# Gravitational Focusing of Wave Dark Matter

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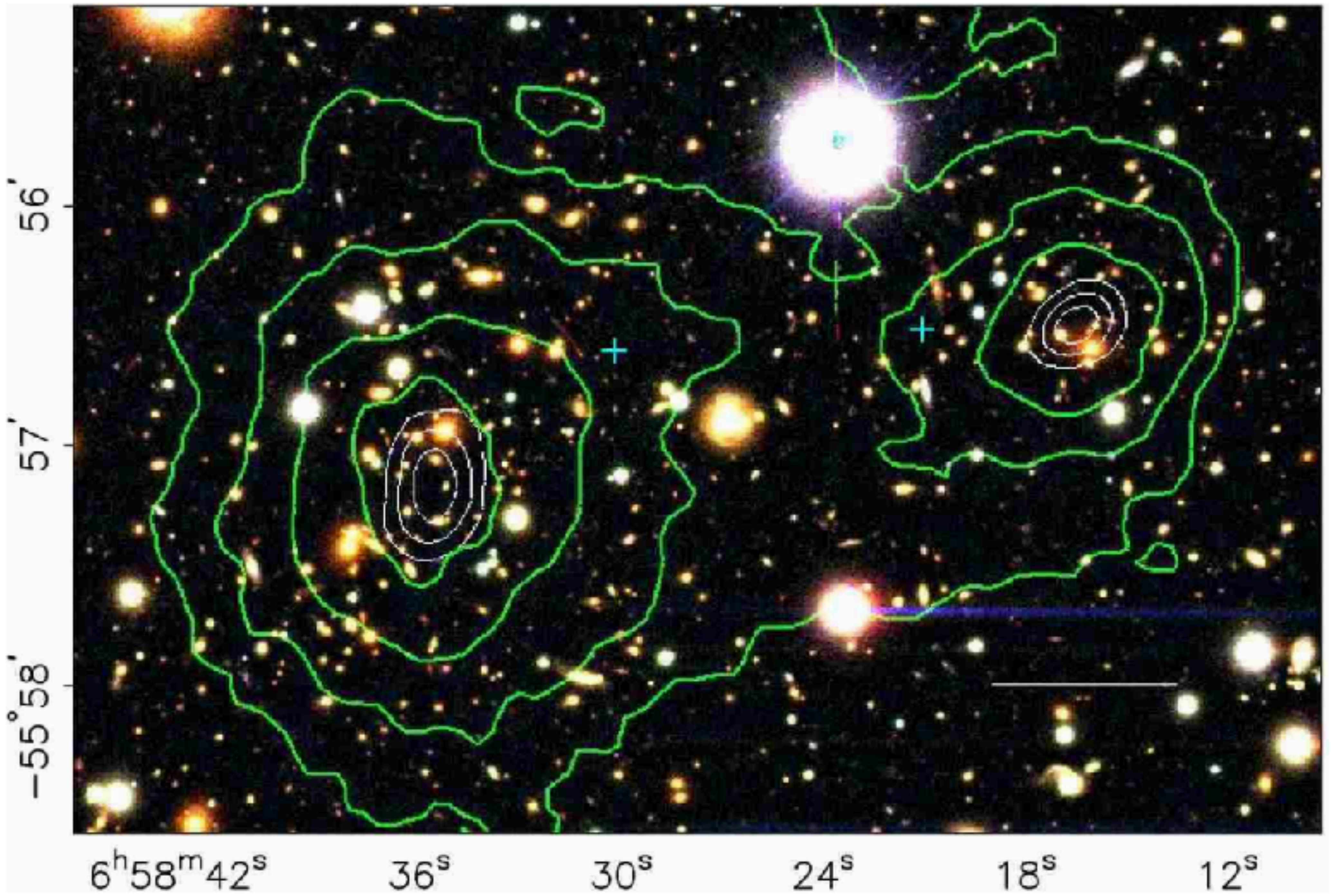
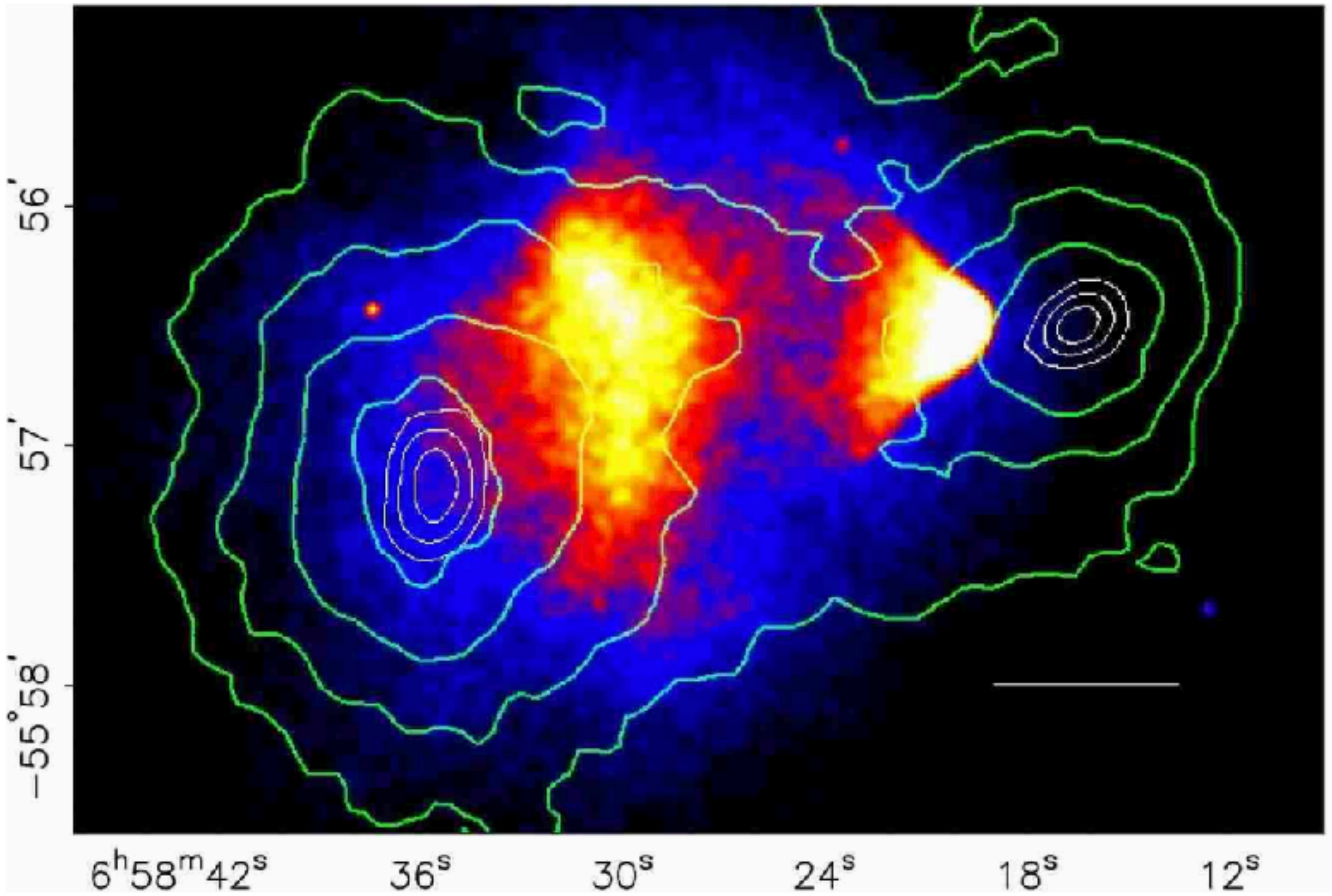
DESY

Based on the work done with Alessandro Lenoci [2112.05718]

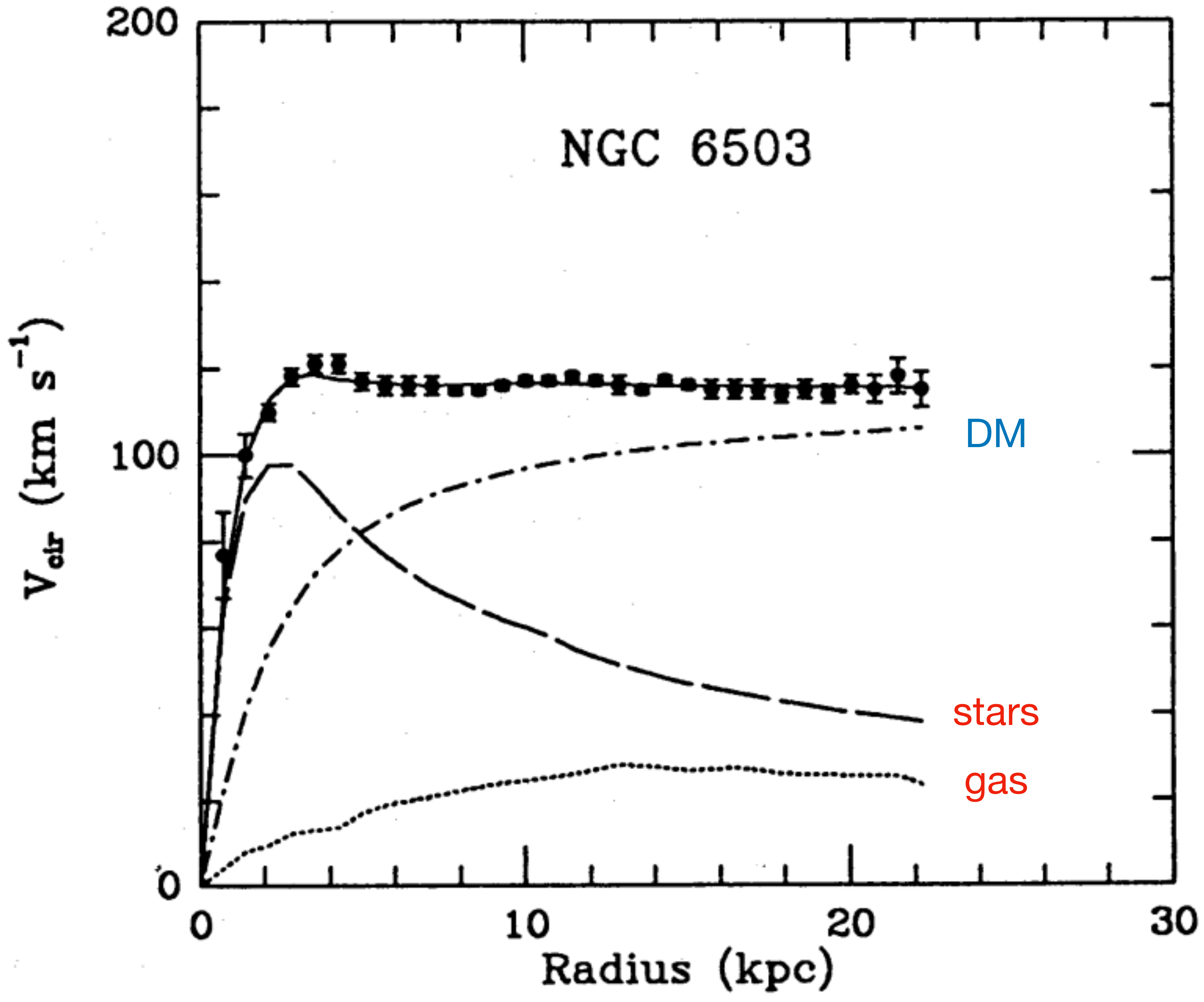
Dark matter is a compelling hypothesis.

Credit: ESA and the Planck Collaboration




[Clowe et al 2006]




[Begeman et al 91]

Yet its nature remains mystery.

What’s its mass?

How does it interaction with SM?

# What's its mass?

ULDM — Black holes

$10^{-22}\,\mathrm{eV}$ — $10^{57}\,\mathrm{GeV}$

Anything in between is possible

# How does it interact with SM?

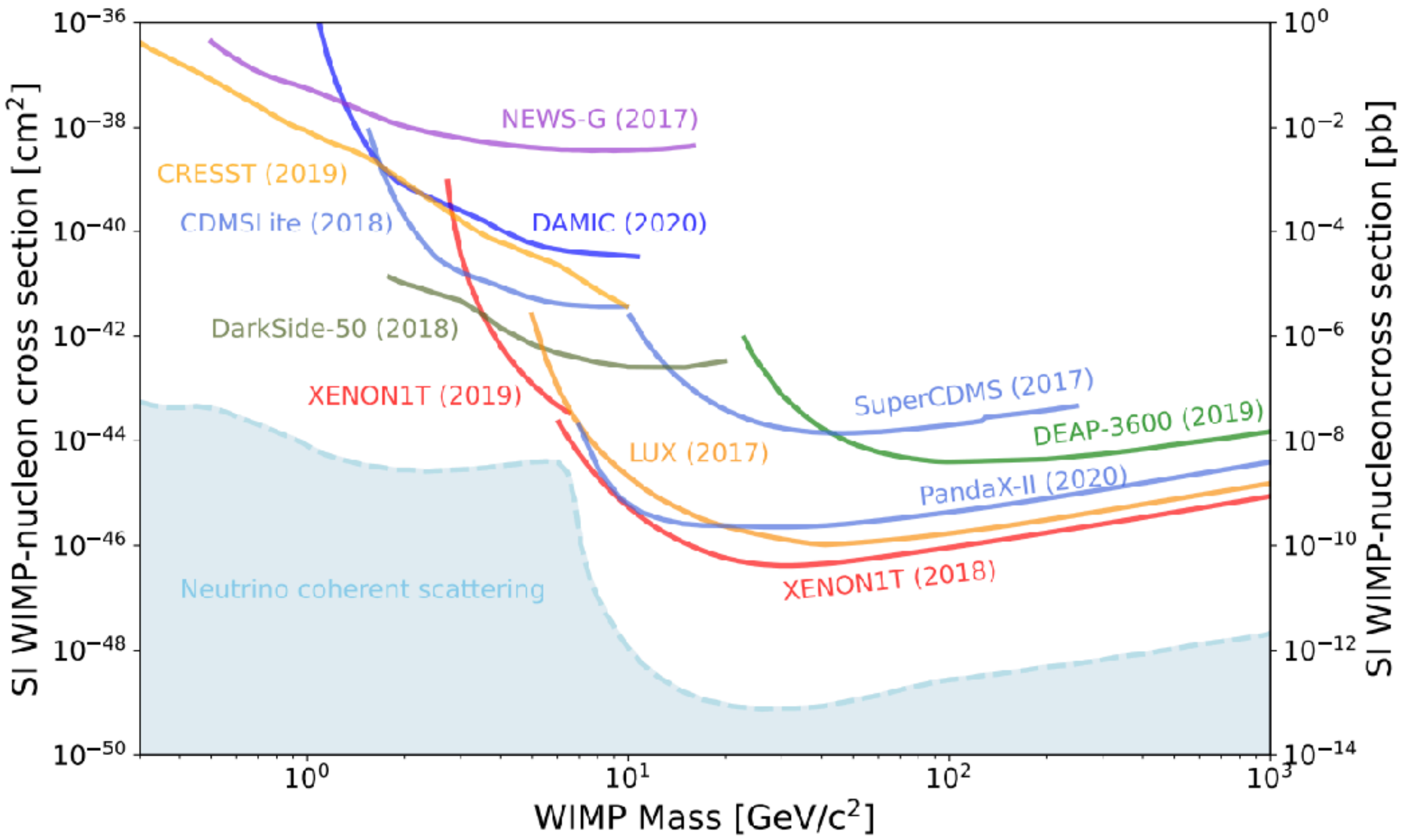
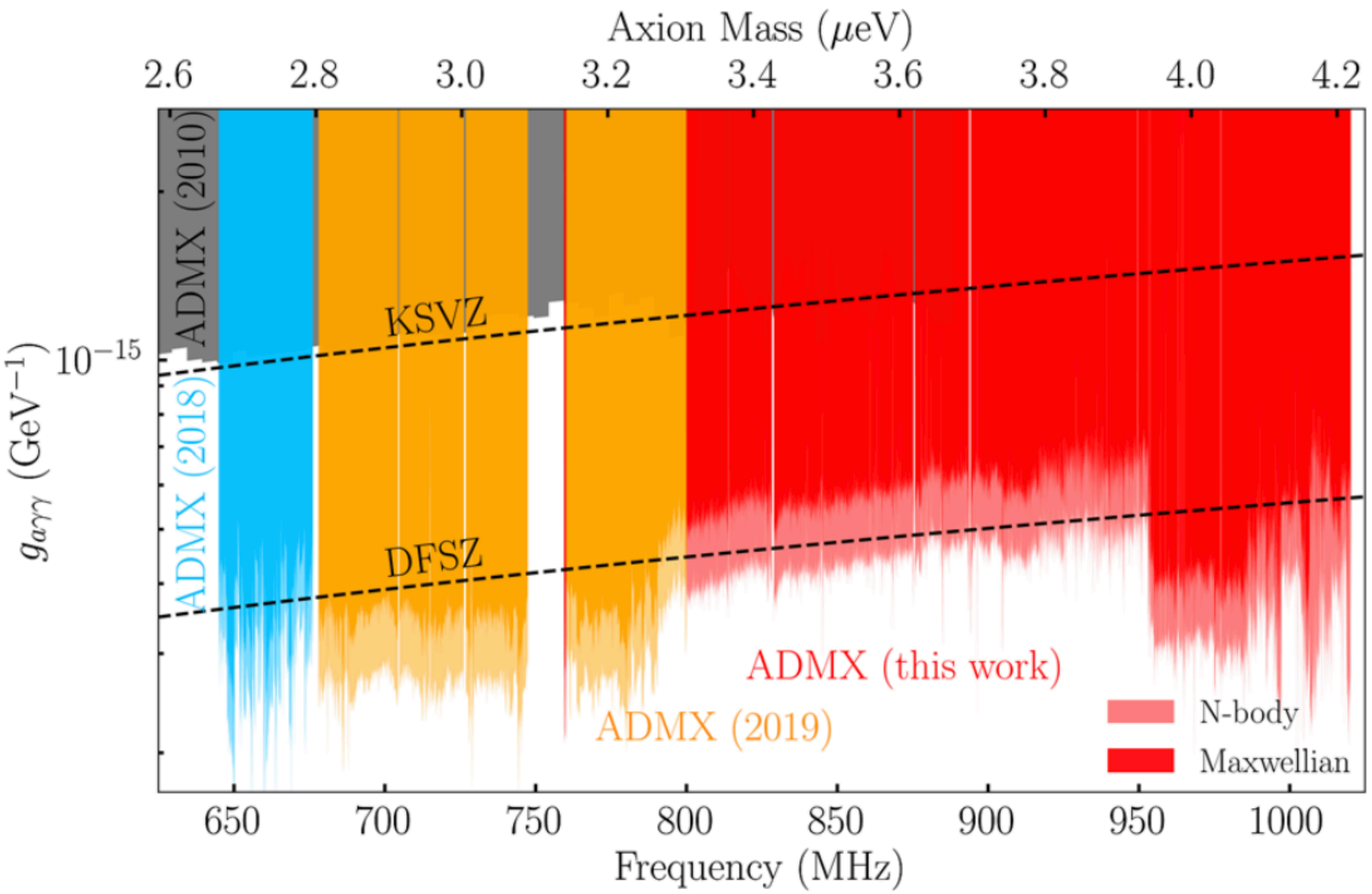


