

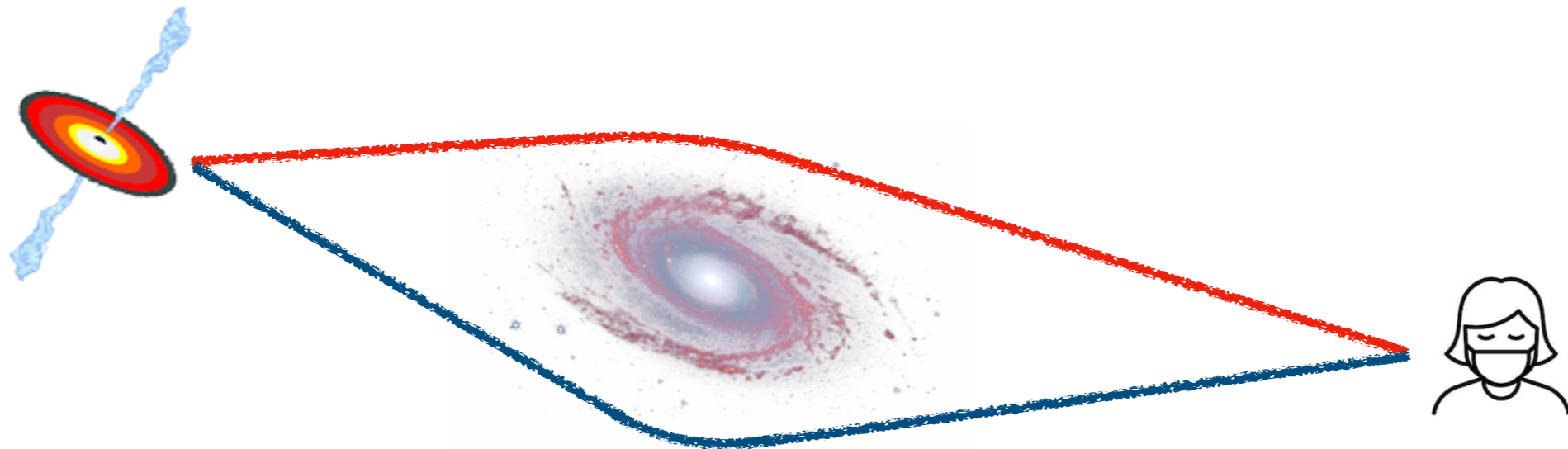
Gravitational lensing measurements of the Hubble parameter: challenges and opportunities

Kfir Blum (Weizmann Institute)

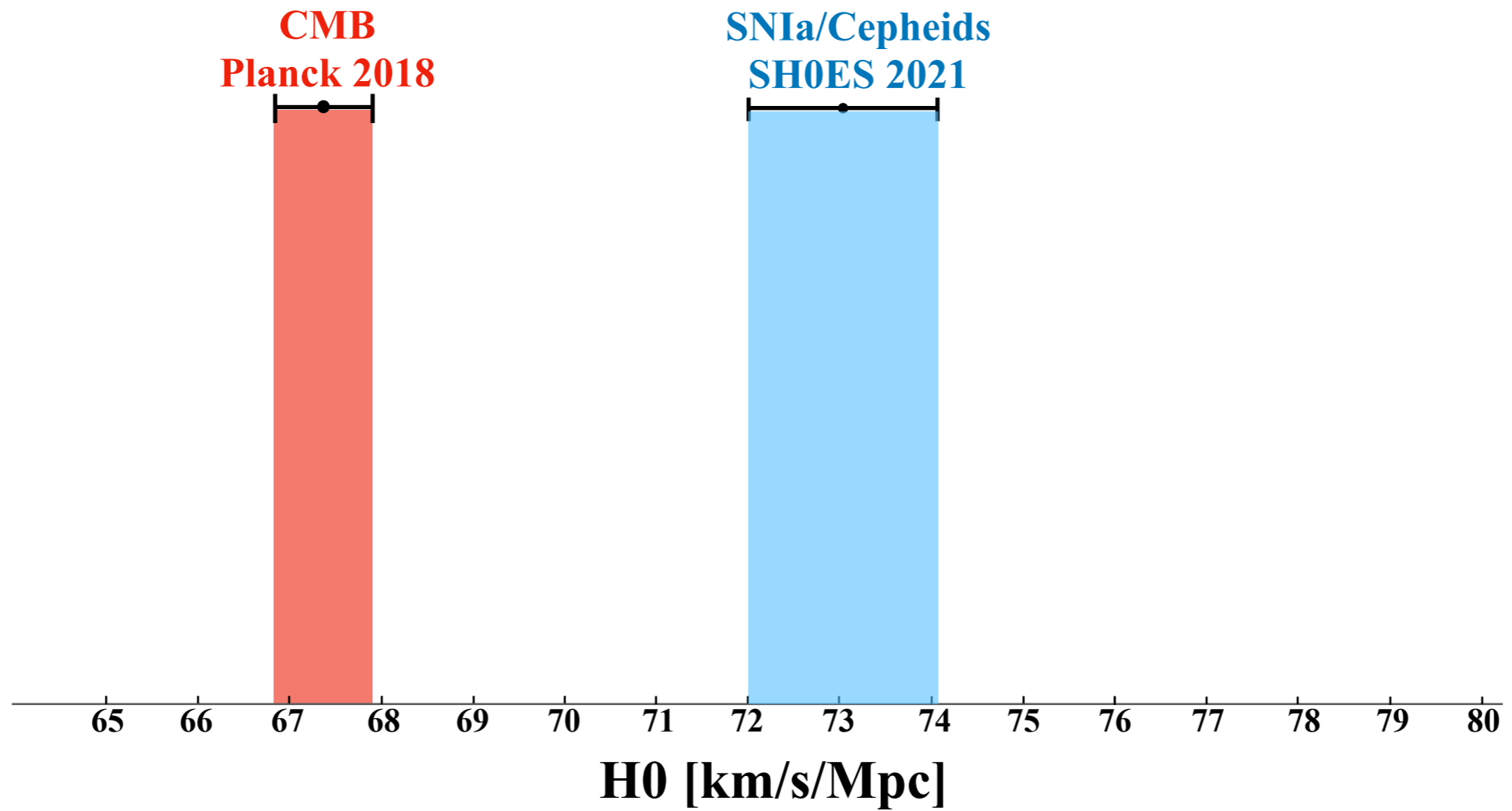
KB, Castorina, Simonović, 2001.07182

KB, Teodori, 2105.10873

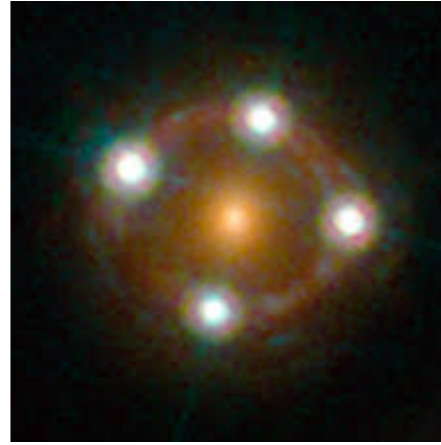
Teodori, KB, Castorina, Simonović, Soreq, 2201.05111



H₀ tension



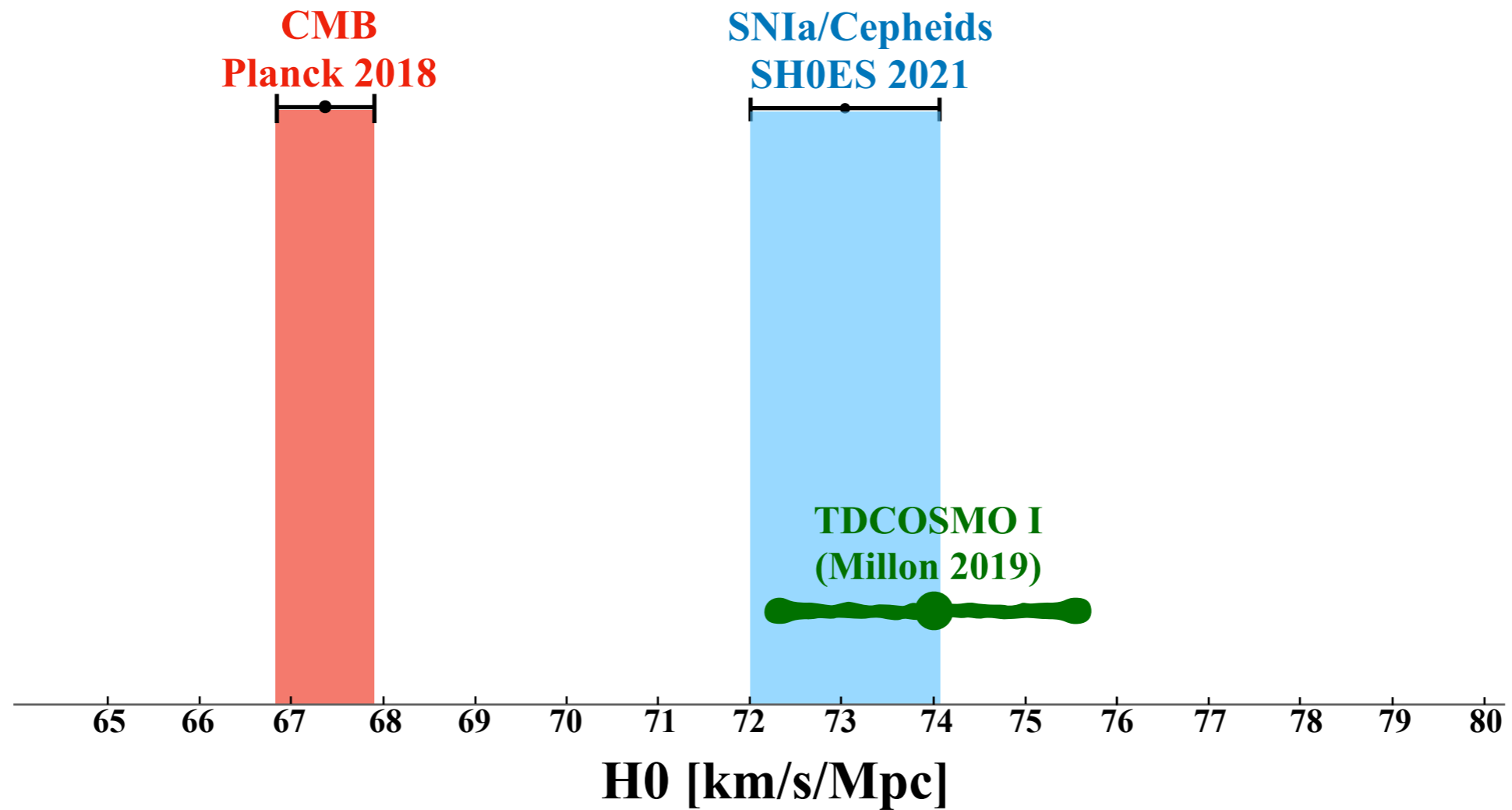
SNIa and **gravitational lensing** agree



TDCOSMO

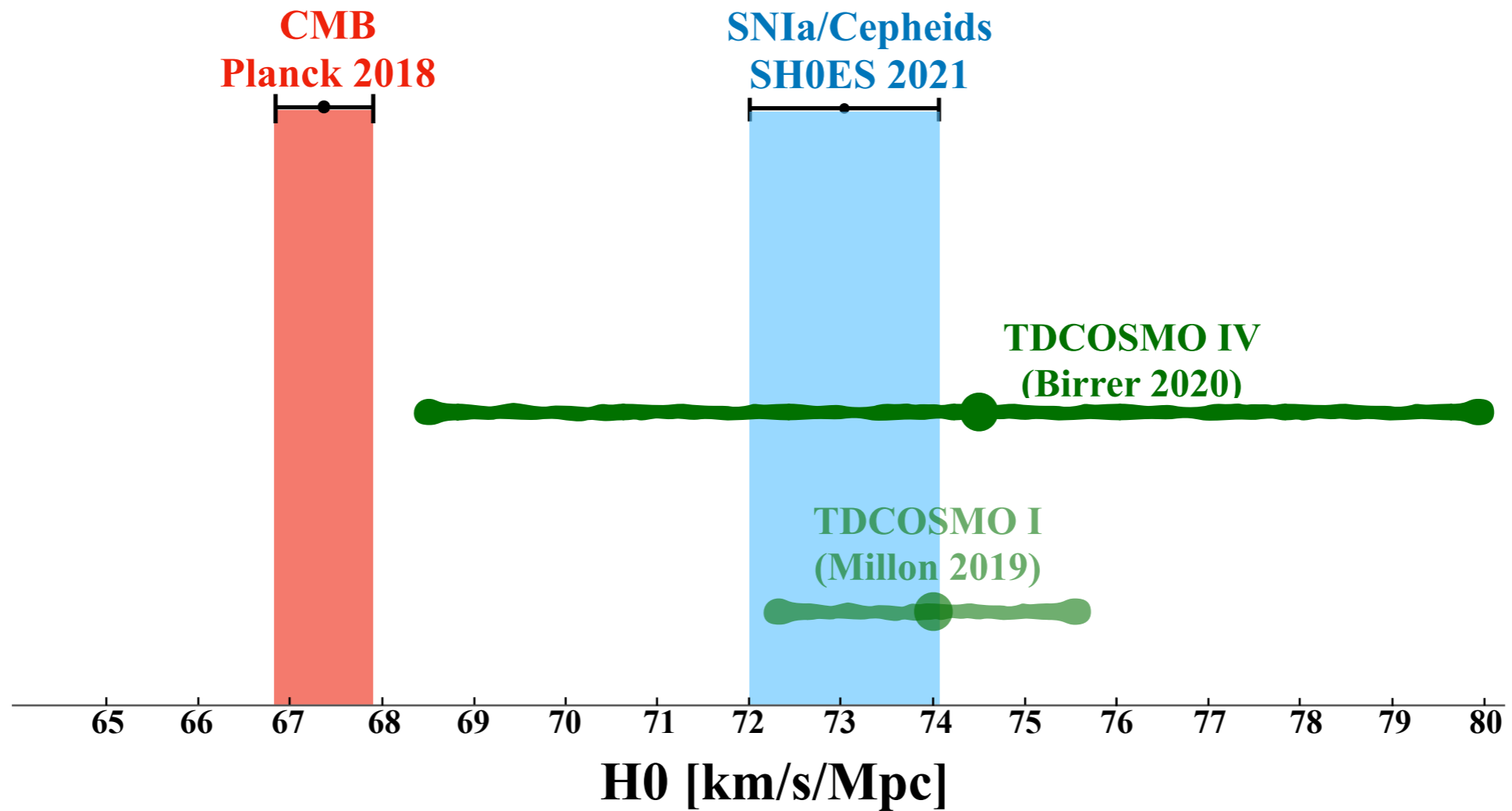
<http://www.tdcosmo.org/projects.html>

- H0LiCOW
- COSMOGRAIL
- STRIDES
- SHARP
- COSMICLENS



SN Ia and gravitational lensing

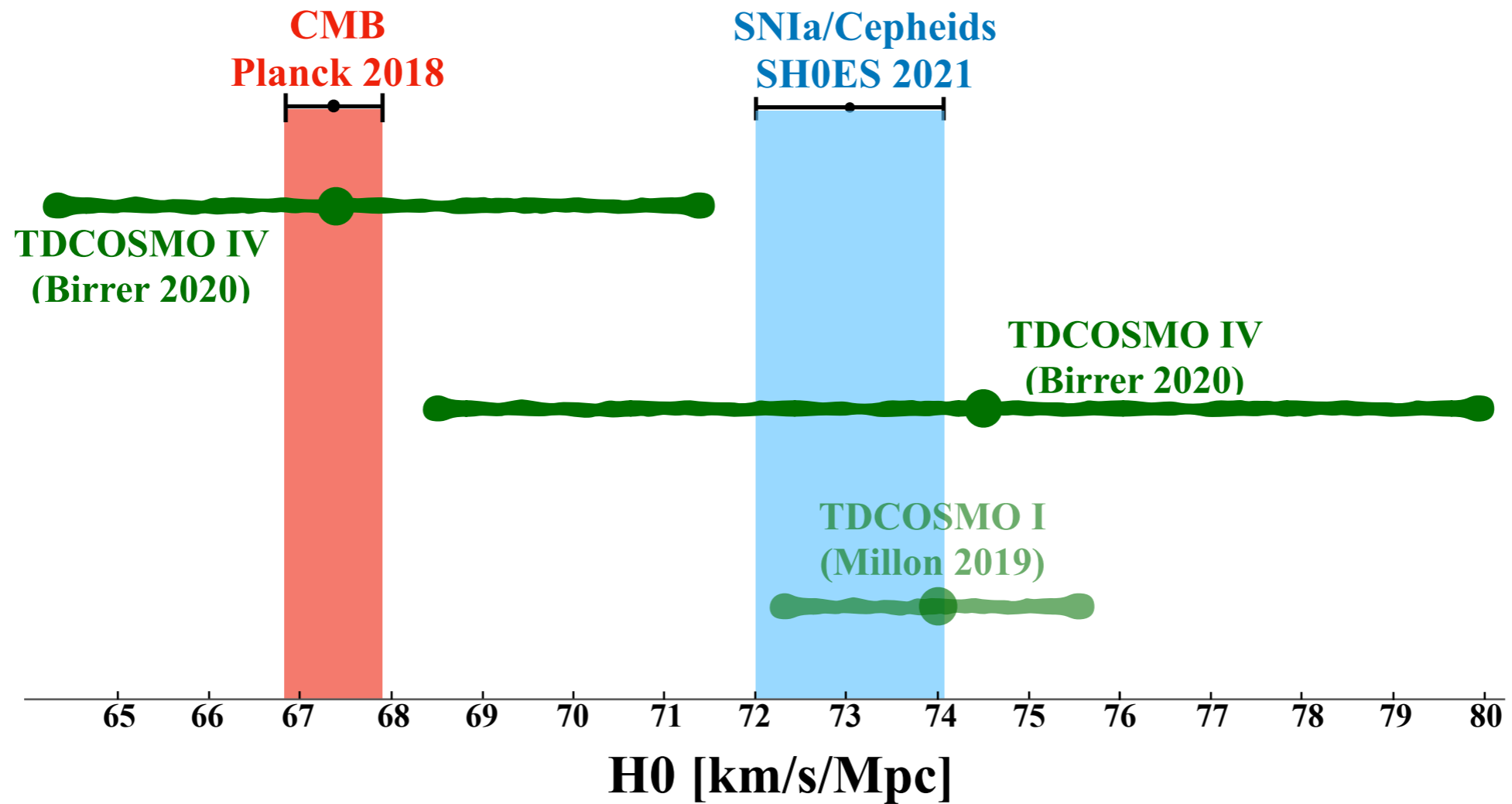
A bit later: lensing out of the game?



SN Ia and gravitational lensing

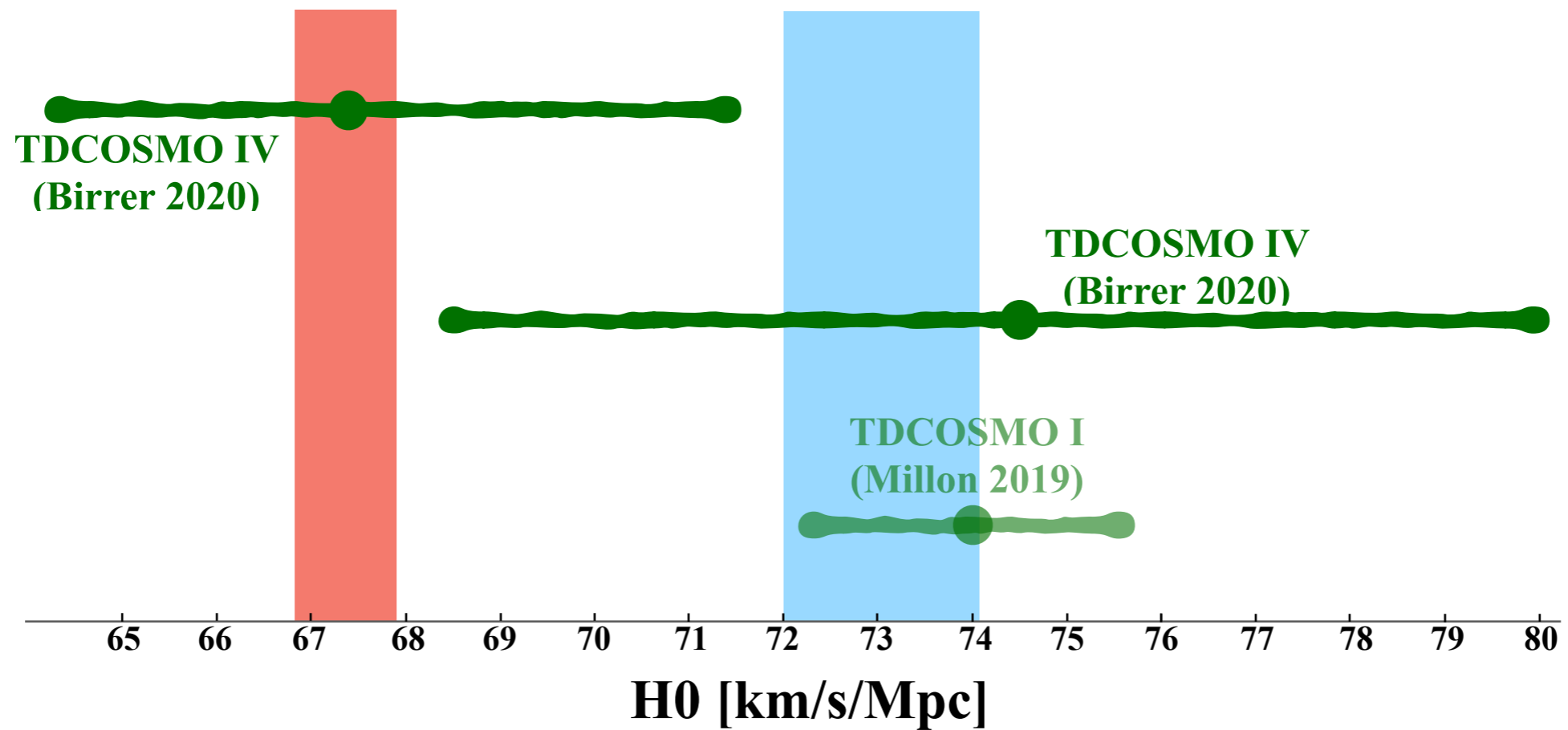
A bit later: lensing out of the game?

... agrees with CMB?



Plan:

1. Recap: how lensing measures H_0
2. Challenges: modeling degeneracy
3. Opportunities: galactic structure



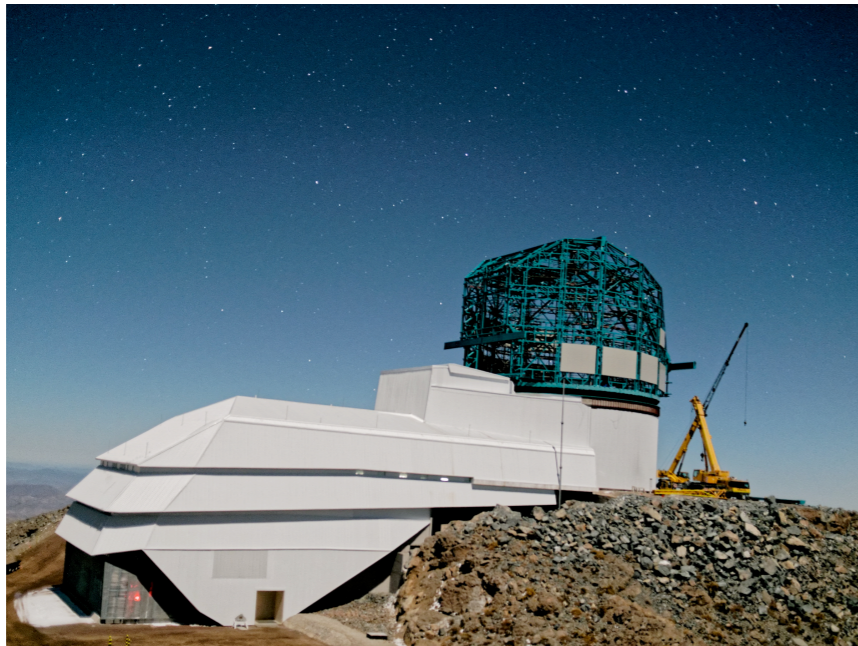
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What is dark matter?

Where are the missing baryons?

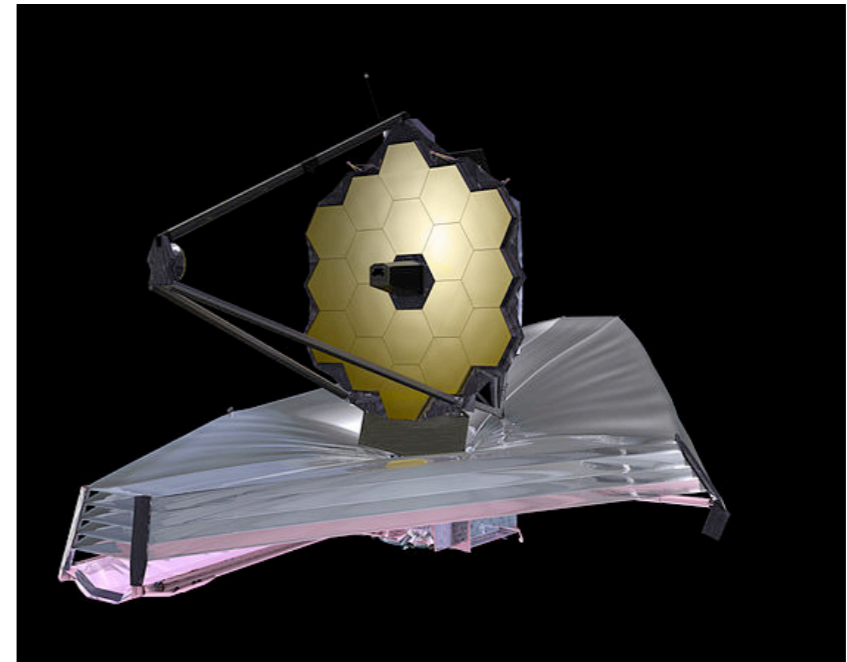
LSST: 100's of strongly lensed variable quasars
Oguri, Marshall, 1001.2037



JWST: improved kinematics

Yıldırım, Suyu, Halkola, 1904.07237

Birrer, Treu, 2008.06157

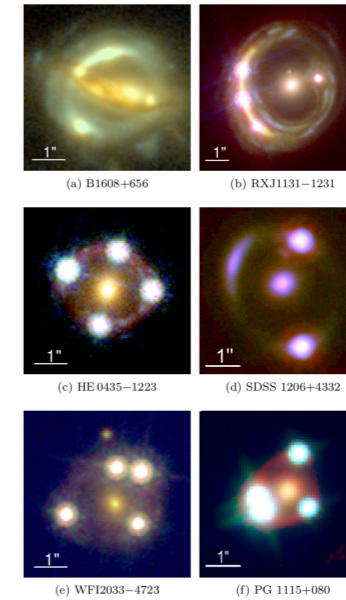


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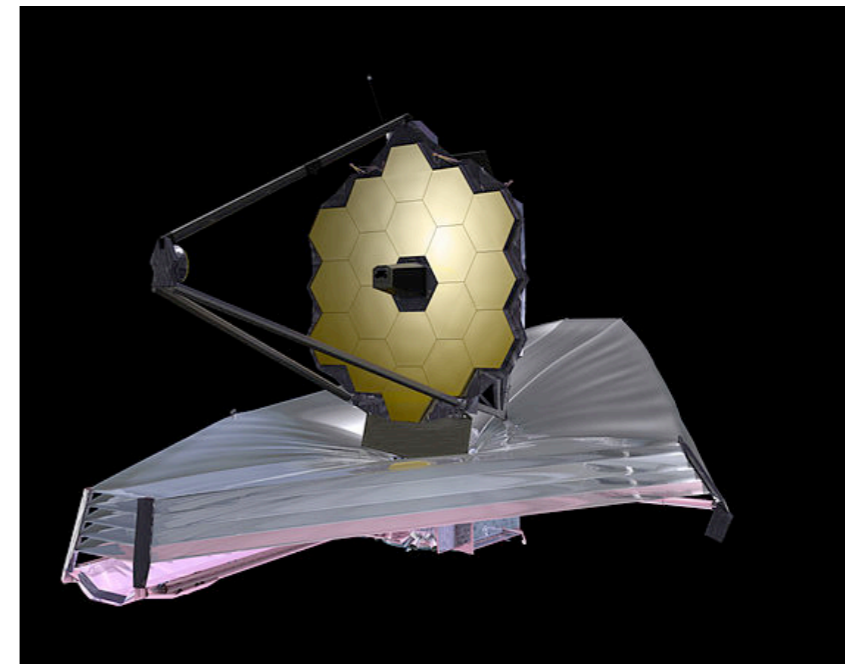
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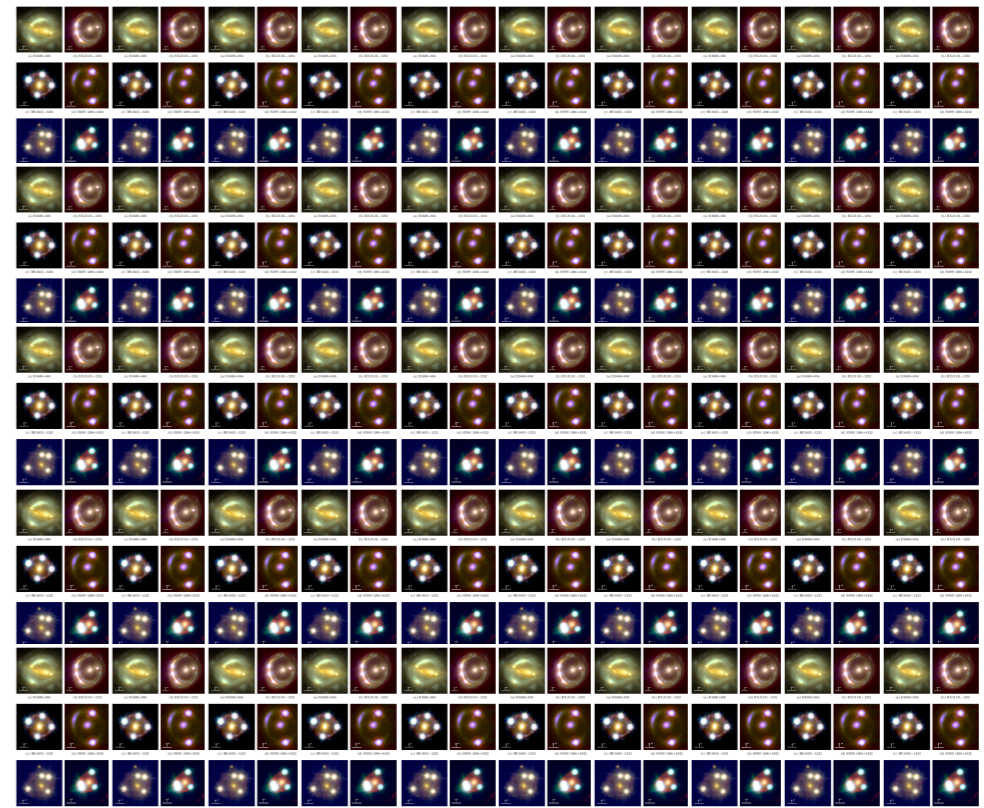


