Higgs Inflation in Einstein-Cartan gravity

Mikhail Shaposhnikov

2022 Chung-Ang University Beyond the Standard Model Workshop

9 February 2022

Based on:

MS, Andrey Shkerin, and Sebastian Zell:

- Standard Model Meets Gravity: Electroweak Symmetry Breaking and Inflation, 2001.09088

- Quantum Effects in Palatini Higgs Inflation, 2002.07105

MS, Andrey Shkerin, Inar Timiryasov and Sebastian Zell:

- Einstein-Cartan gravity, matter, and scale-invariant generalisation, 2007.16158

- Higgs inflation in Einstein-Cartan gravity, 2007.14978

- Einstein-Cartan Portal to Dark Matter, 2008.11686

Outline

- Metric, Palatini and Einstein-Cartan gravities
- Bosonic action in EC gravity with Higgs field
- Higgs Inflation, predictions and challenges
- Conclusions

Short reminder

Flat Minkowski space-time in arbitrary coordinates $x^\mu = f^\mu(\xi^i)$ (ξ^i - Cartesian coordinates). Metric $g_{\mu\nu}(x)$ and connection $\Gamma^\alpha_{\mu\nu}$ (describing the parallel transport of a vector and covariant derivatives), $dV^\mu = -\Gamma^\mu_{\nu\alpha} V^\nu dx^\alpha$ can be found from coordinate transformation $x^\mu = f^\mu(\xi^i)$ and have the following properties:

1. invariant (length) interval, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
2. connection is symmetric, $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$
3. $g_{\mu\nu;\alpha} = 0$: metricity - length of a vector is constant at the parallel transport
4. Covariant derivatives commute, $\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu = 0$

Geometric approach to gravity, Riemann geometry

- distances: symmetric metric tensor
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative:
  $dV^\mu = -\Gamma^\mu_{\nu\alpha} V^\nu dx^\alpha$; symmetric connection $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$.
  Same as 2

- metricity, local Minkowski structure:
  $g_{\mu\nu;\alpha} = 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$. Same as 3.

- Commutator of covariant derivatives: Riemann tensor
  $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu} V_\beta$. Different from 4!

Geometric approach to gravity, Cartan geometry, 1922-1925

- distances: symmetric metric tensor $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative: $dV^\mu = -\Gamma^\mu_{\nu\alpha}V^\nu dx^\alpha$; arbitrary connection $\Gamma^\alpha_{\mu\nu} \neq \Gamma^\alpha_{\nu\mu}$. New object: torsion tensor $T^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu}$. Different from 2!

- metricity, local Minkowski structure (same as 3): $g_{\mu\nu;\alpha} = 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$ and torsion $T^\alpha_{\mu\nu}$

- Commutator of covariant derivatives: Riemann tensor $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu}V_\beta$. Different from 4!

Geometric approach to gravity, non-metricity

- distances: symmetric metric tensor $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative:
  $dV^\mu = -\Gamma^\mu_{\nu\alpha}V^\nu dx^\alpha$; arbitrary connection $\Gamma^\alpha_{\mu\nu} \neq \Gamma^\alpha_{\nu\mu}$. New object: torsion tensor $T^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu}$. Different from 2!

- non-metricity. New object: non-metricity tensor $g_{\mu\nu;\alpha} = Q_{\mu\nu\alpha} \neq 0$
  $\longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$, torsion $T^\alpha_{\mu\nu}$ and non-metricity tensor $Q_{\mu\nu\alpha}$. Length of the vectors changes with parallel transport. Different from 3!

- Commutator of covariant derivatives: Riemann tensor
  $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu}V_\beta$. Different from 4!

Geometric approach to gravity, Weyl theory (1918)

- distances: symmetric metric tensor $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Same as 1.

- Symmetric connection $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$. Same as 2.

- non-metricity. New object - vector field A_μ: non-metricity tensor is reduced to $g_{\mu\nu;\alpha} = Q_{\mu\nu\alpha} = -2A_\alpha g_{\mu\nu} \neq 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$ and vector field A_μ. Different from 3!

- Commutator of covariant derivatives: Riemann tensor $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu} V_\beta$. Different from 4!

Dynamics, Einstein-Hilbert metric action (1915)

- Lowest order action (without cosmological constant) is

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}R$$

- The dynamical variable is $g_{\mu\nu}$, variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations. (We use mostly positive metric.)

Dynamics, Palatini action (1919)

Palatini gravity

Basic structures: metric $g_{\mu\nu}$ (distances) and symmetric connection $\Gamma^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\sigma\nu}$. Riemann curvature tensor is expressed via connection as:

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Same as in metric gravity

Lowest order action (without cosmological constant) is

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}R$$

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives metricity $g_{\mu\nu;\alpha} = 0$, i.e. the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations.

Palatini pure gravity is equivalent to metric gravity

Metric, Palatini and Einstein-Cartan gravities

Einstein-Cartan gravity

Basic structures: metric $g_{\mu\nu}$ (distances) and connection $\Gamma^{\rho}_{\nu\sigma} \neq \Gamma^{\rho}_{\sigma\nu}$. Riemann curvature tensor is expressed via connection as:

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Same as in metric gravity

New object - torsion tensor: $T^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\nu\sigma} - \Gamma^{\rho}_{\sigma\nu}$

Lowest order action (without cosmological constant) is

Holst term

Same as in metric gravity

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}R + \frac{M_P^2}{2\gamma}\int \mathrm{d}^4x\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

Barbero-Immirzi parameter

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations. On the solution $g_{\mu\nu;\alpha} = 0$ and $T^{\rho}_{\nu\sigma} = 0$.

Einstein-Cartan pure gravity is equivalent to metric gravity

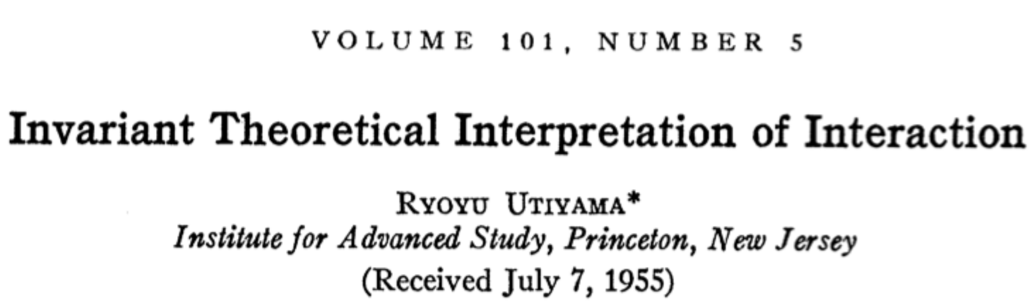
EC gravity as a gauge theory

Existence of electromagnetic field - U(1) global invariance of fermion Lagrangian promoted to be local

Gluons, W^+, W^- Z and γ of the Standard Model - SU(3)xSU(2)xU(1) global invariance of SM fermion Lagrangian promoted to be local

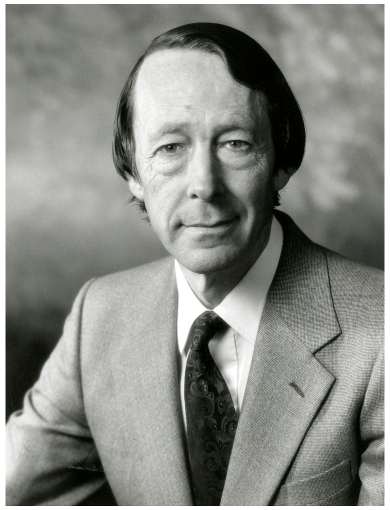
Existence of gravitational field - Poincare invariance of SM fermion Lagrangian promoted to be local?

Gauging of the Poincare group: Utiyama (1956), Kibble (1961) , Sciama (1962,1964).


VOLUME 101, NUMBER 5

Invariant Theoretical Interpretation of Interaction

RYOYU UTIYAMA*
Institute for Advanced Study, Princeton, New Jersey
(Received July 7, 1955)



EC gravity as a gauge theory

Basic gauge fields

- e^I - tetrad one-form (frame field, translations), I=0,1,2,3
- ω^{IJ} - spin connection one form (gauge field of the local Lorentz group). Euclidean: $SO(4) \sim SU(2)_L \times SU(2)_R$
- $F^{IJ} = d\omega^{IJ} + \omega^I_K \omega^{KJ}$: curvature 2-form

Pure gauge action:

$$\frac{M_P^2}{4}\int \epsilon_{IJKL} e^I e^J F^{KL} + \frac{M_P^2}{2\gamma}\int e^I e^J F_{IJ}$$

Again, equivalent to metric gravity

EC gravity with matter fields

Once matter fields are added, the equivalence between different formulation of gravity is lost:

Couplings to

- scalar fields: $\phi^2 \epsilon_{IJKL} e^I e^J F^{KL}$ (or $\phi^2 R$)
- and to fermion fields via covariant derivative $D\Psi = d\Psi + \frac{1}{8}\omega_{IJ}[\gamma^I, \gamma^J]\Psi$

lead to modified relation between the spin connection and tetrad (or Christoffel symbols and the metric). Torsion is non-zero.