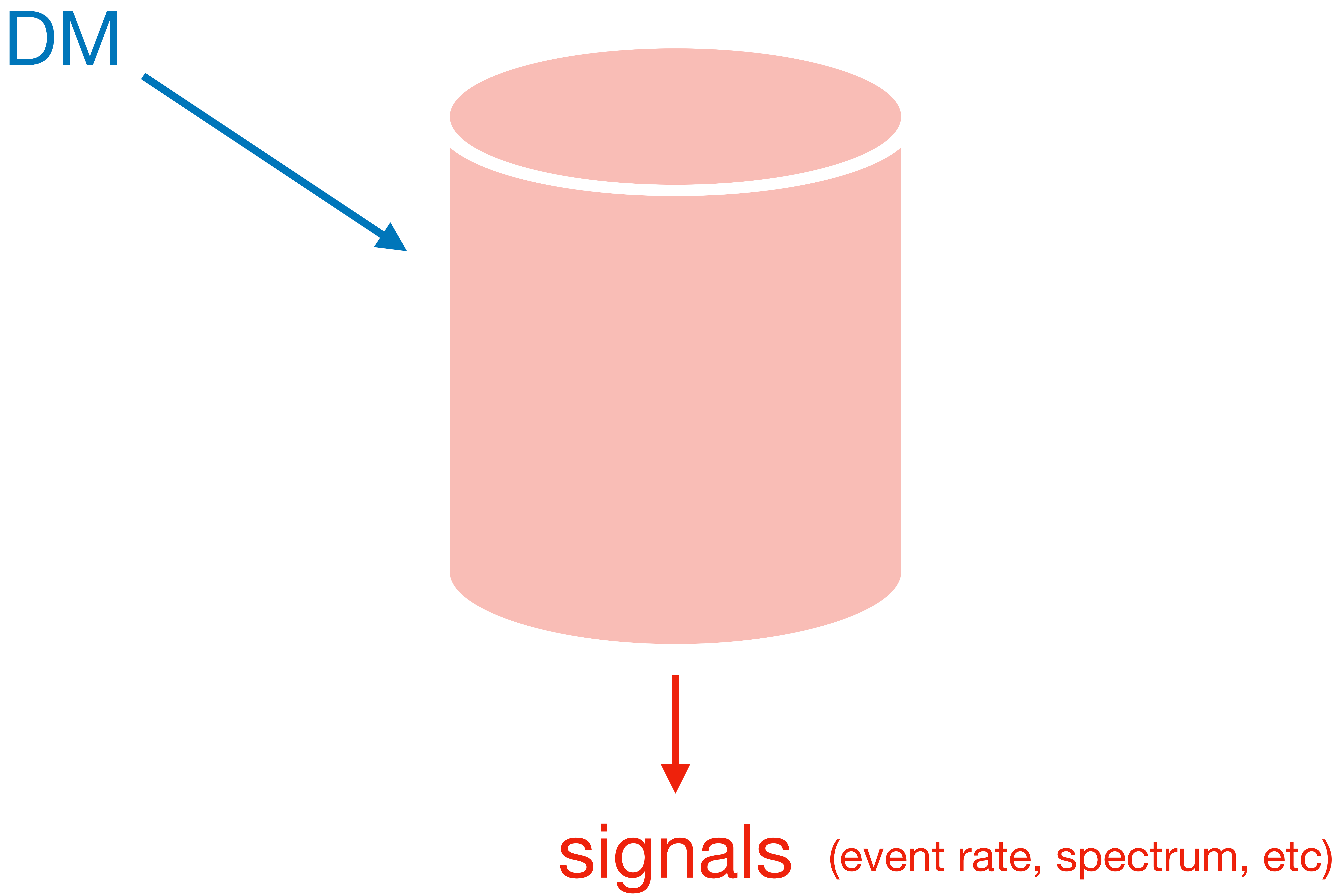

**Figure 27.1:** Upper limits on the SI DM-nucleon cross section as a function of DM mass.

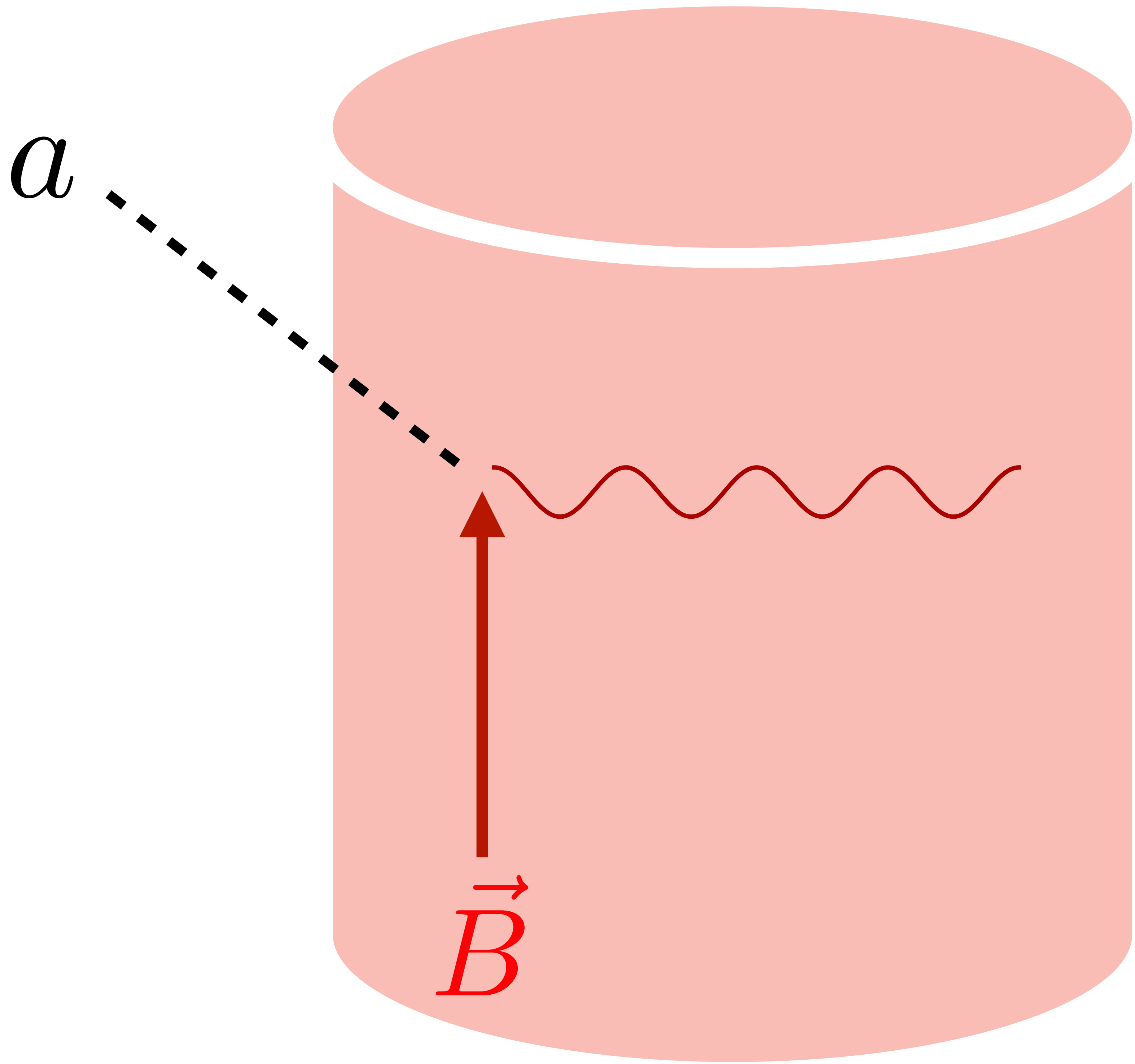
Review of Particle Physics, Particle Data Group 2021

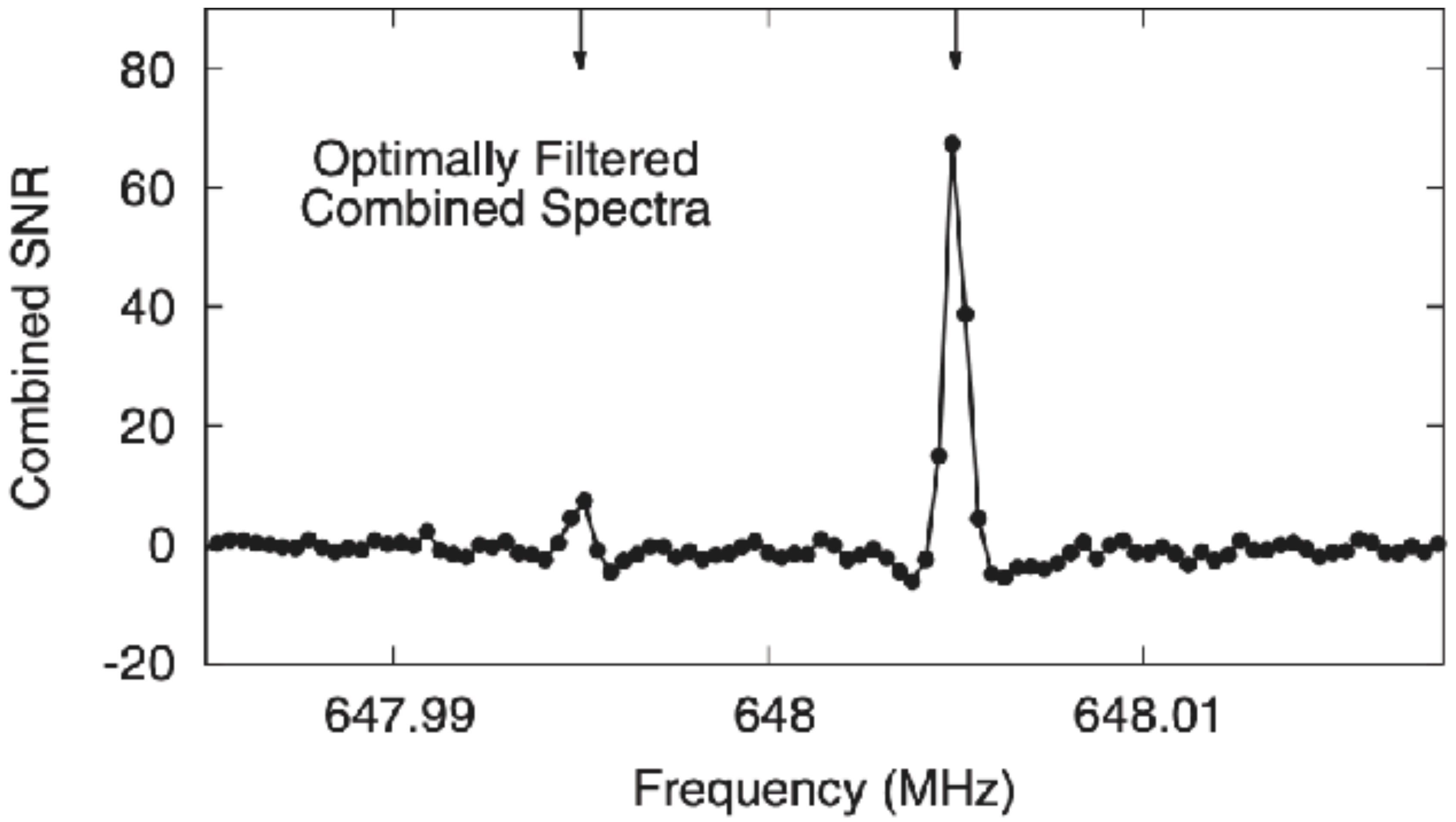
# How does it couples to SM?




