A study on Holographic dark energy model with adiabatic matter creation

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Abstract

- We explore the adiabatic matter creation process in holographic dark energy (HDE) to explain the observed accelerated expansion of the Universe.
- The field equations are solved analytically for cosmological parameters, like Hubble parameter, deceleration parameter and equation of state parameter in terms of redshift by assuming a generalized form of the matter creation rate.
- The evolution of such model is tested by the latest observational datasets.
- It is found that the HDE model with matter creation achieves the phase transition when the Hubble horizon is taken as an IR cut-off.
- Finally, we perform geometrical and cosmographic analyses in order to elucidate the differences and advantages of our model with the other dark energy models like ΛCDM model.

Holographic dark energy model with adiabatic matter creation

- Let us start with the homogeneous and isotropic flat Friedmann-Robertson - Walker(FRW) line element

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)\right], \quad (1)$$

where $a(t)$ is the scale factor of the universe.

- In the background of FLRW metric, the Einstein's Field Equations (EFEs) are given by

$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = T_{\mu\nu}, \quad (2)$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the cosmic fluid and concerning to matter fluids. Notice, that in order to simplify our formalism we use geometrical units $8\pi G = c = 1$. The other symbols have their usual meaning.

- In this work, we assume that the Universe is filled with HDE and pressureless dark matter. In open thermodynamical systems, the matter creation processes can be included phenomenologically in the energy-momentum tensor, which can be written as [I. Prigogine, et al., *Proc. Natl. Acad. Sci.* **85**, 7428 (1988), I. Prigogine, et al., *Gen. Relativ. Grav.* **21**, 8 (1989)].

$$T_{\mu\nu} = (\rho + \bar{P})u_\mu u_\nu + \bar{P}\, g_{\mu\nu}, \tag{3}$$

- where $\rho = \rho_m + \rho_h$ is the total energy density and $\bar{P}$ is the effective pressure which is the sum of pressures of HDE fluid and matter creation, i.e., $\bar{P} = p_h + p_c$.
- Here, ρ_m is the energy density of dark matter, ρ_h the energy density of HDE, while p_h and p_c are the corresponding pressures due to HDE and matter creation.
- In the framework of line element (1) and energy-momentum tensor (3), the basic equations of (2) which govern the global dynamics of the Universe in the matter dominated epoch ($p_m = 0$), are given by

$$3H^2 = \rho = \rho_m + \rho_h, \tag{4}$$

$$\dot{\rho_m} + \dot{\rho_h} + 3H(\rho_m + \rho_h + p_h + p_c) = 0, \tag{5}$$

where $H = \dot{a}/a$ is the Hubble parameter.

- The particle flux vector N^μ is introduced via the definition $N^\mu = nu^\mu$, which obeys the general balance equation $\nabla_\mu N^\mu = \Gamma$, where Γ is the matter production rate, $n = N/V$ is the particle number density and u^μ is the usual particle four velocity.
- Using FLRW metric and comoving observer, $u^\mu = \delta^\mu_i$, in which $u^\mu u_\mu = -1$, we obtain a new balance equation

$$\dot{n} + 3nH = n\Gamma, \tag{6}$$

where the comoving volume is $V = a^3$.

- Let us consider the scenarios driven by adiabatic particle production. In this case, particles and entropy are generated but the entropy per particle does not vary.
- Under such circumstances, Eq.(6) with the second law of thermodynamics, naturally leads to the appearance of a negative pressure associated to the creation rate which is given by [J.A.S. Lima, F.E. Silva and R.C. Santos, *Class. Quantum Grav.* **25**, 205006 (2008), G. Steigman, R.C. Santos and J.A.S. Lima, *J. cosmol. Astropart. Phys.* **06**, 033 (2009) , C.P. Singh and A. Kumar, *Eur. Phys. J. C* **80**, 106 (2020)]

$$p_c = -\frac{\rho_m}{3H}\Gamma. \tag{7}$$

- We suppose that there is no interaction between the two fluids. Therefore, the energy conservation equation (5) for dark matter and HDE conserve separately, which are given by

$$\dot{\rho_m} + 3H\left(1 - \frac{\Gamma}{3H}\right)\rho_m = 0, \tag{8}$$

$$\dot{\rho_h} + 3(\rho_h + p_h)H = 0. \tag{9}$$

- HDE model depends on the choices of IR cutoff L.
- We propose the HDE model with matter creation with same Hubble horizon as an IR cutoff, i.e., assuming the IR cut-off as the inverse of Hubble scale ($L = H^{-1}$).
- Thus, the HDE energy density is given by