Physics is different!

Bosonic action in EC gravity with Higgs field

Inclusion of the scalar field (Higgs field of the Standard Model, unitary gauge)

Scalar action

$$S_h = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\left(\partial_\mu h\right)^2 - U(h)\right), \quad U(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$

Gravity part

Same as in metric gravity

$$S_{\rm grav} = \frac{1}{2}\int d^4x\sqrt{-g}\left(M_P^2 + \xi h^2\right)R$$

Holst term

$$+\frac{1}{2\bar{\gamma}}\int d^4x\sqrt{-g}\left(M_P^2 + \xi_\gamma h^2\right)\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

Nieh-Yan invariant

$$+\frac{1}{2}\int d^4x\,\xi_\eta h^2\partial_\mu\left(\sqrt{-g}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

Three non-minimal couplings: $\xi,\ \xi_\gamma,\ \xi_\eta$

For $1/\bar{\gamma} = \xi_\gamma = \xi_\eta = 0$ we get the Palatini action with non-minimal coupling. Action is polynomial and scale-invariant in the limit $h \to \infty$

Bosonic action in EC gravity with Higgs field

- Torsion is not dynamical
- The same number of degrees of freedom as in the metric gravity + scalar field: 2 (graviton) +1 (scalar)
- Interesting physics: gravity strength, as well as particle masses, are determined by the Higgs field, if $\xi h^2 \gg M_P^2$
- The theory is scale-invariant in this limit, leading to the flat potential for the Higgs field, exactly what is needed for inflation!
- Equivalent metric theory: use the Weyl transformation of the metric field

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Inflation

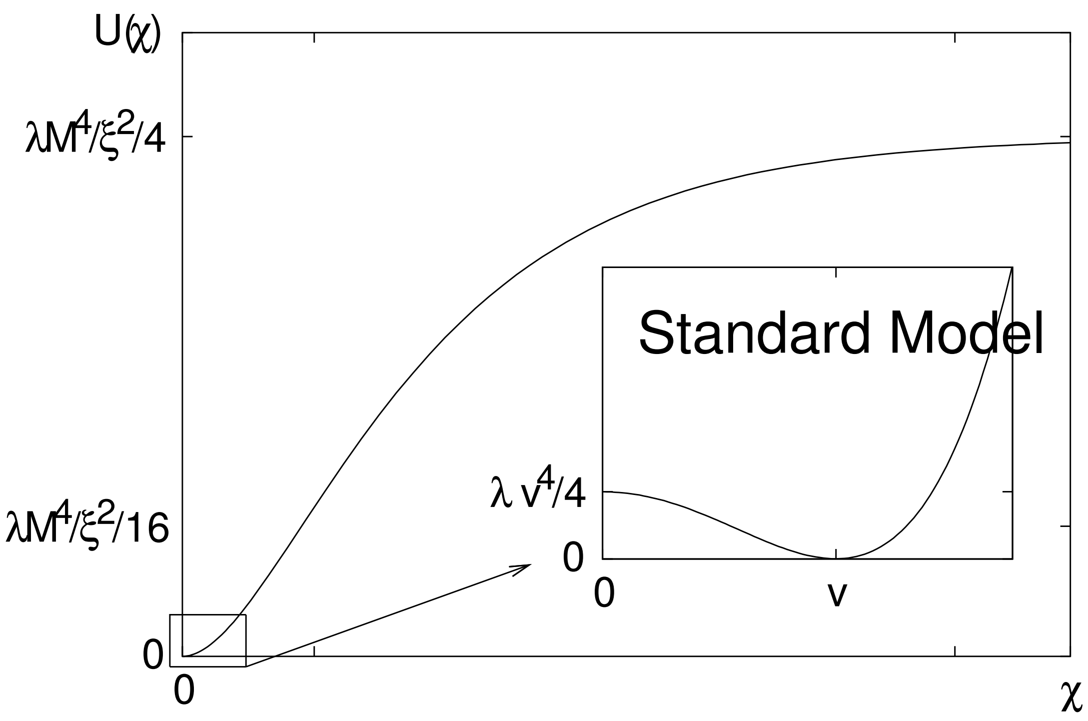
Modified kinetic term: essential for inflation

Metric action:

$$S_{\text{metric}} = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}\left\{R - \left[\frac{1}{2\Omega^2}(\partial_\mu h)^2 + \frac{U}{\Omega^4}\right] - \frac{3M_P^2}{4(\gamma^2+1)}\left(\frac{\partial_\mu \bar{\eta}}{\Omega^2} + \partial_\mu \gamma\right)^2\right\}$$

$$\gamma = \frac{1}{\bar{\gamma}\Omega^2}\left(1 + \frac{\xi_\gamma h^2}{M_P^2}\right), \quad \bar{\eta} = \frac{\xi_\eta h^2}{M_P^2}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Flat potential: essential for inflation



Metric Higgs inflation (Bezrukov, MS): in limit of the vanishing Holst term, $\bar{\gamma} \to \infty, \quad \xi_\gamma = 0,$ take $\quad \xi_\eta = \xi.$

Palatini Higgs inflation (Bauer, Demir): in limit of the vanishing Holst term, $\bar{\gamma} \to \infty, \quad \xi_\gamma = 0,$ take $\quad \xi_\eta = 0$

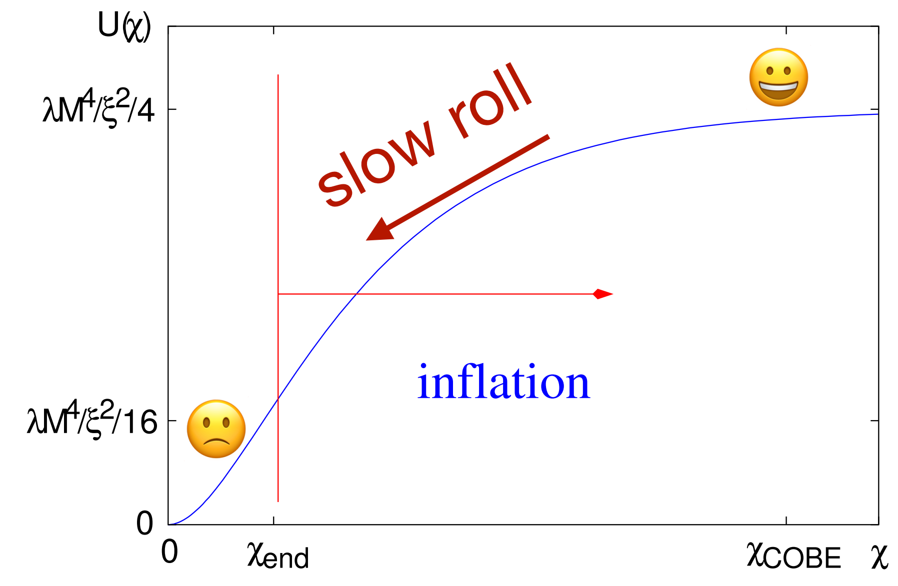
Stages of Higgs Inflation

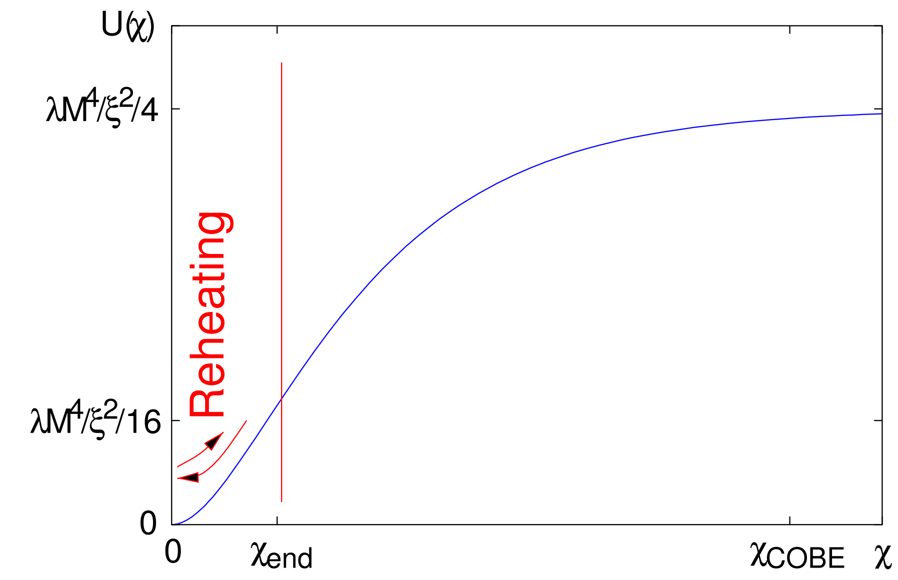
- Chaotic initial conditions: large fields on the plateau inflate, the small fields do not

- Slow roll making the universe flat, homogeneous and isotropic, and producing fluctuations leading to structure formation: clusters of galaxies, etc

- Heating of the Universe : energy stored in the Higgs field goes into the particles of the Standard Model - Higgs makes the Big Bang

