

« Available energy is the main object at stake in the struggle for existence and the evolution of the world. »

L. Boltzmann

(Dark) matter production in the (very) early Universe



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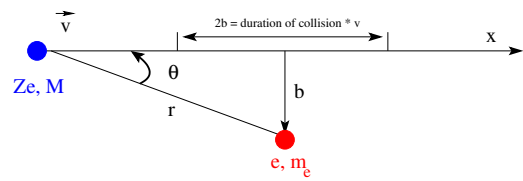


Fig. 5.9 Interaction of a high energy particle of charge Ze with an electron at rest.

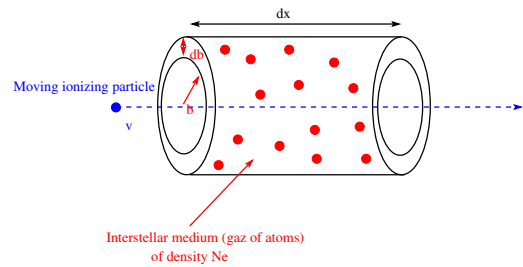


Fig. 5.10 Moving particle in an interstellar medium of density N_e .

distance at which the influence of the traveling particle on the electron is negligible. It corresponds roughly to the time when the orbital period is lower than the typical interaction time. In other words, if the electron takes more time to move around the nucleus than to interact with the moving particle, the electromagnetic influence of the later becomes weak. If one write τ the interacting time and v_0 the frequency of the rotating electron in the atom ($v_0 = \omega_0/2\pi$), it corresponds to

$$\tau \approx \frac{2b}{v} < \frac{1}{v_0} \Rightarrow b < \frac{v}{2v_0} = b_{max} \quad (5.37)$$

The lower limit b_{min} can be obtained if we suppose, by a quantum treatment and the application of the uncertainty principle, that the maximum energy transfer is $\Delta p_{max} = 2m_e v$ (because as we discussed earlier, the maximum velocity transferred to the electron is $2v$) from $\Delta p \Delta x \geq \hbar$ (Heisenberg principle) we have $\Delta x \geq \hbar/2m_e v$. We can then write

The two parts of the Lagrangian one needs to compute the scalar annihilation of Dark Matter $SS \rightarrow h \rightarrow f\bar{f}$ are (see B.235)⁹

$$\begin{aligned} \mathcal{L}_{HSS} &= -\lambda_{HS} \frac{M_W}{2g} hSS \rightarrow C_{HSS} = -i \frac{\lambda_{HS} M_W}{g} \\ \text{and } \mathcal{L}_{Hff} &= -\frac{gm_f}{2M_W} h\bar{f}f \rightarrow C_{Hff} = -i \frac{gm_f}{2M_W} \end{aligned} \quad (B.145)$$

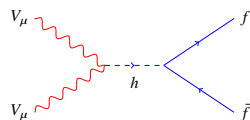
which gives

$$|\mathcal{M}|^2 = \frac{\lambda_{HS}^2 m_f^2 (s/2 - 2m_f^2)}{(s - M_H^2)^2 + \Gamma_H^2 M_H^2} \quad (B.146)$$

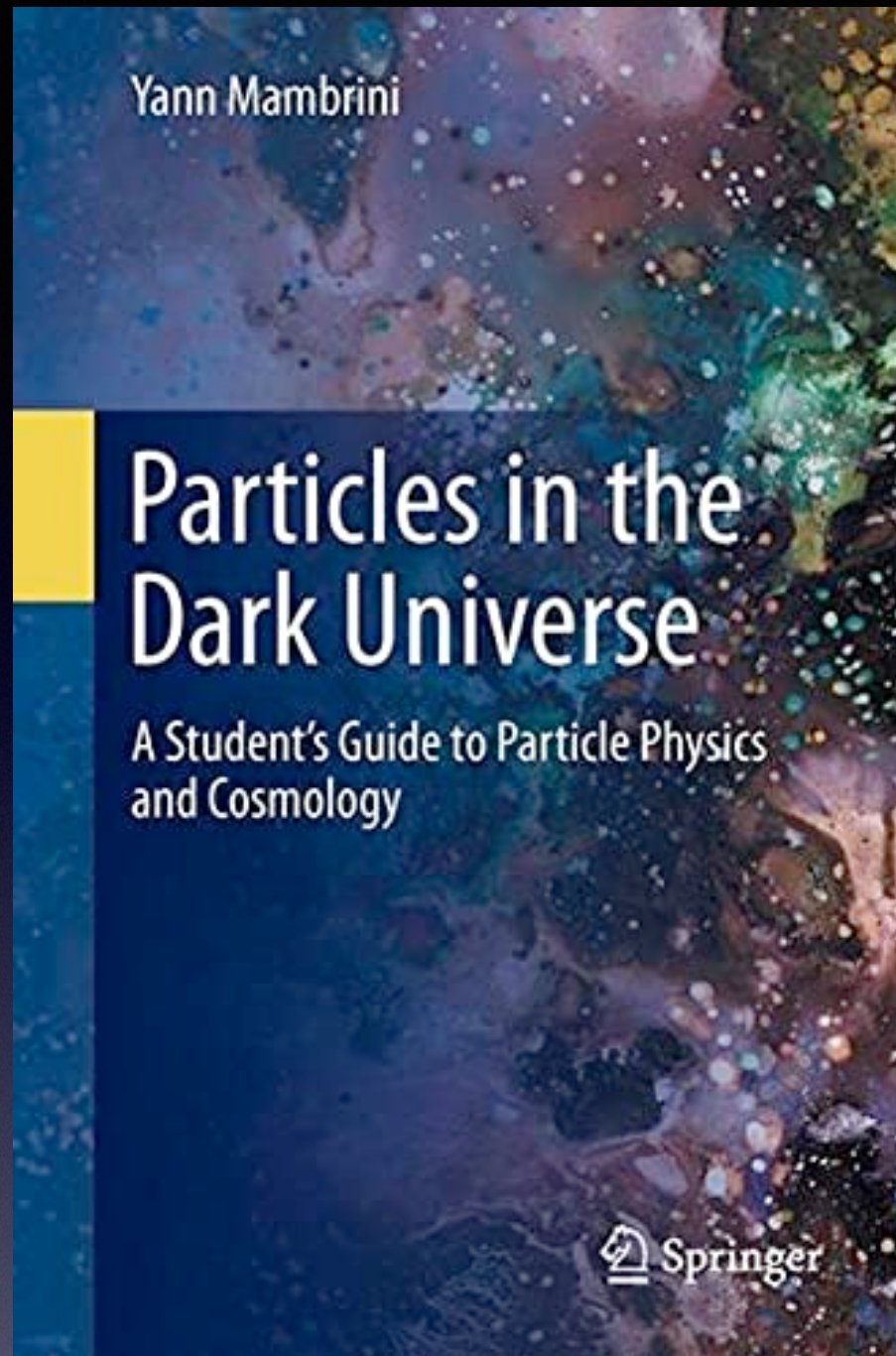
Γ_H being the width of the Higgs boson (including its own decay into SS , see next section). When ones implement this value of $|\mathcal{M}|^2$ into Eq.(B.111) one obtains after simplification

$$\langle \sigma v \rangle_{f\bar{f}}^S = \frac{|\mathcal{M}|^2}{8\pi s} \sqrt{1 - \frac{m_f^2}{M_S^2}} = \frac{\lambda_{HS}^2 (M_S^2 - m_f^2) m_f^2}{16\pi M_S^2 (4M_S^2 - M_H^2)^2} \sqrt{1 - \frac{m_f^2}{M_S^2}} \quad (B.147)$$

B.4.4.11 Annihilation in the case of vectorial Dark Matter to pairs of fermions



One can compute this annihilation cross section by the normal procedure or noticing that a neutral vectorial dark matter of spin 1 corresponds to 3 degrees of freedom. After averaging on the spin one can then write $\langle \sigma v \rangle^V = \frac{3}{3 \times 3} \langle \sigma v \rangle^S = \frac{1}{3} \langle \sigma v \rangle^S$. The academical computation for $V_\mu(p_1) V_\mu(p_2) \rightarrow f\bar{f}$ gives



500+ pages, from inflation to dark matter detection. All what is needed to compute cross-sections, relic abundance, and retrace the history of a Dark Universe.

Preface and forewords by K. Olive, J. Peebles and J. Silk

$$\ddot{S} + 3H\dot{S} + \left[\frac{k^2}{a^2} + \mu\Phi_0 \cos(m_\phi t) \right] S = 0 \quad (2.170)$$

Supposing $a \approx$ constant, we can neglect H . This equation is one form of the Mathieu equation, which is the equation for an oscillator with a time dependant frequency $\omega^2(t) = \frac{k^2}{a^2} + \mu\Phi_0 \cos(m_\phi t)$, and is present in a lot of classic phenomena involving periodical force. It can be shown that for

$$\frac{m_\phi}{2} - \frac{\mu\Phi_0}{2m_\phi} < \frac{k}{a} < \frac{m_\phi}{2} + \frac{\mu\Phi_0}{2m_\phi}, \quad (2.171)$$

we enter in a regime where the solution grows exponentially with time²³. We can understand it easily, from the shape of the Mathieu equation, where, periodically, the coefficient $\cos(m_\phi t)$ becomes negative and drives the evolution of S toward an exponential solution, periodically. The evolution of S is shown in Fig.(2.8). A more refined treatment necessitate to compute the Bogoliubov coefficient to extract the occupation number [10], but we give in the following section a more intuitive view of the phenomena, solving the equation for the density of the ϕ decay products. For the analytical solution of the Mathieu equation (2.170) the reader is directed to [9] which is without doubt the best textbook treating it, and [10] which is (paradoxically) the seminal *research* paper on the subject and one of the clearer and more detailed in the literature.

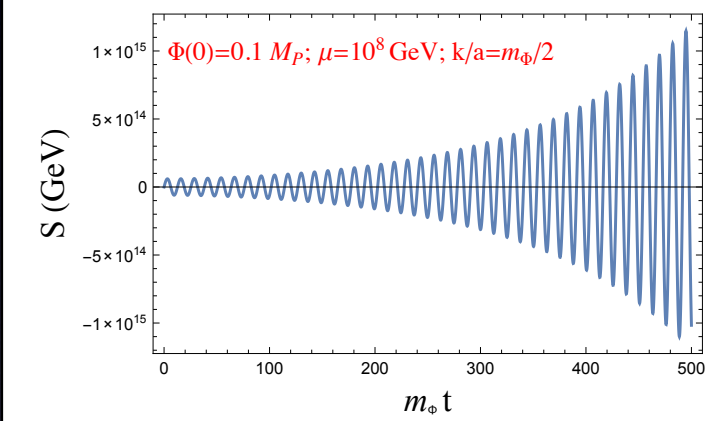
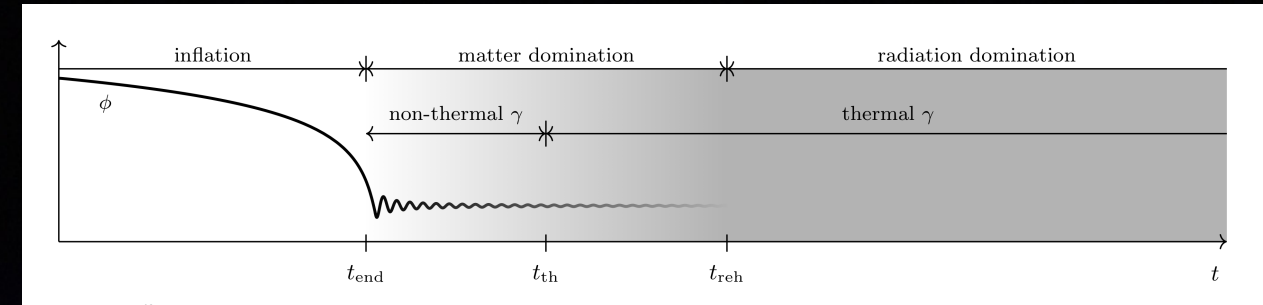
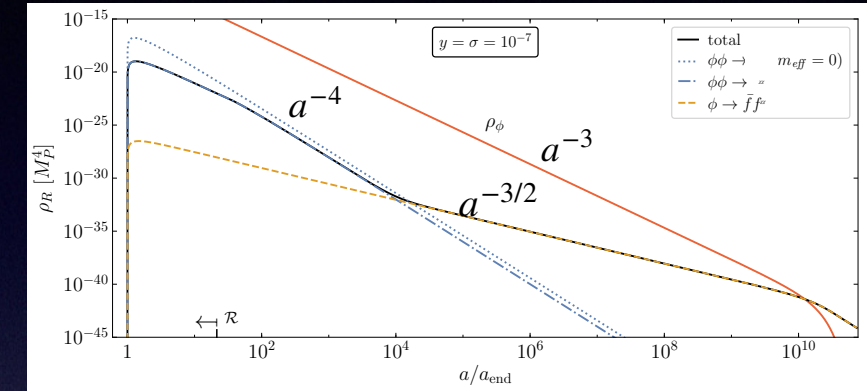


Fig. 2.8 Illustration of the parametric (also called *narrow*) resonance in the context of preheating. We can see clearly the exponential envelop of the periodic solution.

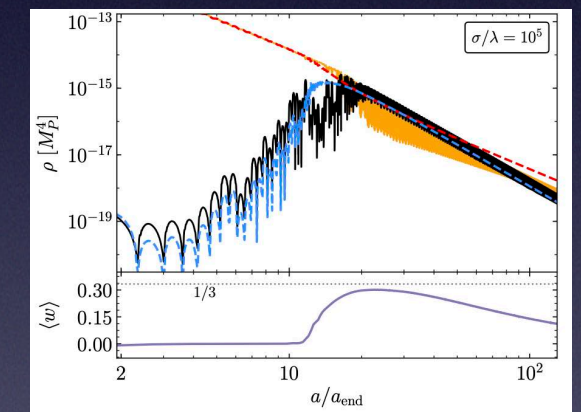
1). Reheating : generalities



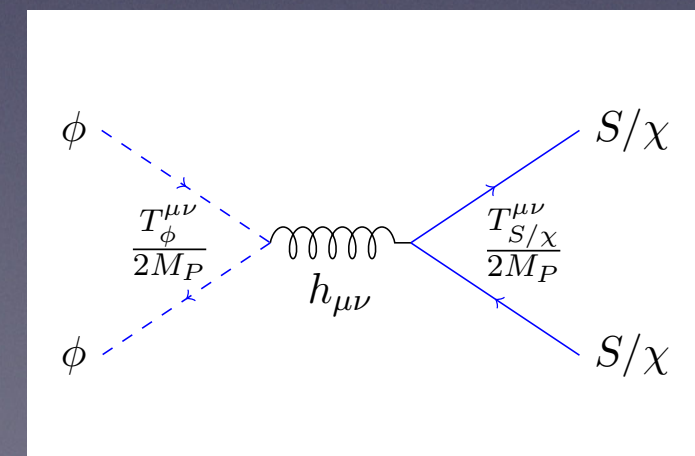
2). Non-instantaneous reheating



3). Preheating phase

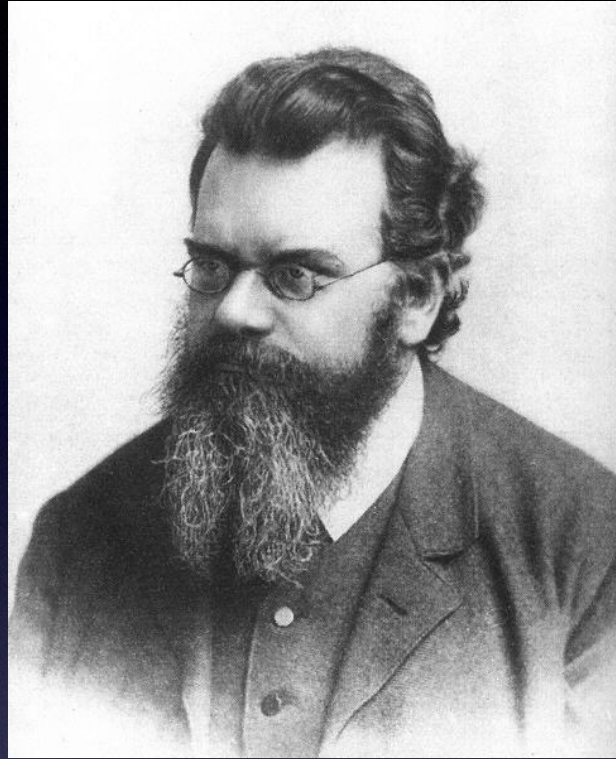


4). Application to gravitational production

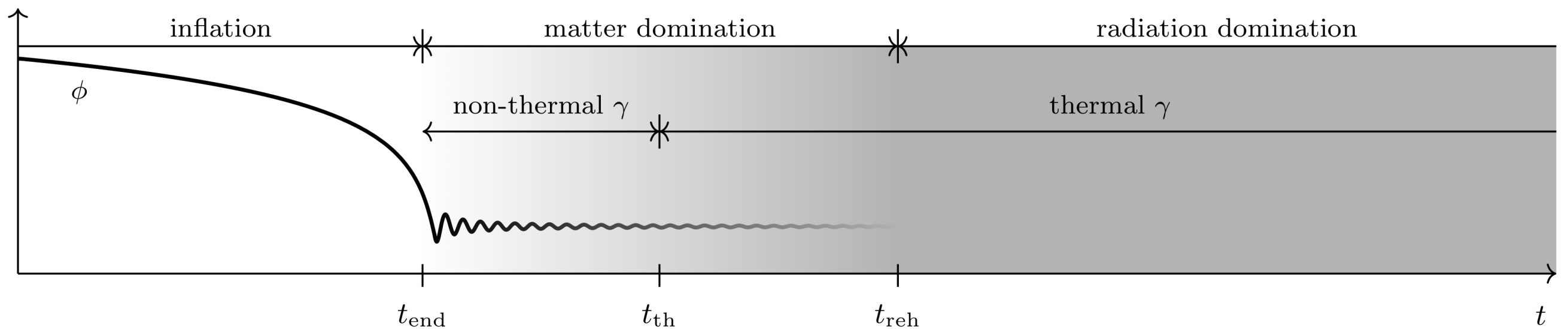


5). Conclusion

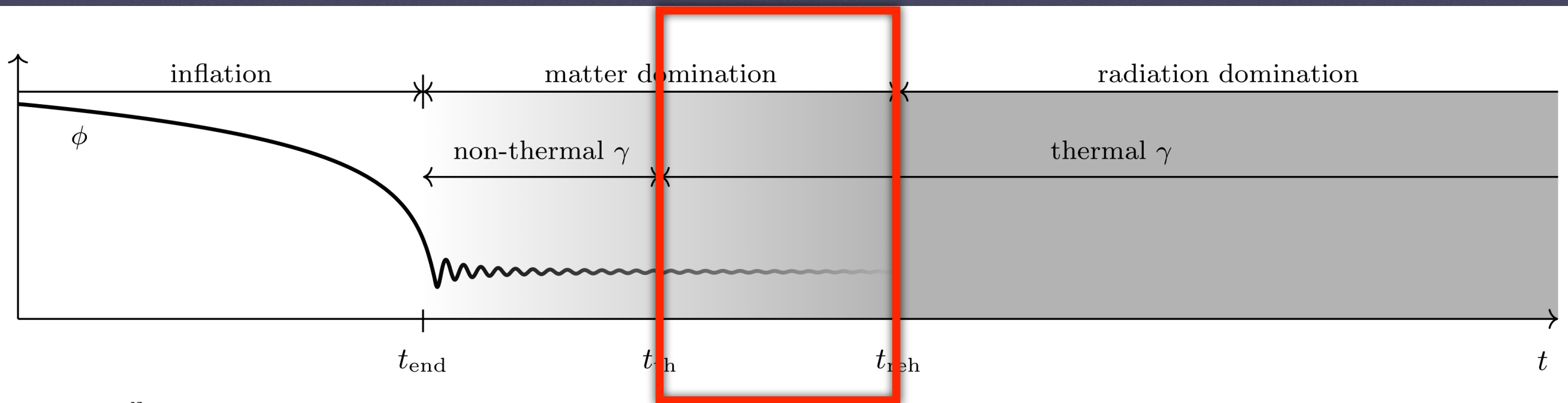
A brief history of the energy in the Early Universe



