Superradiant Instability of Extremal Black Hole in STU Model

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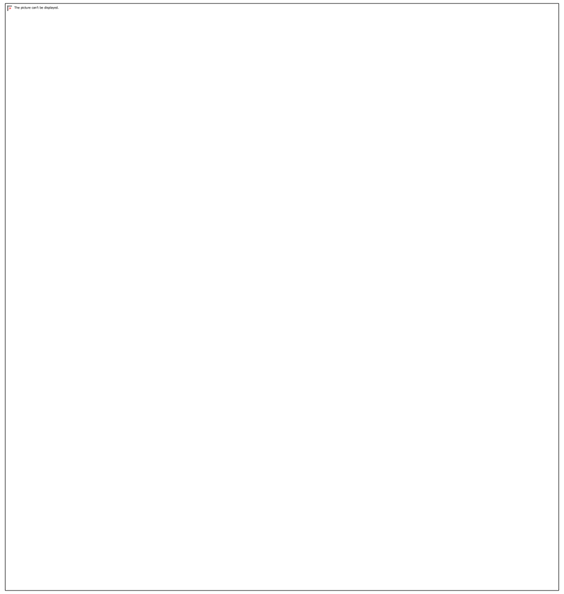
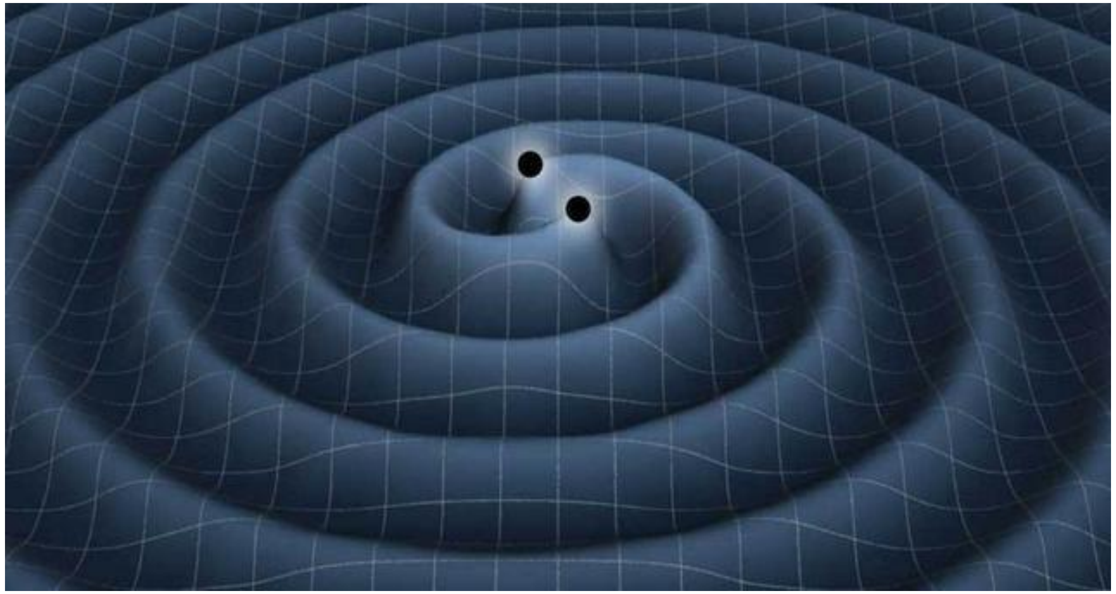
Black Hole Stability

- Black Hole is important objects in gravitational physics.



