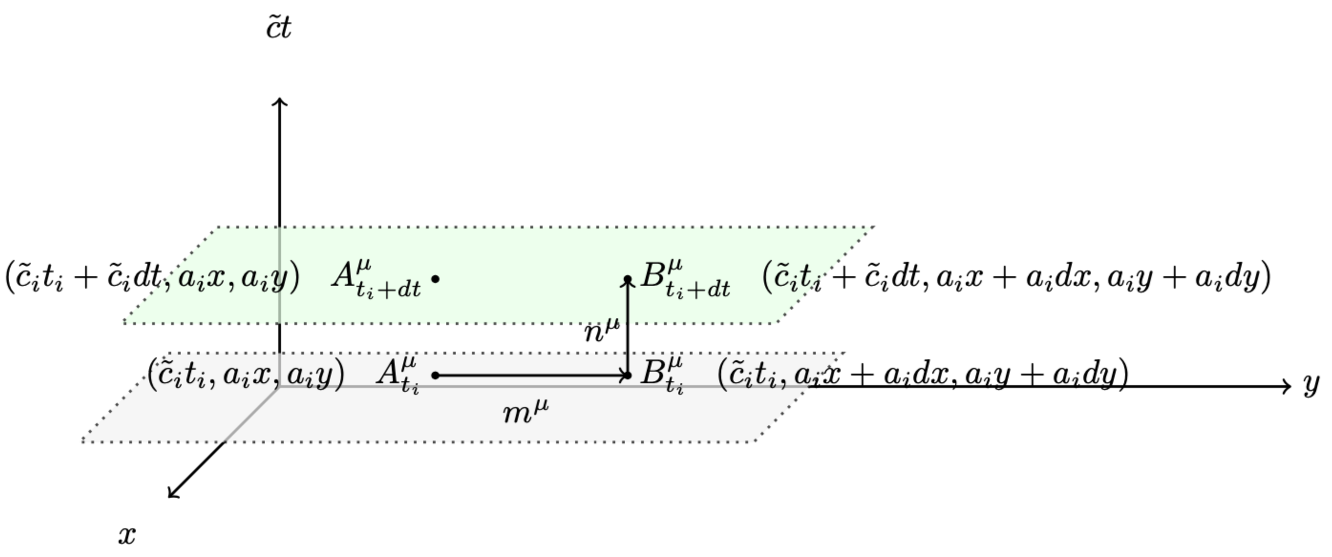
- A set of spacelike hypersuraces orthogonal to cosmic time t : IRF
- $ds_k^2 = -c^2dt^2 + a(t_k)^2dl^2$: RW metric at **each $\mathbf{t_k}$**
- Null interval : $c = a(t_k)\frac{dl}{dt}$
- At the given $\mathbf{t_k}$, c is a constant
- $\frac{dl}{dt}$ is always constant at any given time



Textbooks : Islam p.37-59, Guidry p.365-376, Yvonne p.157-162

Minkowski space at each t (so-called **Weyl postulate**)

This is the textbook definition of RW metric : Islam p.37-59, Guidry p.366, Stephani p.352 etc



IG. 2: Two galaxies A and B at t_1 are separated by $(\tilde{c}_i dt\,, a_i dx\,, a_i dy\,, a_i dz)$ with Lorentz metric to obatin the invariant interval $(\Delta s_i)^2 = -(\tilde{c}_i dt)^2 + a_i^2 dl^2$.

To define `a particular time' in GR, we introduce a series of non-intersectiong spacelike hypersurface where all galaxies lie on the hypersurface. This ***hypersurface*** *is the surface of simultaneity of the* ***local Lorentz frame of any galaxy.***

REVIEWS

- So far, we show **no new facts**
- If one is still confused, then one just needs to read the **definition of RW metric** again
- Existence of a global time, with local LF at each given moment
- Hom + Iso = maximally symmetric 3-d spatial surface : constructed from constant spatial metric not including its derivatives : ${}^{(3)}R_{ijkl} = k(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}),$
- Three space given by

$$\mathrm{d}\sigma'^2 = (1 + \tfrac{1}{4}kr'^2)^{-2}[(\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2],$$
$$r'^2 = (x^1)^2 + (x^2)^2 + (x^3)^2.$$

- 4D metric

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - \frac{R^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)}{[1 + \frac{1}{4}k(x^2 + y^2 + z^2)]^2},$$

FLRW Universe (**Need the correct interpretation**)