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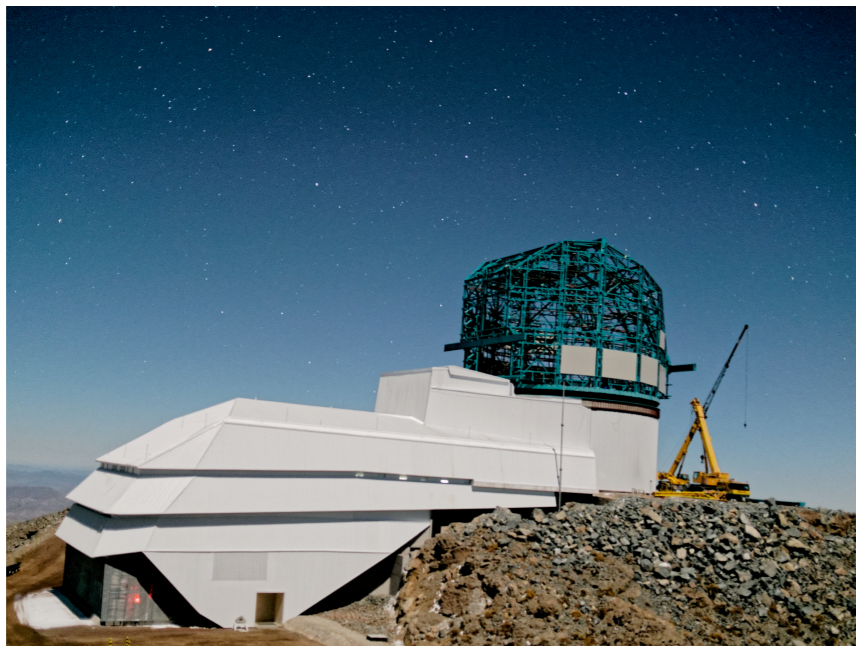
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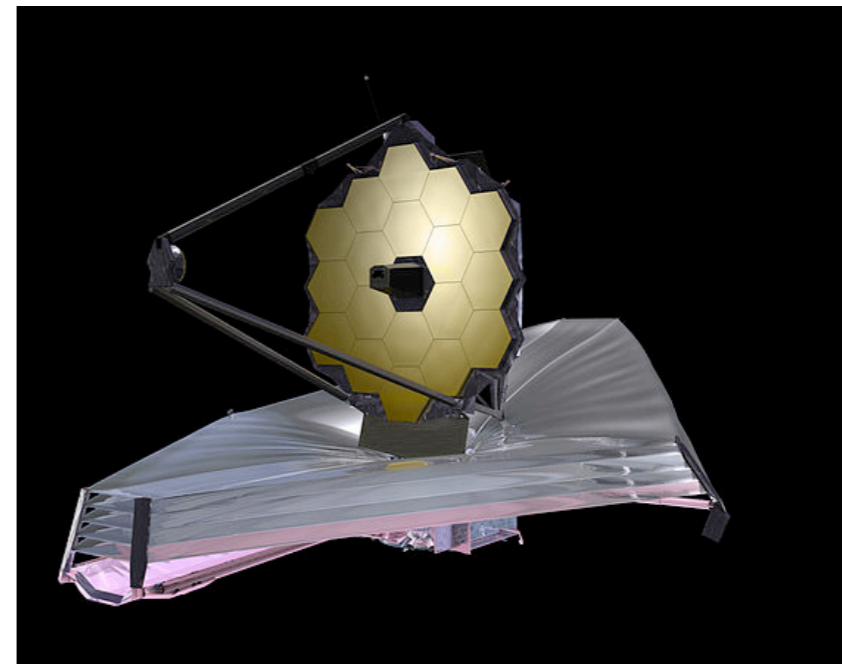
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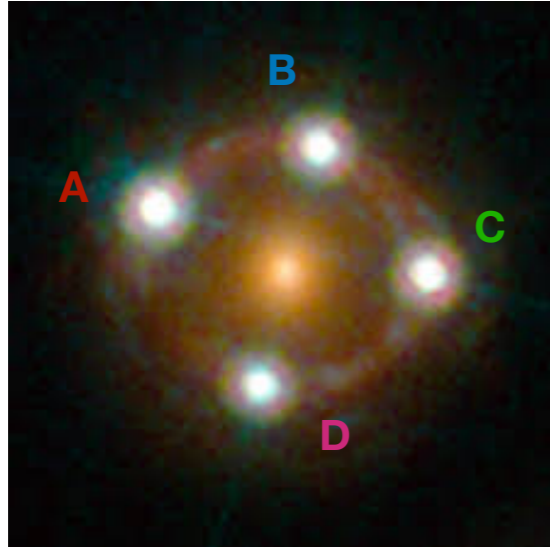
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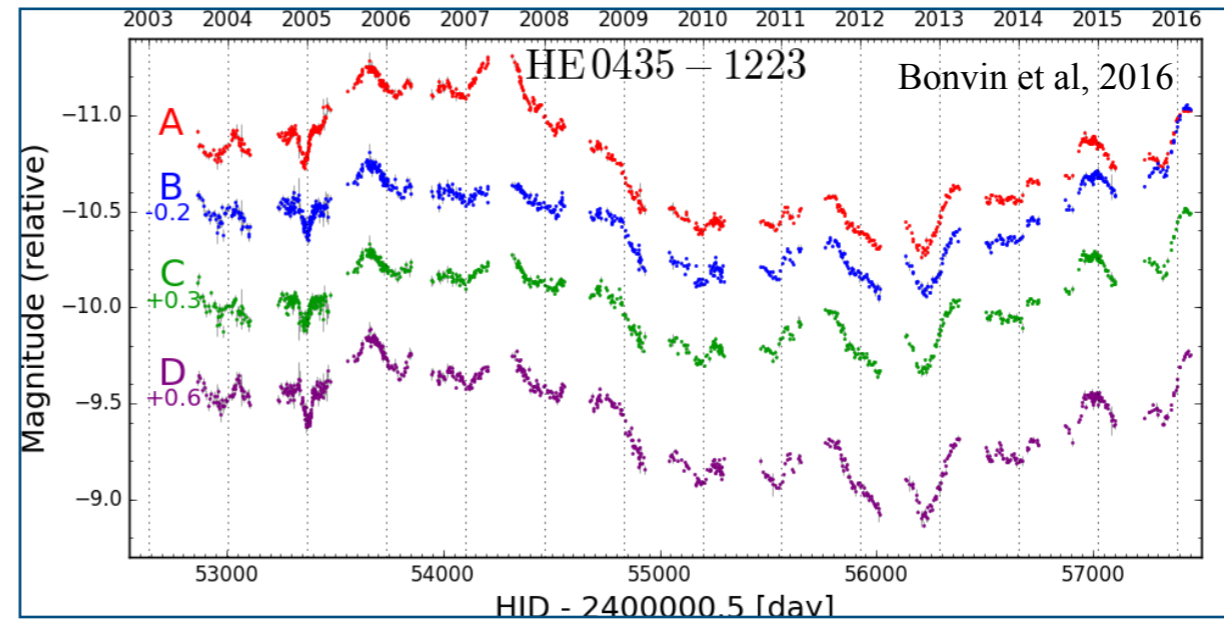
1. Recap: how lensing measures H0

Observables:

- Extended source image



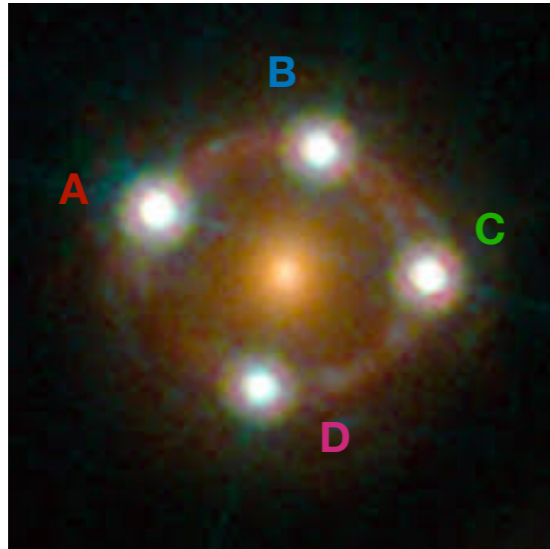
- Time delay Δt_{AB}



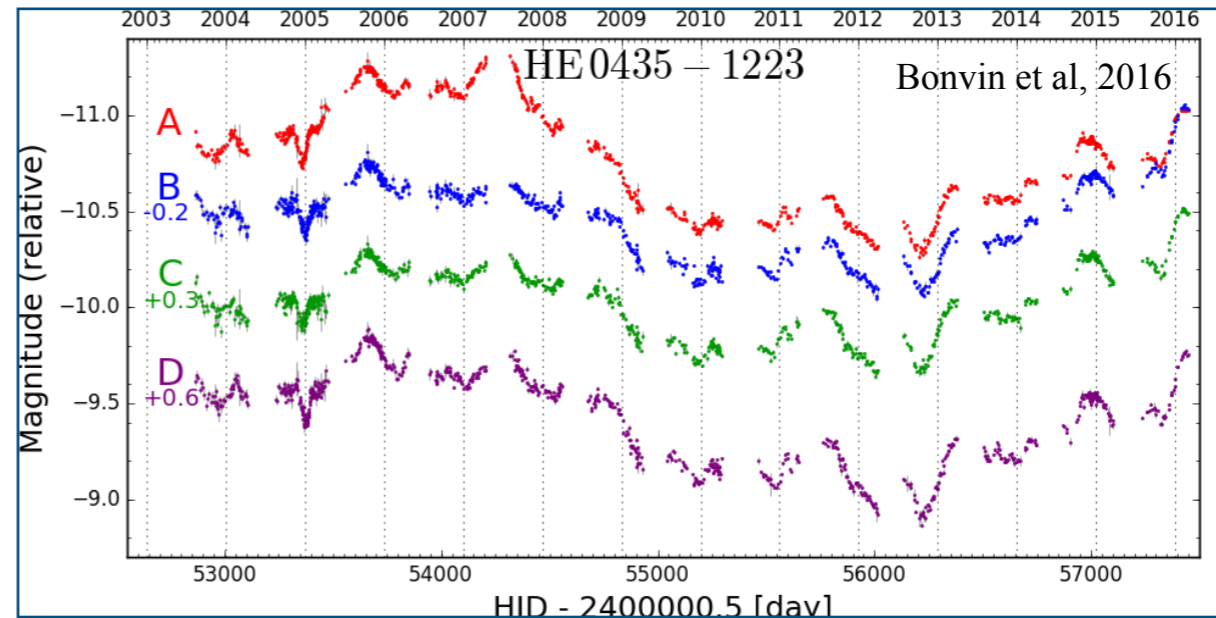
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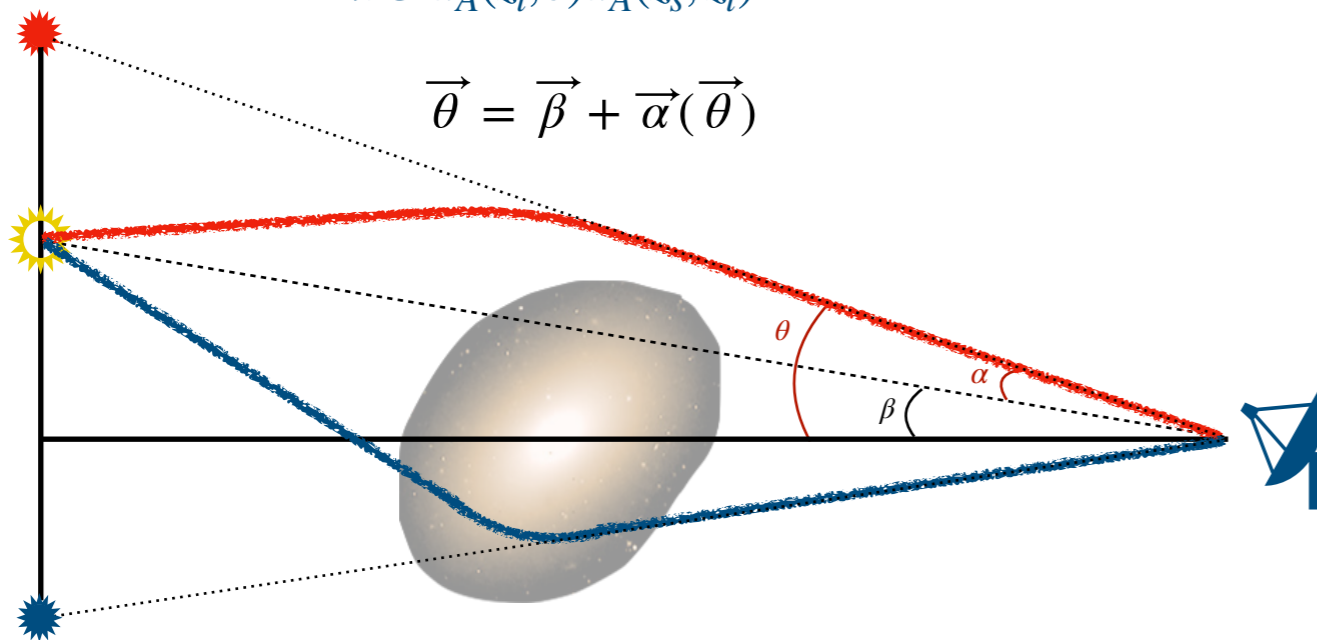
- Time delay Δt_{AB}



$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{crit}}} = \frac{1}{2} \vec{\nabla}_{\theta} \cdot \vec{\alpha} = \frac{1}{2} \vec{\nabla}_{\theta}^2 \psi$$

$$\Sigma_{\text{crit}} = \frac{d_A(z_s, 0)}{4\pi G d_A(z_l, 0) d_A(z_s, z_l)}$$

$$\vec{\theta} = \vec{\beta} + \vec{\alpha}(\vec{\theta})$$



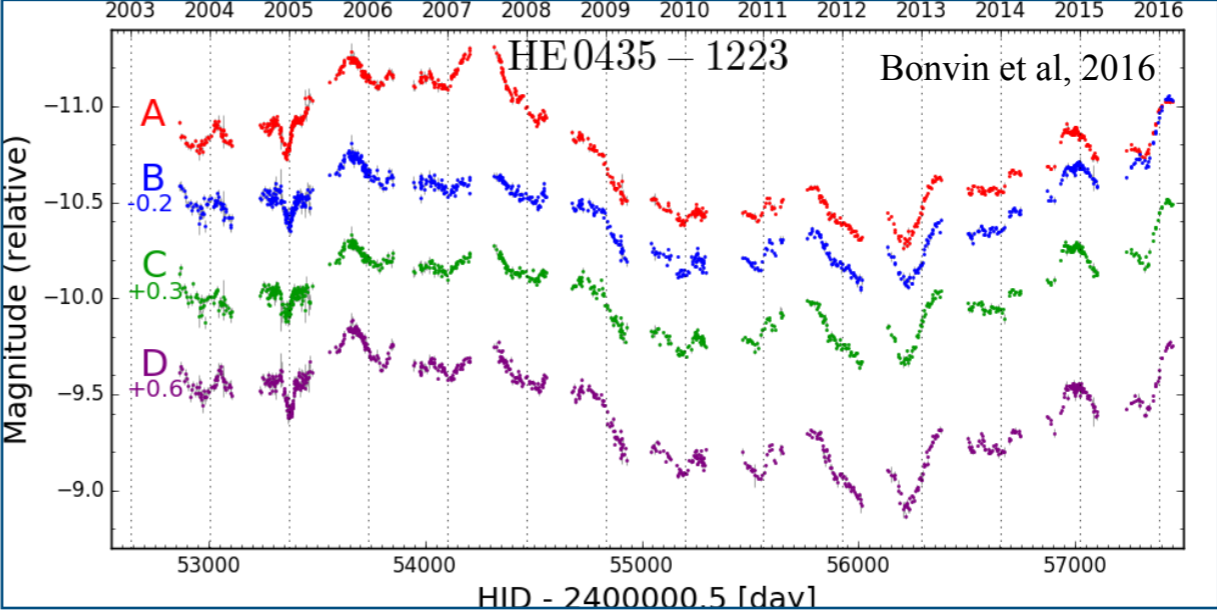
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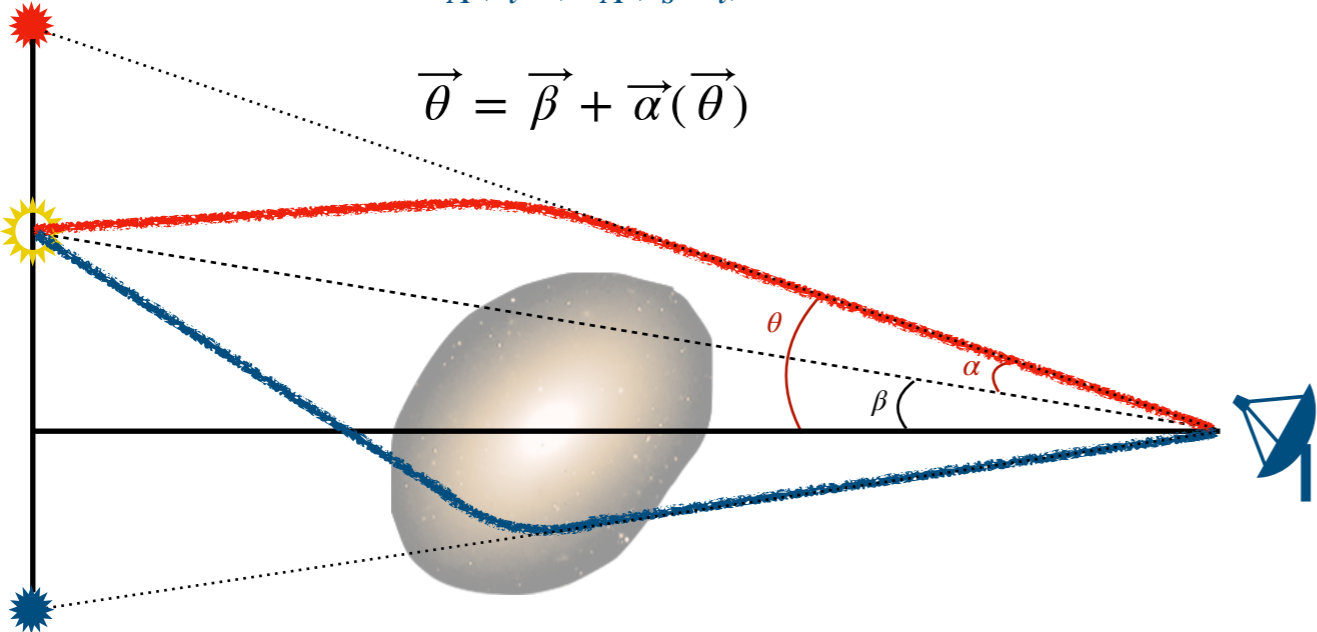
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$$\Delta t_{AB} = D_{\Delta t} \Delta \tau_{AB} \quad \Delta \tau_{AB} = \frac{\vec{\theta}_A^2}{2} - \vec{\beta} \cdot \vec{\theta}_A - \psi(\vec{\theta}_A) - (A \leftrightarrow B)$$

$$\Sigma_{\text{crit}} = \frac{d_A(z_s, 0)}{4\pi G d_A(z_l, 0) d_A(z_s, z_l)}$$

$$D_{\Delta t} = (1 + z_l) \frac{d_A(z_l, 0) d_A(z_s, 0)}{d_A(z_s, z_l)}$$

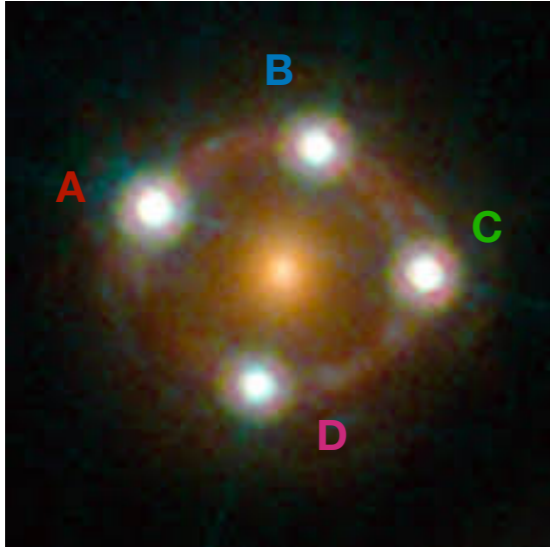
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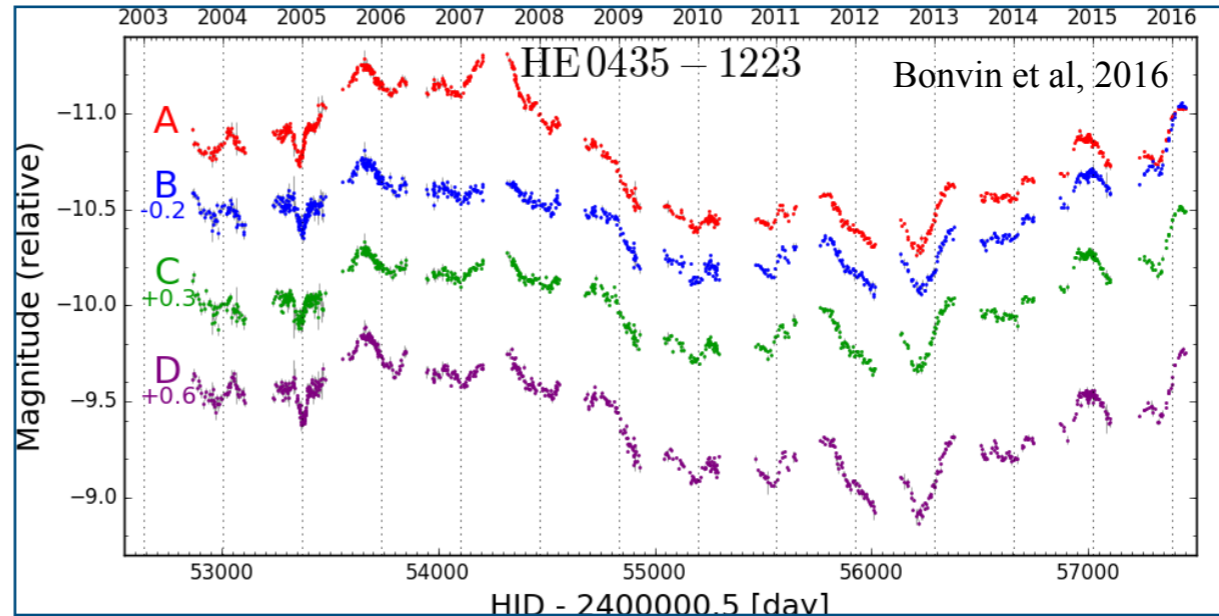
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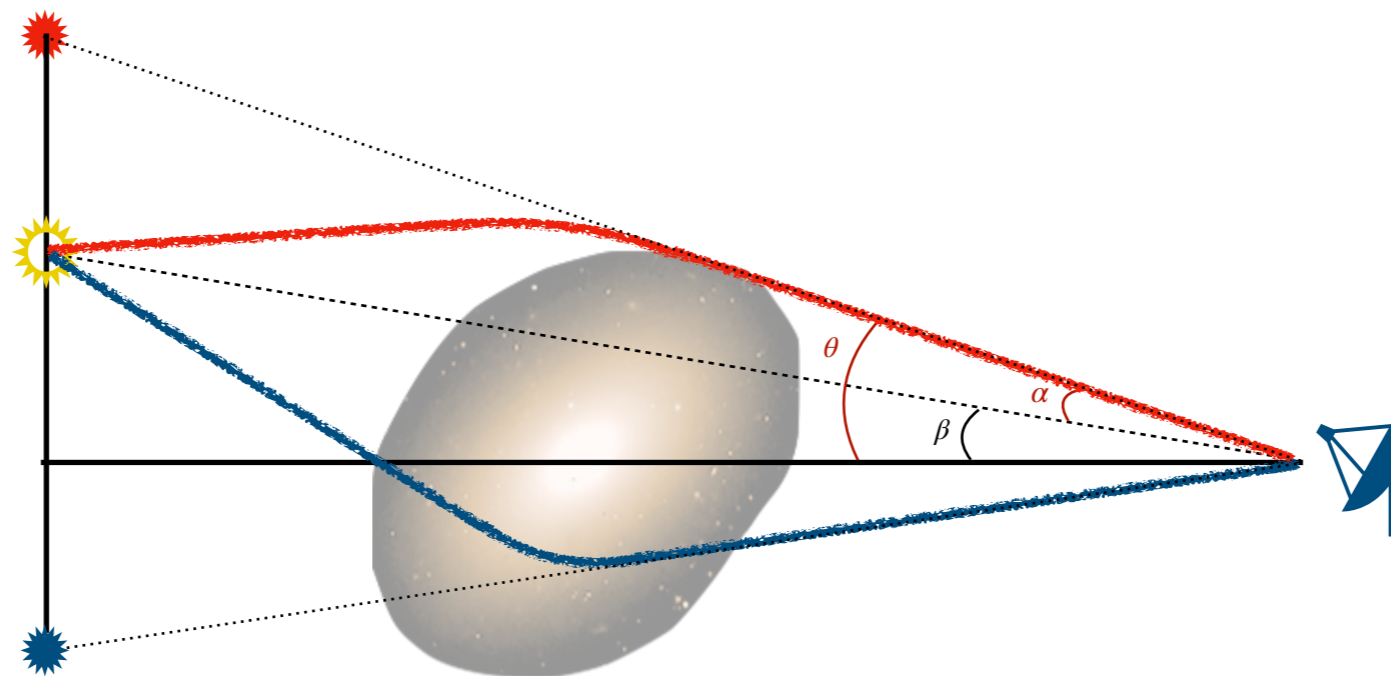
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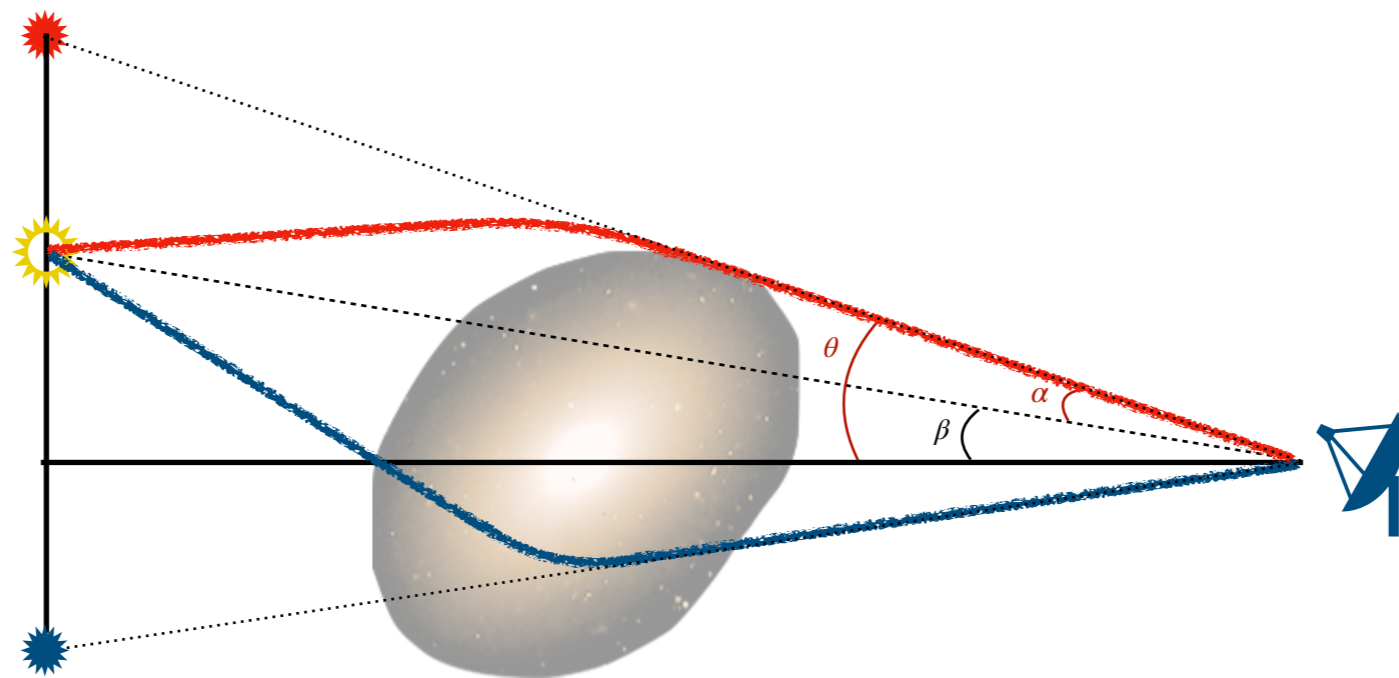
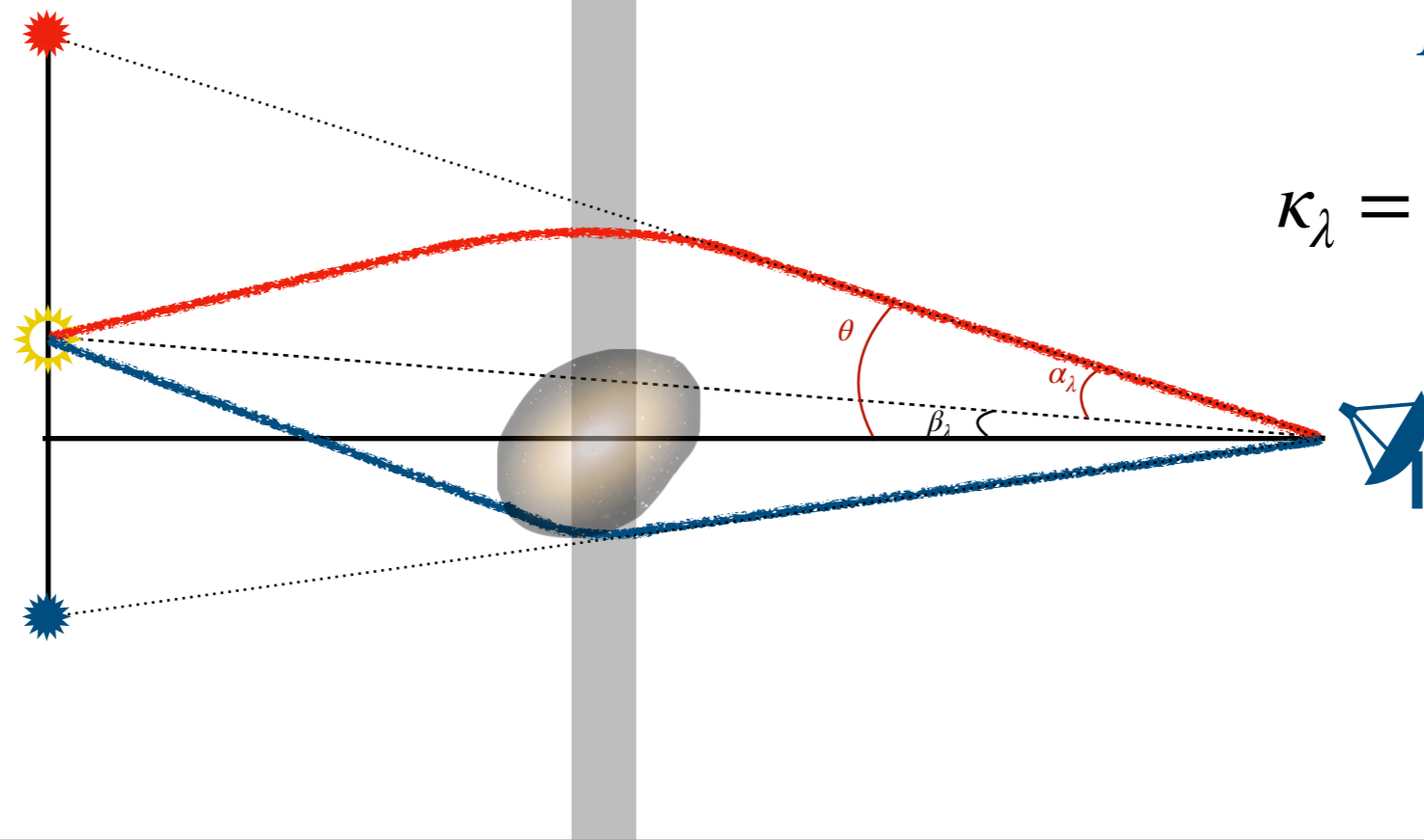
- From the image, reconstruct a model $\kappa(\vec{\theta}), \vec{\beta} \rightarrow \Delta \tau$
- Given the model and Δt , extract $D_{\Delta t} = \frac{\Delta t}{\Delta \tau} \propto 1/H_0$

