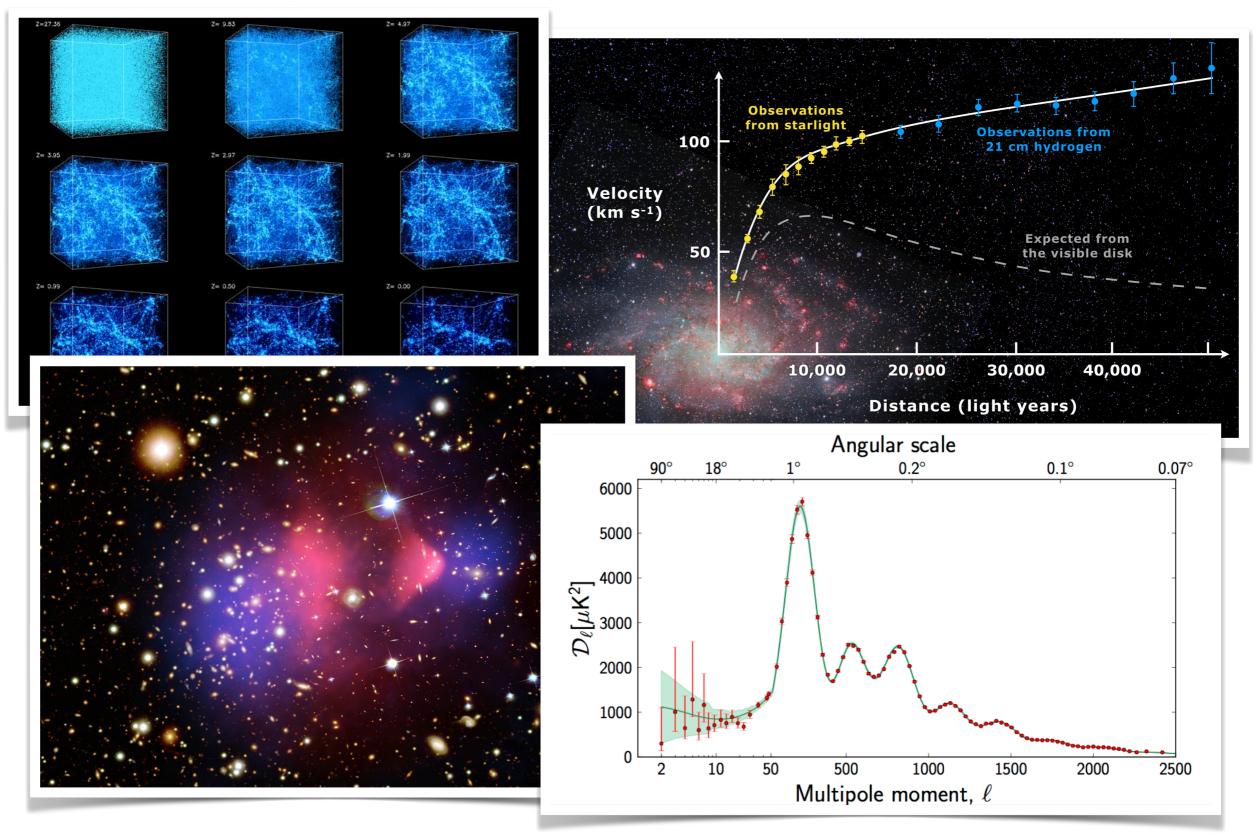
Thermal Misalignment of Scalar Dark Matter

