How to realize inflation and how to use inflation

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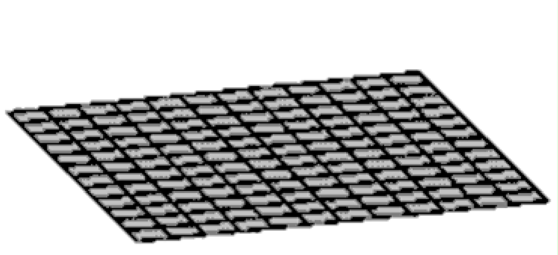
$$c = \hbar = M_G^2 = 1/(8\pi G) = 1$$

Contents

Introduction

Generic predictions of inflation, which is an accelerated expansion in the early Universe

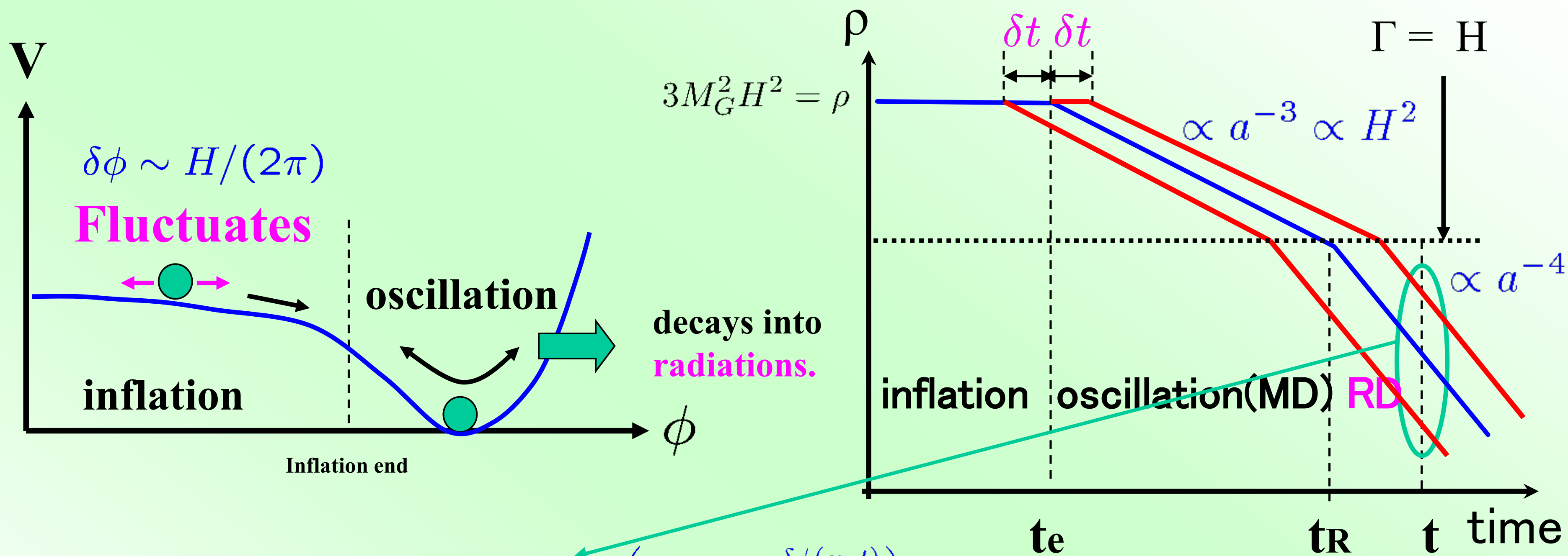
- **Spatially flat universe**



- **Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations**
- **Almost scale invariant and Gaussian primordial tensor fluctuations**

How to generate primordial fluctuations ?

Generation of primordial density fluctuations



Curvature perturbation — Density fluctuation $\left(\delta t(\boldsymbol{x}, t) = \dfrac{\delta\phi(\boldsymbol{x}, t)}{\dot{\phi}_{\rm cl}}\right)$

$$\zeta\Big|_{t_k} \sim \delta\rho/\rho\Big|_{t_k} \sim \delta t/t \sim (\delta\phi/\dot{\phi})/H^{-1} \sim H^2/\dot{\phi}\Big|_{t_*}$$

Almost scale invariant and Gaussian fluctuations are predicted.

(Time translational invariance) (Vacuum fluctuations of a non-interacting field are Gaussian)

The shift symmetry, $\phi \rightarrow \phi + \text{const}$, prohibits the non-linearity of an interaction (without derivatives).

Constraints on scalar and tensor perturbations from the PLANCK satellite

Observational constraints :

$$\begin{cases} \Delta_\zeta(k_0) &= 2.099^{+0.030}_{-0.029} \times 10^{-9}, \\ n_s &= 0.9649 \pm 0.0042, \\ r &< 0.10, \quad \text{(95\% CL TT,TE,EE+lowE+lensing)} \\ & \text{at} \quad k_0 = 0.002\text{Mpc}^{-1}. \end{cases}$$

Theoretical predictions :

$$\begin{cases} \Delta_\zeta(k) &\simeq \dfrac{1}{8\pi^2\epsilon}\left(\dfrac{H}{M_G}\right)^2, \\ n_s - 1 &= \dfrac{d\ln\Delta_\zeta(k)}{d\ln k} \simeq -2\epsilon - 2\eta, \\ \Delta_h(k) &\simeq \dfrac{2}{\pi^2}\left(\dfrac{H}{M_G}\right)^2, \quad n_T = \dfrac{d\ln\Delta_h(k)}{d\ln k} \simeq -2\epsilon, \\ r &\equiv \dfrac{\Delta_h(k)}{\Delta_\zeta(k)} \simeq 16\epsilon\,(= -8n_T)\,. \end{cases}$$