« Available energy is the main object at stake in the struggle for existence and the evolution of the world. »
L. Boltzmann



Mechanisms to produce (dark) matter at the **thermal reheating stage**



After several oscillations ($N \gtrsim 100$) :
The **reheating** phase

$$\rho_\phi = T_{00}^\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$P_\phi = T_{ii}^\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = 0$$

$$\rho_\phi = T_{00}^\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$V(\phi) = \lambda_k \phi^k$$

$$P_\phi = T_{ii}^\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

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$$V(\phi) = \lambda_k \phi^k$$

$$\dot{\rho}_\phi + \frac{6k}{k+2}H\rho_\phi = 0$$

$$\Rightarrow \rho_\phi \propto a^{-\frac{6k}{k+2}} = \langle V(\phi) \rangle$$

$$\rho_\phi = T_{00}^\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

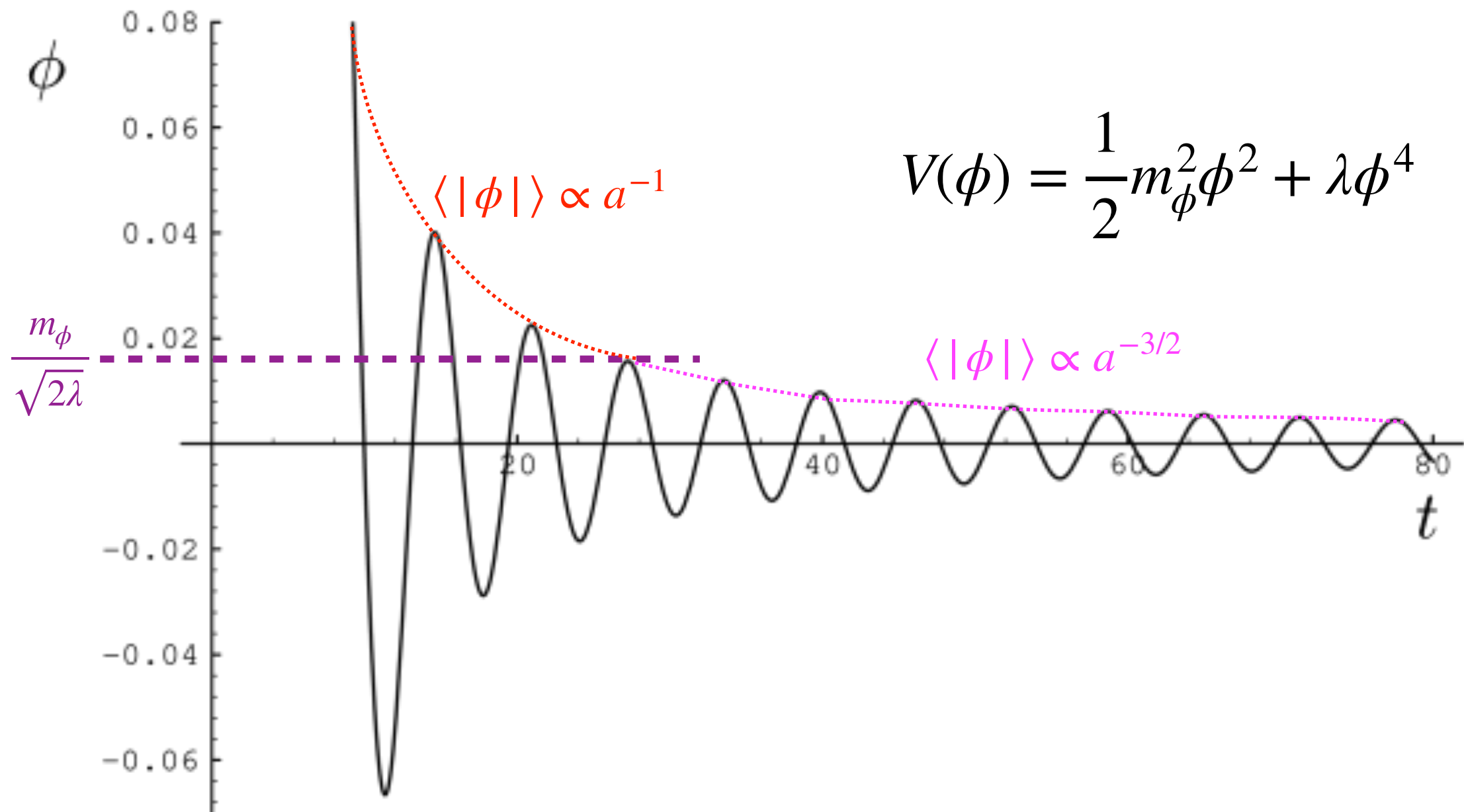
$$P_\phi = T_{ii}^\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

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$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4$$

Adding a coupling to matter (1)

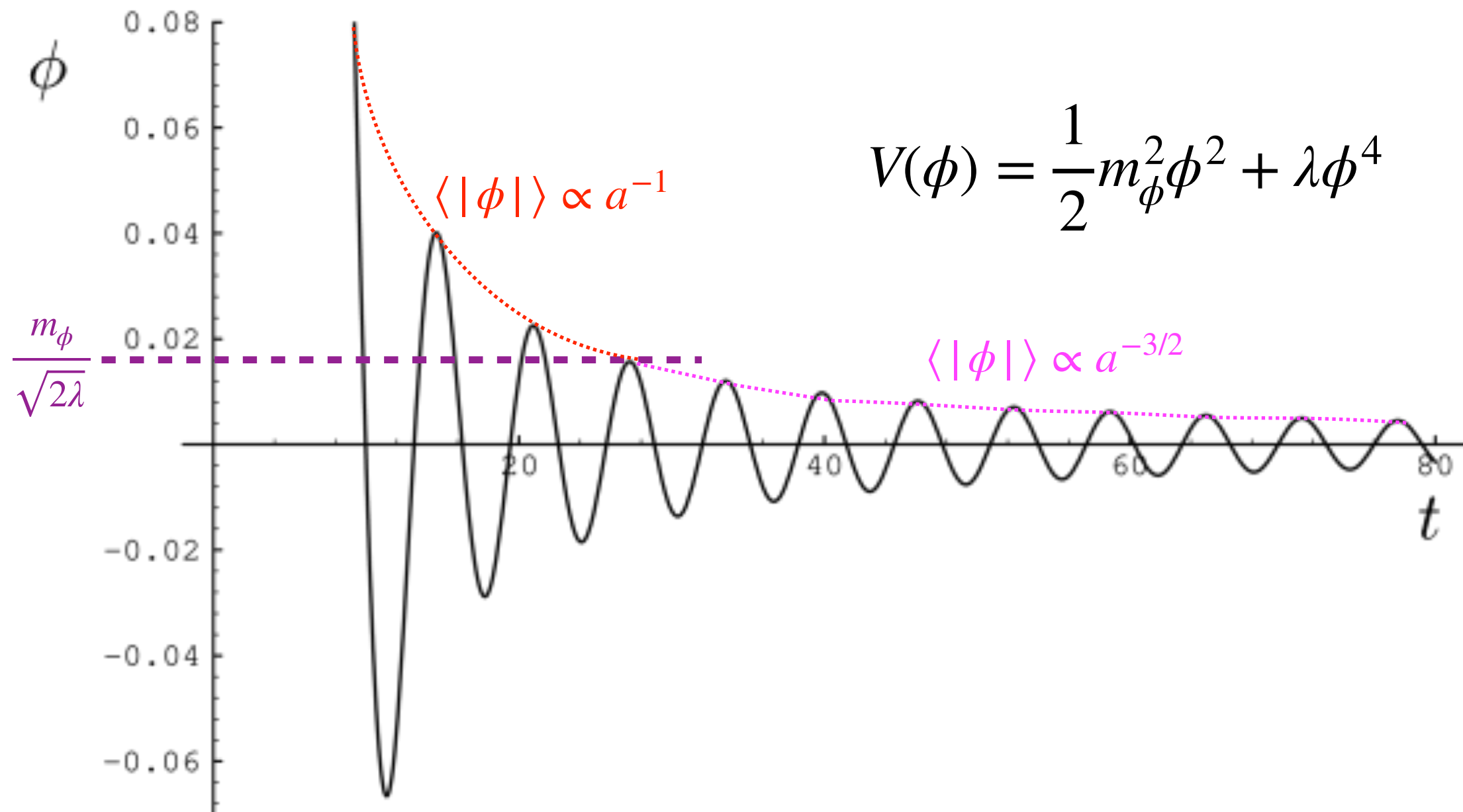
$$V = V(\phi) + y\phi\bar{f}f + \sigma\phi^2\chi^2$$

$$\sigma = 0$$

$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = 0$$

$$\dot{\rho}_\phi + \frac{6k}{k+2}H\rho_\phi = 0$$

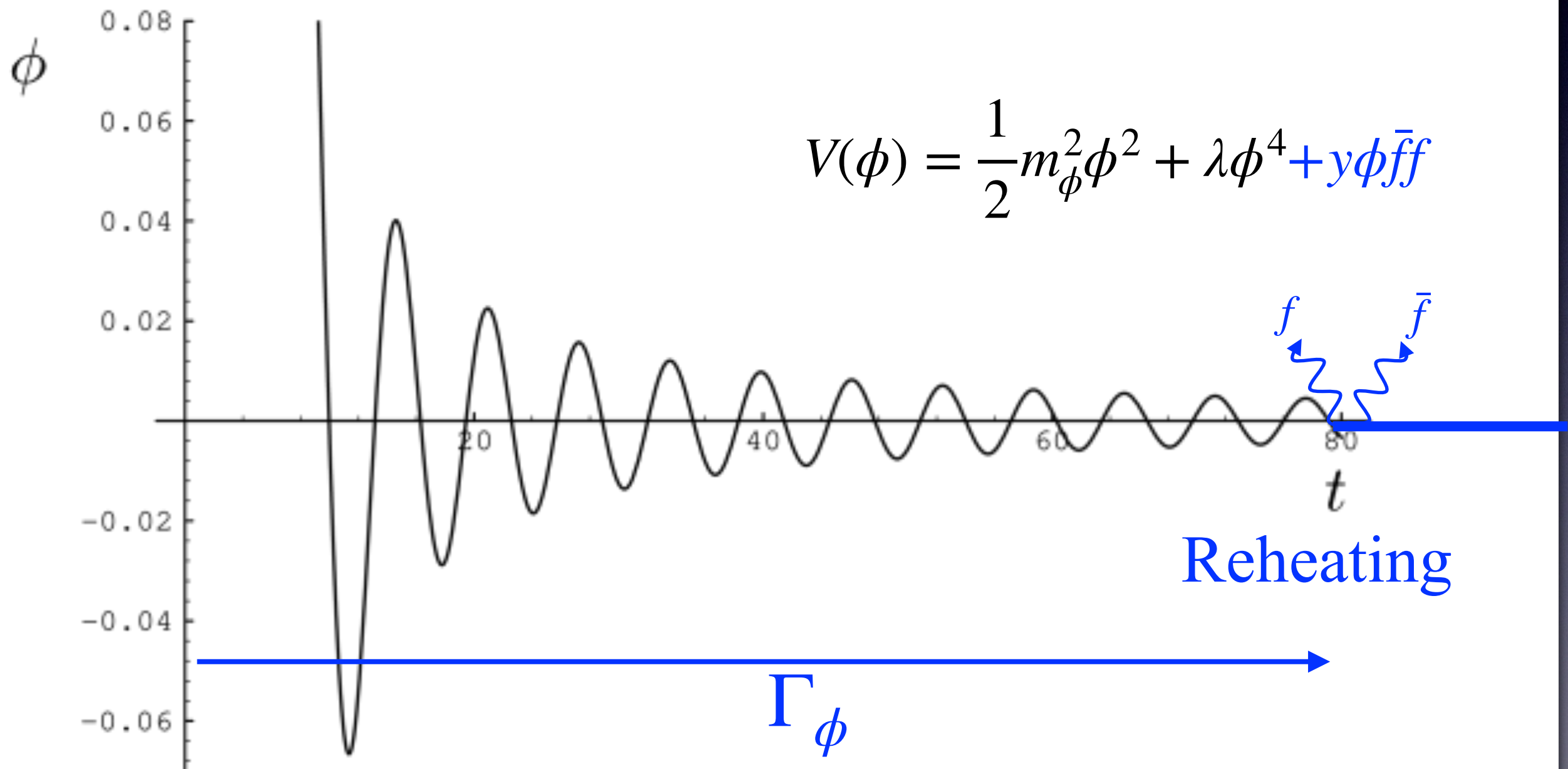
$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4$$



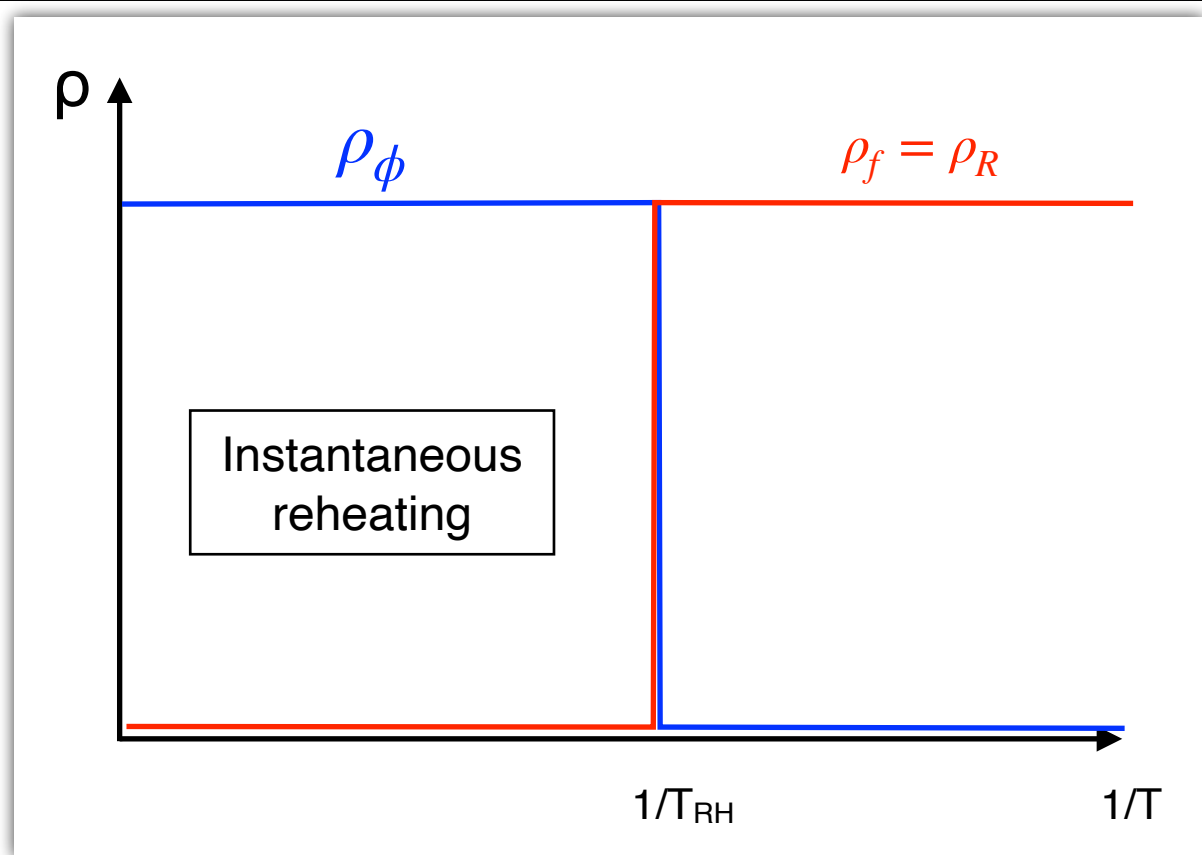
$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = -\Gamma_{\phi}\dot{\phi}$$