ADMX Collaboration 2021

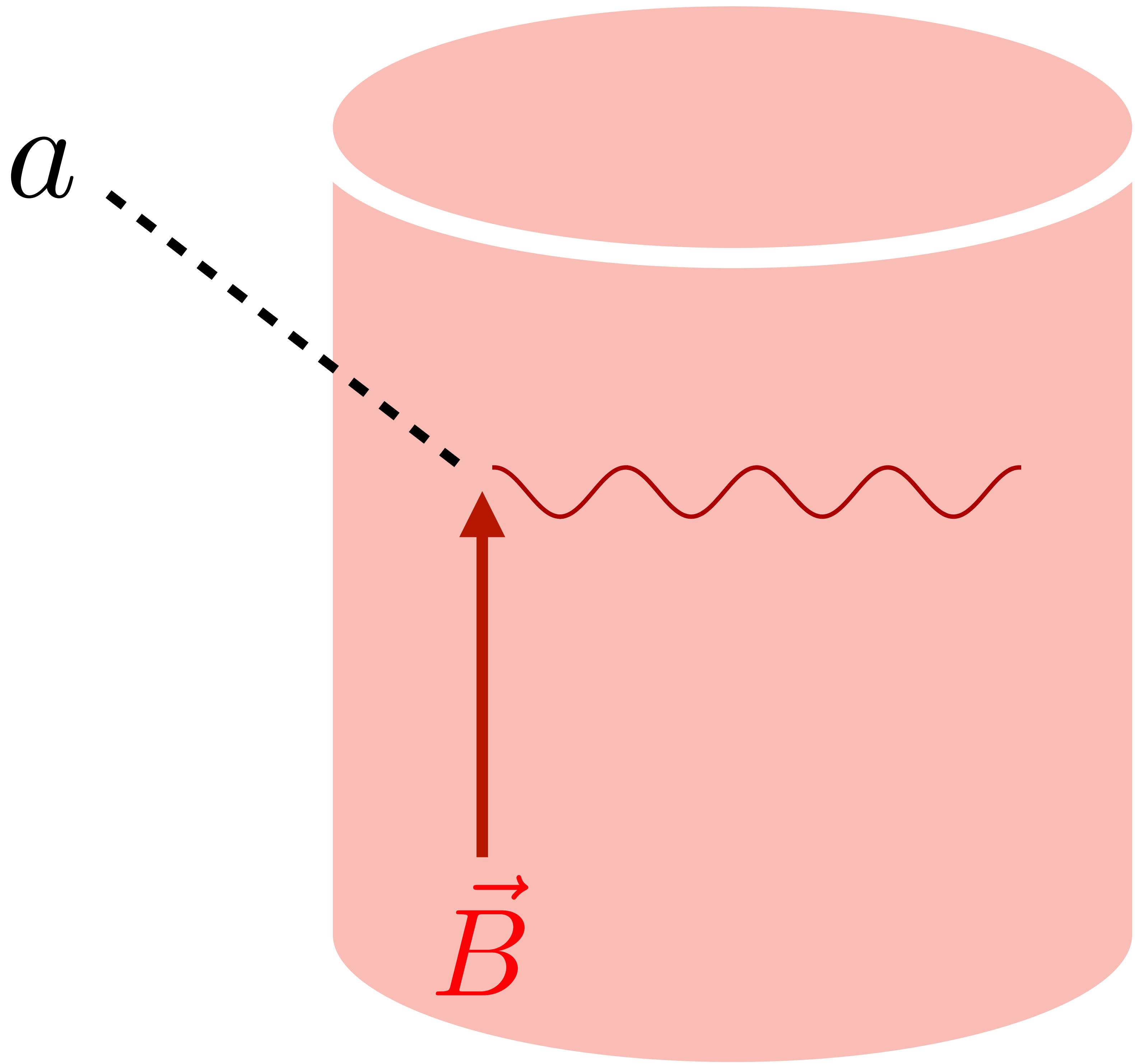

DM
signals (event rate, spectrum, etc)

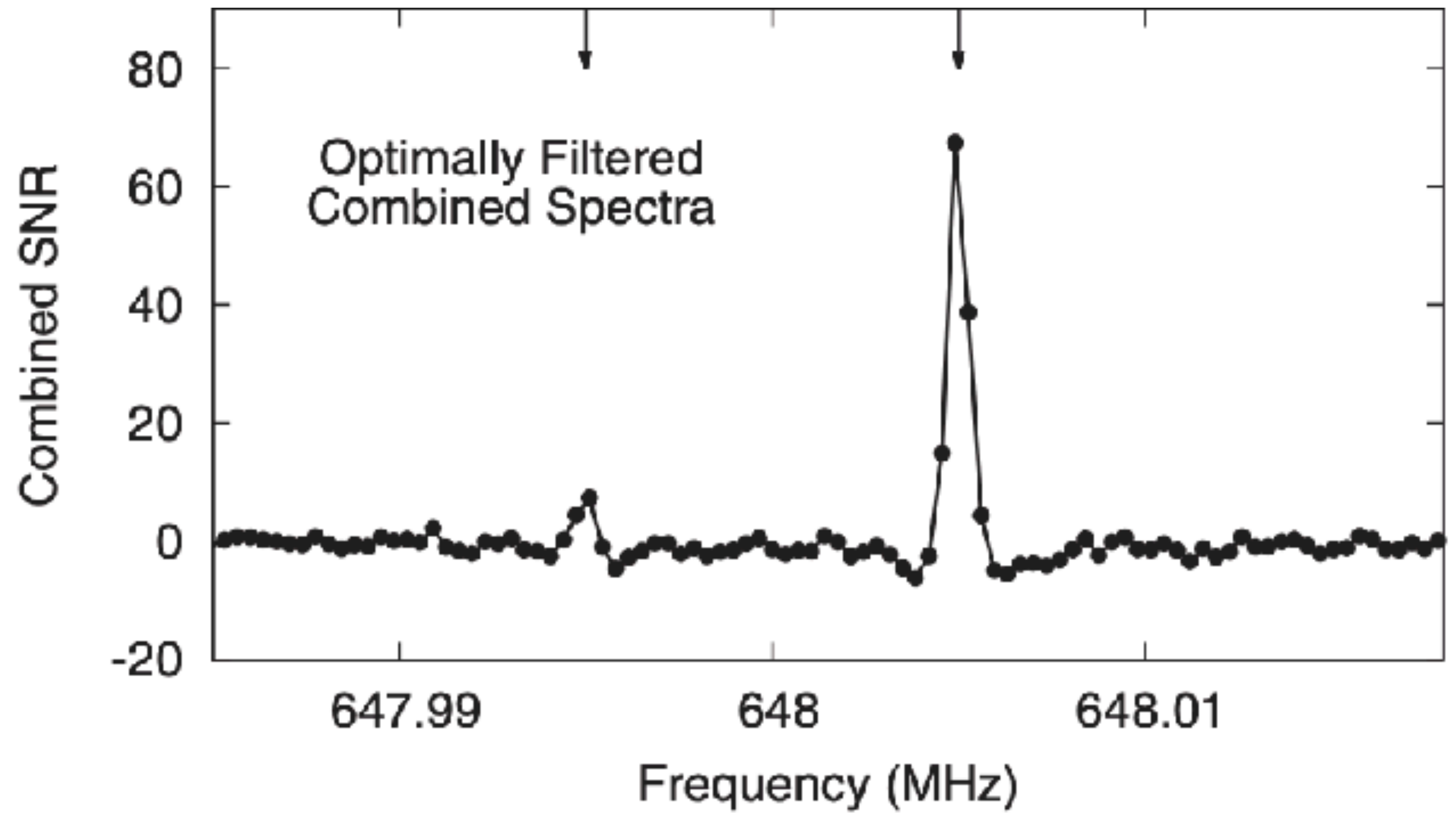



**ADMX Collaboration 2018**

$$S(\omega) \propto \frac{\xi^2}{m} \frac{\rho_{\rm DM} f(v)}{v}$$




ADMX Collaboration 2018

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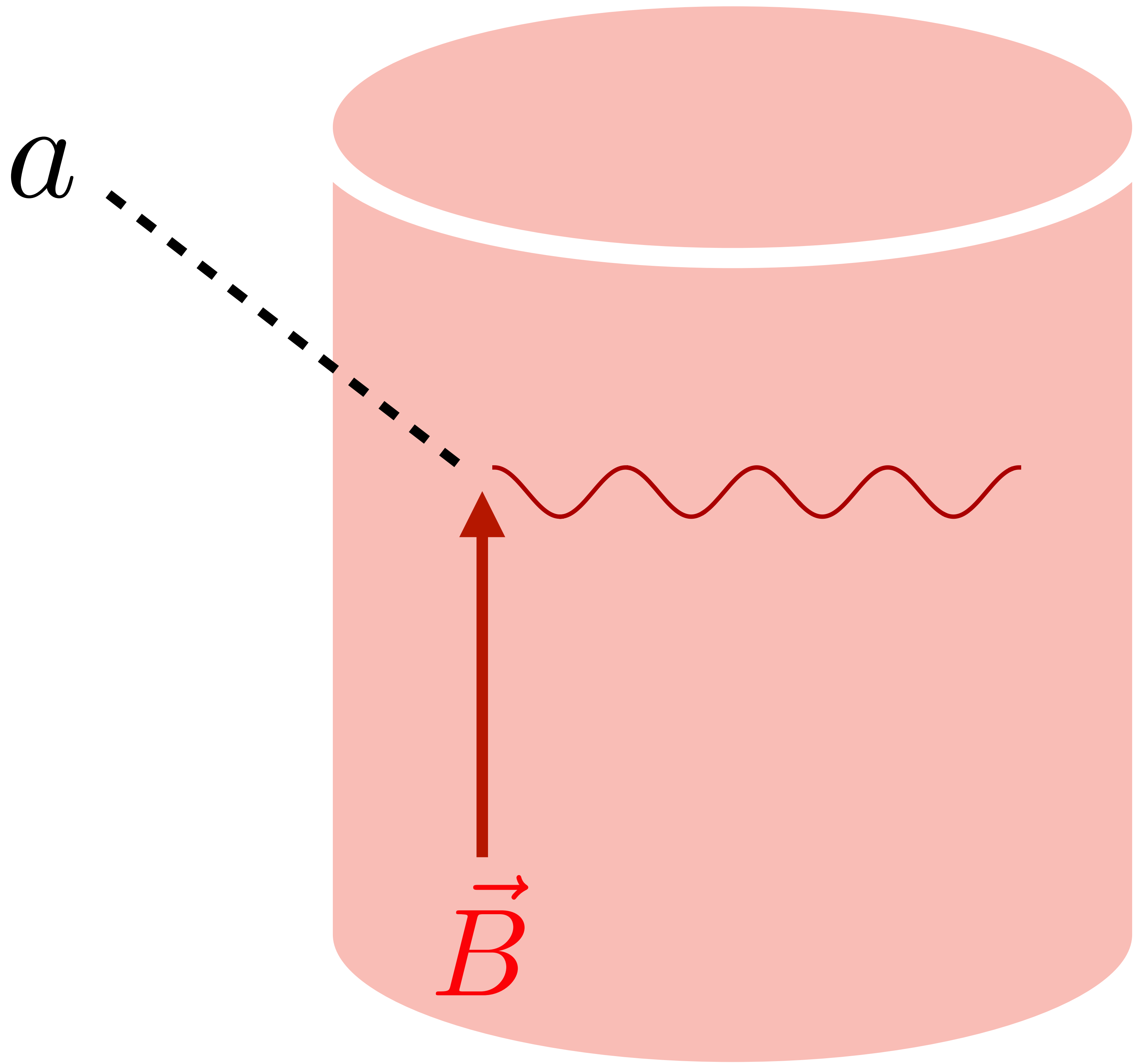
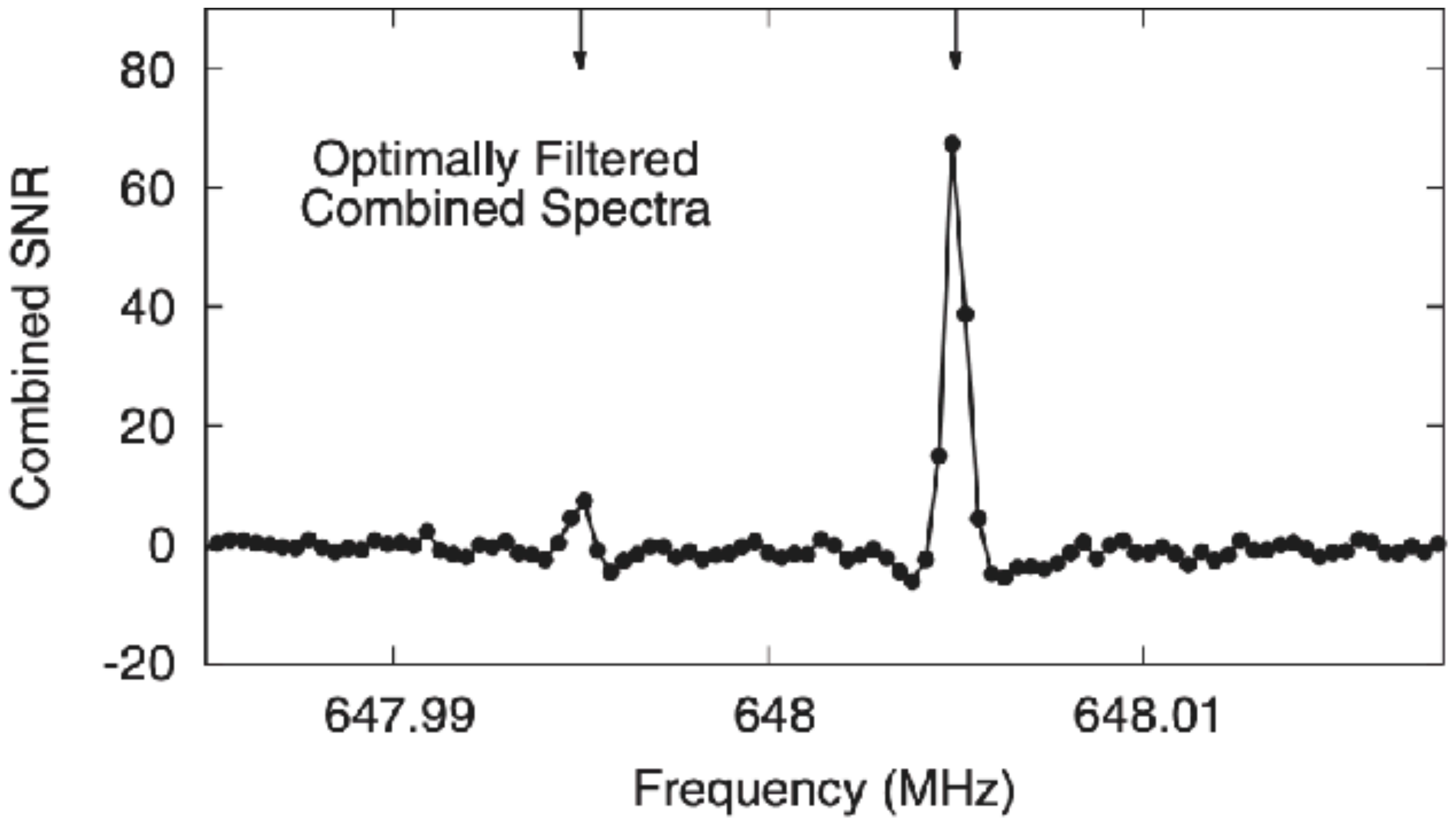



ADMX Collaboration 2018

$$S(\omega) \propto \frac{\xi^2}{m} \frac{\rho_{\mathrm{DM}} f(v)}{v}$$

*A correct modeling of local DM* will be crucial for interpretation of experimental results.