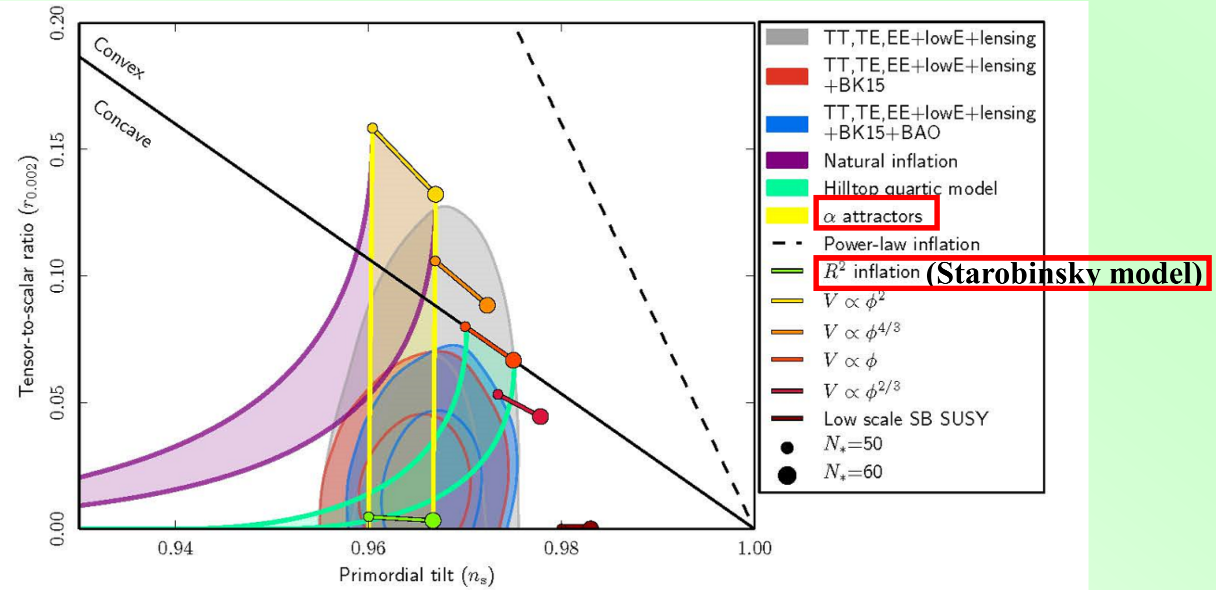


Fig. 8. Marginalized joint 68 % and 95 % CL regions for n_s and r at $k = 0.002\,\text{Mpc}^{-1}$ from *Planck* alone and in combination with BK15 or BK15+BAO data, compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions assume $dn_s/d\ln k = 0$.

Attractor models like Starobinsky model fit the data well.

Planck 2018 results. X 1807.06211

How to realize inflation

General strategies to realize inflation

- **Consider a diverging kinetic term (pole inflation) :**

(Galante et al., Broy et al.)

$$\mathcal{L}_{\mathsf{kin}} = \frac{1}{2}\left(\frac{a_p}{\rho^p} + \cdots\right) g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho$$

As ρ approaches 0, the kinetic term diverges.
After making the canonical normalization,
all of the coupling constants become effectively very weak.
➔ Asymptotically flat potential

- **Introduce a symmetry like the shift symmetry:** (Freese, Frieman, and Olinto, Kawasaki, MY, and Yanagida.)

$$\phi \longrightarrow \phi + C \quad (C \ : \ \mathsf{const})$$

An action depends only on a kinetic term.
➔ A potential becomes flat, or even without a potential.

α-attractor models (pole inflation)

R^2 (Starobinsky) model

(Starobinsky)

($M_G = 1$)

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + \frac{R^2}{12M^2}\right)$$

$$S = \int d^4x\sqrt{-g}f(R) \iff S_{\rm eq} = \int d^4x\sqrt{-g}\left(f(\phi) + \frac{df}{d\phi}(R-\phi)\right)$$

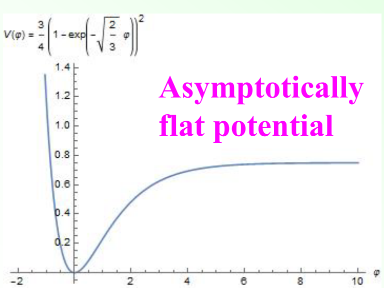
$$\frac{d^2f}{d\phi^2} \neq 0 \qquad \because \frac{\delta S_{\rm eq}}{\delta\phi} = \sqrt{-g}\,\frac{d^2f}{d^2\phi}(R-\phi) = 0.$$

$$S_{\rm eq} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(1+\frac{\phi}{3M^2}\right)R - \frac{\phi^2}{12M^2}\right]$$

Conformal transformation with $\tilde{g}_{\mu\nu} = \Omega(\phi)g_{\mu\nu}, \quad \Omega(\phi) = 1 + \frac{\phi}{3M^2}$

$$S_{\rm eq} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{12M^4}\frac{1}{\left(1+\frac{\phi}{3M^2}\right)^2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\phi^2}{12M^2\Omega^2}\right]$$

$$\left(\frac{d\varphi}{d\phi} = \frac{1}{\sqrt{6}M^2}\frac{1}{1+\phi/(3M^2)} \iff \phi = 3M^2\left(\exp^{\sqrt{2/3}\varphi} - 1\right)\right)$$



$$S_{\rm eq} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{3M^2}{4}\left(1-\exp^{-\sqrt{2/3}\varphi}\right)^2\right]$$

Conformal attractors

(Kallosh & Linde, Ferrara et al., Kallosh et al.)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{12}\left(\chi^2-\phi^2\right)R - \frac{1}{36}\left(\chi^2-\phi^2\right)^2 F\left(\frac{\phi}{\chi}\right)\right]$$

Local (gauge) conformal symmetry :

$$\tilde{g}_{\mu\nu} = e^{-2\sigma(x)}g_{\mu\nu}, \quad \tilde{\chi} = e^{\sigma(x)}\chi, \quad \tilde{\phi} = e^{\sigma(x)}\phi$$

(global SO(1,1) symmetry for constant F(φ/χ))

N.B. χ has wrong sign of kinetic term : compensator field

Gauge fixing with $\chi^2 - \phi^2 = 6$ **:** $\chi = \sqrt{6}\cosh\left(\frac{\varphi}{\sqrt{6}}\right), \quad \phi = \sqrt{6}\sinh\left(\frac{\varphi}{\sqrt{6}}\right)$

(Einstein frame)

$$\Rightarrow \quad S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{6}F\left(\tanh\frac{\varphi}{\sqrt{6}}\right)\right]$$

- **If F is constant, the potential is simply C.C.**
- **If F is smooth, the potential is stretched for large** φ
- **Starobinsky model** ⟺ $F\left(\frac{\phi}{\chi}\right) = \frac{3M^2}{(1+\chi/\phi)^2}$

Conformal attractors II

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{1}{12}\left(\chi^2-\phi^2\right)R-\frac{1}{36}F\left(\frac{\phi}{\chi}\right)\left(\chi^2-\phi^2\right)^2\right]$$