2. Challenges: modeling degeneracy


$$\begin{aligned}\vec{\theta} &= \vec{\beta} + \vec{\alpha} \\ &= \vec{\beta}_\lambda + \vec{\alpha}_\lambda \\ &= \lambda \vec{\beta} + \lambda \vec{\alpha} + (1 - \lambda) \vec{\theta} \\ &= \lambda (\vec{\beta} + \vec{\alpha} - \vec{\theta}) + \vec{\theta} \\ &= \vec{\theta}\end{aligned}$$

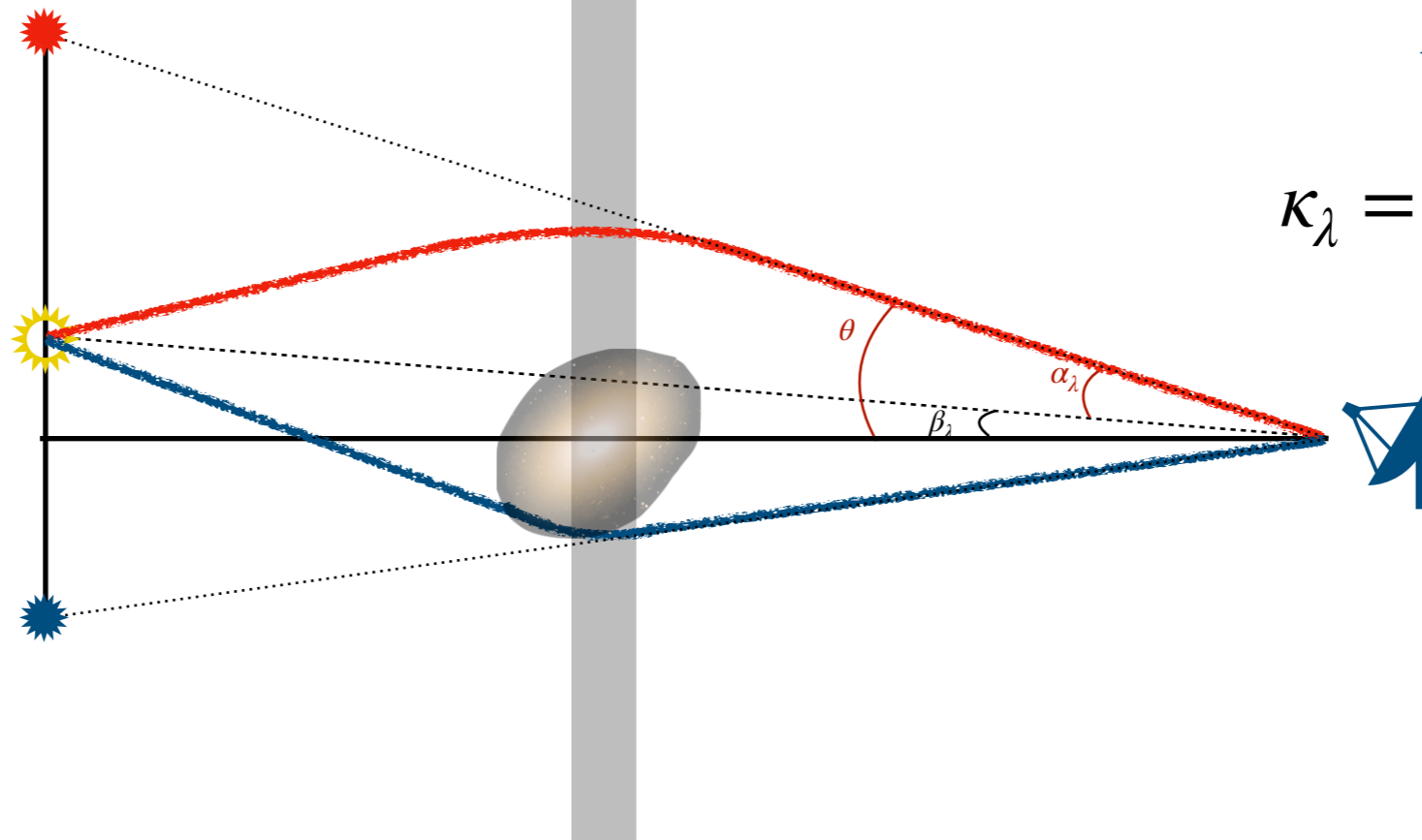


H_0
 κ

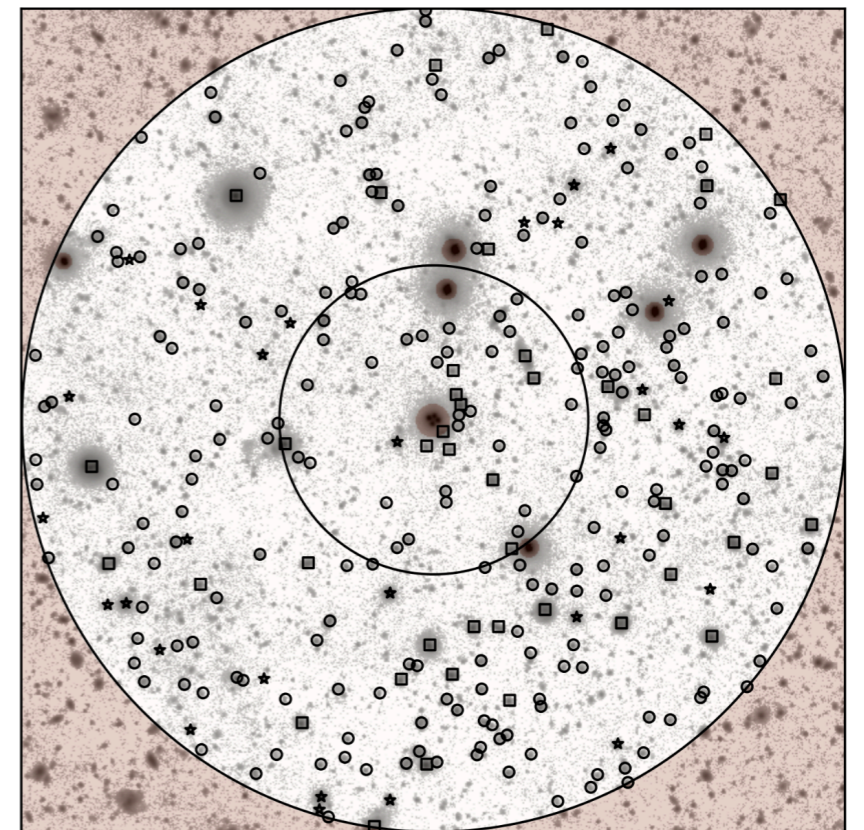
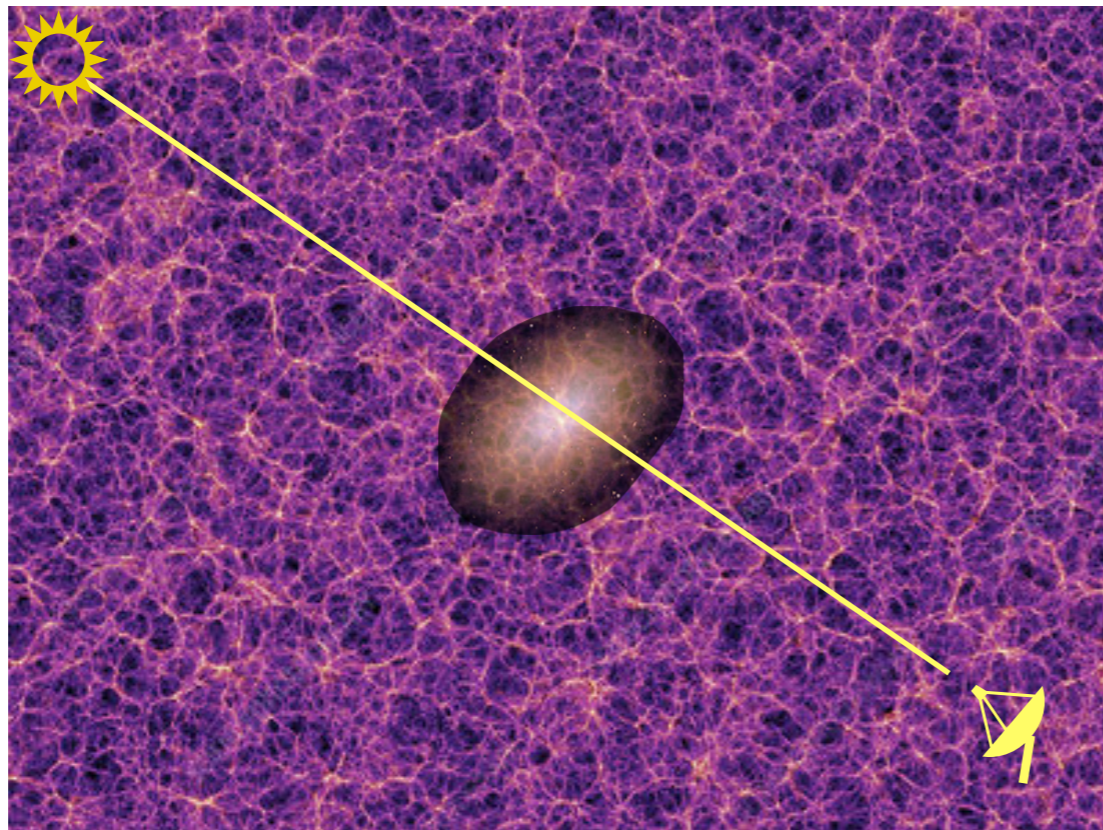


$$H_{0\lambda} = \lambda H_0$$

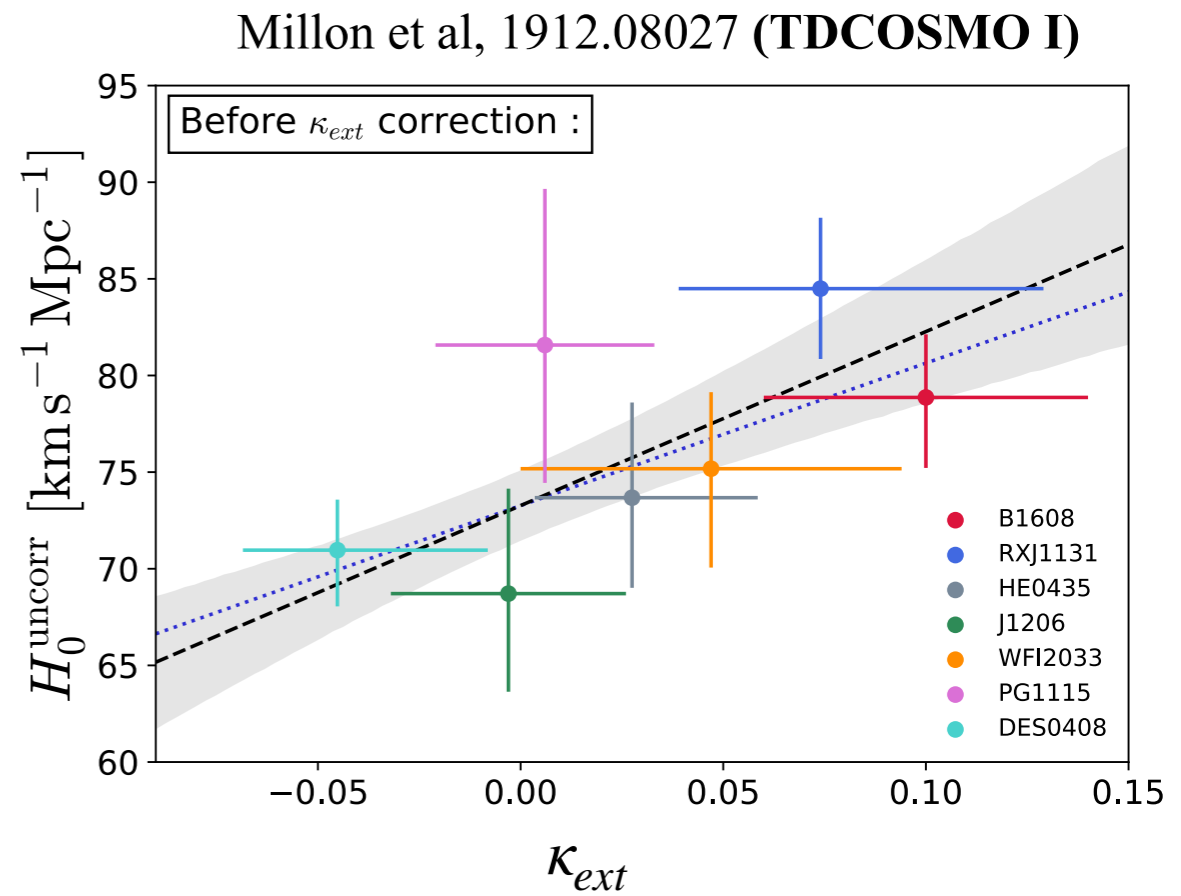
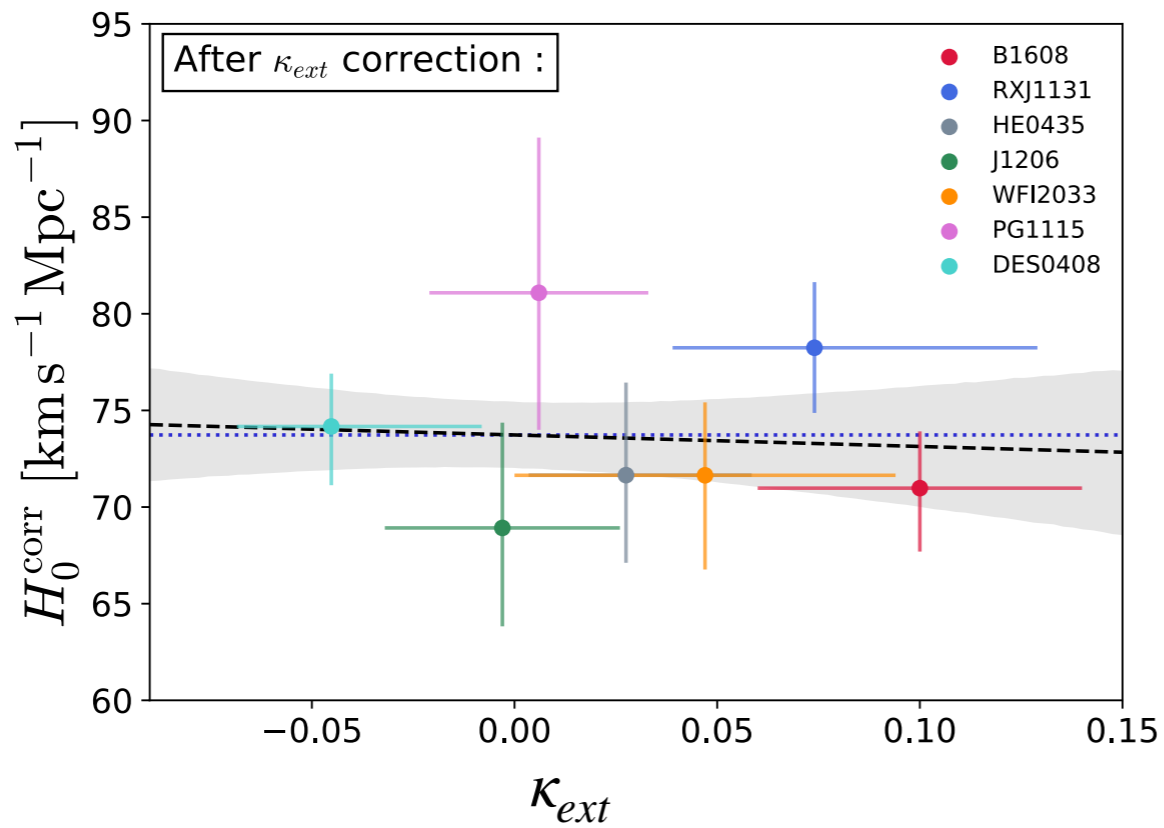
$$\kappa_\lambda = \lambda\kappa + (1 - \lambda)$$



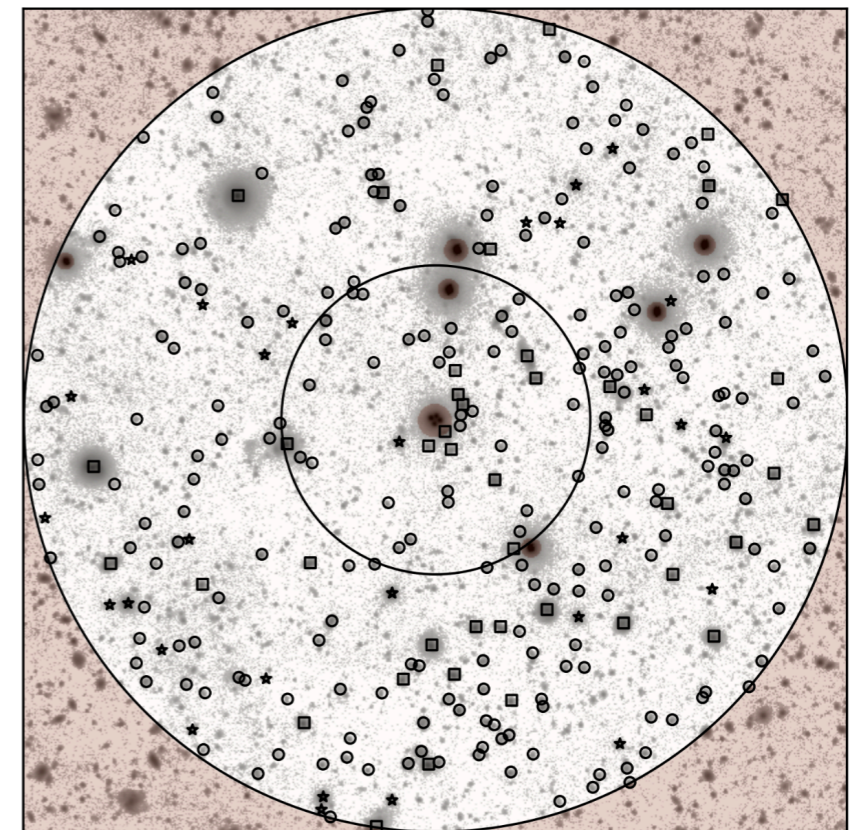
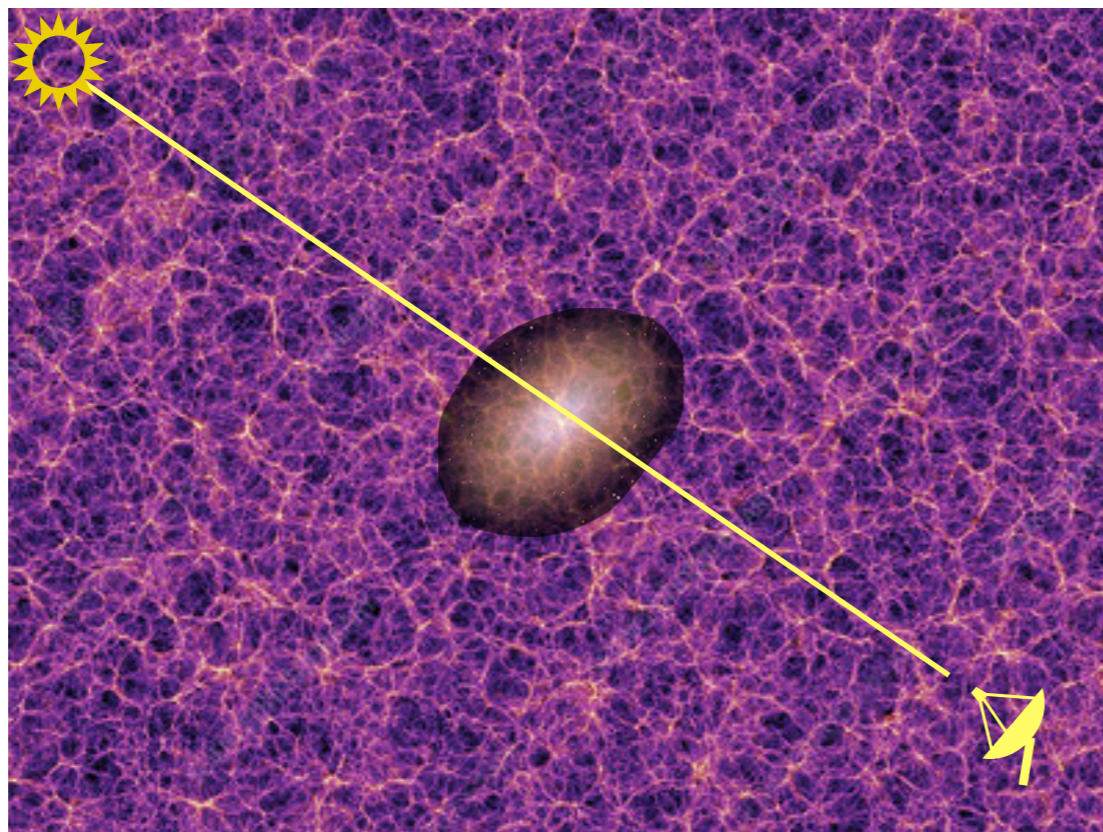
Modelling
external
convergence



Rusu et al, 1607.01047 (H0LiCOW III)