Brian Batell University of Pittsburgh with Akshay Ghalsasi, arXiv:2109.04476

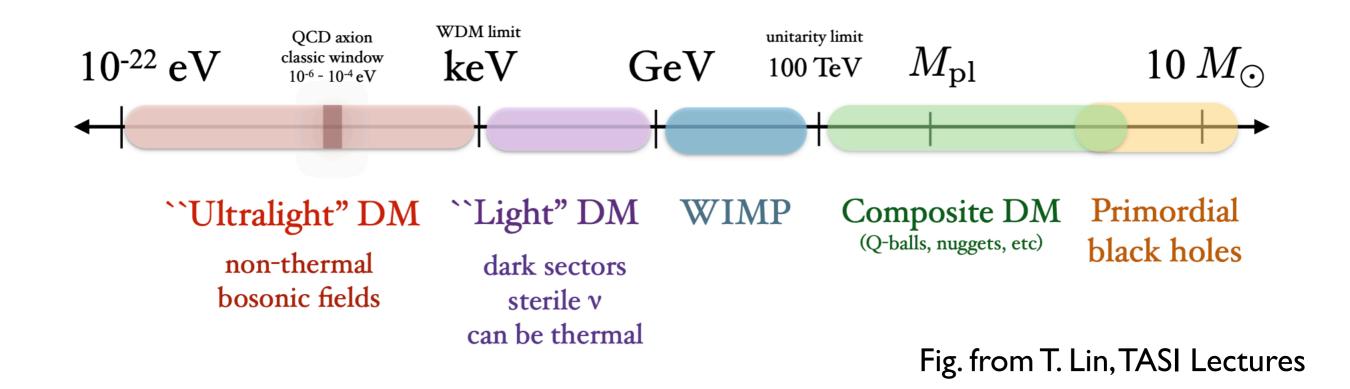


2022 CAU BSM Workshop February 10, 2022

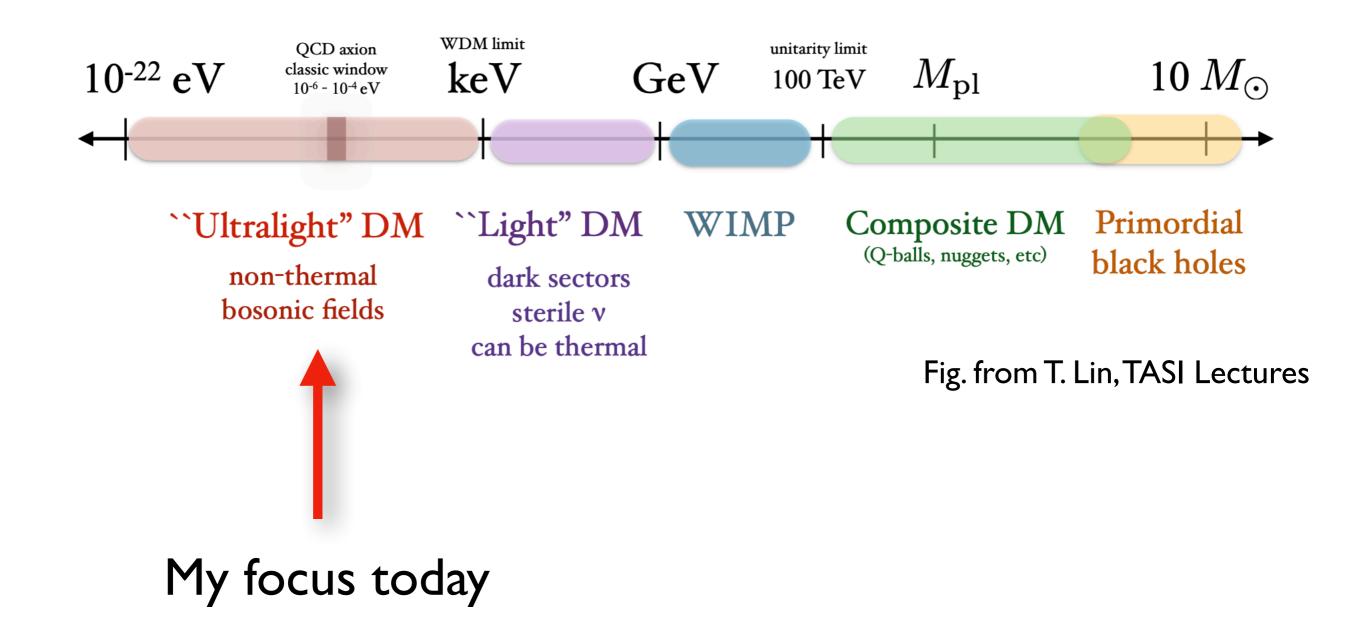
Dark Matter



Dark Matter



Dark Matter



Motivation and Plan

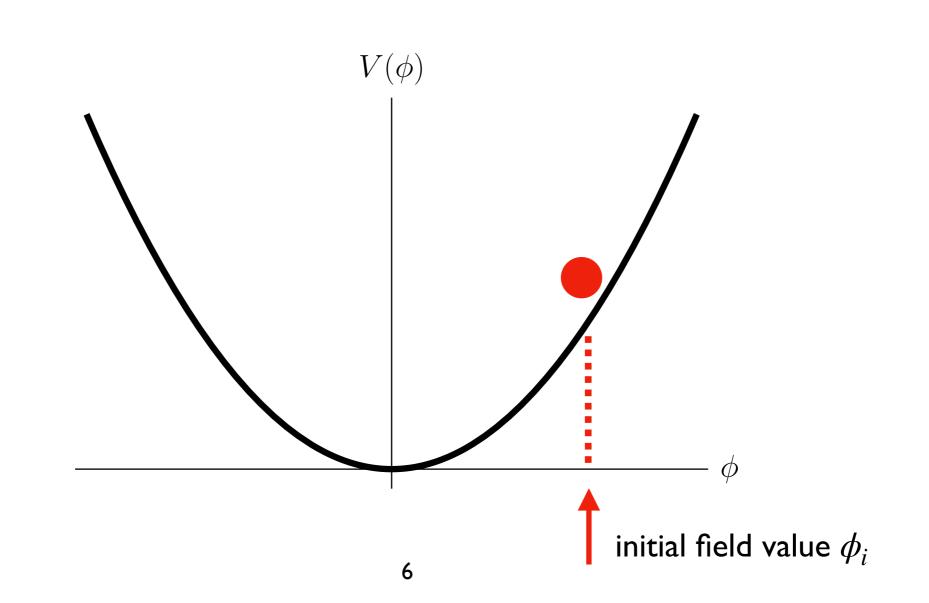
- The misalignment mechanism is a generic dark matter production mechanism for such ultralight dark matter
 - Sensitivity to initial conditions?
 - Connection with short distance particle physics parameters?
- I will describe a new mechanism *thermal misalignment* to dynamically generate large misalignment from generic initial conditions.
- The mechanism relies on a coupling of the scalar dark matter to a fermion in thermal equilibrium and the resulting finite temperature potential
- The phenomenology of a realistic scenario where DM couples to the muon will be discussed

See also talk by Eung Jin Chun for related ideas

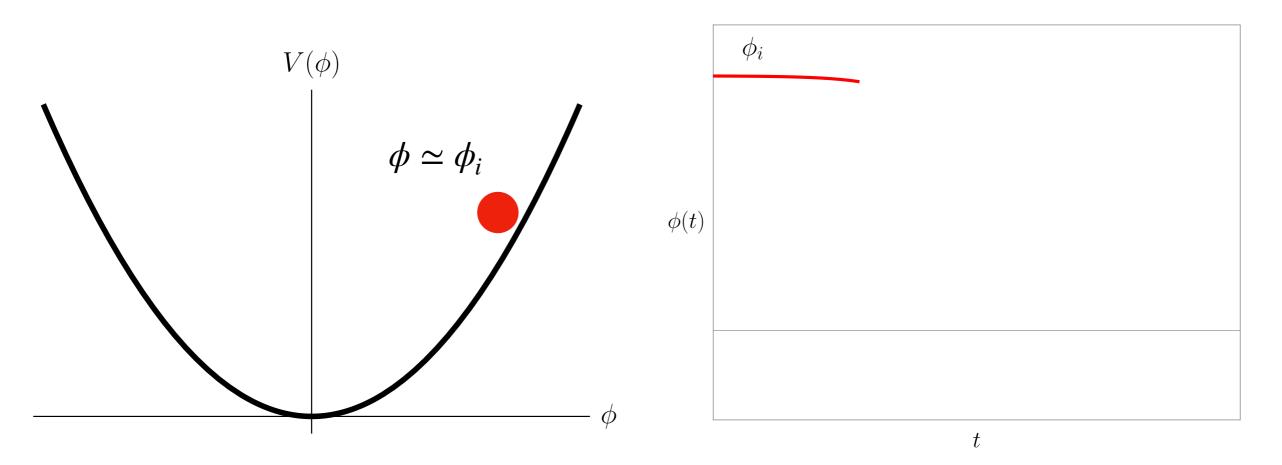
- Consider the dynamics of massive scalar field ϕ in the early universe
- Starting from some initial scalar field value ϕ_i at early times the scalar evolves according to the equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + m_{\phi}^2\phi = 0$$

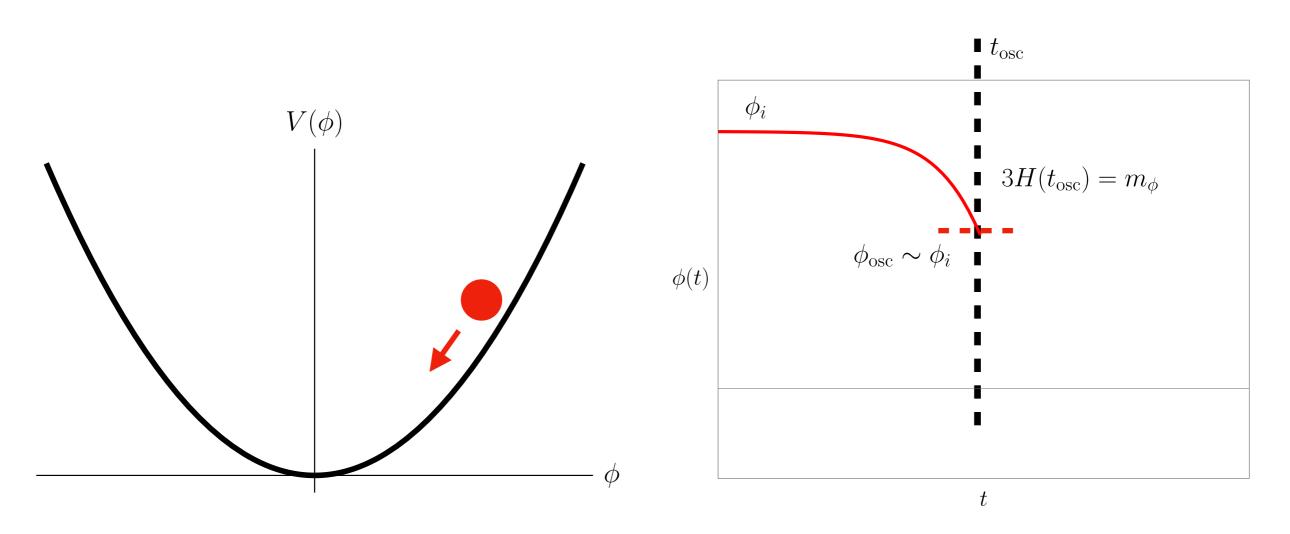
[Preskill, Wise, Wilczek; Abbott, Sikivie; Dine, Fischler, '83]



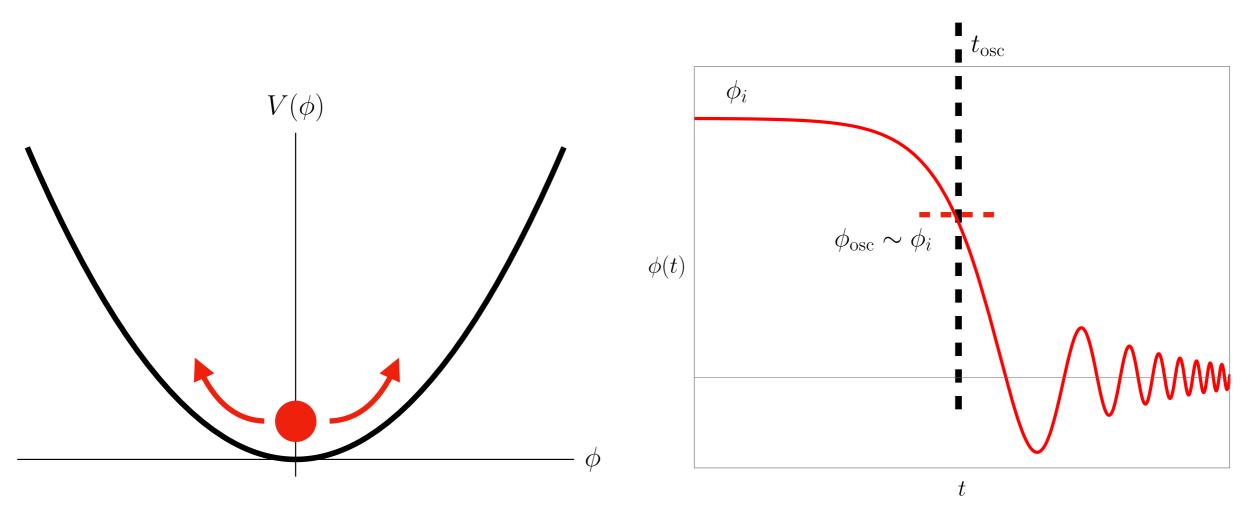
• At early times (high temperature) the scalar is held up by Hubble friction and remains approximately at its initial field value.



