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Scalar dark matter condensed by thermal fermions

Based on 2109.07423

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Outline

- Ultralight scalar field as a **cold dark matter** candidate.
- Its huge population can be driven by a misaligned initial amplitude forming a **coherent state** (e.g., QCD axion with instanton potential)

$$\frac{n_\phi}{s} \sim 10^{19} \frac{10^{-20} \text{eV}}{m_\phi} \quad \phi_i \sim 10^{16} \text{ GeV} \left(\frac{10^{-20} \text{eV}}{m_\phi} \right)^{1/4}$$

- New idea: **Coherent DM state** driven by tiny couplings to **thermal fermions**.

Coherent scalar DM from misalignment

- Coherent state of a scalar field = classical wave

$$\hat{\phi}(x) = \int_k [a_k e^{-ik \cdot x} + a_k^+ e^{+ik \cdot x}]$$

$$\langle \hat{\phi}(x) \rangle_c = \phi_0 e^{-ik_0 \cdot x} + \phi_0^+ e^{+ik_0 \cdot x}$$

with $k_0 \approx (m_\phi, m_\phi \vec{v})$

$$|\phi_0| \sim \sqrt{\rho_\phi / m_\phi}$$

- Free field in the FRW universe:

$$\langle \hat{\phi}(x) \rangle_T = \phi(t)$$

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + m_\phi^2 \phi(t) = 0$$

$$H \gg m_\phi \quad (m_\phi t \ll 1) : \phi = \phi_i; \dot{\phi} = 0$$

$$H \ll m_\phi \quad (m_\phi t \gg 1) : \phi \sim \phi_i \frac{\sin(m_\phi t + \frac{\pi}{8})}{(m_\phi t)^{3/4}}$$

$$m_\phi \gg H_{eq} \approx 3 \cdot 10^{-27} \text{ eV}$$

$$\text{CDM density: } \rho_\phi(t) \sim \frac{m_\phi^2 \phi_i^2}{(t/t_i)^{3/2}} \Rightarrow \phi_i \sim 0.01 M_p \left(\frac{10^{-20} \text{ eV}}{m_\phi} \right)^{1/4}$$

Scalar field in thermal background

- Scalar field interacting with thermal fermions:

$$\mathcal{L}' = y_\phi \hat{\phi} (\bar{f}_R f_L + \bar{f}_L f_R)$$

$$\rightarrow V_{T,\text{eff}}(\phi) = -\frac{g_f}{2\pi^2} T^4 J_F \left(\frac{(m_f + y_\phi \phi)^2}{T^2} \right)$$

Dolan+Jackiw,
Weinberg, 1974

- Leading thermal effect in cosmological evolution:

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + (m_\phi^2 + m_T^2)\phi(t) \approx \frac{\partial}{\partial \phi} \langle \mathcal{L}' \rangle_T$$

$$m_T^2 = \frac{g_f}{24} y_\phi^2 T^2, \quad \langle \mathcal{L}' \rangle_T = y_\phi \phi \frac{g_f m_f T^2}{24}$$

$$g_f = 4N_c \quad (2) \quad \text{for } f = q, l (v)$$

Esteban+Salvado, 2101.05804

Batell+Ghalsasi, 2109.04476

General features I

- Evolution from $T_{ew} \approx 100$ GeV down to $T_{eq} \approx 0.8$ eV:

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \left(1 + \frac{x_1}{x}\right) \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x_1 \equiv y_\phi^2 \frac{c_t^2 g_f M_P}{48 m_\phi}, \quad x_S \equiv y_\phi \frac{c_t^2 g_f m_f}{48 m_\phi}$$

(Notation) $T = c_t \sqrt{M_P/2t}$ $c_t = 1.74/g_*^{1/4}$

$$x = m_\phi t = \frac{c_t^2 m_\phi M_P}{2T^2},$$

$$\tilde{\phi} = \phi/M_P, \quad m_{20} \equiv m_\phi/10^{-20} \text{ eV}$$

- Vanishing initial condition at x_{ew} : $\phi = 0, \phi' = 0$.
- Nontrivial evolution from $T_{ew}(x_{ew})$ to $T_f = m_f(x_f)$, and then free evolution from $T_f(x_f)$ to $T_{eq}(x_{eq})$ for $f = q, l$.
- Evolution from T_{ew} to T_{eq} for $f = \nu$.