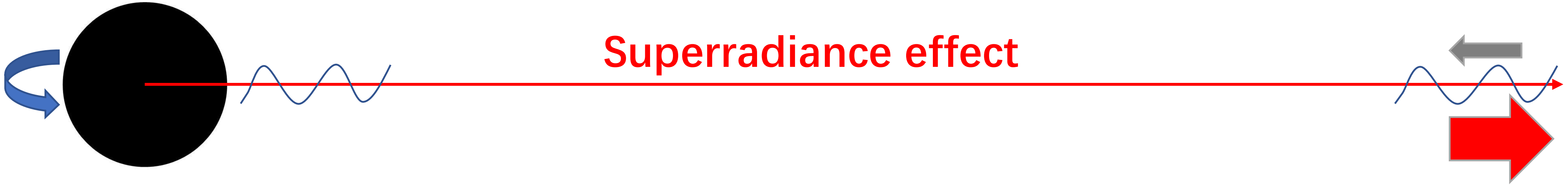
Black Hole image



Gravitational Wave

Motivation

What is superradiant Instablity?
Can it arise in static black hole ?



- ⭕ **Kerr black hole** [A.A. Starobinskil and S.M. Churilov (1974)] [S.M.Dolan (2007)]
- ⭕ **Kerr-Newmann Black Hole** [H. Furuhashi and Y. Nambu (2004)] [Y.Huang (2018)]
- ❌ **RN Black Hole** [S. Hod (2012)][S. Hod(2015)][J. H. Huang and Z. F. Mai (2016)]

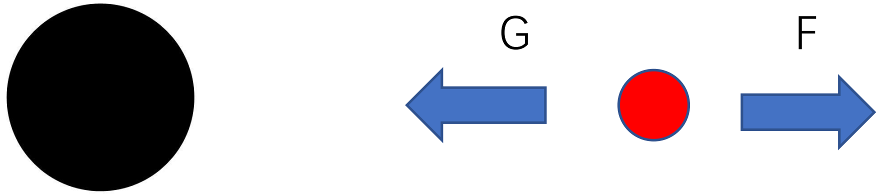
Motivation: Why Extremal black hole in STU

$$ds^2 = -(\tilde{H}_1\tilde{H}_2\tilde{H}_3\tilde{H}_4)^{-\frac{1}{2}}dt^2 + (\tilde{H}_1\tilde{H}_2\tilde{H}_3\tilde{H}_4)^{\frac{1}{2}}(dr^2 + r^2 d\Omega_2^2)\,,$$

$$e^{\frac{1}{2}\vec{a}_i\cdot\vec{\varphi}} = \tilde{H}_i^{-1}(\tilde{H}_1\tilde{H}_2\tilde{H}_3\tilde{H}_4)^{\frac{1}{4}}\,, \qquad \tilde{A}_i = (\tilde{H}_i^{-1} - 1)dt\,, \qquad \tilde{H}_i = 1 + \frac{4\tilde{Q}_i}{r^2}\,.$$

Remark:

1. **No Hawking Radiation.**
2. **It is a static black hole.**
3. **When four charges are equal, it reduce to Extremal RN black hole.**



STU: rich structure

Linear Perturbation

KG equation:

$$\left(g^{\mu\nu}D_\mu D_\nu - m_p^2\right)\Phi = 0\,, \qquad D_\mu := \nabla_\mu - i\tilde{q}_1\tilde{A}_{1\mu} - i\tilde{q}_2\tilde{A}_{2\mu} - i\tilde{q}_3\tilde{A}_{3\mu} - i\tilde{q}_4\tilde{A}_{4\mu}\,,$$