- Radiation dominated stage of the Universe expansion starts, leading to baryogenesis, dark matter production, nucleosynthesis…





Metric Higgs Inflation

Jordan frame action:

$$S_G = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{\xi h^2}{2}R - \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}h^4\right)$$

Kinetic term in the Einstein frame:

$$\left(\frac{1}{\Omega^2} + \frac{6\xi^2 h^2}{M_P^2\Omega^4}\right)(\partial h)^2,\ \ \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Cobe normalisation: $\xi \simeq 5 \times 10^4\sqrt{\lambda}$

Palatini Higgs Inflation

Jordan frame action:

$$S_G = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{\xi h^2}{2}R - \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}h^4\right)$$

Kinetic term in the Einstein frame:

$$\left(\frac{1}{\Omega^2} + \frac{6\xi^2 h^2}{M_P^2\Omega^4}\right)(\partial h)^2,\ \ \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Cobe normalisation: $\xi \simeq 1.5 \times 10^{10}\lambda$

Metric and Palatini Higgs inflations

$$S_{\text{grav.+h.}} = \int d^4x\sqrt{-\hat{g}}\left\{-\frac{M_P^2}{2}\hat{R} + \frac{1}{2}K(h)\left(\partial_\mu h\right)^2 - \frac{\lambda}{4\Omega^4}h^4\right\}$$

$$\frac{dh}{d\chi} = \frac{1}{\sqrt{K(h)}}$$

$$S_{\text{grav.+h.}} = \int d^4x\sqrt{-\hat{g}}\left\{-\frac{M_P^2}{2}\hat{R} + \frac{1}{2}\left(\partial_\mu \chi\right)^2 - U(\chi)\right\}$$

$$U(\chi) = \frac{\lambda}{4}F(\chi)^4$$

UV cutoff at $h = 0$: $\Lambda \sim M_P/\xi$

UV cutoff at $h = 0$: $\Lambda \sim M_P/\sqrt{\xi}$

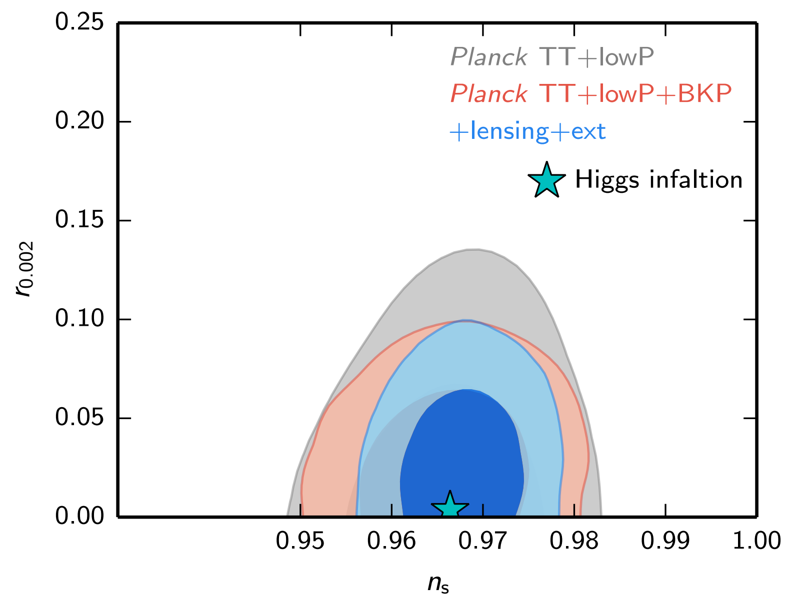
	Metric	Palatini
$K(h)$	$\frac{1}{\Omega^2} + \frac{6\xi^2 h^2}{M_P^2\Omega^4}$	$\frac{1}{\Omega^2}$
$h(\chi)$	$\frac{M_P}{\sqrt{\xi}}\exp\frac{\chi}{\sqrt{6}M_P}$ for $h \gg \frac{M_P}{\sqrt{\xi}}$	$\frac{M_P}{\sqrt{\xi}}\sinh\frac{\sqrt{\xi}\chi}{M_P}$
$F(\chi)$	$\begin{cases}\chi & \chi < \frac{M_P}{\xi} \\ \sqrt{\frac{\sqrt{2}M_P\chi}{\sqrt{3}\xi}} & \frac{M_P}{\xi} < \chi < M_P \\ \frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & M_P < \chi\end{cases}$	$\frac{M_P}{\sqrt{\xi}}\tanh\frac{\sqrt{\xi}\chi}{M_P}$
T_{reh}	$3 \cdot 10^{15}\,\text{GeV}$	$\approx 4 \cdot 10^{13}\,\text{GeV}$
N	55.4	$54.9 - \frac{1}{4}\ln\xi \approx 50.9$
n_s	$1 - \frac{2}{N} = 0.964$	$1 - \frac{2}{N} \approx 0.961$
r	$\frac{12}{N^2} = 3.9 \cdot 10^{-3}$	$\frac{2}{\xi N^2} \approx 7.7 \cdot 10^{-11}$

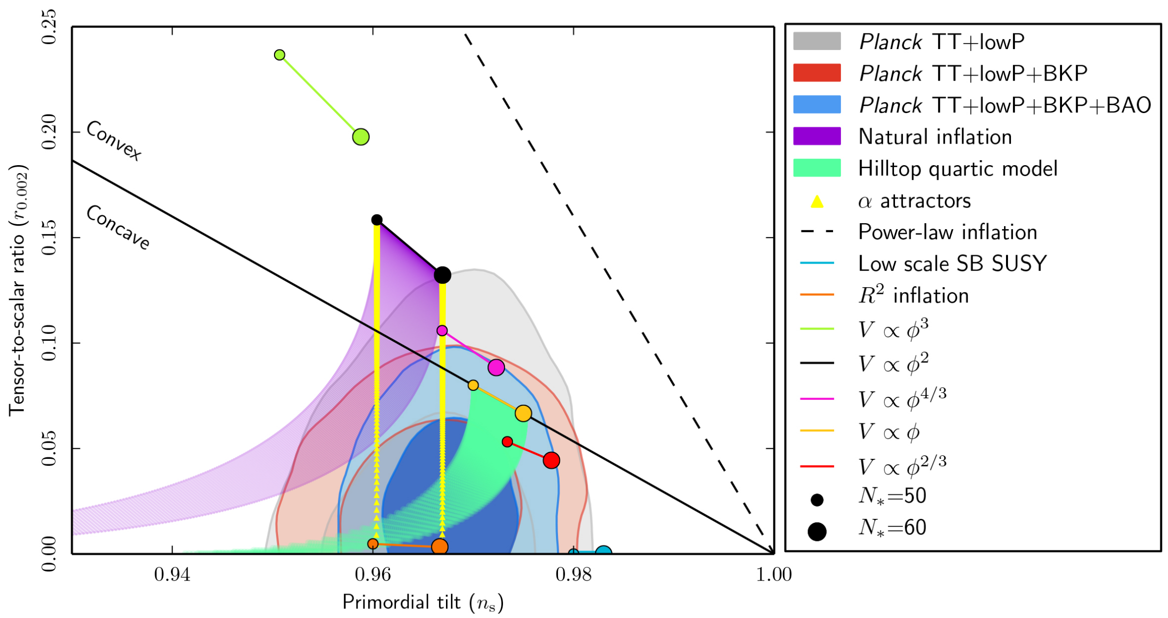
Table 1: Comparison between metric and Palatini Higgs inflation at tree level and for $\xi \gg 1$. In the Palatini case, we use $\xi = 10^7$. Since the analysis of this paper shows that ξ can deviate from this value by an order of magnitude, we use the symbol "$\approx$" when displaying numerical values that depend on ξ. Expressions for n_s and r are given to the leading order in N^{-1} and ξ^{-1}.

