



Dark matter filtering-out effect during a first-order phase transition

Chang Sub Shin (Chungnam National University)

*based on 1912.04238 (Phys.Rev.D101,095019(2020)) done with
Dongjin Chway (IBS), Tae Hyun Jung (Florida State U.)*

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Outline

Heavy Dark Matter Scenarios

Basic Idea of Filtering-Out Mechanism

Realization

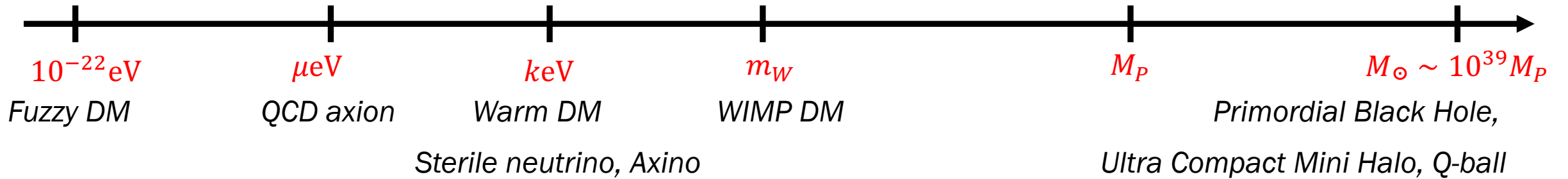
Implications for Model Building

Heavy Dark Matter Scenarios

Dark Matter Landscape - Mass

The nature of dark matter is still shrouded in mystery

$$\rho_{\text{DM}} \simeq 1.2 \times 10^{-6} \text{ GeV/cm}^3 = m_{\text{DM}} n_{\text{DM}}$$



Wave-like ($n_{\text{DM}} \gg m_{\text{DM}}^3$)

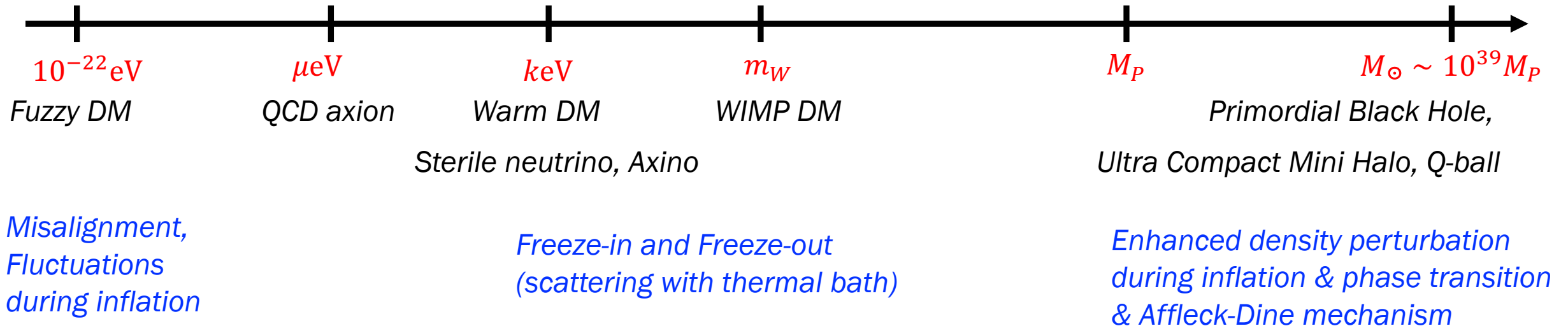
Particle-like (boson, fermion)

Macroscopic objects

Dark Matter Landscape - Production Mechanism

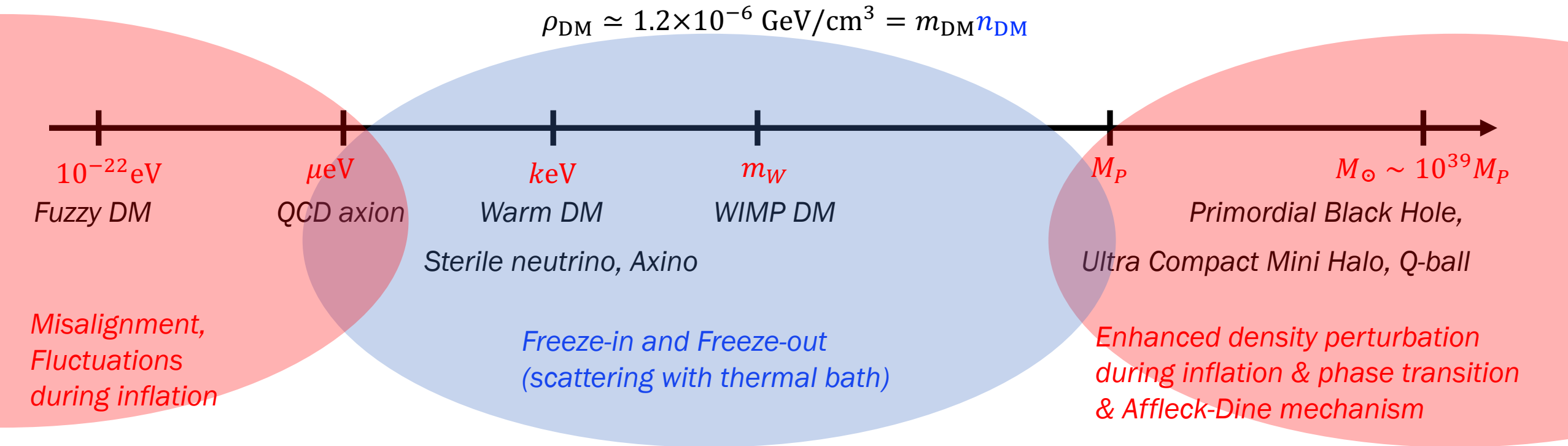
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Dark Matter Landscape - Production Mechanism

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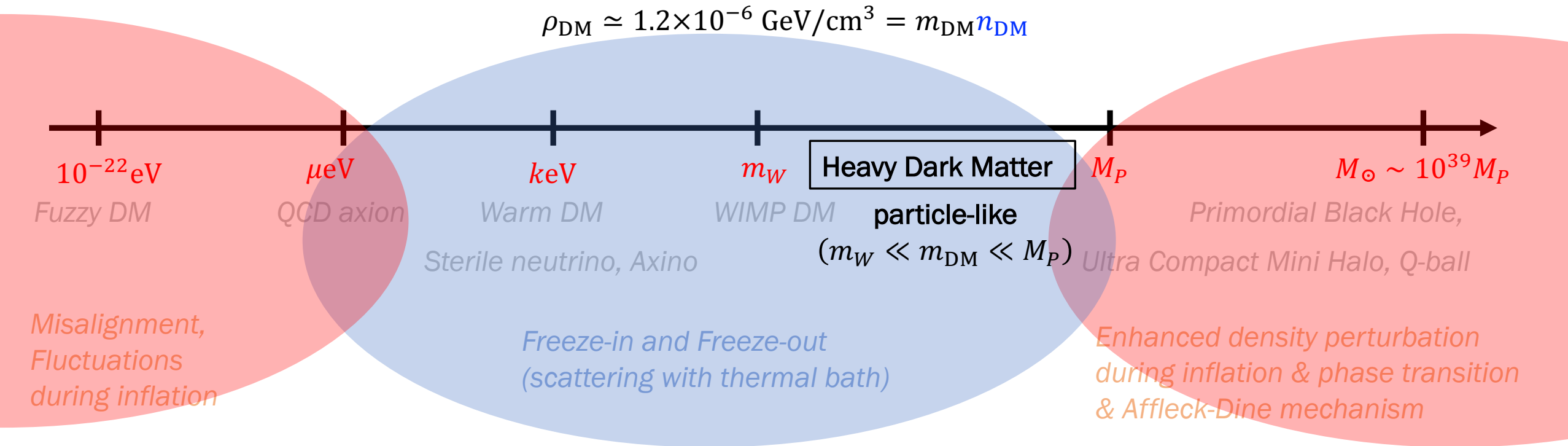


Interactions between DM and thermal bath are more crucial for DM production

Gravitational interactions are more crucial for DM production

Dark Matter Landscape - Production Mechanism

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Gravitational interactions are more crucial for DM production

Unitarity Bound for Thermal Heavy DM

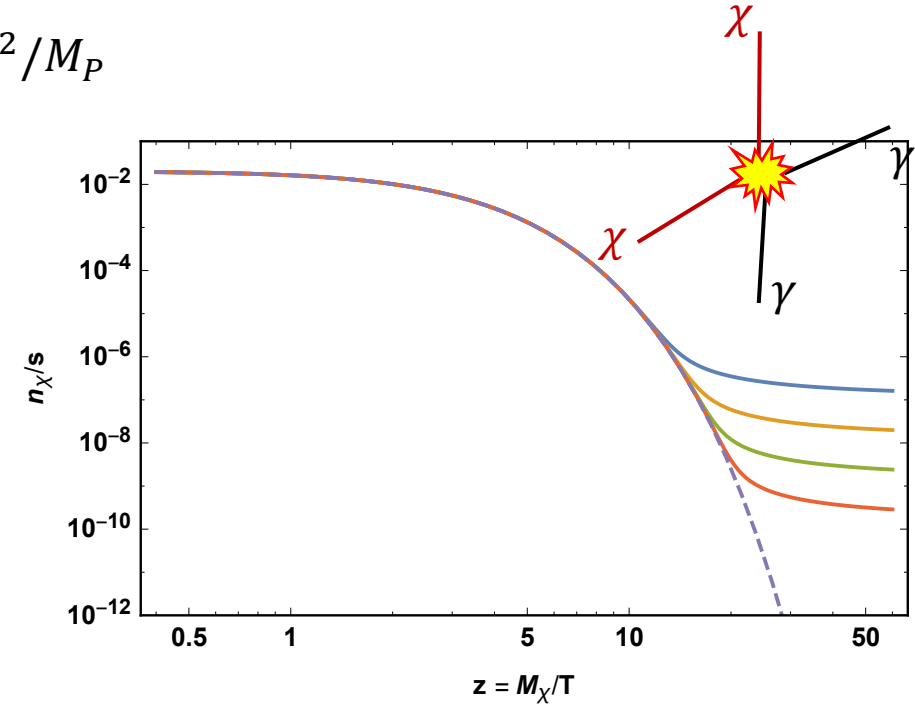
DM annihilation cross-section becomes *cosmologically ineffective* when the ann. rate becomes smaller than the Hubble rate.