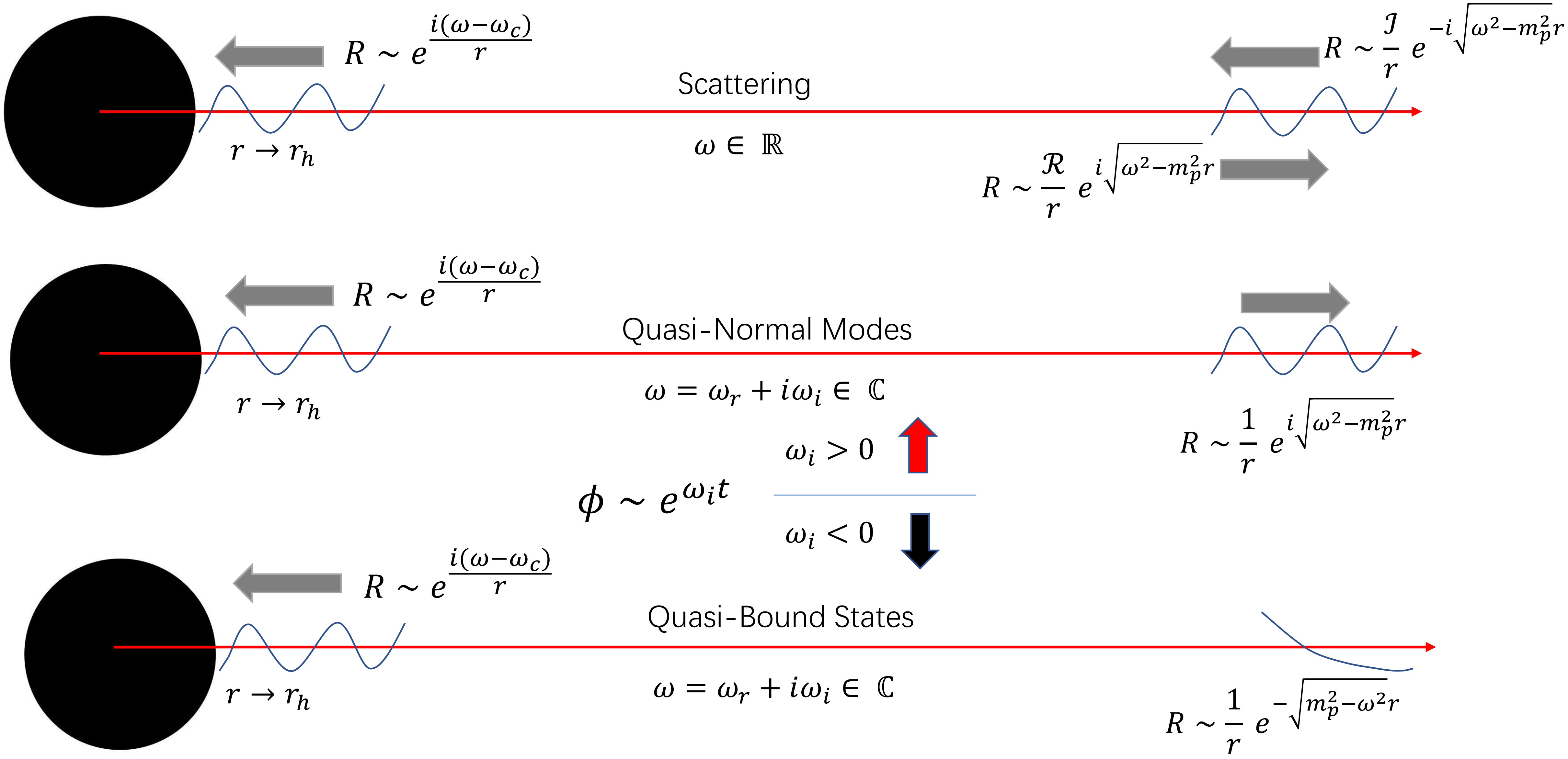
Anstaz:

$$\Phi = \mathrm{e}^{-i\omega t}R(r)\mathrm{Y}_{\ell m}(\theta,\varphi)$$

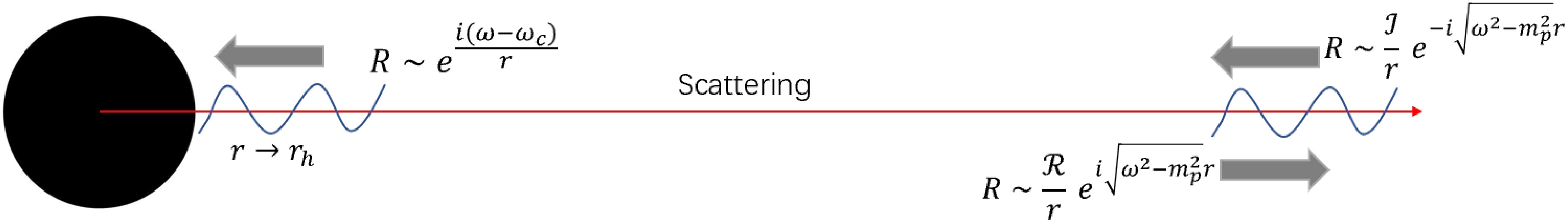
Radial Equation

$$-r^2\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + UR = 0\,,$$

Boundary Condition



Superradiant Scattering



Wroskain Determinant

$$|\mathcal{R}|^2 = |\mathcal{I}|^2 - \frac{\omega - \omega_c}{\sqrt{\omega^2 - m_p^2}} |\mathcal{T}|^2$$

Conserved Current

Energy Momentum Tensor

$$\tilde{T}_{\mu\nu} = \sum_{j=r,i} 2\nabla_\mu \Phi_j \nabla_\nu \Phi_j - g_{\mu\nu}\left(\nabla^\rho \Phi_j \nabla_\rho \Phi_j + \left(\sum_i^4 \tilde{q}_i \tilde{A}_i^\rho\right)^2 \Phi_j^2\right) - m_p^2 \Phi_j^2$$

Energy Current: $J_{\mathrm{E}}^{\mu} = \tilde{T}_{\mu\nu} (\frac{\partial}{\partial t})^{\nu}$