$$\rho_h = 3b^2 H^2. \tag{10}$$

- By assuming the usual equation of state for HDE, $p_h = \omega_h \rho_h$, where ω_h is the equation of state parameter of HDE, it is readily checked from Eqs. (4) and (8)- (10) that the Hubble function is governed by the differential equation

$$\dot{H} + \frac{3}{2}\left[(1-b^2)\left(1-\frac{\Gamma}{3H}\right) + (1+\omega_h)b^2\right]H^2 = 0. \tag{11}$$

- We can observe that the above evolution equation will be fully determined once Γ is specified.
- In this paper, we consider a more generalized form of Γ which is a linear combination of three terms: first term is constant, second term is proportional to Hubble parameter and the third term as the combination of Hubble parameter and it's derivative [C.P. Singh and A. Kumar, *Eur. Phys. J. C* **80**, 106 (2020)], that is,

$$\Gamma = 3\left[\alpha H_0 + \beta H + \gamma\left(\frac{\dot{H}}{H} + H\right)\right] \tag{12}$$

where the parameters α, β and γ are dimensionless parameters lying in the interval $[0, 1]$ and H_0 is the present value of Hubble parameter.

- Using (12), the dynamical equation (11) becomes

$$H' + 3k\frac{H}{a} = \frac{3\alpha(1-b^2)H_0}{[2-3(1-b^2)\gamma]a} \quad (13)$$

where $k = \frac{3\left((1-\beta-\gamma)+(\beta+\gamma+\omega_h)b^2\right)}{2-3(1-b^2)\gamma}$ and a prime denotes the derivative with respect to lna.

- On solving (13), we get the Hubble parameter as

$$H = H_0\left[\frac{3\alpha(1-b^2)}{k[2-3\gamma(1-b^2)]} + \left(1 - \frac{3\alpha(1-b^2)}{k[2-3\gamma(1-b^2)]}\right)a^{-k}\right] \quad (14)$$

- We can find the solution of matter-dominated epoch $H = H_0a^{-3/2}$ by setting all the parameters equal to zero.

- The solution for the scale factor is given by

$$a(t) = \left[1 + \frac{(2-3(1-b^2)\gamma)k}{3\alpha(1-b^2)}\{e^{\frac{3\alpha(1-b^2)H_0}{2-3(1-b^2)\gamma}(t-t_0)} - 1\}\right]^{1/k} \quad (15)$$

- From (15)), it can be noticed that the scale factor varies as a power-law $a(t) \propto [1 + kH_0(t-t_0)]^{1/k}$ in early time where as it varies as an exponential form $a(t) \propto e^{\frac{(1-b^2)\alpha}{k(2-3(1-b^2)\gamma)}H_0(t-t_0)}$ in late epoch, which corresponds to de Sitter universe.
- Thus, the HDE model with matter creation shows transition from decelerated epoch to late time accelerated expansion.
- Now, by considering a well-known relation between the scale factor and redshift, $a = (1+z)^{-1}$, Eq. (14) can be rewritten as

$$H(z) = H_0\left[\frac{3\alpha(1-b^2)}{k(2-3\gamma(1-b^2))} + \left(1 - \frac{3\alpha(1-b^2)}{k(2-3\gamma(1-b^2))}\right)(1+z)^k\right] \quad (16)$$

Observational Analysis

- For the estimation of parameters of the model (16), we briefly present a statistical analysis of HDE model parameters using observational data obtained from Type Ia Supernovae (Pantheon data), Hubble parameter $H(z)$ observation, baryon acoustic oscillations/cosmic microwave background and latest local H_0 by SHOES.
- We use the Markov Chain Monte Carlo (MCMC) package EMCEE to perform the numerical analysis of our model [D. Foreman-Mackey, D. Hogg, D. Lang, *J. Goodman, Publ. Astron. Soc. Pac.* **125**, 306 (2012)] to put constraints on the free parameters.