- **Gauge fixing with $\chi=\sqrt{6}$:**

(Jordan frame)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\left(1-\frac{\phi^2}{6}\right)R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{36}F\left(\frac{\phi}{\sqrt{6}}\right)\left(6-\phi^2\right)^2\right]$$

Conformal transformation with $\tilde{g}_{\mu\nu}=\Omega(\phi)g_{\mu\nu},\quad \Omega(\phi)=1-\frac{\phi^2}{6}$

(Einstein frame)

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R}-\frac{1}{2}\frac{1}{\left(1-\frac{\phi^2}{6}\right)^2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-F\left(\frac{\phi}{\sqrt{6}}\right)\right]$$

$$\left(\frac{d\varphi}{d\phi}=\frac{1}{1-\phi^2/6}\iff\frac{\phi}{\sqrt{6}}=\tanh\left(\frac{\varphi}{\sqrt{6}}\right)\right)\qquad\left(\phi\to\sqrt{6}\iff\varphi\to\infty\right)$$

Same action

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R}-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{6}F\left(\tanh\frac{\varphi}{\sqrt{6}}\right)\right]$$

The pole structure of the kinetic term stretch the potential effectively !!

- **α attractors :**

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}\frac{\alpha}{\left(1-\frac{\phi^2}{6}\right)^2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-F\left(\frac{\phi}{\sqrt{6}}\right)\right]$$

(Conformal attractors including Strobinsky model correspond to α= 1.)

Pole inflation

(Galante et al., Broy et al.)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}K(\rho)g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho-V(\rho)\right]$$

- **K(ρ) has a pole at ρ= 0 in Laurent series :**

$$K(\rho)=\frac{a_p}{\rho^p}+\cdots,$$

canonical field φ →

$$\rho\simeq\begin{cases}\rho_0 e^{-\frac{\varphi}{\sqrt{a_2}}} & \text{for } p=2\\ \left(\frac{(p-2)}{2\sqrt{a_p}}\varphi\right)^{-\frac{2}{p-2}} & \text{for } p>2\end{cases}$$

- **V(ρ) is regular at ρ= 0 :**

$$V(\rho)=V_0\left(1-\rho+\cdots\right)$$

→

$$V(\varphi)\simeq\begin{cases}V_0\left[1-\rho_0 e^{-\frac{\varphi}{\sqrt{a_2}}}\right] & \text{for } p=2\\ V_0\left[1-\left(\frac{(p-2)}{2\sqrt{a_p}}\varphi\right)^{-\frac{2}{p-2}}\right] & \text{for } p>2\end{cases}$$

(asymptotically flat)

Primordial perturbations :

$$\begin{cases}n_s-1\simeq-\frac{p}{p-1}\frac{1}{N},\\ r\simeq\frac{8}{a_p}\left[\frac{a_p}{(p-1)N}\right]^{\frac{p}{p-1}}.\end{cases}$$

α attractors ←→ p=2, a_2 = 3α/2

a_p dependence appears only in r.

Subleading terms yield higher order corrections.

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Observational constraints :

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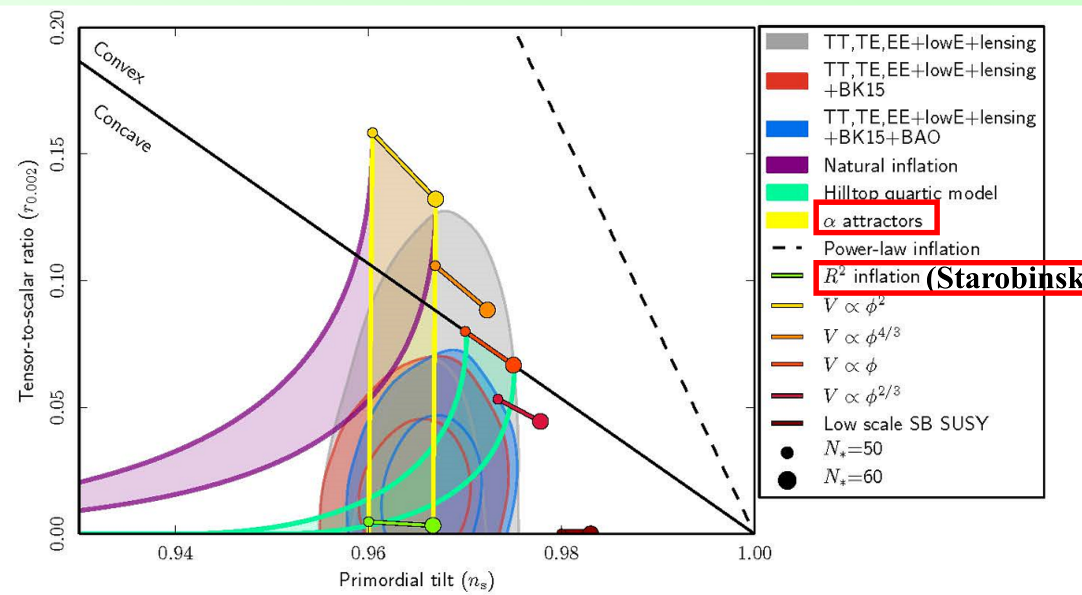

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What kind of physics is behind pole structure with the order 2 ???

Planck 2018 results. X 1807.06211

Pole structure of Higgs inflation

(Futamase & Maeda, Cervantes-Cota & Dehnen, Bezrukov & Shaposhnikov)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(1+\xi h^2\right)R - \frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h - \frac{\lambda}{4}\left(h^2-v^2\right)^2\right].$$

($\xi > 0$)

$$\tilde{g}_{\mu\nu} = \Omega(h)g_{\mu\nu}, \quad \Omega(h) = 1+\xi h^2.$$

$$\Rightarrow \quad \mathcal{L}_K^E = \left(\frac{3\Omega'^2}{2\Omega^2}+\frac{1}{\Omega}\right)(\partial h)^2 = \left(\frac{3}{2\Omega^2}+\frac{1}{\Omega\Omega'^2}\right)(\partial\Omega)^2 = \left(\frac{3}{2\rho^2}+\frac{\rho}{\rho'^2}\right)(\partial\rho)^2$$

$$\left(\rho = \frac{1}{\Omega} = \frac{1}{1+\xi h^2}\right)$$

$$\rho \to 0 \iff \Omega(h) \to \infty$$

$$\Rightarrow \quad K_E(\rho) = \left(\frac{3}{2\rho^2}+\frac{\rho}{\rho'^2}\right) = \frac{3}{2}\frac{1}{\rho^2} + \frac{1}{4\xi}\frac{1}{\rho^2(1-\rho)} = \frac{3\alpha}{2}\frac{1}{\rho^2} + \frac{1}{4\xi}\frac{1}{\rho} + \cdots.$$

Leading term coincides with α attractors !! $\left(\alpha = 1+\frac{1}{6\xi}\right)$

(Density perturbations ➔ $\xi \sim 10^4$ ➔ $\alpha \sim 1$)

Shift symmetry and extension

$$\phi \longrightarrow \phi + C \quad (C \text{ : const})$$

Shift symmetry and k-inflation

$$\phi \longrightarrow \phi + C \quad (C \;:\; \text{const})$$

$$\Longrightarrow \quad \mathcal{L}_\phi = K(X), \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

To end an inflation, slight φ dependence is necessary.