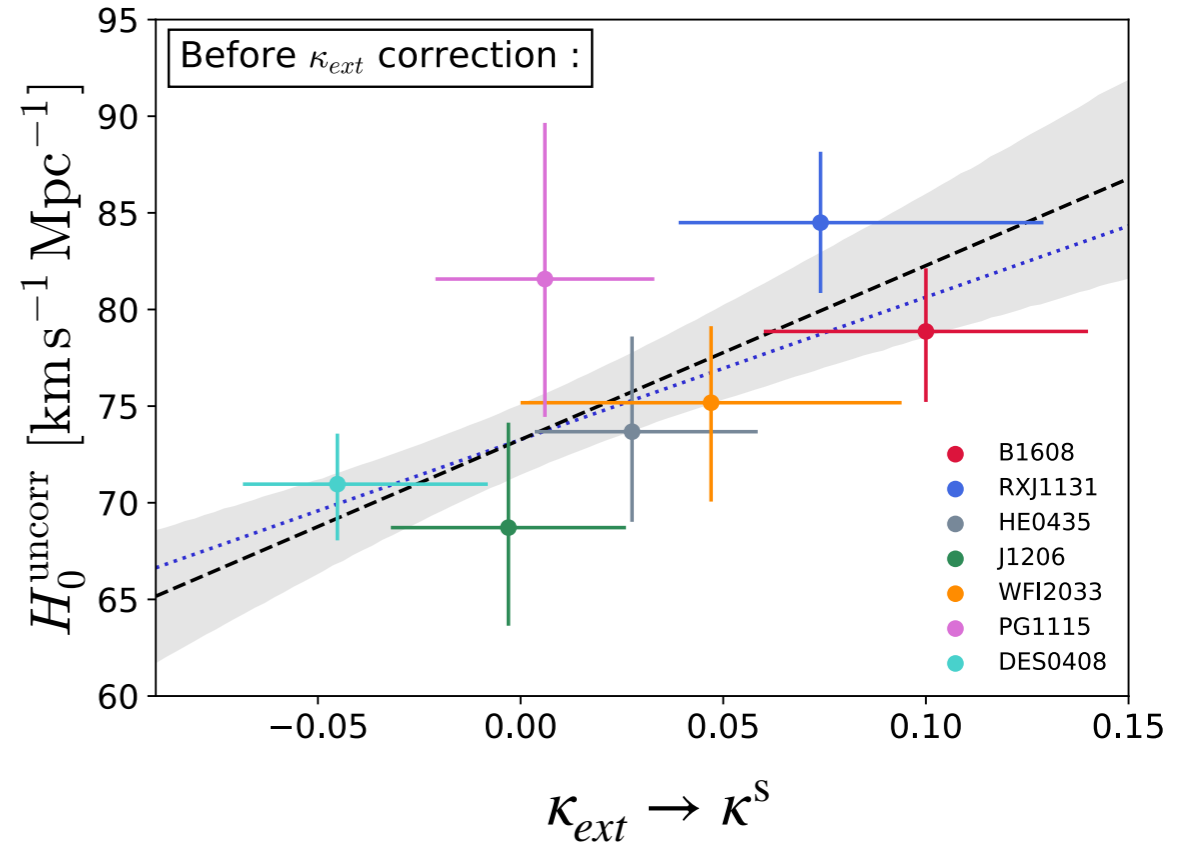
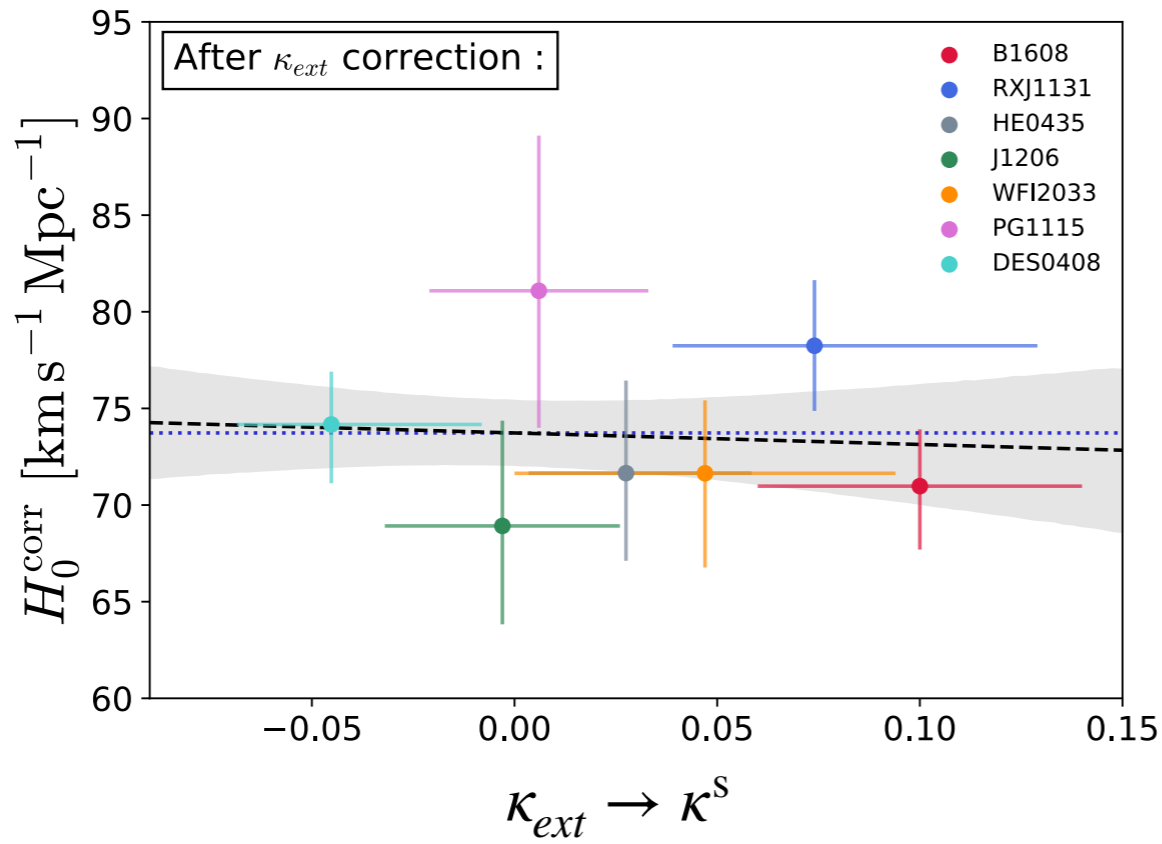


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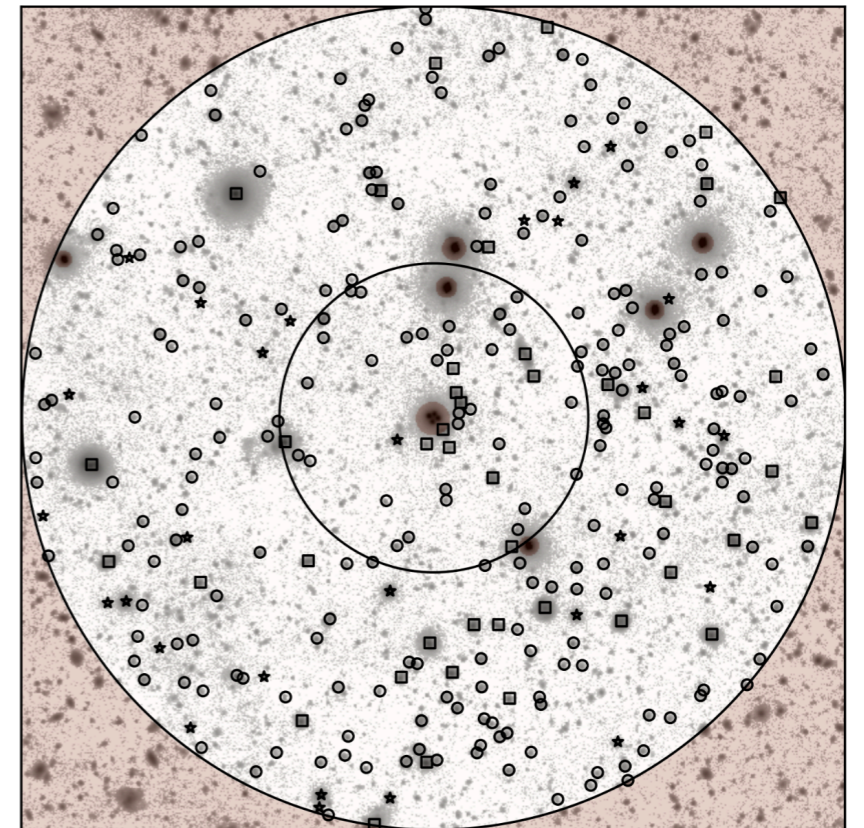
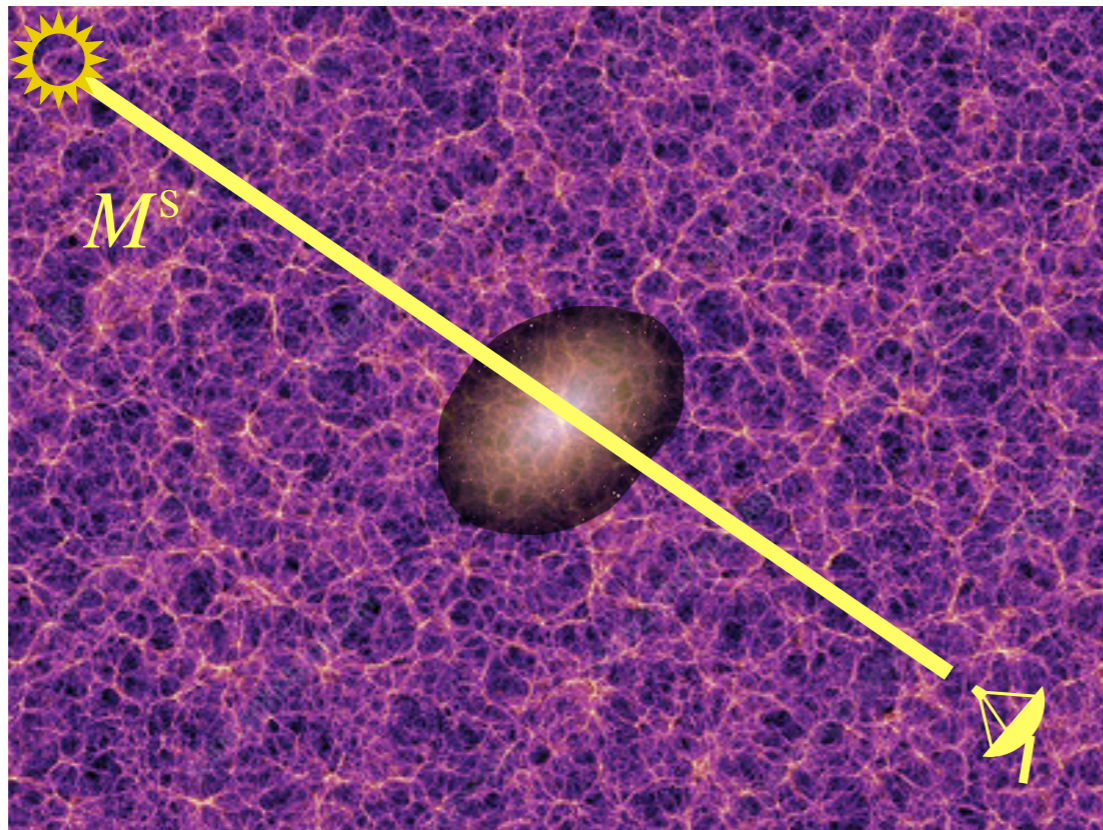


Rusu et al, 1607.01047 (H0LiCOW III)

Millon et al, 1912.08027 (TDCOSMO I)



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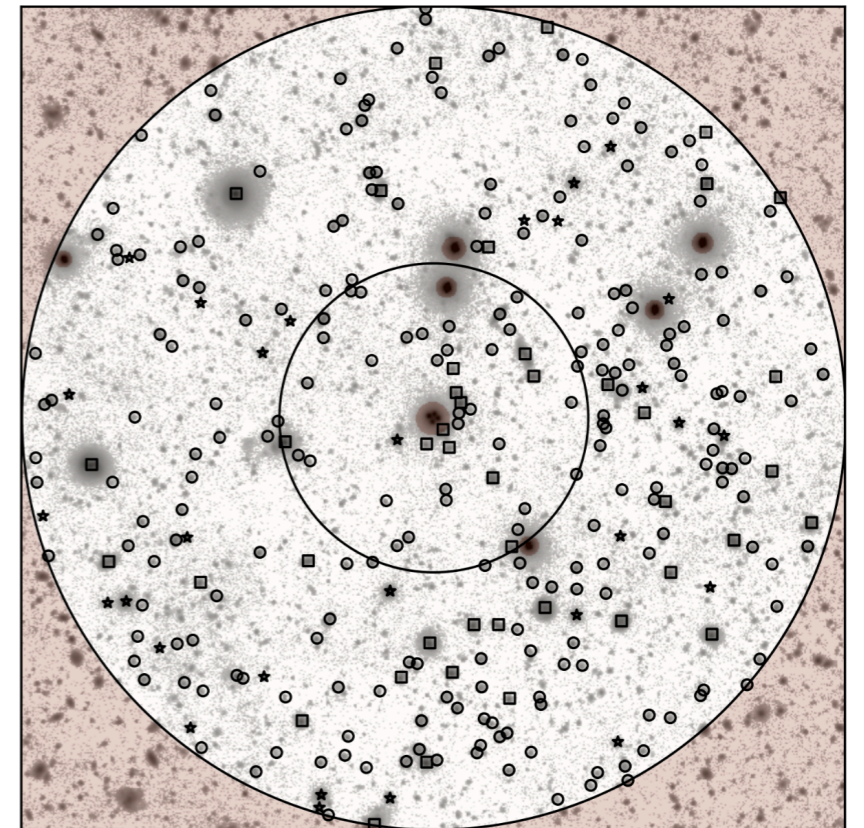
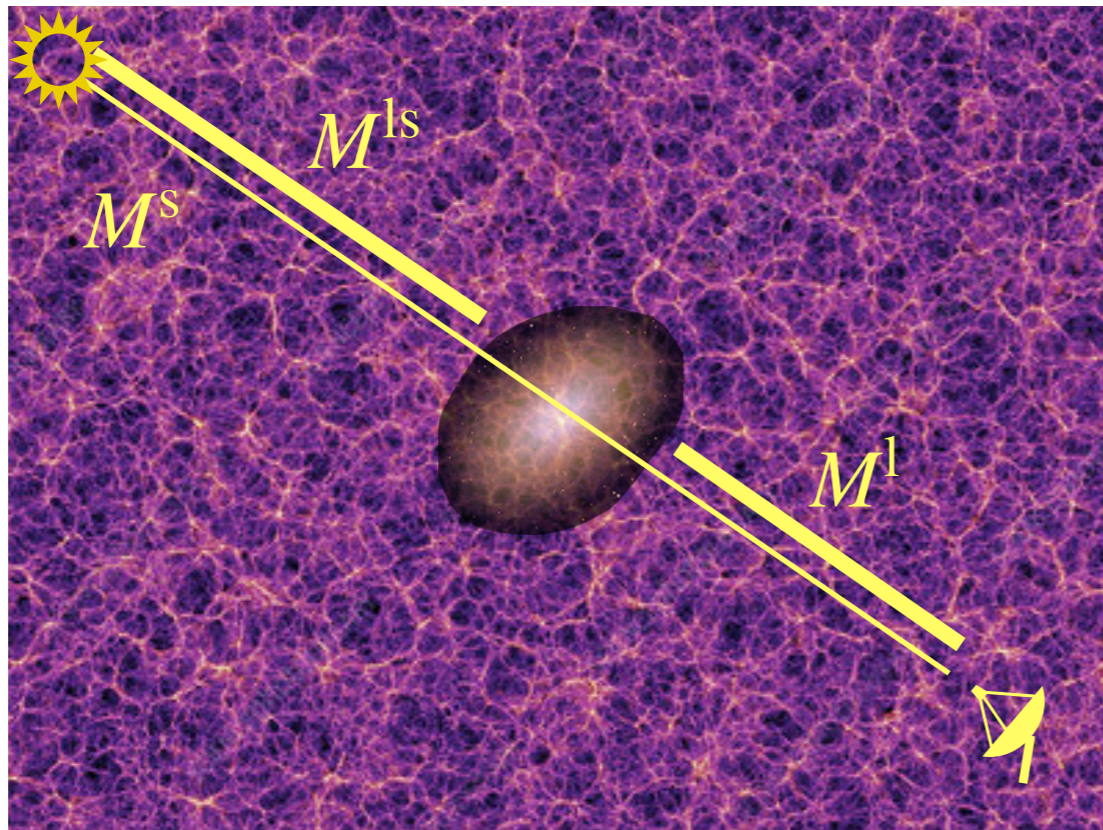
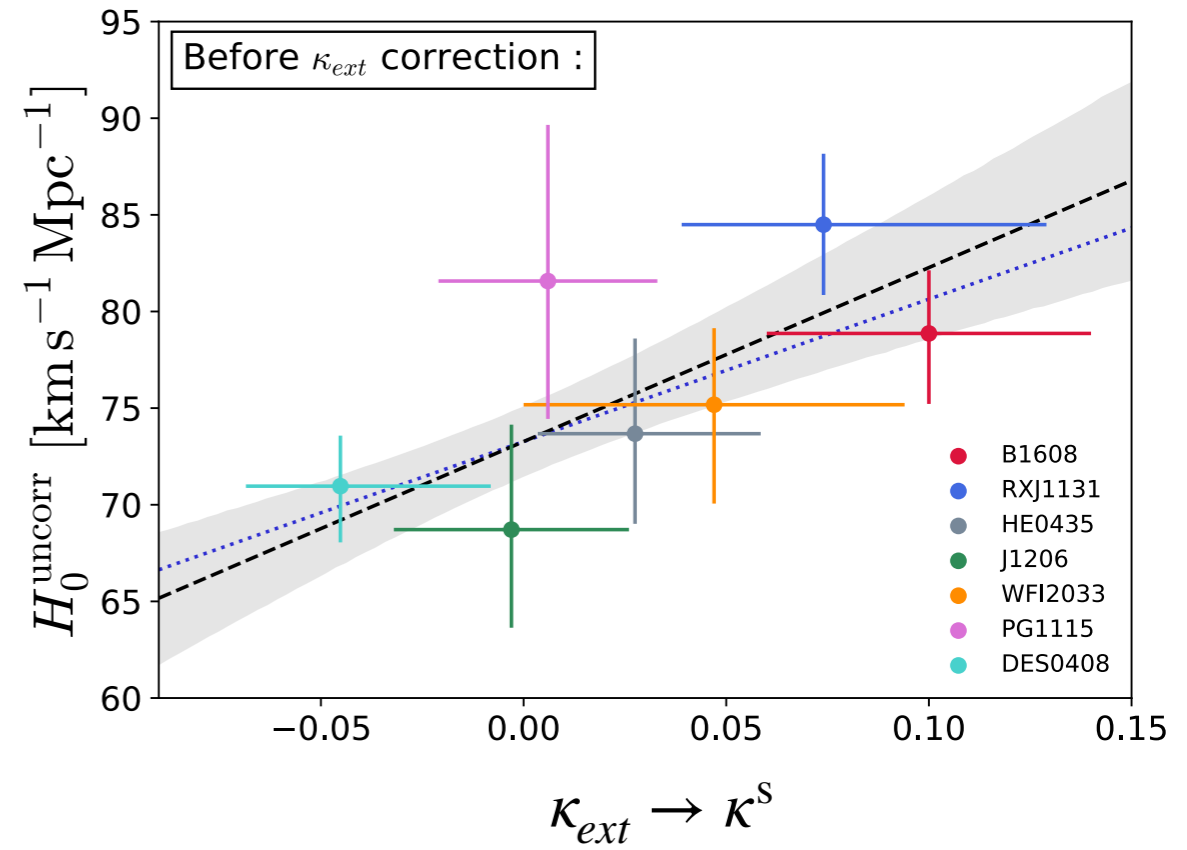


Rusu et al, 1607.01047 (H0LiCOW III)

Weak lensing degeneracy:

$$H_0^{\text{uncorr}} = \frac{1 - \kappa^{\text{ls}}}{1 - \kappa^{\text{l}}} \frac{1}{1 - \kappa^{\text{s}}} H_0$$

Millon et al, 1912.08027 (TDCOSMO I)



Rusu et al, 1607.01047 (H0LiCOW III)

Weak lensing degeneracy:

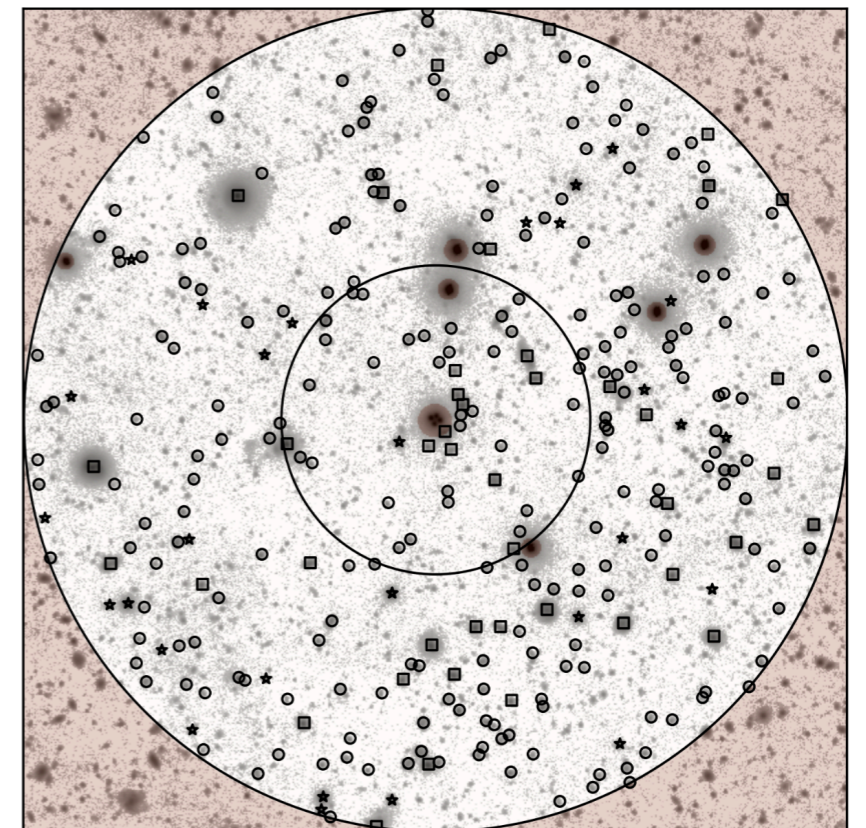
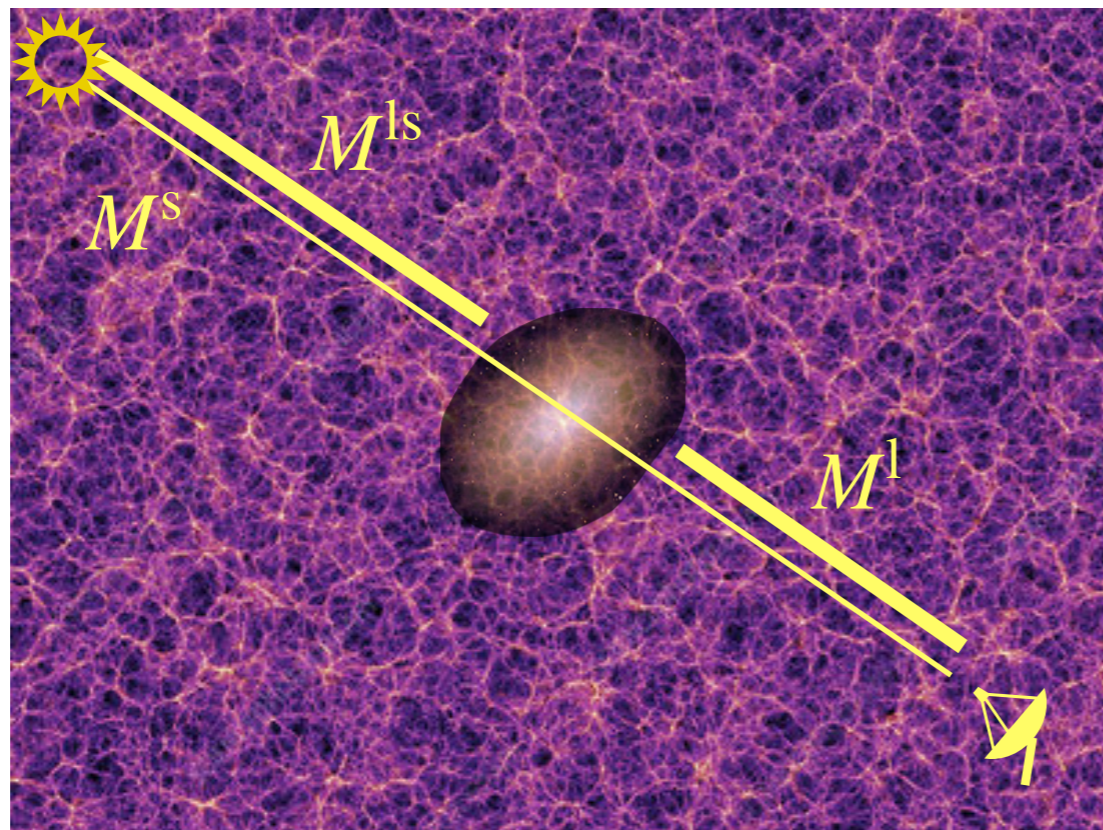
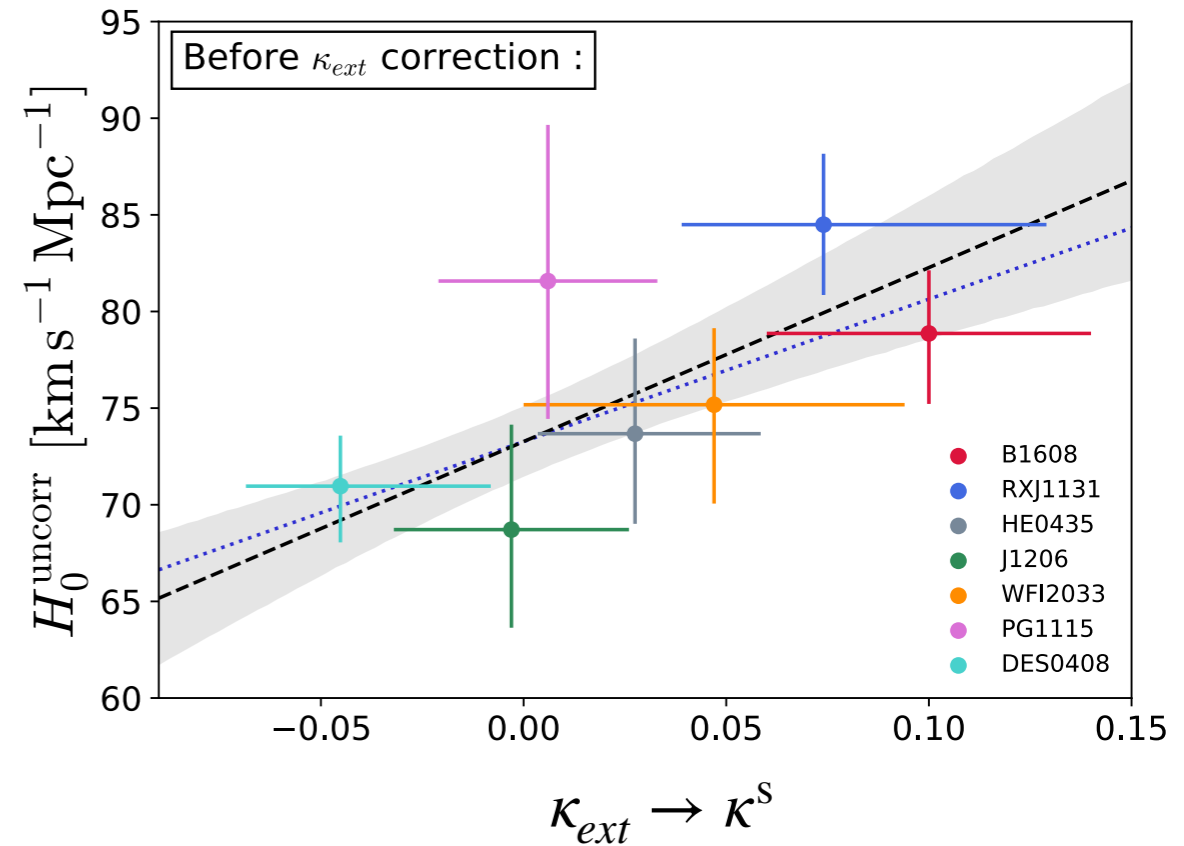
$$H_0^{\text{uncorr}} = \frac{1 - \kappa^{\text{ls}}}{1 - \kappa^{\text{l}}} \frac{1}{1 - \kappa^{\text{s}}} H_0$$

Weak lensing correction in H0LiCOW / TDCOSMO is probably *a little bit off*.