$$\dot{\rho}_{\phi} + \frac{6k}{k+2}H\rho_{\phi} = -\Gamma_{\phi}\rho_{\phi}$$

$$V(\phi) = \frac{1}{2}m_{\phi}^2\phi^2 + \lambda\phi^4 + y\phi\bar{f}f$$



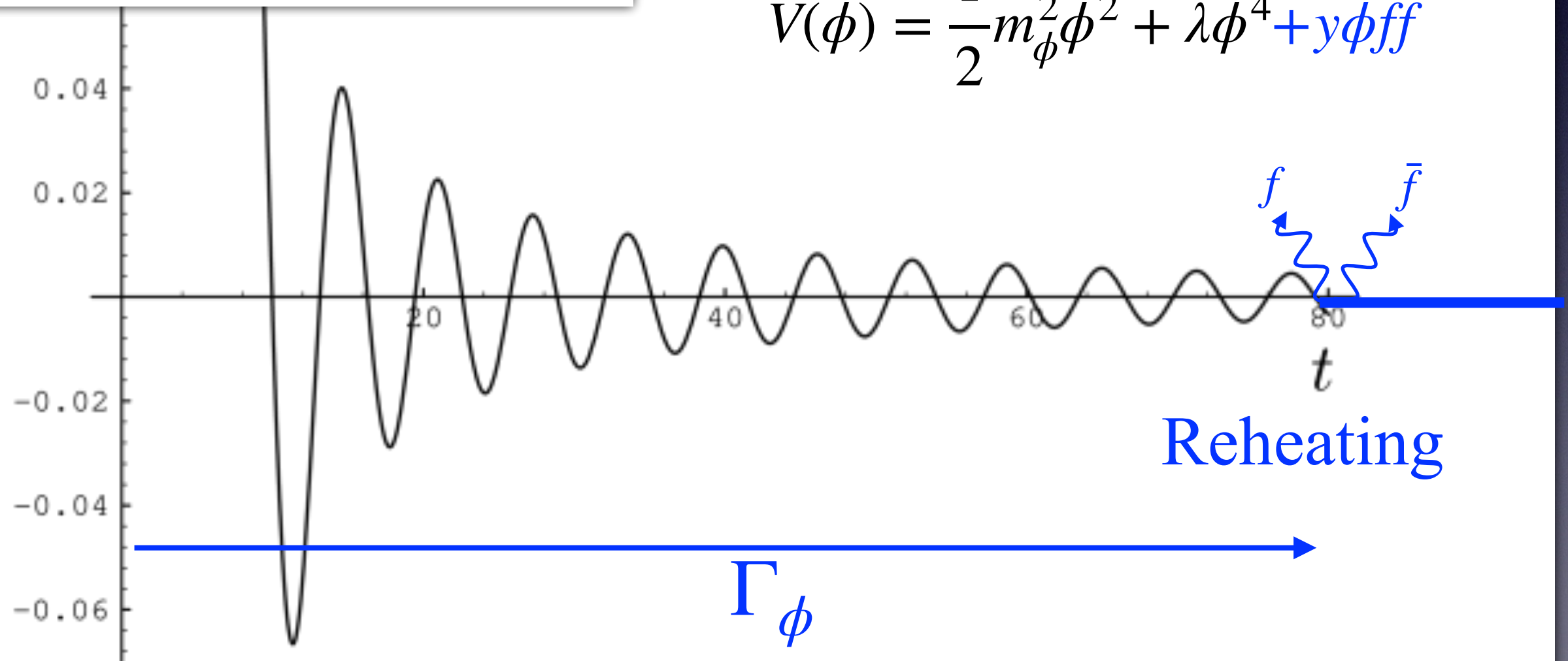
Reheating

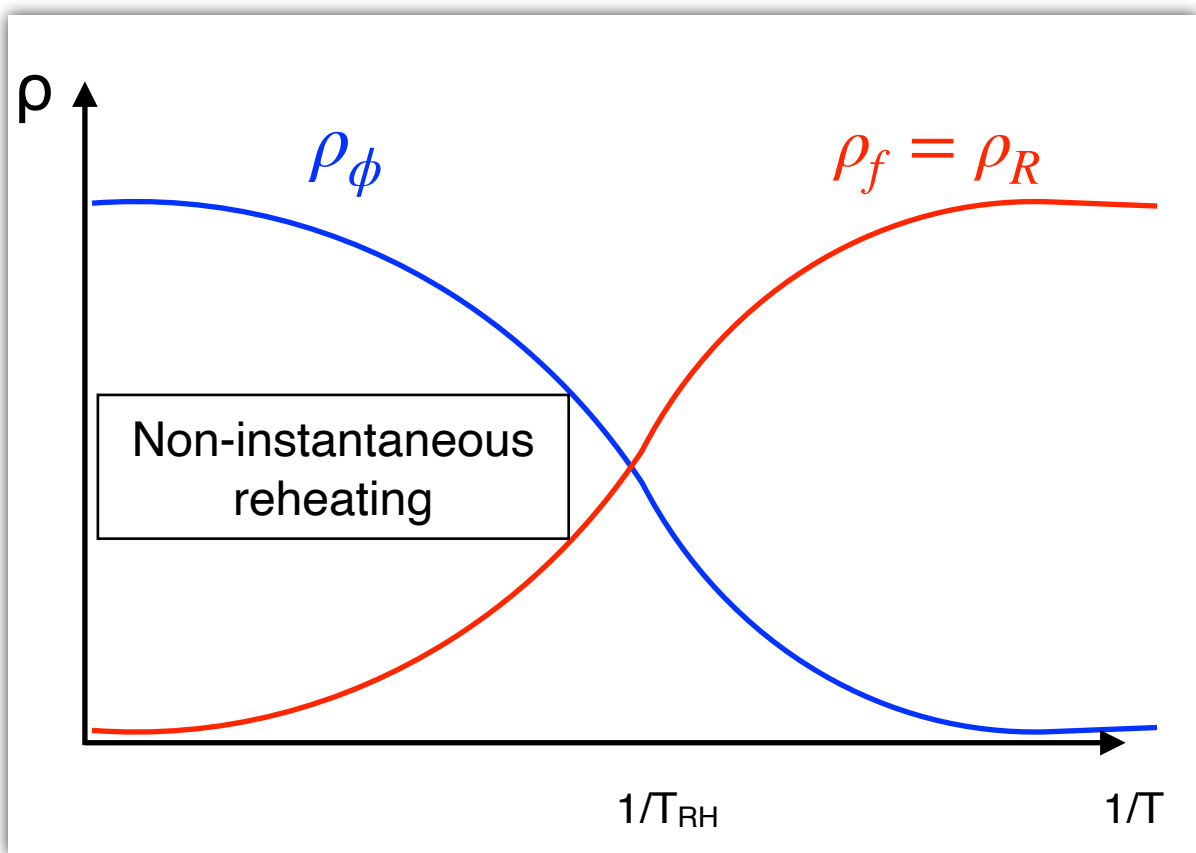


$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = -\Gamma_\phi\dot{\phi}$$

$$\dot{\rho}_\phi + \frac{6k}{k+2}H\rho_\phi = -\Gamma_\phi\rho_\phi$$

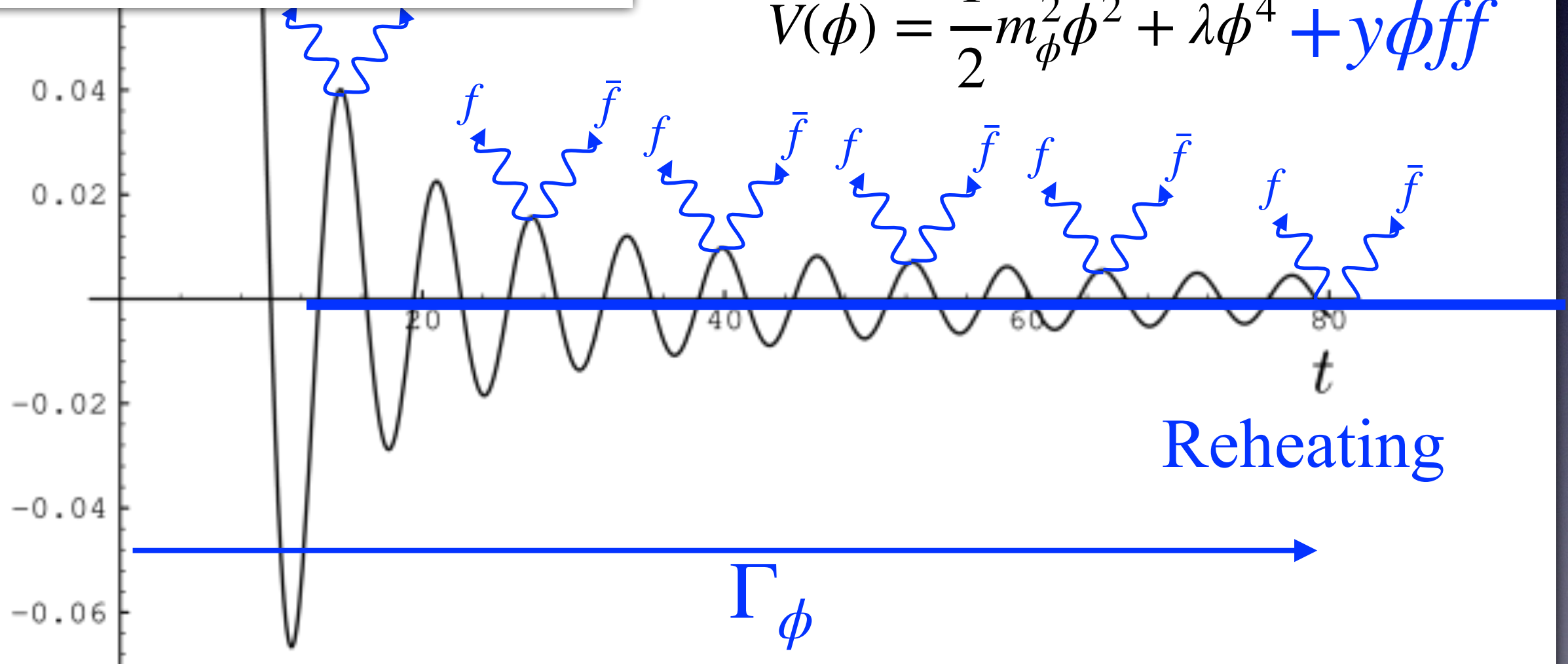
$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4 + y\phi\bar{f}f$$





$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = -\Gamma_\phi\dot{\phi}$$

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4 + y\phi f\bar{f}$$

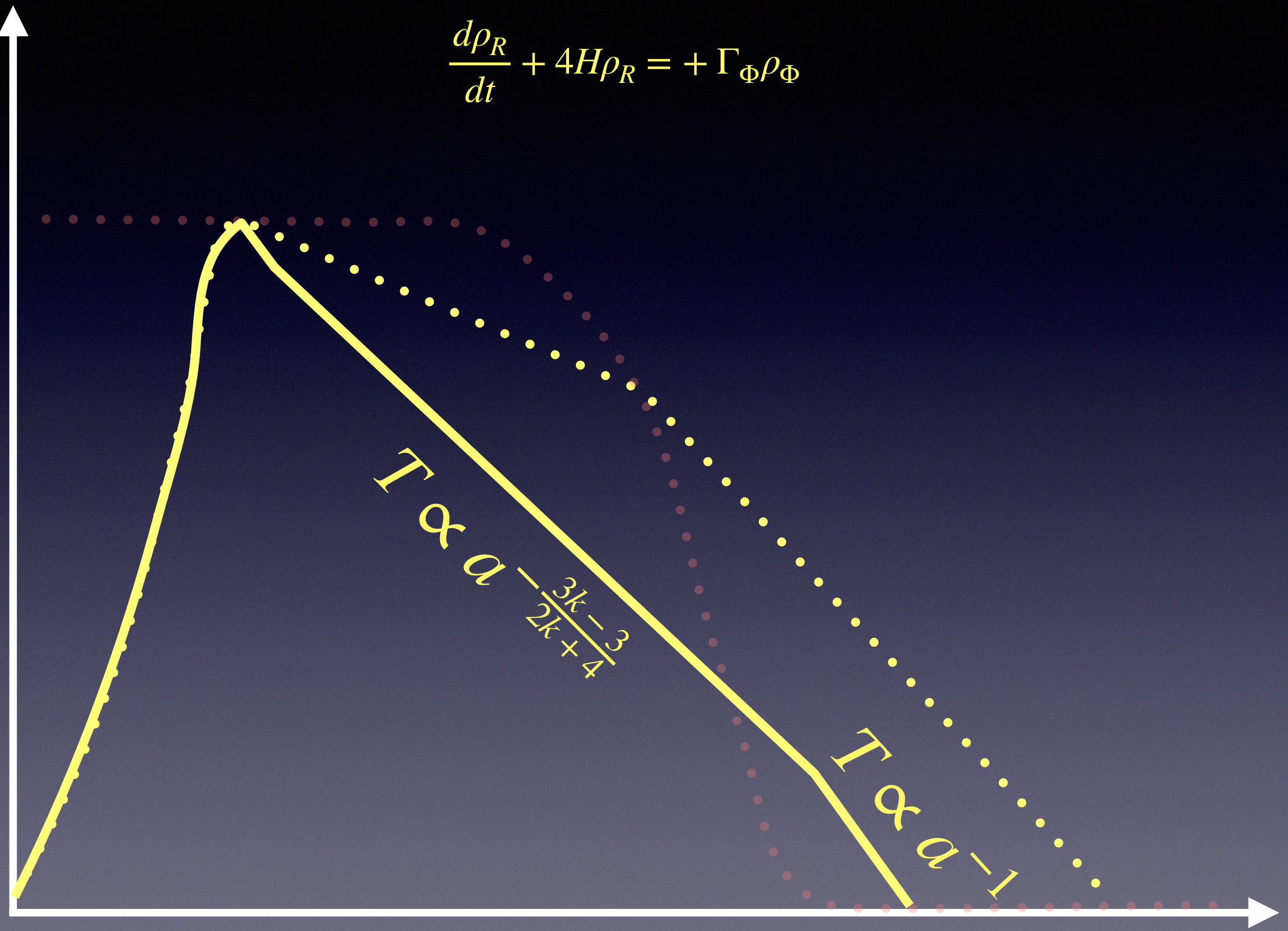


Temperature
(T)

$$\frac{d\rho_\Phi}{dt} + \frac{6k}{k+2}H\rho_\Phi = -\Gamma_\Phi\rho_\Phi$$

$$V(\phi) = \lambda_k\phi^k$$

$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma_\Phi\rho_\Phi$$



$$T \propto a^{-\frac{3k-3}{2k+4}}$$

$$T \propto a^{-1}$$

Scaling factor (a)

Adding a coupling to matter (2)

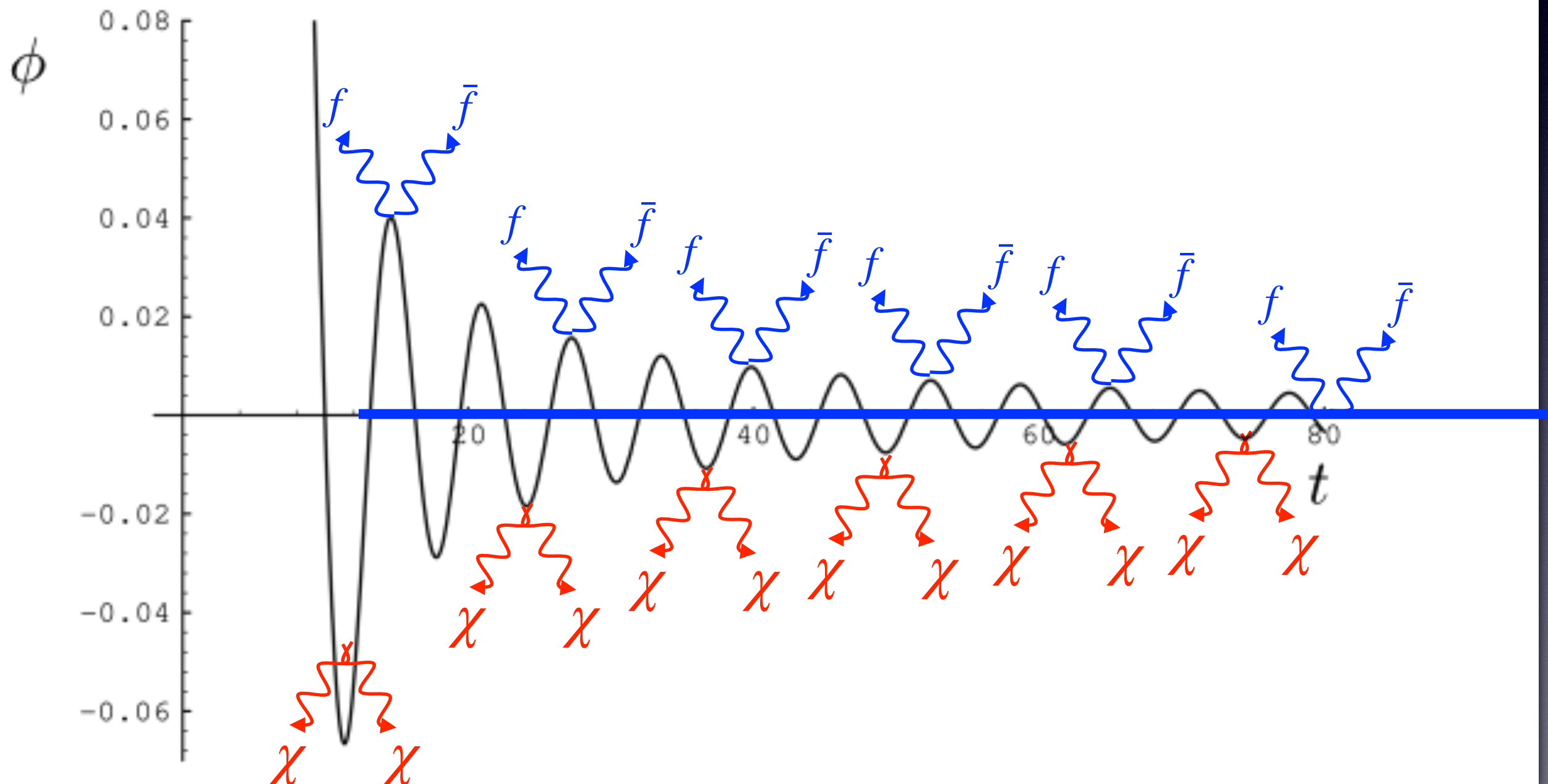
$$V = V(\phi) + y\phi\bar{f}f + \sigma\phi^2\chi^2$$

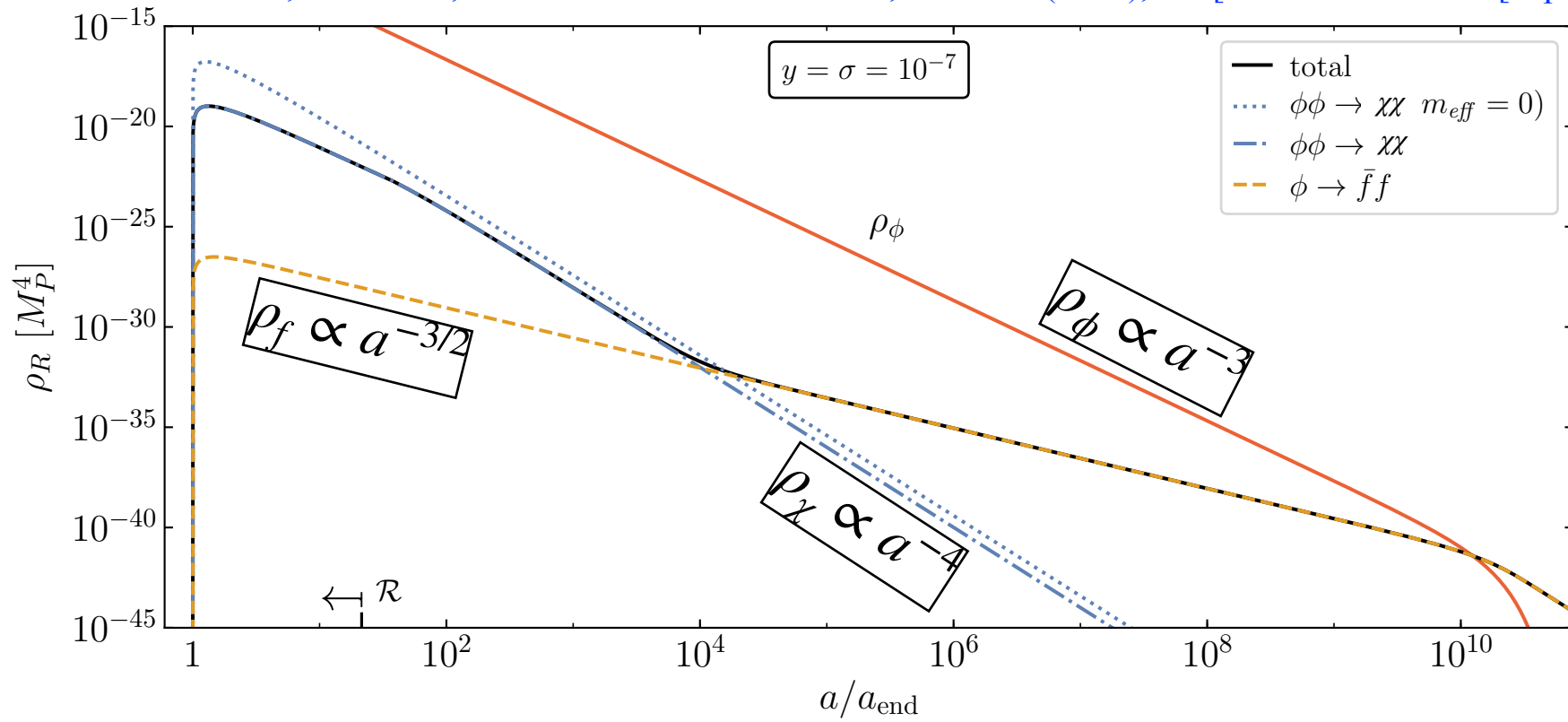
$$\sigma \neq 0, \quad \sigma \times \phi_{\text{end}}^2 \ll m_\phi^2$$