Total Energy: $E = \int_V \sqrt{-g}\, J_E^0$

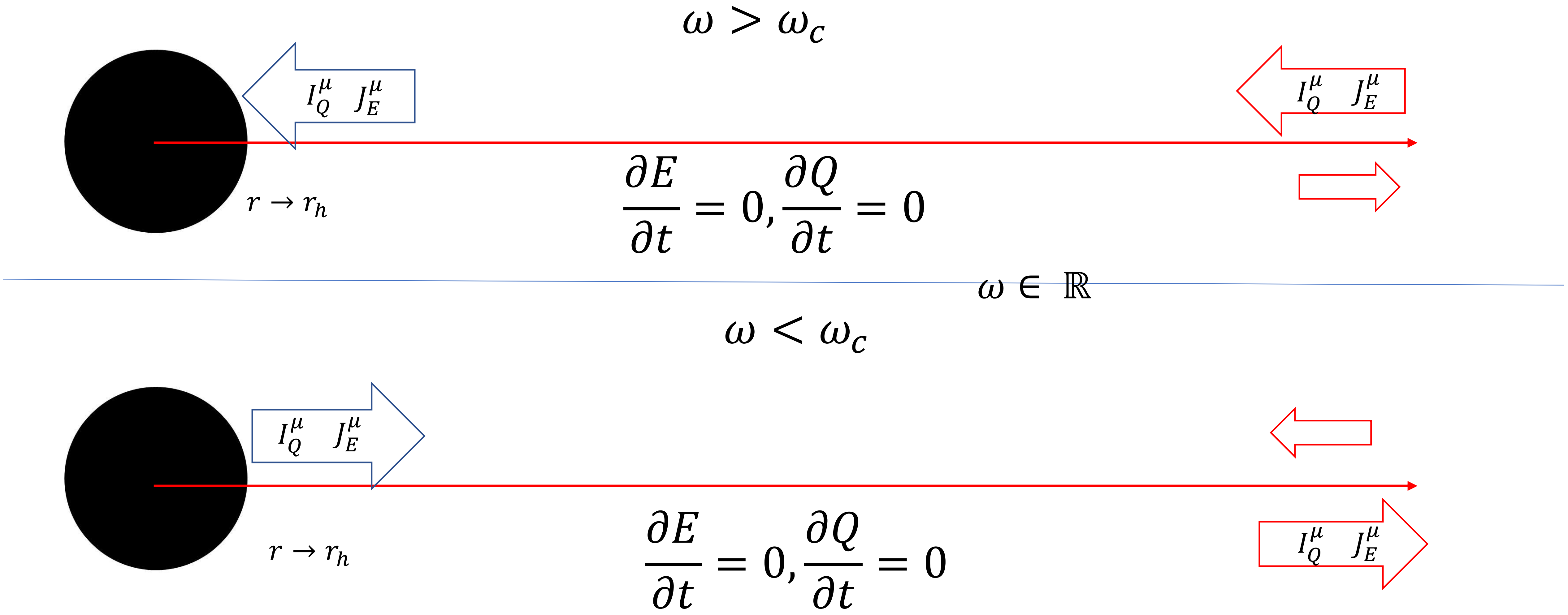
Energy Flux: J_E^r

Charge Current $iI_Q^\mu = \Phi^\dagger D^\mu \Phi - \Phi (D^\mu \Phi)^\dagger$

Total Charge $Q = \int_V \sqrt{-g}\, I_Q^0$

Charge Current: I_Q^r

Current (Superradiant Scattering)



Superradiant Condition for Complex Frequency

$$\frac{\partial E}{\partial t} = 2\omega_i\, E, \qquad \frac{\partial Q}{\partial t} = 2\,\omega_i Q$$

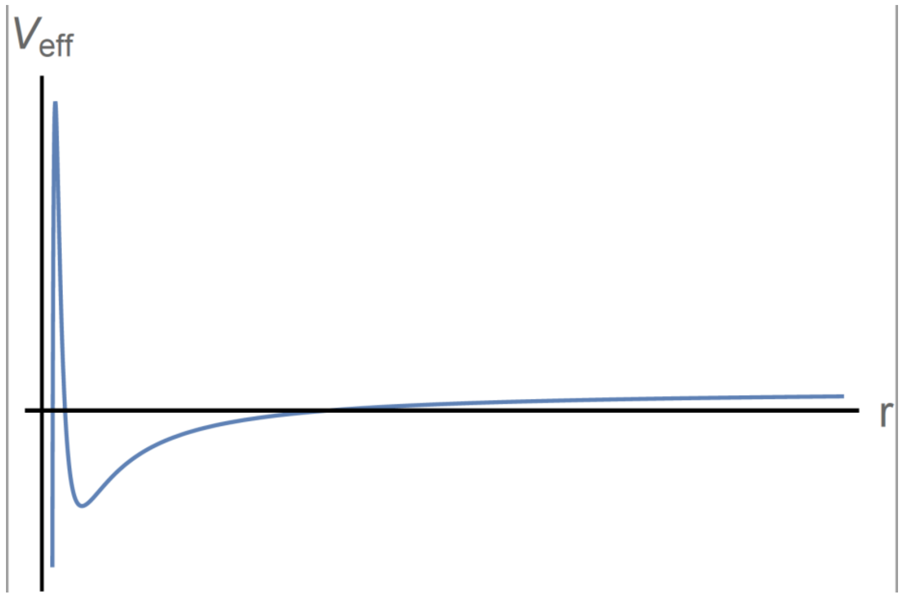
$\omega_i > 0$	E, Q	↑	**Unstable**
$\omega_i < 0$	E, Q	↓	**Stable**

Conservation Law:

$$\frac{\partial E}{\partial t} = \int_S J_E^r \Big|_{r\to\infty}^{r\to 0} \qquad \frac{\partial Q}{\partial t} = \int_S I_Q^r \Big|_{r\to\infty}^{r\to 0}$$

Superradiant Condition for instability : $\omega_i > 0 \rightarrow \omega_r < \omega_c$