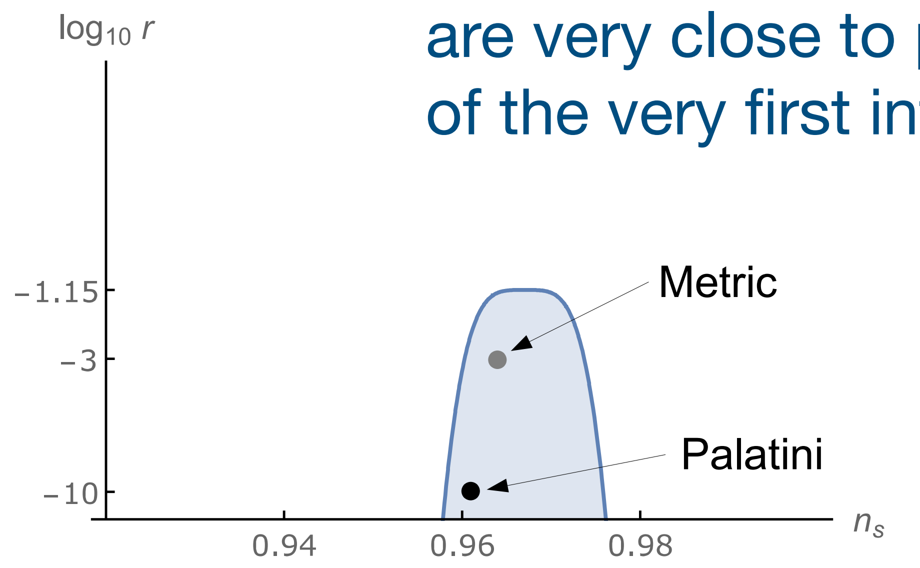
Advantages of the Palatini formulation:

- Parametrically larger UV cutoff
- More robust relation between low energy and high energy parameters
- Natural relation between Fermi and Planck scales

Predictions of metric and Palatini Higgs inflations







Predictions of metric Higgs inflation
are very close to predictions
of the very first inflationary model by Starobinsky

Generic EC Higgs inflation

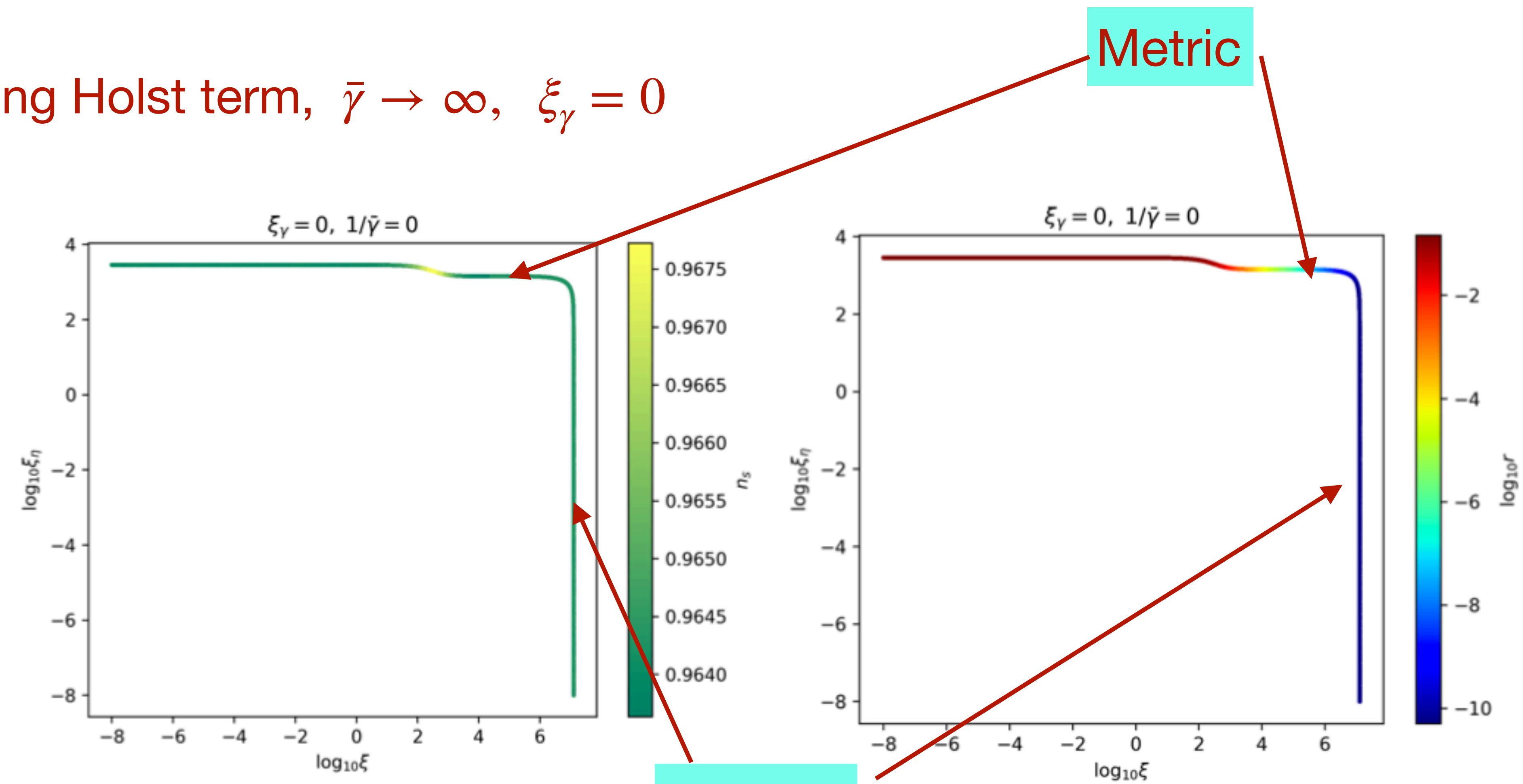
By varying continuously the Higgs coupling to the Nieh-Yan term, one can deform one model into another!

New types of the Higgs inflation (see also Langvik, Ojanpera, Raatikainen and Rasanen, 2007.12595):

- "Nieh-Yan Higgs inflation": vanishing Holst term, $\bar{\gamma} \to \infty, \quad \xi_\gamma = 0.$
- "Holst Higgs inflation", $\xi_\gamma = \xi_\eta = 0.$
- Generic Einstein-Cartan Higgs inflation.

Nieh-Yan Higgs inflation

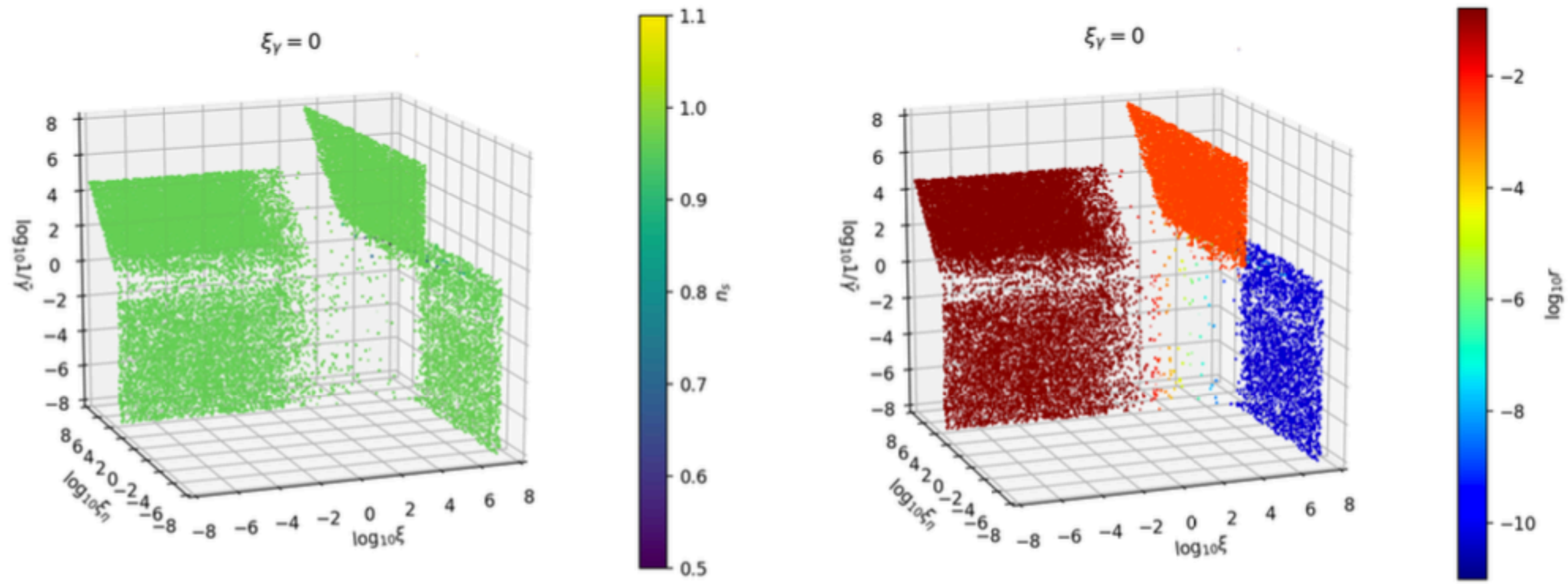
Vanishing Holst term, $\bar{\gamma} \to \infty, \ \ \xi_\gamma = 0$



(a) (b)

Figure 1. Spectral tilt (a) and tensor-to-scalar ratio (b) in Nieh-Yan inflation. We take $N_\star = 55$ and $\lambda = 10^{-3}$. The regions of Palatini (the right vertical segment) and metric (the "ankle" at which n_s and r vary considerably) Higgs inflation are clearly distinguishable. The transition between the two regions is smooth and stays within the observational bounds. The left horizontal segment has $r > 0.1$ and is not compatible with observations.