A kinetic term of an inflaton is not necessarily canonical.

$$\mathcal{L} = X - V(\phi), \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \quad \Longrightarrow \quad \mathcal{L} = K(\phi, X)$$

(k-inflation)

(Armendariz-Picon et.al. 1999)

In fact, an action may include even higher derivatives.

(Nicolis et.al. 2009)

$$\mathcal{L} = K(\phi, X) \quad \Longrightarrow \quad \Delta\mathcal{L} = G(\phi, X) \Box \phi$$

Galileon field

Nicolis et al. 2009
Deffayet et al. 2009

The theory has Galilean shift symmetry in flat space :

$$\partial_\mu \phi \longrightarrow \partial_\mu \phi + b_\mu$$

$$\begin{cases} \mathcal{L}_1 = \phi \\ \mathcal{L}_2 = (\partial\phi)^2 \\ \mathcal{L}_3 = (\partial\phi)^2 \, \Box\phi \\ \mathcal{L}_4 = (\partial\phi)^2 \left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2 \right] \\ \mathcal{L}_5 = (\partial\phi)^2 \left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3 \right] \end{cases}$$

$$(\partial_\mu\partial_\nu\phi)^2 = \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi,$$
$$(\partial_\mu\partial_\nu\phi)^3 = \partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi$$

Lagrangian has second order derivatives, but EOM is second order.

Why do we consider higher derivative terms ???

It is impossible to break null energy condition stably within k-inflation.

That is, primordial tensor perturbations (GW) have always red spectrum.
The equation of state, $w = p / \rho$, of dark energy is always larger than -1.

Null energy condition (NEC)

$$T_{\mu\nu}\xi^\mu\xi^\nu \geq 0$$ **for any null vector ξ^μ.**

$$(g_{\mu\nu}\xi^\mu\xi^\nu = 0)$$

This is the weakest among all of the local classical energy conditions.

For a perfect fluid : $T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + g_{\mu\nu} p$

⇨ **NEC** ⇔ $\rho + p \geq 0$ ⇔ **$w \geqq -1$**

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$$

⇨ $\dot{\rho} = -3H(\rho + p).$

As long as $\rho + p \geq 0$ **(and H > 0 for an expanding Universe like inflation)**

⇨ $\dot{\rho} \leq 0.$

How robust is the NEC ?

- **Canonical kinetic term with potential:**

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi).$$

$$\Rightarrow \begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases} \Rightarrow \rho + p = \dot{\phi}^2 \geq 0.$$

(NEC is conserved)

- **How about k-inflation ?** **(Armendariz-Picon, Damour, Mukhanov 1999)**

$$\mathcal{L} = K(\phi, X), \qquad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.$$

$$\Rightarrow \begin{cases} \rho = 2XK_X - X \\ p = K \end{cases} \Rightarrow \rho + p = 2XK_X.$$

$$(K_X \equiv \partial K / \partial X)$$

Apparently, it looks that, if $K_X < 0$, it can violate the NEC.
But, this is not the case.

Primordial density fluctuations

Garriga & Mukhanov 1999

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2 e^{2\zeta} d\boldsymbol{x}^2$$

Comoving gauge :

$$\phi = \phi(t), \quad \delta\phi = 0.$$

Prescription:

- **Expand the action up to the second order**
- **Eliminate α and β by use of the constraint equations**
- **Obtain quadratic action for ζ**

$$\Rightarrow \quad S_S^{(2)} = \int dt d^3x\, a^3\, M_G^2 \frac{\epsilon}{c_s^2}\left(\dot{\zeta}^2 - \frac{c_s^2}{a^2}\zeta_{,k}\zeta_{,k}\right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{XK_X}{M_{\rm pl}^2 H^2}, \qquad c_s^2 = \frac{K_X}{K_X + 2XK_{XX}}$$

(sound velocities of curvature perturbations)

In order to avoid the ghost and gradient instabilities, ε > 0 & c_s^2 > 0.

(Hsu et al. 2004)

(See also Dubovsky et al. 2006)

$$\Rightarrow \quad \rho + p = 2XK_X > 0.$$

Stable violation of NEC is impossible within k-inflation

It is impossible to break the NEC stably within k-inflation.

- **Background solutions can break NEC apparently.**
- **But, the perturbations around them always become unstable for such background solutions.**

This is quite reasonable in some sense because violation of NEC must pay some price. (see Sawicki & Vikman 2013, Easson, Sawicki, Vikman 2013)
e.g. An observer with almost speed of light observes arbitrary negative energy.

N.B. k-inflation is the most general action coming from phi and its first derivatives. $\left(\mathcal{L} = K(\phi, X), \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.\right)$

→ **One may wonder how about introducing higher order derivative terms.**

Galilean Genesis

(Creminelli et al.,
Nicolis et al.,
Kobayashi et al. G-inflation)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}M_G^2R+f^2e^{2\phi}(\partial\phi)^2+\frac{f^3}{\Lambda^3}(\partial\phi)^2\Box\phi+\frac{f^3}{2\Lambda^3}(\partial\phi)^4\right]$$

(In the flat spacetime limit, this theory has conformal symmetry SO(4,2).)

- **Energy-momentum tensor :**

$$\begin{cases}\rho &= -f^2\left(e^{2\phi}\dot\phi^2-\frac{3}{2}\frac{f}{\Lambda^3}\dot\phi^4-6H\frac{f}{\Lambda^3}\dot\phi^3\right),\\ p &= -f^2\left(e^{2\phi}\dot\phi^2-\frac{1}{2}\frac{f}{\Lambda^3}\dot\phi^4+2\frac{f}{\Lambda^3}\dot\phi^2\ddot\phi\right).\end{cases}$$

- **A background solution, (t : -∞ -> 0) :** **Starts from Minkowski in infinite past.**

$$e^{\phi}\simeq\frac{1}{\sqrt{2Y_0}}\frac{1}{(-t)},\qquad H\simeq\frac{h_0}{(-t)^3},\qquad\left(a(t)\simeq 1+\frac{h_0}{2(-t)^2}\right).$$

$$\left(Y_0\equiv\frac{\Lambda^3}{3f},\qquad h_0\equiv\frac{1}{2M_G^2}\frac{f^3}{\Lambda^3}\right)$$

$$\Rightarrow\quad \rho+p\simeq-\frac{f^3}{\Lambda^3}\frac{4}{(-t)^4}<0.$$

(Actually, you can verify that H increases.)