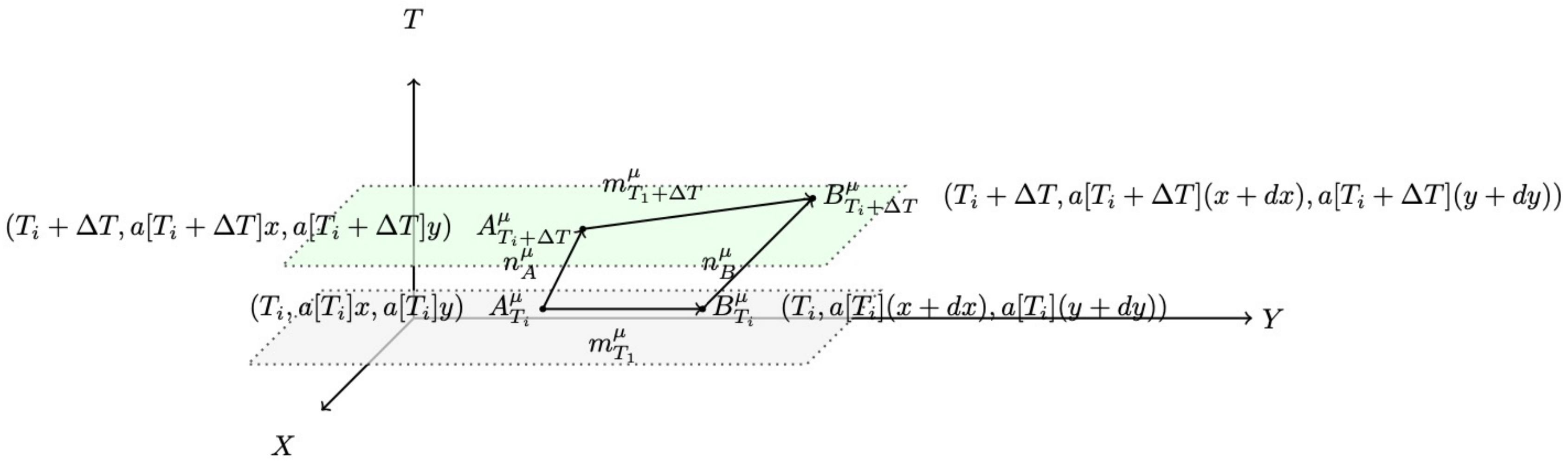
Rewrite the interval at t_k as $ds_k^2 = -c_k^2 dt^2 + a(t_k)^2 dl^2$

- By solving Einstein's field eq : expanding Univ
- $a(t_2) > a(t_1)$ if $t_2 > t_1$
- Speed of light : $c = a(t_k)\frac{dl}{dt}$
- The distance need to be traveled by a light increases as a(k) increases (Expanding of the Universe requires the increasing the value of c)
- Then c at t_2 , t_1 are inequal
- What's wrong ? & How to solve this problem?
- Ignore or overlook because $ds^2 = -c^2 dt^2 + a(t)^2 dl^2$
- RW metric (1935), Friedmann eq (1922,1924), Lemaitre (1927)
- Einstein's static univ : $\dot{a} = \ddot{a} = 0$

Speed of light

- Rewrite the interval at t_k as $ds_k^2 = -c_k^2 dt^2 + a(t_k)^2 dl^2$ instead of $ds^2 = -c^2 dt^2 + a(t)^2 dl^2$
- Then from null interval : $c_k = a(t_k)\frac{dl}{dt} \equiv a_k v$
- How can do this ?
- $x^0 \equiv c[a[t]]t \ , dx^0 = \left(\frac{d \ln c}{d \ln a} Ht + 1\right) cdt \equiv \tilde{c}dt$
- Correct RW interval :$ds_k^2 = -\tilde{c}_k^2 dt^2 + a(t_k)^2 dl^2$
- VSL : $\tilde{c}_k = a(t_k)\frac{dl}{dt} \equiv a_k v$: ok
- This is nothing new but we already use T_k, P_k, etc
- One cannot specify the form of c[a] : various possible models. But should satisfy isotropy and homogeneity

Can be cured by new defintion of time?
$T = c[t]t$ **(Skip)**



'IG. 3: Two galaxies A and B separated by infinitesimally (dx, dy, dz) in the comoving coordinate at T_i and at $T_i + \Delta T$.