- The most probable values of the model free parameters are estimated correspond to the minimal χ^2_{total} values of the three different combined datasets, namely

  1. DS1: $\chi^2_{DS1} = \chi^2_{SNe} + \chi^2_{H(z)} + \chi^2_{BAO/CMB} + \chi^2_{H_0}$,
  2. DS2: $\chi^2_{DS2} = \chi^2_{SNe} + \chi^2_{H(z)} + \chi^2_{H_0}$ and
  3. DS3: $\chi^2_{DS3} = \chi^2_{SNe} + \chi^2_{H(z)} + \chi^2_{BAO/CMB}$.

Table 1: The best-fit results of different model parameters obtained from joint analysis of different data sets.

Parameters	$DS1$	$DS2$	$DS3$
H_0	$70.880^{+0.996}_{-1.038}$	$70.586^{+1.382}_{-0.902}$	$68.646^{+1.250}_{-1.669}$
α	$0.452^{+0.067}_{-0.126}$	$0.392^{+0.101}_{-0.116}$	$0.441^{+0.087}_{-0.107}$
β	$0.054^{+0.085}_{-0.033}$	$0.085^{+0.078}_{-0.052}$	$0.071^{+0.060}_{-0.060}$
γ	$0.310^{+0.099}_{-0.079}$	$0.391^{+0.106}_{-0.104}$	$0.283^{+0.103}_{-0.074}$
ω_h	$-0.793^{+0.534}_{-0.671}$	$-0.790^{+1.030}_{-0.826}$	$-0.807^{+2.008}_{-1.690}$
b	$0.097^{+0.105}_{-0.064}$	$0.075^{+0.140}_{-0.133}$	$0.012^{+0.075}_{-0.070}$

Table 2: The values of z_{tr}, q_0, $\omega_{\rm eff}(z=0)$ and t_0 (Gyr) using different combinations of data sets.

Values	DS1	DS2	DS3
z_{tr}	$0.94^{+0.479}_{-0.937}$	$0.90^{+0.503}_{-0.892}$	$0.97^{+0.231}_{-0.681}$
q_0	$-0.51^{+0.053}_{-0.045}$	$-0.48^{+0.061}_{-0.047}$	$-0.51^{+0.043}_{-0.036}$
$\omega_{\rm eff}(z=0)$	$-0.77^{+1.7}_{-1.6}$	$-0.73^{+1.9}_{-1.4}$	$-0.75^{+1.9}_{-1.5}$
t_0 (Gyr)	14.27	14.10	14.40

Deceleration Parameter & Equation of state parameter

- We know that a cosmological parameter is required to describe the transition from decelerated phase to accelerated phase.
- In this context, the first cosmological parameter is the deceleration parameter, which is a dimensionless measure of the cosmic acceleration of the Universe, plays an important role in describing the transition phase.
- It is defined as $q = \frac{-a\ddot{a}}{\dot{a}^2}$. which takes the form in terms of redshift for our model as

$$q(z) = -1 + \frac{k - \frac{3\alpha(1-b^2)}{2-3(1-b^2)\gamma}}{1 + \frac{3\alpha(1-b^2)}{(2-3(1-b^2)\gamma)k}\{(1+z)^{-k} - 1\}}. \qquad (17)$$

- The transition redshift z_{tr}, which is defined at $q = 0$, is given by

$$z_{tr} = -1 + \left[\frac{3\alpha(1-b^2)}{3\alpha(1-b^2) + k(1 - 3\alpha - 3\beta + 3\alpha b^2 + 3\beta b^2 + 3\omega_h b^2)} \right]^{1/k} \tag{18}$$

- It is noted that if $\alpha = 0$, there is no transition from an early decelerating to a late time accelerating Universe. The present value of q at $z = 0$ is

$$q_0 = \frac{\left(1 - 3\alpha - 3\beta + 3\alpha b^2 + 3\beta b^2 + 3b^2\omega_h\right)}{(2 - 3(1-b^2)\gamma)}. \tag{19}$$

- The second cosmological parameter is the equation of state (EoS) parameter which is used to describe the evolution of the Universe filled with matter.
- It is defined as $\omega = \bar{P}/\rho$. This can also be represented as $\omega = -1 - \frac{2a}{3H}\frac{dH}{da}$. In this we calculate the effective EoS ω_{eff}, which is given by

$$\omega_{eff} = -1 + \frac{2}{3}\left[\frac{k - \frac{3\alpha(1-b^2)}{2-3(1-b^2)\gamma}}{1 + \frac{3\alpha(1-b^2)}{(2-3(1-b^2)\gamma)k}\{(1+z)^{-k} - 1\}}\right] \tag{20}$$