(The NEC is violated !!)

Primordial density fluctuations

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2 e^{2\zeta} d\boldsymbol{x}^2$$

Comoving gauge :

$$\phi = \phi(t), \quad \delta\phi = 0.$$

$$\Rightarrow \quad S_S^{(2)} = \int dt d^3x\, a^3 \left(\mathcal{G}_s \dot{\zeta}^2 - \frac{\mathcal{F}_s}{a^2} \zeta_{,k}\zeta_{,k} \right)$$

In order to avoid the ghost and gradient instabilities, Gs > 0 & Fs > 0.

$$\mathcal{G}_s = \mathcal{F}_s \simeq 6 M_G^4 \frac{\lambda^3}{f^3}(-t)^2 > 0.$$

(The NEC is violated stably !!)

Higher order derivative terms open a new window with the safe violation of NEC (blue spectrum of GW !!)

General strategies to realize inflation

- **Consider a diverging kinetic term (pole inflation) :**

(Galante et al., Broy et al.)

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\left(\frac{a_p}{\rho^p} + \cdots\right) g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho$$

- **Introduce a symmetry like the shift symmetry:** (Freese, Frieman, Olinto, Kawasaki, MY, Yanagida.)

$$\phi \longrightarrow \phi + C \quad (C\ :\ \text{const})$$

- **A pole structure of a kinetic term or A symmetry like the shift symmetry or is a key idea to realize inflation.**
- **Is there yet another key idea to realize inflation naturally ?**

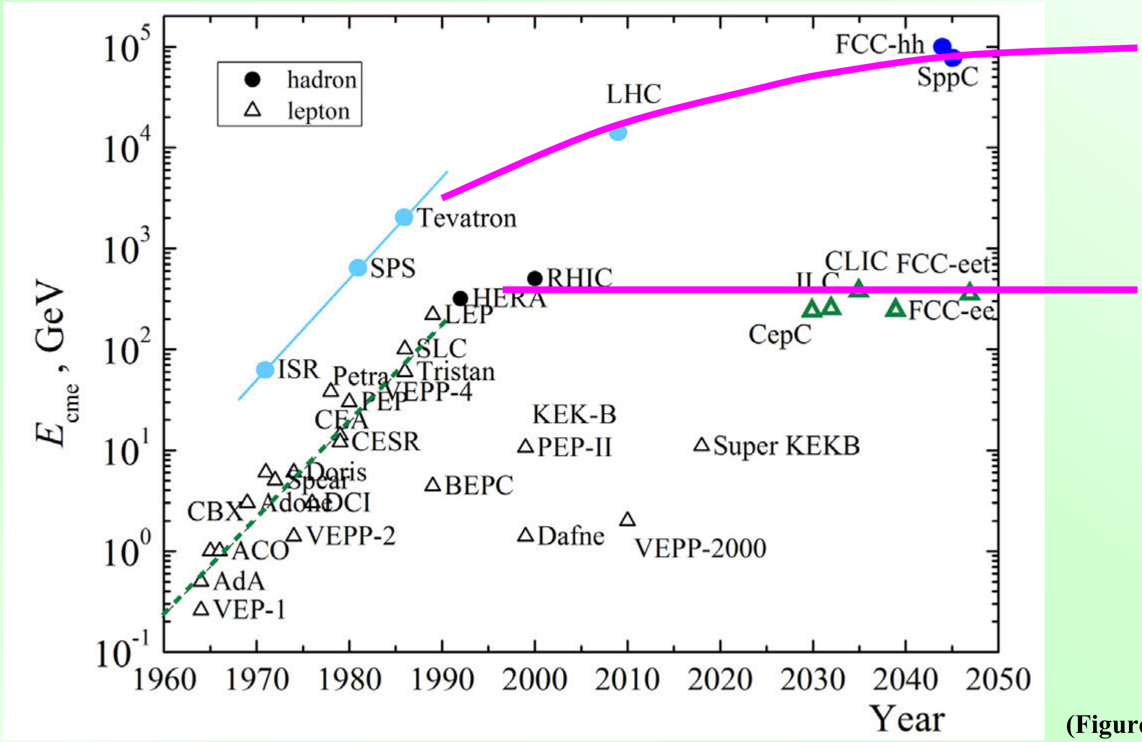
Anyway, inflation is now strongly supported observationally.

↓

How to use inflation ?

Cosmology offers alternative collider

History of colliders on earth



saturate

saturate

(Figure taken from Gray 2021)

The energy scale of colliders on earth is going to saturate and then we need alternative.

Cosmology would be the unique place for alternative collider.

A higher energy state is realized in early universe, in particular, during inflation, which can excite particles !!

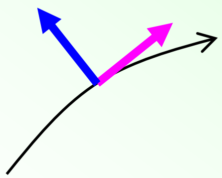
Excitations during inflation

- **Single field inflation :**

  Scalar field φ ⇐==⇒ curvature perturbation ζ
  ($m_\varphi \ll H$) (adiabatic mode)

- **Multiple field inflation :**

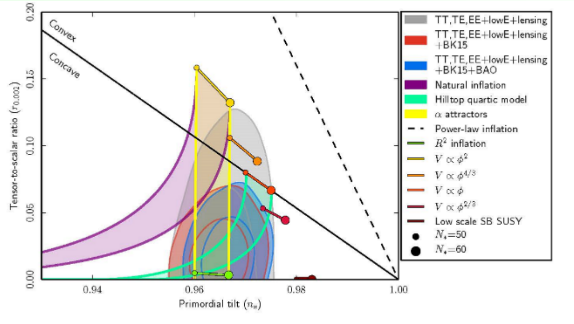
  Scalar fields φ_i ⇐==⇒ curvature perturbation ζ
  ($m_{\varphi i} \ll H$) (adiabatic mode)
  + isocurvature perturbations S_{ij}



If they are related to ⇕ baryon number, DM abundance, …

Severely constrained from Planck

- Up to now, we have mainly paid attention to **powerspectrum (two-point correlation functions)** to identify e.g. an inflaton.