Adiabatic Expansion I

- If only c varies as a function of scale factor "a", then heat flow will break isotropy
- Thus, **need adiabatic condition to keep hom + iso**
- $\varepsilon_\gamma = \frac{\pi^2}{15}\frac{(k_B T_\gamma)^4}{(\tilde{\hbar}\tilde{c})^3} \equiv \tilde{\sigma}_\gamma T_\gamma^4$, $P_\gamma = \frac{1}{3}\varepsilon_\gamma$
- Adiabaticity : $dQ = E_\gamma d \ln \tilde{\sigma}_\gamma = 0 \;\rightarrow\; \tilde{\hbar} = \tilde{\hbar}_0 a^{-b/4}$ for $\tilde{c} = \tilde{c}_0 a^{b/4}$ and $\tilde{k}_B = \tilde{k}_{B0}$ **(One possible model)**
- This is minimally extended Varying speed of Light (meVSL) : arXiv:2011.09274

Adiabatic Expansion II

local physics laws	Special Relativity	Electromagnetism	Thermodynamics
quantities	$\tilde{m} = \tilde{m}_0 a^{-b/2}$	$\tilde{e} = \tilde{e}_0 a^{-b/4}\,, \tilde{\lambda} = \tilde{\lambda}_0 a\,, \tilde{\nu} = \tilde{\nu}_0 a^{-1+b/4}$	$\tilde{T} = \tilde{T}_0 a^{-1}$
constants	$\tilde{c} = \tilde{c}_0 a^{b/4}\,, \tilde{G} = \tilde{G}_0 a^{b}$	$\tilde{\epsilon} = \tilde{\epsilon}_0 a^{-b/4}\,, \tilde{\mu} = \tilde{\mu}_0 a^{-b/4}\,, \tilde{c} = \tilde{c}_0 a^{b/4}$	$\tilde{k}_{\mathrm{B}0}\,, \tilde{\hbar} = \tilde{\hbar}_0 a^{-b/4}\,, \tilde{c} = \tilde{c}_0 a^{b/4}$
energies	$\tilde{m}\tilde{c}^2 = \tilde{m}_0 \tilde{c}_0^2$	$\tilde{h}\tilde{\nu} = \tilde{h}_0 \tilde{\nu}_0$	$\tilde{k}_{\mathrm{B}} \tilde{T} = \tilde{k}_{\mathrm{B}0} \tilde{T}_0 a^{-1}$

TABLE I: Summary for cosmological evolutions of physical constants and quantities of meVSL model. These relations satisfy all known local physics laws including special relativity, thermodynamics, and electromagnetic force.

meVSL (arXiv:2011.09274 and series 2101.09862,2104.09690,2108.096902110.08809)

- Consequence
- D_A , $\frac{H}{c}$, z (**redshift**)are same as traditional method (TM)
- D_L, H are different from TM
- No effect on BBN
- Effect on CMB, SL

$$
\begin{aligned}
H^2 &= \left(\frac{8\pi \tilde{G}_0}{3} \sum_i \rho_{i0} a^{-3(1+\omega_i)} - k \frac{\tilde{c}_0^2}{a^2} \right) a^{\frac{b}{2}} \equiv \left(\frac{8\pi \tilde{G}_0}{3} \rho_{\mathrm{cr}} - k \frac{\tilde{c}_0^2}{a^2} \right) a^{\frac{b}{2}} \equiv H^{(\mathrm{GR})2} a^{\frac{b}{2}} \,, \qquad (5.19) \\
\frac{\ddot{a}}{a} &= \left(-\frac{4\pi \tilde{G}_0}{3} \sum_i (1+3\omega_i)\, \rho_{i0} a^{-3(1+\omega_i)} \right) a^{\frac{b}{2}} + \frac{b}{4} H^2 \\
&= \left(-\frac{4\pi \tilde{G}_0}{3} \sum_i (1+3\omega_i)\, \rho_{i0} a^{-3(1+\omega_i)} + \frac{b}{4} H^{(\mathrm{GR})2} \right) a^{\frac{b}{2}} \equiv \left(\left(\frac{\ddot{a}}{a} \right)^{(\mathrm{GR})} + \frac{b}{4} H^{(\mathrm{GR})2} \right) a^{\frac{b}{2}} \,,
\end{aligned}
$$

So Far I

$$(\Delta s_1)^2 \equiv -(ca d\eta)^2 + a(\eta_1)^2\left(dx^2+dy^2+dz^2\right) = \left[-(cd\eta)^2 + \frac{a_1^2}{a^2}dl^2\right] a^2(\eta) = \text{const}\,, \tag{36}$$

$$(\Delta s_2)^2 \equiv -(ca d\eta)^2 + a(\eta_2)^2\left(dx^2+dy^2+dz^2\right) = \left[-(cd\eta)^2 + \frac{a_2^2}{a^2}dl^2\right] a^2(\eta) = \text{const}\,. \tag{37}$$

From the null interval, one obtains

$$\Delta s_1 = \Delta s_2 = 0 \quad \Rightarrow c = \frac{a_1}{a}\frac{dl}{d\eta} = \frac{a_2}{a}\frac{dl}{d\eta}\,. \tag{38}$$

Thus, we can regard the infinitesimal interval at a given cosmic time t_i, $ds(t_i)^2 = - c^2 dt^2 + a^2(t_i) dl^2 $ where dt is infinitesimal interval of the coordinate time (not a worldline)

> Use proper time τ along the world lines, as is standard in cosmology. Then $c^2 = 1$ in the above.

and dl is the comoving distance. Both are quantities on a given hypersurface as one does for speed of light on Minkowski spacetime.