Generic Einstein-Cartan Higgs inflation



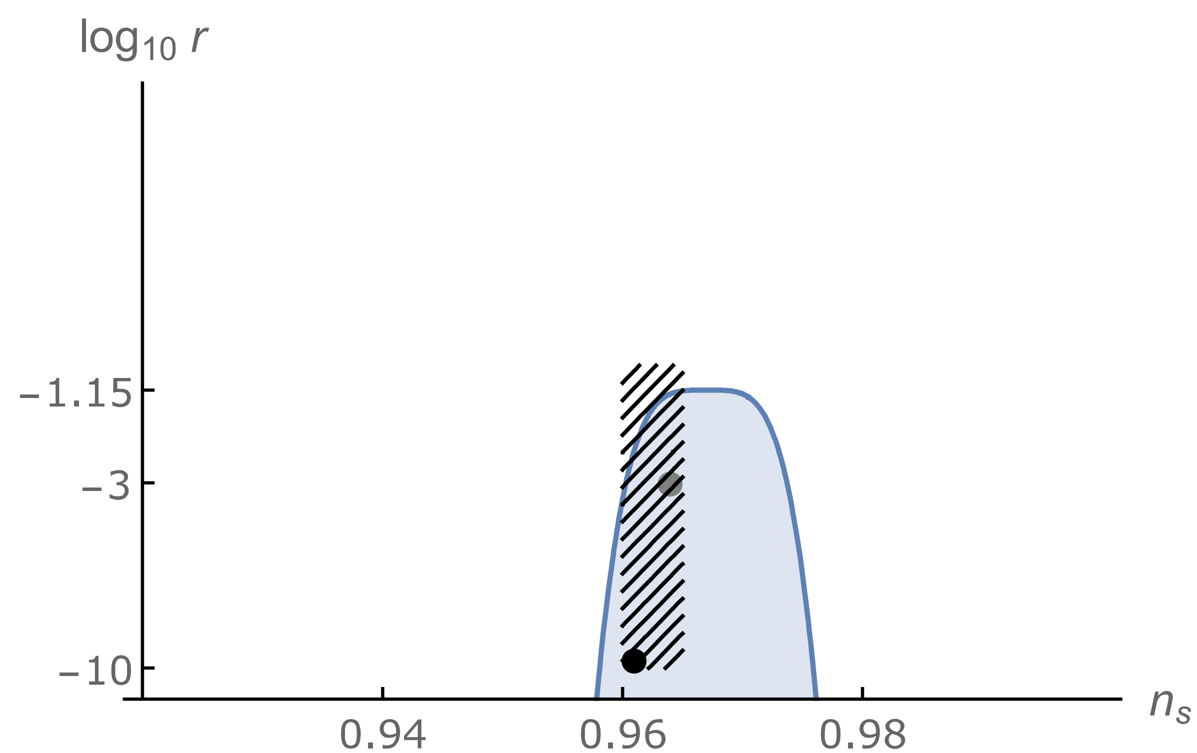
(a) (b)

Figure 5. Spectral tilt (a) and tensor-to-scalar ratio (b) in the case $\xi_\gamma = 0$. One can see that two regions in the right part of the plots reproduce metric and Palatini Higgs inflation. The left region is completely new. Note that due to the large values of the tensor-to-scalar ratio, this region is observationally excluded.

Generic Einstein-Cartan Higgs inflation

Observations:

- Inflation is a generic phenomenon.
- Large parts of the parameter space reproduce the predictions of either metric or Palatini Higgs inflation.
- The spectral index n_s is mostly independent of the choice of couplings and lies very close to $n_s = 1 - 2/N$.
- The tensor-to-scalar ratio r can vary between 1 and 10^{-10}. Detection of r in near future?



Challenges in Higgs inflation

1. Classical theory of the Higgs inflation should be promoted to the quantum theory of Higgs inflation. Any theory of inflation involves gravity which is non-renormalisable. An approach to every type of inflation (and HI in particular) should be formulated if the framework of some effective theory and be self-consistent.

2. The SM parameters are measured at small energies ~ 100 GeV, whereas inflation takes place at high energies: radiative corrections and RG running must be accounted for. What happens if the SM vacuum is metastable?

Challenge 1: Higher dimensional operators

Important: the energy domain of perturbative validity of the scalar-gravity theory with non-minimal coupling

$$S_G = \int d^4x \sqrt{-g}\left(-\frac{M_P^2}{2}R - \frac{\xi h^2}{2}R + \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}h^4\right)$$

Take some background field h, and consider all sorts of scattering reactions at energy E. Define the “cut-off” scale E = Λ at which the perturbative expansion breaks down.

This depends on the formulation of gravity, for example:

- For zero h background for the metric theory $\Lambda \sim M_P/\xi$ due to kinetic mixing of the Higgs and the metric (Burgess, Lee, Trott; Barbon and Espinosa). This may be dangerous for the Higgs inflation, as the typical scale of it is $M_P/\sqrt{\xi} \gg \Lambda,\ \xi \gg 1$.

- For zero h background for the Palatini theory $\Lambda \sim M_P/\sqrt{\xi}$ - no problem!

Several ways to proceed for the metric theory:

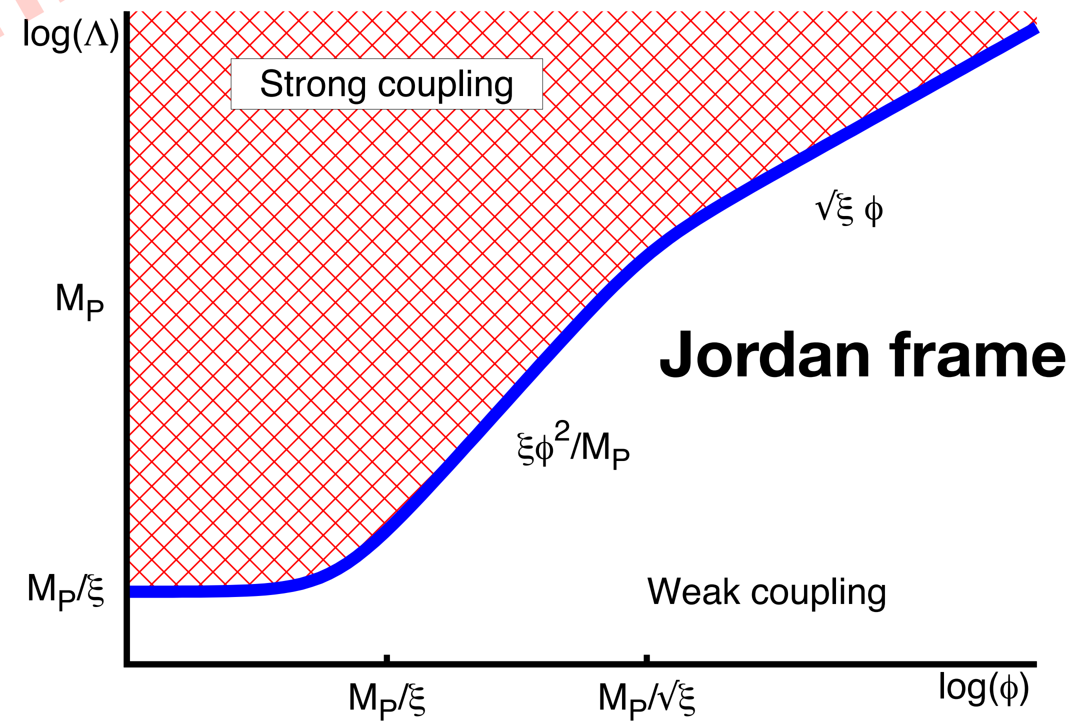
- Add new physics (new fields) and construct a theory in such a way that for zero Higgs and other fields backgrounds the inflation occurs in a weak coupling regime (Giudice, Lee, and many generalisations, e.g. for Higgs-Starobinsky inflation: Bezrukov, Gorbunov, Shepherd, Tokareva)

- Go beyond naive power counting: a refined effective field theory of Higgs Inflation

Beyond naive power counting: effective field theory of Higgs Inflation

In fact, the cutoff is background dependent (Bezrukov, Magnin, M.S., Sibiryakov; see also Ferrara, Kallosh, Linde, A. Marrani, Van Proeyen).