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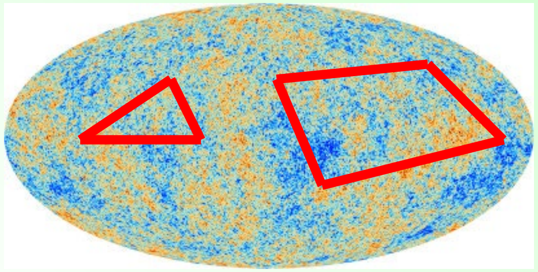
- Why we don't go into non-Gaussianities (connected **higher order correlation functions) like bispectrum** to obtain additional information ?



Soft limits of correlation functions

Soft limits

Figures taken from Assassi et al. 2012

- **squeezed limit :**

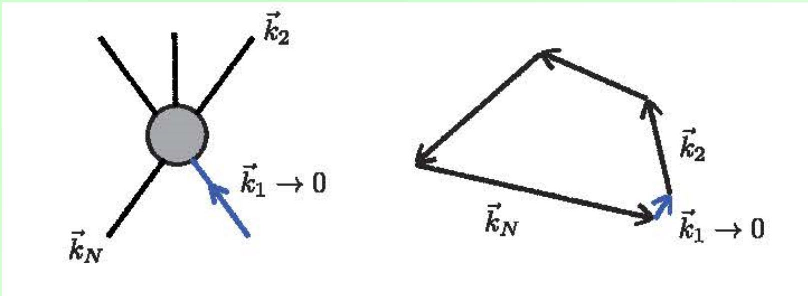
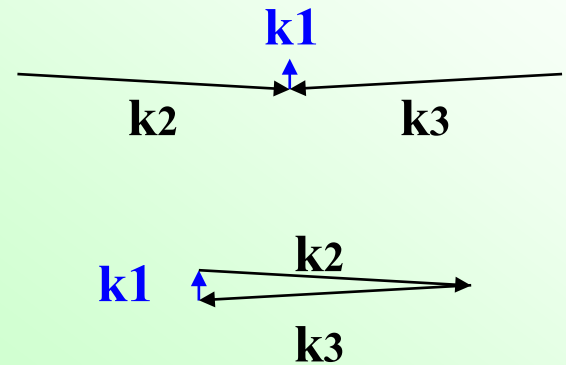


Figure 1. Soft external momentum or the 'squeezed limit'.



e.g. $$\lim_{k_1 \to 0} \langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \delta^3 (\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)(1 - n_s) P_{k_1} P_{k_3}.$$

(Maldacena 2003, Creminelli & Zaldarriaga 2004)

- **collapsed limit :**

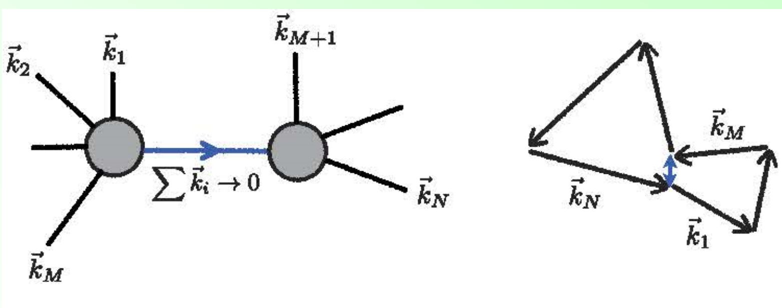
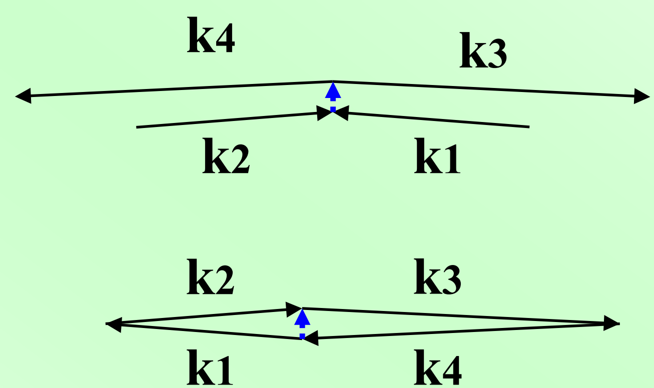


Figure 2. Soft internal momentum or the 'collapsed limit'.



e.g. So-called Suyama-Yamaguchi inequality, $\tau_{NL} \geq \left(\frac{6}{5} f_{NL}\right)^2$ **(Suyama & MY 2008)**

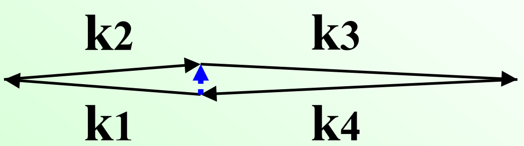
Example of collapsed limit

Suyama & MY 2008

$$\tau_{NL} \geq \left(\frac{6}{5} f_{NL}\right)^2$$

Information of **four-point functions** (for τ_{NL})

Information of **three-point functions** (for f_{NL})



$$\begin{cases} f_{NL} \equiv \dfrac{5}{12} \lim\limits_{k_1 \to 0} \dfrac{\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle}{P(k_1)P(k_2)} \\ \tau_{NL} \equiv \dfrac{1}{4} \lim\limits_{|\boldsymbol{k}_1 - \boldsymbol{k}_2| \to 0} \dfrac{\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \zeta_{\boldsymbol{k}_4} \rangle}{P(k_1)P(k_3)P(|\boldsymbol{k}_1 - \boldsymbol{k}_2|)}. \end{cases}$$

① This relation becomes **equality** **only when single field** contributes to primordial density (curvature) perturbations.

② This relation becomes **strict inequality** **when multiple fields** contribute.

③ If **this relation** would be shown to be **violated** from future observations, it suggests that **this generation mechanism itself might be wrong**.
(It can serve as **the test of an inflationary scenario**.)

→ **Discriminates single or multiple sources.**

Excitations during inflation

- **Single field inflation :**

  Scalar field φ ⟸==⟹ curvature perturbation ζ
  ($m_{\varphi} \ll H$) (adiabatic mode)

- **Multiple field inflation :**

  Scalars field φi ⟸==⟹ curvature perturbation ζ
  ($m_{\varphi i} \ll H$) (adiabatic mode)
  + isocurvature perturbations S_{ij}

There is another interesting class of inflation (excitation) !!