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + y\phi f\bar{f} + \sigma\phi^2\chi^2$$

$$\ddot{\chi}(t, x) + 3H\dot{\chi} - \frac{\nabla^2}{a^2}\chi(t, x) + 2\sigma\phi^2\chi = 0$$

$$m_\chi^{\text{eff}} = \sqrt{2\sigma}\phi$$

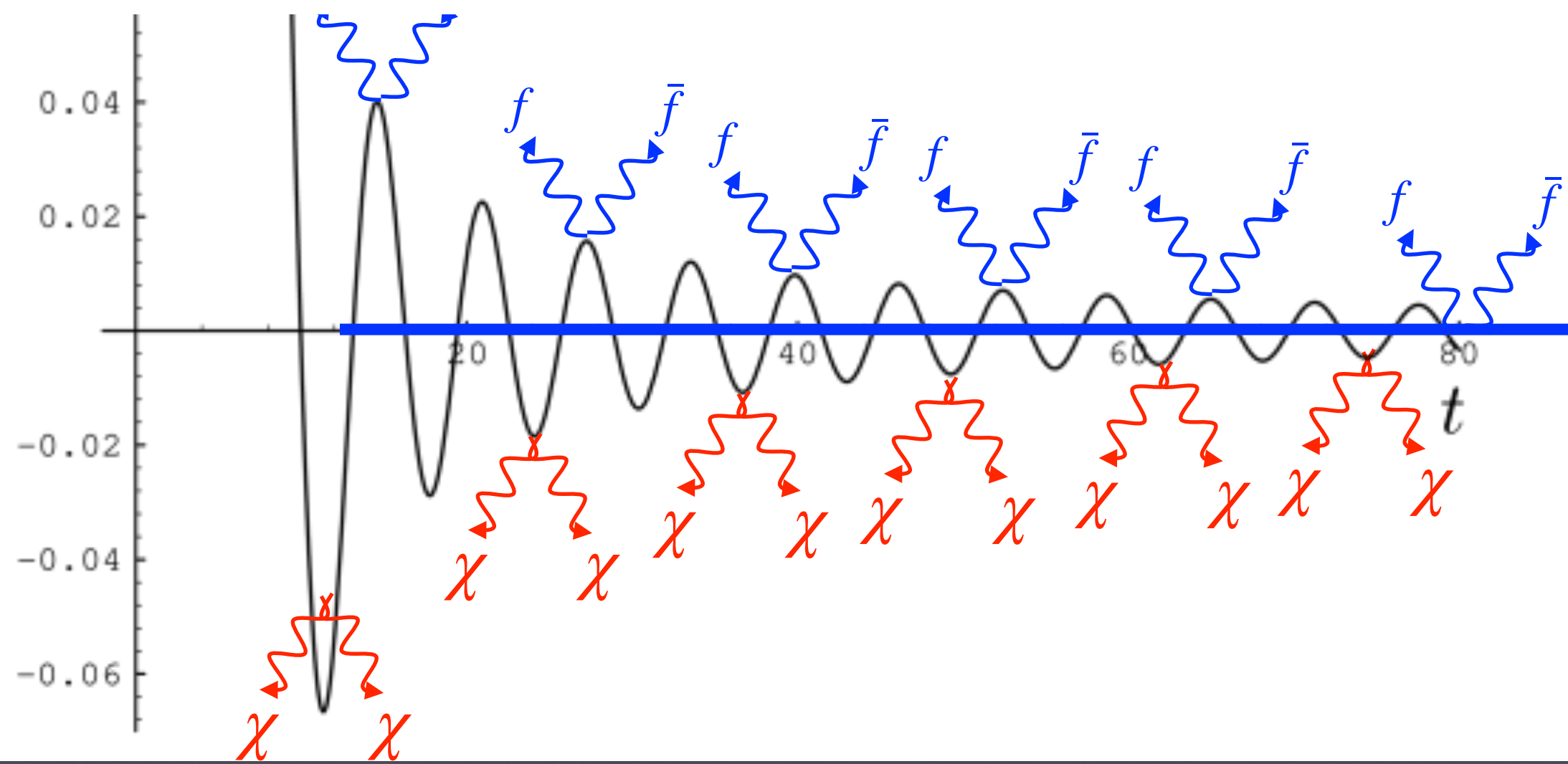




$$\frac{1}{2}m_\phi^2\phi^2 + y\phi f\bar{f} + \sigma\phi^2\chi^2$$

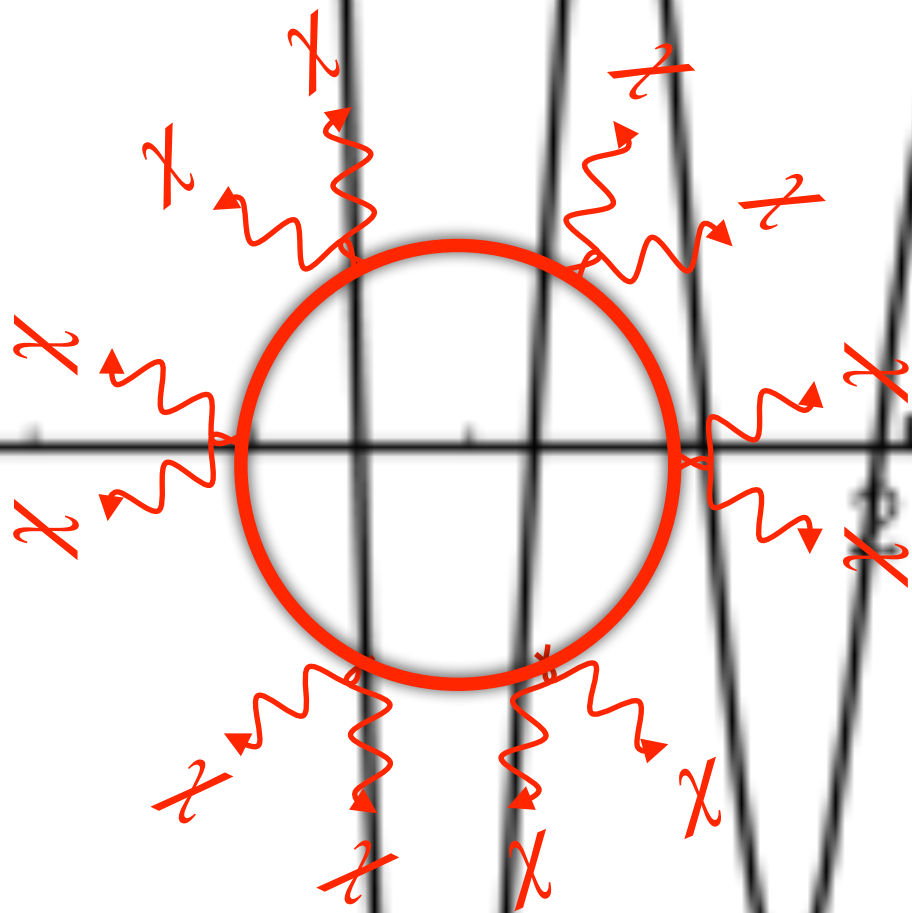
$$3H\dot{\chi} - \frac{\nabla}{a^2}\chi(t, x) + 2\sigma\phi^2\chi = 0$$

$$m_\chi^{\text{eff}} = \sqrt{2\sigma\phi}$$



$$\ddot{\chi}(t, x) + 3H\dot{\chi}(t, x) - \frac{\nabla^2}{a^2}\chi(t, x) + 2\sigma\phi^2\chi(t, x) = 0$$

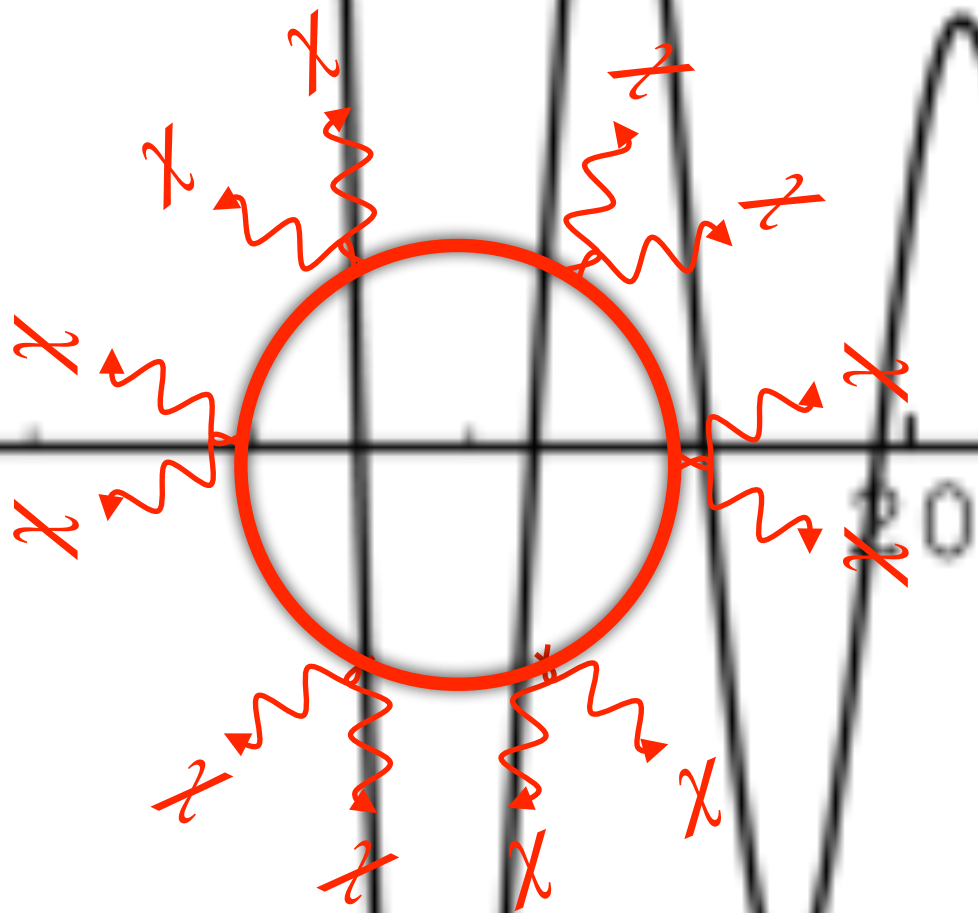
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Resonant
production

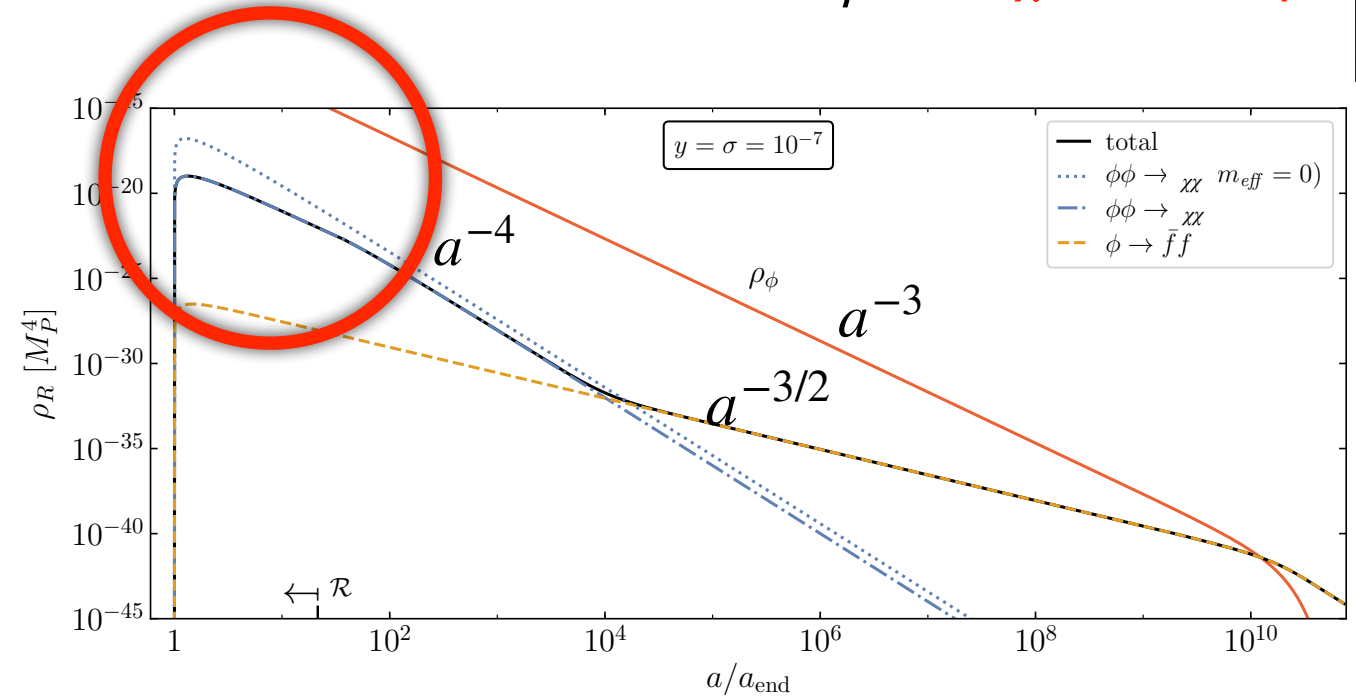
$$\ddot{\chi}(t, x) + 3H\dot{\chi}(t, x) - \frac{\nabla^2}{a^2}\chi(t, x) + 2\sigma\phi^2\chi(t, x) = 0$$

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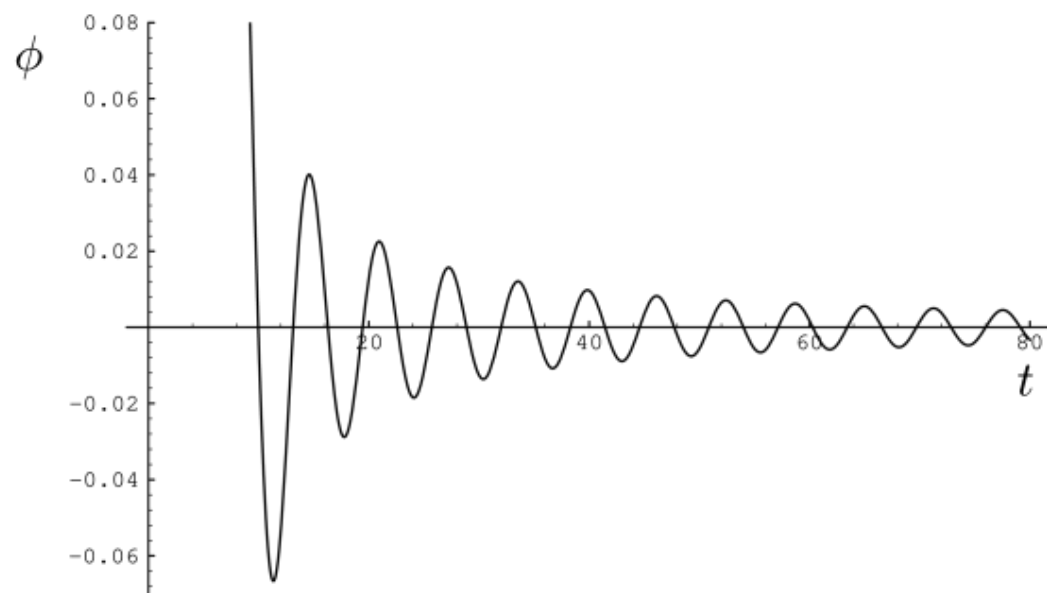
$$\sigma \neq 0, \quad \sigma \times \phi_{\text{end}}^2 \gtrsim m_\phi^2 \quad [m_\chi^{\text{eff}} \gtrsim m_\phi]$$



$$\ddot{\chi}(t, x) + 3H\dot{\chi}(t, x) - \frac{\nabla^2}{a^2}\chi(t, x) + 2\sigma\phi^2\chi(t, x) = 0 \quad m_\chi^{\text{eff}} = \sqrt{2\sigma\Phi}$$

$$\chi = \int \frac{d^3p}{(2\pi)^{3/2}} \left[e^{-ipx} \chi_p(t) a_p + e^{ipx} \chi_p^*(t) a_p^\dagger \right]$$

$$\ddot{\chi}_p(t) + \left[\frac{p^2}{a^2} + (m_\chi^{\text{eff}})^2 + \sigma\Phi^2(t) \times \cos 2m_\phi t \right] \chi_p(t) = 0 \quad \text{Mathieu equation}$$



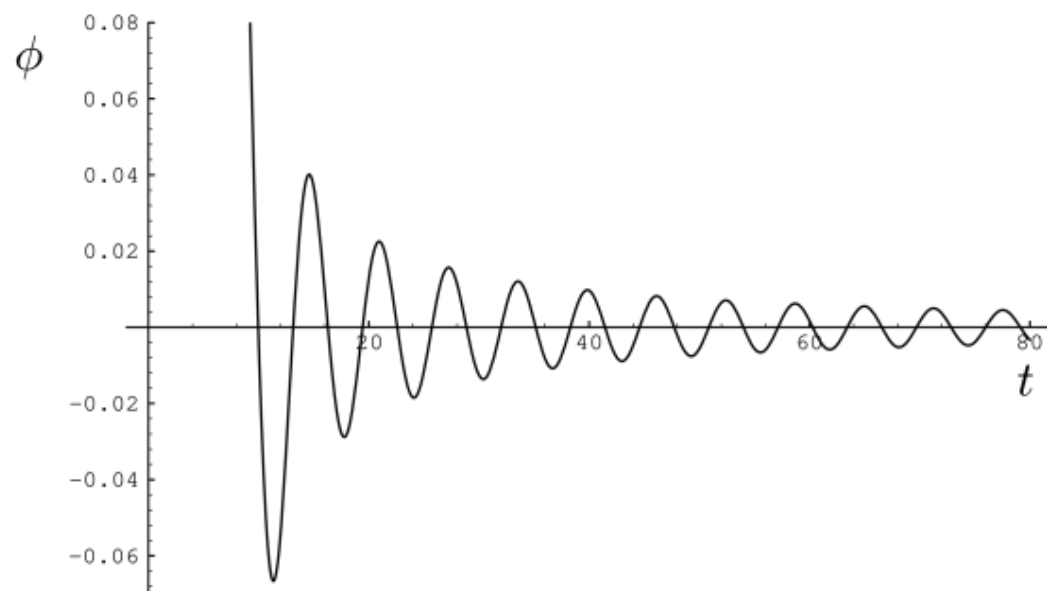
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The Mathieu equation is present in any system with a periodical source of energy.

From electric circuit to mechanical balance, spring excitations...