General features II

- Different types of solutions depending on the location of $T_1(x_1)$:

$$x_{ew} \xrightarrow{x_1} x_f \xrightarrow{x_1} x_{eq}$$

- For $f = q, l$:
- i) $x_1 < x_{ew} < x_f \rightarrow x_{eq}$
 - ii) $x_{ew} < x_f < x_1 \rightarrow x_{eq}$
 - iii) $x_{ew} < x_1 < x_f \rightarrow x_{eq}$

For $f = \nu$: $x_{ew} < x_1 < x_{eq}$

- The resulting DM density is to be described by

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2 m_\phi^2 M_P^2}{\pi x_{eq}^{3/2}}$$

$$\rho_\phi = \rho_{DM} \Rightarrow c_f^2 x_S^2 \approx 2.5 \times 10^{-4} m_{20}^{-1/2}$$

$$x_{ew} \approx 10^{-15} m_{20} \qquad x_{eq} \approx 2 \times 10^7 m_{20}$$

$$x_f \approx 5 \times 10^{-4} m_{20} (m_e/m_f)^2$$

$$x_1 \approx 10^{47} y_\phi^2 m_{20}^{-1} \qquad x_S \approx 2 \cdot 10^{24} y_\phi m_{20}^{-1} \left(\frac{m_f}{m_e} \right)$$

Asymptotic solutions in various regimes

$$x \gg x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x \ll x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \frac{x_1}{x} \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\begin{aligned} \tilde{\phi}(x) = & \frac{\pi x_S}{(2x)^{1/4}} \left(\frac{x}{\Gamma(3/4)} J_{1/4}(x) F_{1,2} \left(\frac{1}{2}; \frac{3}{4}, \frac{3}{2}; -\frac{x^2}{4} \right) \right. \\ & \left. - \frac{x^{3/2}}{3\Gamma(5/4)} (J_{1/4}(x) - Y_{1/4}(x)) F_{1,2} \left(\frac{3}{4}; \frac{5}{4}, \frac{7}{4}; -\frac{x^2}{4} \right) \right) \\ & + C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}} \end{aligned}$$

$$\tilde{\phi}(x) = \frac{x_S}{x_1} + C_1 \frac{e^{2i\sqrt{xx_1}}}{\sqrt{x}} + iC_2 \frac{e^{-2i\sqrt{xx_1}}}{\sqrt{xx_1}}$$

$$\tilde{\phi}(x) = C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}}$$

Scalar DM condensed by q/l

$$\text{i) } x_1 < x_{ew} < x_f$$

$$x < x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

Solution

$$\tilde{\phi}(x) = \begin{cases} x_S \left(\frac{\pi}{\Gamma(\frac{3}{4})} \frac{J_{1/4}(x)}{(2x)^{1/4}} [G_1(x) - G_1(x_{ew})] - \frac{4\Gamma(\frac{3}{4})}{3} \frac{J_{-1/4}(x)}{(2x)^{1/4}} [G_2(x) - G_2(x_{ew})] \right) \\ C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}} \end{cases}$$

$$G_1(x) = x F_{1,2} \left(\frac{1}{2}; \frac{3}{4}, \frac{3}{2}; -\frac{x^2}{4} \right), \quad G_2(x) = x^{3/2} F_{1,2} \left(\frac{3}{4}; \frac{5}{4}, \frac{7}{4}; -\frac{x^2}{4} \right)$$

$$\frac{C_1}{x_S} = \frac{\pi}{2^{\frac{1}{4}} \Gamma(\frac{3}{4})} G_1(x_f) - \frac{2^{\frac{5}{4}} \Gamma(\frac{3}{4})}{3} G_2(x_f)$$