Quasi-single field inflation

Excitations during inflation II

- **Single field inflation :**

Scalar field φ ⟸==⟹ curvature perturbation ζ
($m_\varphi \ll H$) **(adiabatic mode)**

- **Multiple field inflation :**

Scalar fields φ_i ⟸==⟹ curvature perturbation ζ
($m_{\varphi i} \ll H$) **(adiabatic mode)**
+ isocurvature perturbations S_{ij}

- **Quasi-single field inflation :**

Scalar field φ ⟸==⟹ curvature perturbation ζ
($m_\varphi \ll H$) **(adiabatic mode)**
Scalar field σ **+ isocurvature perturbation**
($m_\sigma \sim H$) **with $m \sim H$**

Natural Hubble mass in supergravity

In supergravity,

$V \neq 0$ → SUSY breaking

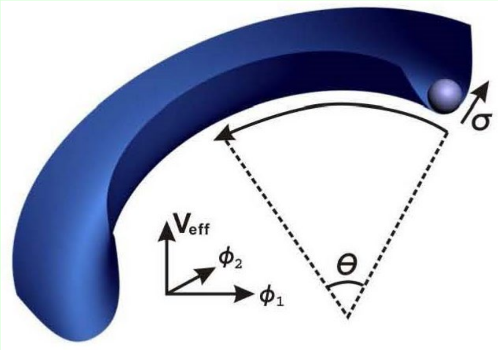
(inflation)

(mediated by gravity) ↓

soft breaking masses : $m^2 \sim V\,G \sim H^2$

In supergravity, a situation naturally happens, in which there is only one light field and the masses of other fields are comparable to the Hubble parameter.

(Note also that a non-minimal coupling $R\varphi^2$ and dimension 6 operators easily lead to $m \sim H$)



(Credit: Chen & Wang)

This model is called quasi-single field inflation

(Chen & Wang 2009)

(Heavy particles inaccessible to colliders on earth can be excited during inflation !!)

Squeezed limits

- **Single field inflation :**

  Scalar field φ

  ($m_\varphi \ll H$)

$$\lim_{k_1 \to 0} \langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle' \propto \frac{1}{k_1}$$

- **Multiple field inflation :**

  Scalar fields φi

  ($m_{\varphi i} \ll H$)

$$\lim_{k_1 \to 0} \langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle' \propto \frac{1}{k_1^3}$$

$$\left(\zeta(\boldsymbol{x}) = \zeta_G(\boldsymbol{x}) + \frac{3}{5} f_{NL} \zeta_G^2(\boldsymbol{x}) \quad \rightarrow \quad \lim_{k_1 \to 0} \langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle' \propto f_{NL} P_\zeta(k_1) \right)$$

- **Quasi-single field inflation :**

  Scalar field φ

  ($m_\varphi \ll H$)

  Scalar fields σi

  ($m_{\sigma i} \sim H$)

$$\lim_{k_1 \to 0} \langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle' \propto \frac{1}{k_1^\alpha}$$

$$\left(\alpha = \frac{3}{2} + i \sqrt{\frac{m_\sigma^2}{H^2} - \frac{9}{4}} \right)$$

$m_\sigma / H > 3/2$ => oscillation

Squeezed limit of bispectrum

The presence of a new single particle of mass m > 3H/2 and spin s leads to

$$\lim_{k_1\to 0}\frac{\langle\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle'}{P_1P_2}\sim\epsilon e^{-\pi\mu}|c(\mu)|\left[e^{i\delta(\mu)}\left(\frac{k_1}{k_2}\right)^{\frac{3}{2}+i\mu}+e^{-i\delta(\mu)}\left(\frac{k_1}{k_2}\right)^{\frac{3}{2}-i\mu}\right]P_s(\cos\theta)$$

(Noumi, MY, Yokoyama 2013, Arkani-Hamed & Maldacena 2015)

$$\mu=\sqrt{\frac{m^2}{H^2}-\frac{9}{4}}$$

angle between **k**1 & **k**2

N.B.

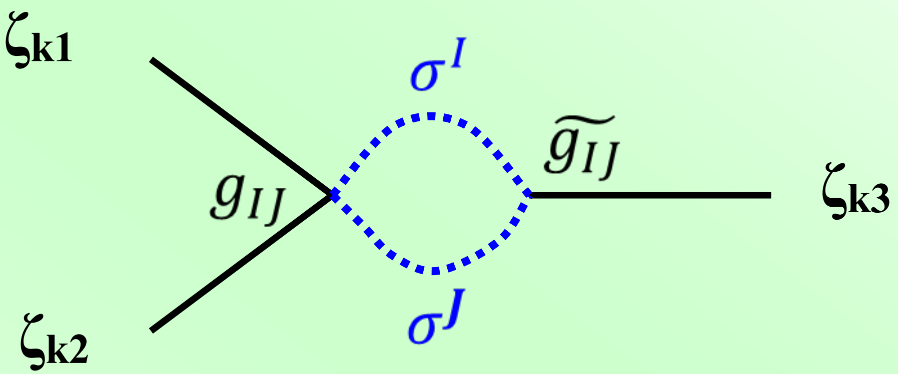
- **The phase δ depends only on mass m.**
- **The oscillatory behavior represents the quantum interference between the (usual) inflaton fluctuation and the decay effect of a massive particle, which is pair created and subsequently decays into the inflaton.**
- **3/2 can be understood as the square of the wavefunction decays as exp(-3N) ~ 1/Volume with N = log(k_2/k_1).**
- **exp(- πμ) is the suppression factor representing the interference effect for creating a pair of massive particles.**

What happens if there are multiple heavy particles, which often would appear ??? → **Shuntaro Aoki and MY 2020**

Multiple isocurvatons σ^I with m ~ H

(Aoki & MY 2020,
Pinol, Aoki, Renaux-Petel, MY 2021)

e.g.



$$\left(\subset S_{\rm int}(\phi,\sigma) = \int d^4x \mathcal{L}_{\rm int} = -\int d^4x\sqrt{-g}f(\phi)c_{IJ}\sigma^I\sigma^J\right)$$

- σ^I (I=1, …, n) : massive isocurvatons
- g_{IJ}, $\widetilde{g_{IJ}}$: couplings, non-diagonal in general
- ($g_{IJ} = \widetilde{g_{IJ}}$ after the normalization by H for simplicity)

Squeezed limits of bispectra

$$\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle' = (2\pi)^4 \mathcal{P}_\zeta^2 \frac{1}{(k_1k_2k_3)^2} S(k_1,k_2,k_3), \qquad S = \sum_{I,J} S_{IJ}$$