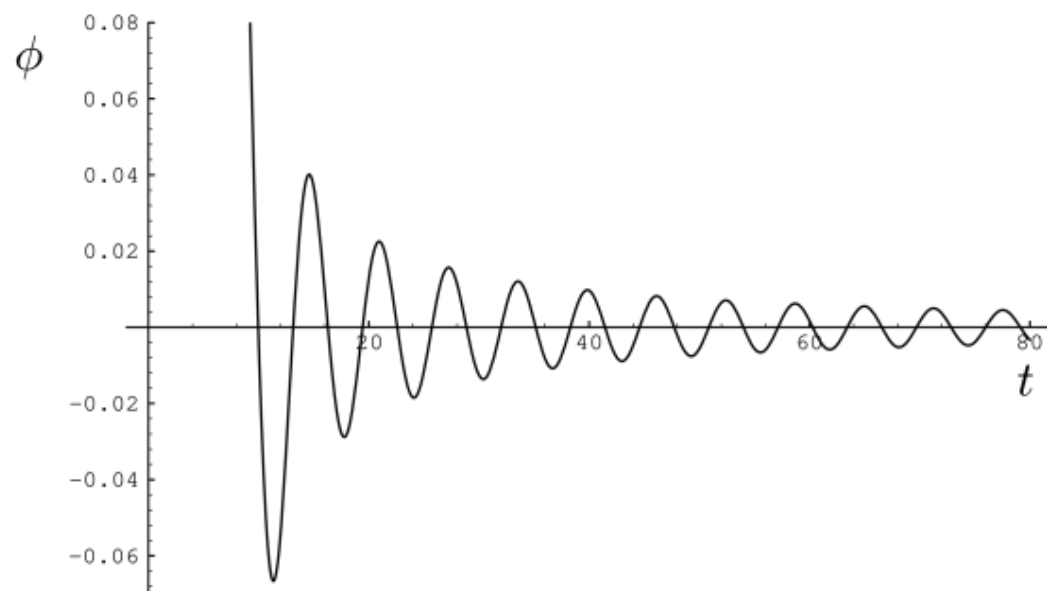
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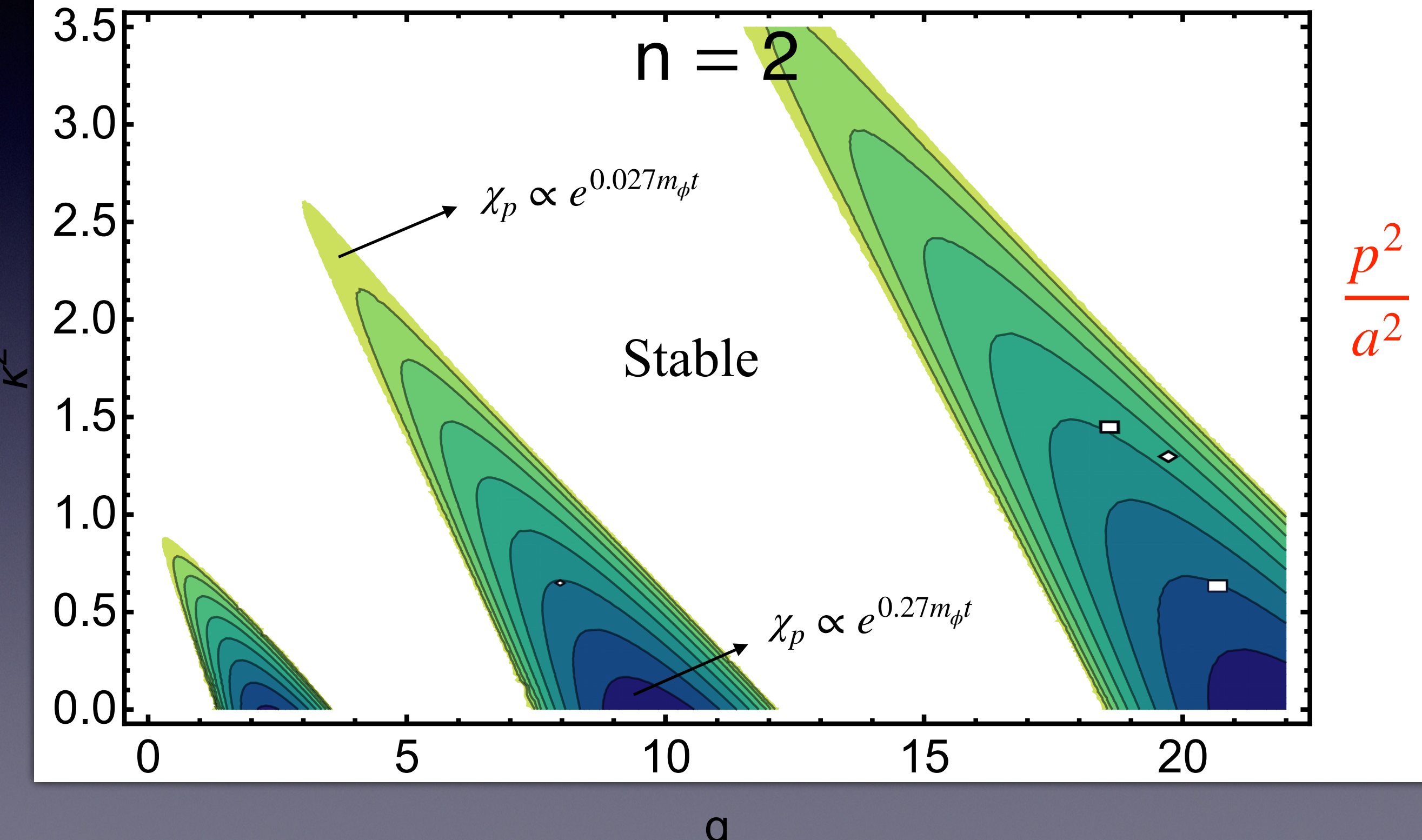
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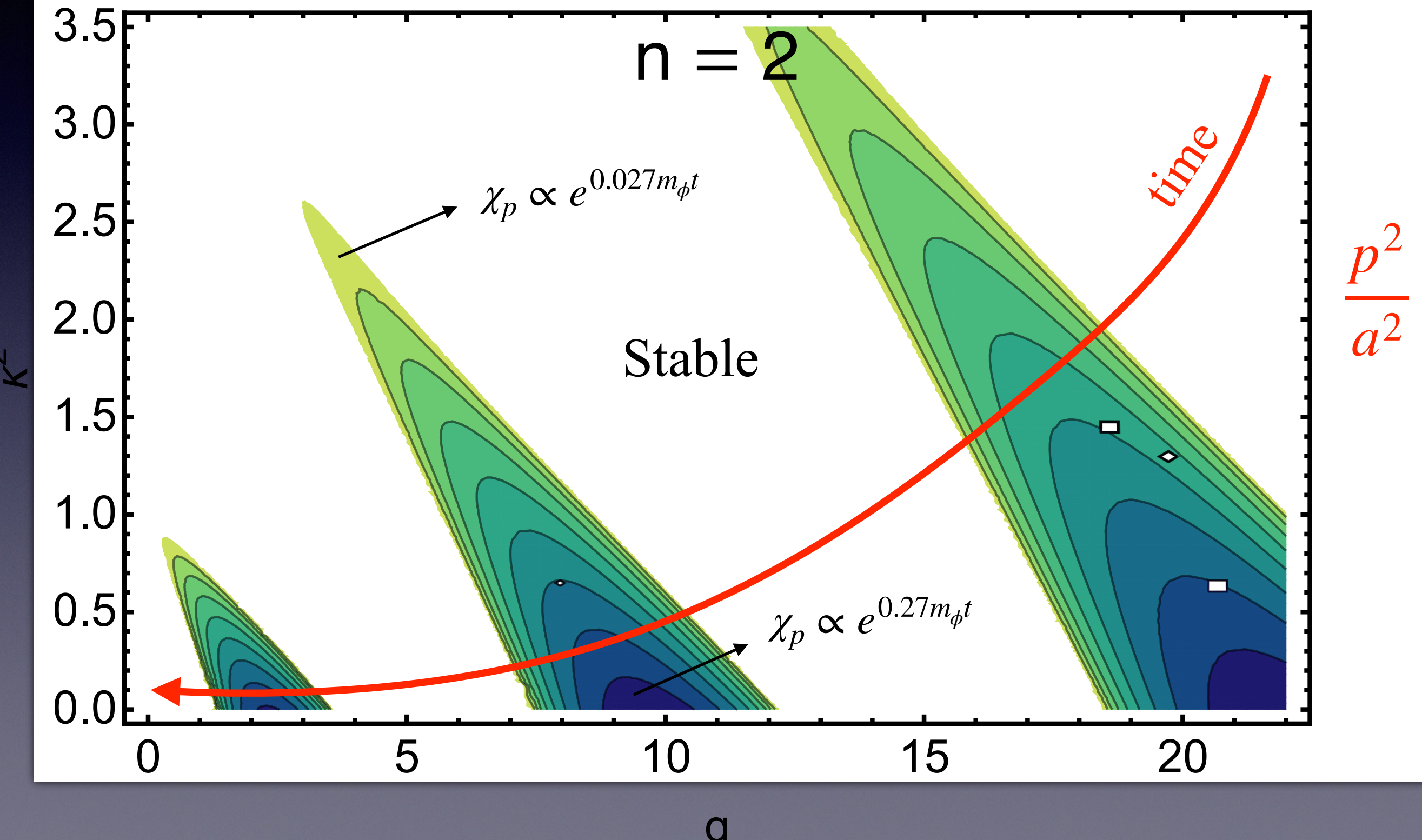


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 $\sigma\Phi^2$


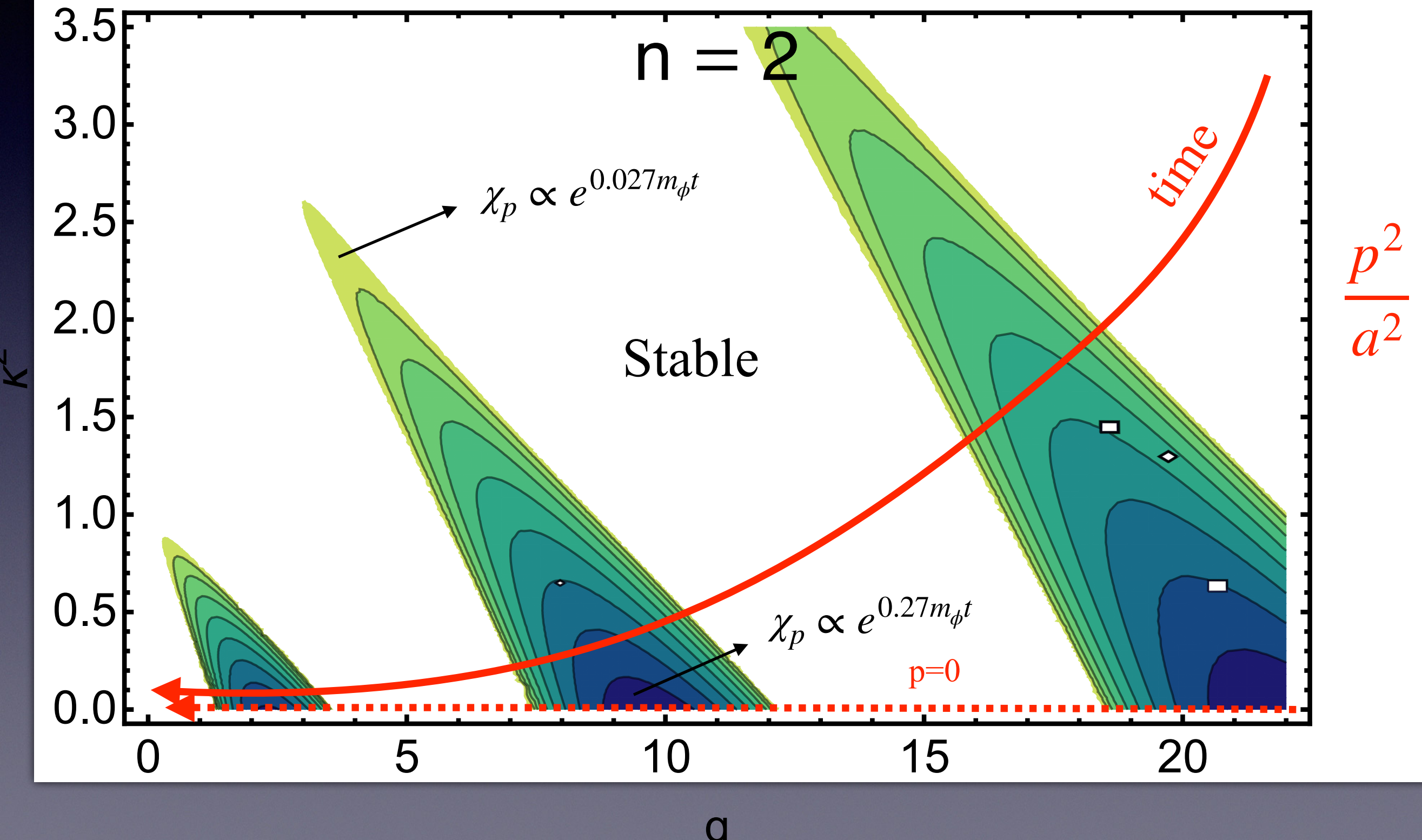
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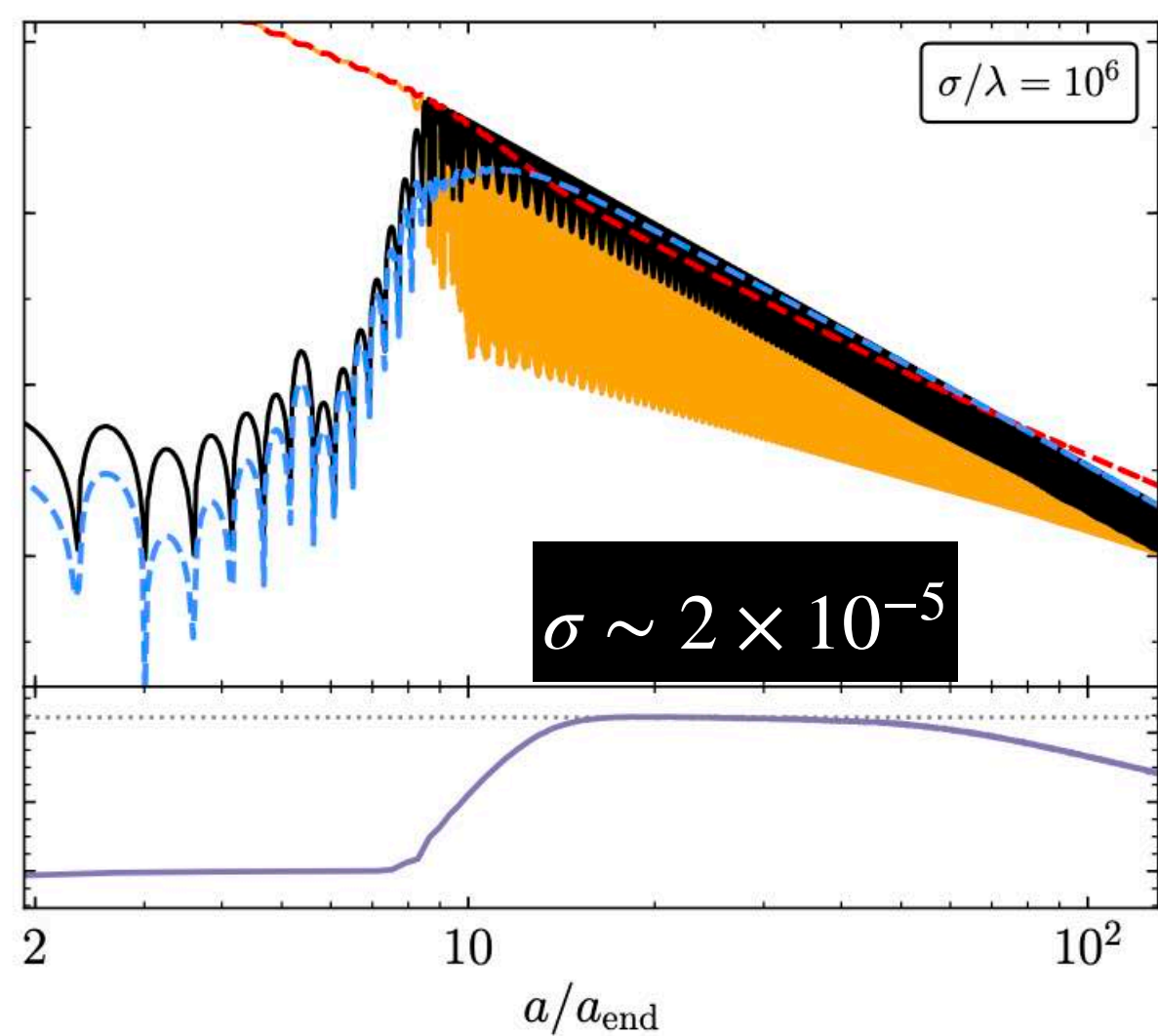
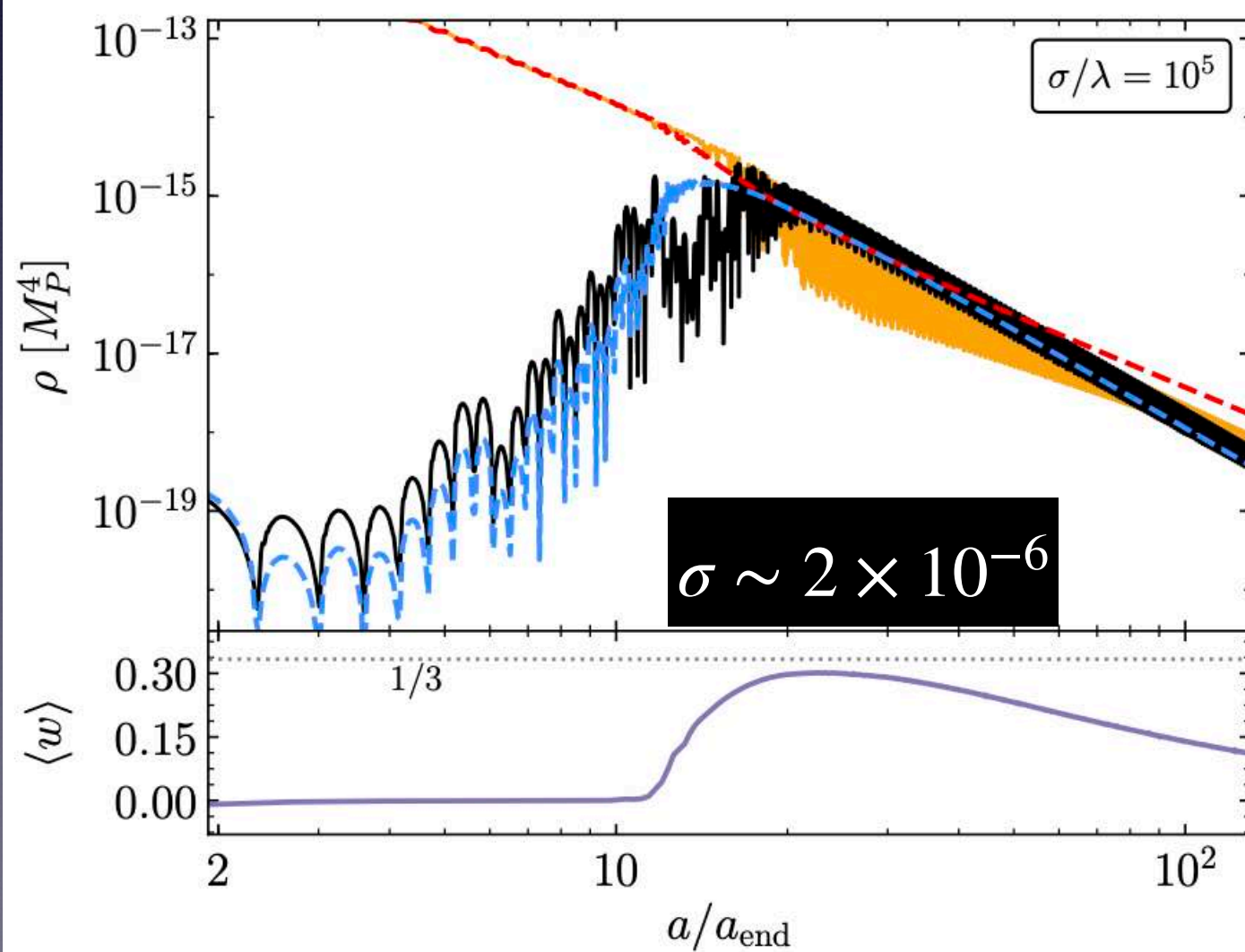
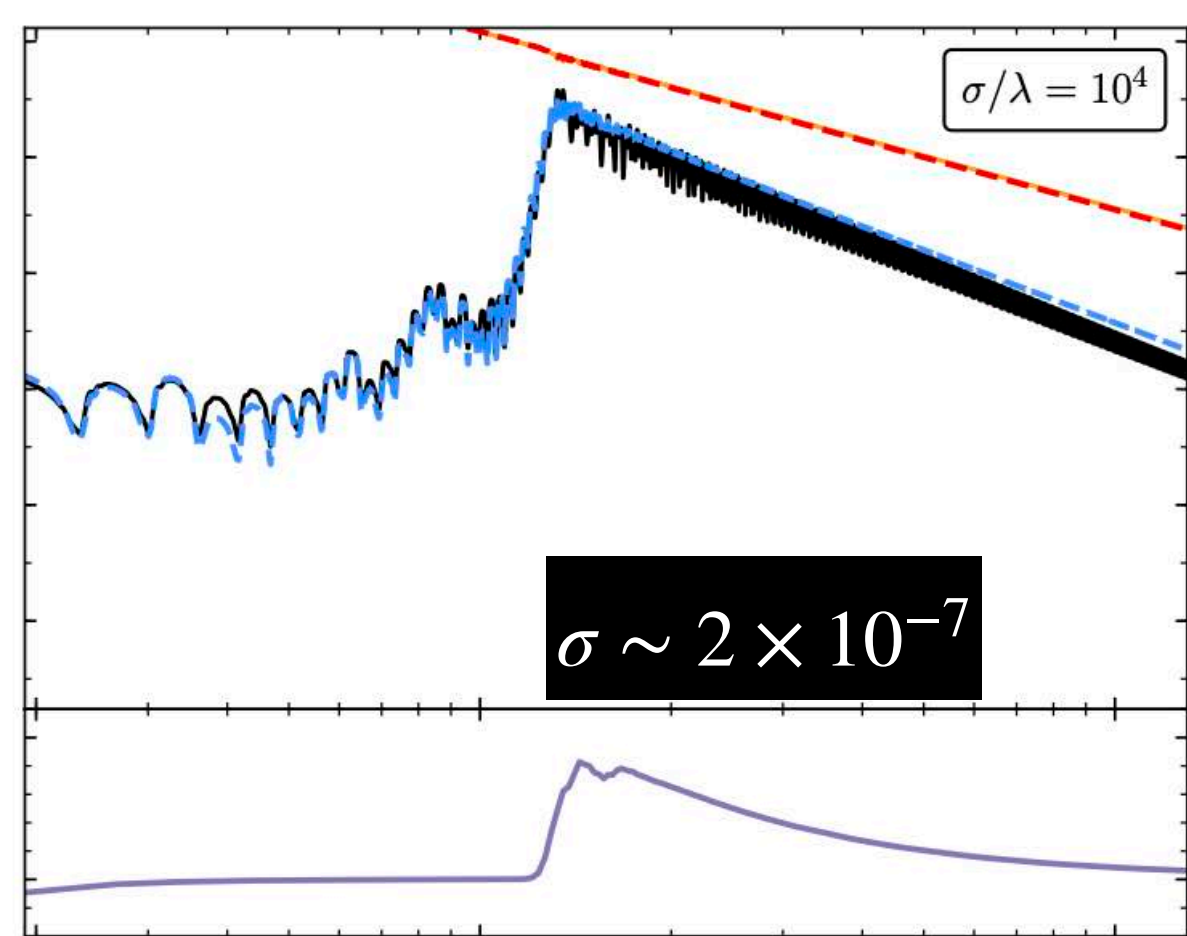
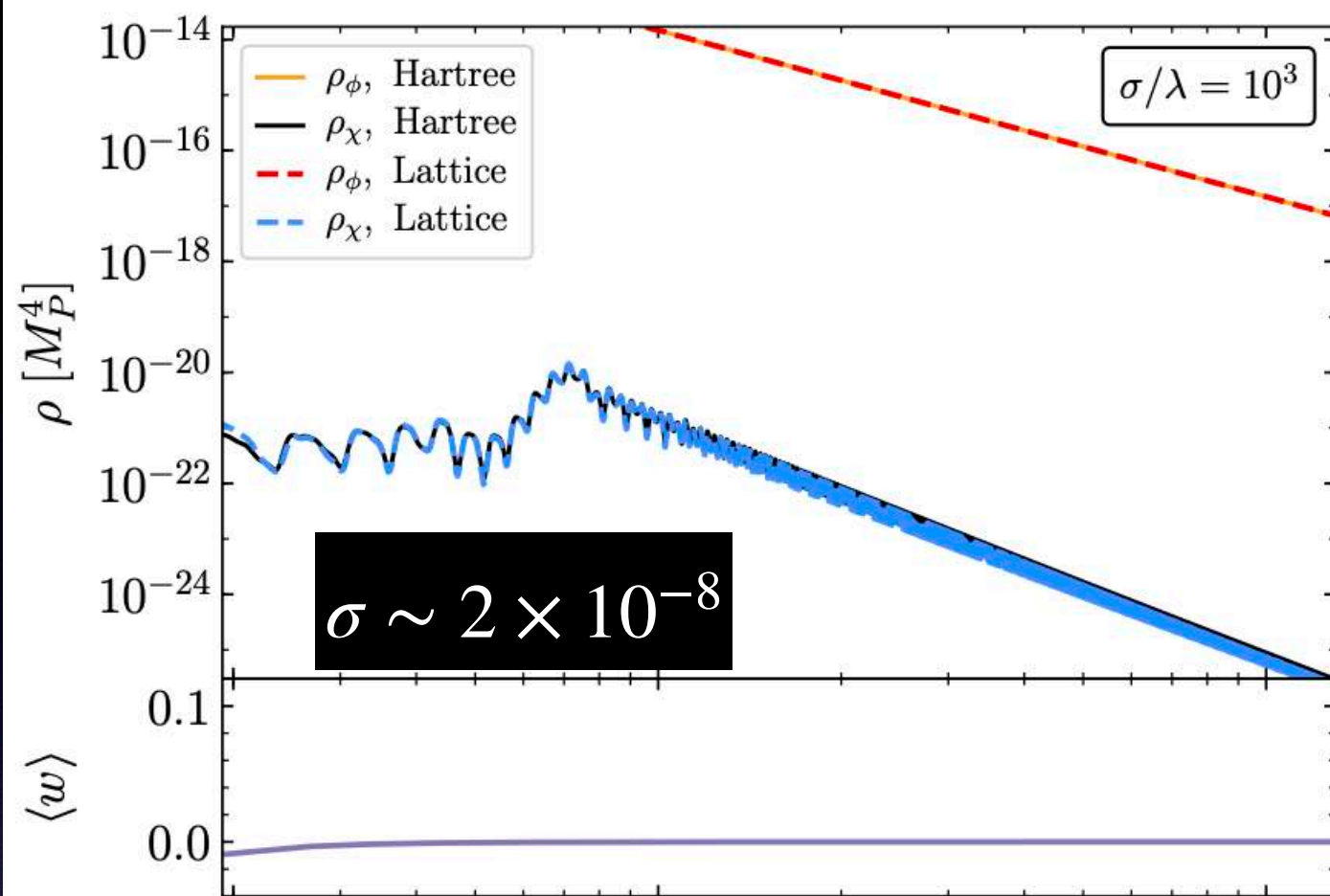
$\sigma\Phi^2$