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- As the universe cools, the Hubble eventually drops below the scalar mass. This signals the onset of scalar oscillations.

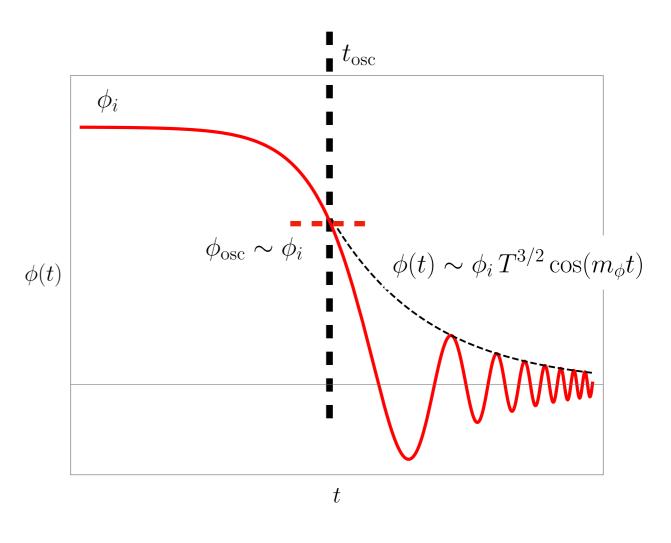


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- The energy density stored in the scalar field redshifts like matter

$$\rho_{\phi} = \frac{1}{2} m_{\phi} \langle \phi^2(t) \rangle \sim a(t)^{-3} \sim t^{-3/2} \sim T^3$$

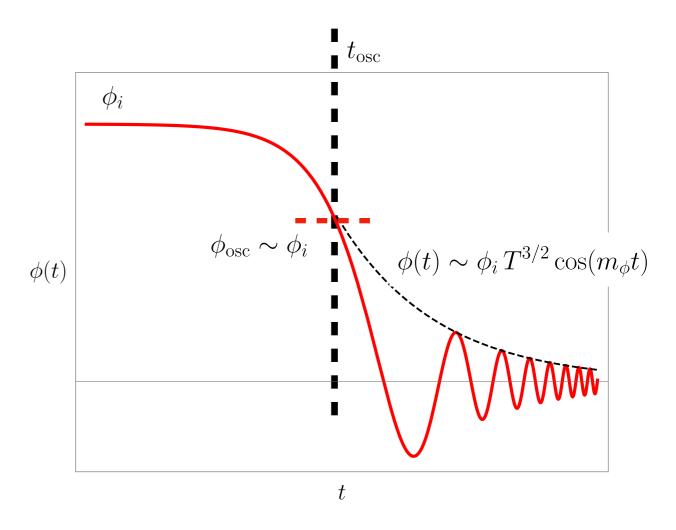


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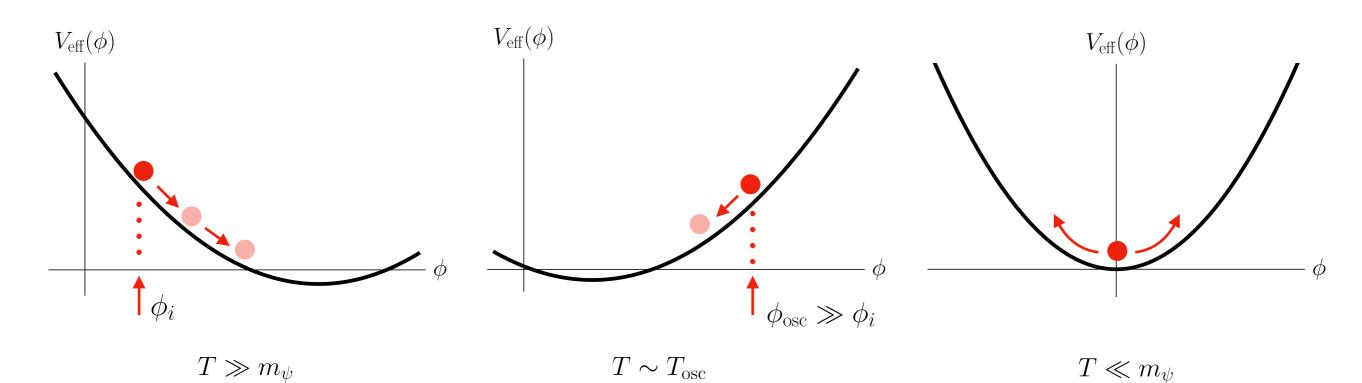
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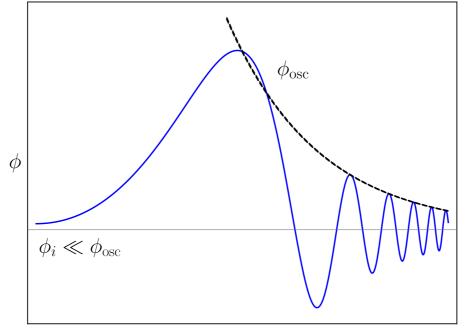
• The initial value of the field sets the amplitude of oscillations and controls the late time relic abundance

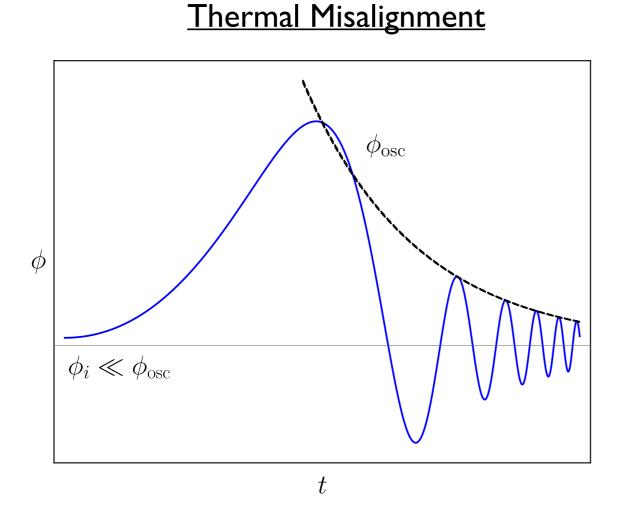
$$\Omega_{\phi}\big|_{0} = \frac{\rho_{\phi,0}}{\rho_{c,0}} \simeq \frac{\frac{1}{2}m_{\phi}^{2}\phi_{\rm osc}^{2}(T_{0}/T_{\rm osc})^{3}(g_{*S}^{0}/g_{*S}^{\rm osc})}{\rho_{c,0}}$$
$$\approx 0.2\left(\frac{m_{\phi}}{10^{-11}\,{\rm eV}}\right)^{1/2}\left(\frac{\phi_{i}/M_{\rm pl}}{10^{-4}}\right)^{2}$$