- The present value of ω_{eff} is calculated as

$$\omega_{eff}(z=0) = -1 + \frac{2}{3}\left(k - \frac{3\alpha(1-b^2)}{2-3(1-b^2)\gamma}\right) \tag{21}$$

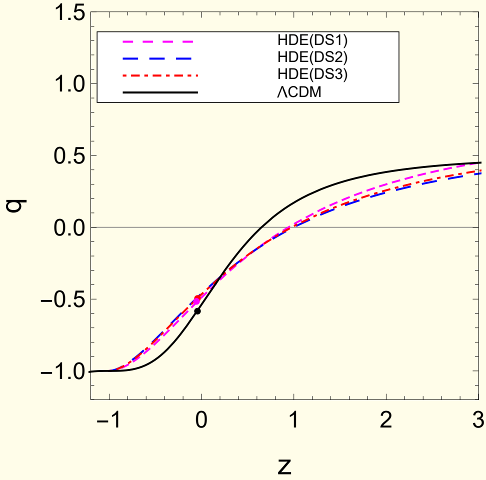

Figure 1: Plot of the deceleration parameter q as a function of redshift z for best-fit values of parameters obtained from DS1, DS2 and DS3 data sets.

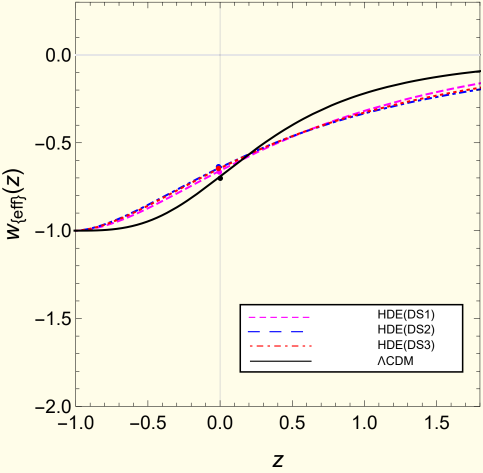

Figure 2: The evolution of effective EoS parameter ω_{eff} with respect to the redshift z for best-fit values of parameters obtained from DS1, DS2 and DS3 data sets.

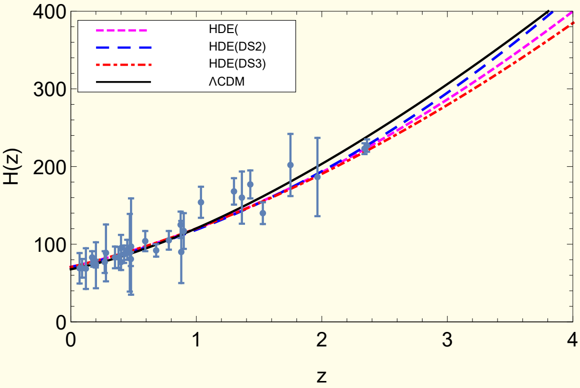

Figure 3: The Hubble function as a function of redshift for HDE model with matter creation from DS1, DS2 and DS3 data sets. The solid curve corresponds to the ΛCDM model. The $H_{obs}(z)$ data are also plotted with their error bars.

Geometric and Cosmographic Analyses

- We study various geometric and cosmographic parameters which are used to discriminate between the various contenders of DE models.
- Firstly, we discuss geometrical analysis of our model.[V. Sahni, et al., *JETP Lett.* **77**, 201 (2003), U. Alam, et al., *Mon. Not. R. Astron. Soc.* **344**, 1057 (2003)] proposed two geometrical diagnostic parameters $\{r, s\}$, called statefinder parameters, defined as

$$r = \frac{1}{aH^3}\frac{d^3a}{dt^3} \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)}. \tag{22}$$

- The fixed points $\{r, s\} = \{1, 0\}$ represents the ΛCDM model of the Universe where as $\{r, s\} = \{1, 1\}$ represents the standard cold dark matter (SCDM) model.
- We plot the $\{r, s\}$ and $\{r, q\}$ evolutions of trajectory in $s-r$ and $q-r$ planes for the best fit values of parameters obtained by observational data sets DS1, DS2 and DS3 which are shown in Figs. 4 and 5, respectively.