# The Standard Halo Model

(isothermal sphere with isotropic velocity distribution)

$$f(\vec{v}) = \frac{1}{(2\pi)^{3/2}} \exp\left[-\frac{v^2}{2\sigma^2}\right]$$

$$v_c(R) = \sqrt{2}\sigma$$

**Drukier, Freese, Spergel 86**

It’s *simple*

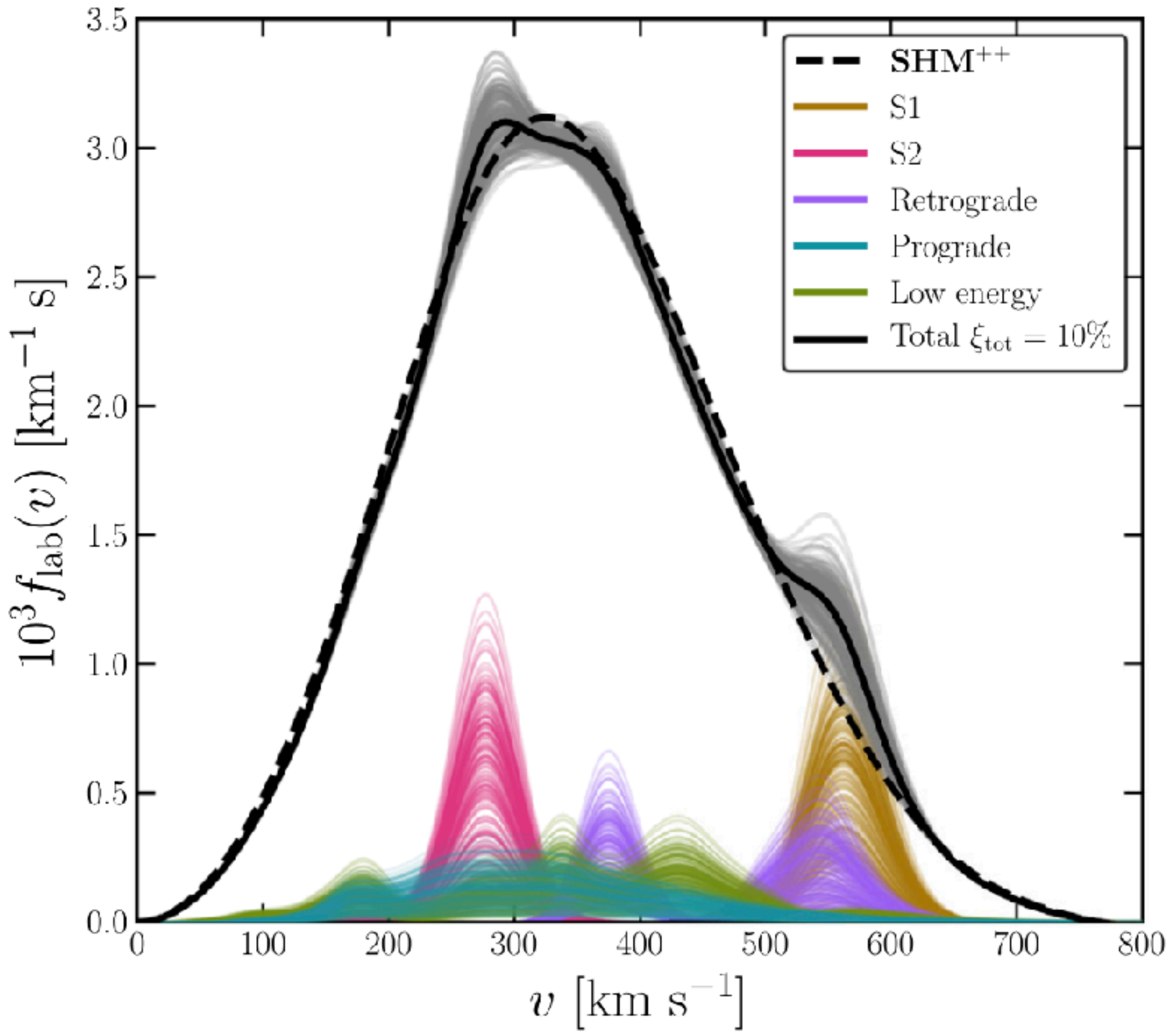
and *a reasonable approx. for the bulk halo*

But it does not fully reflect reality:

1. Dark matter substructures

2. Gravitational deformation due to the sun

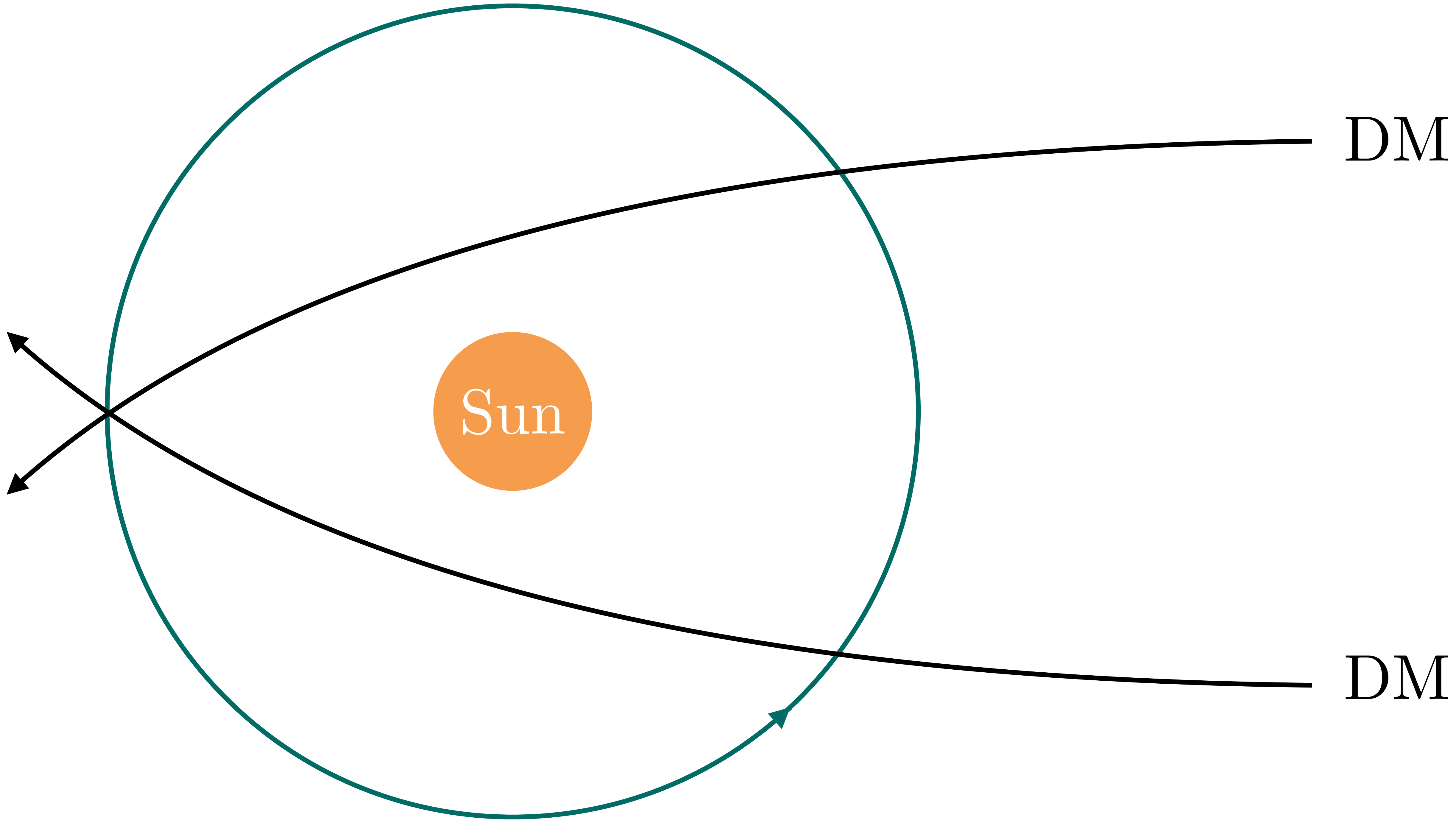
# And many other substructures near solar neighborhood




| Name | | Number of stars | $(X, Y, Z)$ kpc | $(\Delta X, \Delta Y, \Delta Z)$ kpc | $(v_R, v_\phi, v_z)$ km s$^{-1}$ | $(\sigma_R, \sigma_\phi, \sigma_z)$ km s$^{-1}$ | $\langle$[Fe/H]$\rangle$ | $P(\mathbf{x}_\odot)$ $(\sigma)$ |
|---|---|---|---|---|---|---|---|---|
| S1 | | 28 | (8.4, 0.6, 2.6) | (0.7, 1.8, 2.2) | (−34.2, −306.3, −64.4) | (81.9, 46.3, 62.9) | $-1.9 \pm 0.3$ | 1.2 |
| S2 | a | 46 | (8.7, 0.4, 0.1) | (0.7, 1.2, 6.9) | (5.8, 163.6, −250.4) | (45.9, 13.8, 26.8) | $-2.0 \pm 0.2$ | 0.3 |
| | b | 8 | (10.1, 0.2, 3.3) | (4.9, 0.7, 1.4) | (−50.6, 138.5, 183.1) | (90.8, 25.0, 43.8) | $-2.0 \pm 0.3$ | 1.5 |
| Retrograde | Rg2 | 13 | (8.9, 0.3, 4.4) | (0.8, 2.1, 2.7) | (44.5, −248.4, 185.2) | (105.9, 23.1, 63.5) | $-1.6 \pm 0.2$ | 1.4 |
| | Rg5a | 15 | (8.4, 0.8, 1.1) | (1.0, 1.3, 3.3) | (6.4, −74.5, −159.5) | (32.4, 17.5, 31.7) | $-2.2 \pm 0.3$ | 0.7 |
| | Rg5b | 14 | (8.1, −0.2, 2.2) | (1.1, 1.2, 2.4) | (−37.6, −83.8, 178.1) | (47.5, 16.8, 31.1) | $-2.1 \pm 0.3$ | 0.9 |
| | Rg6a | 17 | (8.3, 0.2, 3.3) | (1.8, 1.4, 2.0) | (105.1, −230.2, 202.4) | (73.7, 16.8, 86.6) | $-1.6 \pm 0.2$ | 1.1 |
| | Rg6b | 12 | (8.5, 0.9, 3.2) | (1.5, 1.5, 2.2) | (−233.2, −221.8, 51.6) | (32.7, 14.4, 115.7) | $-1.7 \pm 0.3$ | 0.6 |
| | Rg7a | 5 | (8.2, 0.5, 3.3) | (2.1, 1.5, 3.3) | (309.0, −191.3, −83.4) | (66.7, 17.1, 102.7) | $-1.5 \pm 0.1$ | 1.1 |
| | Rg7b | 9 | (8.9, −0.0, 5.1) | (1.9, 1.3, 2.0) | (−288.7, −158.1, −105.5) | (78.7, 65.8, 111.8) | $-1.5 \pm 0.3$ | 1.8 |
| Prograde | Cand8a | 31 | (9.9, −0.1, 2.4) | (2.1, 2.5, 4.4) | (−6.7, 207.7, −186.4) | (114.6, 20.8, 73.5) | $-1.8 \pm 0.4$ | 0.4 |
| | Cand8b | 18 | (8.4, 0.6, 1.1) | (1.5, 2.2, 3.6) | (33.6, 213.9, 214.1) | (96.5, 22.7, 37.7) | $-1.8 \pm 0.2$ | 0.1 |
| | Cand9 | 43 | (9.2, −0.2, 1.7) | (1.1, 1.4, 3.4) | (11.0, 177.5, −251.4) | (120.6, 13.9, 132.2) | $-1.8 \pm 0.2$ | 0.5 |
| | Cand10 | 38 | (8.6, −0.0, 2.0) | (1.7, 1.3, 2.5) | (−37.4, 20.0, 192.3) | (161.5, 18.2, 195.0) | $-2.0 \pm 0.2$ | 0.2 |
| | Cand11a | 14 | (9.1, −0.3, 2.7) | (2.5, 1.4, 3.8) | (36.8, 116.5, −271.5) | (96.1, 27.9, 95.4) | $-2.1 \pm 0.3$ | 0.3 |
| | Cand11b | 23 | (9.0, −0.1, 2.4) | (1.9, 1.1, 2.8) | (−152.7, 80.2, 258.2) | (122.1, 21.0, 38.9) | $-2.0 \pm 0.3$ | 0.5 |
| | Cand12 | 36 | (9.6, −0.8, 3.7) | (2.0, 2.4, 4.2) | (−43.3, 102.4, 50.0) | (172.8, 21.2, 197.8) | $-1.6 \pm 0.2$ | 0.6 |
| | Cand13 | 36 | (9.1, 1.0, 3.1) | (2.5, 2.0, 4.1) | (−2.1, −13.2, 202.2) | (215.7, 28.1, 215.9) | $-1.4 \pm 0.2$ | 0.4 |
| | Cand14a | 24 | (11.9, 0.2, 1.8) | (1.8, 1.7, 3.6) | (−168.0, 166.7, −25.1) | (29.1, 27.9, 82.7) | $-1.4 \pm 0.2$ | 1.2 |
| | Cand14b | 12 | (10.7, 0.3, 1.4) | (1.8, 2.1, 3.5) | (193.6, 202.9, −5.7) | (14.3, 13.5, 51.8) | $-1.5 \pm 0.1$ | 0.7 |
| | Cand15a | 12 | (10.5, 1.4, 4.0) | (1.9, 2.1, 3.9) | (−297.4, 220.0, −49.9) | (29.6, 23.5, 79.3) | $-1.5 \pm 0.1$ | 1.2 |
| | Cand15b | 7 | (10.3, −0.3, 2.4) | (1.8, 2.3, 5.9) | (291.3, 207.3, 48.3) | (20.2, 10.4, 68.7) | $-1.4 \pm 0.1$ | 0.5 |
| | Cand16a | 12 | (8.7, 0.5, 3.9) | (1.6, 1.5, 3.9) | (315.2, 109.2, −12.5) | (30.9, 4.6, 67.2) | $-1.4 \pm 0.2$ | 0.7 |
| | Cand16b | 5 | (8.9, 2.8, −1.3) | (1.3, 2.1, 3.2) | (−360.7, 147.5, 81.7) | (26.7, 9.2, 76.3) | $-1.4 \pm 0.1$ | 0.9 |
| | Cand17 | 10 | (9.5, −0.4, 2.0) | (1.0, 0.9, 2.5) | (127.6, 68.0, 339.4) | (157.4, 8.0, 54.8) | $-2.1 \pm 0.2$ | 0.7 |