Freeze-out happens at around T_{fo} which gives $\Gamma_{\text{ann}} = \langle \sigma_{\text{ann}} v \rangle_T n_\chi = H(T) \sim T^2/M_P$

$$\frac{z^2}{M_P} \frac{d}{dz} \left(\frac{n_\chi}{s} \right) \simeq \langle \sigma_{\text{ann}} v \rangle_T M_\chi \left(\left(\frac{n_\chi^{\text{eq}}}{s} \right)^2 - \left(\frac{n_\chi}{s} \right)^2 \right)$$

$$\Omega_\chi h^2 = 2.8 \times 10^8 \left(\frac{M_\chi}{\text{GeV}} \right) \left(\frac{n_\chi}{s} \right)_{T_{\text{fo}}} \simeq 0.11 \left(\frac{M_\chi/T_{\text{fo}}}{20} \right) \left(\frac{10^{-10} \text{GeV}^{-2}}{\langle \sigma_{\text{ann}} v \rangle_{T_{\text{fo}}}} \right)$$

$$\simeq 0.11 \left(\frac{M_\chi}{100 \text{ GeV}} \right) \left(\frac{(M_\chi/T_{\text{fo}})^{3/2} e^{-M_\chi/T_{\text{fo}}}}{e^{-21.6}} \right)$$



Unitarity Bound for Thermal Heavy DM

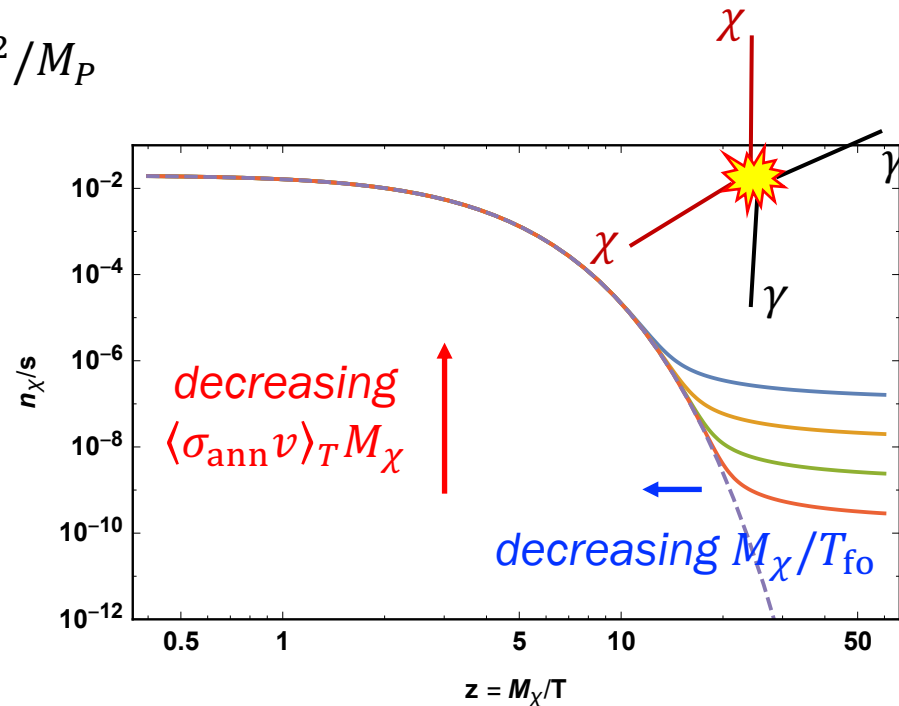
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Unitarity bound for (point-like) DM annihilation process

[Griest, Kamionkowski PRL64 (615) 1990] $(\sigma v_{rel})_{total}^J < (\sigma v_{rel})_{max}^J = \frac{4\pi(2J+1)}{M_\chi^2 v_{rel}} \sim \frac{1}{M_\chi^2} \left(\frac{M_\chi}{T} \right)^{1/2}$

means that generically $\langle \sigma_{ann} v \rangle_T M_\chi \lesssim 1/M_\chi \rightarrow \left(\frac{n_\chi}{s} \right)_{T_{fo}}$ increases as M_χ increases.

For thermal DM with $M_\chi \gg O(100 \text{ TeV})$, $M_\chi/T_{fo} \sim O(10)$ and $\Omega_\chi h^2 \gg O(1) \rightarrow$ **Thermal heavy DM is not favored**

(c.f. there are several references which include Sommerfeld effect and bound state effect like [Smirnov, Beacom 1904.11503])

Personal Motivations for Thermal Heavy DM

* As a compelling source of unidentified high energy cosmic rays: *Decaying DM*

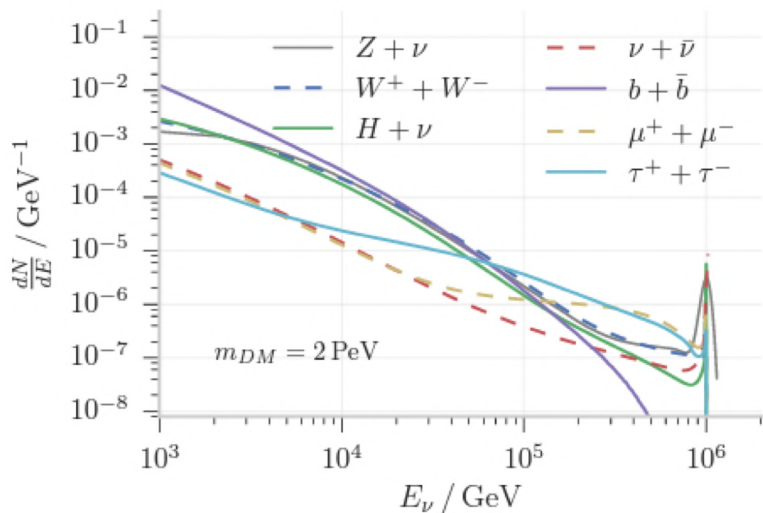


Fig. 1 Neutrino yield per decay as a function of neutrino energy (flavour-averaged): All considered decay channels (BR = 100%) are presented for an assumed dark matter mass of 2 PeV.

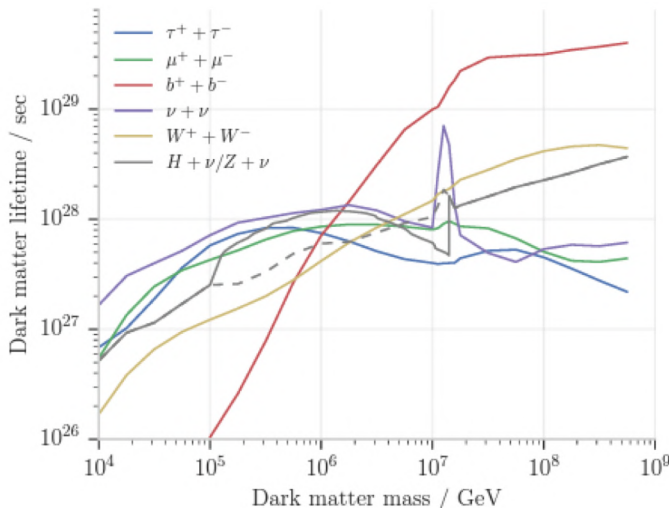
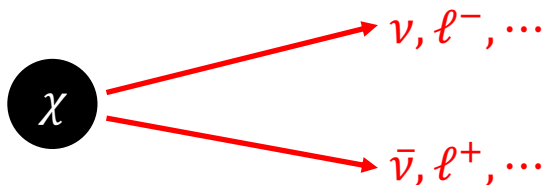


Fig. 7 Dark matter lifetime limits for all considered decay channels. For the $Z + \nu/H + \nu$ channel, the limit was combined (solid grey line) as described in the text. Between $m_{DM} \sim 10^5$ GeV and $m_{DM} = 1.5 \times 10^7$ GeV the limit is obtained from the more sensitive track analysis. The limit from the cascade analysis is shown as a dashed line and turns out to be stronger above $m_{DM} \sim 5 \times 10^7$ GeV.

[IceCube Collaboration, 1804.03848]

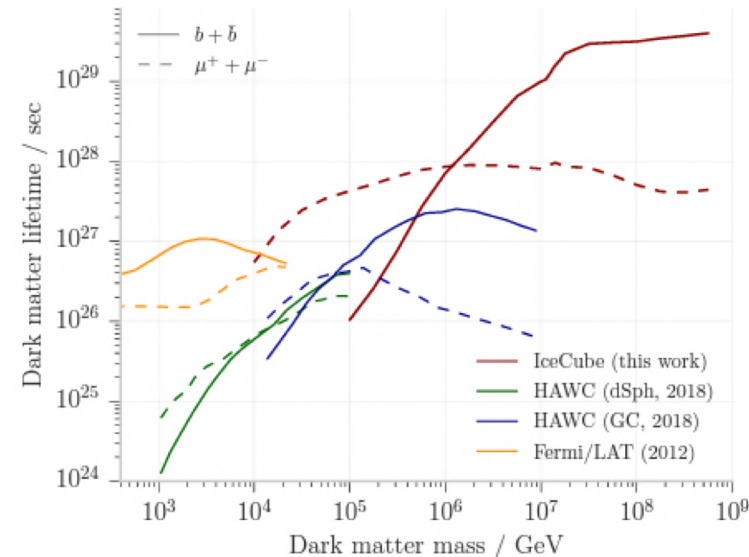
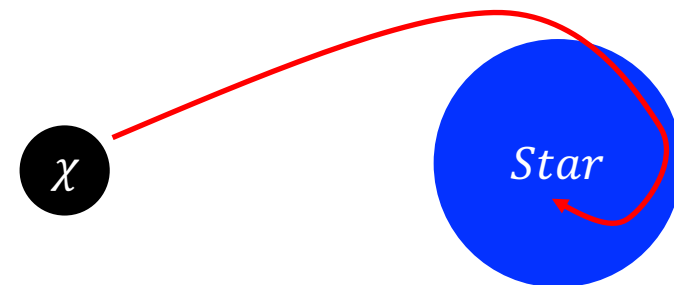


Fig. 8 Comparison of the lower lifetime limits with results obtained from gamma-ray telescopes: HAWC (Dwarf Spheroidal Galaxies) [44], HAWC (Galactic Halo/Center) [45] and Fermi/LAT [47].



* *Even if DM is absolutely stable, an interesting effect on the star evolution could exist when there is a sizable interaction between DM and the star*

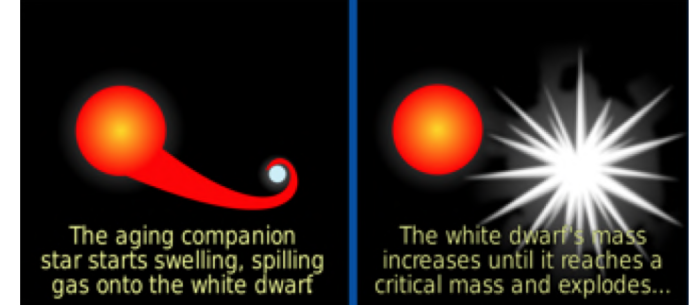
Personal Motivations for Thermal Heavy DM

* An interesting role for the star evolution with a sizable scattering cross-section between heavy DM and stars

[1904.11993]

Supernovae Sparked By Dark Matter in White Dwarfs

Javier F. Acevedo^δ and Joseph Bramante^{δ,†}



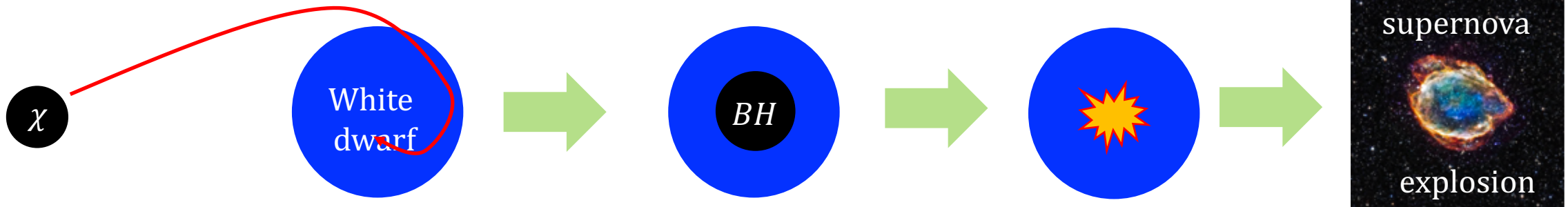
Standard Type Ia SN explosion

find that for dark matter more massive than 10^{11} GeV, Type Ia supernova ignition can proceed through the Hawking evaporation of a small black hole formed by the collapsed dark matter.