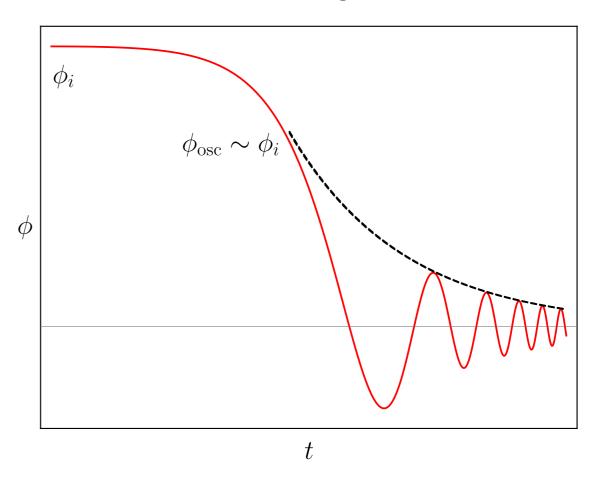
[BB,Ghalsasi, 2109.04476]





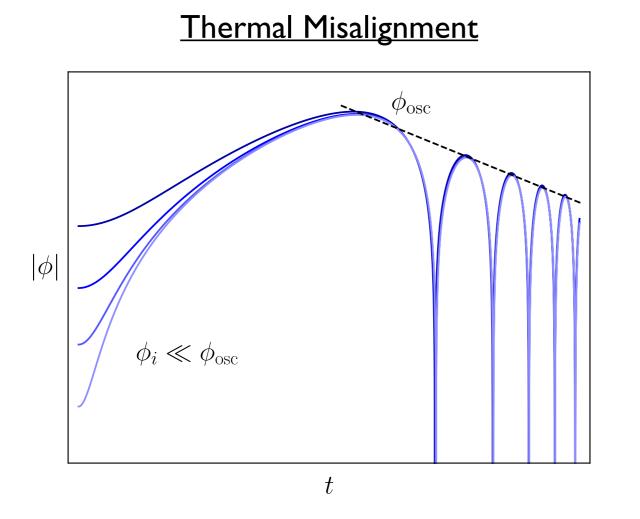


• At high temperatures, ϕ is dynamically misaligned from a small initial value to its oscillation amplitude



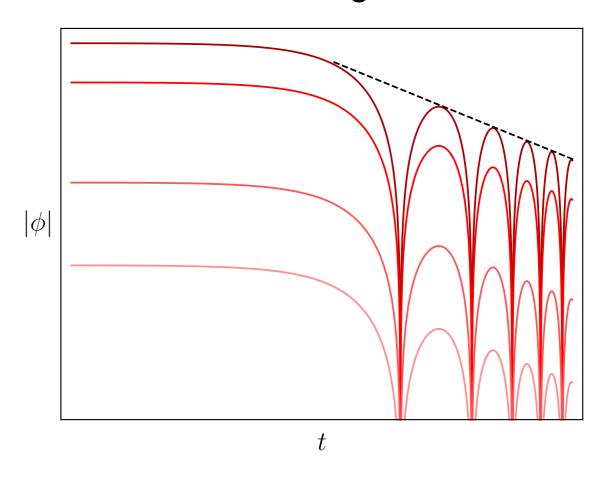
Classic Misalignment

- ϕ oscillation amplitude and abundance dictated by initial conditions

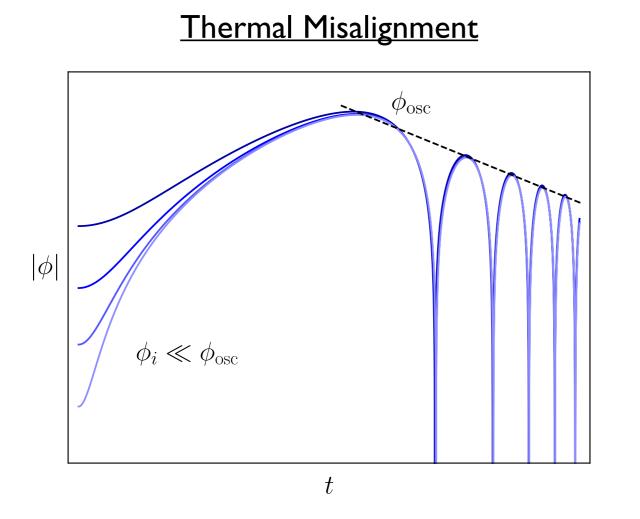


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- The oscillation amplitude is an attractor for $\phi_i \ll \phi_{\rm osc}\,$ insensitive to initial conditions

Classic Misalignment

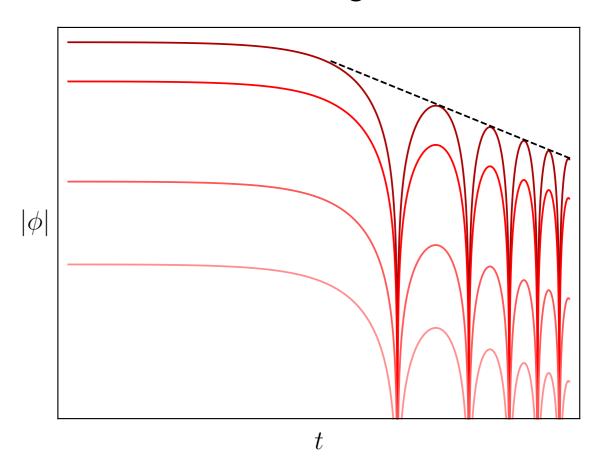


- ϕ oscillation amplitude and abundance dictated by initial conditions



- At high temperatures, ϕ is dynamically misaligned from a small initial value to its oscillation amplitude
- The oscillation amplitude is an attractor for $\phi_i \ll \phi_{\rm osc}\,$ insensitive to initial conditions
- The oscillation amplitude and resulting abundance is dictated by microscopic particle physics

Classic Misalignment



- ϕ oscillation amplitude and abundance dictated by initial conditions

Basic model

- Consider a toy model with a scalar DM ϕ and a Dirac fermion ψ .
- The fermion is in thermal equilibrium with the plasma in the early universe
- The DM couples to the fermion through a Yukawa interaction. The Lagrangian is

$$\begin{split} -\mathcal{L} \supset \frac{1}{2} m_{\phi}^{2} \phi^{2} + m_{\psi} \,\overline{\psi} \,\psi + \lambda \,\phi \,\overline{\psi} \,\psi \\ &= \frac{1}{2} m_{\phi}^{2} \phi^{2} + m_{\psi} \,\left(1 - \frac{\beta \,\phi}{M_{\rm pl}}\right) \overline{\psi} \,\psi \end{split} \qquad \lambda \equiv -\beta \frac{m_{\psi}}{M_{\rm pl}} \end{split}$$

• We will be interested in couplings β that are within a few orders of magnitude of unity. This means the Yukawa coupling is very feeble.

• Effective fermion mass in the scalar field background:
$$m_\psi(\phi) = m_\psi + \lambda \, \phi = m_\psi \left(1 - rac{\beta \, \phi}{M_{
m pl}}\right)$$

• Fermion free energy density depends on scalar field background, giving rise to scalar effective potential

Effective potential

• To investigate the cosmological evolution of ϕ , we require the effective potential:

$$V_{\rm eff}(\phi, T) = V_0(\phi) + V_1^0(\phi) + V_1^T(\phi)$$

- There are three contributions:
 - Tree level potential

$$V_0(\phi)=rac{1}{2}\,m_\phi^2\phi^2$$

 Coleman-Weinberg (zero temperature)

$$V_1^0(\phi) = -g_\psi \frac{[m_\psi^2(\phi)]^2}{64\pi^2} \left[\log\left(\frac{m_\psi^2(\phi)}{\mu^2}\right) - \frac{3}{2} \right]$$
 (MS scheme)

$$\begin{split} V_1^T(\phi) &= -\frac{g_{\psi}}{2\pi^2} T^4 J_F \left[\frac{m_{\psi}^2(\phi)}{T^2} \right] \\ \text{Integral:} \quad J_F(w^2) &= \int_0^\infty dx \, x^2 \, \log[\,1 + e^{-\sqrt{x^2 + w^2}}\,] \end{split}$$