$$\frac{C_2}{x_S} = \frac{2^{\frac{5}{4}} \Gamma(\frac{3}{4})}{3} G_2(x_f)$$

Scalar DM condensed by q/l

i) $x_1 < x_{\text{ew}} < x_f$

$$x < x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \frac{x_1}{x} \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\rho_\phi(x_{\text{eq}}) \approx \frac{c_f^2 x_S^2 m_\phi^2 M_P^2}{\pi x_{\text{eq}}^{3/2}} \text{ with } c_f^2 = C_1^2 + C_2^2$$

$$c_f^2 = \begin{cases} 4\sqrt{2}\Gamma\left(\frac{5}{4}\right)^2 x_f^2 & \text{for } x_f \ll 1 \\ \frac{\pi\Gamma\left(\frac{3}{4}\right)^2}{2\sqrt{2}} & \text{for } 1 \ll x_f \end{cases}$$

$$y_\phi \approx 3.6 \cdot 10^{-24} \frac{m_{20}^{\frac{1}{4}}}{N_c} \left(\frac{m_f}{m_e}\right) \text{ with } \frac{550}{N_c^{\frac{2}{5}}} \left(\frac{m_f}{m_e}\right)^{\frac{4}{5}} \ll m_{20} \ll 2 \cdot 10^3 \left(\frac{m_f}{m_e}\right)^2 \text{ for } x_f \ll 1$$

$$y_\phi \approx 3 \cdot 10^{-27} \frac{m_{20}^{\frac{3}{4}}}{N_c} \left(\frac{m_e}{m_f}\right) \text{ with } m_{20} \gg \text{Max} \left[2 \frac{10^{16}}{N_c^2} \left(\frac{m_f}{m_e}\right)^4, 2 \cdot 10^3 \left(\frac{m_f}{m_e}\right)^2 \right] \text{ for } 1 \ll x_f$$

(*) Neglecting Freeze-In:
 $m_\phi \ll 46 \text{ eV (5.7 MeV)}$ for $f = e (b)$,
 $y_\phi \ll 5.3 (1.4) \times 10^{-11}$.

Scalar DM condensed by q/l

ii) $x_{ew} < x_f < x_1$

$x < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \frac{x_1}{x} \tilde{\phi}(x) = \frac{x_S}{x}$$

$x > x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

Solution

$$\tilde{\phi}(x) = \begin{cases} \frac{x_S}{x_1} \left(1 - \sqrt{\frac{x_{ew}}{x}} \cos[2(\sqrt{xx_1} - \sqrt{x_{ew}x_1})] - \frac{1}{2\sqrt{xx_1}} \sin[2(\sqrt{xx_1} - \sqrt{x_{ew}x_1})] \right) \\ C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}} \end{cases}$$

$$\frac{C_1}{x_S} = -\frac{\pi x_f^{1/4}}{4x_1} \left[2x_f Y_{5/4}(x_f) - Y_{1/4}(x_f) \cos(2\sqrt{x_1 x_f}) + \sqrt{\frac{x_f}{x_1}} Y_{-3/4}(x_f) \sin(2\sqrt{x_1 x_f}) \right],$$

$$\frac{C_2}{x_S} = \frac{\pi x_f^{1/4}}{4x_1} \left[2x_f J_{5/4}(x_f) - J_{1/4}(x_f) \cos(2\sqrt{x_1 x_f}) + \sqrt{\frac{x_f}{x_1}} J_{-3/4}(x_f) \sin(2\sqrt{x_1 x_f}) \right].$$