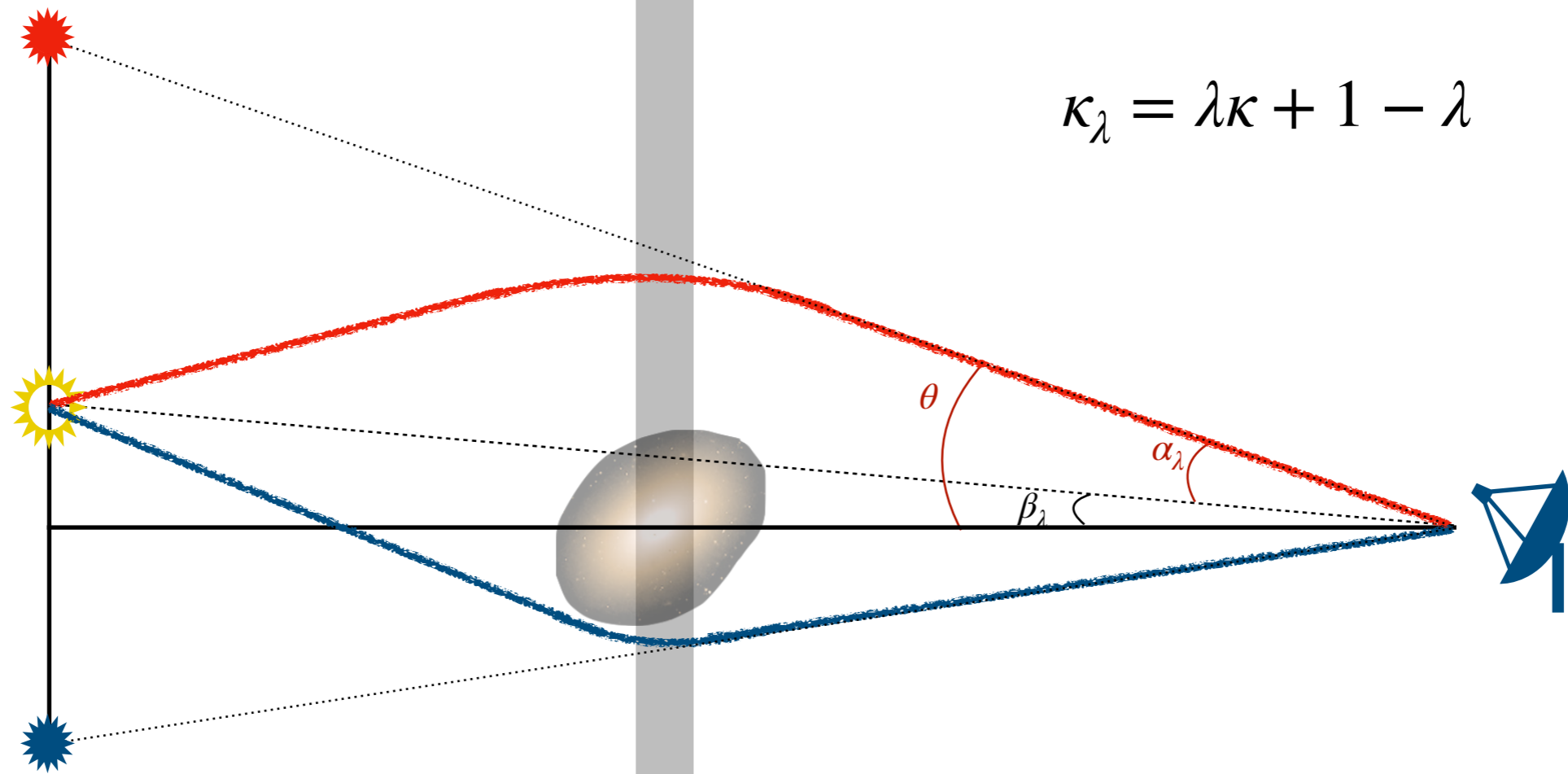
Birrer et al, 2007.02941 (TDCOSMO IV)

Teodori, et al, 2201.05111

Millon et al, 1912.08027 (TDCOSMO I)

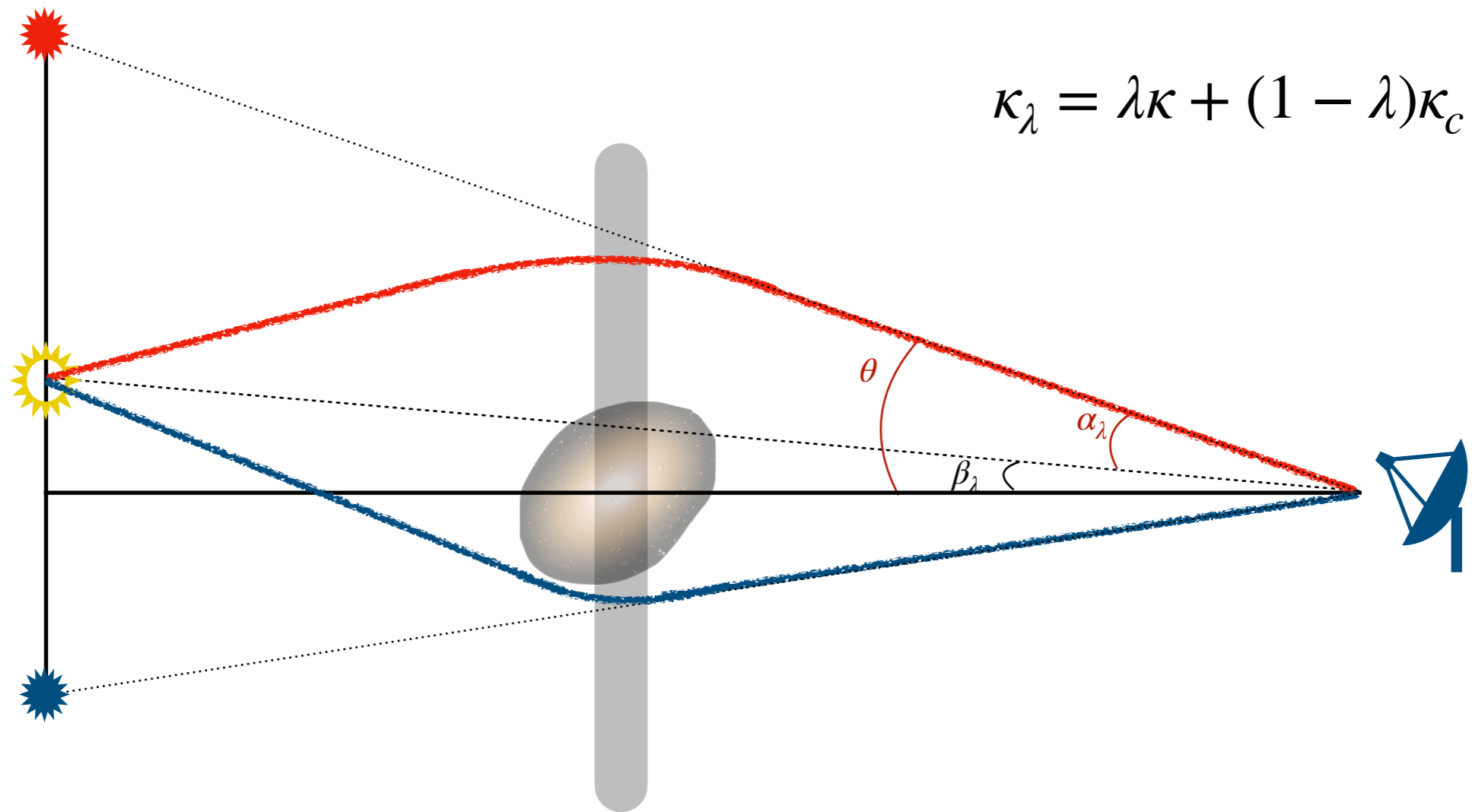


Rusu et al, 1607.01047 (H0LiCOW III)



Internal vs. External Convergence

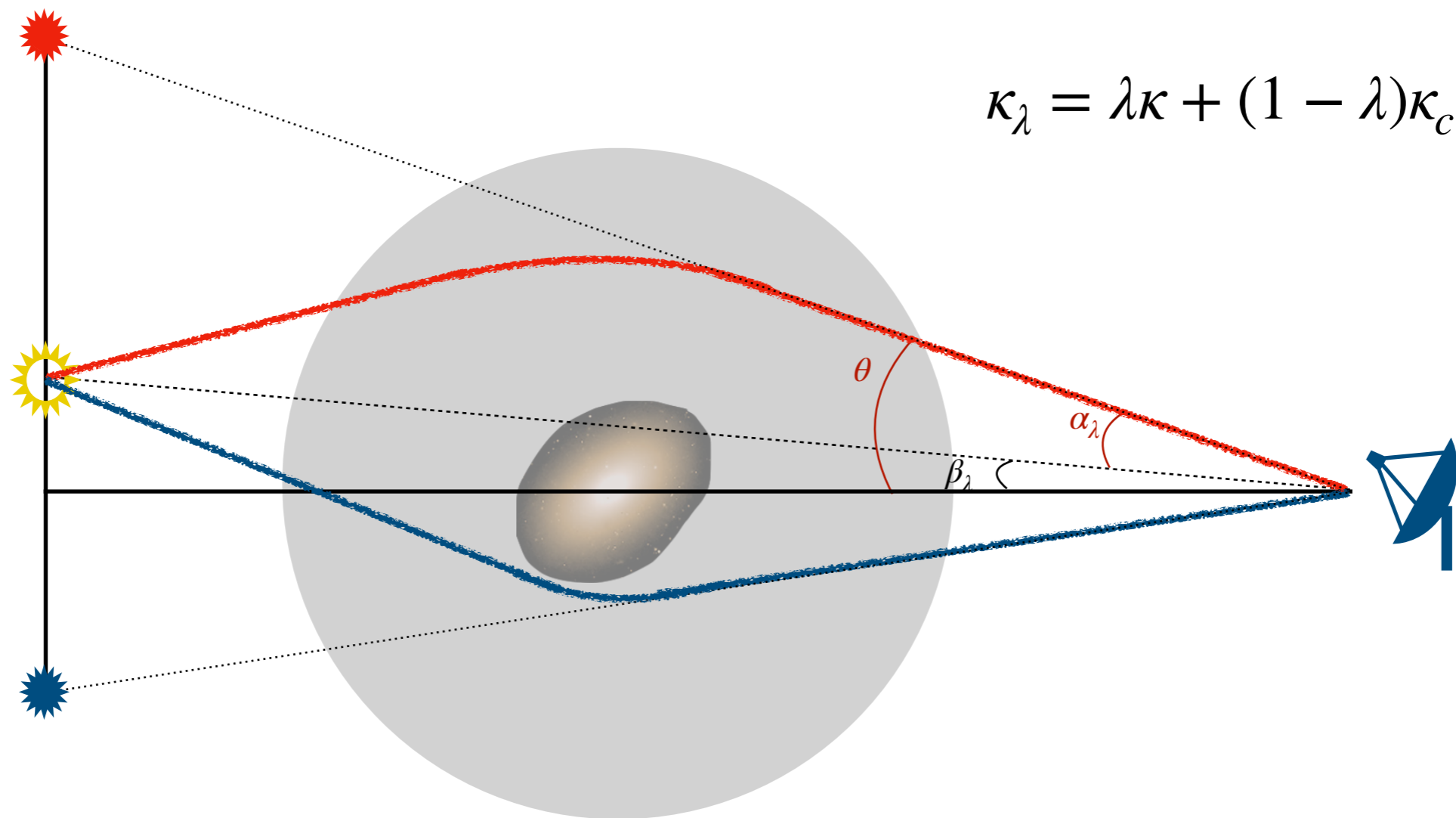
Internal vs. External Mass Sheet Degeneracy



$$\kappa_\lambda = \lambda\kappa + (1 - \lambda)\kappa_c$$

Internal vs. External Convergence

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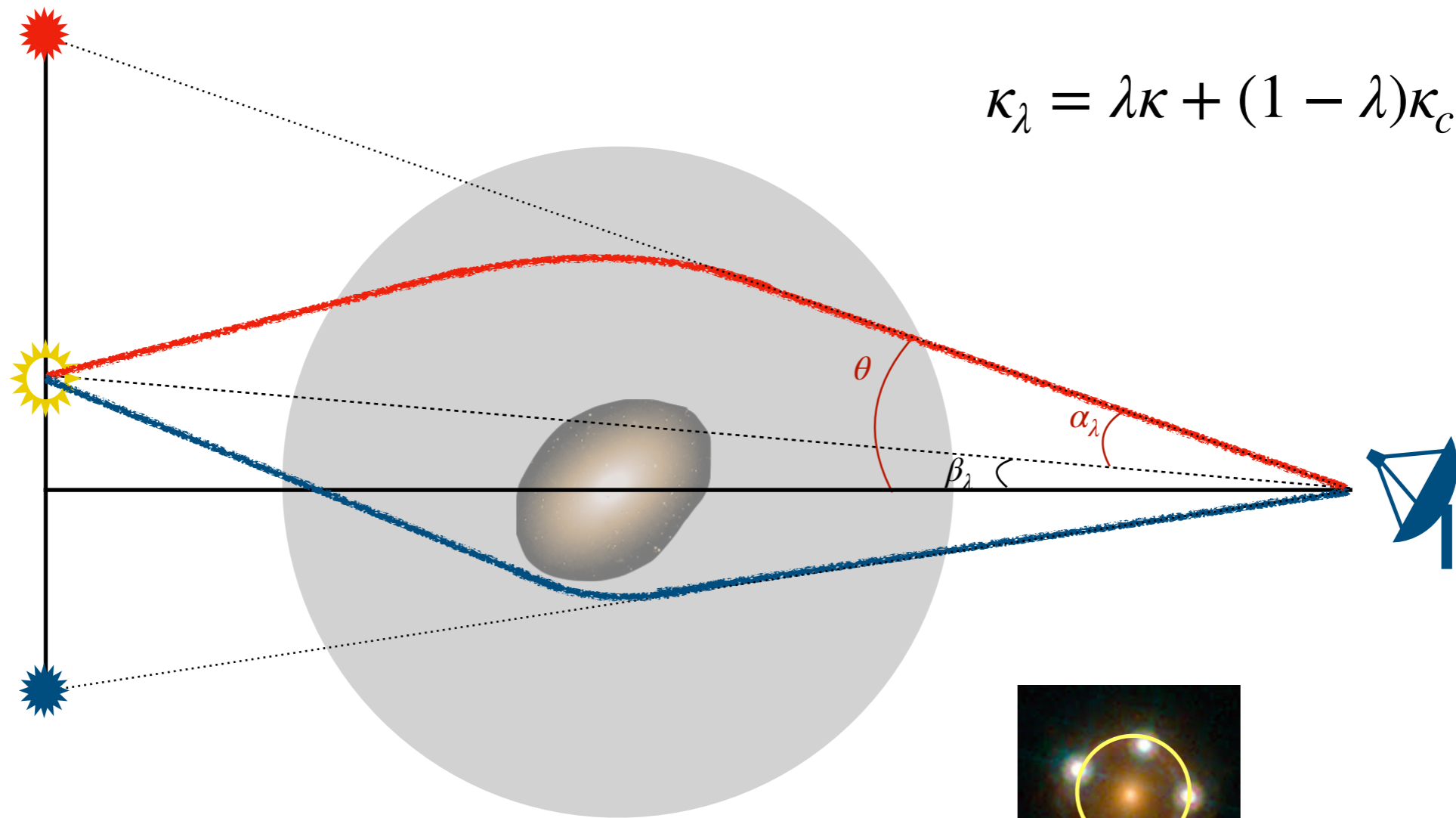


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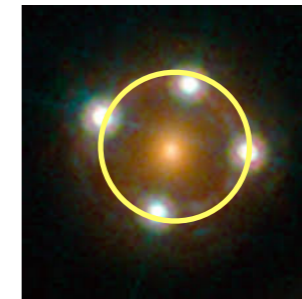
Internal vs. External Mass Sheet Degeneracy

KB, Castorina, Simonović, 2001.07182

A core component in lens halos could solve lensing H0 tension.



$$\kappa_\lambda = \lambda\kappa + (1 - \lambda)\kappa_c$$



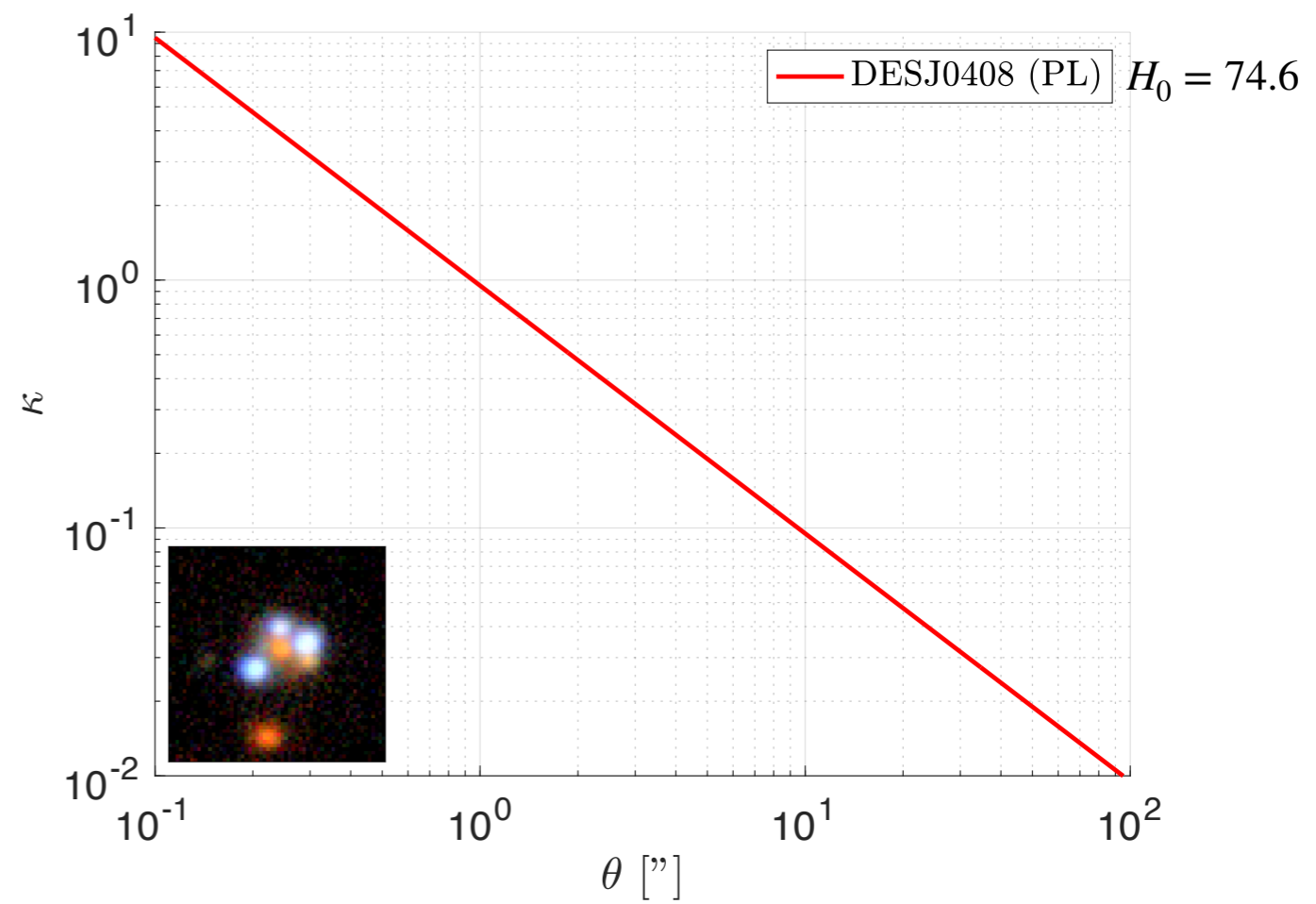
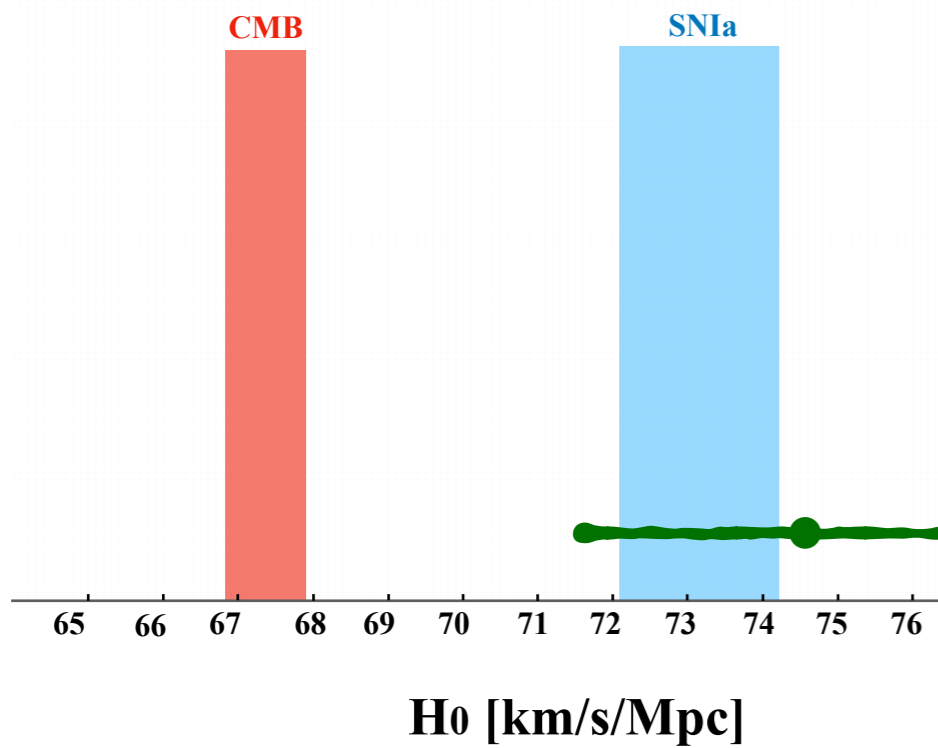
Example:
$$\rho_c(r) = \frac{1}{\sqrt{\pi}\Gamma\left(\frac{3}{2}\right)} \frac{\Sigma_c R_c^3}{(R_c^2 + r^2)^2}$$



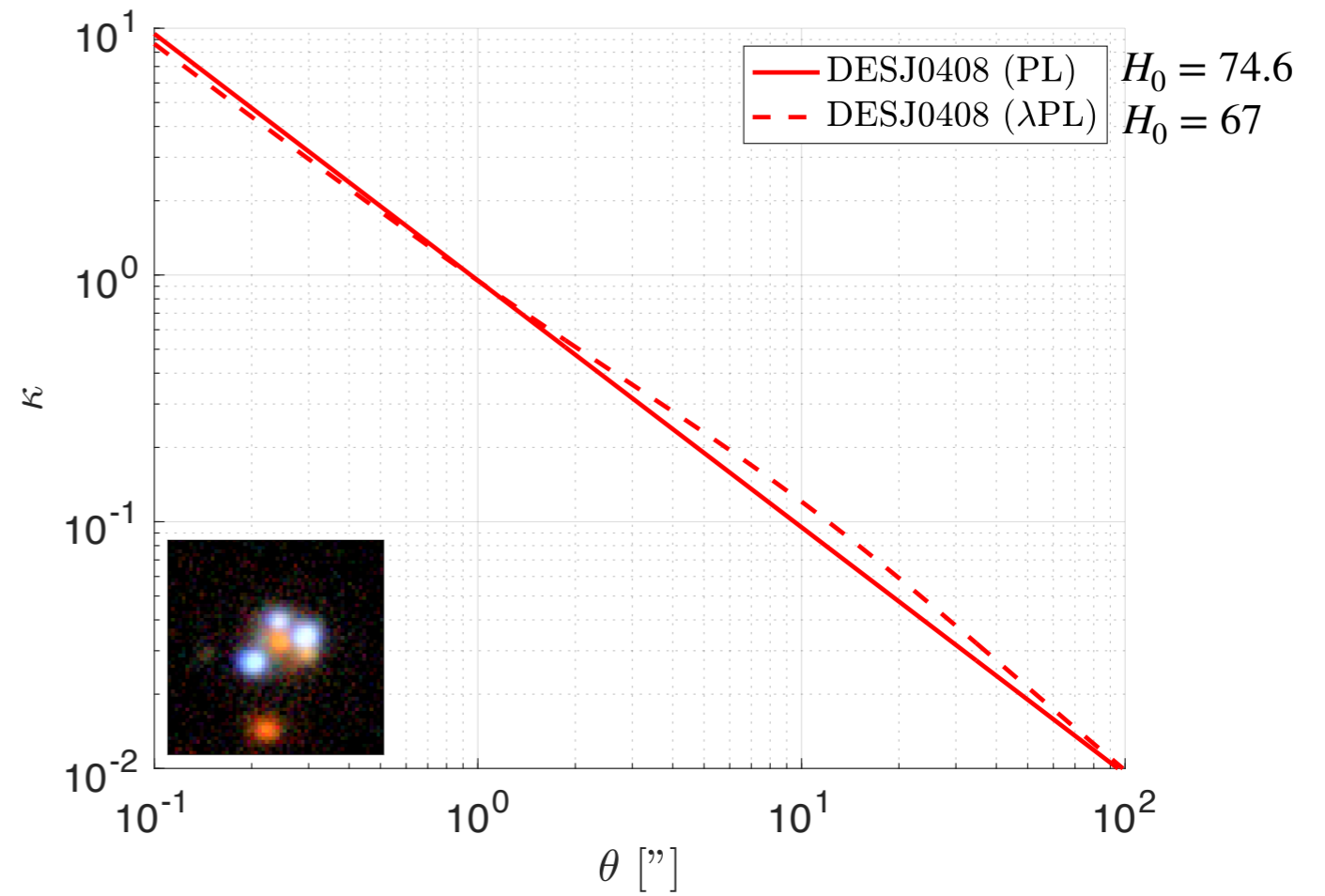
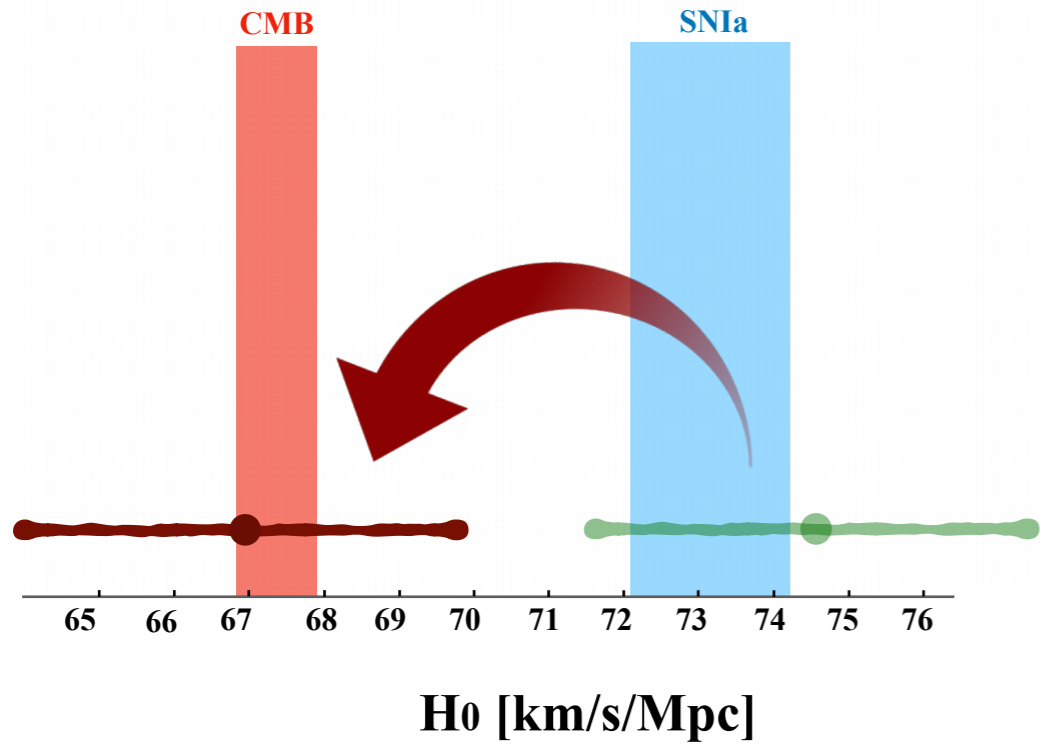
$$\theta_{E\lambda} = \theta_E(1 + \delta)$$

$$\delta = -\frac{3}{4(\gamma-1)} \frac{1-\lambda}{\lambda} \frac{\theta_E^2}{\theta_c^2} + \mathcal{O}\left(\frac{\theta_E^4}{\theta_c^4}\right)$$