Accumulated DM by thermalization \rightarrow form a mini blackhole inside the white dwarf

\rightarrow Hawking evaporation with $T_{BH} \sim 6 \text{ TeV} (M_\chi/10^{13} \text{ GeV})^{3/2}$

\rightarrow Spark the fusion reaction for Type Ia SN explosion

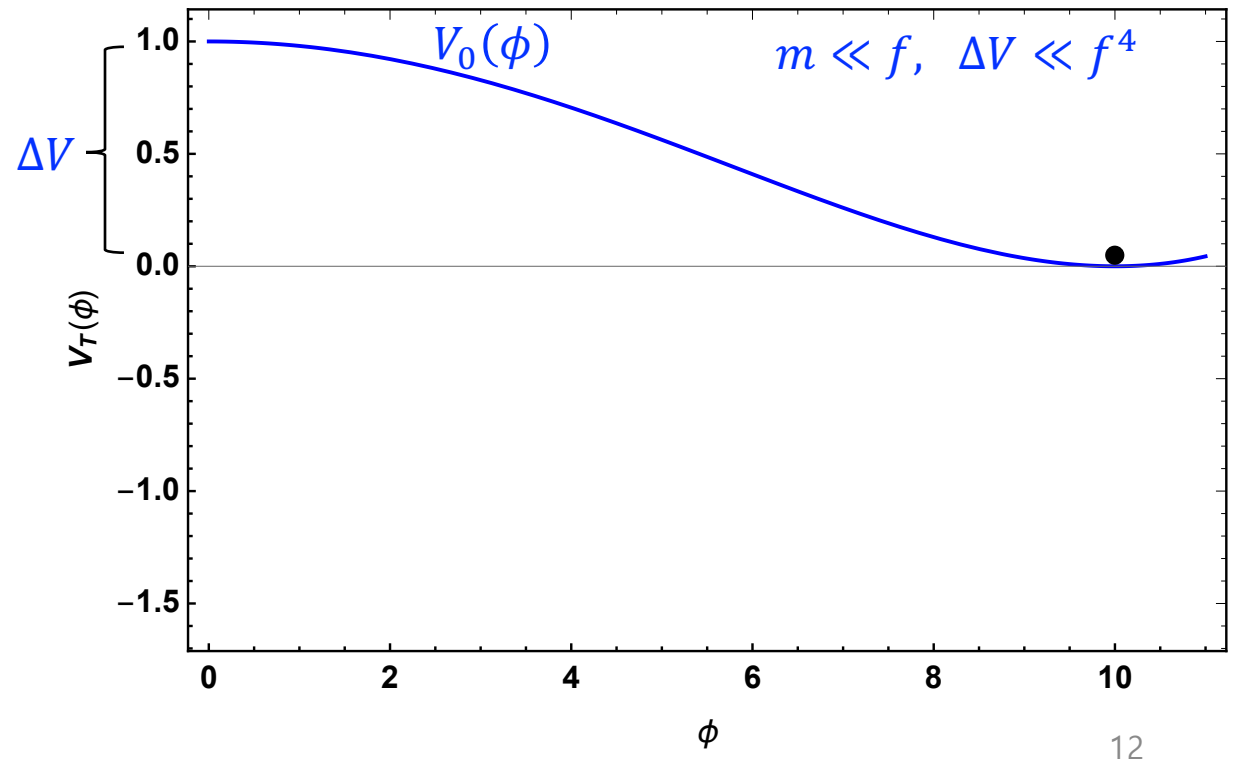


Personal Motivations for Thermal Heavy DM

* Some development of theoretical ideas for heavy DM with a correct relic density

From the old story...

In SUSY models, it is natural for a scalar ϕ whose VEV f is much larger than its mass m



Personal Motivations for Thermal Heavy DM

* Some development of theoretical ideas for heavy DM with a correct relic density

From the old story...

In SUSY models, it is natural for a scalar ϕ whose VEV f is much larger than its mass m

At high temperatures, ϕ can be trapped at the origin due to the thermal effect.

As the temperature drops, eventually there is the phase transition from $\phi = 0$ to $\phi = v_\phi \neq 0$ at T_{ph}

For $\Delta V \ll f^4$, generically $T_{\text{ph}}^4 \ll \Delta V$

the Universe experiences “supercooled period”

(mini-inflation period) \rightarrow reheating process

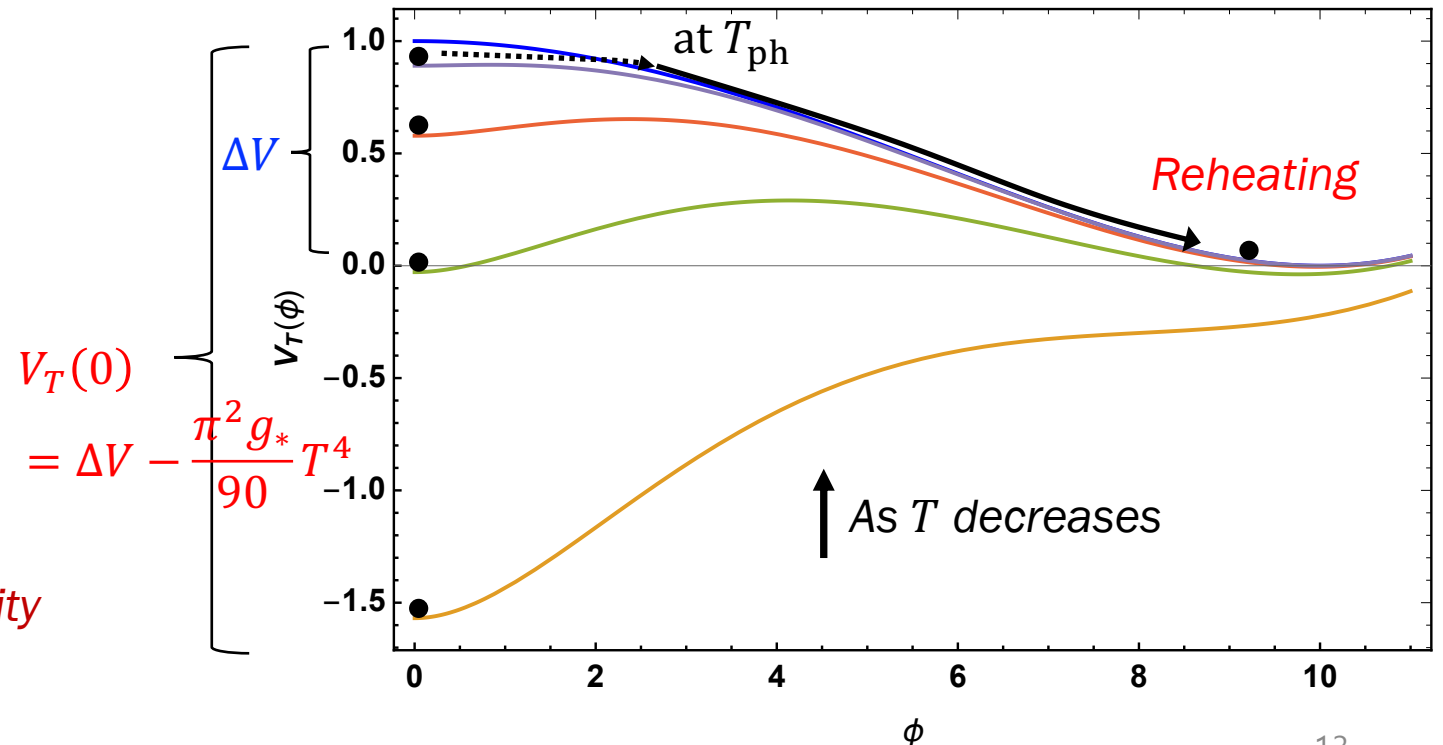
\rightarrow large entropy production

\rightarrow DM abundance is diluted as (roughly)

$$\left(\frac{n_\chi}{s}\right) \rightarrow \left(\frac{n_\chi}{s}\right)_{\text{ini}} \left(\frac{T_{\text{ph}}}{T_{\text{rh}}}\right)^3$$

Heavy DM can naturally have a correct relic density

(but not special for thermal heavy dark matter)



$$V_T(\phi) = V_0(\phi) + \text{Thermal corrections : the free energy density}$$

Personal Motivations for Thermal Heavy DM

Evolution of the DM mass is related with phase transition at the early Universe

A mass of the heavy DM is identified as $M_\chi = y\phi \rightarrow \left(\frac{n_\chi}{s}\right)_{\text{ini}} = \left(\frac{n_\chi}{s}\right)_{\text{thermal}} \sim \frac{1}{g_{\text{SM}}} \sim 0.01$

[hep-ph/9812345]

Superheavy Dark Matter from Thermal Inflation

Lam Hui* and Ewan D. Stewart†
NASA/Fermilab Astrophysics Group
Fermi National Accelerator Laboratory
Box 500, Batavia, IL 60510-0500

It is quite plausible that the mass of the dark matter particle increases significantly after its freeze-out, due to a scalar field rolling to large values. We describe a realization of this scenario in the context of thermal inflation which naturally gives a cold dark matter particle with the correct cosmological abundance and a mass around 10^{10} GeV, evading the conventional upper bound of 10^5 GeV. We also discuss another realization which could produce a cosmologically interesting abundance of near Planck mass, possibly electromagnetically charged, particles. The detection and observational consequences of superheavy cold dark matter or WIMPZILLAs are briefly examined.

$$\left(\frac{n_\chi}{s}\right) = \left(\frac{n_\chi}{s}\right)_{\text{ini}} \left(\frac{T_{\text{ph}}}{T_{\text{rh}}}\right)^3 + \left(\frac{n_\chi}{s}\right)_{\text{subthermal}}$$

[1805.01473]

Super-cool Dark Matter

Thomas Hambye^a, Alessandro Strumia^{b,c,d}, Daniele Teresi^a

^a Service de Physique Théorique, Université Libre de Bruxelles, Brussels, Belgium

^b CERN, Theory Division, Geneva, Switzerland

^c Dipartimento di Fisica dell'Università di Pisa

^d INFN, Sezione di Pisa, Italy

$$\left(\frac{n_\chi}{s}\right)_{\text{subth}} \simeq M_P (\langle \sigma_{\text{ann}} v \rangle_T M_\chi) \int \frac{dz}{z^2} \left(\frac{n_\chi^{\text{eq}}}{s}\right)^2 \propto e^{-2M_\chi/T_{\text{rh}}}$$

$$\text{from } \frac{z^2}{M_P} \frac{d}{dz} \left(\frac{n_\chi}{s}\right) \simeq \langle \sigma_{\text{ann}} v \rangle_T M_\chi \left(\left(\frac{n_\chi^{\text{eq}}}{s}\right)^2 - \left(\frac{n_\chi}{s}\right)^2 \right)$$

Personal Motivations for Thermal Heavy DM

* Considering a standard thermal history and a constant DM mass

[1906.00981]

Super heavy thermal dark matter

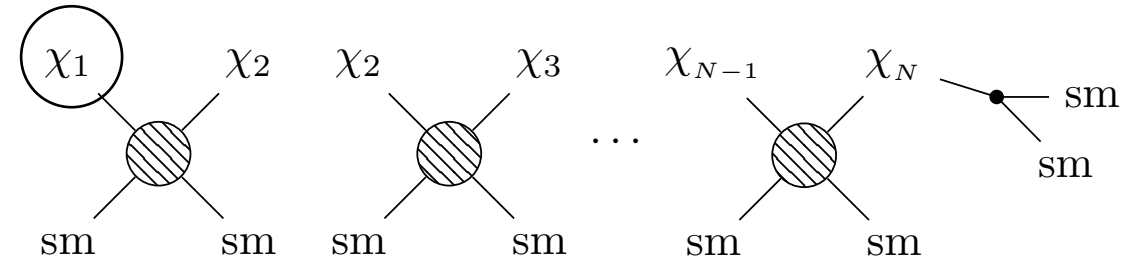
Hyungjin Kim

Department of Particle Physics and Astrophysics,
Weizmann Institute of Science, Rehovot 7610001, Israel

Eric Kuflik

Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel

We propose a mechanism of elementary thermal dark matter with mass up to 10^{14} GeV, within a standard cosmological history, whose relic abundance is determined solely by its interactions with the Standard Model, without violating the perturbative unitarity bound. The dark matter consists of many nearly degenerate particles which scatter with the Standard Model bath in a nearest-neighbor chain, and maintain chemical equilibrium with the Standard Model bath by in-equilibrium decays and inverse decays. The phenomenology includes super heavy elementary dark matter and heavy relics that decay at various epochs in the cosmological history, with implications for CMB, structure formation and cosmic ray experiments.



$$\chi_i + \text{sm} \leftrightarrow \chi_{i+1} + \text{sm},$$

With series of co-scattering, the DM abundance is effectively reduced (keeping its metastability).