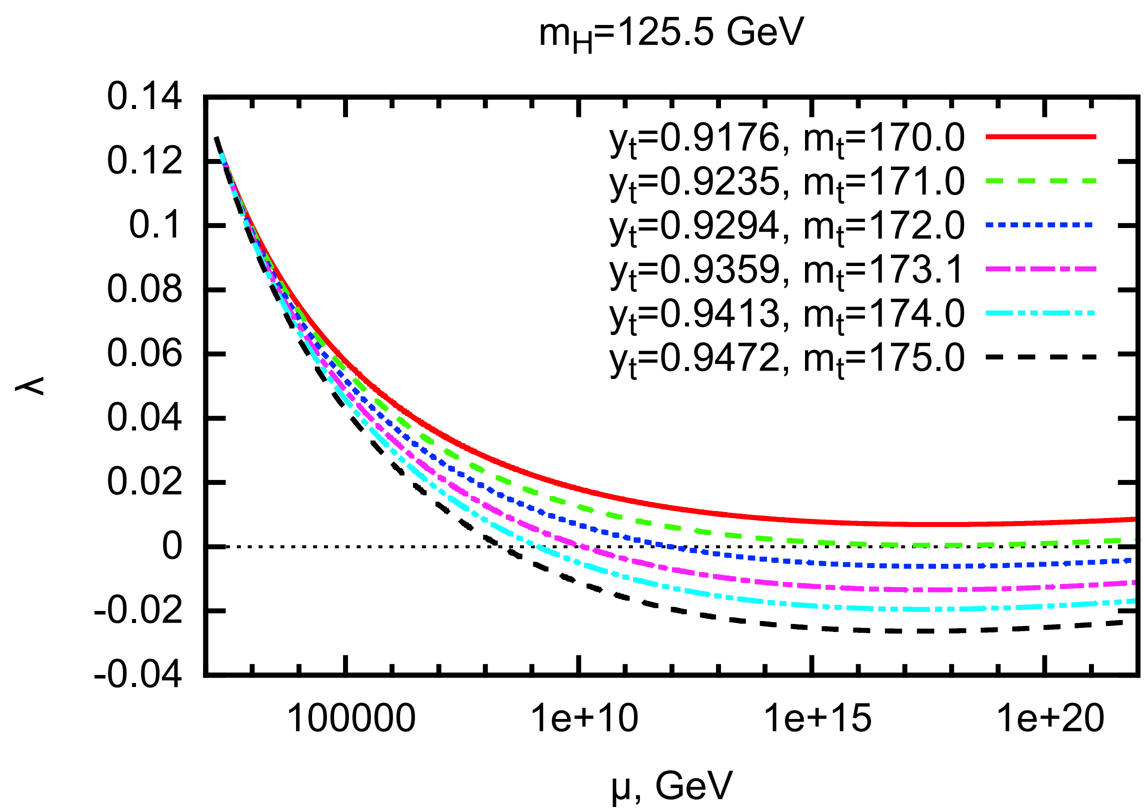
$$\Lambda(h) \simeq \begin{cases} \frac{M_P}{\xi}\,, & \text{for } h \lesssim \frac{M_P}{\xi}\,, \\ \frac{h^2\xi}{M_P}\,, & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}}\,, \\ \sqrt{\xi}h\,, & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}}\,. \end{cases}$$



Cutoff is higher than the relevant dynamical scales throughout the the inflationary epoch. The Higgs-inflation occurs in the weakly coupled regime.

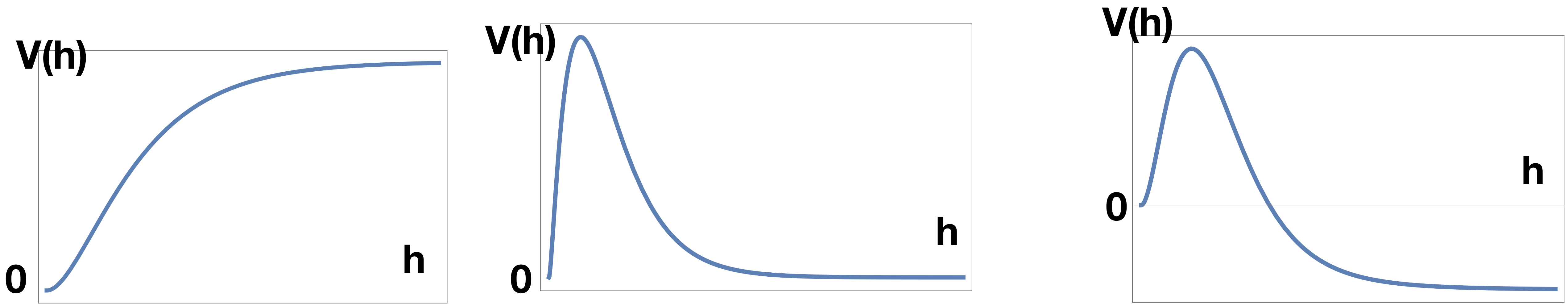
Challenge 2: possible vacuum metastability

Behaviour of the scalar self-coupling λ: depending on the top quark Yukawa coupling, λ may cross zero at energies as small as 10^{11} GeV (for larger m_t) or never cross it (for smaller m_t). For all admissible SM parameters, $|\lambda| \sim 0.01$ in inflationary region, much smaller than at low energies



This behaviour may change the form of the Higgs potential at large h

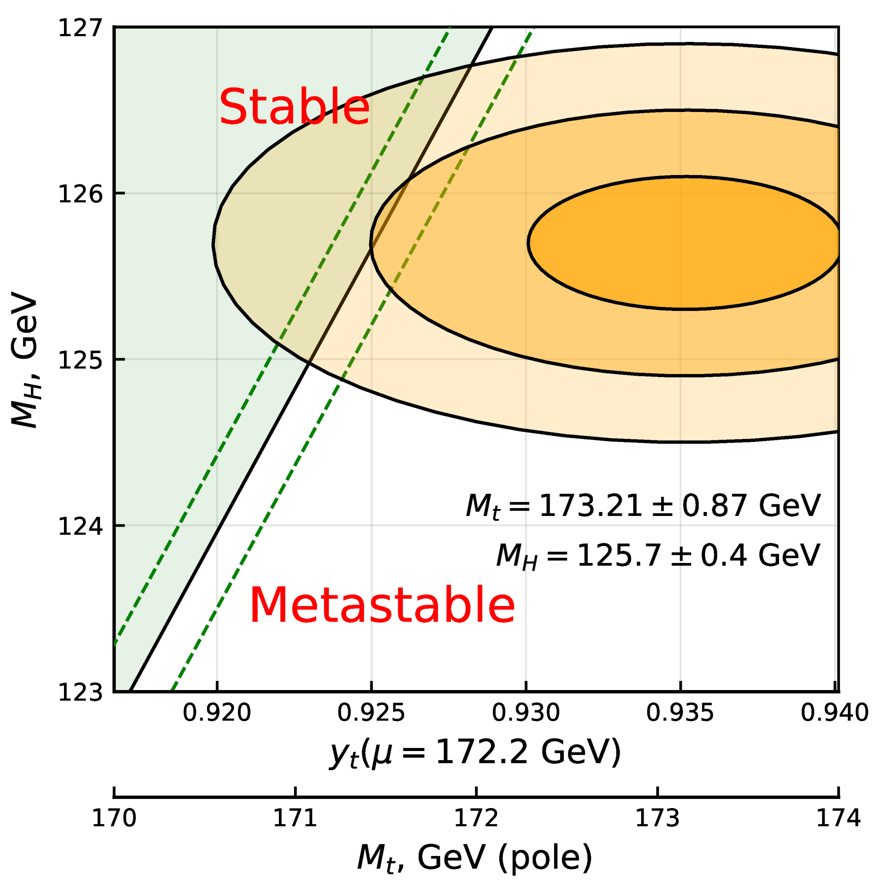
Naive RG: $V(h)=\lambda(h)h^4$



Stability **Criticality** **Metastability**

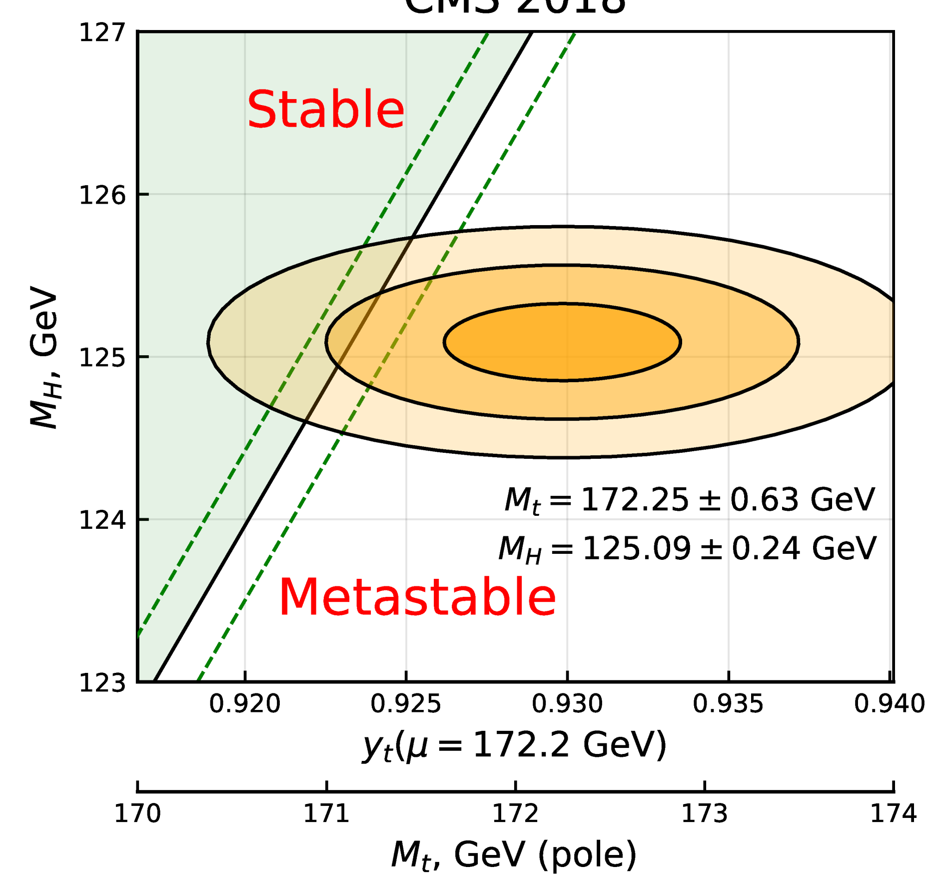
- Marginal evidence (less than 2σ) for the SM vacuum metastability given uncertainties in relation between Monte-Carlo top mass and the top quark Yukawa coupling

2015



Time evolution of SM vacuum metastability

CMS 2018



Radiative corrections in Higgs Inflation

The minimal setup, working for renormalisable theories: add to the Lagrangian all counter-terms necessary to make the theory finite. The theory is predictable: everything is expressed via few parameters.