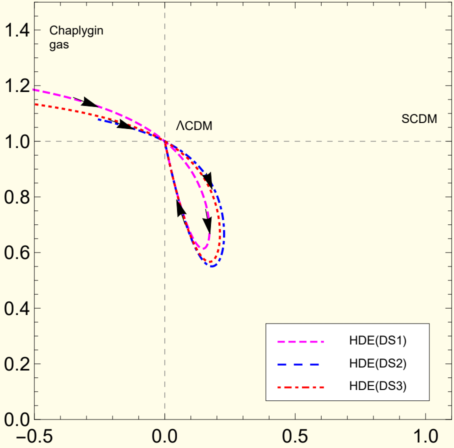

Figure 4: The evolutions of $\{r, s\}$ in $s - r$ plane corresponding to best fit values of model parameters obtained from data sets DS1, DS2 and DS3. The arrow shows the direction of the evolution of each trajectory.

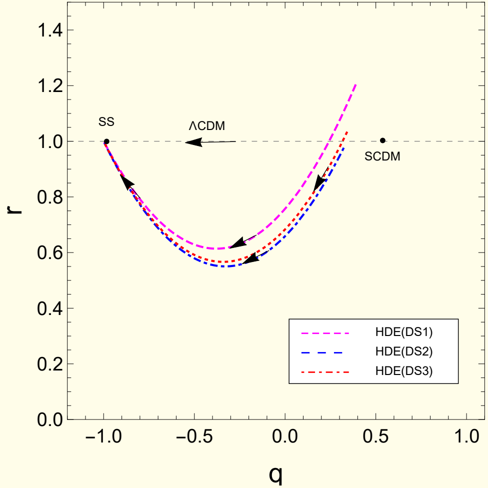

Figure 5: The evolutions of $\{r, q\}$ in $q - r$ plane corresponding to best fit values of model parameters for data sets DS1, DS2 and DS3. The arrow shows the direction of the evolution of each trajectory.

- We plot the $\{r, s\}$ and $\{r, q\}$ evolutions of trajectory in $s - r$ and $q - r$ planes for the best fit values of parameters obtained by observational data sets DS1, DS2 and DS3 which are shown in Figs. 4 and 5, respectively.
- It can be observed that the all the trajectory of $\{r, s\}$ begin from chaplygin gas type DE ($r > 1$, and $s < 0$) [Y.-B. Wu, S. Li, M.-H. Fu and J. He, *Gen. Relativ. Grav.* **39**, 653 (2007)] in early times and go to the quintessence region ($r < 1$ and $s > 0$) [V. Sahni, et al., *JETP Lett.* **77**, 201 (2003)], U. Alam, et al., *Mon. Not. R. Astron. Soc.* **344**, 1057 (2003)] in intermediate phase, and finally approach to ΛCDM model ($r = 1$ and $s = 0$) in late time.
- In Fig. 5, the fixed point $\{r, q\} = \{1, 0.5\}$ corresponds to standard cold dark matter (SCDM) and $\{r, q\} = \{1, -1\}$ to steady state (SS) or de Sitter.
- The horizontal line $r = 1$ corresponds to the trajectory of ΛCDM model.

- The evolutions of the trajectory corresponding to data sets $DS1$, $DS2$ and $DS3$ show that the model starts from decelerated phase in early time and after transition to accelerated phase, the model with each data sets approach to de Sitter model.
- Apart from the above mentioned two geometric parameters, the comparing of models can also be extracted by cosmography.
- The fundamental rule of cosmography is based on the Taylor series expansion of the scale factor about the present time [M. Visser, *Class. Quantum. Grav.* **21**, 2603 (2004), M. Visser, *Class. Quantum Grav.* **37**, 1541 (2005).].
- From it one obtains the cosmographic parameter (CP) usually referred to as *jerk* (j), *snap* ($\mathbf{s}$), *lerk* (l) and m parameters which are respectively defined as (see, [M. Dunajski, G. Gibbons, *Class. Quantum Grav.* **25**, 235012 (2008)]).

$$j = \frac{1}{aH^3}\frac{d^3a}{dt^3}\ ,\ \mathbf{s} = \frac{1}{aH^4}\frac{d^4a}{dt^4}\ ,\ l = \frac{1}{aH^5}\frac{d^5a}{dt^5}\ ,\ m = \frac{1}{aH^6}\frac{d^6a}{dt^6} \quad (23)$$