- For now, we neglect the influence of the Coleman-Weinberg potential
 - This implies a fine-tuning when the scalar is very light (usual hierarchy problem)
 - As we will see, much of the cosmologically motivated parameter space is not tuned

$$V_{\text{eff}}(\phi, T) \simeq V_{0}(\phi) + V_{1}^{T}(\phi) \longrightarrow V_{0}(\phi) = \frac{1}{2} m_{\phi}^{2} \phi^{2}$$

$$V_{1}^{T}(\phi) = -\frac{g_{\psi}}{2\pi^{2}} T^{4} \int_{0}^{\infty} dx \, x^{2} \log \left[1 + \exp\left(-\sqrt{x^{2} + \frac{m_{\psi}(\phi)^{2}}{T^{2}}}\right) \right]$$

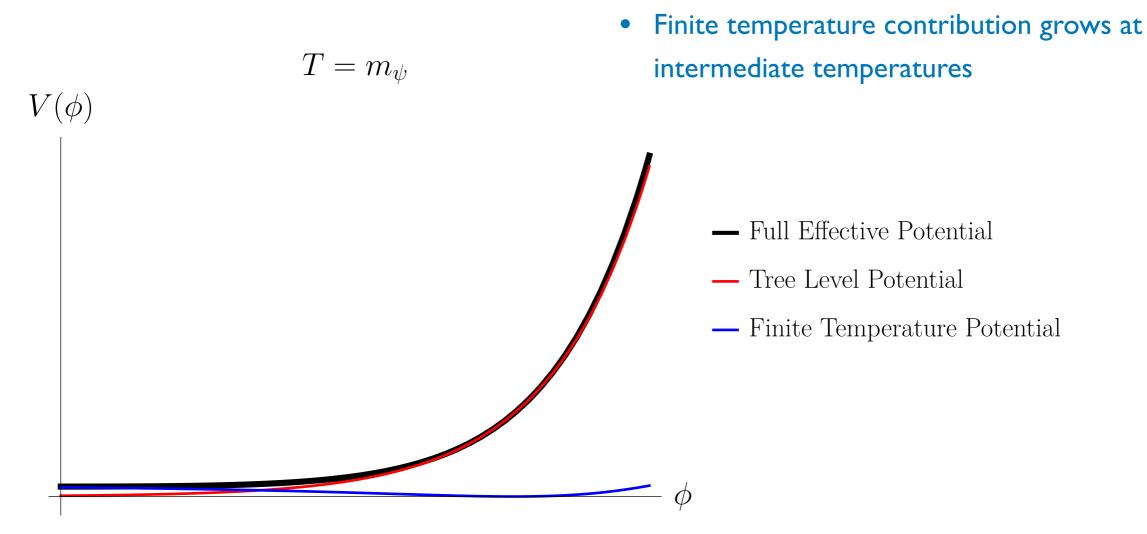
$$\sim e^{-m_{\psi}/T} \quad \text{for} \quad T \ll m_{\psi} \qquad \begin{array}{c} \text{Boltzmann} \\ \text{suppressed} \end{array}$$

$$T = 0.1 \, m_{\psi} \qquad \bullet \quad \text{Tree level dominates at low temperature} \\ \bullet \quad \text{Finite temperature contribution suppressed} \end{aligned}$$

$$V(\phi) \qquad - \quad \text{Full Effective Potential} \\ - \quad \text{Tree Level Potential} \\ - \quad \text{Finite Temperature Potential} \\ - \quad \text{Finite Temperature Potential}$$

$$V_{\text{eff}}(\phi, T) \simeq V_0(\phi) + V_1^T(\phi) \longrightarrow V_0(\phi) = \frac{1}{2} m_{\phi}^2 \phi^2$$
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Intermediate temperature



$$V_{\text{eff}}(\phi, T) \simeq V_{0}(\phi) + V_{1}^{T}(\phi) \longrightarrow V_{0}(\phi) = \frac{1}{2} m_{\phi}^{2} \phi^{2}$$

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$$\xrightarrow{T^{2} m_{\psi}^{2}} \left(1 - \frac{\beta \phi}{M_{\text{pl}}} \right)^{2} \quad \text{for} \quad T \gg m_{\psi}$$

$$\longrightarrow \text{Minimum at } \phi \sim M_{\text{pl}} / \beta$$

$$V(\phi) \longrightarrow \text{Finite temperature piece dominates at high temperatures}$$

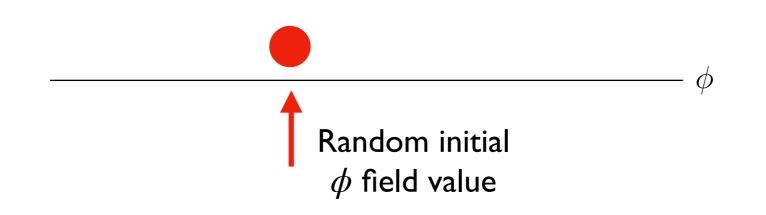
$$\bullet \text{Potential minimum located at large scalar field values}$$

$$- \text{Full Effective Potential}$$

$$- \text{Finite Temperature Potential}$$

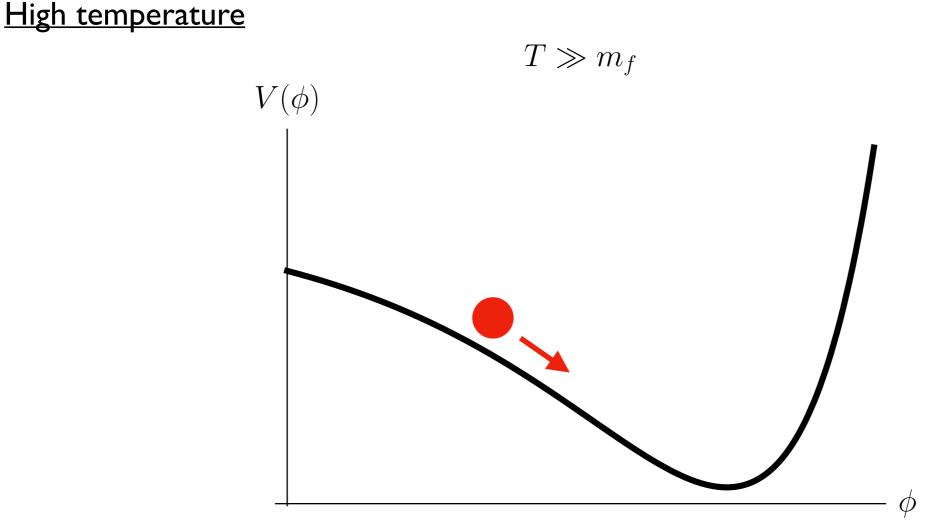
$$- \text{Finite Temperature Potential}$$

- Assume nonzero homogeneous scalar field with arbitrary^{*} initial condition after inflation
- In the case of a Standard Model fermion, initial condition set after electroweak phase transition