Signatures from DM decay products, etc.

$$\left(\frac{n_\chi}{s}\right) \propto \exp\left(-f(N) \frac{M_\chi}{T_{\text{fo}}}\right)$$

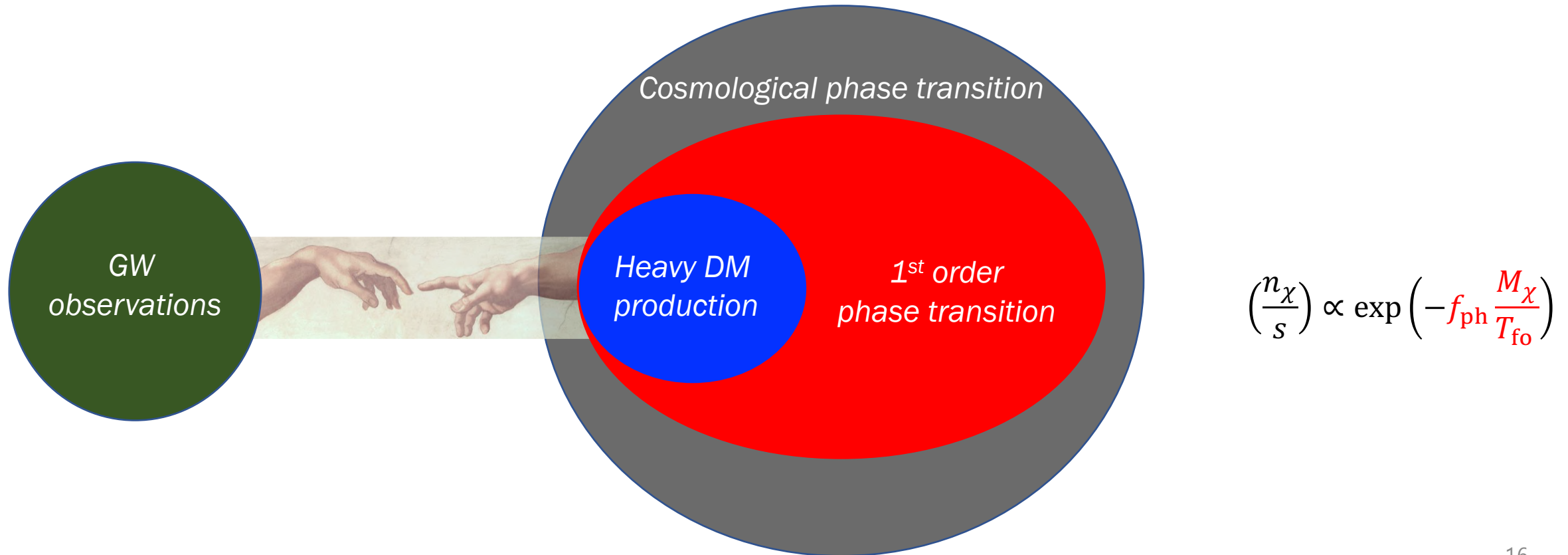
The instability of decaying DM : Originating from its production mechanism

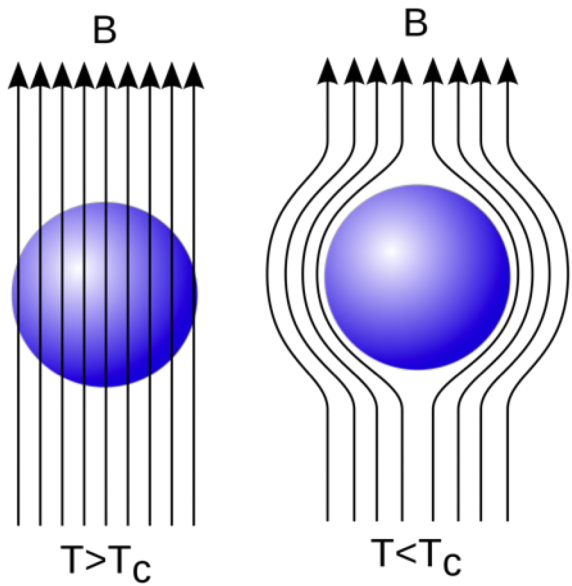
Personal Motivations for Thermal Heavy DM

Thermal Heavy DM scenario can provide interesting phenomenology.

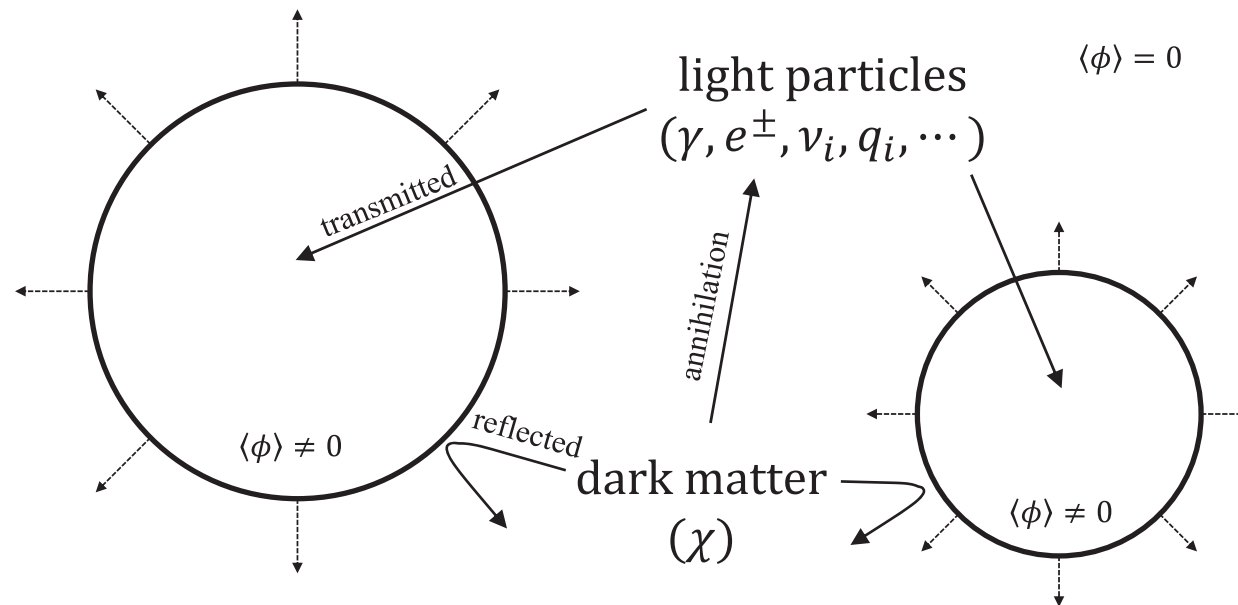
*In this talk we want to focus more on the relation between phase transition and the **stable** DM mass & abundance highlighting the role of **the first order phase transition** for thermal DM : **filtering-out effect during the phase transition***

*→ finding evidences of thermal heavy DM scenario from **gravitational probes** like Gravitational Wave Observations*



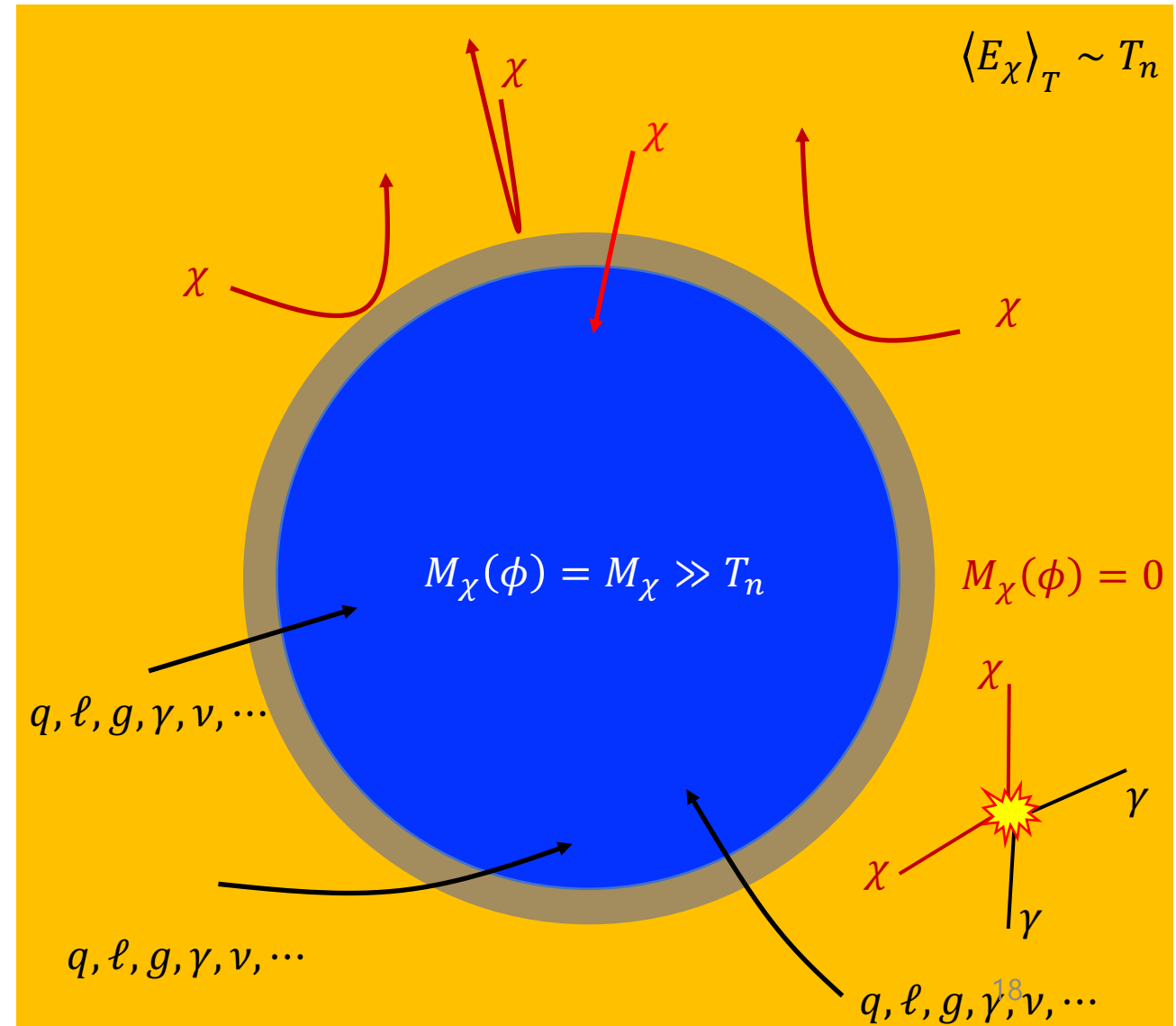
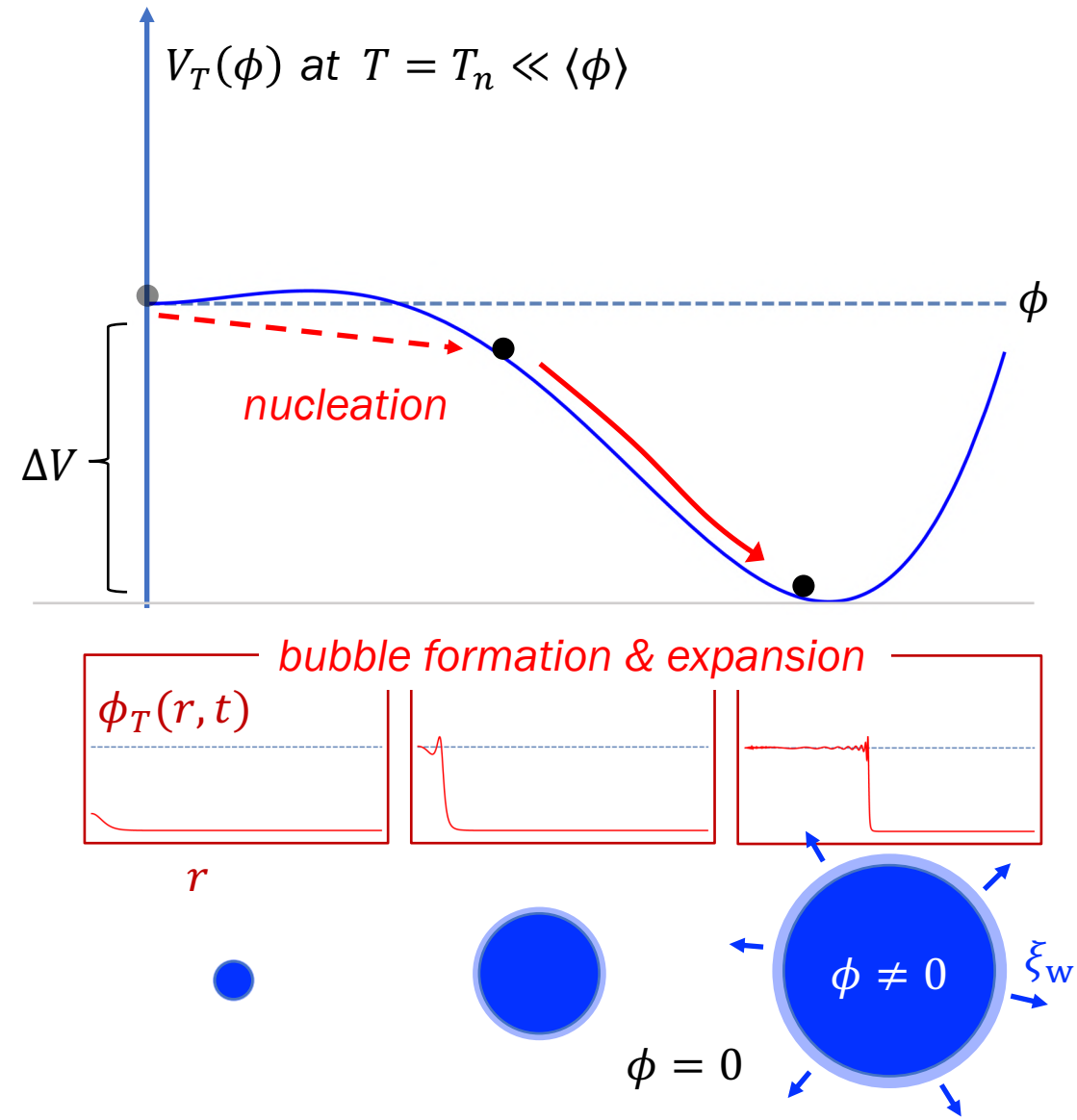