Scalar DM condensed by q/l

ii) $x_{ew} < x_f < x_1$

$$x < x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \frac{x_1}{x} \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

$$\rho_\phi(x_{eq}) \approx \frac{c_f^2 x_S^2 m_\phi^2 M_P^2}{\pi x_{eq}^{3/2}} \text{ with } c_f^2 = C_1^2 + C_2^2$$

$$c_f^2 = \begin{cases} 4\sqrt{2}\Gamma\left(\frac{5}{4}\right)^2 x_f^2 & \text{for } x_f \ll x_1 \ll 1 \\ \frac{\pi x_f^3}{2x_1^2} & \text{for } 1 \ll x_f \ll x_1 \end{cases}$$

$$y_\phi \approx 3.6 \cdot 10^{-24} \frac{m_{20}^{\frac{1}{4}}}{N_c} \left(\frac{m_f}{m_e}\right) \text{ with } m_{20} \ll 31 N_c^{-\frac{2}{5}} \left(\frac{m_f}{m_e}\right)^{8/5} \text{ for } x_f \ll x_1$$

$$\ll 1$$

$$y_\phi \approx 5.3 \cdot 10^{-24} m_{20} \left(\frac{m_e}{m_f}\right)^{\frac{1}{2}} \text{ with } m_{20} \gg 2 \cdot 10^3 \left(\frac{m_f}{m_e}\right)^2 \text{ for } 1 \ll x_f \ll x_1$$

(*) BBN constraint: $m_\phi \ll 10^{-6} (2.5 \times 10^{-3}) \text{ eV}$ for $f = e (b)$,

$$y_\phi \ll 5.7 \times 10^{-10} (1.5 \times 10^{-8}).$$

Scalar DM condensed by q/l

iii) $x_{ew} < x_1 < x_f$

$x < x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \frac{x_1}{x} \tilde{\phi}(x) = \frac{x_S}{x}$$

$x_1 < x < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$x_f < x$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = 0$$

(i) $x_1 \ll x_f \ll 1$:

$$y_\phi \approx 3.6 \times 10^{-24} \frac{m_{20}^{-\frac{1}{4}} m_f}{N_c m_e} \quad \text{with} \quad \frac{28}{N_c^{\frac{2}{5}}} \left(\frac{m_f}{m_e}\right)^{\frac{18}{5}} \ll m_{20} \ll \text{Min} \left[2 \times 10^3 \left(\frac{m_f}{m_e}\right)^2, \frac{6 \times 10^5}{N_c^{\frac{2}{5}}} \left(\frac{m_f}{m_e}\right)^{\frac{4}{5}} \right]$$

(ii) $x_1 \ll 1 \ll x_f$:

$$y_\phi \approx 3 \times 10^{-27} \frac{m_{20}^{\frac{3}{4}} m_e}{N_c m_f} \quad \text{with} \quad 2 \times 10^3 \left(\frac{m_f}{m_e}\right)^2 \ll m_{20} \ll \text{Min} \left[3.2 \times 10^{11} N_c^2 \left(\frac{m_f}{m_e}\right)^4, \frac{2.1 \times 10^{16}}{N_c^2} \left(\frac{m_e}{m_f}\right)^4 \right]$$

(iii) $1 \ll x_1 \ll x_f$:

$$y_\phi \approx 1.9 \times 10^{-22} \frac{m_{20}^{\frac{1}{3}}}{N_c^{\frac{1}{6}}} \left(\frac{m_f}{m_e}\right)^{\frac{2}{3}} \quad \text{with} \quad 1.2 \times 10^8 N_c^{\frac{5}{4}} \left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \ll m_{20} \ll 4.4 \times 10^{11} N_c^2 \left(\frac{m_f}{m_e}\right)^4 \quad \text{for } x_1^5 \ll x_f$$

$$y_\phi \approx 4.5 \times 10^{-28} \frac{m_{20}}{N_c} \left(\frac{m_e}{m_f}\right)^{\frac{3}{2}} \quad \text{with} \quad m_{20} \gg 4.9 \times 10^8 N_c^{\frac{5}{4}} \left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \quad \text{for } x_f \ll x_1^5.$$

(*) BBN+Feeze-In: $y_\phi \ll 5.7 (1.2) \times 10^{-10}$,

$m_\phi \ll 1.3 \times 10^{-2} (6.1 \times 10^3) \text{ eV} \quad \text{for } f = e (b)$

Scalar DM condensed by ν

- $x_{ew} < x_1 < x_{eq} < x_\nu$

$$x < x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \frac{x_1}{x} \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