The HI theory is not renormalisable - how to deal with radiative corrections? The minimal approach: add to the Lagrangian all counter-terms necessary to make the theory finite. The theory is not predictable for all energy scales, as the number of appearing structures - counter-terms is infinite. However, if the energy scale is well below the dynamical cutoff $\Lambda(h)$, the reliable computations can be done and ignorance of UV completion can be parametrised by the unknown coefficients - finite part of counter-terms (Bezrukov, Magnin, MS, Sibiryakov; Burgess, Patil, Trott …). RG evolution of coupling constants from the Fermi to inflation scale is possible. Studies of RG evolution: Simone, Hertzberg, Wilczek; Barvinsky, Kamenshchik, Kiefer, Starobinsky, Steinwachs…

Summary of the outcome of this program:

Example: metric Higgs inflation

- The inflationary Higgs potential is well defined for $h \lesssim M_P/\xi$ and is expressed via low energy SM parameters

- The inflationary Higgs potential is well defined for $h \gtrsim M_P/\sqrt{\xi}$ and is expressed via SM low energy parameters and unknown matching coefficients for all coupling constants describing the "jumps" of couplings at the field value $h \simeq M_P/\xi$

- The predictions of HI for $n_s = 0.97$ and r=0.0033 remain the same almost for all parameters (for a detailed study of the parameter space see Enckell, Enkvist, Nurmi; Bezrukov, Pauly, Rubio) except for one very specific point corresponding to the "critical" Higgs inflation. Studies of radiative corrections: Fumagalli, Mooij, Postma,…

- For some choice of these matching coefficients HI can be realised even for metastable vacuum

Symmetries of UV completion?

- The successful models of inflation (Starobinsky, Higgs, **α** - attractors) all share the same feature: constant potential in the Einstein frame at large values of the canonically normalised scalar field:

$$S = \int d^4x \sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}M_P^4\right)$$

- The theory has a shift symmetry, $\phi \to \phi$+const. In the Jordan frame the action is scale-invariant,

$$S = \int d^4x \sqrt{-g}\left(-\frac{\xi h^2}{2}R + \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}h^4\right)$$

- Perhaps, the Nature is scale-invariant? The scale-invariant Higgs-dilaton inflation (scale invariance is broken spontaneously) requires dynamical generation of the Planck scale and is based on the action

$$S = \int d^4x \sqrt{-g}\left(-\frac{\xi_h h^2 + \xi_\chi \chi^2}{2}R + \frac{1}{2}(\partial h)^2 + \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}h^4\right)$$

For cosmology of scale-invariant theory see Garcia-Bellido, Rubio, MS, Zenhausern; Trashorras, Nesseris, Garcia-Bellido; Ferreira, Hill, Ross,…

Conclusions

Einstein-Cartan gravity is an interesting theory with the following properties:

- It has the same number of degrees of freedom as the metric gravity.

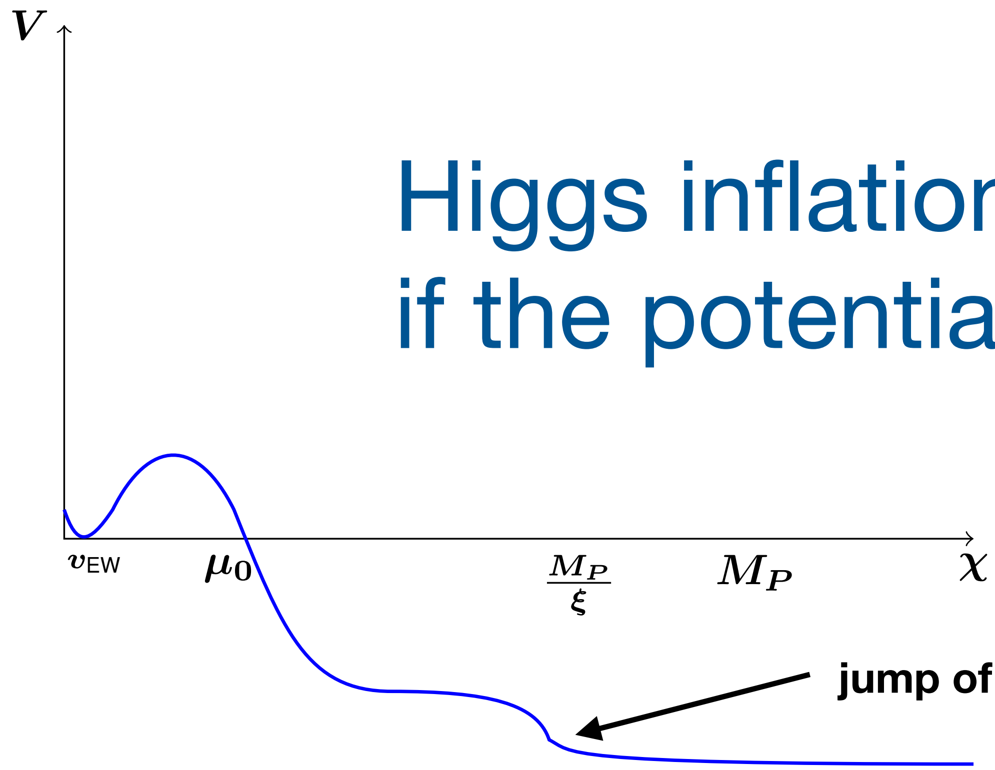
- Higgs inflation and the hot big bang with $T_{\rm reh} \gtrsim 4 \times 10^{13}$ GeV are the natural consequence of EC gravity with a universal prediction for $n_s = 1 - 2/N \simeq 0.96$, and with r which can be as small as 10^{-10} and as large as the present experimental limit.

Topics not covered:

- EC gravity leads to a new universal mechanism for fermion dark matter production operating for masses as small as few keV and as large as $(3-6) \times 10^8$ GeV.

- EC gravity may lead to an explanation why the Fermi scale is much smaller than the Planck scale, using non-perturbative semiclassical effects - a new type of singular instanton