Basic Idea of Filtering-Out Mechanism



Bubble Formation from 1st order Phase Transition

1st order phase transition at the nucleation temp. T_n . Dark matter χ gets a mass by $\langle\phi\rangle$, e.g. $M_\chi(\phi) = y\phi$



DM Filtering-out Effect

1st order phase transition at the nucleation temp. T_n . Dark matter χ gets a mass by $\langle \phi \rangle$, e.g. $M_\chi(\phi) = y\phi$

DMs in chemical equilibrium outside bubbles

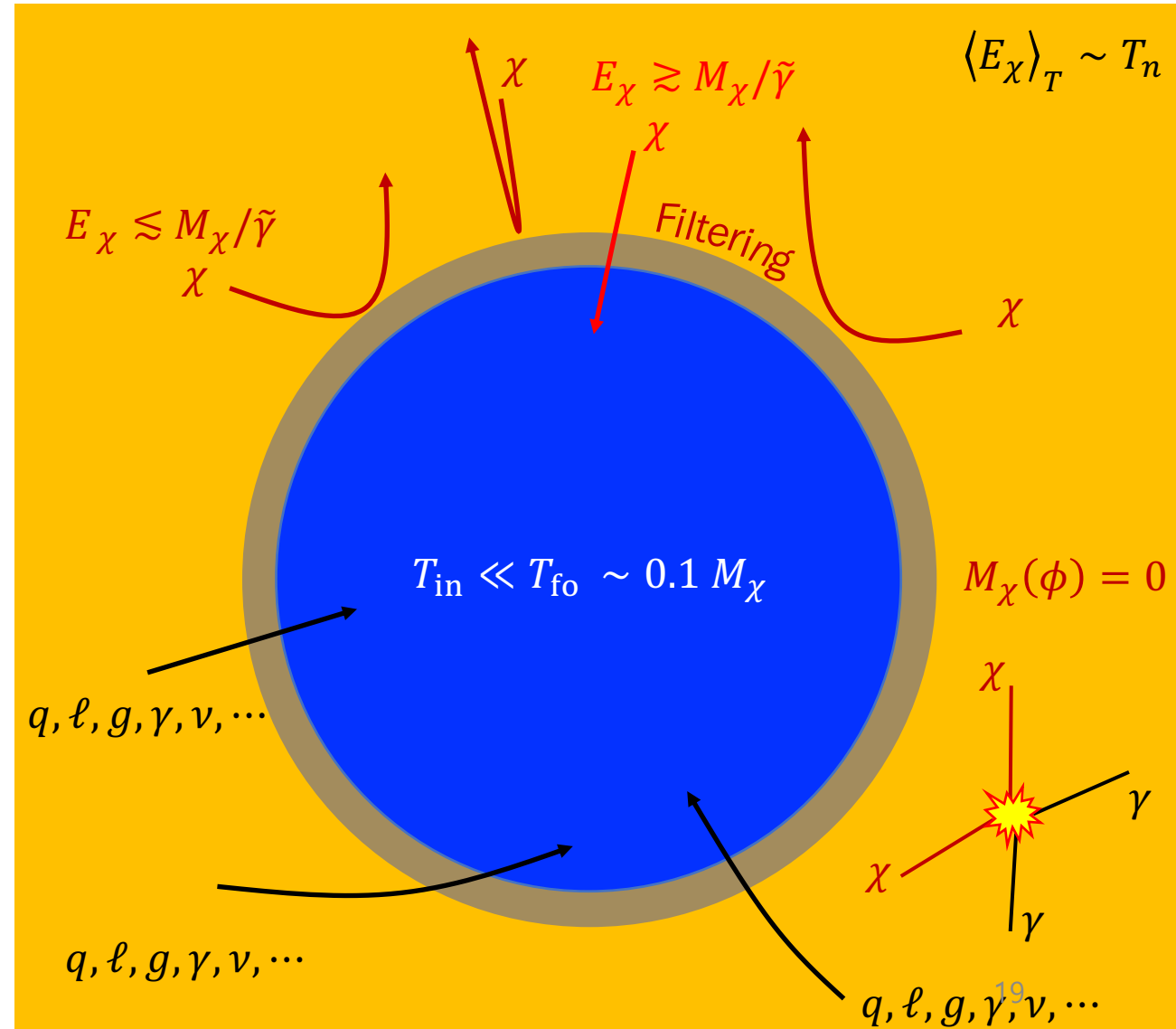
$$(n_\chi)_{\text{out}} \simeq \frac{g_\chi}{\pi^2} T^3$$

Most of DMs are reflected against the bubble wall:

$$(n_\chi)_{\text{in}} \ll (n_\chi)_{\text{out}}$$

For $\tilde{v}(T)$, incoming DM fluid velocity in the wall rest frame

(temperature of DM), and $\tilde{\gamma} = 1/\sqrt{1 - \tilde{v}^2}$,



DM Filtering-out Effect

1st order phase transition at the nucleation temp. T_n . Dark matter χ gets a mass by $\langle\phi\rangle$, e.g. $M_\chi(\phi) = y\phi$

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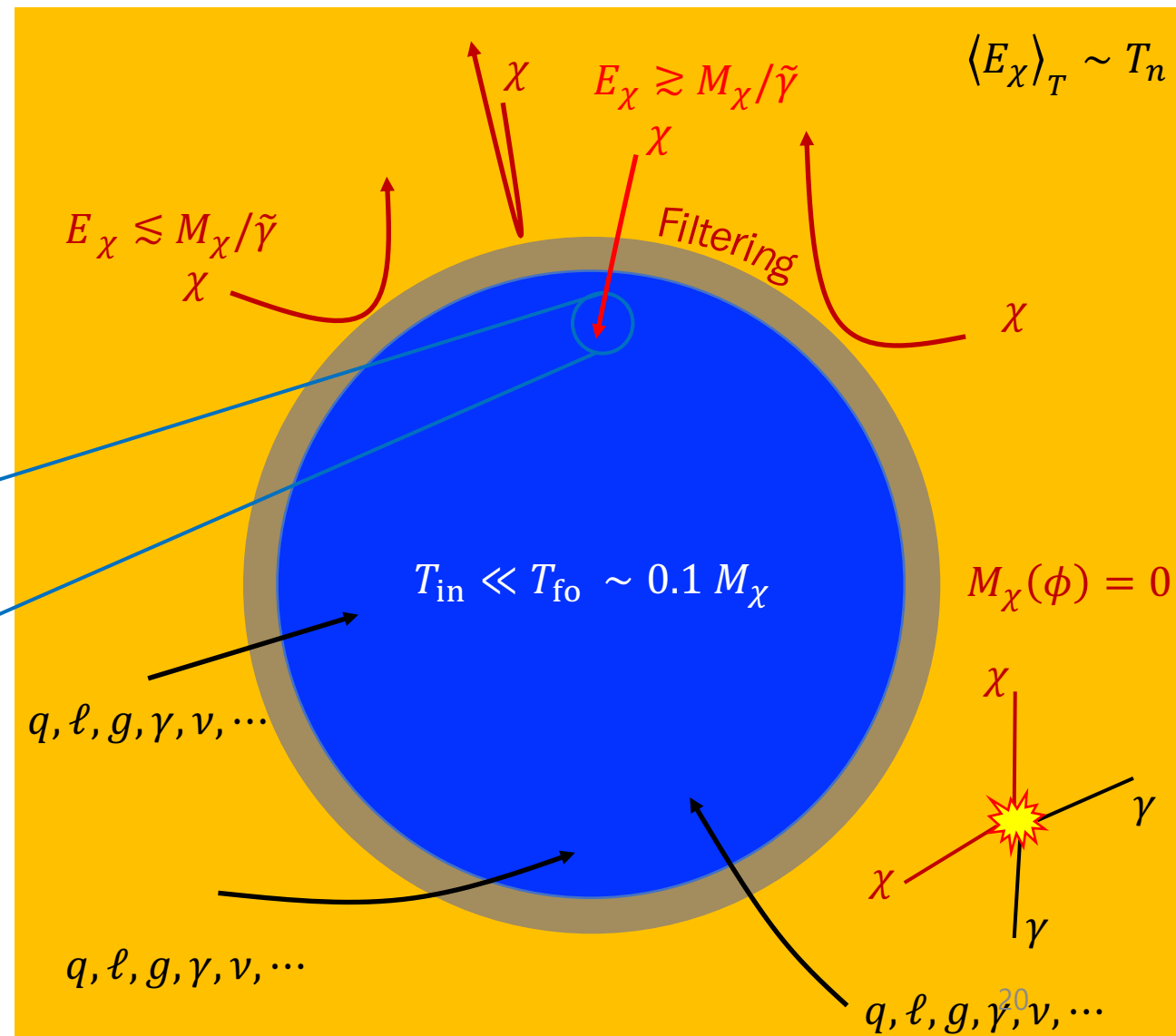
$$(n_\chi)_{\text{in}} \simeq \left(\frac{g_\chi T^3}{\gamma_w \xi_w} \right) \left(\frac{\tilde{\gamma}(1 - \tilde{v})M_\chi/T + 1}{4\pi^2 \tilde{\gamma}^3 (1 - \tilde{v})^2} \right) e^{-\tilde{\gamma}(1 - \tilde{v})M_\chi/T}$$