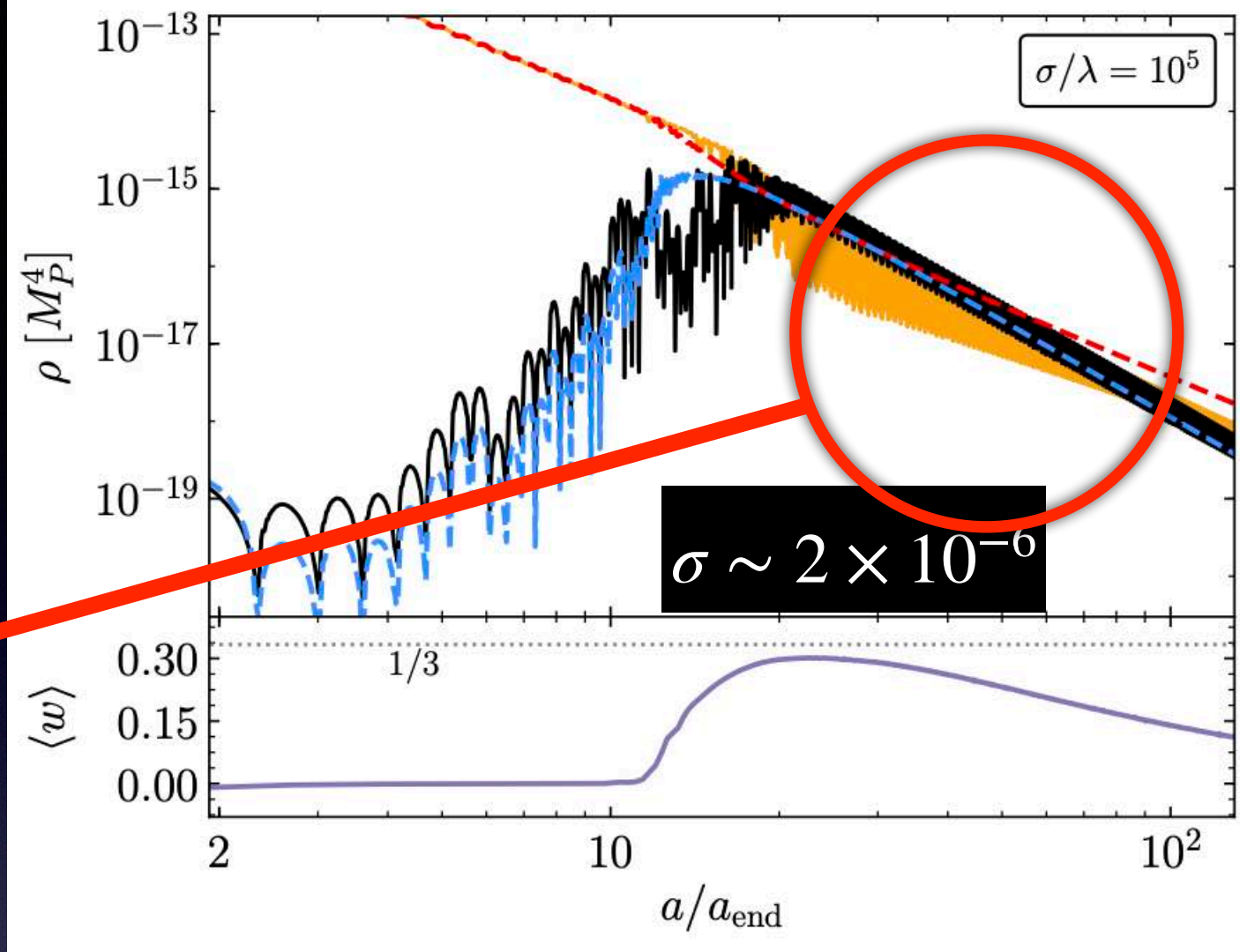
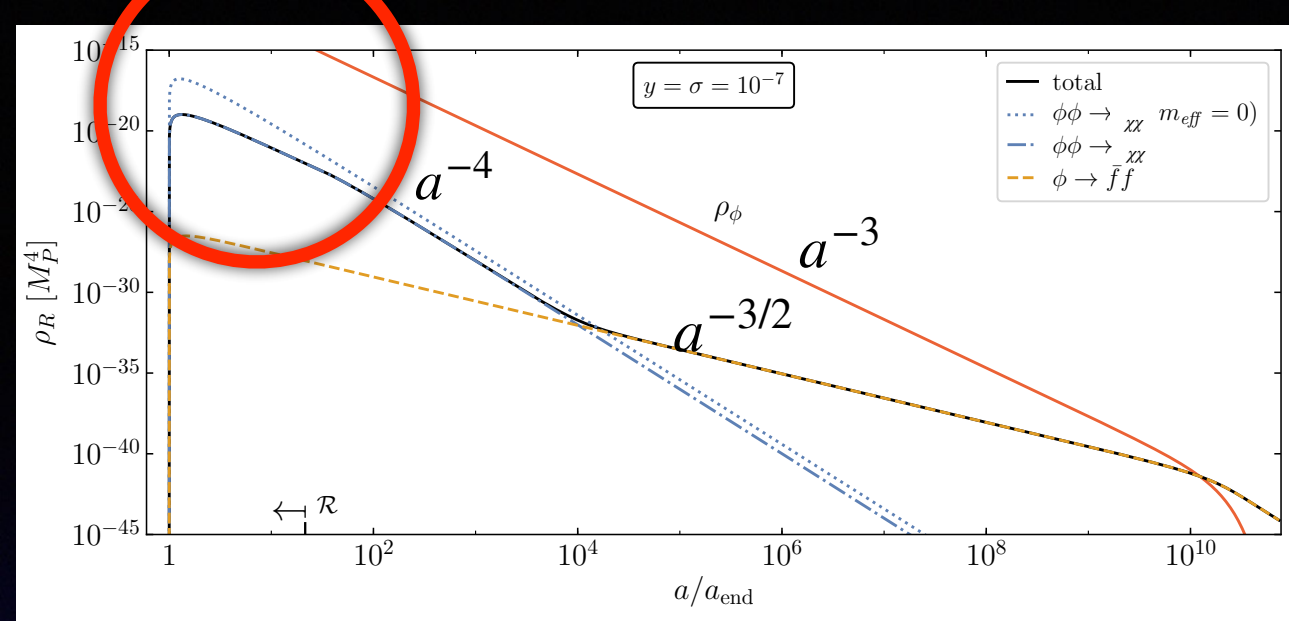


$$\ddot{\chi}_p(t) + \left[\frac{p^2}{a^2} + (m_\chi^{\text{eff}})^2 + \sigma\Phi^2(t) \times \cos 2m_\phi t \right] \chi_p(t) = 0 \quad \text{Mathieu equation}$$

$\sigma\Phi^2$

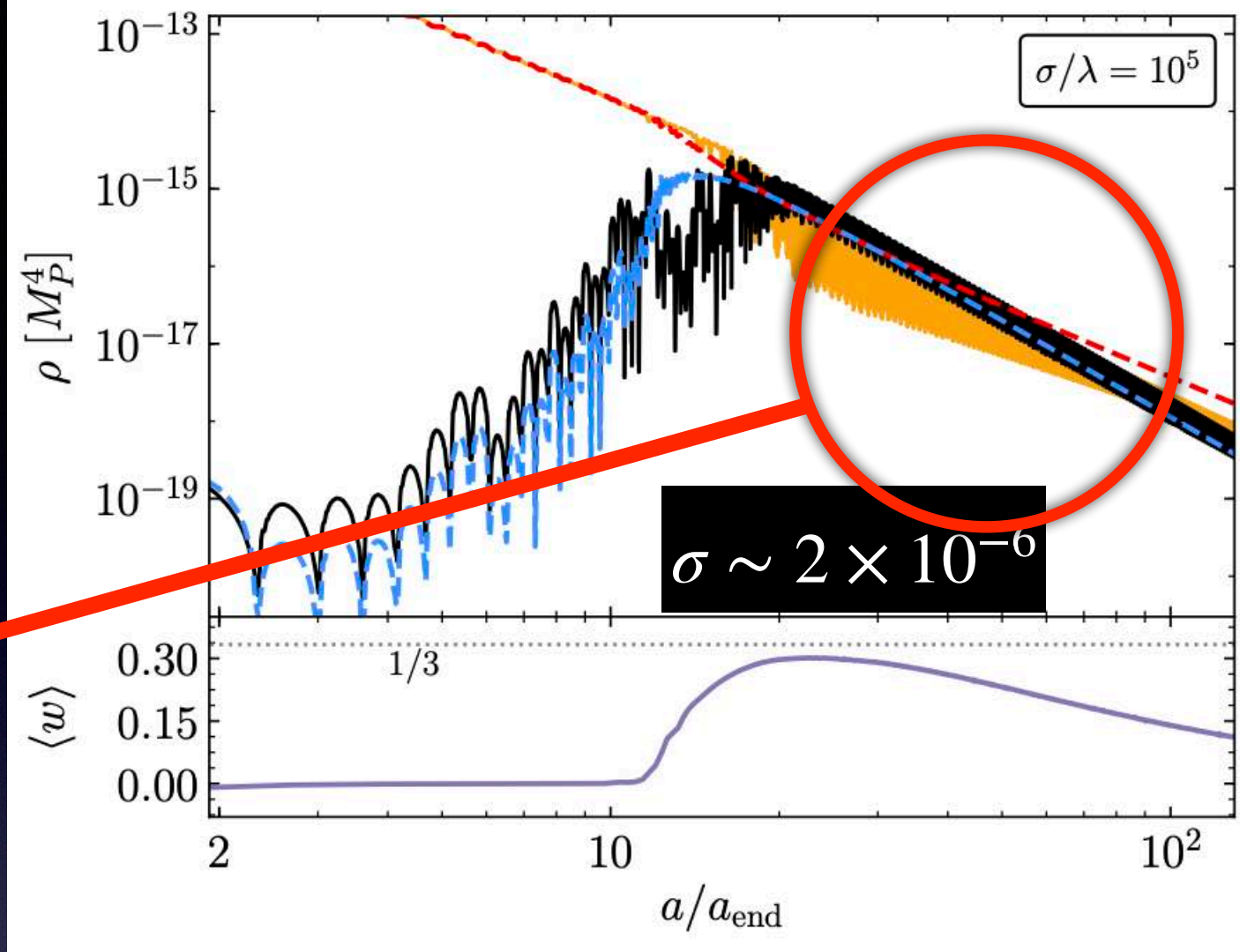
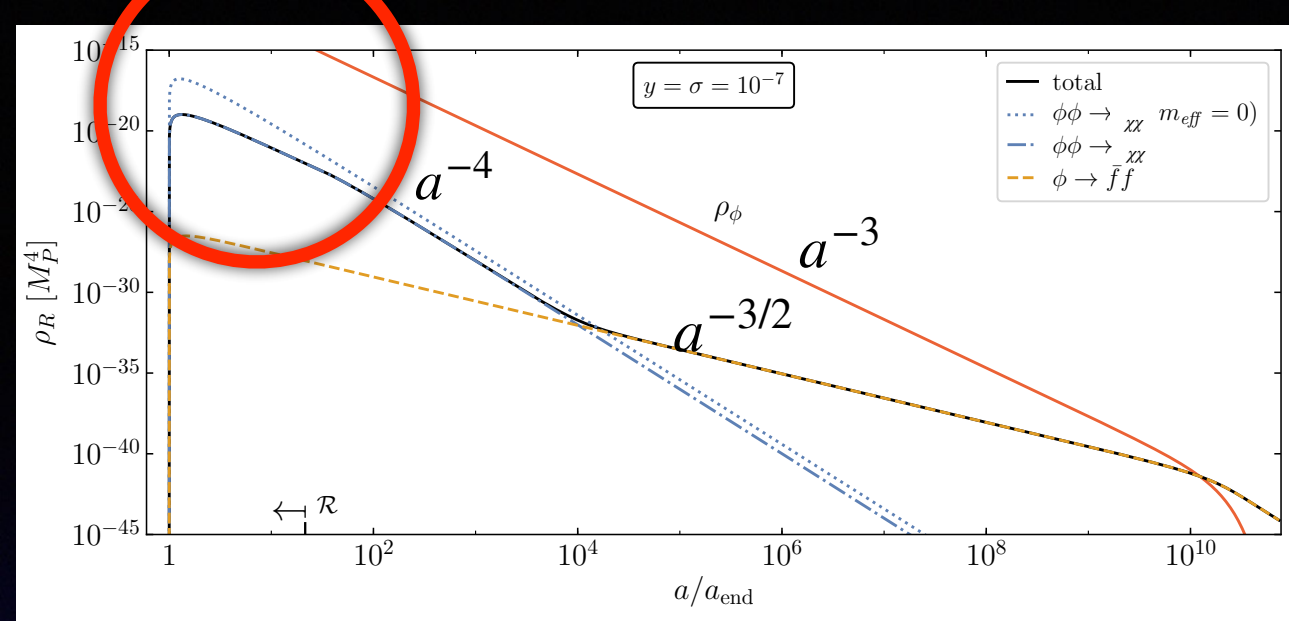






$$\sigma \Phi^2 \lesssim H^2$$

$$\dot{\chi}(t) + 3H\dot{\chi} - \frac{\nabla}{a^2}\chi(t) + 2\sigma\phi^2\chi = 0$$