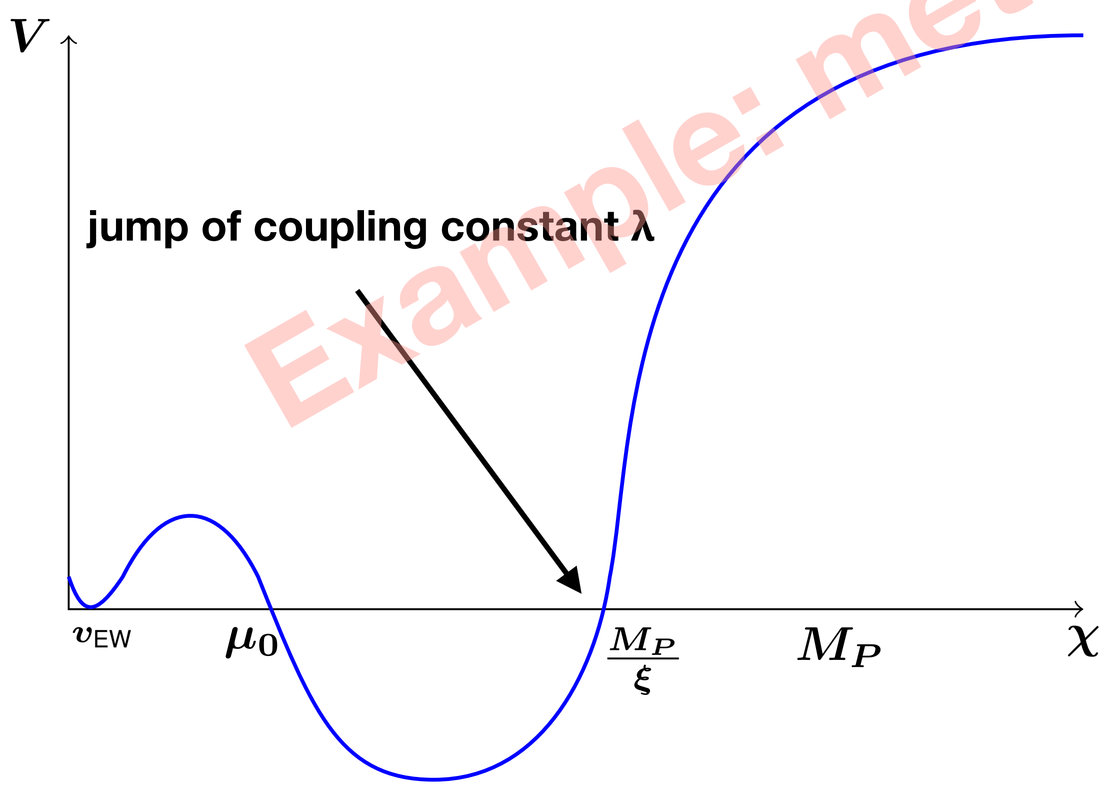
Back up slides



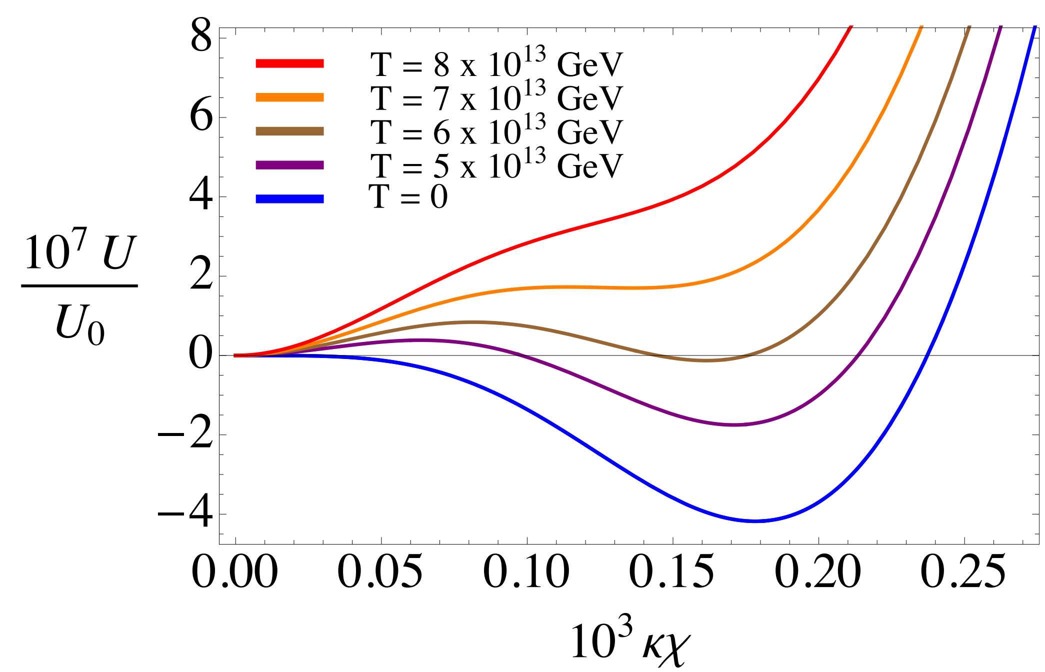
Higgs inflation does not work if the potential has this form

Bezrukov, Rubio, MS

Higgs inflation still works if the potential has this form, as reheating brings the Higgs field to the origin.

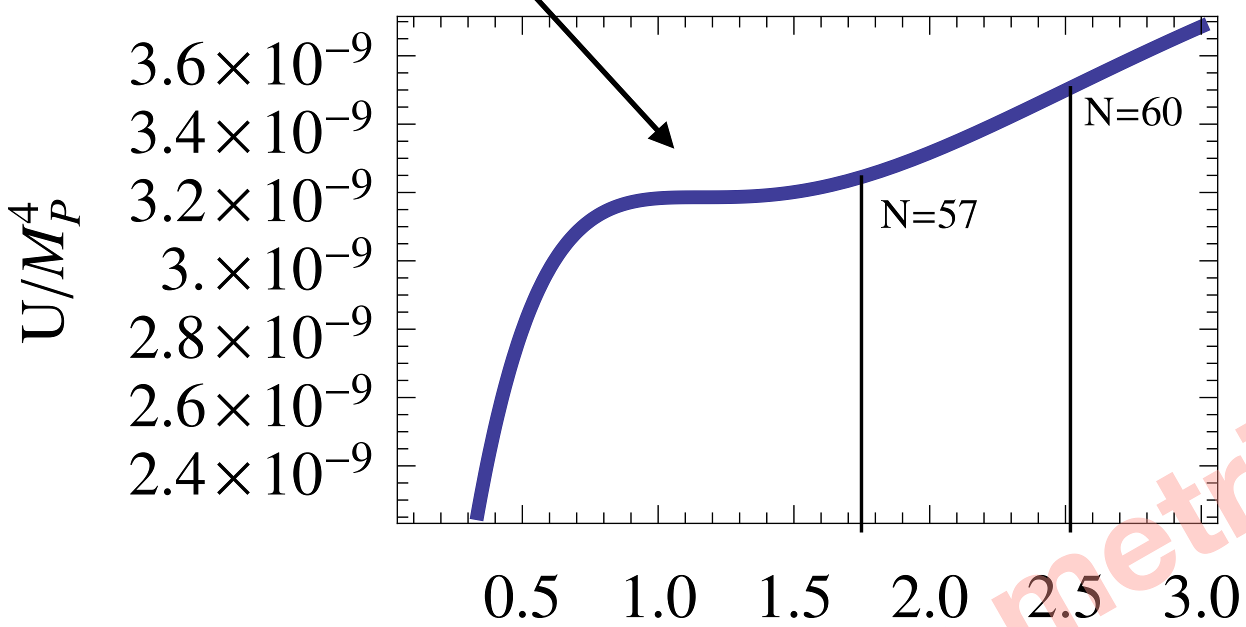


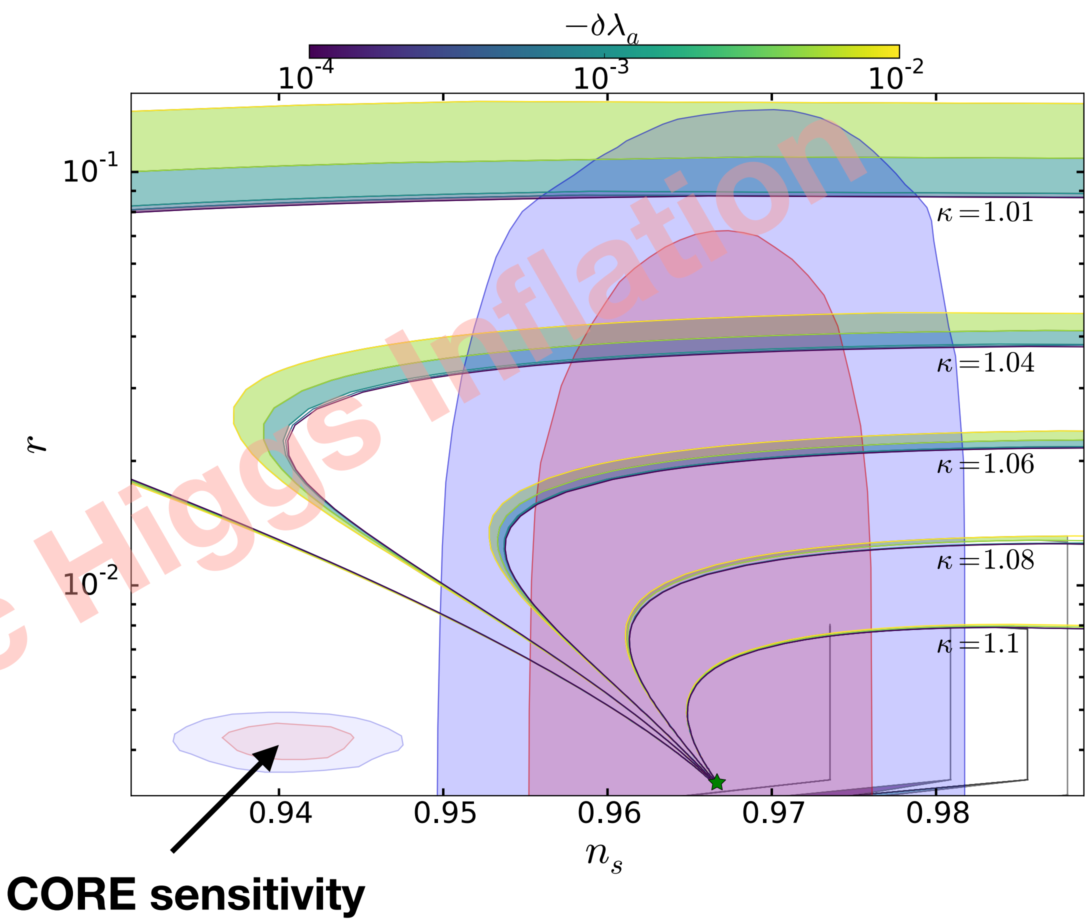
Symmetry restoration



Critical Higgs Inflation

For a very particular choice of top and Higgs masses, and of the matching coefficients the Higgs potential can develop an inflection point: $V'(h_0) = V''(h_0) = 0$





Example: metric Higgs Inflation

Hamada, Kawai, Oda, Park; Bezrukov, MS; Bezrukov, Pauly, Rubio, …

Primordial Black Holes? Garcia-Bellido; Rubio,…