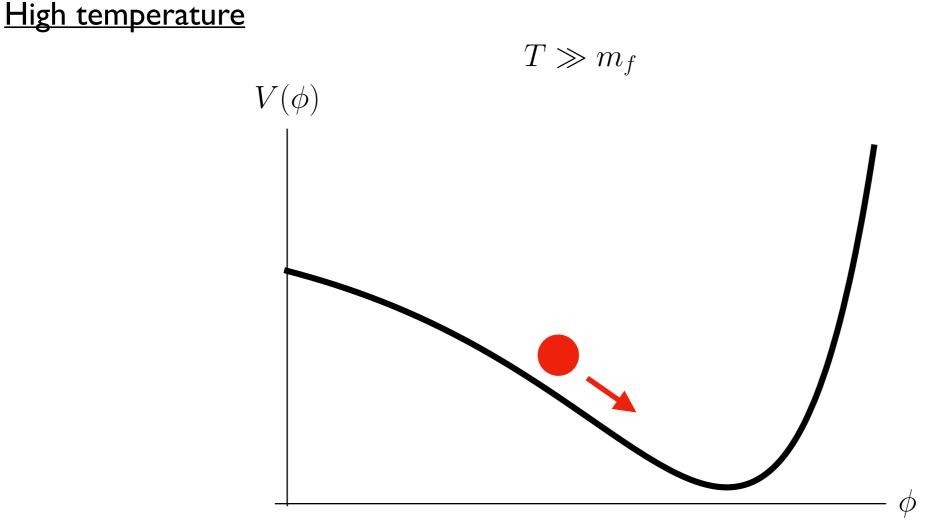


* Caveats will be discussed below

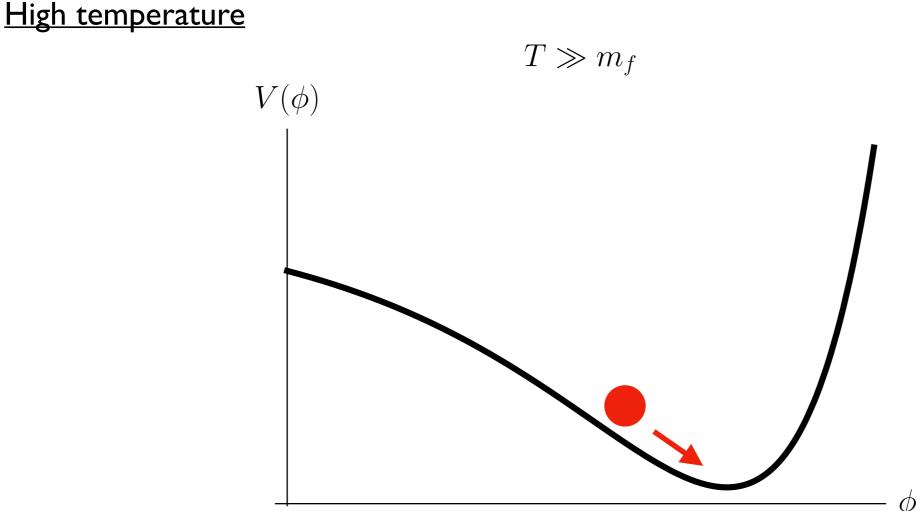
- Inflation ends and reheating occurs, creating the thermal plasma.
- The finite temperature potential dominates at this stage.
- ϕ rolls toward the minimum at large field values, generating misalignment



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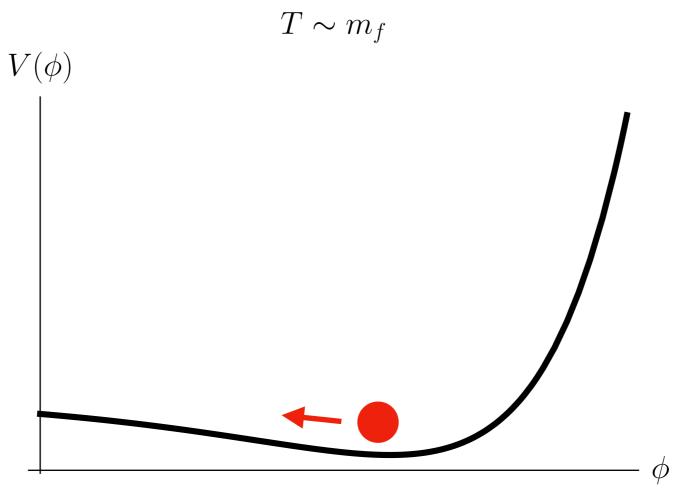


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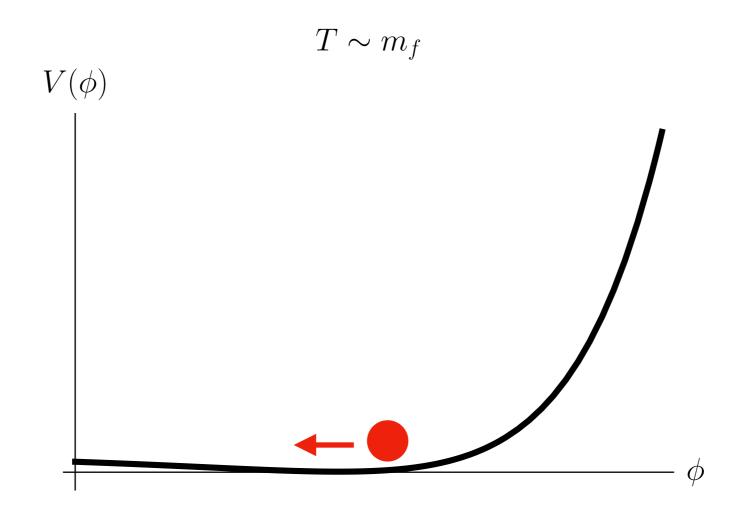


- At intermediate temperatures of order the fermion mass, the finite temperature pieces becomes smaller.
- The minimum moves toward the origin

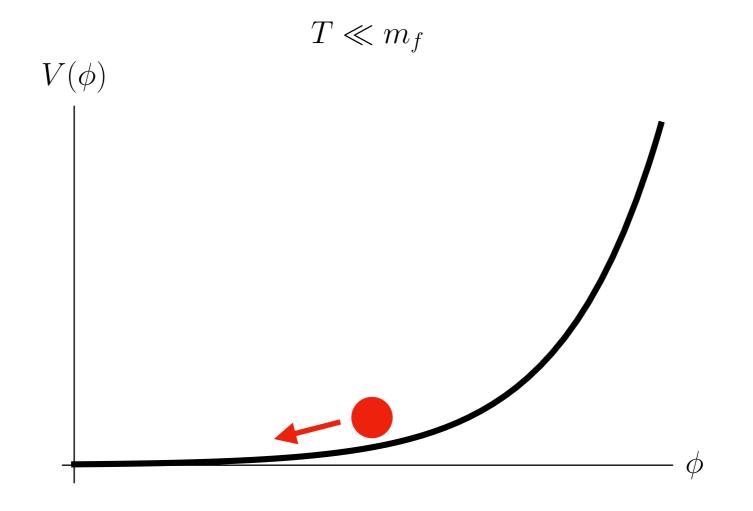
Intermediate temperature



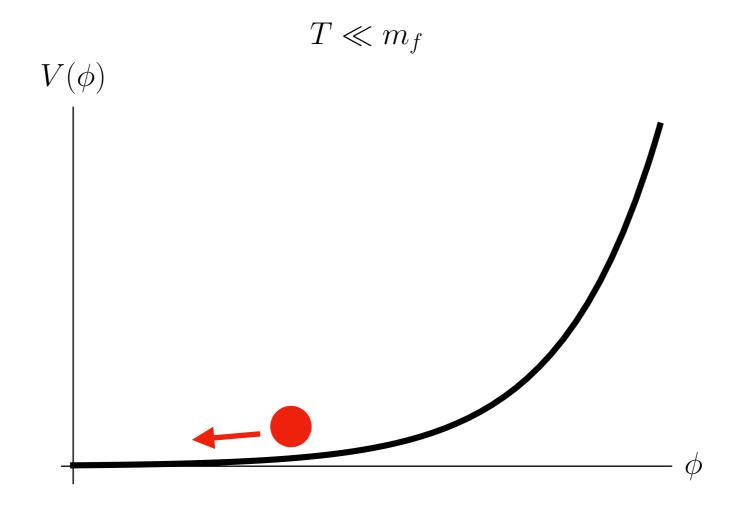
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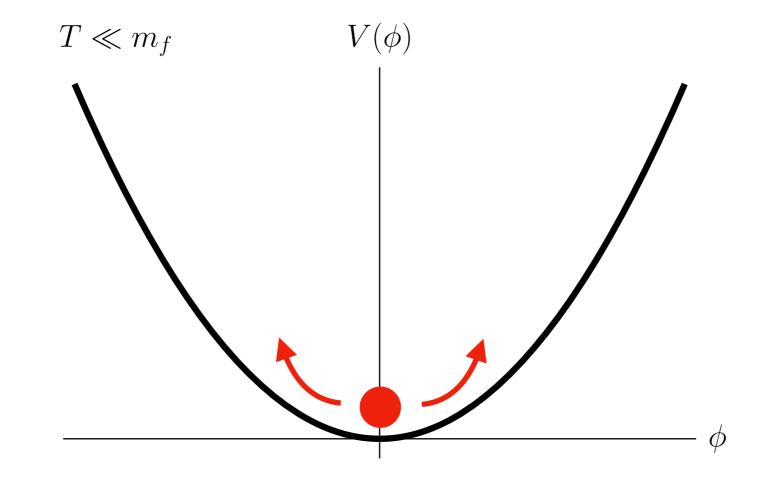
- At low temperatures the tree level potential dominates
- The minimum is located at the origin
- Eventually ϕ oscillates and behaves as dark matter