- **I = J**

$$S_{II} \propto g_{II}^2 C(\mu_I,\mu_I)\left(\frac{k_l}{k_s}\right)^{2i\mu_I} + \text{const.} + \text{c.c.}$$

$$C(\mu_I,\mu_I) \propto e^{-2\pi\mu_I^2}$$

(Boltzman suppression factor)

$2i\mu_I$ → **High frequency**

$$\mu^I \equiv \sqrt{\left(\frac{m^I}{H}\right)^2 - \frac{9}{4}}$$

- **I≠J**

$$S_{IJ} \propto g_{IJ}^2 C(\mu_I,\mu_J)\left(\frac{k_l}{k_s}\right)^{i(\mu_I+\mu_J)} + g_{IJ}^2 C(\mu_I,-\mu_J)\left(\frac{k_l}{k_s}\right)^{i(\mu_I-\mu_J)} + \text{c.c.}$$

mixing (new effect)

$i(\mu_I+\mu_J)$ → **High frequency**

$i(\mu_I-\mu_J)$ → **low frequency (modulation)**

- **easily distinguished**
- **specific to multi particles**

Two field case with mixing ($g_{11} = g_{22} = g_{12}$)

$$S_{12} \propto g_{12}^2 \underline{C(\mu_1, \mu_2)} \left(\frac{k_l}{k_s}\right)^{\boxed{i(\mu_1+\mu_2)}} + g_{12}^2 \underline{C(\mu_1, -\mu_2)} \left(\frac{k_l}{k_s}\right)^{\boxed{i(\mu_1-\mu_2)}} + \text{c.c.}$$

High frequency

low frequency (modulation)

In degenerate limit ($\mu_1 \sim \mu_2 = \mu$),

$$\left|\underline{C(\mu_1, \mu_2)} / \underline{C(\mu_1, -\mu_2)}\right| \sim 2 \times 10^{-2} \times \mu^{-5/2} \ll 1$$

The total signal

~ characterized by the low frequency mode (large wavelength)

Two field case with mixing ($g_{11} = g_{22} = g_{12}$) II

$S = S_{11} + S_{22} + S_{12}$

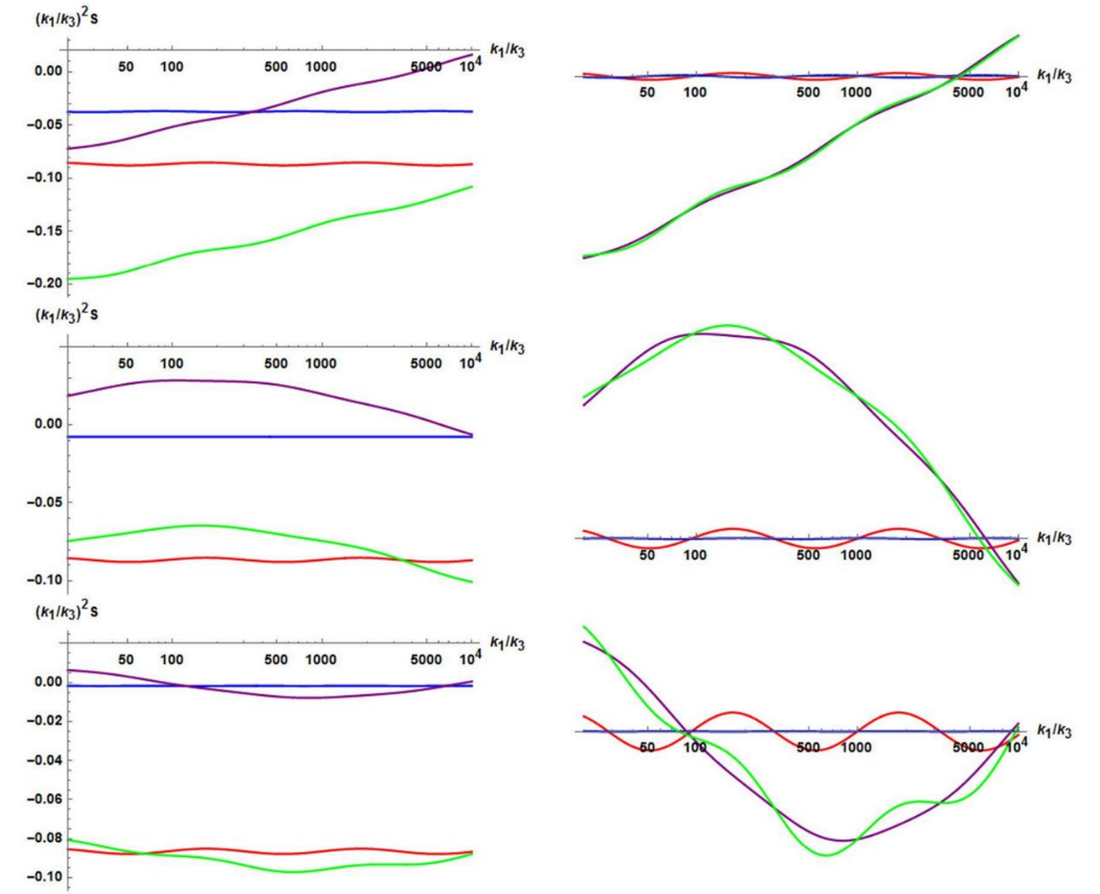


Figure 4. The same figure with figure 2 but the mixing term S_{12} included (purple line). We set $(m_1, m_2)/H = (2, 2.1)$, $(m_1, m_2)/H = (2, 2.3)$, and $(m_1, m_2)/H = (2, 2.5)$ from top to bottom. The couplings are taken universally, $g_{IJ} = \tilde{g}_{IJ} = 1$ for $I, J = 1, 2$. The right figures show that the waveforms (momentum dependence) of the total signal are mainly determined by the mixing term S_{12}.

The waveform is mainly determined by S12 !!

Small modulations on the large waveform

↓

Easily disentangle the mass spectra

Summary

- A pole structure of a kinetic term or a symmetry like the shift symmetry is a key idea to realize inflation.
- Is there yet another key idea to realize inflation naturally ?
- Theories with higher order derivatives can open a new possibility of the predictions of inflation.
- Cosmology, in particular, inflation offers colliders alternative to those on earth.
- Many particles including heavy ones can be excited during inflation and can be probed through primordial perturbations.
- We need to prepare theoretical predictions for future observations.