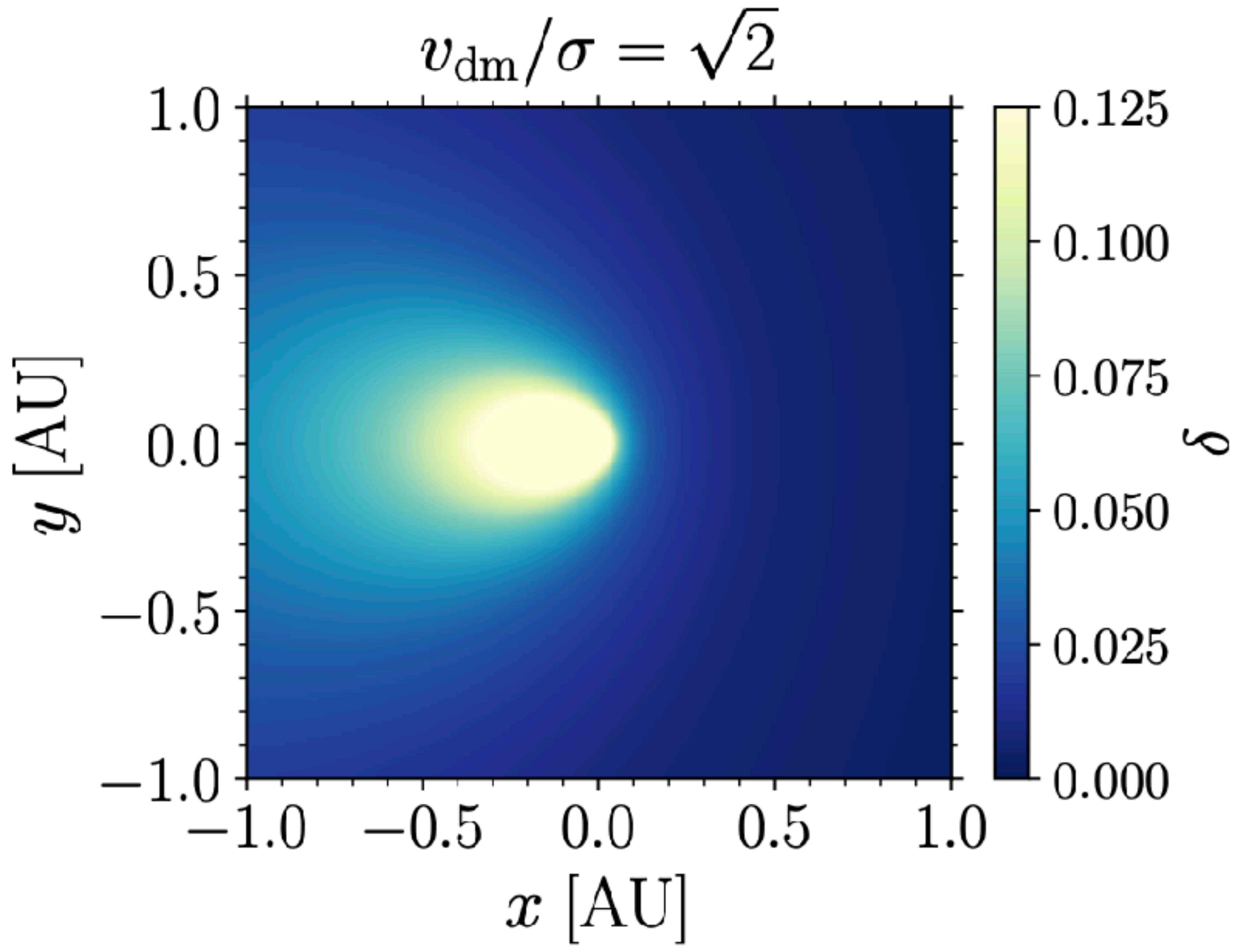
**O'Hare et al 20**

# Gravitational influence of the sun




Gravity distorts a local map of DM

# Gravitational influence of the sun




The sun *gravitationally focus* dark matter particles

A relevant question would be:

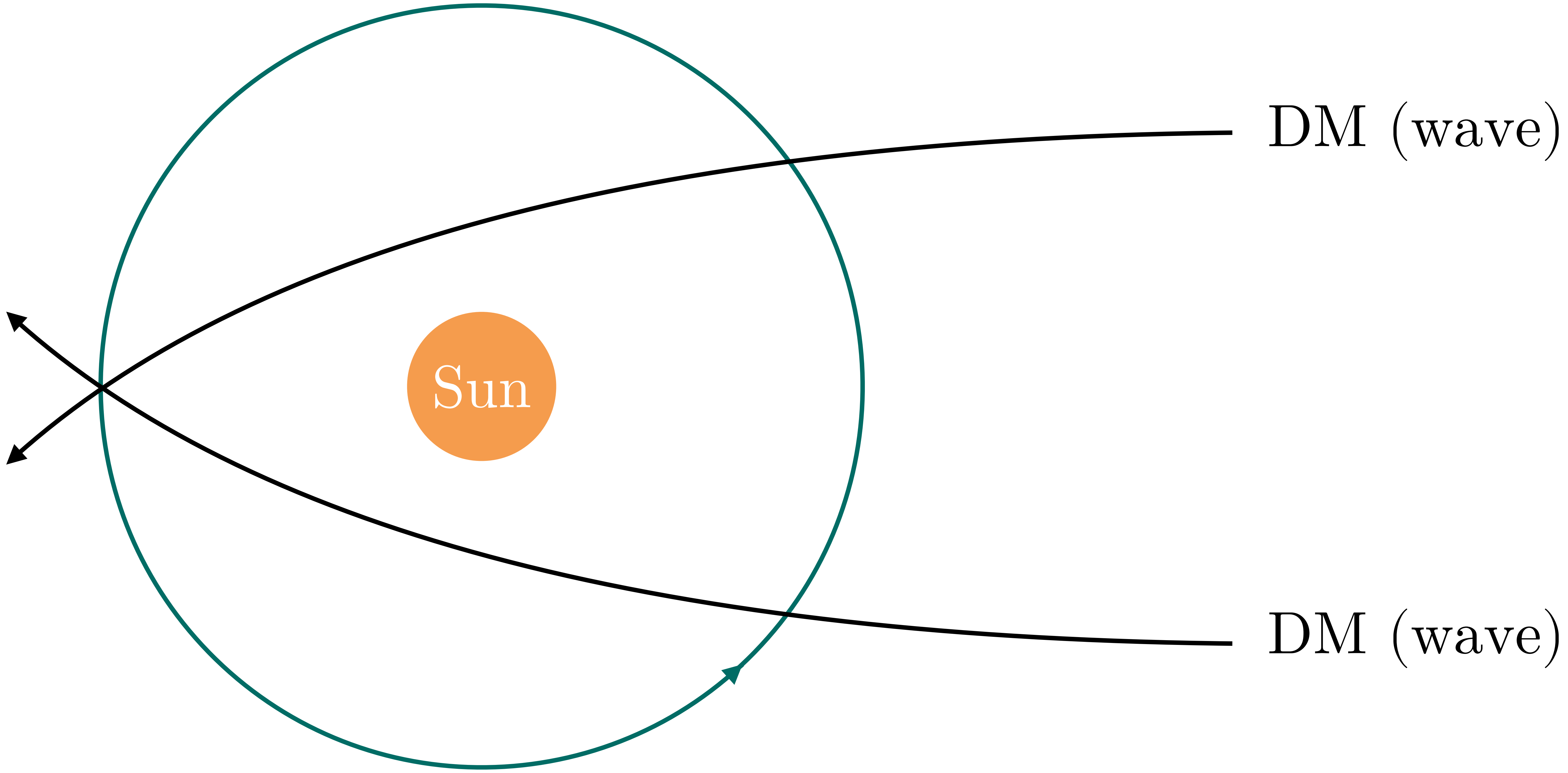
to which degree

these effects would modify signals in DM detectors?

# Gravitational focusing of wave dark matter

**HK, Alessandro Lenoci [2112.05718]**

# An action for the following situation?



# Action

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$$

Lagrangian for wave dark matter

$$ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Phi)dx^2$$

Gravitational potential

$$\Phi = -\frac{GM}{r}$$

# Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[ \hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

# Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[ \hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

$$(\Box + m^2)\phi = 0 \quad \rightarrow \quad i\dot{\psi} = \left[ -\frac{\nabla^2}{2m} - \frac{GMm}{r} \right] \psi$$

# Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[ \hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

# Gravitational Response

$$i\dot{\psi} = \left[-\frac{\nabla^2}{2m} - \frac{GMm}{r}\right]\psi$$

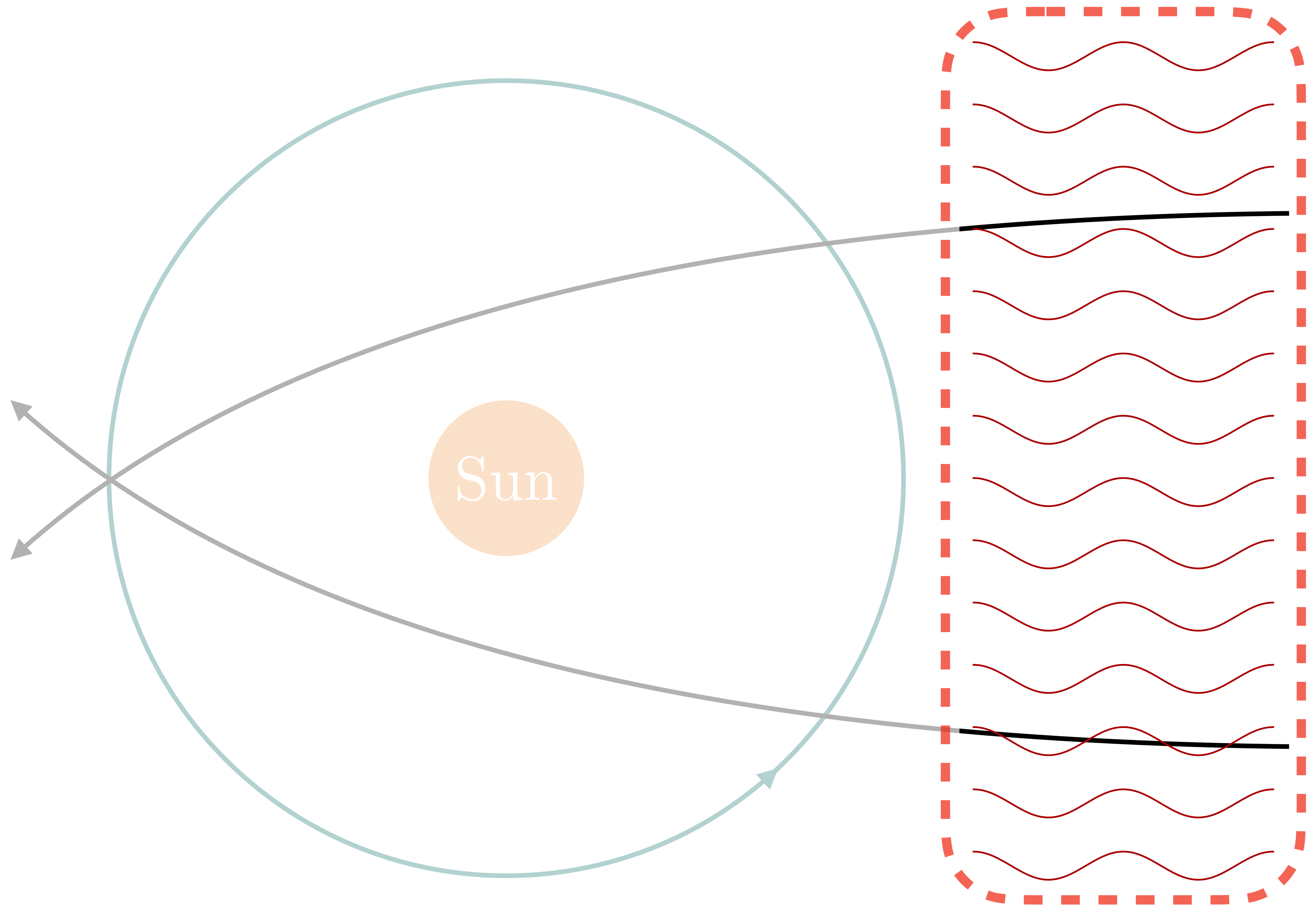
a solution is known

$$\psi_{\vec{k}}(t,\vec{x}) = e^{-iE_k t} e^{i\vec{k}\cdot\vec{x}} \psi_{\vec{k}}(0)\, M[i\alpha_G/v, 1, ikr(1-\hat{k}\cdot\hat{x})]$$

# Remarks on the solution

$$\psi_{\vec{k}}(t,\vec{x}) = e^{-iE_k t} e^{i\vec{k}\cdot\vec{x}} \psi_{\vec{k}}(0)\, M[i\alpha_G/v, 1, ikr(1-\hat{k}\cdot\hat{x})]$$