Solution

$$\tilde{\phi}(x) = \begin{cases} \frac{x_S}{x_1} \left(1 - \sqrt{\frac{x_{ew}}{x}} \cos[2(\sqrt{xx_1} - \sqrt{x_{ew}x_1})] - \frac{1}{2\sqrt{xx_1}} \sin[2(\sqrt{xx_1} - \sqrt{x_{ew}x_1})] \right) \\ C_1 \frac{J_{1/4}(x)}{x^{1/4}} + C_2 \frac{Y_{1/4}(x)}{x^{1/4}} + \frac{x_S}{(2x)^{1/4}} \left(\frac{\pi}{\Gamma(\frac{3}{4})} J_{1/4}(x) G_1(x) - \frac{4\Gamma(\frac{3}{4})}{3} J_{-1/4}(x) G_2(x) \right) \end{cases}$$

(* $G_{1,2}(x) \rightarrow \text{constant for } x \rightarrow \infty$)

$$\frac{C_1}{x_S} = -\frac{\pi}{2^{1/4}\Gamma(\frac{3}{4})} G_1(x_1) + \frac{2^{5/4}\Gamma(\frac{3}{4})}{3} G_2(x_1) - \frac{\pi Y_{1/4}(x_1) \sin^2(x_1) - Y_{-3/4}(x_1)[x_1 - \frac{1}{2} \sin(2x_1)]}{x_1^{3/4}}$$

$$\frac{C_2}{x_S} = -\frac{2^{5/4}\Gamma(\frac{3}{4})}{6} G_2(x_1) + \frac{\pi J_{1/4}(x_1) \sin^2(x_1) - J_{-3/4}(x_1)[x_1 - \frac{1}{2} \sin(2x_1)]}{x_1^{3/4}}$$

Scalar DM condensed by ν

- $x_{ew} < x_1 < x_{eq} < x_\nu$

$$x < x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \frac{x_1}{x} \tilde{\phi}(x) = \frac{x_S}{x}$$

$$x > x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x} \tilde{\phi}'(x) + \tilde{\phi}(x) = \frac{x_S}{x}$$

$$\rho_\phi(x_{eq}) \approx \frac{c_\nu^2 x_S^2 m_\phi^2 M_P^2}{\pi x_{eq}^{3/2}} \text{ with } c_\nu^2 = C_1^2 + C_2^2$$

$$c_\nu^2 = \frac{\pi \Gamma\left(\frac{3}{4}\right)^2}{2\sqrt{2}} + \frac{2^{\frac{3}{4}} \pi^{\frac{3}{2}} C_1}{\Gamma\left(\frac{1}{4}\right) x_S} + \frac{C_1^2 + C_2^2}{x_S^2} \approx 3.34 \text{ for } x_1 \gg 1$$

$$y_\phi \approx 4.2 \cdot 10^{-20} m_{20}^{\frac{3}{4}} \left(\frac{0.05\text{eV}}{m_\nu}\right) \text{ with } m_{20} \gg 710 \left(\frac{0.05\text{eV}}{m_\nu}\right)^4$$

$$\frac{x_S}{x_1} \sim 4.8 \times 10^{-11} m_{20}^{-3/4}$$

$$\frac{x_1}{x_{eq}} \sim 8.8 m_{20}^{-1/2}$$

$$(*) \text{ BBN: } y_\phi \lesssim 7 \times 10^{-6} \quad m_\phi \lesssim 0.092 \text{ eV} \left(\frac{m_\nu}{0.05 \text{ eV}}\right)^{4/3}$$

Conclusion

- An ultralight scalar can form a coherent DM state through its tiny coupling to SM fermions.
- Considering the ϕqq or ϕll coupling, it works for wide ranges of $m_\phi \sim (10^{-22}, 10^6)\text{eV}$ and $y_\phi \sim (10^{-28}, 10^{-8})$.
- For the $\phi\nu\nu$ coupling, DM genesis requires $m_\phi \sim (10^{-17}, 10^{-1})\text{eV}$ and $y_\phi \sim (10^{-17}, 10^{-5})$, and can occur **very late** at around $T > 10T_{eq}$.