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Backreaction effects of χ on ϕ

Backreactions

Formally speaking, this is the effect of χ on the condensate ϕ through the equation of motion

The net effect is the destruction of the ϕ -condensate into ϕ -particles

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Backreactions

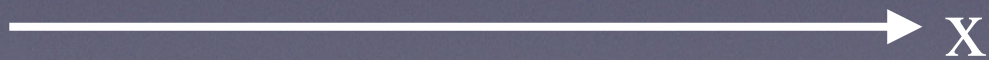
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The net effect is the destruction of the ϕ -condensate into ϕ -particles

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2}{a^2}\chi + 2\sigma\phi^2\chi = 0$$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_\phi^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_\phi^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$



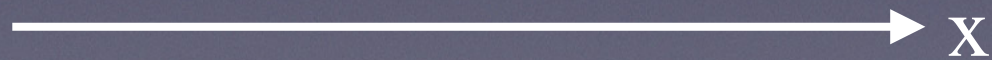
Frequency $\frac{1}{2}m_\phi$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_\phi^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$



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Frequency $\frac{1}{2}m_\phi$

\Leftrightarrow N coherent oscillators of density

$n_\phi = \frac{\rho_\phi}{m_\phi}$ and frequency $\frac{1}{2}m_\phi$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_\phi^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$

$\chi(p_1)$ $\chi(p_2)$ $\chi(p_3)$ $\chi(p_4)$

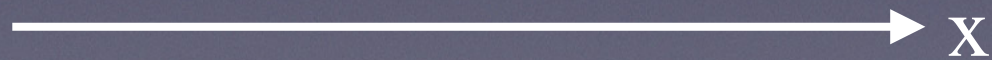


Frequency m_ϕ

\Leftrightarrow N coherent oscillators of density

$$n_\phi = \frac{\rho_\phi}{m_\phi} \text{ and frequency } m_\phi$$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_\phi^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$



Frequency m_ϕ

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Frequency $\frac{1}{2}m_\phi$

\Leftrightarrow N **incoherent** oscillators of **mean**
momentum $p_* = \sqrt{m_\phi m_\chi}$

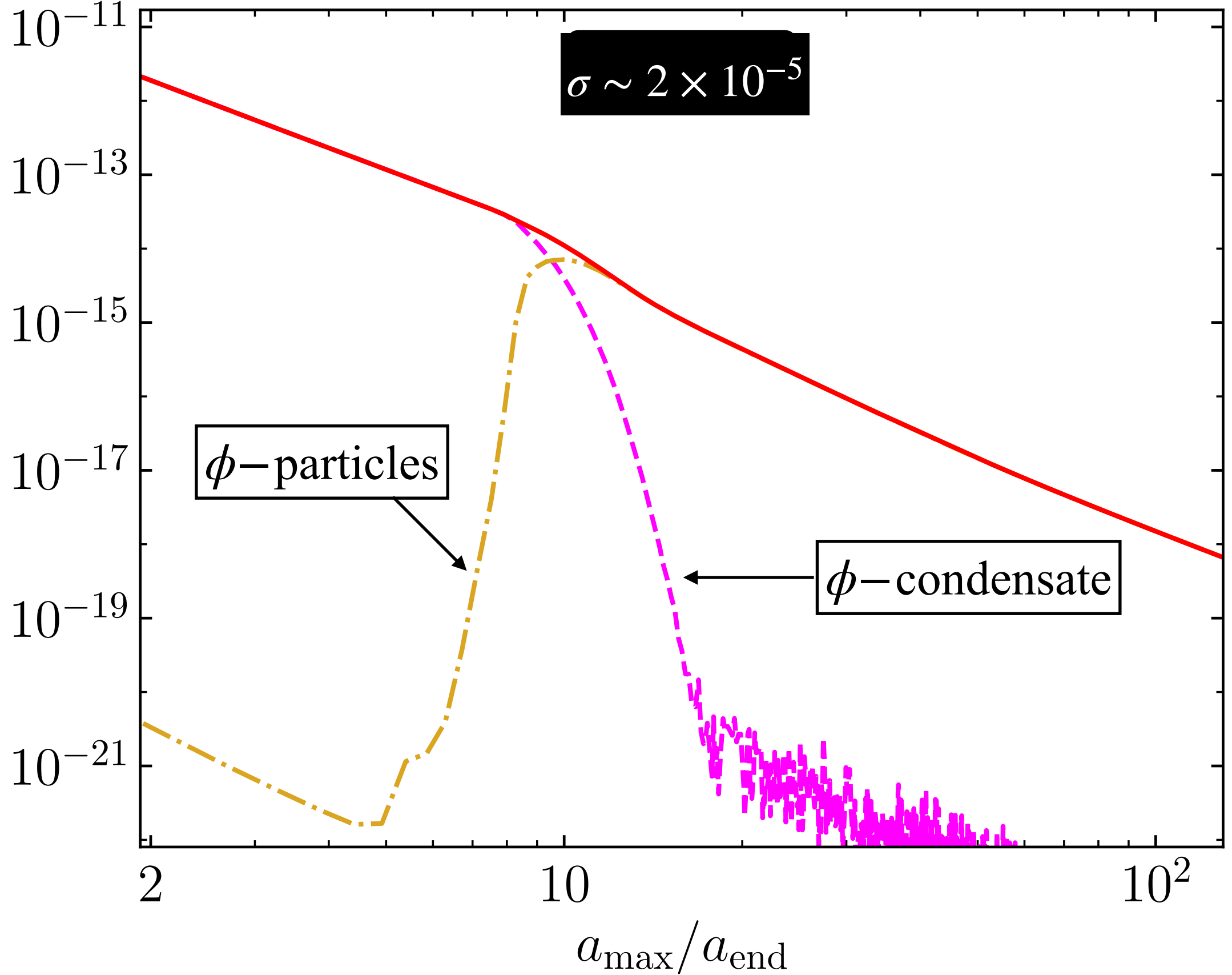
$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_\phi^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$



Frequency $\frac{1}{2}m_\phi$

\Leftrightarrow N **incoherent** oscillators of **mean**
momentum $p_* = \sqrt{m_\phi m_\chi}$

$\ddot{\phi}(t, \mathbf{x})$



an

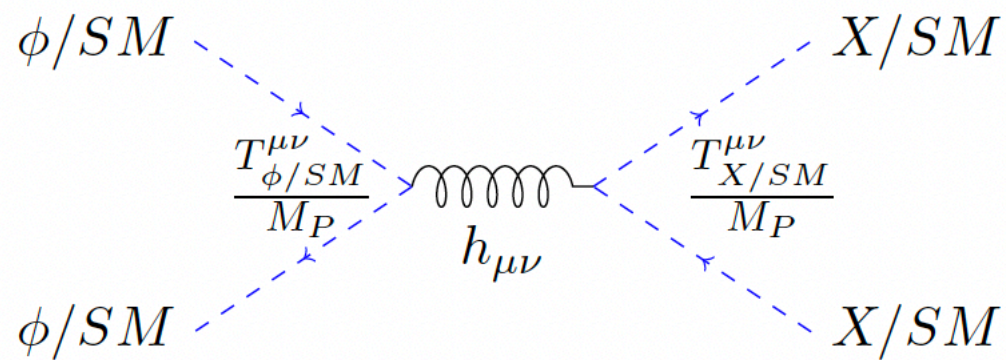
Example :

(dark) matter from gravitational
scattering of the inflaton

$$\mathcal{L} = \frac{1}{M_P} h_{\mu\nu} T_{\phi}^{\mu\nu} + \frac{1}{M_P} h_{\mu\nu} T_S^{\mu\nu} + y\phi\bar{f}f + \sigma\phi^2\chi^2$$

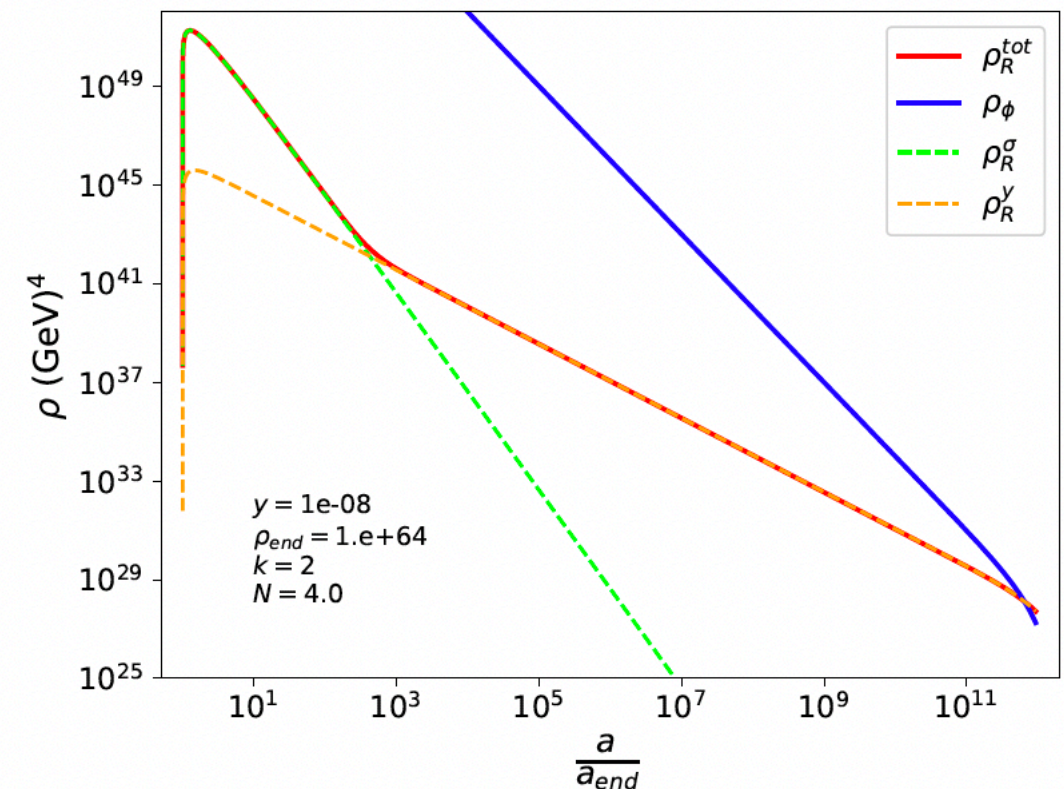
Y. Mambrini and K. A. Olive, Phys. Rev. D **103** (2021) no.11, 115009 [arXiv:2102.06214].

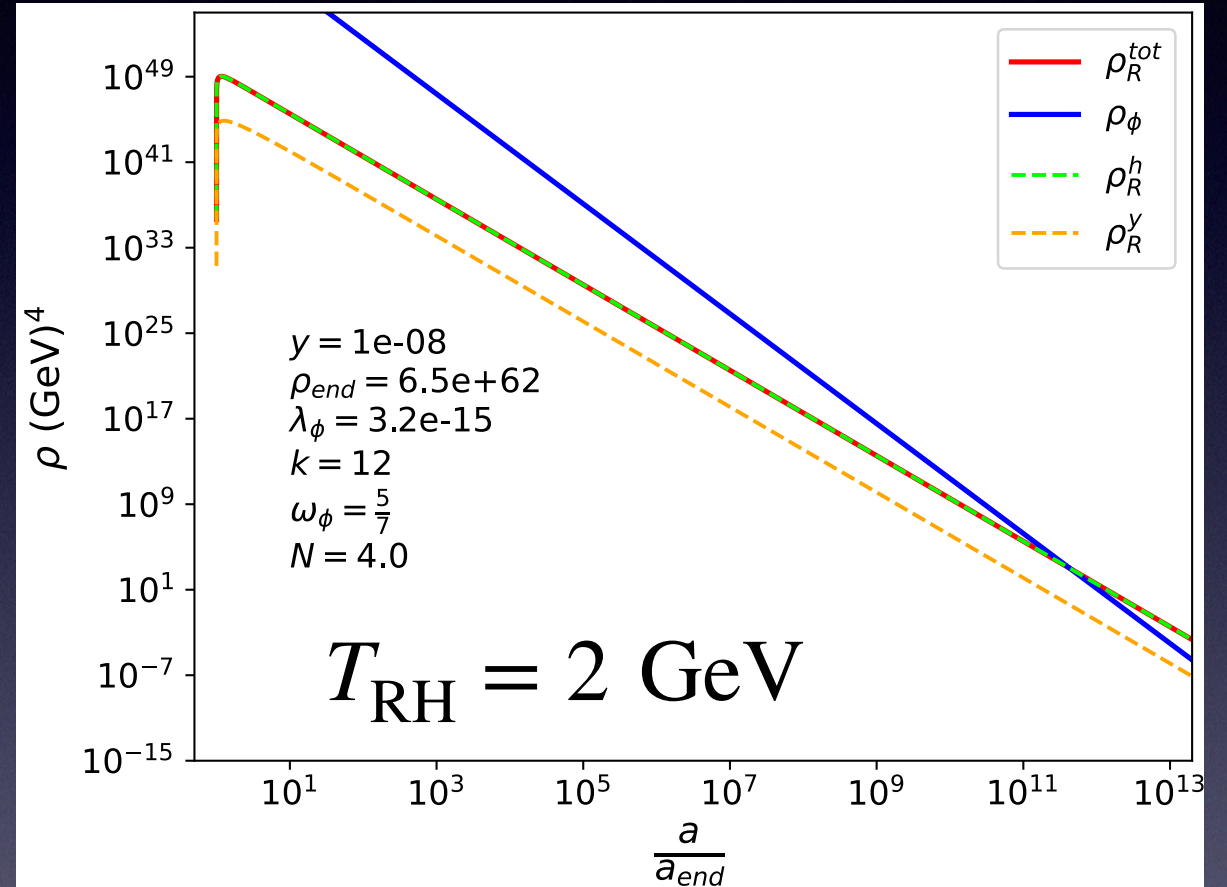
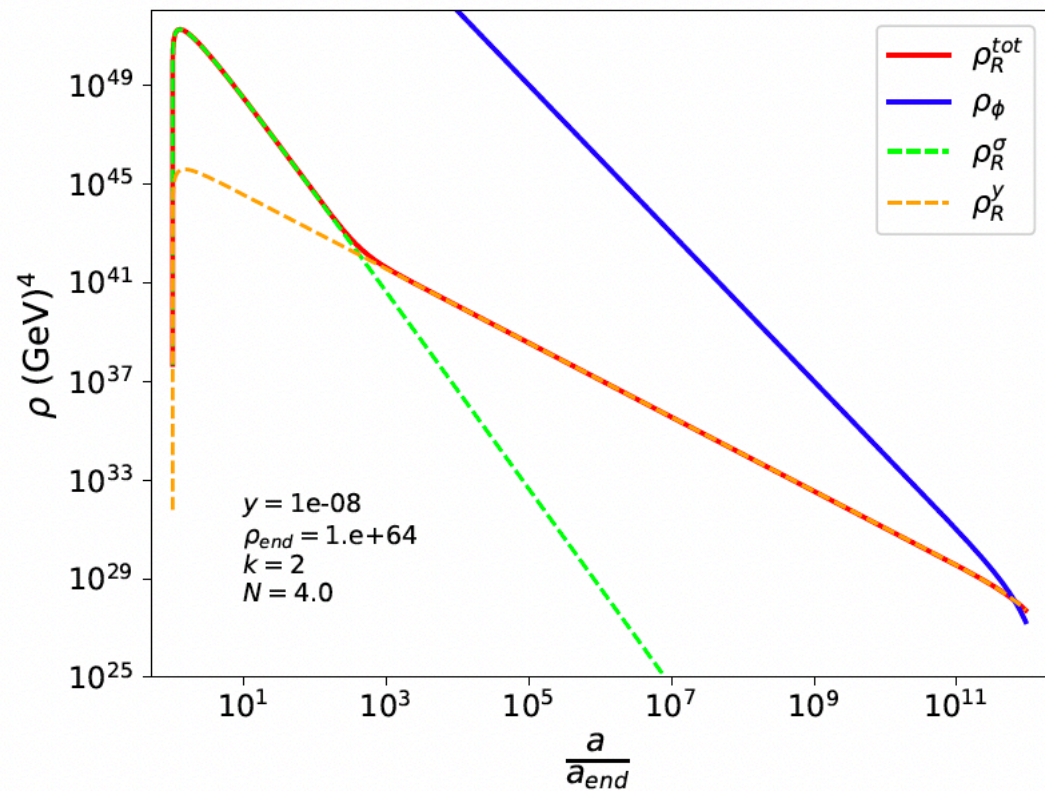
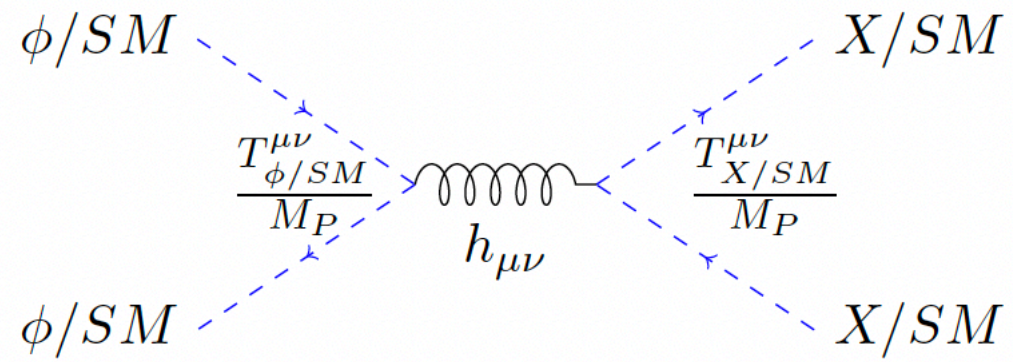
S. Clery, Y. Mambrini, K. A. Olive, and S. Verner, [arXiv:2112.15214].