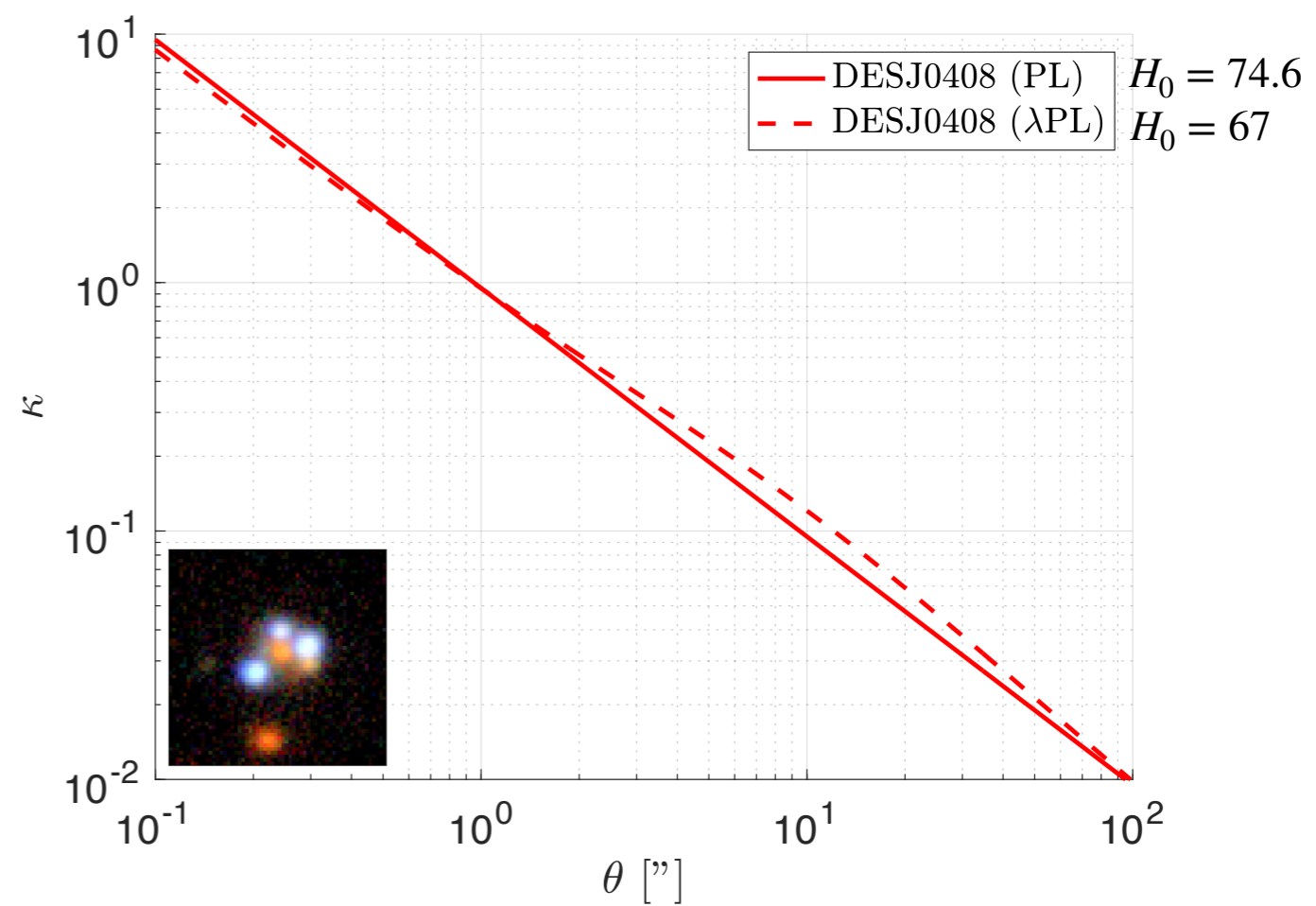
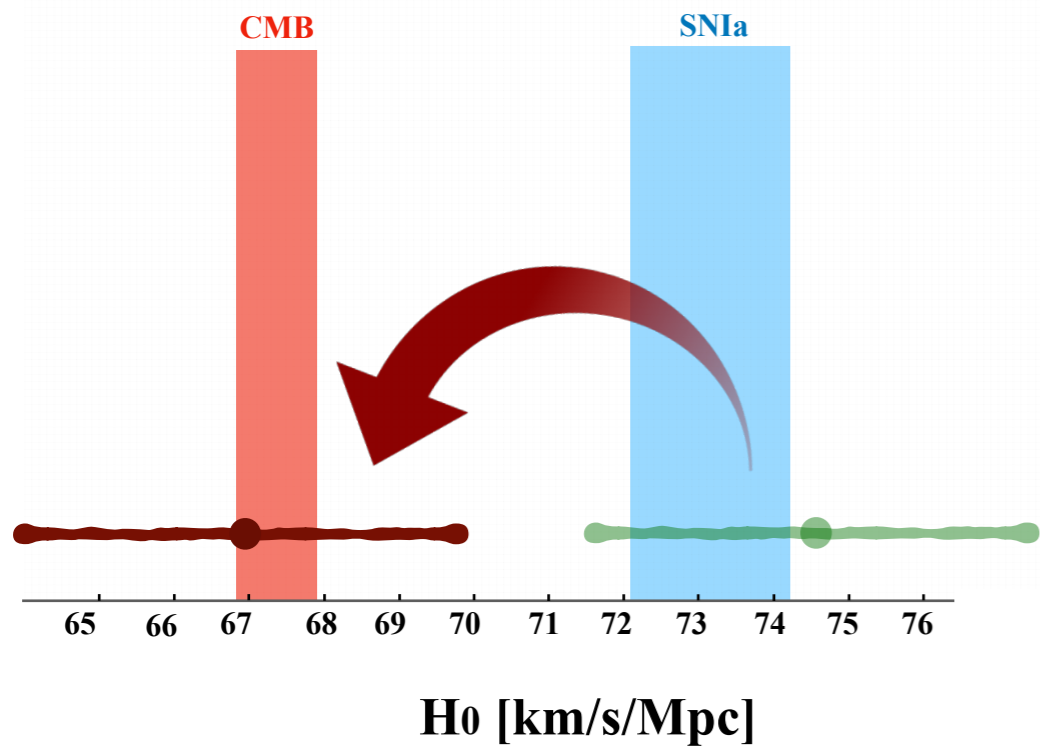
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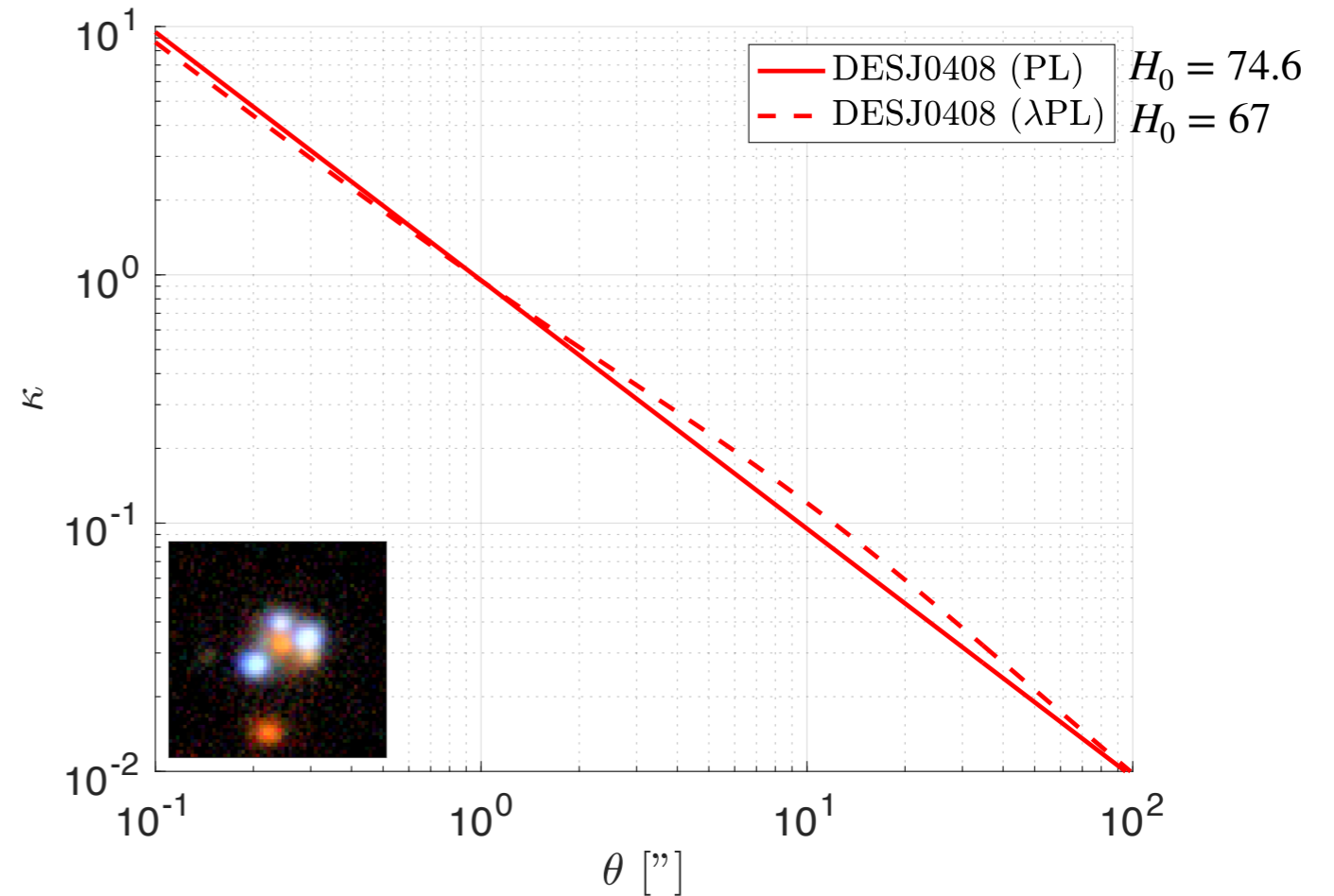
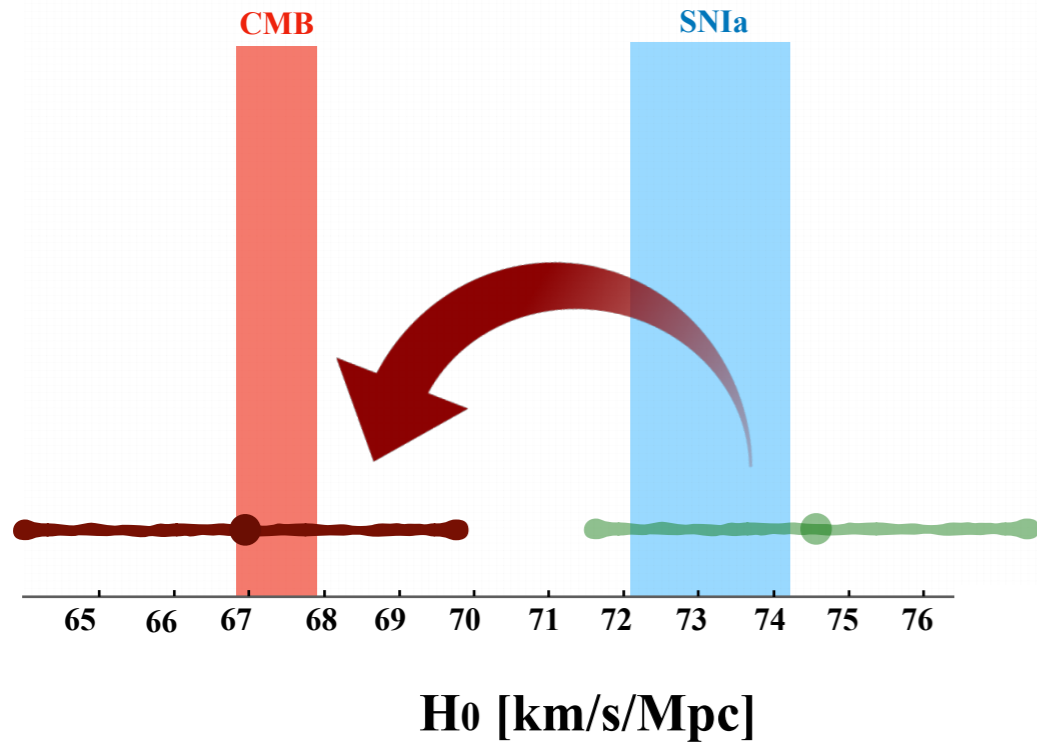


3. Opportunities: galactic structure



3. Opportunities: galactic structure

What do we learn about galaxies if we add CMB/LSS prior?



	H_0	$\lambda = 67/H_0$	γ	θ_E ["]	θ_s ["]	lens redshift z_l	ref
RXJ1131	$76.1^{+3.6}_{-4.3}$	$0.88^{+0.06}_{-0.04}$	1.98	1.6	19	0.295	Chen et al. (2016)
PG1115	$83.0^{+7.8}_{-7.0}$	$0.81^{+0.07}_{-0.07}$	2.18	1.1	17	0.311	Chen et al. (2019)
HE0435	$71.7^{+5.1}_{-4.6}$	$0.93^{+0.07}_{-0.06}$	1.87	1.2	10	0.4546	Chen et al. (2019)
DESJ0408	$74.6^{+2.5}_{-2.9}$	$0.9^{+0.03}_{-0.03}$	2	1.9	13	0.6	Shajib et al. (2019)
WFI2033	$72.6^{+3.3}_{-3.5}$	$0.92^{+0.05}_{-0.04}$	1.95	0.9	11	0.6575	Rusu et al. (2019)
J1206	$67.0^{+5.7}_{-4.8}$	$1^{+0.08}_{-0.08}$	1.95	1.2	4.7	0.745	Birrer et al. (2019)

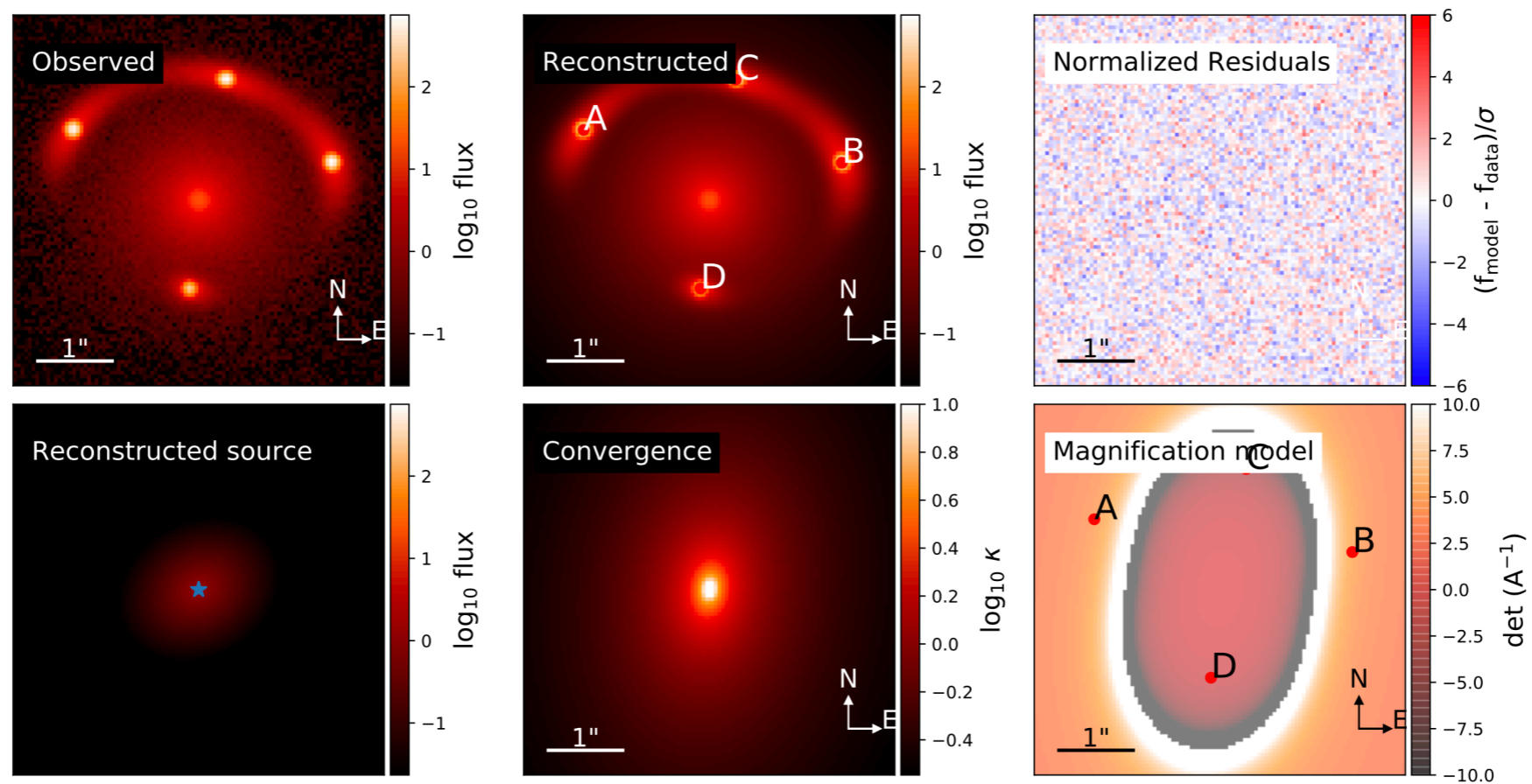
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Expect evidence for core component, reflecting precision on H_0

KB, Castorina, Simonović, 2001.07182

KB, Teodori, 2105.10873



3. Opportunities: galactic structure

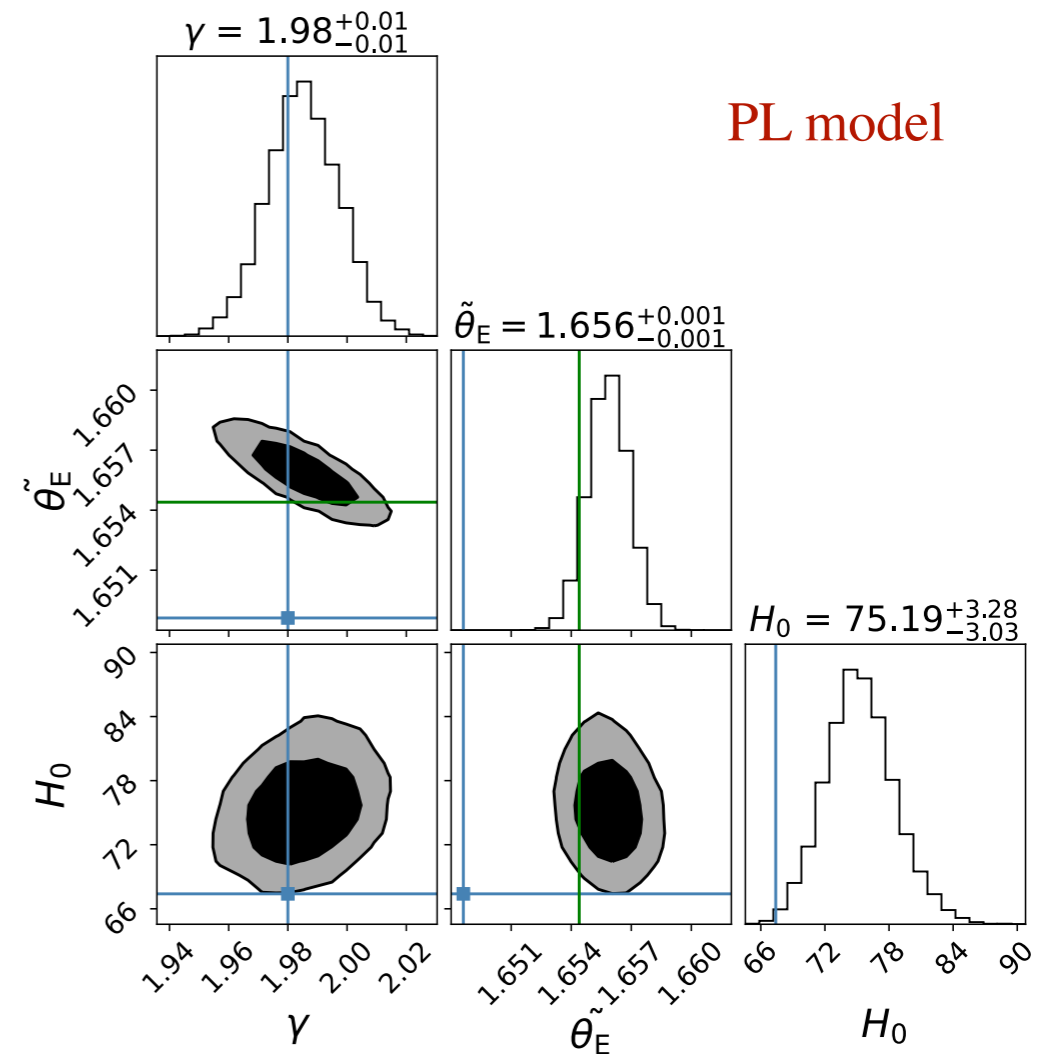
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Mock inference using
power-law model.

Truth has $H_0=67.4$ km/s/Mpc,
and a 10% core!

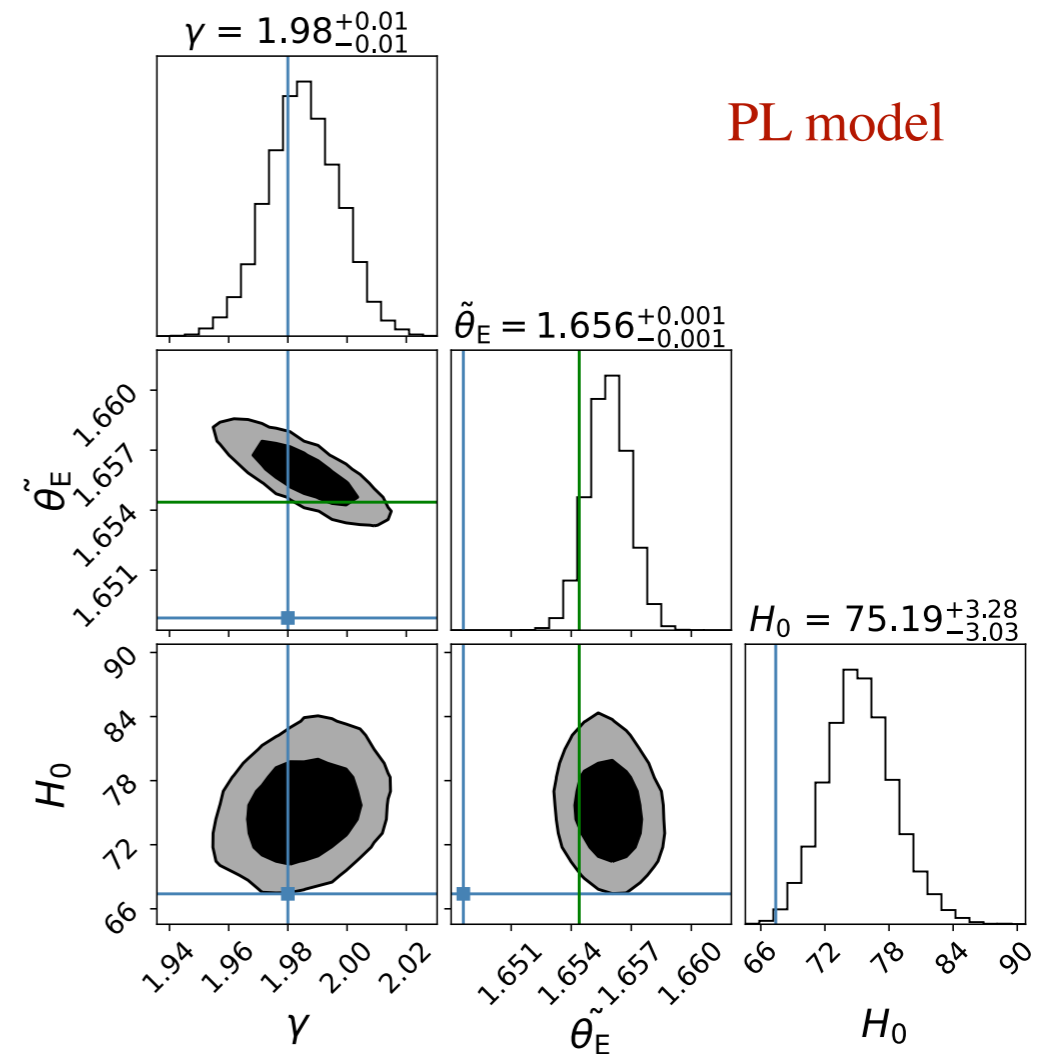
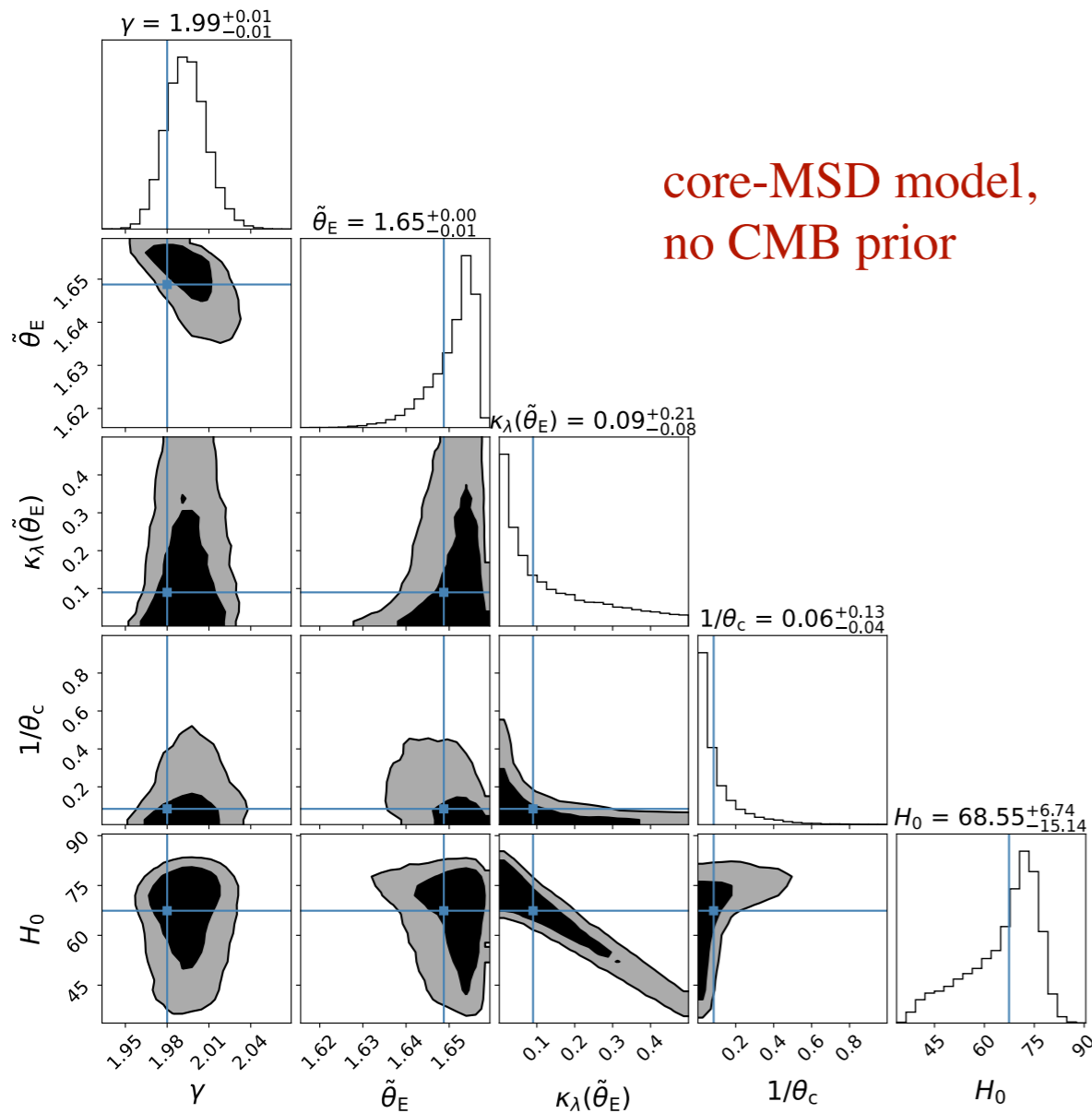


3. Opportunities: galactic structure

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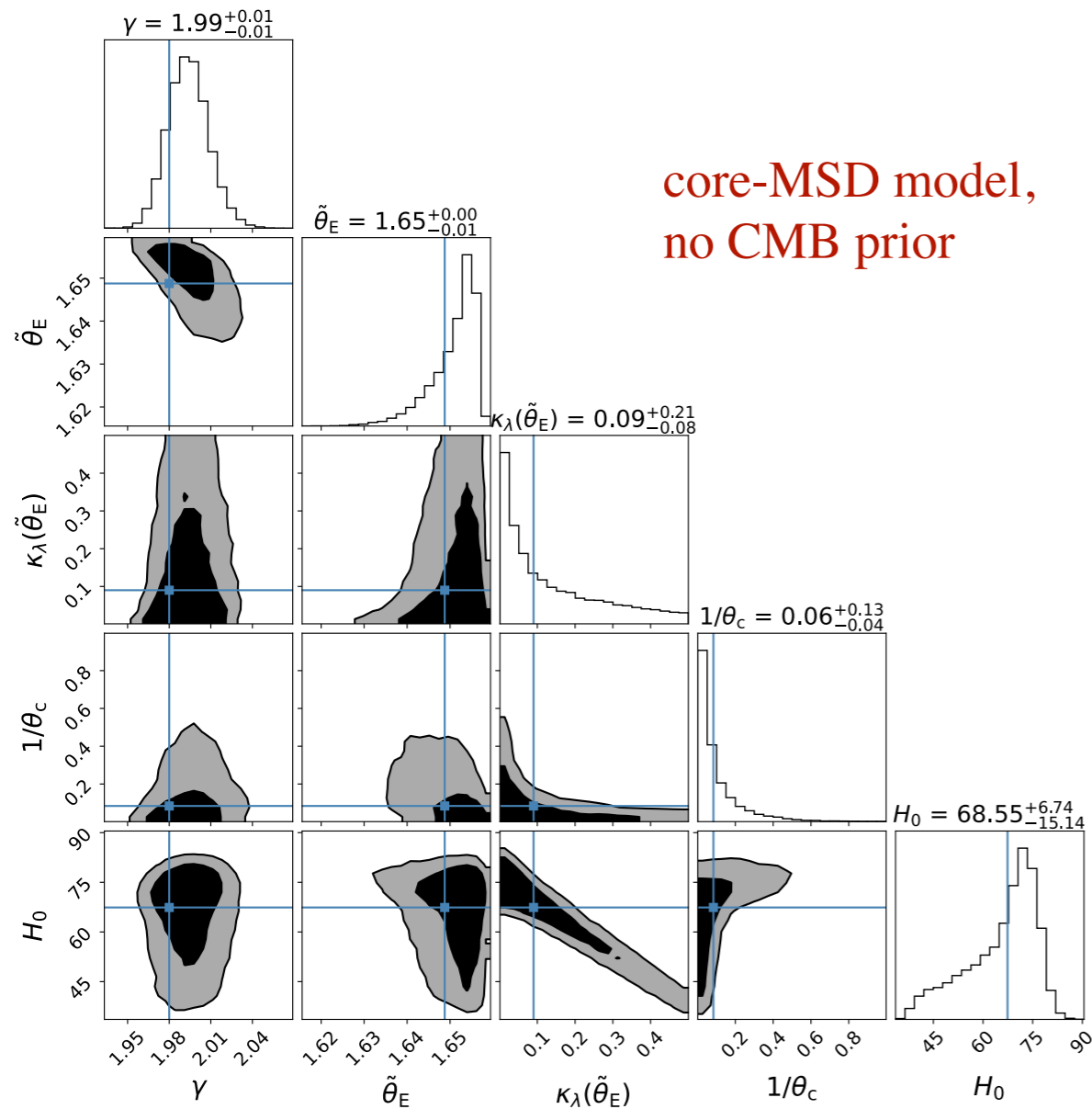


3. Opportunities: galactic structure

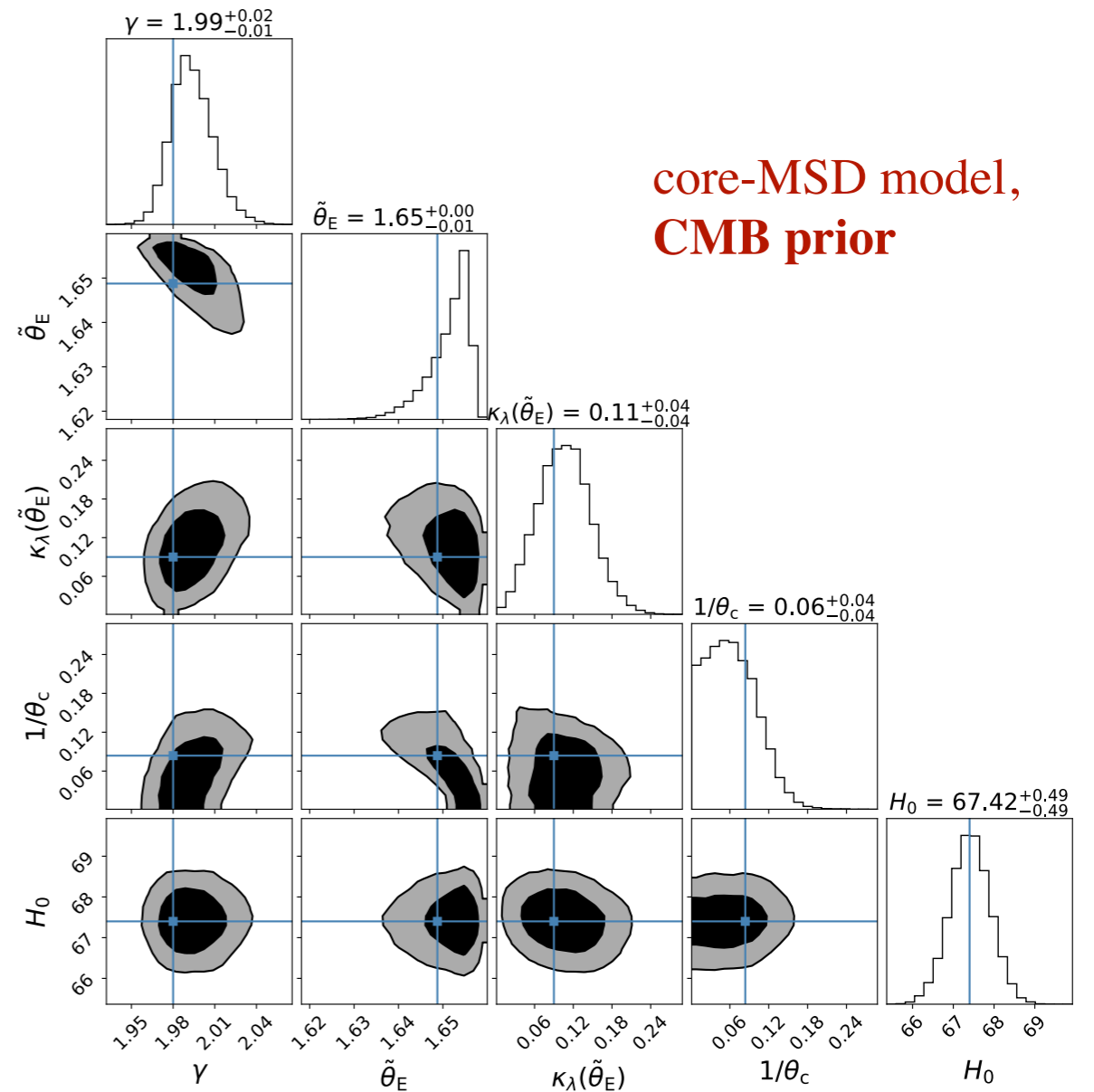
What do we learn about galaxies if we add CMB/LSS prior?