(i) it's a plane wave asymptotically far away from the sun



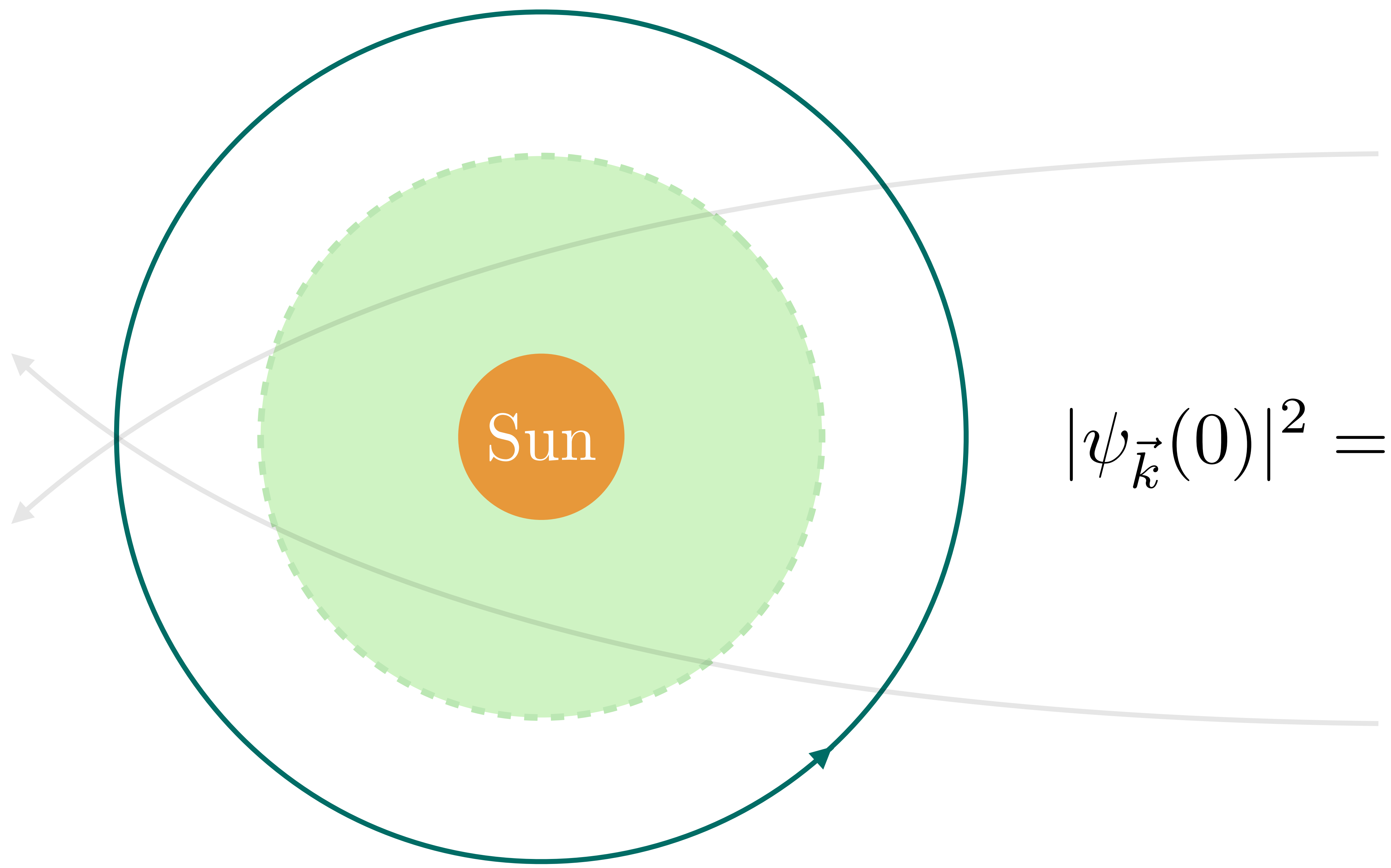

$$\psi_{\vec{k}}(t,\vec{x}) \sim e^{-iE_k t} e^{i\vec{k}\cdot\vec{x}}$$

$$E_k = \frac{\vec{k}^2}{2m}$$

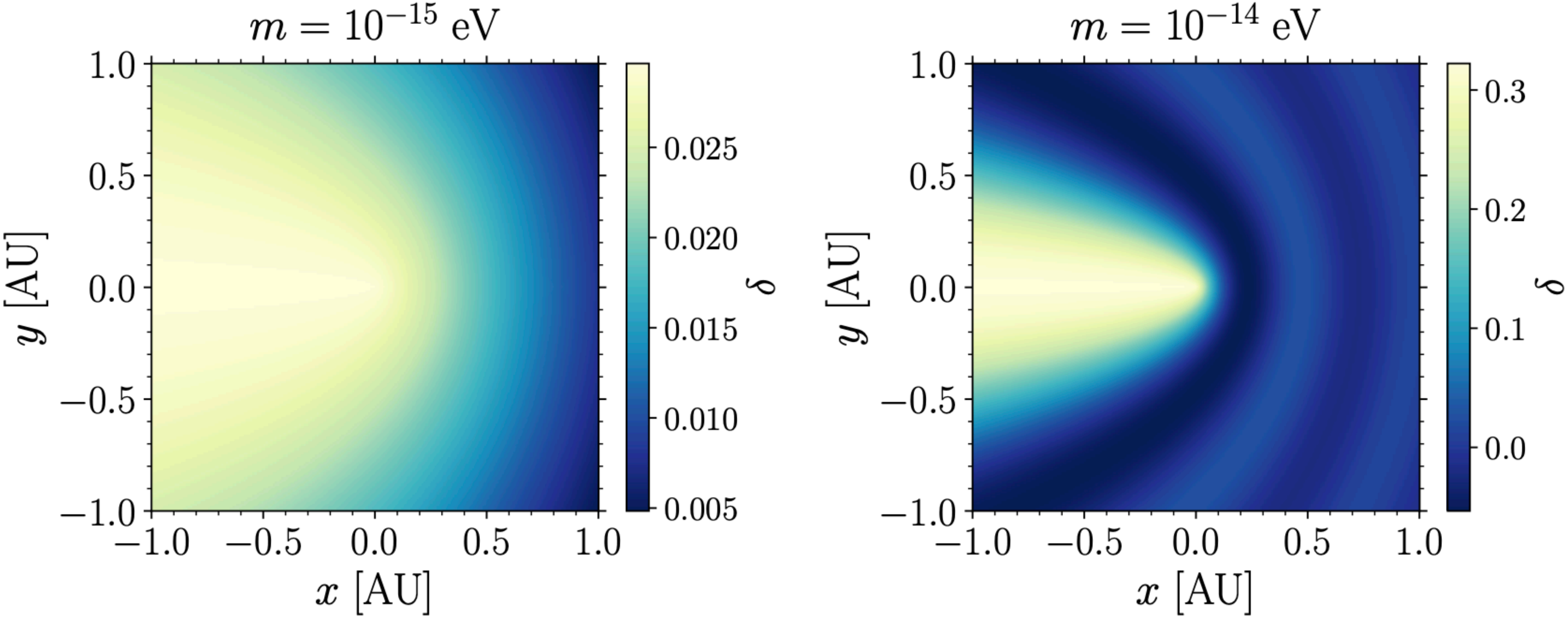
# Remarks on the solution

$$\psi_{\vec{k}}(t,\vec{x}) = e^{-iE_k t} e^{i\vec{k}\cdot\vec{x}} \psi_{\vec{k}}(0)\, M[i\alpha_G/v, 1, ikr(1-\hat{k}\cdot\hat{x})]$$

(ii) within de Broglie wavelength the wave function saturates




$$|\psi_{\vec{k}}(0)|^2 = \frac{2\pi\alpha_G/v}{1-e^{-2\pi\alpha_G/v}}$$




$$1 + \delta = \frac{\langle \phi^2 \rangle}{\phi_0^2}$$

# Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[ \hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

# Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[ \hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

*density operator* must be specified

to determine statistical properties

$$\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}\,(\hat{\rho}\hat{\mathcal{O}})$$

we take

$$\hat{\rho} = \frac{1}{1+\langle n\rangle}\sum_n \left(\frac{\langle n\rangle}{1+\langle n\rangle}\right)^n |n\rangle\langle n|$$

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- this holds for canonical ensemble $\hat{\rho} \propto e^{-\beta \hat{H}}$

$$\langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$$

# we take

$$\hat{\rho} = \frac{1}{1 + \langle n \rangle} \sum_n \left( \frac{\langle n \rangle}{1 + \langle n \rangle} \right)^n |n\rangle\langle n|$$

- this holds for canonical ensemble $\hat{\rho} \propto e^{-\beta \hat{H}}$
- this also holds for a wide range of system

(maximizes the *entropy* for a given mean occupation number)

This form is not quite convenient for our analysis

in particular when the mean occupation number is very large

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in particular when the mean occupation number is very large

We express the density matrix in terms of coherent state

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

This form is not quite convenient for our analysis

in particular when the mean occupation number is very large

We express the density matrix in terms of coherent state

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

$$\alpha \in \mathbb{C}$$

In this case

$$\hat{\rho} = \int d^2\alpha \, P(\alpha) |\alpha\rangle\langle\alpha|$$

$$P(\alpha) = \frac{1}{\pi\langle n\rangle} \exp\left[-\frac{|\alpha|^2}{\langle n\rangle}\right]$$

can be though of as *probability distribution!*

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$$\hat{\rho} = \int d^2\alpha \, P(\alpha) |\alpha\rangle\langle\alpha|$$

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modulus ~ Rayleigh distribution

phase ~ uniform distribution

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$$\hat{\rho} = \int d^2\alpha \, P(\alpha) |\alpha\rangle\langle\alpha|$$

$$P(\alpha) = \frac{1}{\pi\langle n\rangle} \exp\left[-\frac{|\alpha|^2}{\langle n\rangle}\right]$$

can be though of as *probability distribution!*

modulus ~ Rayleigh distribution

phase ~ uniform distribution

**Derevianko 16**

**Foster et al 18**

**Centers et al 19**

**Gramolin et al 21**

$$\langle a^\dagger a \rangle = \langle n \rangle$$

$$\langle a a^\dagger \rangle = \langle n \rangle + 1$$

corrections arising from commutation relation are $\mathcal{O}(1/\langle n \rangle)$

$$\langle \mathcal{O}(\hat{a}, \hat{a}^\dagger) \rangle \approx \int d^2\alpha \, P(\alpha) \mathcal{O}(\alpha, \alpha^*)$$

to the leading order in the occupation number

we may replace all operator with a complex random number

$$\hat{\phi}(t, \vec{x}) \to \phi(t, \vec{x}) = \sum_i \frac{1}{\sqrt{2mV}} \left[\alpha_i \psi_i e^{-imt} + \text{h.c.}\right]$$

arbitrary moments of fields and derivatives can be computed with

$$P(\alpha_i) = \frac{1}{\pi \langle n_i \rangle} \exp\left[-\frac{|\alpha_i|^2}{\langle n_i \rangle}\right]$$

# Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[ \hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

How would this affect signals in a detector?

Axion (and scalar) dark matter searches based on

$$\mathcal{L} \supset g\phi\mathcal{O}, \;\; g\partial_\mu\phi\mathcal{O}^\mu, \;\; \cdots$$

signals can be generally written as

$$s(t) = \xi\phi(t, \vec{x}_{\rm dec})$$

coupling constants + detector specific parameters

the power and spectrum of signal is proportional to that of field *density contrast* for wave dark matter

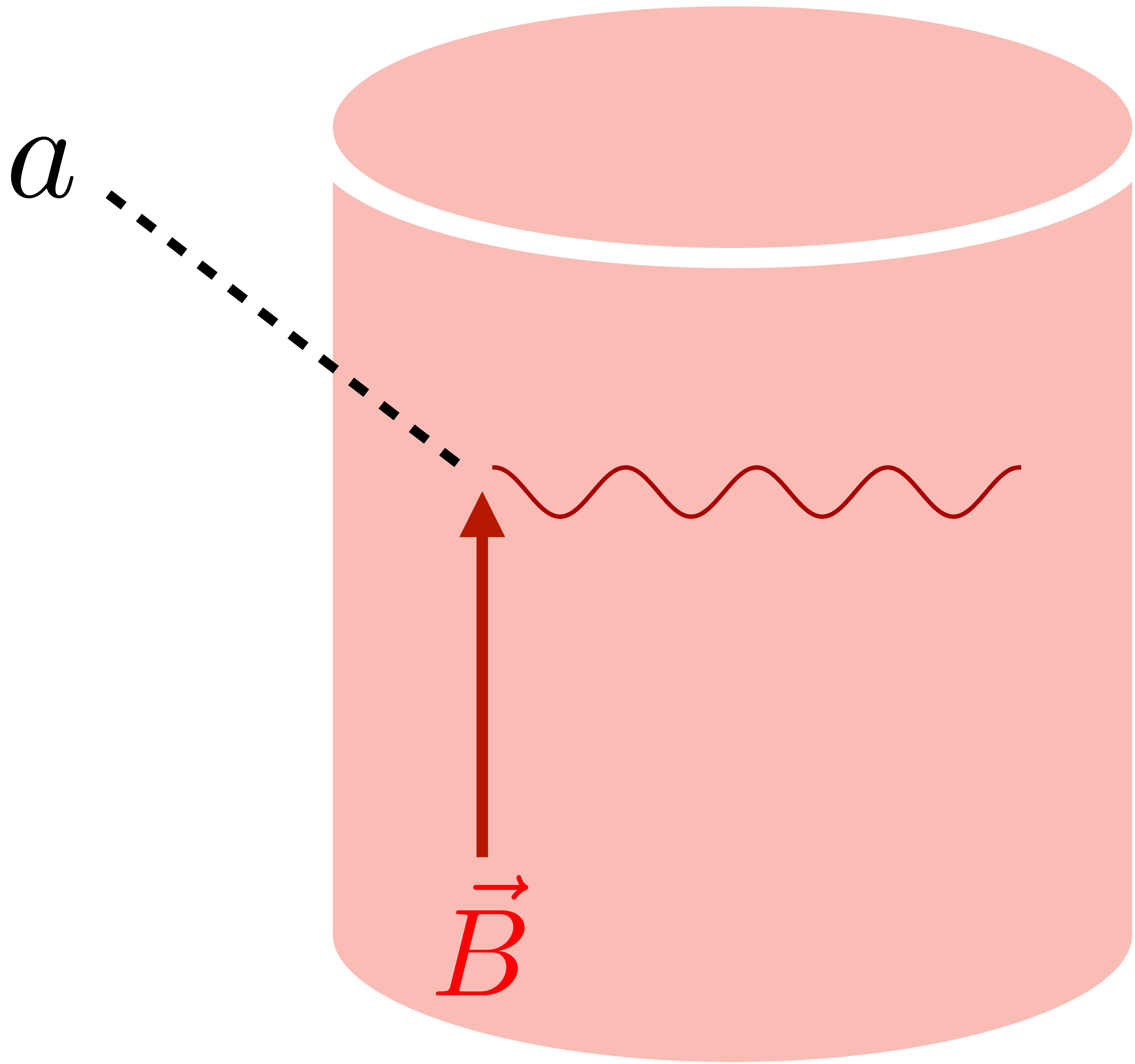
$$1 + \delta = \frac{\langle \phi^2 \rangle}{\phi_0^2} = \int d^3v \, f(\vec{v}) |\psi_{\vec{v}}|^2$$

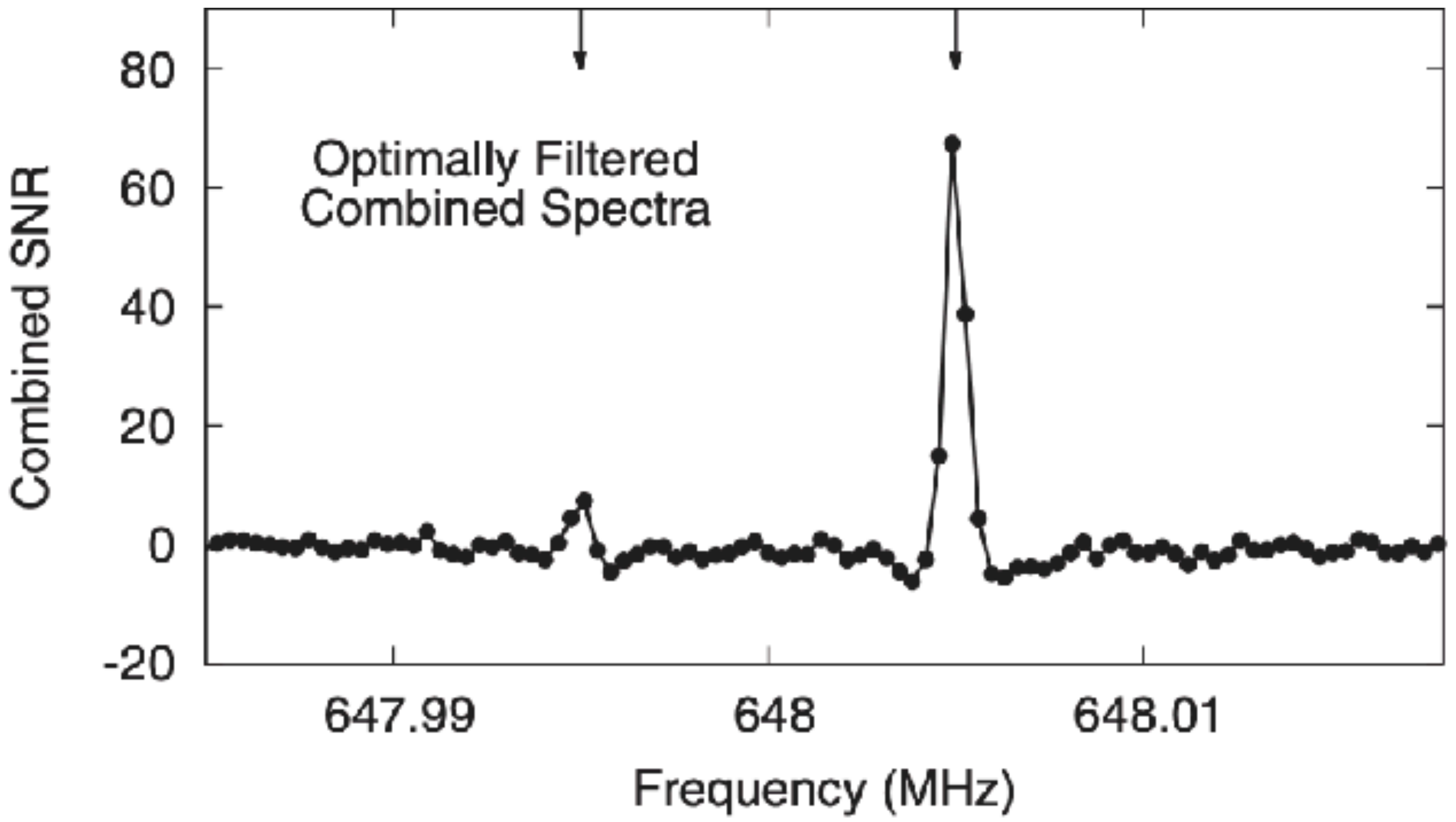
*density contrast* for wave dark matter

$$1 + \delta = \frac{\langle \phi^2 \rangle}{\phi_0^2} = \int d^3v \, f(\vec{v}) |\psi_{\vec{v}}|^2$$