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Evolution of scalar dark matter

• Equation of motion for ϕ : $\ddot{\phi} + 3H\dot{\phi} + \frac{dV_{\text{eff}}}{d\phi} = 0$

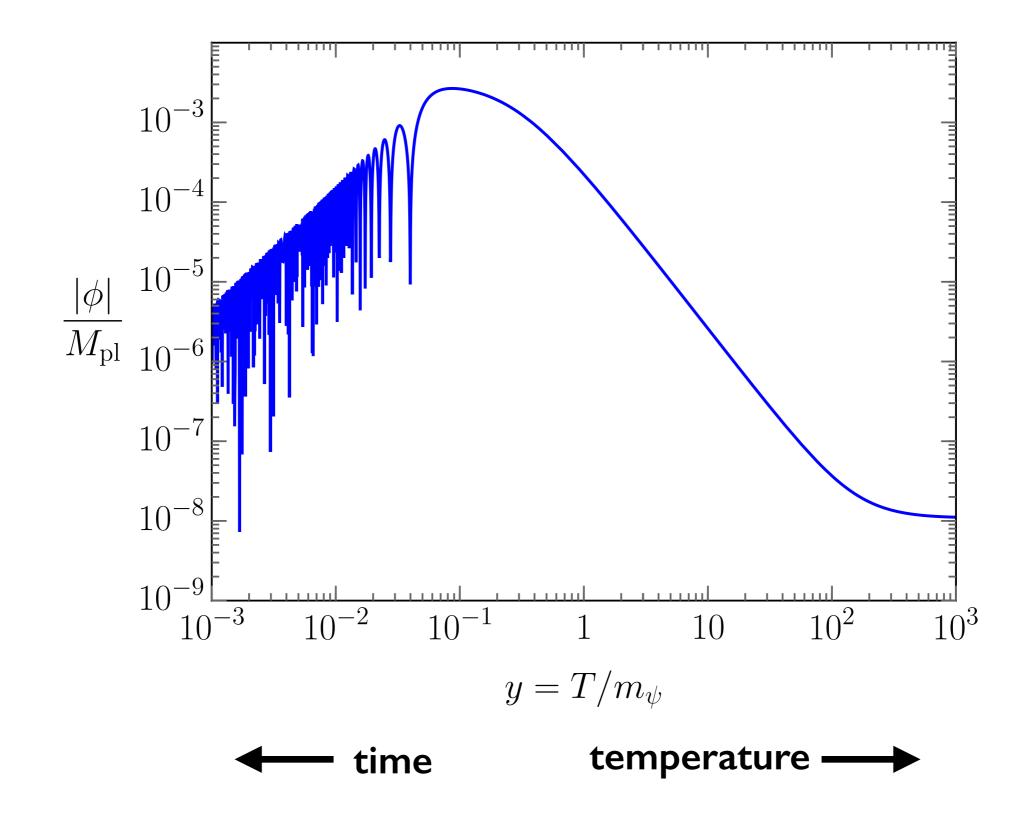
•
$$V_{\text{eff}}$$
 depends on $\frac{m_{\psi}(\phi)}{T} = \frac{m_{\psi}}{T} \left(1 - \frac{\beta \phi}{M_{\text{pl}}} \right)$
• Change dependent variable to $y \equiv \frac{T}{m_{\psi}}$ (proxy for time or temperature)
• Introduce dimensionless parameter $\kappa \equiv \frac{m_{\phi}M_{\text{pl}}}{m_{\psi}^2}$ (proxy for the DM mass)
 $\frac{d^2\phi}{dt^2} = \kappa^2 \left[1 - \frac{q_{\psi}}{T} \frac{y^2 M_{\text{pl}}}{T} \left(1 - \frac{\beta \phi}{T} \right) \int_{0}^{\infty} \frac{x^2}{T} \right]$

• Equation of motion: $\frac{d^{2}\phi}{dy^{2}} + \frac{\kappa^{2}}{\gamma^{2}y^{6}} \left[\phi - \frac{g_{\psi}}{2\pi^{2}} \frac{y^{2}M_{\text{pl}}}{\kappa^{2}} \beta \left(1 - \frac{\beta \phi}{M_{\text{pl}}} \right) \int_{0}^{\infty} dx \frac{x^{2}}{(1+e^{\xi})\xi} \right] = 0$

Equation of motion depends only on model parameters β and κ

 $\xi = \sqrt{x^2 + \frac{1}{y^2} \left(1 - \frac{\beta \, \phi}{M_{\rm pl}}\right)^2}$

• Given the model parameters and initial conditions, we can solve the equation of motion to determine $\phi(y)$



Effective potential analysis

• Effective potential at high temperatures:

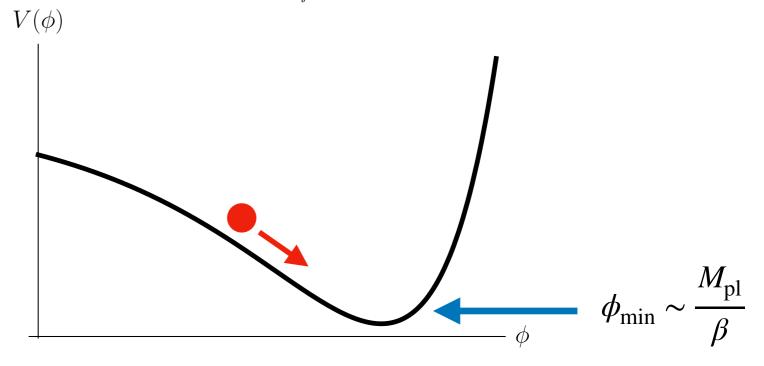
$$V_{\rm eff} \simeq \frac{1}{2} m_{\phi}^2 \phi^2 + \frac{T^2 m_{\psi}^2}{12} \left(1 - \frac{\beta \, \phi}{M_{\rm pl}} \right)^2 = m_{\psi}^4 \left\{ \frac{1}{2} \kappa^2 \left(\frac{\phi}{M_{\rm pl}} \right)^2 + \frac{y^2}{12} \left[1 - \beta \left(\frac{\phi}{M_{\rm pl}} \right) \right]^2 \right\}$$