Expect evidence for core component, reflecting precision on H_0

KB, Teodori, 2105.10873



core-MSD model,
no CMB prior



core-MSD model,
CMB prior

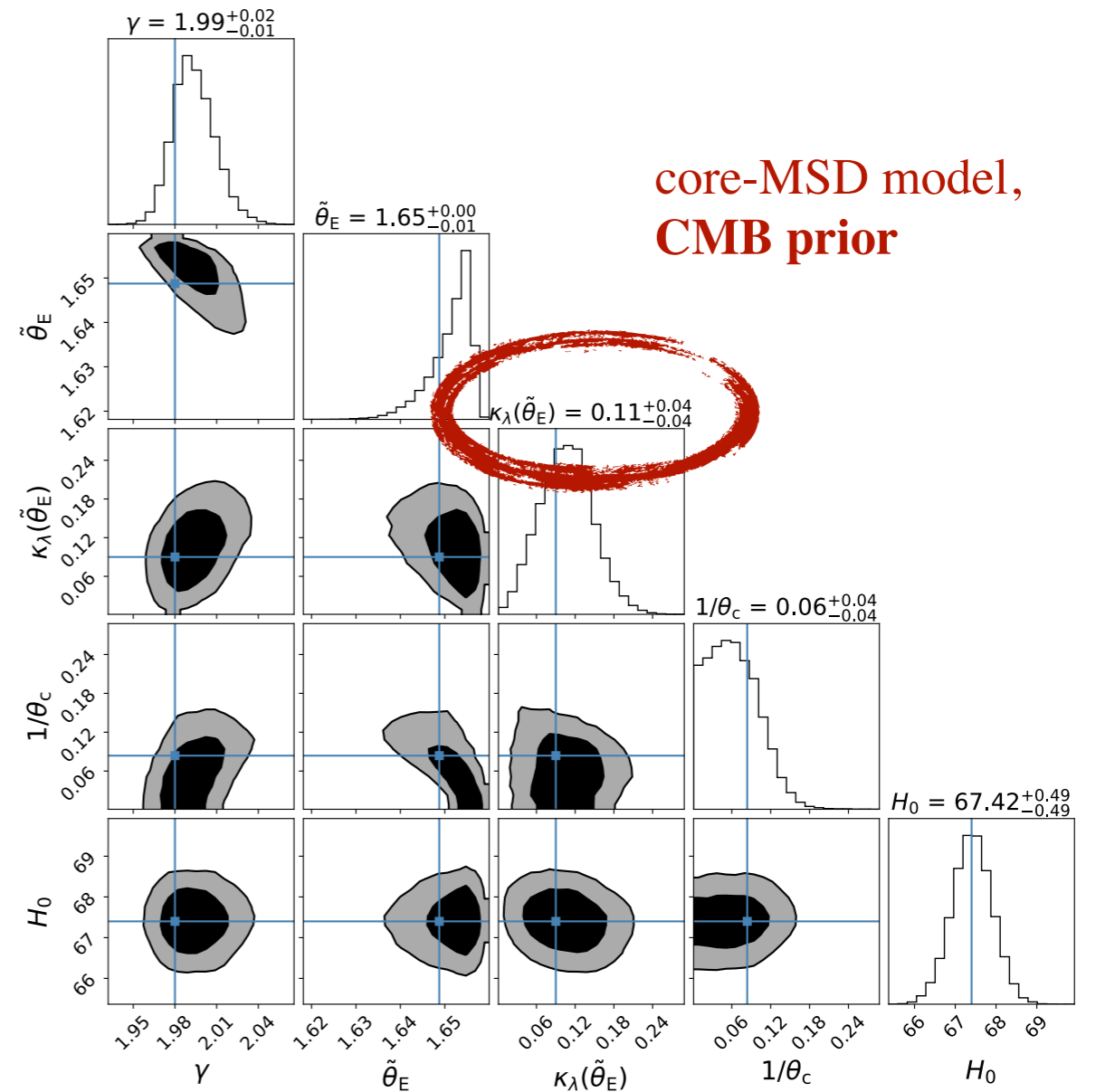
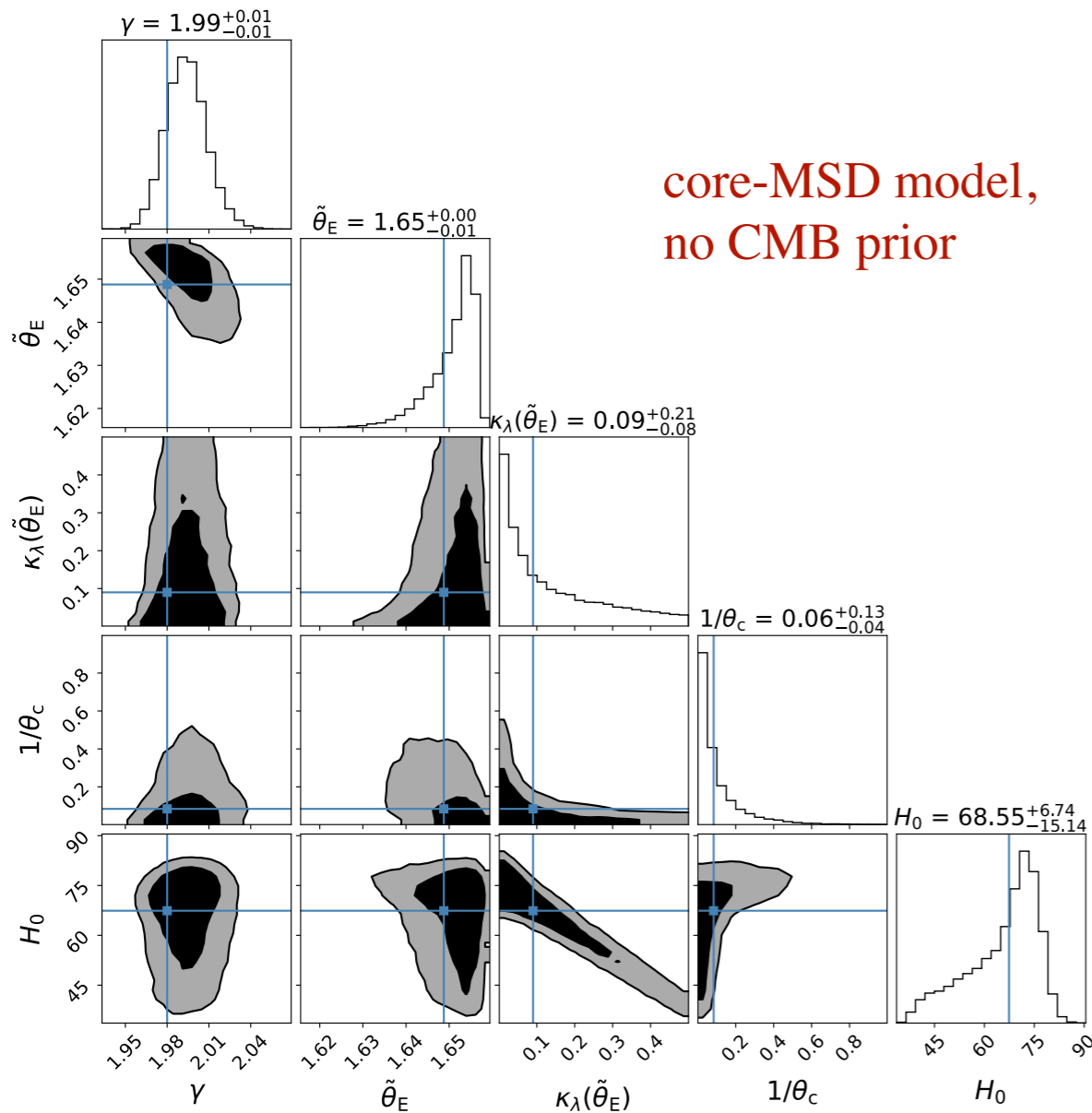
3. Opportunities: galactic structure

What do we learn about galaxies if we add CMB/LSS prior?

Expect evidence for core component, reflecting precision on H_0

KB, Teodori, 2105.10873

Could reach $>2\sigma$ per system



A step towards covering internal MSD (no CMB prior) :

Birrer et al 2020 (TDCOSMO IV)

Data sets	H_0 [km s ⁻¹ Mpc ⁻¹]	$\lambda_{\text{int},0}$	α_λ	$\sigma(\lambda_{\text{int}})$	a_{ani}	$\sigma(a_{\text{ani}})$	$\sigma_{\sigma^{\text{P}},\text{sys}}$
TDCOSMO-only	$74.5^{+5.6}_{-6.1}$	$1.02^{+0.08}_{-0.09}$	$0.00^{+0.07}_{-0.07}$	$0.01^{+0.03}_{-0.01}$	$2.32^{+1.62}_{-1.17}$	$0.16^{+0.50}_{-0.14}$	-
TDCOSMO + SLACS _{IFU}	$73.3^{+5.8}_{-5.8}$	$1.00^{+0.08}_{-0.08}$	$-0.07^{+0.06}_{-0.06}$	$0.07^{+0.09}_{-0.05}$	$1.58^{+1.58}_{-0.54}$	$0.15^{+0.47}_{-0.13}$	-
TDCOSMO + SLACS _{SDSS}	$67.4^{+4.3}_{-4.7}$	$0.91^{+0.05}_{-0.06}$	$-0.04^{+0.04}_{-0.04}$	$0.02^{+0.04}_{-0.01}$	$1.52^{+1.76}_{-0.70}$	$0.28^{+0.45}_{-0.25}$	$0.06^{+0.02}_{-0.02}$
TDCOSMO + SLACS _{SDSS+IFU}	$67.4^{+4.1}_{-3.2}$	$0.91^{+0.04}_{-0.04}$	$-0.07^{+0.03}_{-0.04}$	$0.06^{+0.08}_{-0.04}$	$1.20^{+0.70}_{-0.27}$	$0.18^{+0.50}_{-0.15}$	$0.06^{+0.02}_{-0.02}$

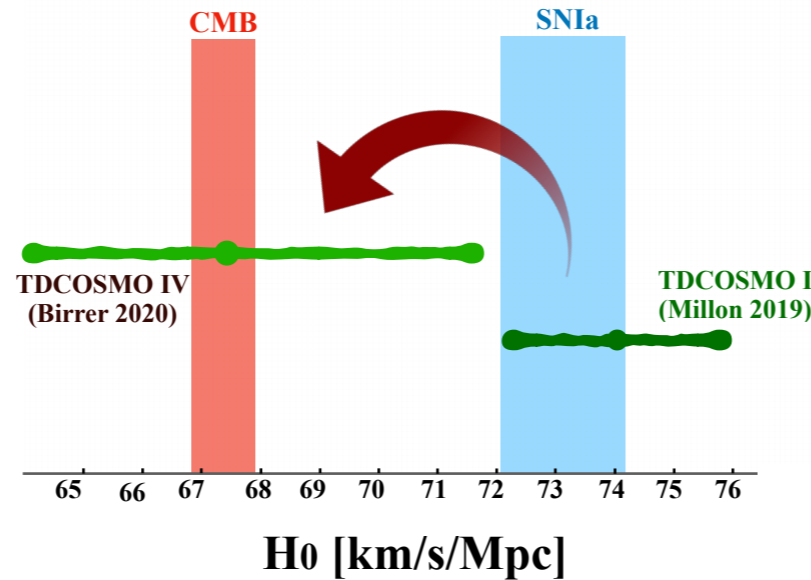
KB, Castorina, Simonovic 2020

	H_0	$\lambda = 67/H_0$	γ	θ_E ["]	θ_s ["]	lens redshift z_l	ref
RXJ1131	$76.1^{+3.6}_{-4.3}$	$0.88^{+0.06}_{-0.04}$	1.98	1.6	19	0.295	Chen et al. (2016)
PG1115	$83.0^{+7.8}_{-7.0}$	$0.81^{+0.07}_{-0.07}$	2.18	1.1	17	0.311	Chen et al. (2019)
HE0435	$71.7^{+5.1}_{-4.6}$	$0.93^{+0.07}_{-0.06}$	1.87	1.2	10	0.4546	Chen et al. (2019)
DESJ0408	$74.6^{+2.5}_{-2.9}$	$0.9^{+0.03}_{-0.03}$	2	1.9	13	0.6	Shajib et al. (2019)
WFI2033	$72.6^{+3.3}_{-3.5}$	$0.92^{+0.05}_{-0.04}$	1.95	0.9	11	0.6575	Rusu et al. (2019)
J1206	$67.0^{+5.7}_{-4.8}$	$1^{+0.08}_{-0.08}$	1.95	1.2	4.7	0.745	Birrer et al. (2019)

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Hint from SLACS kinematics?

(but see comments in KB, Teodori, 2105.10873)

KB, Castorina, Simonovic 2020

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“Internal MSD” (and a CMB prior) may require a **non-minimal** density profile.

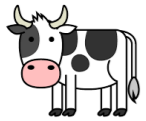


Why would galaxies be non-minimal?



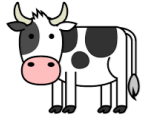
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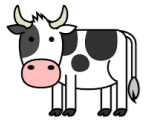
Why not? (What is dark matter?)



Why should galaxies have a core component?

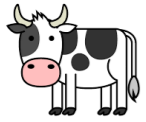
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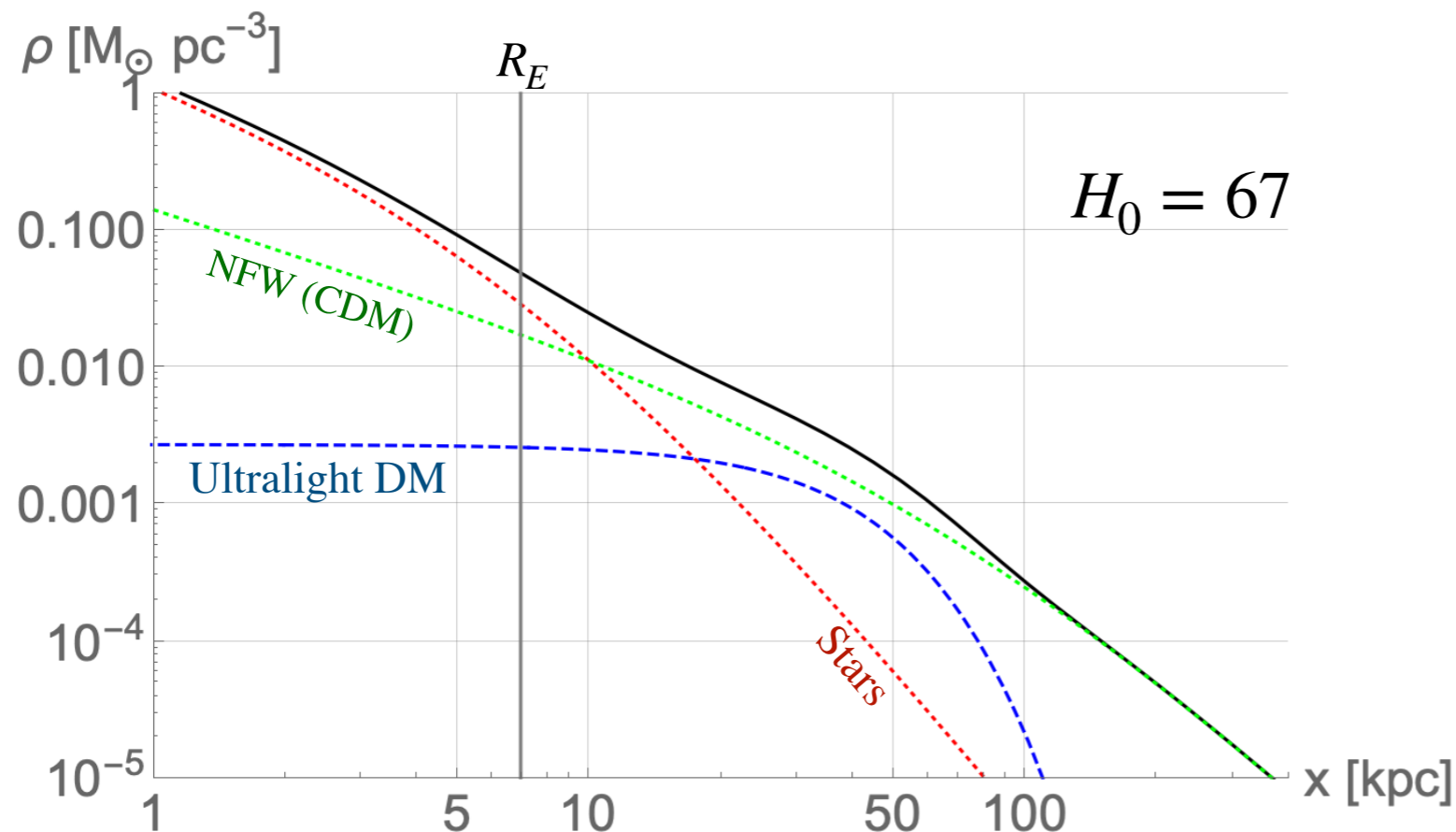
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Why should galaxies have a core component?

Can think of several reasons.

Dark matter not boring NFW? ...a little bit of ultralight dark matter?



KB, Teodori, 2105.10873

20 % of total DM, $m = 2.5 \times 10^{-25}$ eV

Dynamical relaxation consistent at O(1).
Cosmology OK.

Summary

Lensing H_0 sensitive to galaxy profile at few % level:

Unexpected feature in the galaxy profile,
or fundamental breakdown of Λ CDM?

Weak lensing: include all segments of line of sight.
Lacking in published results. Likely few % bias on H_0 .

Teodori, et al, 2201.05111

Adding a core to a density profile is an approximate MSD.
10% core solves the lensing H_0 tension?

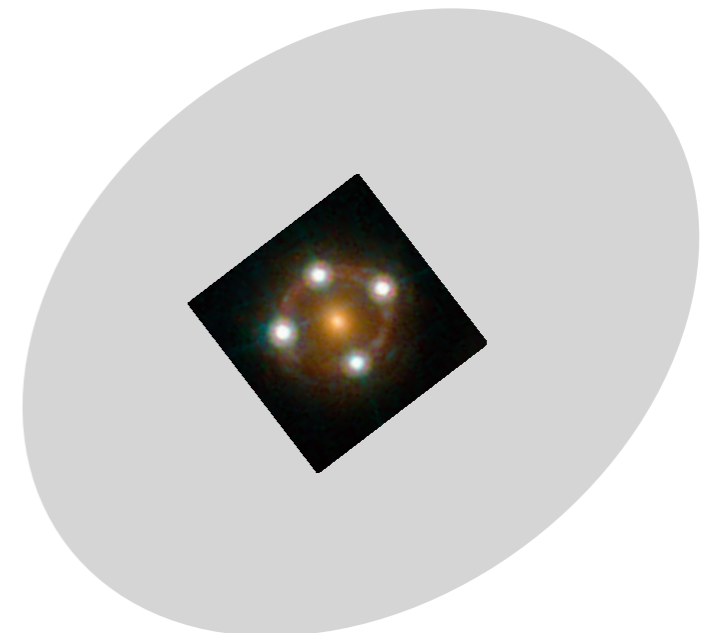
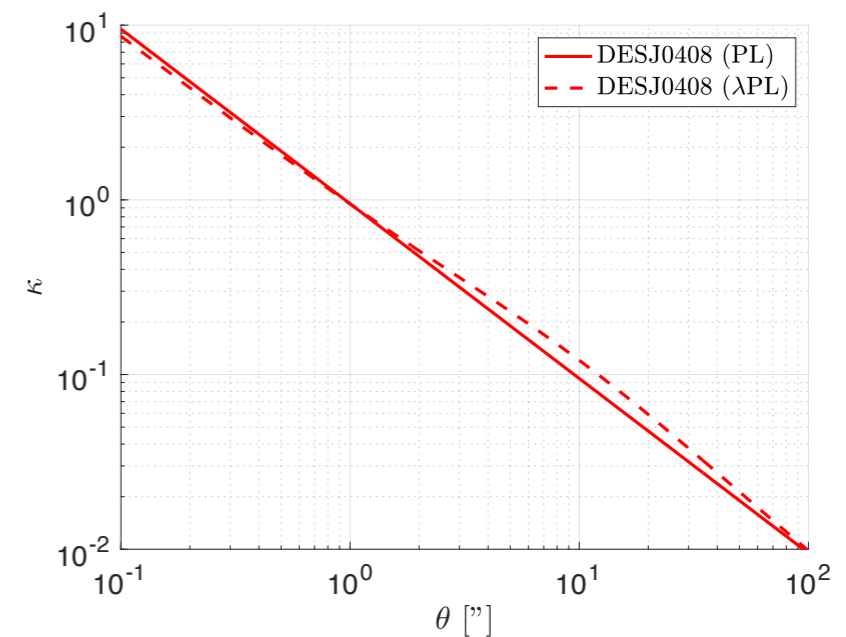
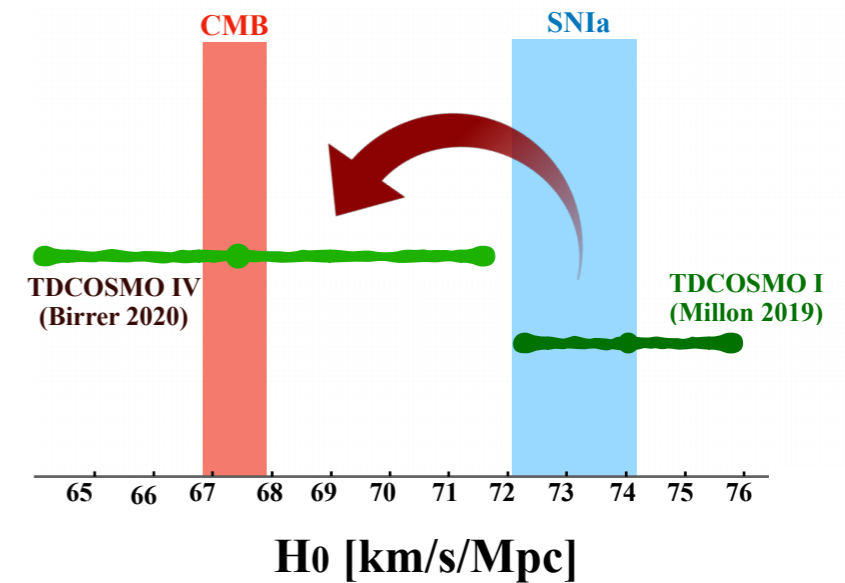
KB, Castorina, Simonović 2001.07182

Could point to interesting dark matter dynamics.
If we go there, may as well adopt CMB (or SN_{Ia} !) prior on H_0 .

Ultralight DM (axion-like):

Vanilla vacuum misalignment. Dynamically makes a core.
Correct ballpark to solve lensing H_0 tension, if $10^{-25} \text{ eV} \lesssim m \lesssim 10^{-24} \text{ eV}$
Dynamical relaxation consistent at O(1) level.

KB, Teodori 2105.10873



Thank you!