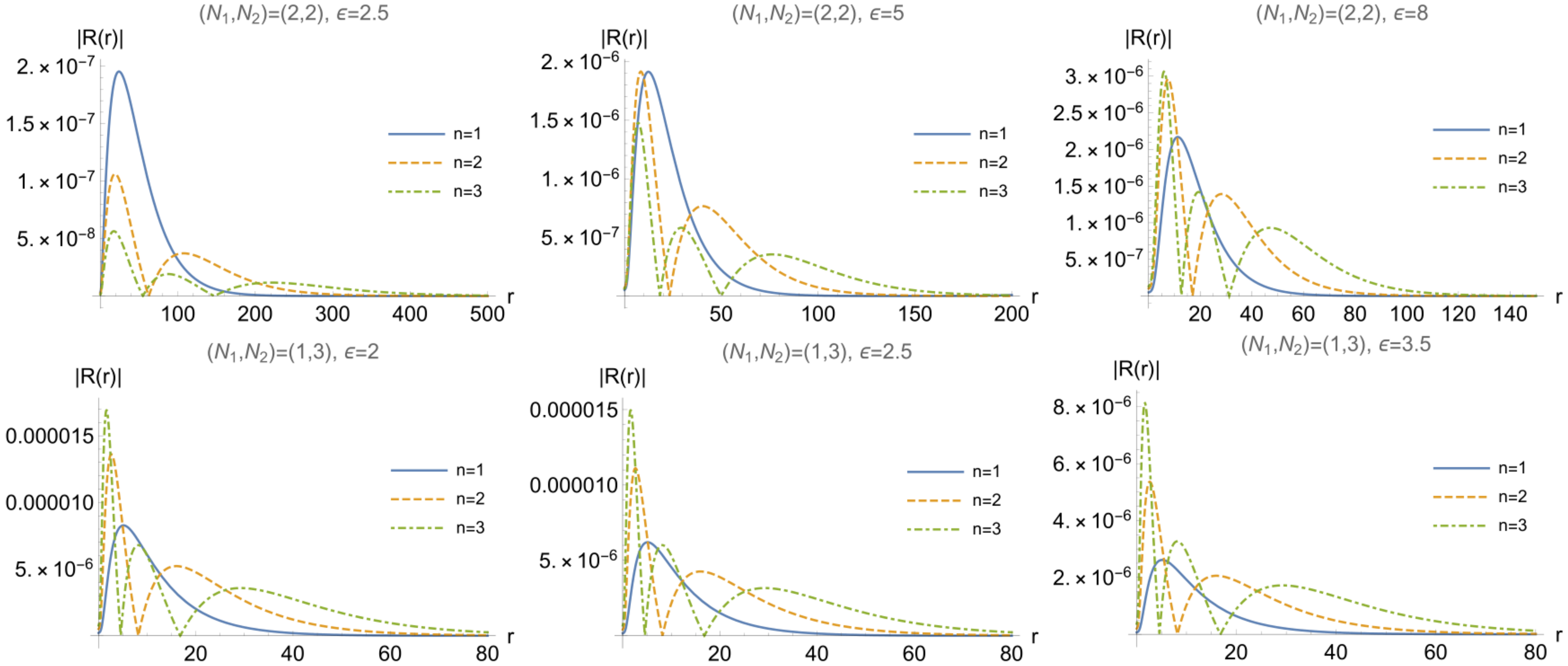
Numerical Result: Single Peak (Only $(N_1, N_2) = (2,2)$)



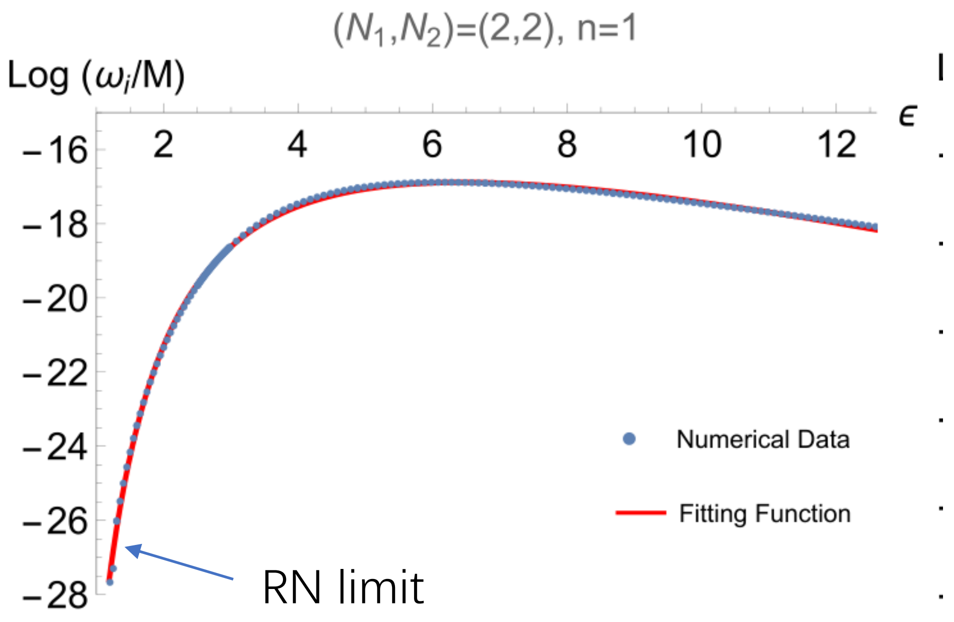
Parameters Choice:

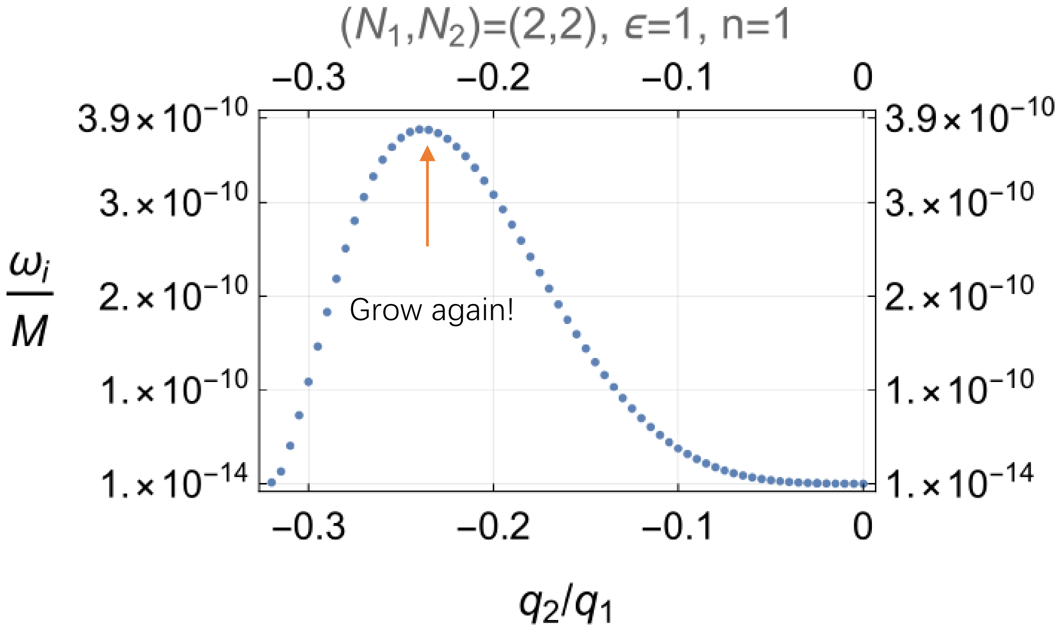
$$\epsilon = \frac{\sqrt{\mathrm{N}_1}\mathrm{Q}_2}{\sqrt{N_2}Q_1} - 1, \ell = 1, Q_1 = 1, q_2 = 0, q_1 = \frac{2}{10}, m_p = \frac{2}{10}$$

Numerical Result: Single Peak



Result





$\epsilon = 0.01, Q_1 = 1, m_p = \frac{2}{10}, q_1 = 20.2, q_2 = -20,$

Superradiantly stable RN black hole is a fine-tuning case

$$\omega_i = 0.1927 + 3.752 \times 10^{-8} i$$

Conclusion

- The relation between superradiant instability and unstable quasi-bound states or quasi-normal modes.

- RN black hole is a fine-tuning case in superradiant instability in the framework of STU supergravity model.

Thank you for your attention !