[Chway, Jung, CSS 1912.04238]

$$\text{For } \tilde{v} \ll 1 (\tilde{\gamma} \simeq 1), (n_\chi)_{\text{filtered}} \sim e^{-M_\chi/T}$$

$$\text{For } \tilde{v} \rightarrow 1 (\tilde{\gamma} \gg 1), (n_\chi)_{\text{filtered}} \sim e^{-M_\chi/2\tilde{\gamma}T}$$

The abundance of DM is **frozen** after penetration



Realization

A List of Questions

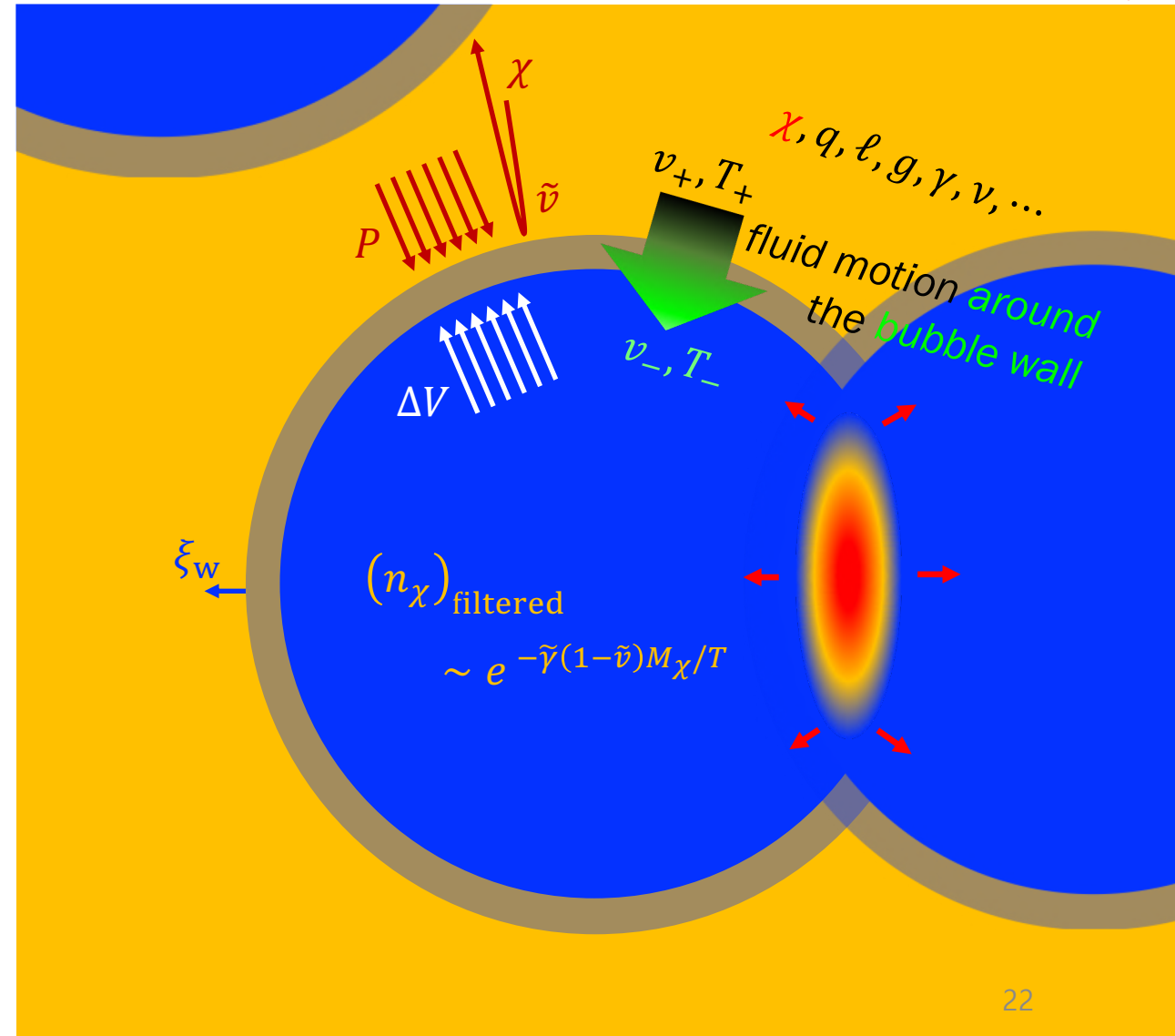
1. How can we determine \tilde{v} , T and the relation with ξ_w , T_n , T_{rh} ?

2. What is the fate of the reflected DMs against the wall?

3. Is the filtering-out process safe from bubble collisions?

4. What are the conditions for the scalar potential of ϕ ?

$$T_n(\text{global}) \rightarrow T_{\text{max}}(\text{local}) \rightarrow T_{\text{rh}}(\text{global}) \ll M_\chi$$

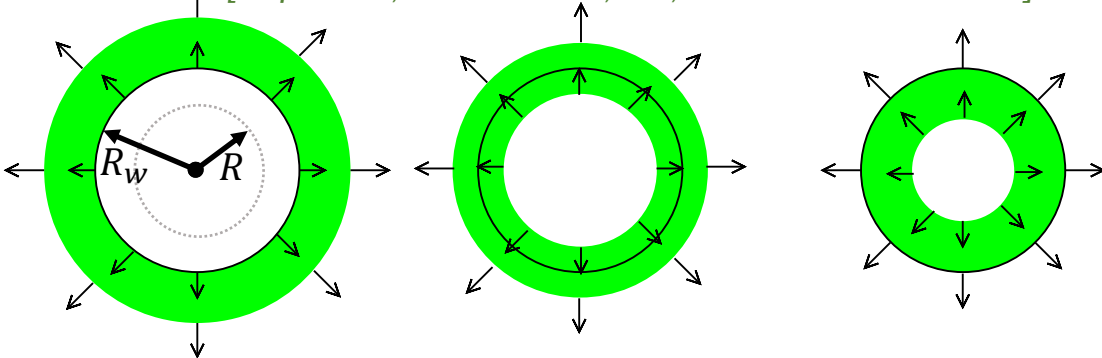


Expansion of Bubbles with Hydrodynamics

1. How can we determine \tilde{v} , T and the relation with ξ_w , T_n , T_{rh} ?
2. What is the fate of the reflected DMs against the wall?

The assumption: particles interacting with bubble walls can instantly (*microscopic time scale*) exchange their energy and momentum with the ambient thermal plasma. Before bubble collisions, perfect fluid approximation can be used for the plasma.

[Espinosa, Konstandin, No, Servant 1004.4187]



deflagration

$$\xi_w < c_s$$

deflagration

hybrid

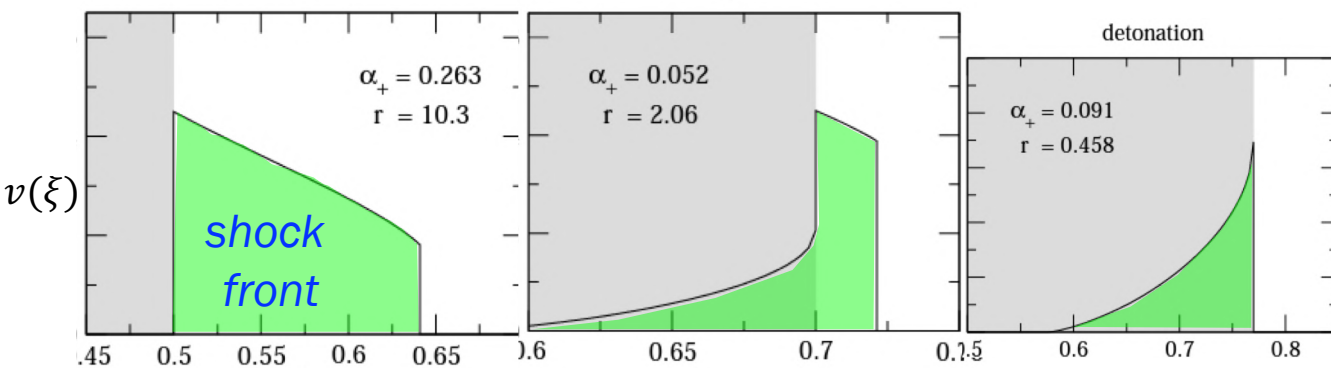
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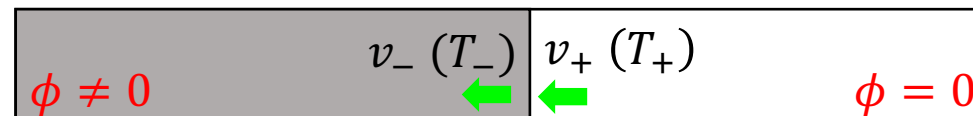
detonation

$$\xi_w > c_s$$

detonation



$$\xi_w = R_w/t$$



$v_{\pm}(T_{\pm})$ is the fluid velocity (temperature) of the plasma near the wall in the wall rest frame

Deflagration: $v_+ < v_- = \xi_w < c_s$, $T_- < T_n < T_+$

Detonation: $c_s < v_- < v_+ = \xi_w$, $T_n = T_+ < T_-$

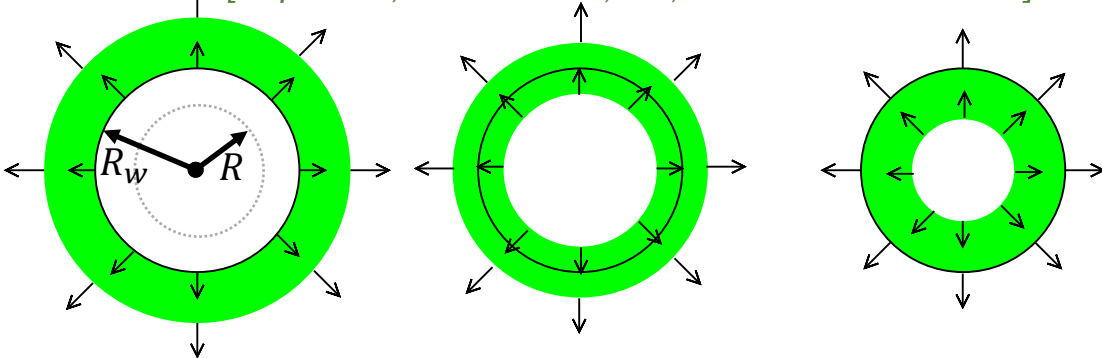
$\xi = R/t$, $v(\xi)$ is the fluid velocity in the plasma rest frame

Expansion of Bubbles with Hydrodynamics

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2. What is the fate of the reflected DMs against the wall?