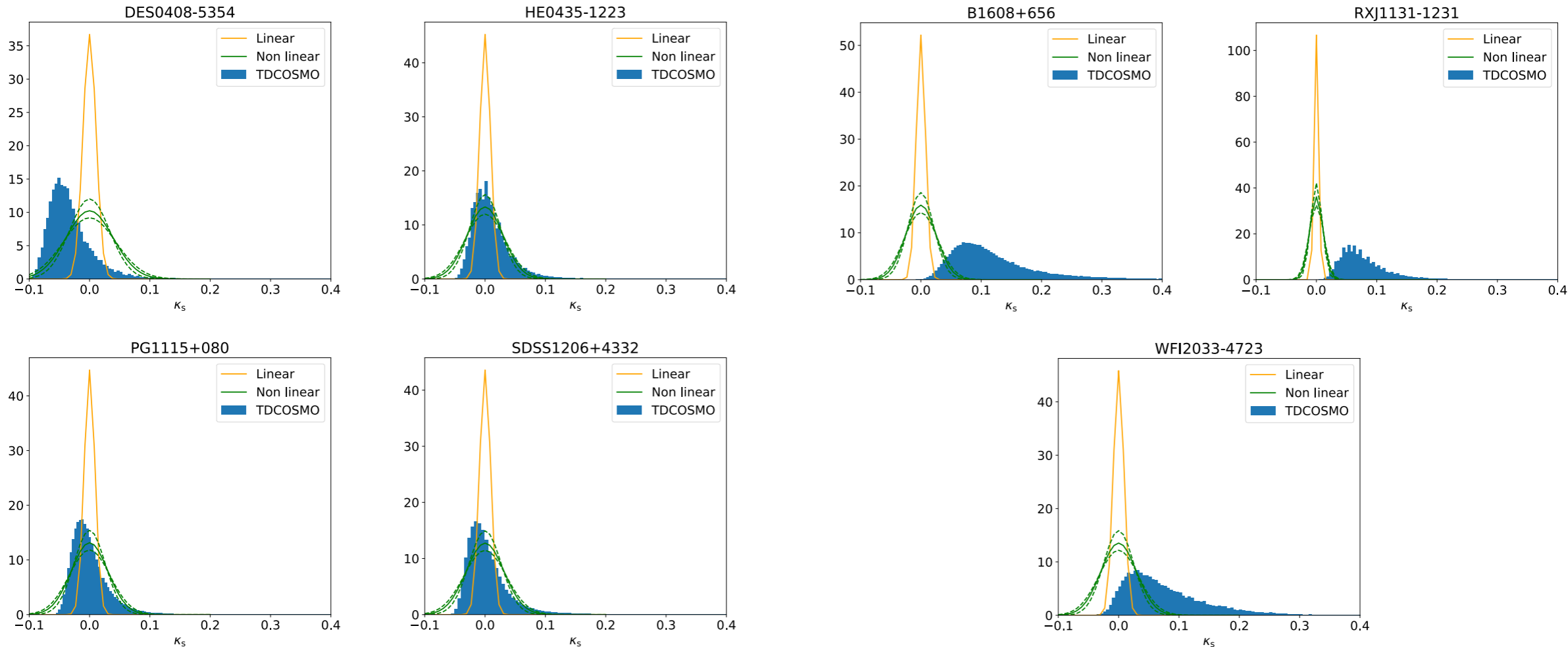
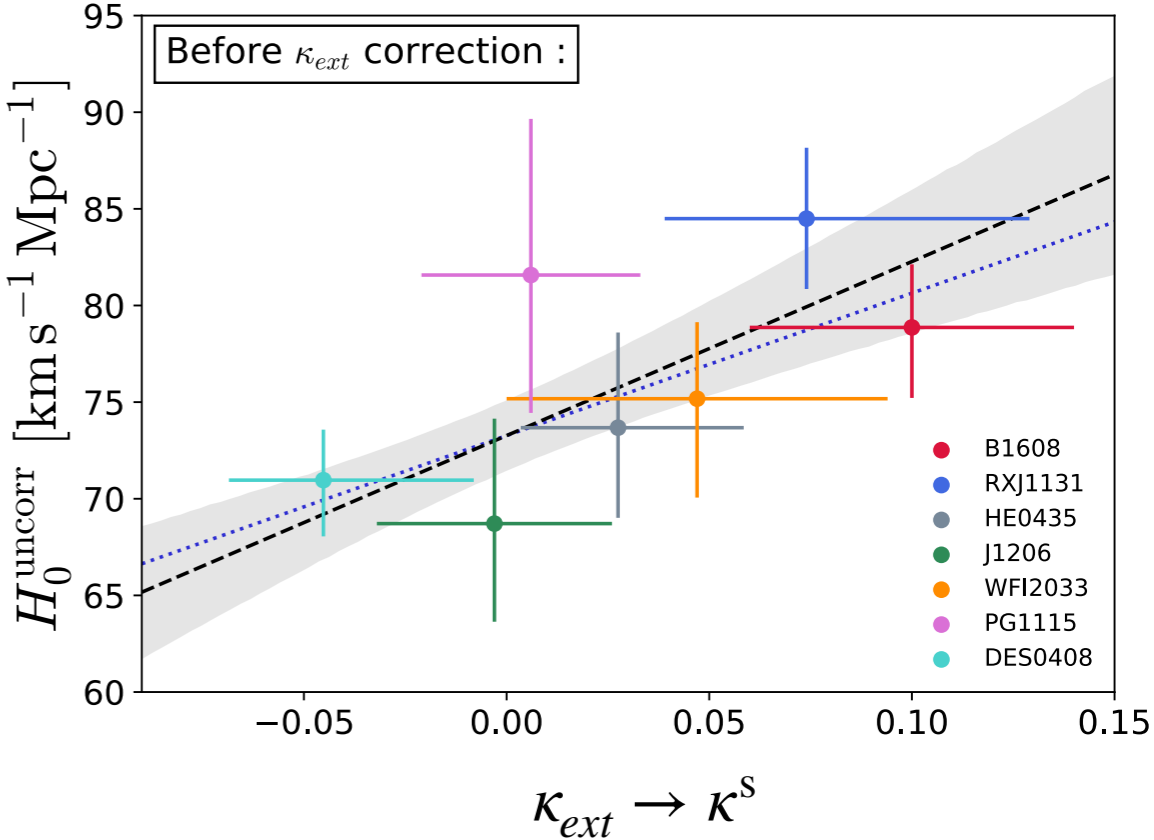
Xtra

Weak lensing degeneracy:

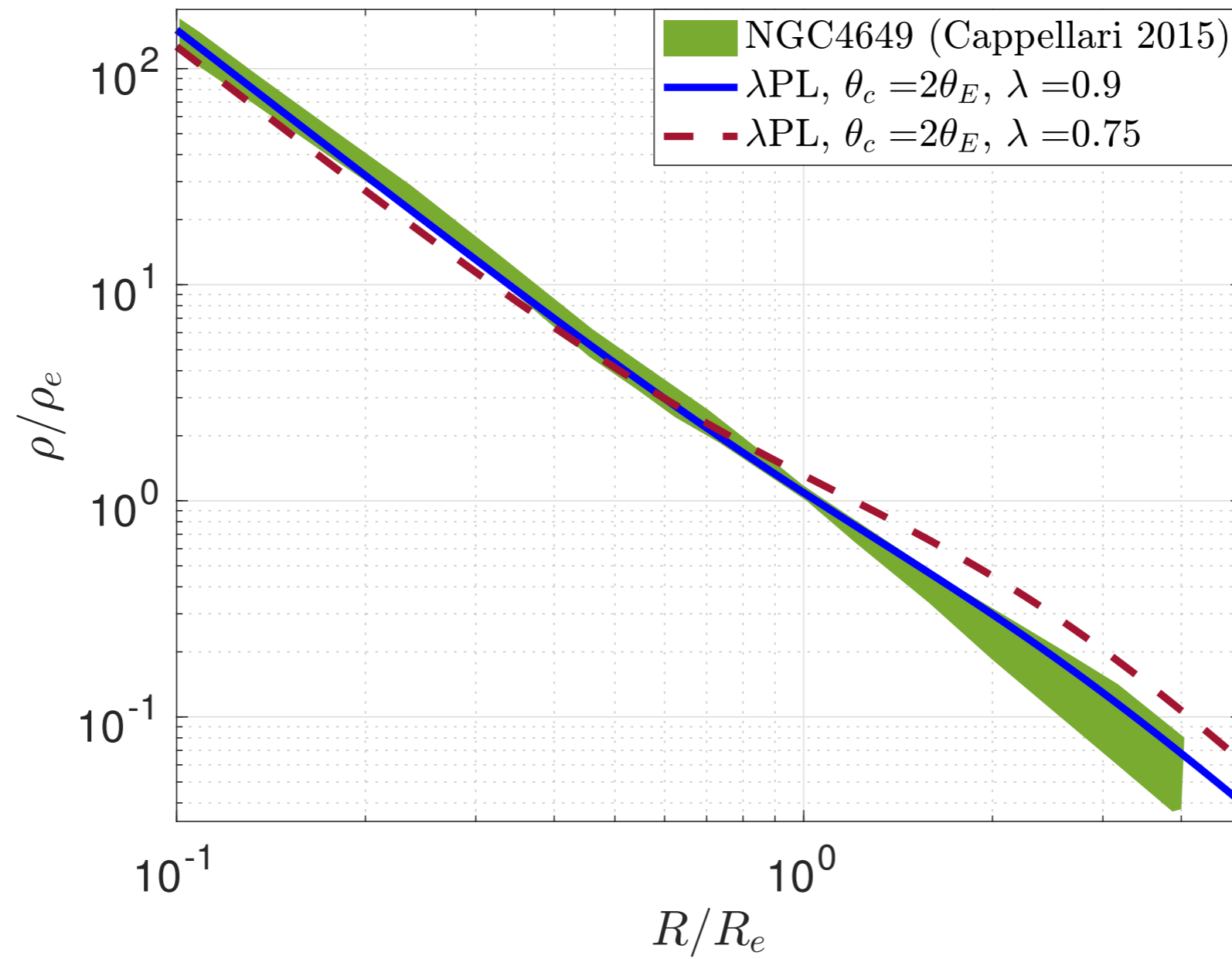
$$H_0^{\text{uncorr}} = \frac{1 - \kappa^{\text{ls}}}{1 - \kappa^{\text{l}}} \frac{1}{1 - \kappa^{\text{s}}} H_0$$

Weak lensing correction in H0LiCOW / TDCOSMO is probably a little bit off.

Birrer et al, 2007.02941 (TDCOSMO IV)
 Teodori, et al, 2201.05111



Stellar kinematics (of *other* elliptical galaxies)

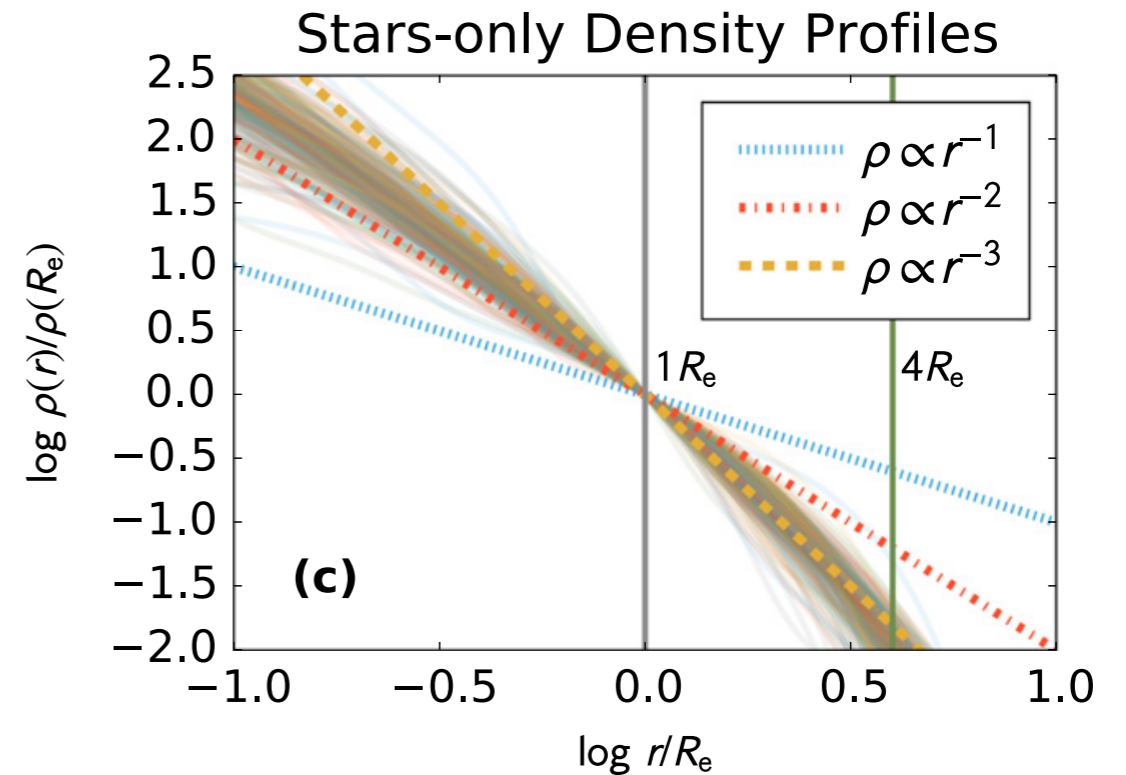
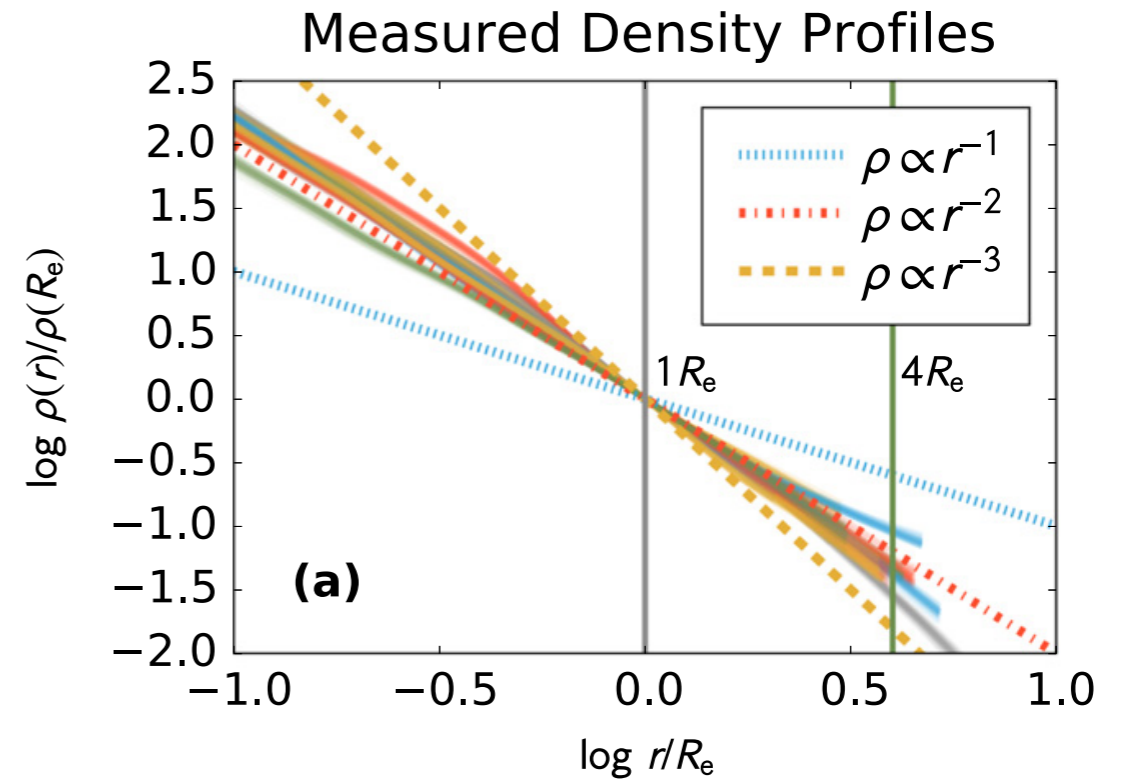


Stellar kinematics (of *other* elliptical galaxies)

Cappellari et al, 1504.00075

$$\rho_{\text{DM}}(r) = \rho_s \left(\frac{r}{r_s} \right)^\alpha \left(\frac{1}{2} + \frac{1}{2} \frac{r}{r_s} \right)^{-\alpha-3}. \quad (3)$$

Our models have seven free parameters. Some are poorly constrained but are not of interest here. They are just “nuisance parameters,” marginalized out to derive the total mass profiles studied here. The parameters are (i) the inclination i ; (ii) the anisotropy $\beta_z \equiv 1 - \sigma_z^2 / \sigma_R^2$, with σ_z and σ_R the stellar dispersion in cylindrical coordinates, for the MGE Gaussians with $\sigma_j < R_e$; (iii) the anisotropy for the remaining Gaussians at larger radii; (iv) the stellar $(M/L)_{\text{stars}}$; (v) the break radius of the dark halo, constrained to be $10 < r_s < 50$ kpc; (vi) the halo density ρ_s at r_s ; and (vii) the dark halo slope α for $r \ll r_s$.

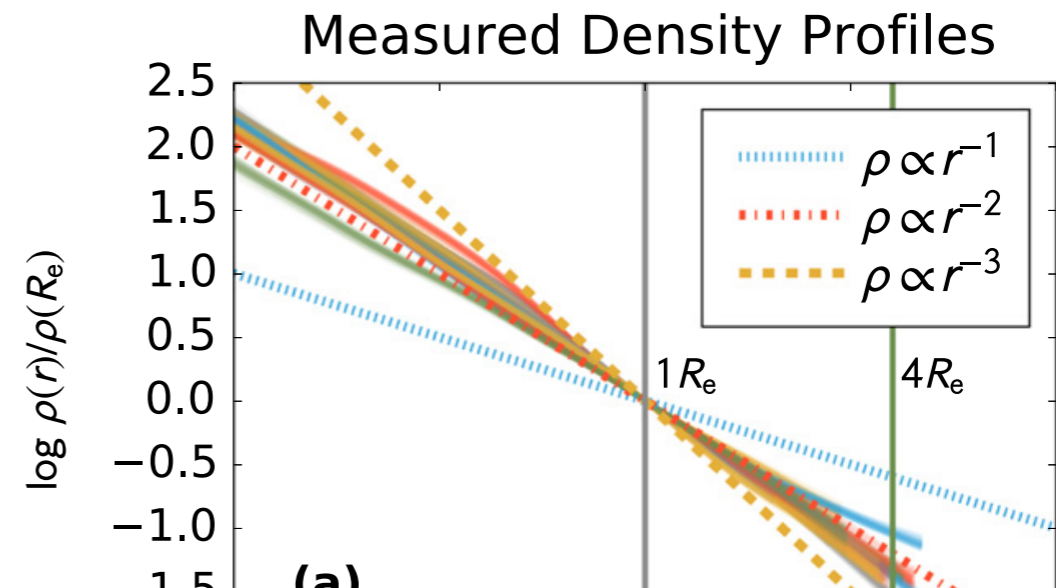


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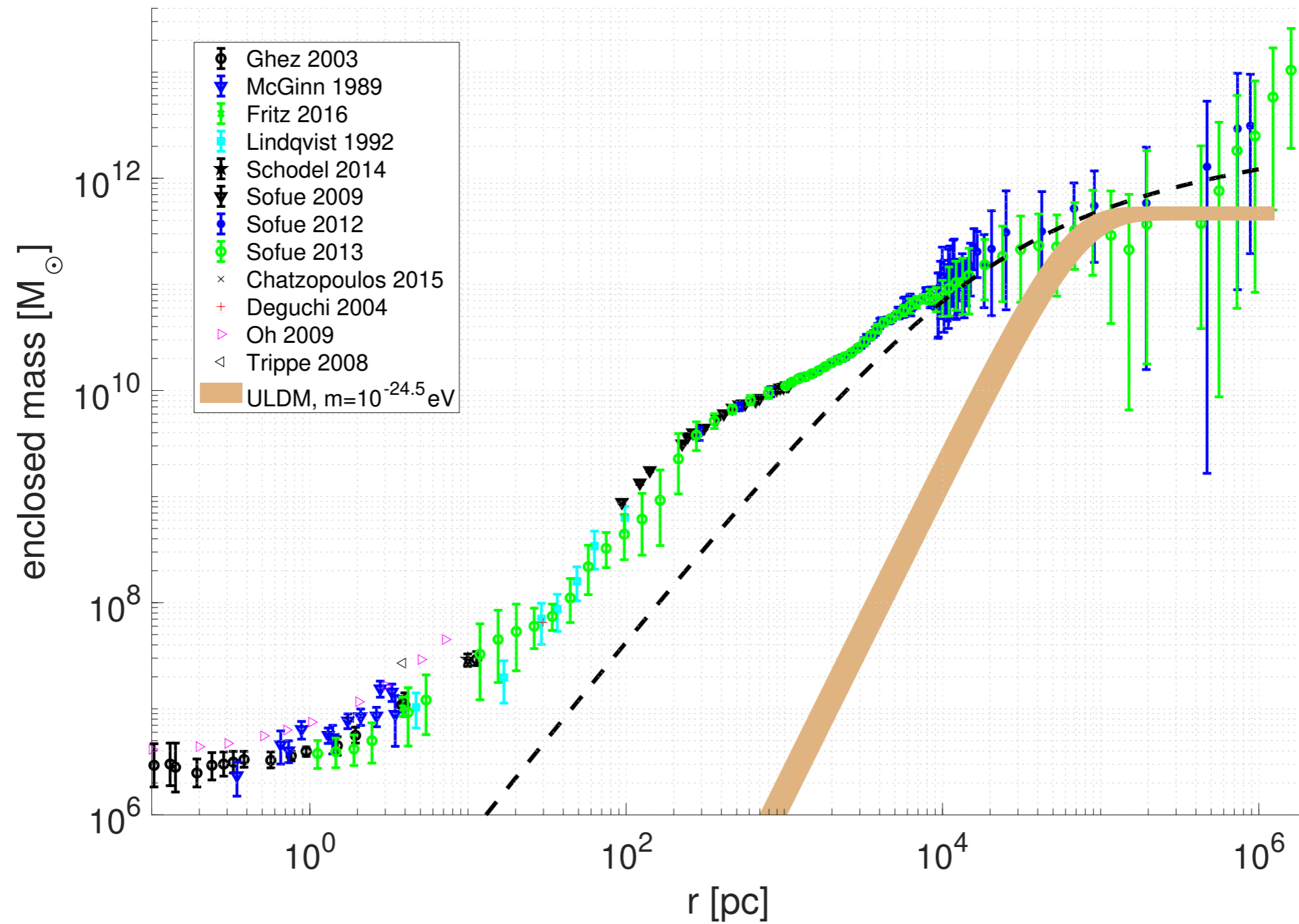


A friend:

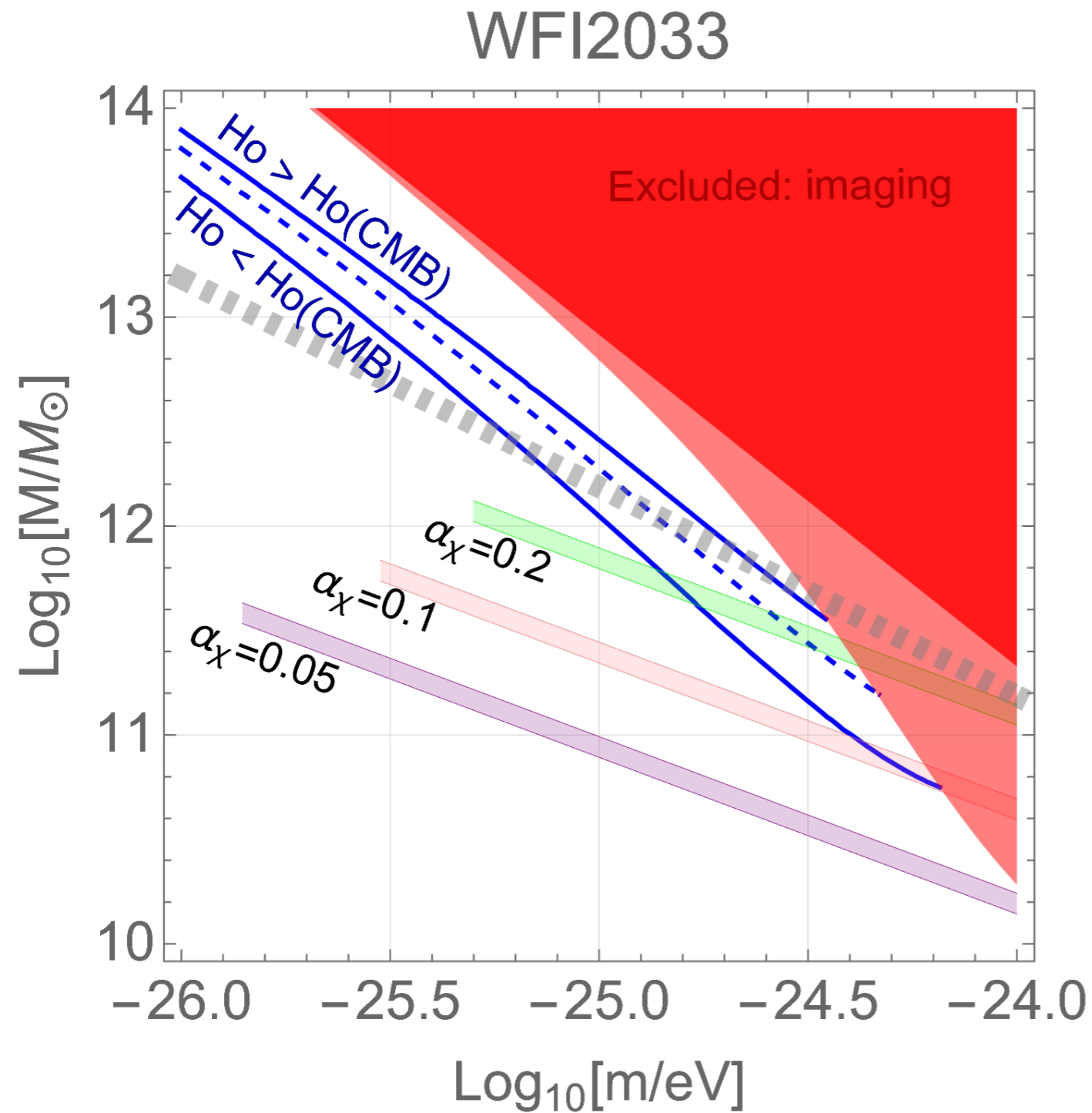
...a cored structure of the kind you propose would be difficult to exclude from measurements of the stellar kinematics. Part of the reason is the mass profile-velocity anisotropy degeneracy. Another part is simply that no one has tried: most modelers fit the system to a small number of components (stars, gas, dark matter, central black hole) with constant mass-to-light ratio and none of these look like the core you propose. It would be straightforward for some of the modelers to try adding cores.

I suppose some critics will say that your **cores are ad hoc**, but I think they are **less ad hoc than most of the modifications to cosmology needed to explain the Hubble discrepancy!**

Stellar kinematics (of *other* galaxies, e.g., Milky Way)



Ultralight DM as a solution of the lensing H0 tension:

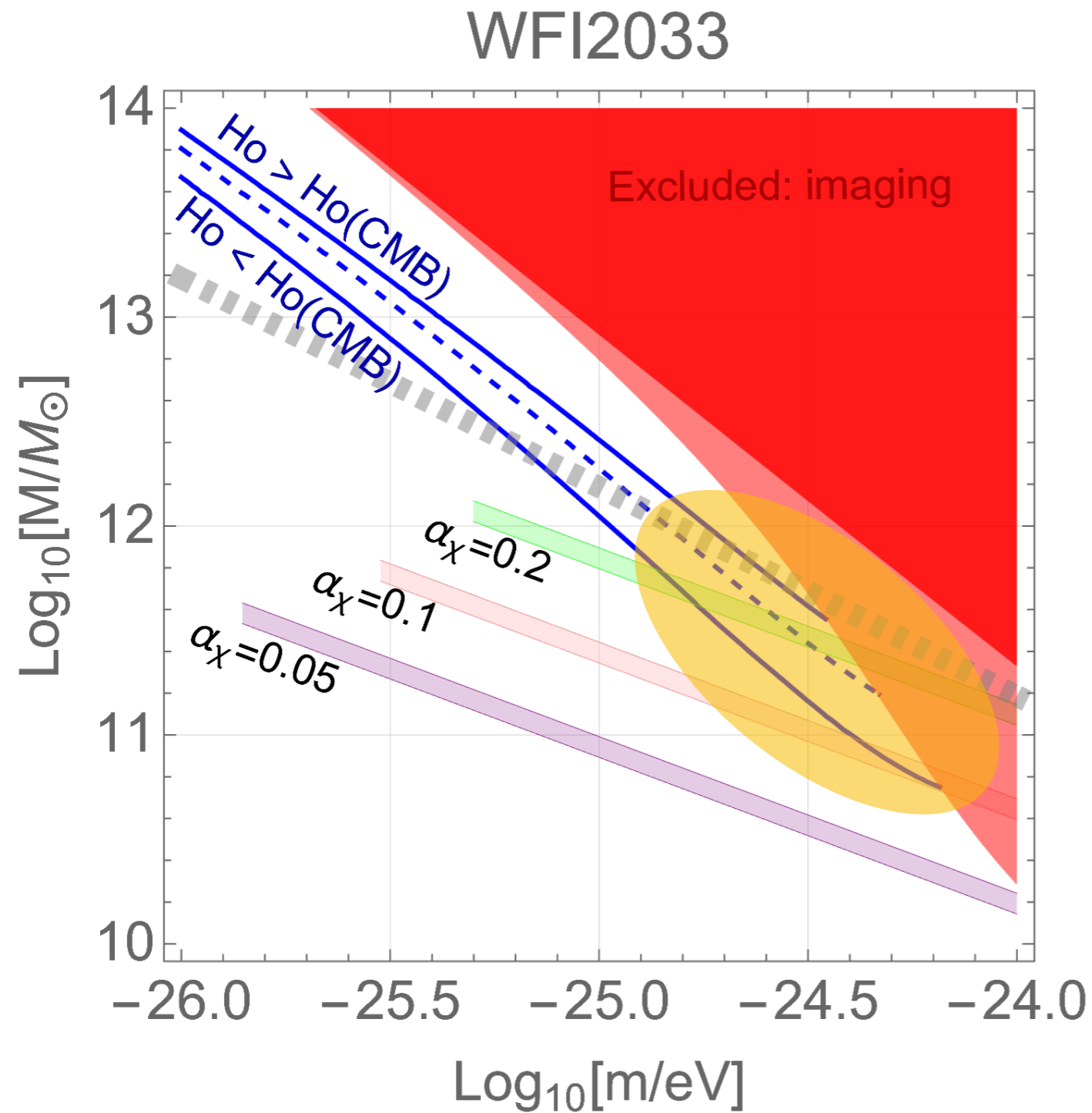


Dynamical relaxation
consistent at $O(1)$,
can become a bottleneck:

$$\tau \sim \frac{\sqrt{2}}{12\pi^3} \frac{m^3 \sigma^6}{G^2 \rho^2 \ln \Lambda}$$

(But see Eggemeier, Niemeyer 2019,
Chen et al 2020, Schwabe et al 2020;
for effect of background density.)

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H0 tension