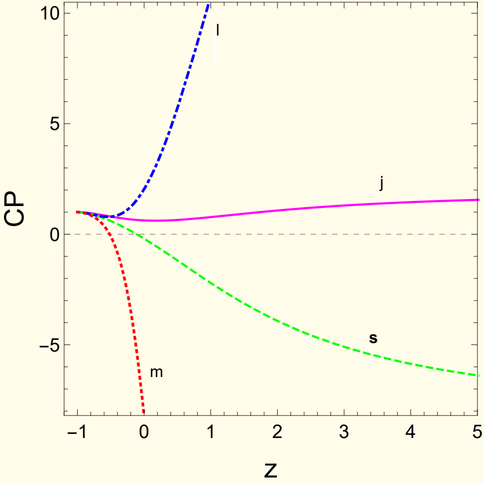

Figure 6: The cosmographic parameters j, $\mathbf{s}$, l and m are plotted for best fit values of model parameters for DS1.

- We find that j, $\mathbf{s}$, l and $m \to 1$ as $z \to -1$, i.e., in late time evolution, which is in good agreement with the observations of standard ΛCDM model.
- It is also observed that *jerk* and *lerk* parameters are positive throughout the cosmic evolution. The jerk parameter gives information about inflection points in the expansion history of the Universe.
- A positive *jerk* and *lerk* implies that the Universe has gone through a change of sign of the acceleration parameter in the past, which means that there has been a transition from deceleration to acceleration.
- On the other hand, it is well known that the *snap* and m parameters do not have a well established physical meaning, however they are important part of the Taylor series of the Hubble parameter in cosmography, giving us more precision in the preferred model.
- Figure 6 shows that *snap* and m transit from initial negative values to later positive ones.

Conclusion

- HDE model with Hubble horizon as an IR cutoff does not explain the transition phase.
- In order to overcome this drawback we have presented the HDE model using same IR cutoff but with gravitationally induced matter creation mechanism to observe the possible transition during the evolution of the Universe.
- We have considered the most general phenomenological form of Γ in terms of H and $\dot{H}$ and investigated the evolution equation by three different latest observational data sets; DS1 (SNe Pantheon, $H(z)$, BAO/CMB and local H_0 by SH0ES), DS2 (SNe Pantheon, $H(z)$ and local H_0) and DS3 (SNe Pantheon, H(Z) and BAO/CMB) to observe how they depict present scenario of the Universe.
- We have plotted the trajectories of Hubble parameter and other cosmographic parameters like deceleration parameter, effective EoS parameter, *jerk*, *snap*, *lerk* and m parameters with respect to the redshift.

- We know that the current observational data seem to favour an EoS for DE greater than -1.
- The present values q_0 and $\omega_{eff}(z=0)$ for HDE model are very much compatible with the present value of these parameters of ΛCDM model.
- The evolutions of these parameters almost match with the evolution of the ΛCDM from the data sets of DS1, DS2 and DS3.
- We find that the evolution of the Universe begins from higher redshift, from a decelerating phase to low redshift, i.e. to accelerating phase. In late time, the HDE model with matter creation approaches to standard ΛCDM model.
- The present values of effective EoS parameter are comparatively higher than that predicted by the joint analysis of WMAP, BAO, H_0 and SNe [E. Komatsu, et al., *Astrophys. J. Suppl.* **192**, 18 (2011)]. The age of Universe with each data set is found to be slightly higher than that of ΛCDM model.

- We have calculated the chi-squared minimum and reduced chi-squared for the model using DS1, DS2 and DS2 data sets. We have observed that the χ^2_{red} is less than unity with each data set which shows that the model gives the best fit values of model parameters and shows the good support to ΛCDM model.
- We have also discussed the statefinder parameters and have plotted the trajectories of {r, s} in $s - r$ plane and {r, q} in $q - r$ plane using best fit values of model parameters for all three combination of data sets.
- It can be observed that the each trajectory starts from chaplygin gas region in early times, enters into quintessence region in medieval time and finally approaches to ΛCDM in late times.

- In conclusion, we have investigated (analytically and numerically) the overall dynamics of HDE model with matter creation along with the Hubble horizon as an IR cutoff.
- This dynamical HDE model provides a transition phase from decelerating to accelerating phase of the cosmic expansion. The HDE model with matter creation provides good quality fits of the cosmological parameters at all redshifts and it resembles the global dynamics of the standard ΛCDM model.

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THANKYOU!