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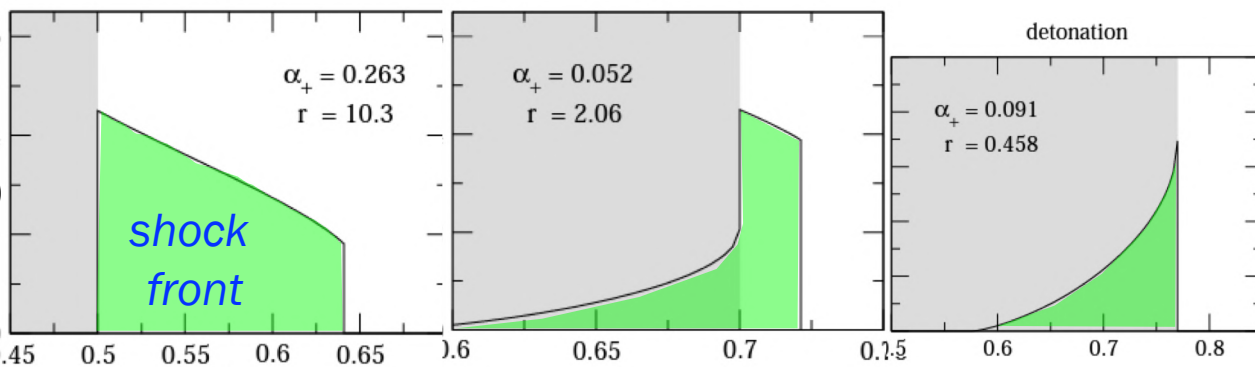
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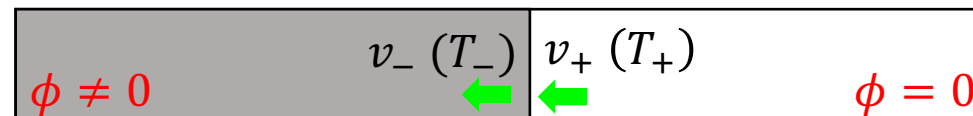
detonation

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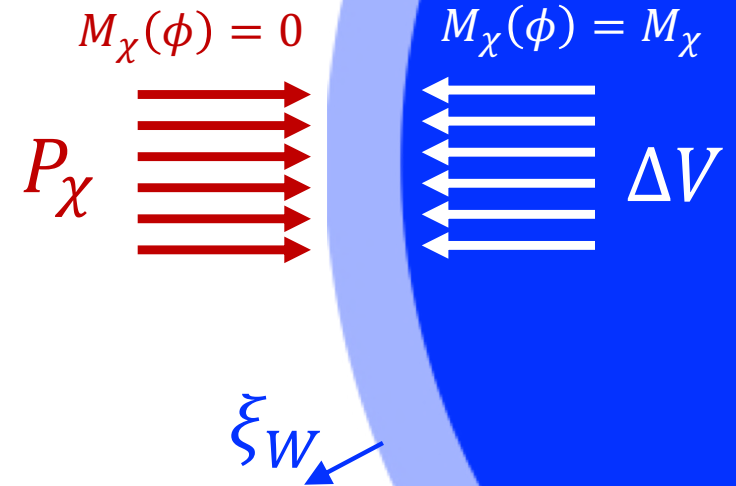
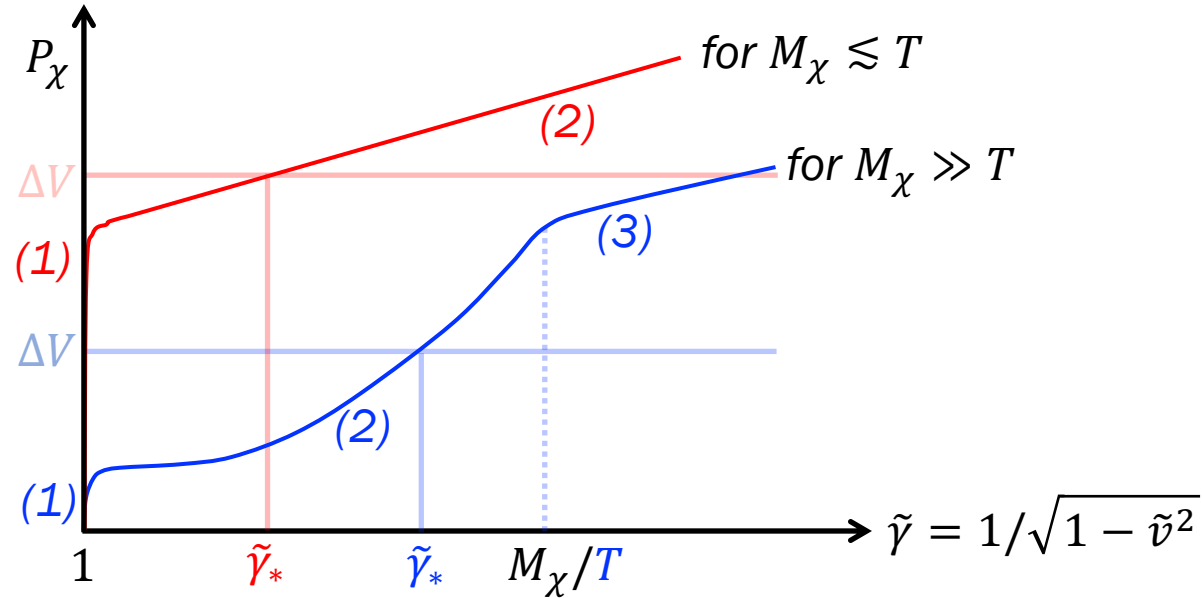
Our key assumption for dark matter:

$$\tilde{v} = v_+, T = T_+$$

Dynamics for the Wall-Velocity (minimal consideration)

The terminal wall velocity ξ_w (also \tilde{v}) by force balancing condition: $P = \Delta V$

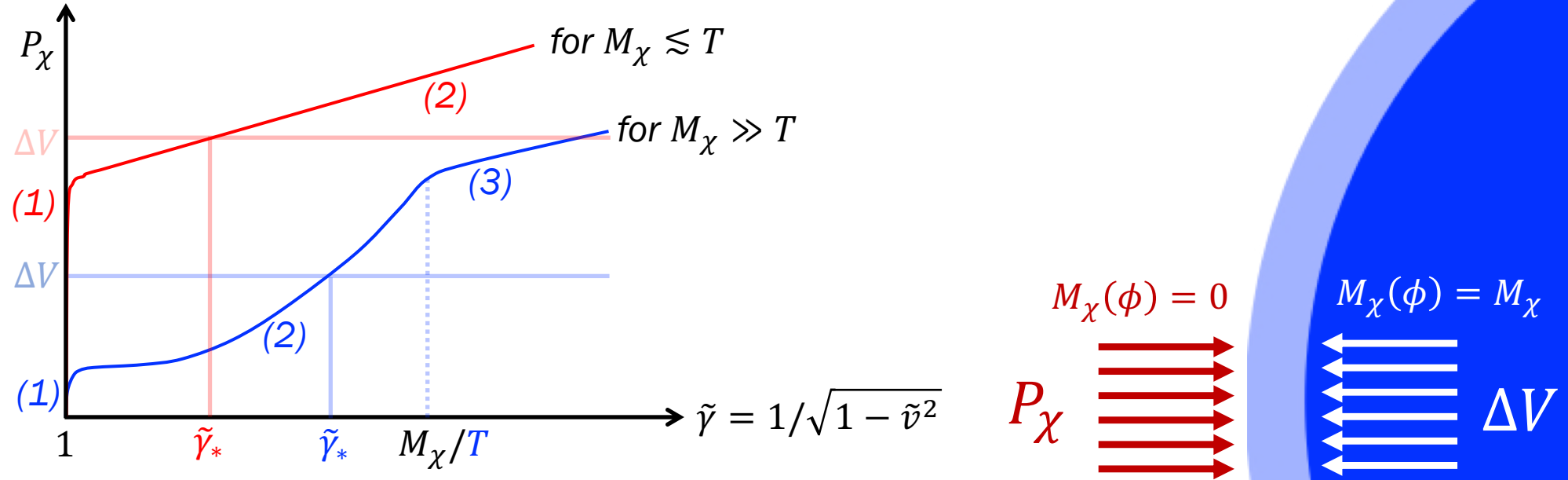
Filtered dark matters are the main source of the pressure (No direct effect from SM particles)



Dynamics for the Wall-Velocity (minimal consideration)

The terminal wall velocity ξ_w (also \tilde{v}) by force balancing condition: $P = \Delta V$

Filtered dark matters are the main source of the pressure (No direct effect from SM particles)



Using the approximation of reflection and transmission of thermal particles at the wall

[Arnold hep-ph/9302258, ...] [Bodeker, Moore 1703.08215, and some interesting recent debates...]

(1): $(g_\chi/24) M_\chi^2 T^2$, (2): $\alpha_{\chi \rightarrow \nu\chi} \tilde{\gamma} m_\nu T^3$ from soft radiation of DM

(1)&(2): $(\pi^2 g_\chi/90)(1+\tilde{v})^3 \tilde{\gamma}^2 T^4$ since most DMs are reflected ($M_\chi > \tilde{\gamma}T$), (3) ($M_\chi \lesssim \tilde{\gamma}T$) \rightarrow (2)

[Chway, Jung, CSS 1912.04238]

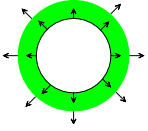
If $P = \Delta V$ happens at $\tilde{\gamma}_* \ll M_\chi/T$, DM filtering is effective

In most working parameter space, $\tilde{\gamma}_* \gg 1$ i.e. detonation profile ($\tilde{v} \simeq \xi_w \simeq 1$, $T \simeq T_n$)

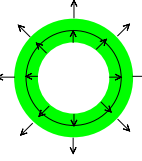
DM Production from Bubble (plasma) Collisions

Depends on the plasma profile around the bubble wall

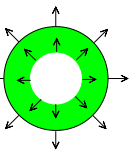
For deflagration, no problem



For hybrid, spoils the filtering effect



For detonation, several possibilities



For detonation, the energy density of the plasma just behind the bubble wall

$$\rho_{\text{pl}} \sim \gamma_w^2 T_{\text{pl}}^4, \quad \text{where } T_{\text{pl}} \simeq T_- \sim \sqrt{\gamma_w} T_n \sim T_{\text{rh}} \text{ (reheating temp.)}$$

During collisions, plasma is assumed to be in local thermal equilibrium

$$\rho_{\text{local}}(\Delta L \rightarrow 0) \sim T_{\text{max}}^4, \quad \text{where } T_{\text{max}} = O(0.1)\gamma_w T_n$$

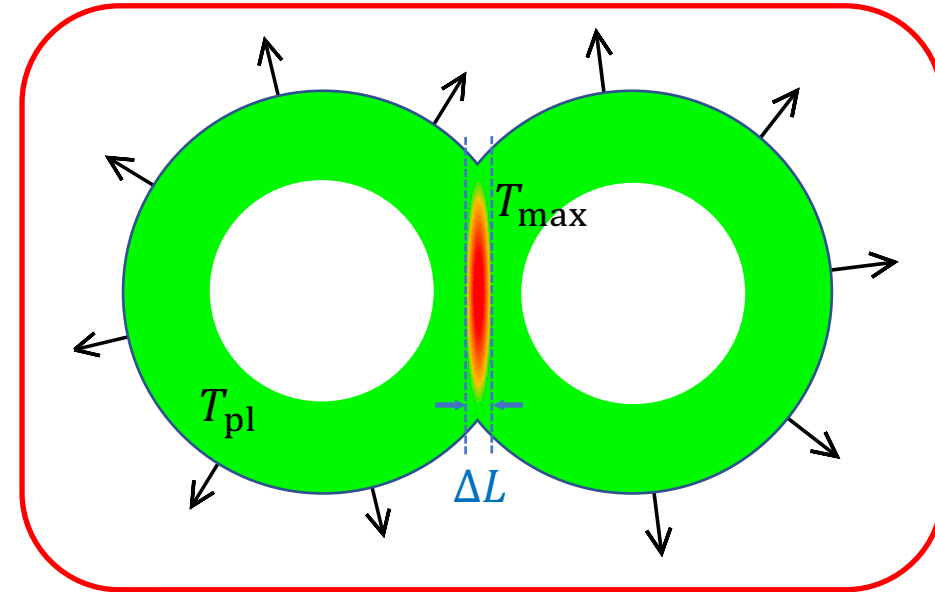
and such local temperature will decrease as the collision volume increases:

$$T_{\text{local}}(\Delta L \rightarrow 0) = T_{\text{max}}, \quad T_{\text{local}}(\Delta L \rightarrow R_w) = T_{\text{rh}}$$