There exists a minimal maximal temperature in the Universe $\sim 10^{12}$ GeV

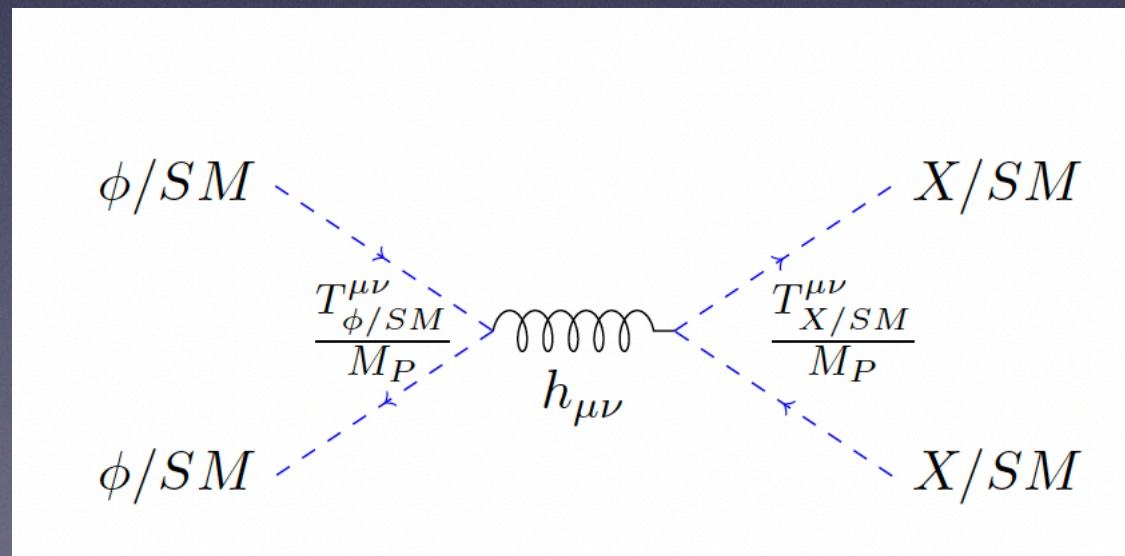
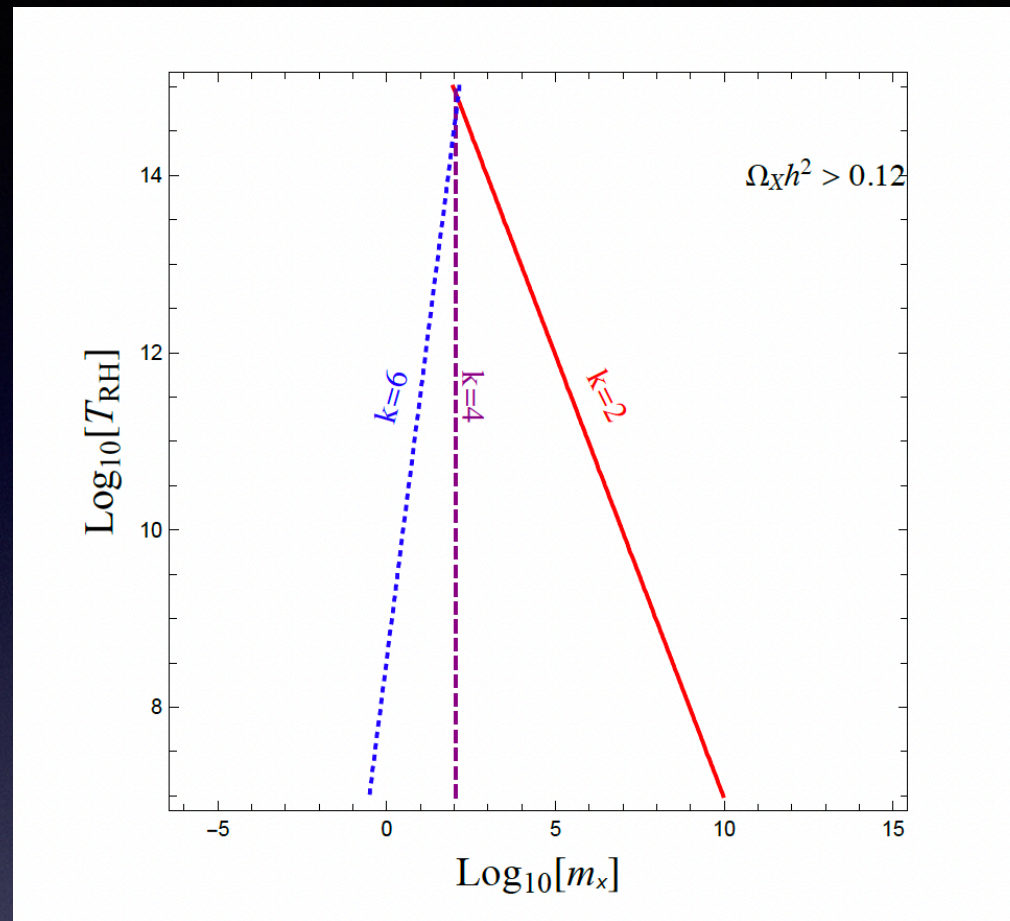
	$k = 2$	$k = 4$	$k = 6$
T_{\max}	1.0×10^{12} GeV	7.5×10^{11} GeV	6.5×10^{11} GeV
y_{\max}	1.8×10^{-6}	1.4×10^{-6}	1.1×10^{-6}
$T_{RH_{\max}}$	7.9×10^8 GeV	470 GeV	9.7×10^{-4} GeV



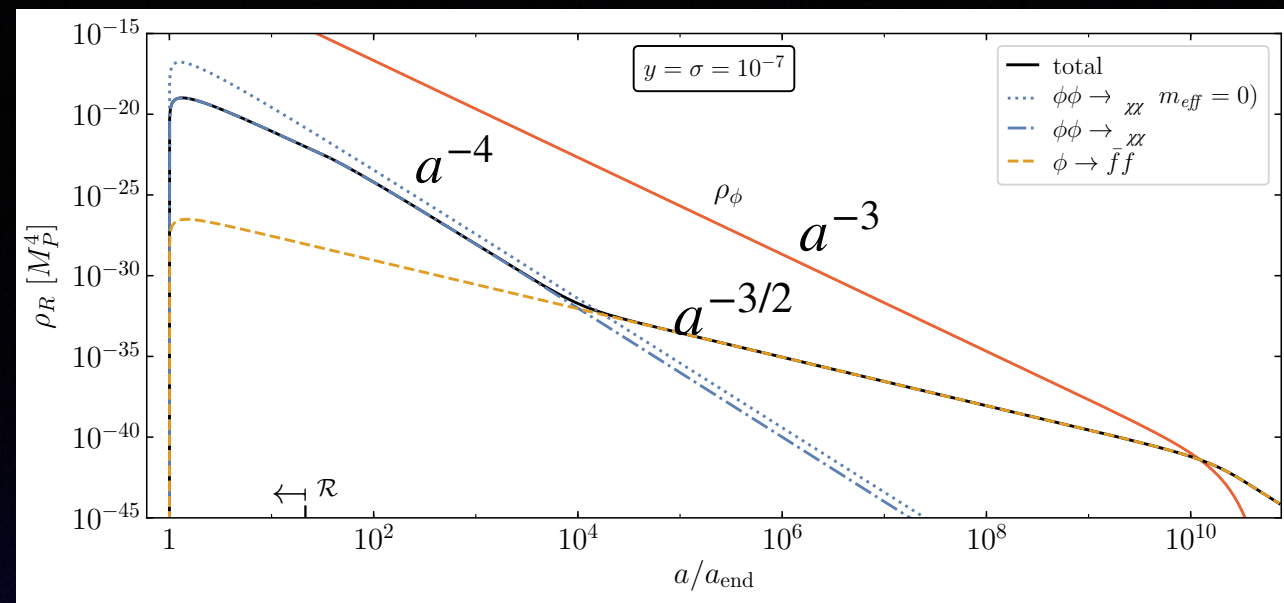
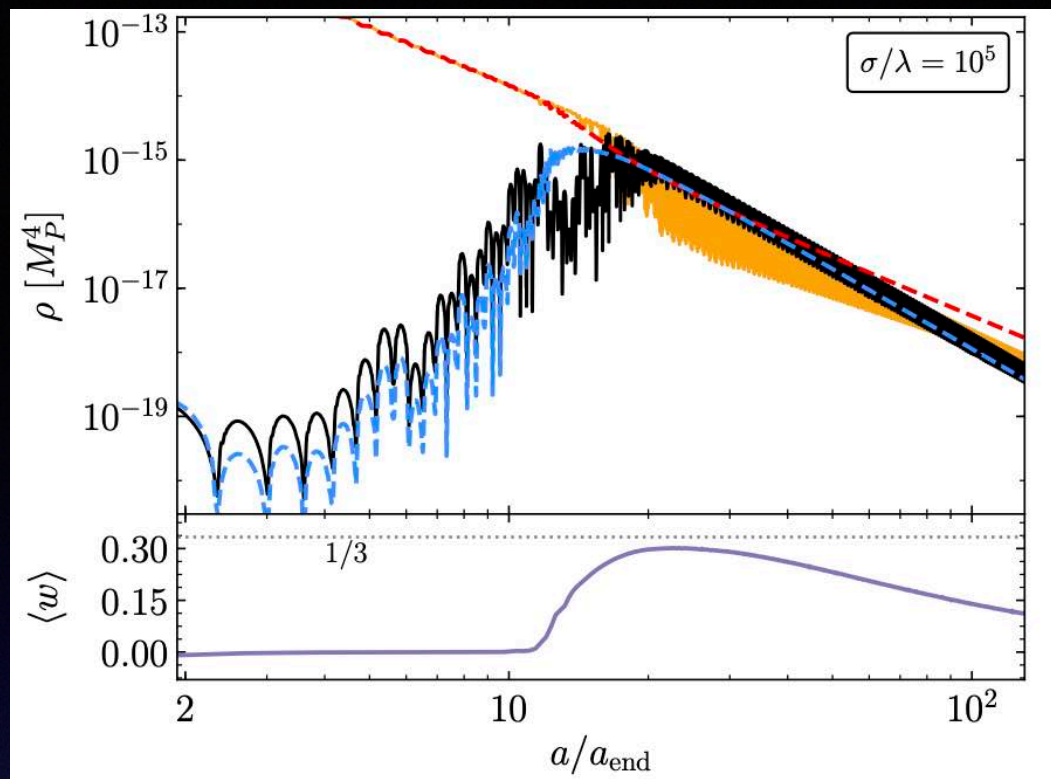


We can even gravitationally reheat!!

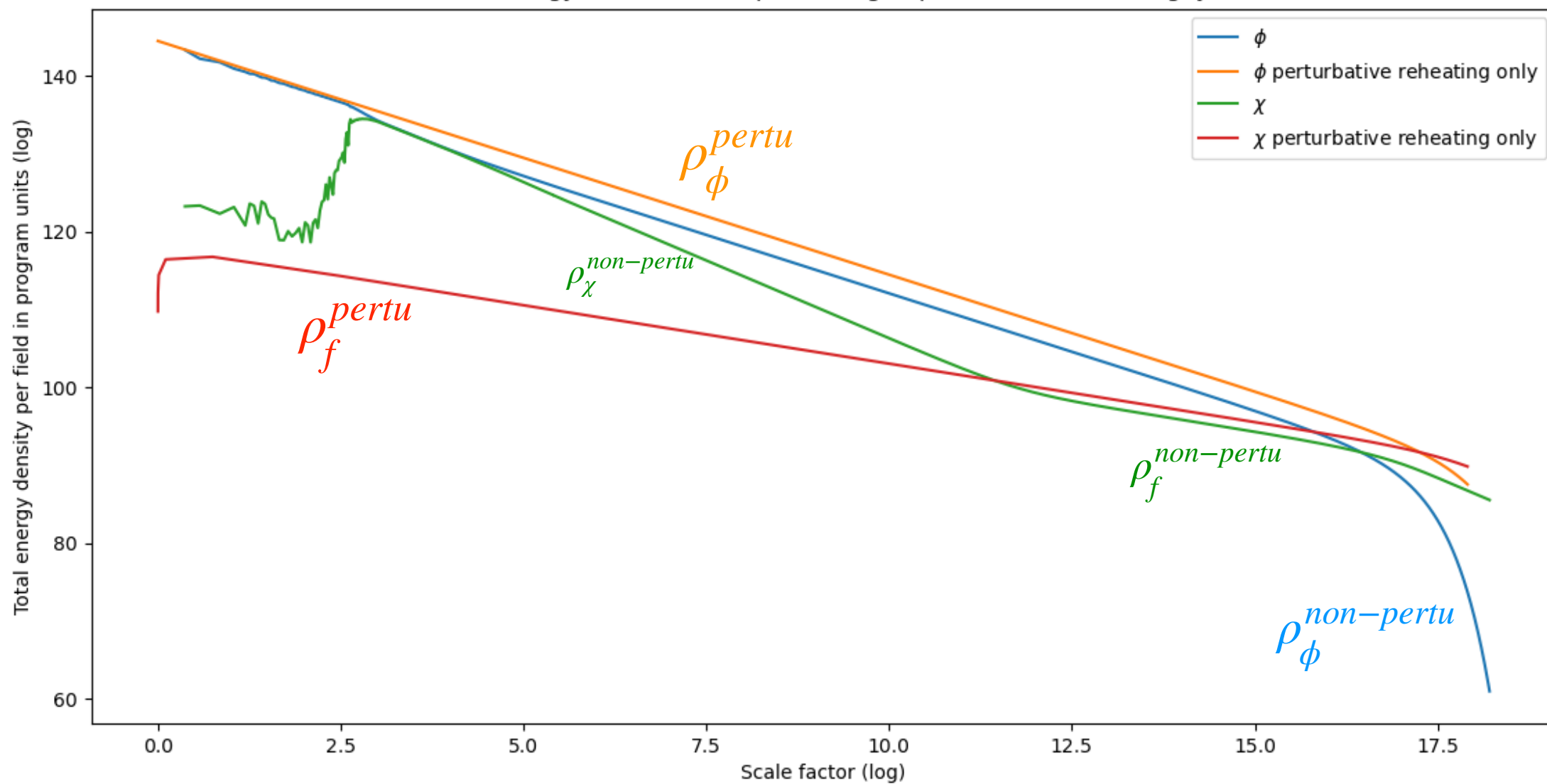
Also working for Dark Matter

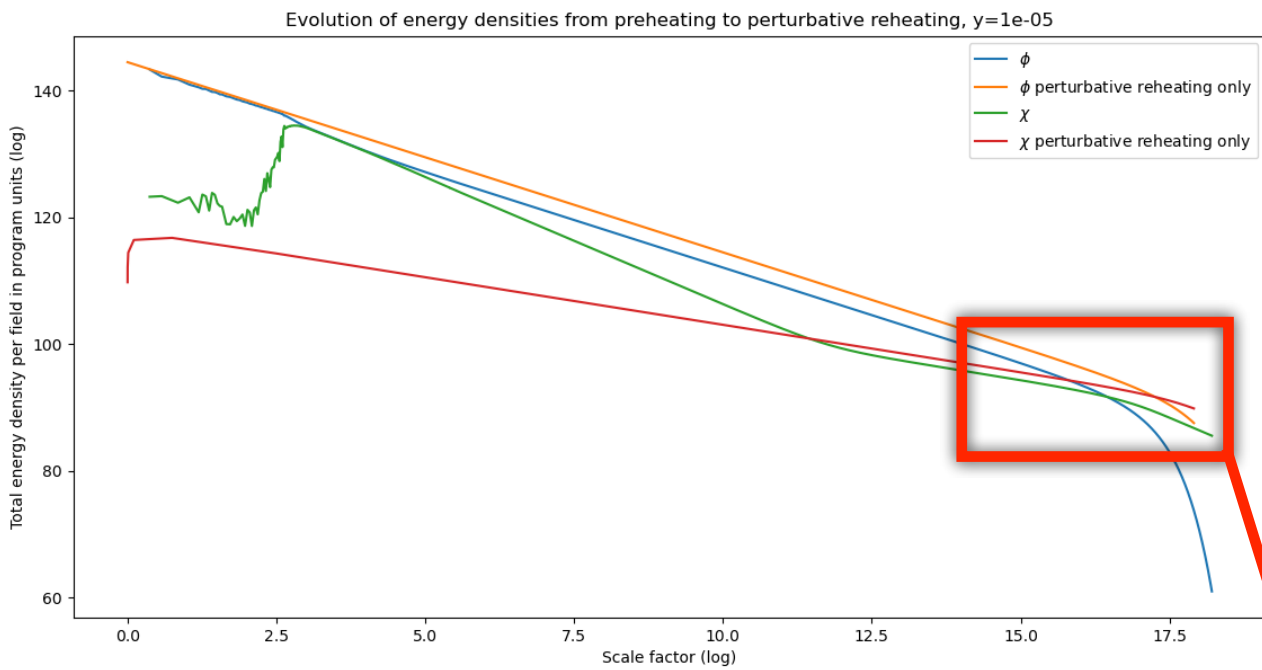


Summary



Evolution of energy densities from preheating to perturbative reheating, $y=1e-05$



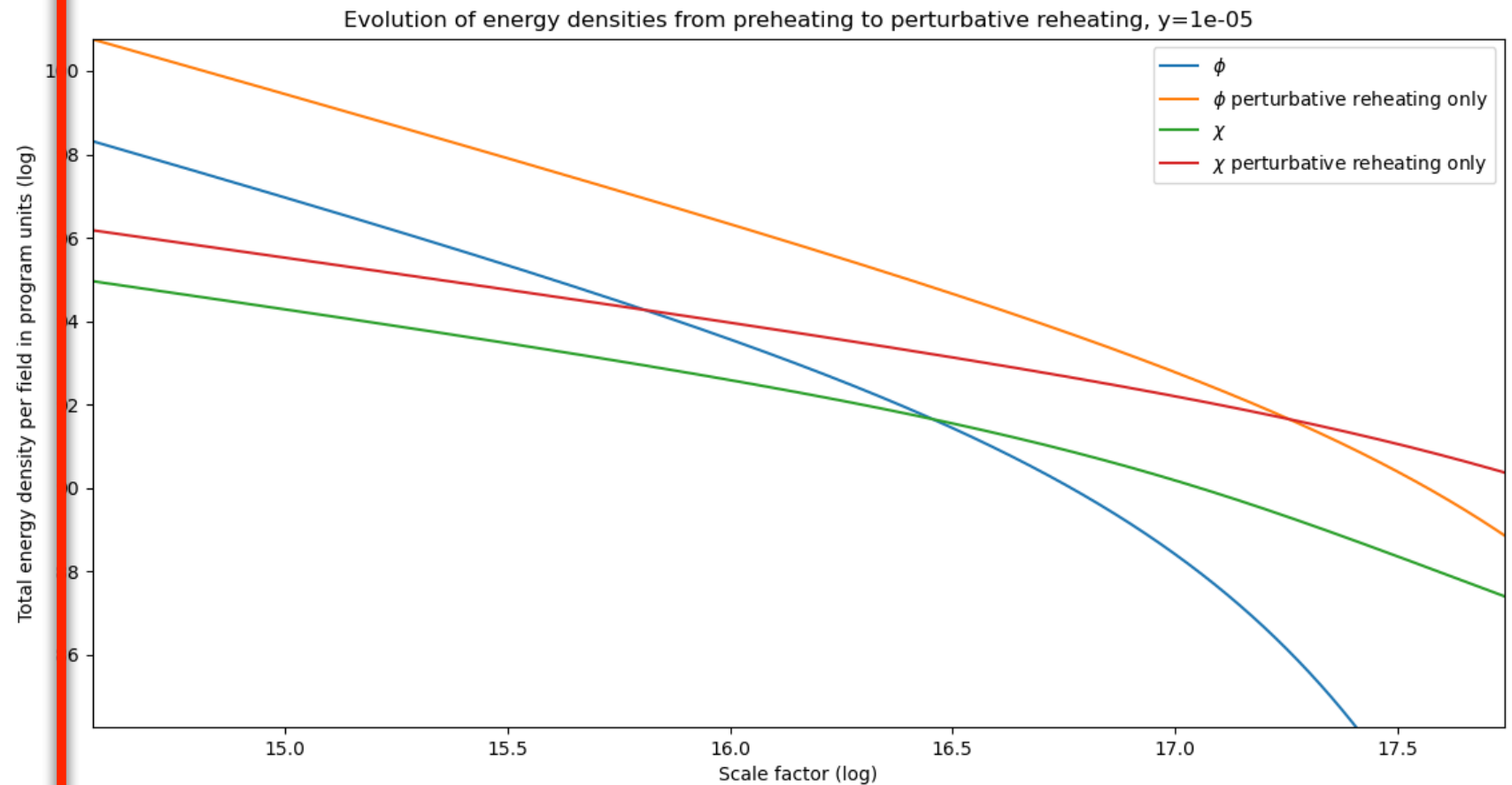


Important remark : the preheating *says nothing about reheating*, and especially gives the same reheating temperature than a perturbative treatment.

This comes from the fact that the reheating happens for

$$\rho_R = \rho_\phi \simeq \sqrt{\Gamma_\phi M_P} \sim T_{RH}^4$$

It just happens *before*.



Conclusion

Studying the details of dark matter production in the earliest phase of the Universe is important for DM production in **freeze-in** scenario