> Minkowski space time is the special case when a^2(t) =1. *No, Minkowski spacetime is the special cases when a(t)s = constants*

This is just an extension of Minkowski space with changing physical distance as $a(t_i) \vec{r}$. Thus, one obtains different physical local speeds of the null ray by putting $ds(t_i)^2 =0$ for the different values of $a(t_i)$ at each hypersurface. SR requires the invariant local speed of light only and this is what I meant.

> As I already stated, the physical speed of light is given by using conformal time η given by $d\eta = a(t) dl$. Then $ds^2 = 0$ is the same as

No, the RW metric is true at each t slice, there's no global conformal factor in RW metric as shown in Eq.(36) and (37)

$a^2(t)(d\eta^2 - dl^2) =0$ i.e. $d\eta = \pm dl$ locally, as in special relativity .

GE

So Far II

Dear author,

Thank you for submitting your work to arXiv. We regret to inform you that arXiv's moderators have determined that your submission will not be accepted and made public on [1]arXiv.org.

Our moderators determined that your submission does not contain sufficient original or substantive scholarly research and is not of interest to arXiv.

For more information, please see [2]https://arxiv.org/help/moderation.

arXiv moderators strive to balance fair assessment with decision speed. We understand that this decision may be disappointing, and we apologize that, due to the high volume of submissions arXiv receives, we cannot offer more detailed feedback. Some authors have found that asking their personal network of colleagues or submitting to a conventional journal for peer review are alternative avenues to obtain feedback.

Mairi Sakellariadou
Editor-in-Chief
General Relativity and Gravitation

Reviewer comments:

Associate Editor:

The submission does not have the required degree of novelty and as such is not appropriate for GRG.

So Far III

Referee's report of JKPS

Of course, the conclusion is in strike contradiction to the standard lore in general relativity. Nevertheless, the author's solid publication record forces the referee to sift through the manuscript. The referee has concluded that the author's argument is fallacious, and the conclusion results from a series of mistakes. The referee, therefore, cannot recommend this manuscript for publication in JKPS.

Adding m^mu and n^mu is legitimate only in the Minkowski spacetime but not in the FRW spacetime. That is, Eq. (9) cannot be extended to Eq. (19). The reason is that, in curved spacetime, the vector m^mu and n^mu are defined in different tangent spaces.

While it is true that one can always set up a local Minkowski spacetime around any geodesic observer, for general curved spacetime, there must be a correction (proportional to the Riemann tensor) to the local Minkowski spacetime metric. This coordinate system is called Fermi Normal Coordinate (Manasse and Misner, 1962). The correction term is quadratic in the FNC coordinate and more significant when looking far from the central geodesic. Therefore, FRW spacetime at a fixed time is not the same as the local Minkowski spacetime.

Finally, to show that the speed of light varies in time, the author must show that dl/dt between A and B connected by null geodesic does not depend on time. While the author state that following equation (21), there is no proof to that statement.

In conclusion, while the referee agrees that speed of light could have a time variation, he disagrees that the time variability must be a generic consequence of the FLRW spacetime. That must rather come from the new physics mechanism.

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Meaning of speed of light in the FLRW Universe

December 2021

DOI: 10.13140/RG.2.2.11798.88644

Project: Varying speed of light

Seokcheon Lee

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Review of the speed of light in the local Lorentz frame

January 2022

DOI: 10.13140/RG.2.2.12470.63049

Project: Varying speed of light

Seokcheon Lee

Summary

- Cosmic time = proper time
- The value of c in a IRF determined by maximum distance travelled by a massless particle at a given time (If the distance need to be traveled increases, then the value of c is also increased)
- c should vary as a function of “a” in order to make RW metric consistent with Hom+Iso
- Other quantities (and/or constants) also need to evolve as a function of “a” in order to keep isotropy. (adiabatic expansion)
- Hom + Iso = observational assumption. c[a] is output of this assumption with expansion of the Univ.
- Static Univ : c = constant
- **'Everyone is entitled to his own opinion, but not to his own facts.'**

Future works

- Need to specify c as a function of the scale factor a[t]
- **Any mechanism** for this ? RW metric is not , Need new explanation
- Need to solve Einstein eqs properly. Traditional methods are approximations.
- Tensions between geometric and dynamical quantities might be solved
- Non-minimally extended models?
- Need Perturbations (using so-callled geometrical clocks method)