We require $T_{\text{max}} < T_{\text{fo}}$ which is conservative bound on T_{max}

Hierarchies of various temperatures:

$$T_n < T_{\text{rh}} < T_{\text{max}} < \gamma_w T_n < O(0.1) M_\chi$$



plasma collision happens in the broken phase
 Massive DMs can be produced by pair creation processes
 $(q, \ell, g..) + (q, \ell, g..) \rightarrow \chi\chi$
 in local hot thermal bath₂₇

Summary of the Set-up

1st order phase transition at T_n

→ Bubble formations and expansions → DM filtering

→ Bubble collisions → Reheating the Universe with the temp T_{rh}

Mini (thermal) inflation can happen for $\Delta V \gg \rho_{\text{rad}}(T_n) \sim g_{\text{SM}} T_n^4$. In such a case, $g_{\text{SM}} T_{\text{rh}}^4 \simeq \Delta V \rightarrow T_{\text{rh}}^4 \gg T_n^4$

Including all the effects

$$\Omega_{\text{DM}} h^2 = 2.8 \times 10^8 \left(\frac{M_\chi}{\text{GeV}} \right) \left(\frac{n_\chi}{s} \right) \text{filtering out}$$

$$\simeq 0.11 \left(\frac{M_\chi}{10^{10} \text{GeV}} \right) \left(\frac{g_\chi}{0.01 g_{\text{SM}}(T_n)} \right) \left(\frac{T_n}{0.1 T_{\text{rh}}} \right)^3 \left(\frac{\frac{M_\chi}{2\gamma_w T_n} \exp\left(-\frac{M_\chi}{2\gamma_w T_n}\right)}{e^{-33.6}} \right)$$

dilution effect
by entropy production

filtering-out effect
by 1st order phase transition

Working Parameter Space

Defining two useful parameters

$$\alpha_n \equiv \frac{\Delta V}{\rho_{\text{rad}}(T_n)}$$

how much the Universe is supercooled

$$\lambda_{\text{eff}} \equiv \frac{\Delta V}{M_\chi^4}$$

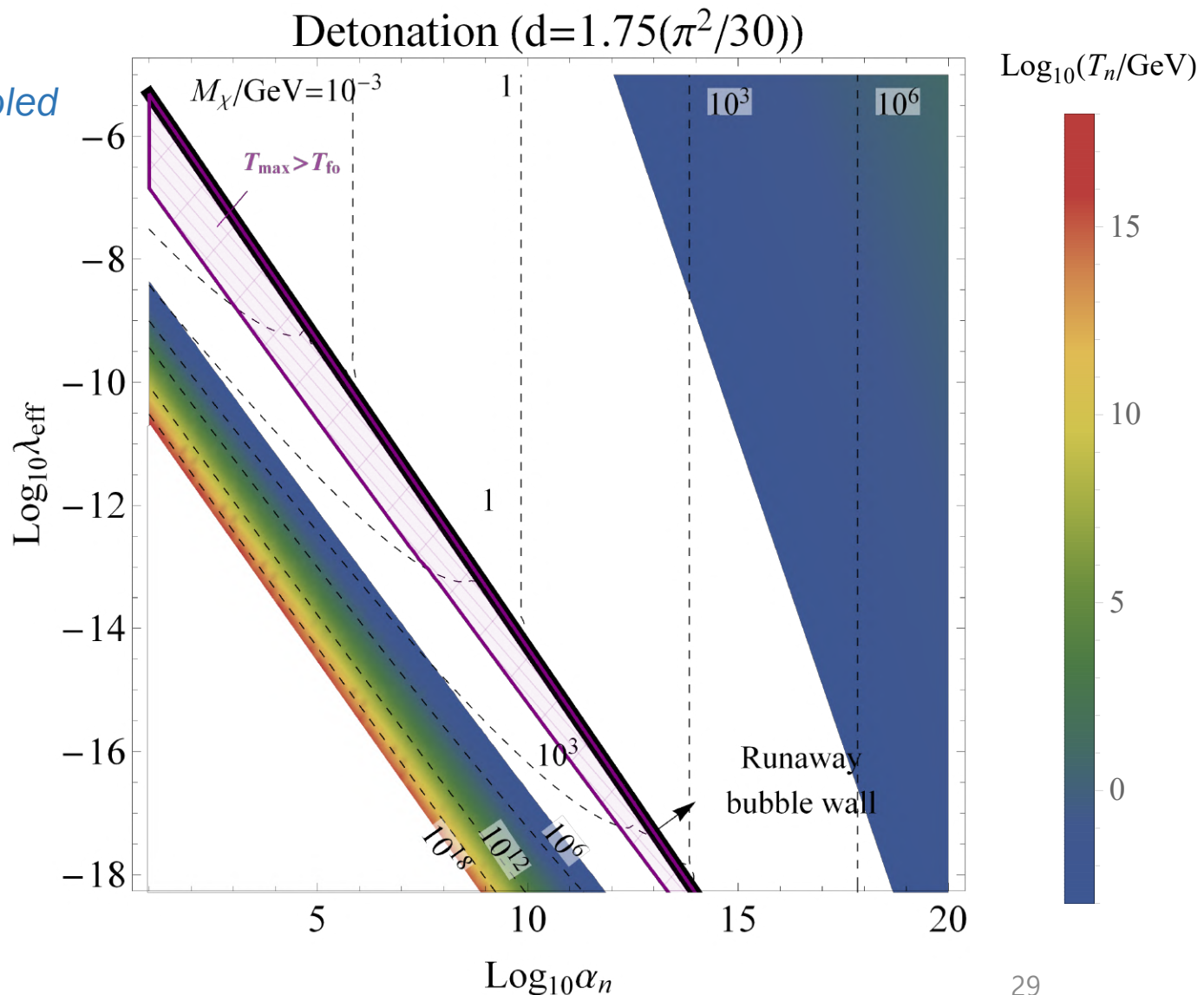
how small ΔV in the unit of DM mass

Rainbow colored region: filtering-out effect is dominant

Parametrically, $\exp\left(-\frac{M_\chi}{2\gamma_w T_n}\right) \sim \exp\left(-\frac{O(1)}{(\lambda_{\text{eff}}\alpha_n)^{1/4}}\right)$

Numerically, $\lambda_{\text{eff}}\alpha_n \sim 10^{-9}$ for correct density

Top-right corner: bubbles run away (dilution is dominant)
difficult to estimate the effect of bubble collision



Implications for Model Building

About the Scalar Potential of ϕ ($\lambda_{eff}\alpha_n \sim 10^{-9}$)

$\lambda_{eff}\alpha_n$ should be a very small value for the successful DM filtering-out mechanism

Ex1) $V(\phi) = -m^2\phi^2 + \lambda\phi^4$, $M_\chi = y_\chi\phi$: $\langle\phi\rangle \sim m/\sqrt{\lambda}$, $\Delta V \sim m^2\langle\phi\rangle^2$

Then, $\lambda_{eff} \sim \lambda/y_\chi^4$, $T_n \sim m/y_\chi$, $\alpha_n \sim 10^{-4}y_\chi^4/\lambda$

$\rightarrow \lambda_{eff}\alpha_n \sim 10^{-4} \gg 10^{-9}$

Ex2) Inspired by gauge mediation in SUSY axion model (the messenger scale M)

$$V_1(\phi) = -m^2\phi^2 \quad \text{for } \phi \ll M$$

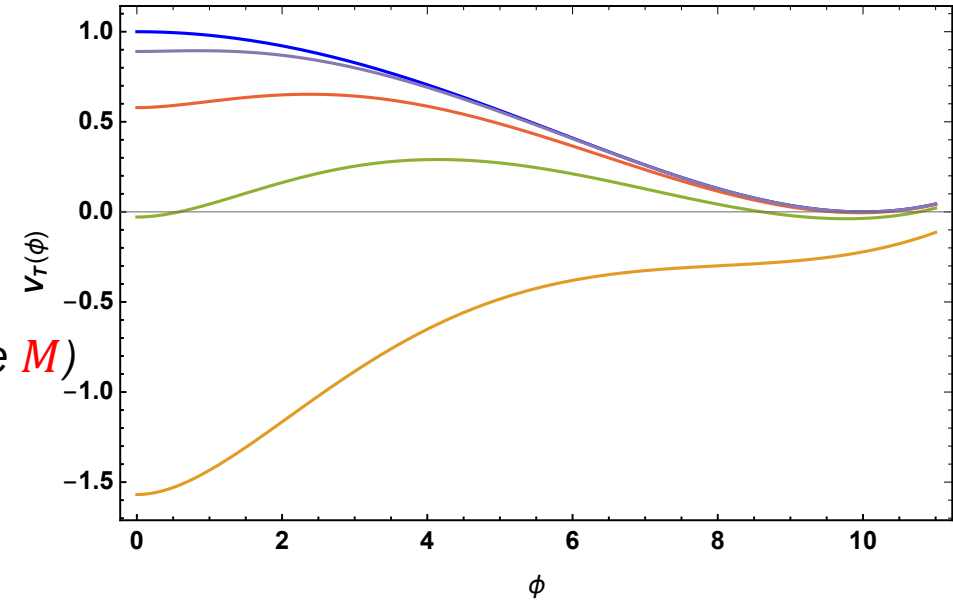
$$= -m^2M^2 \ln^n(\phi/M) \quad \text{for } \phi \gg M$$

The scalar potential of ϕ is

$$V(\phi) = V_1(\phi) + m_{3/2}^2\phi^2 \quad \text{with } m_{3/2} \ll m: \quad \langle\phi\rangle \sim M(m/m_{3/2}) \gg M, \quad \Delta V \sim m^2M^2 \ll m^2\langle\phi\rangle^2$$

$\rightarrow \lambda_{eff}\alpha_n \sim (m_{3/2}/m)^4 \sim 10^{-9}$ can be easily obtained

$$\alpha_n \equiv \frac{\Delta V}{\rho_{\text{rad}}(T_n)} \quad \lambda_{eff} \equiv \frac{\Delta V}{M_\chi^4}$$



Summary

Although there is the Unitarity upper bound on the mass of thermal DM, the thermal heavy DM scenario still can be considered as an interesting possibility due to its phenomenological implication

The mass of DM could result from a symmetry-breaking effect, whose abundance is closely related to the cosmic phase transition at the early Universe.

The first-order phase transition can provide the filtering-out effect for the heavy DM relic abundance, allowing a wide range of DM mass beyond the Unitarity bound.

GW generated from the first-order phase transition is one of the observable results from which the detailed information about DM mass and its interaction strength can be inferred.

There are still many questions about the transport of DM around the ultra-relativistic bubble wall, the fate of DMs confined locally in the symmetric phase before the bubble collisions, the possible DM evolution after the bubble collisions, and the concrete model buildings