*DM speed distribution*

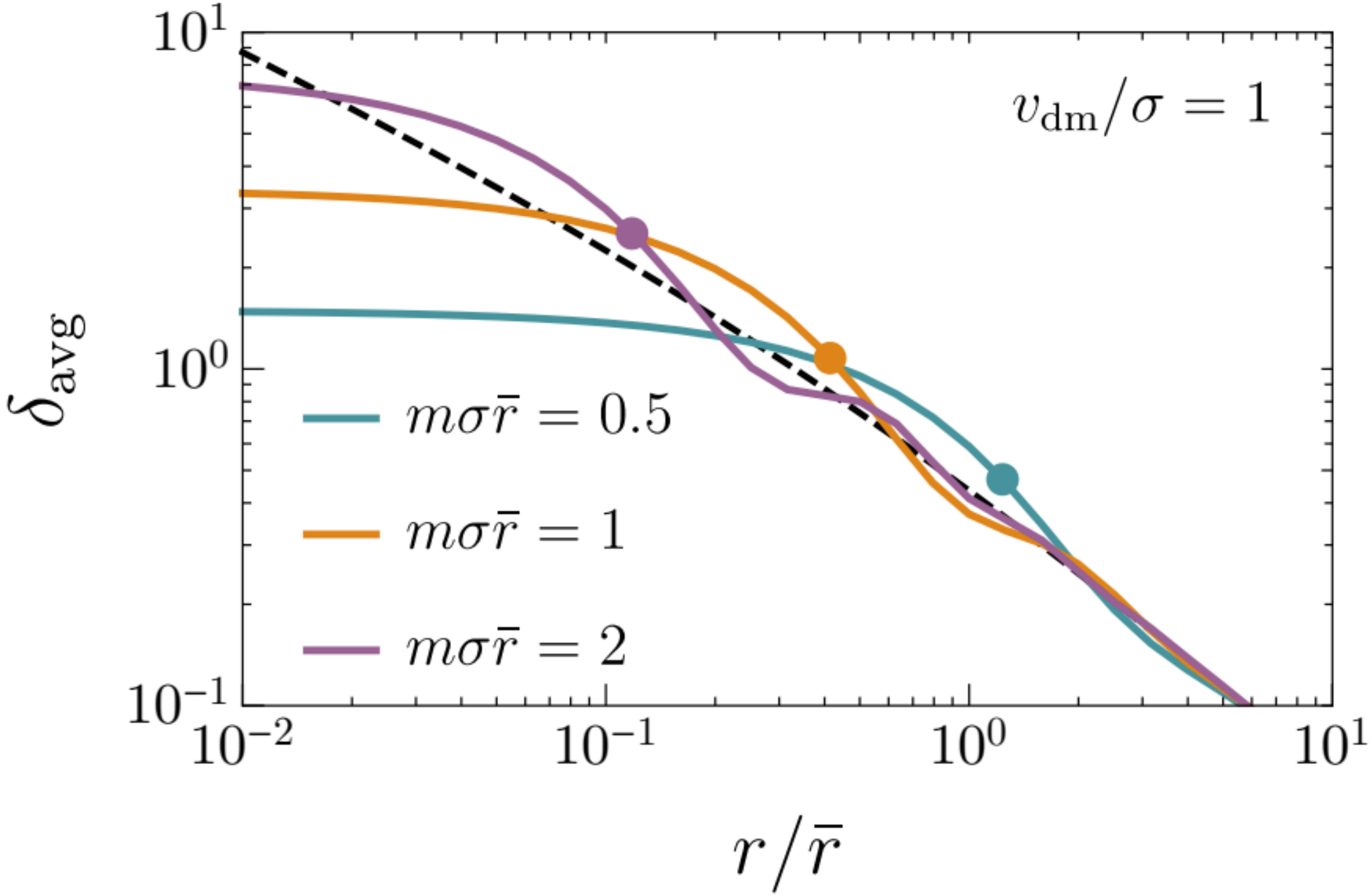
$$f(v) \to \tilde{f}(v) = v^2 \int d\Omega f(\vec{v}) |\psi_{\vec{v}}|^2$$




**ADMX Collaboration 2018**

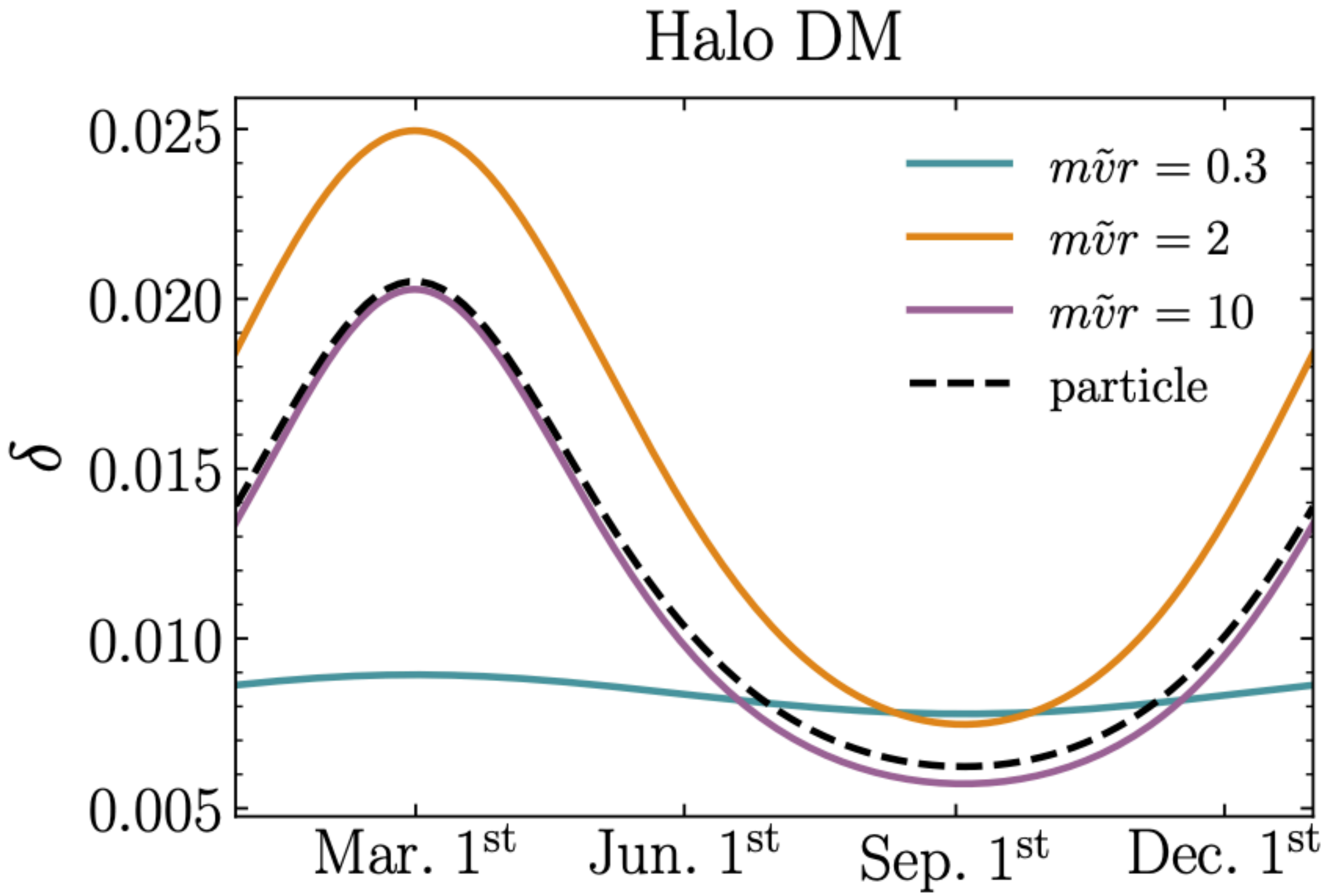
$$S(\omega) \propto \frac{\xi^2}{m} \frac{\rho_{\rm DM} f(v)}{v} \rightarrow \tilde{f}(v)$$



$v_{\rm dm}/\sigma = 1$
$m\sigma\bar{r} = 0.5$
$m\sigma\bar{r} = 1$
$m\sigma\bar{r} = 2$
$\delta_{\rm avg}$
$r/\bar{r}$
$10^{1}$
$10^{0}$
$10^{-1}$
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$

# Applications:

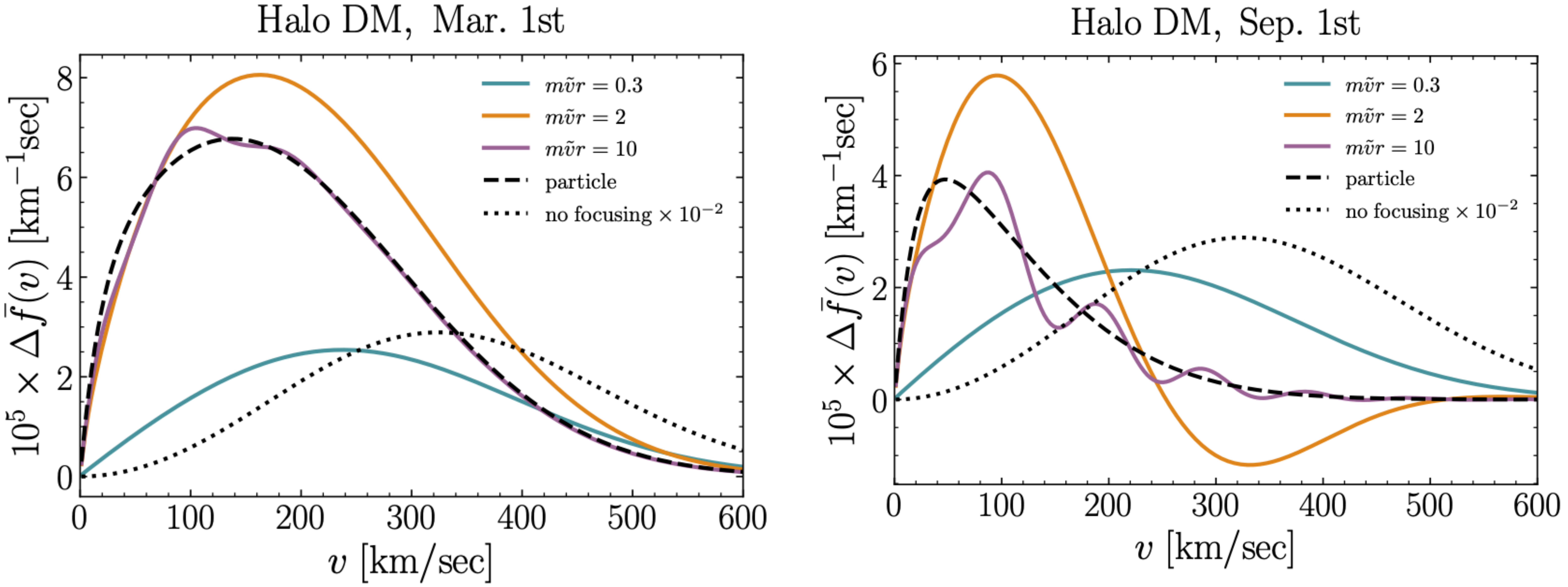
Gravitational response of local DM substructures




$$f(\vec{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\vec{v}-\vec{v}_{\rm dm})^2}{2\sigma^2}\right]$$

$$v_{\rm dm} \simeq 230\,{\rm km/sec}$$

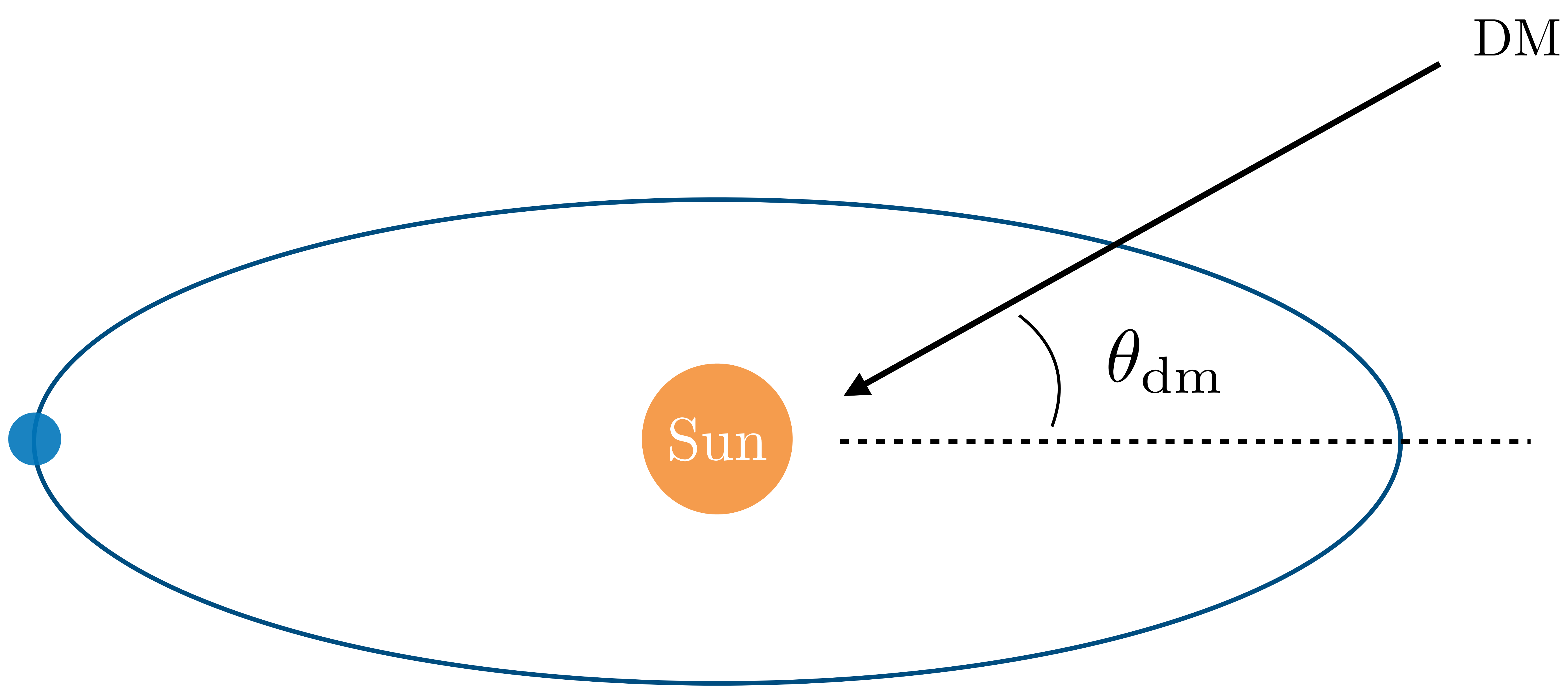
$$\sigma \simeq 160\,{\rm km/sec}$$

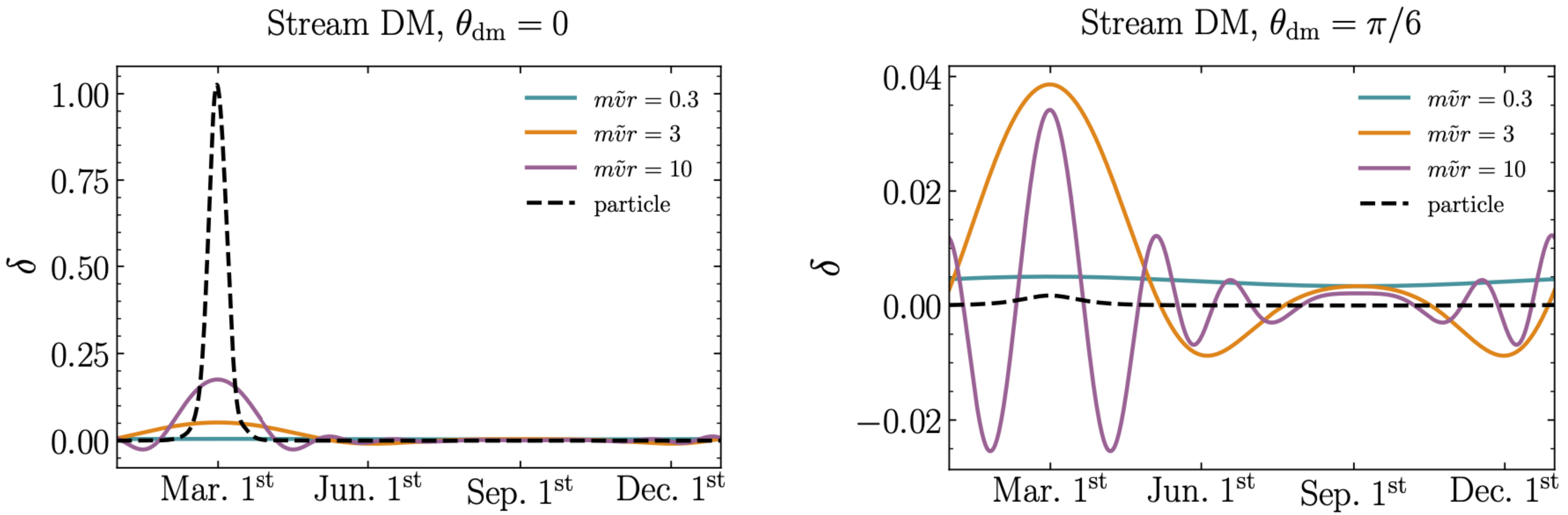


Halo DM, Mar. 1st
$10^5 \times \Delta\bar{f}(v)$ [km$^{-1}$sec]
$v$ [km/sec]
$m\tilde{v}r = 0.3$
$m\tilde{v}r = 2$
$m\tilde{v}r = 10$
particle
no focusing $\times 10^{-2}$
Halo DM, Sep. 1st
$10^5 \times \Delta\bar{f}(v)$ [km$^{-1}$sec]
$v$ [km/sec]
$m\tilde{v}r = 0.3$
$m\tilde{v}r = 2$
$m\tilde{v}r = 10$
particle
no focusing $\times 10^{-2}$

Finally we are going to consider a stream-like object

with a large velocity and small velocity dispersion






$$f(\vec{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\vec{v}-\vec{v}_{\rm dm})^2}{2\sigma^2}\right]$$

$$v_{\rm dm} = 400\,{\rm km/sec}$$

$$\sigma = 30\,{\rm km/sec}$$

# Conclusions

- Gravitational focusing / stochasticity are considered
- Model-independent; purely gravitational effects

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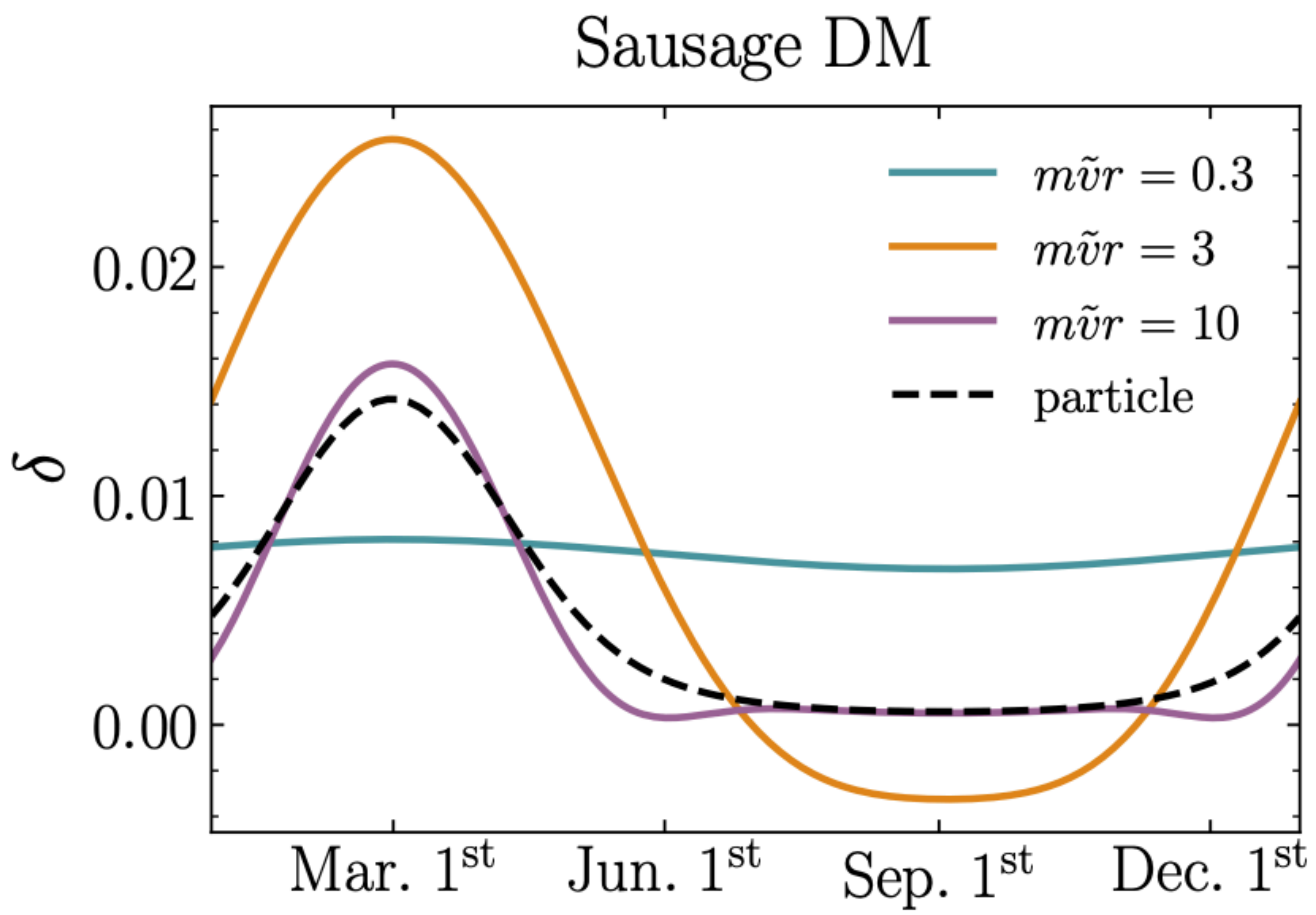
- Effects are more dramatic for local DM substructures

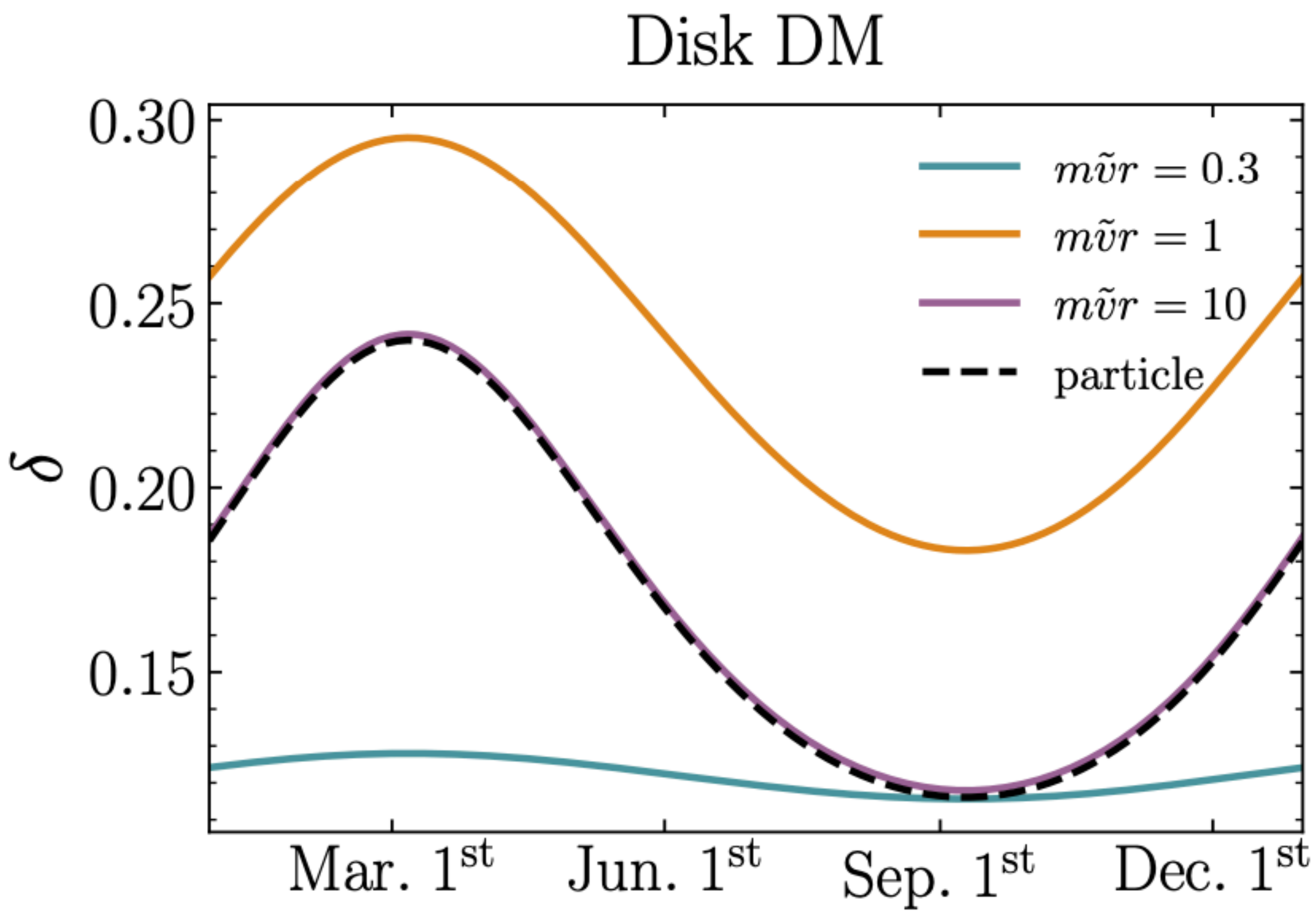
- Important for the correct modeling of local DM structure
- Important for constructing a local map of DM (upon detection)

# Back-up



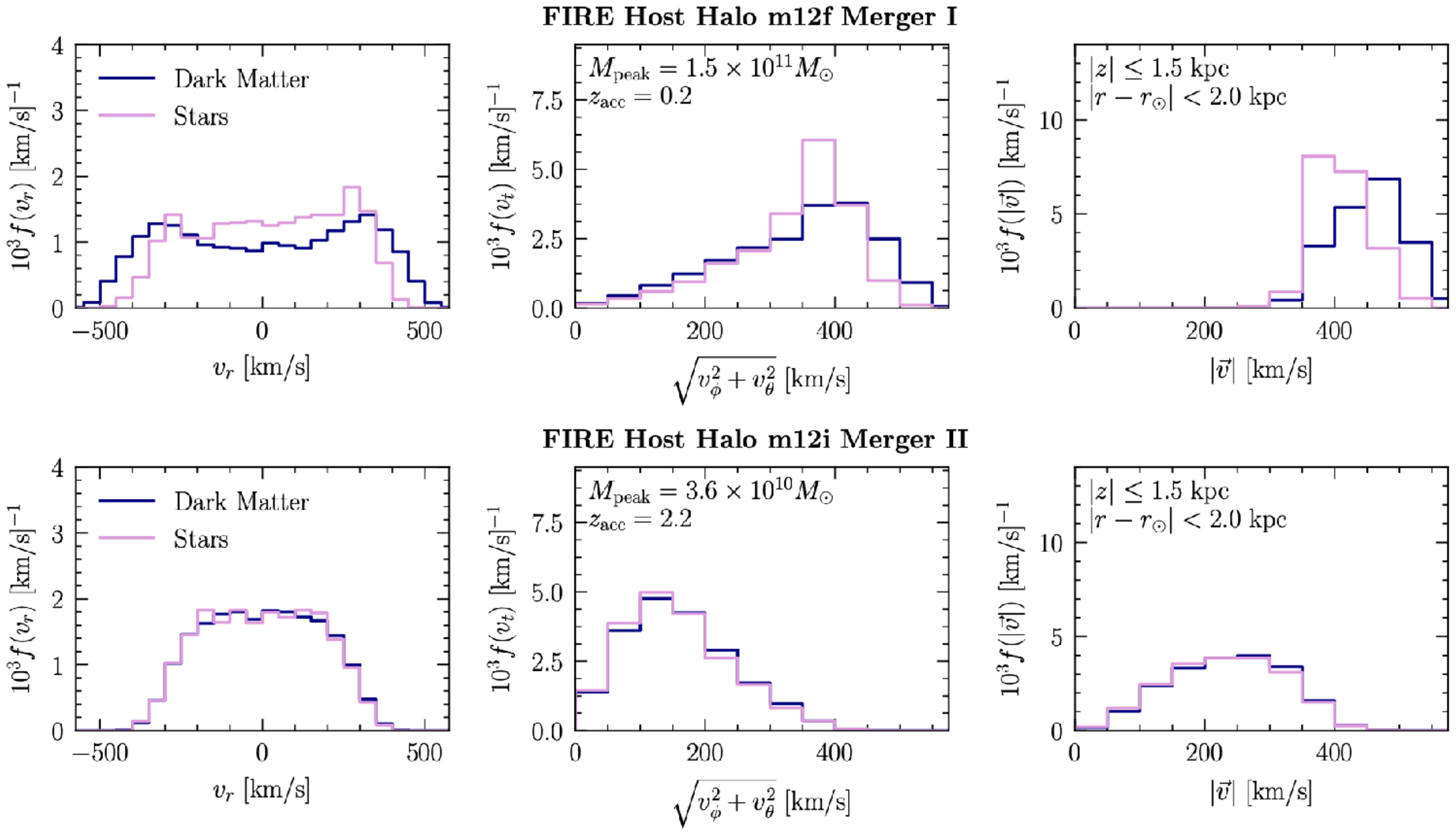

$$f(\vec{v}) = \frac{1}{(2\pi)^{3/2}\sqrt{\det \Sigma}} \exp\left[-\frac{1}{2}(\vec{v} - \vec{v}_{\rm dm}) \cdot \Sigma^{-1} \cdot (\vec{v} - \vec{v}_{\rm dm})\right]$$

$$\sigma_r = 256\,\text{km/sec}, \quad \sigma_\theta = \sigma_\phi = 81\,\text{km/sec},$$




$$f(\vec{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\vec{v}-\vec{v}_{\rm dm})^2}{2\sigma^2}\right]$$

$$v_{\rm dm} = \sigma \simeq 50\,\mathrm{km/sec}$$

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# An example from QM

consider a harmonic oscillator

$$\hat{H} = \omega a^\dagger a$$

density matrix for canonical ensemble is

$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{\mathrm{tr}(e^{-\beta \hat{H}})}$$

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$$= (1 - e^{-\beta\omega}) \sum e^{-n\beta\omega} |n\rangle\langle n|$$

$$= \frac{1}{1 + \langle n \rangle} \sum_n \left( \frac{\langle n \rangle}{\langle n \rangle + 1} \right)^n |n\rangle\langle n|$$

$$\text{with} \quad \langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$$

# Equation of motion

## For scalar field

$$(\Box + m^2)\phi = 0$$

## For potential

$$\nabla^2 \Phi = 4\pi G(\rho_{\rm ext} + \rho_\phi)$$

# Assumption / approximation

*(i) self-gravity due to DM can be ignored*

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$$\Phi \approx -GM/r$$

*(iii) DM is non-relativistic*

$$\phi(t,x) \to \psi(t,x)e^{-imt} + {\rm h.c.}$$