• Minimize potential

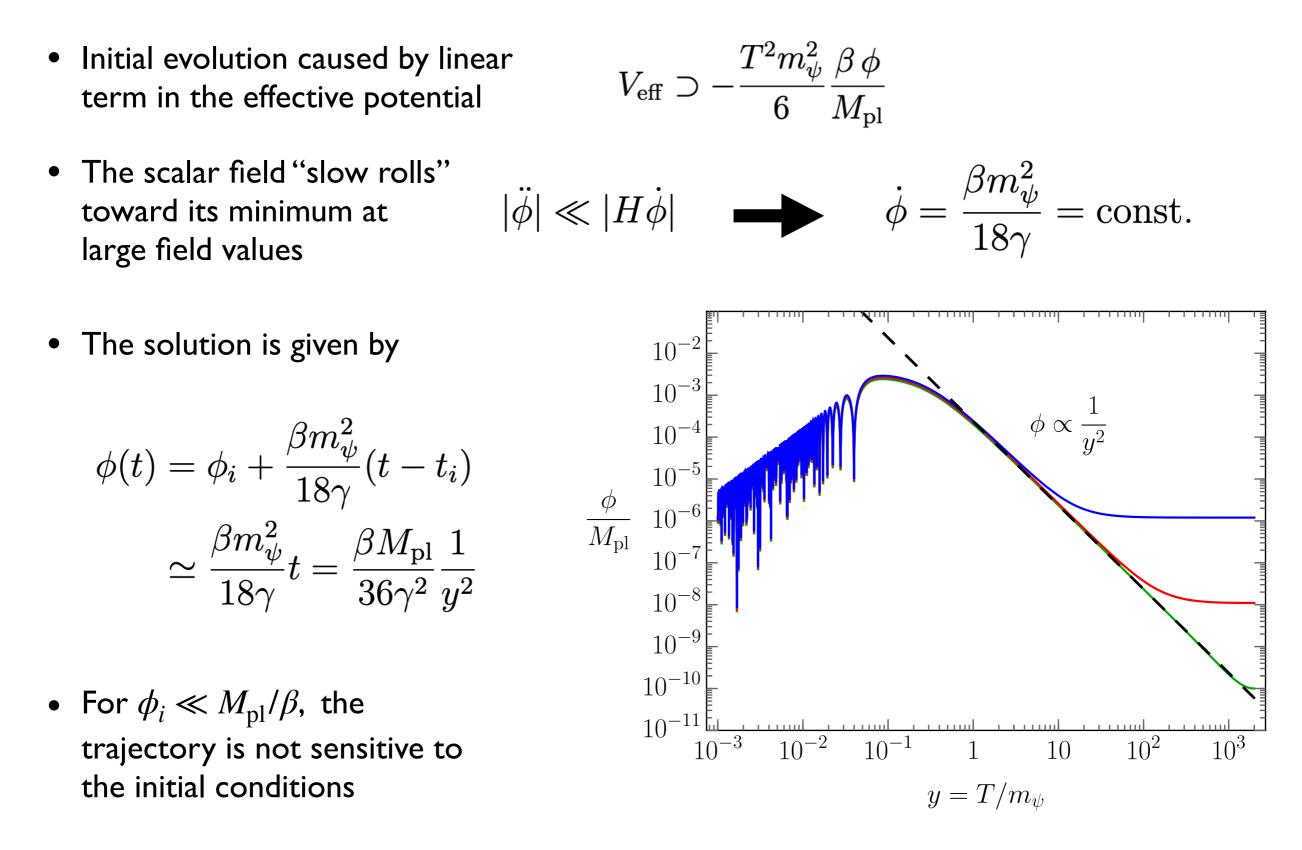
$$\phi_{\min} \simeq M_{\rm pl} \left[\frac{\beta y^2}{\beta^2 y^2 + 6 \kappa^2} \right] \simeq \begin{cases} \frac{M_{\rm pl}}{\beta} & \text{for} \quad y \gg \frac{\sqrt{6} \kappa}{\beta} & \text{High } T \\ M_{\rm pl} \frac{\beta y^2}{6 \kappa^2} & \text{for} & y \ll \frac{\sqrt{6} \kappa}{\beta} & \text{Lower } T, \text{but } y_{\rm eff} > 1 \end{cases}$$

 $T \gg m_f$

 The scalar will initially evolve towards the minimum at large field values



Initial trajectory

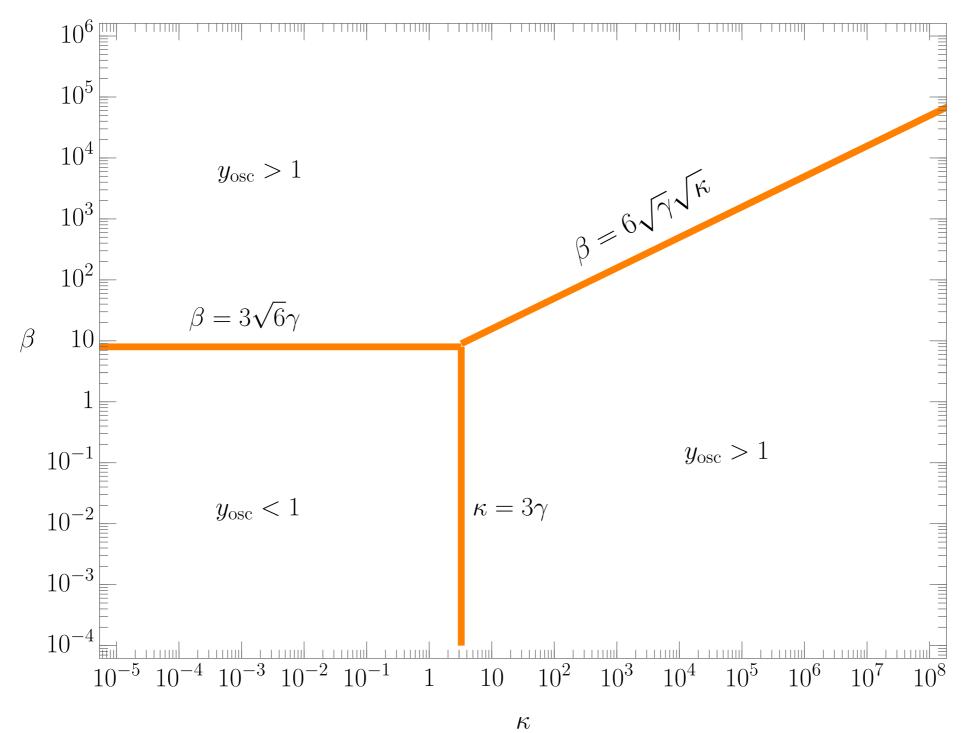


Onset of scalar oscillations

• The onset of oscillations occurs when the effective temperature-dependent scalar mass is equal to the Hubble rate. For $T_{\rm osc} \gg m_{\psi}$,

- For $T_{\rm osc} \ll m_{\psi}$, the bare mass dominates and $y_{\rm osc} \simeq \sqrt{\kappa/3\gamma}$
- This suggests there are three qualitatively distinct regions of $\kappa \beta$ parameter space.
- The boundaries of these regions are defined by the lines $y_{\rm osc} = 1$ and $\beta = 6\sqrt{\gamma}\sqrt{\kappa}$

Parameter space



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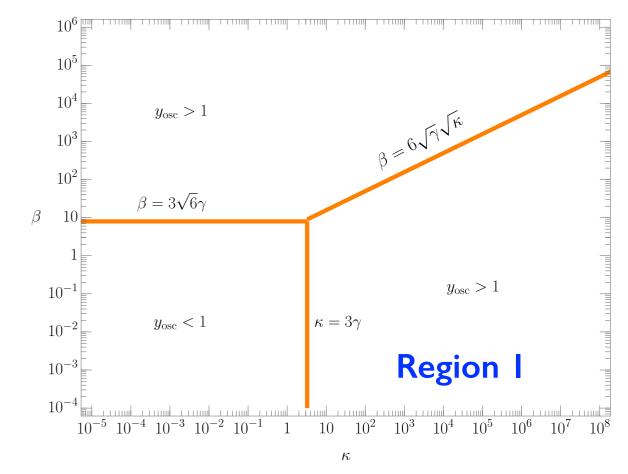
Region I (small β large κ , $y_{osc} \gg 1$)

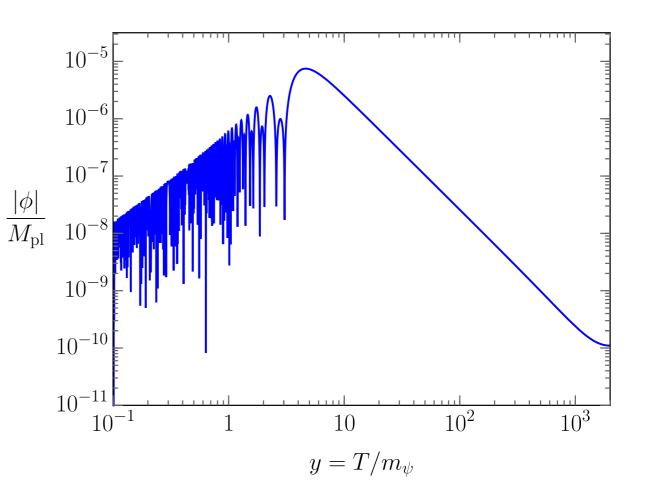
- This region is defined by $y_{osc} > 1$ and $\beta < 6\sqrt{\gamma}\sqrt{\kappa}$, which implies $\kappa > 3\gamma$
- Initial trajectory: $\phi(y) \simeq \frac{\beta M_{\rm pl}}{36\gamma^2} \frac{1}{y^2}$
- As temperature decreases, scalar field grows until the $y_{\rm osc} \simeq \sqrt{$ onset of oscillations

Oscillation
$$\phi_{
m osc}\equiv\phi(y_{
m osc})\simeqrac{eta M_{
m pl}}{12\gamma\kappa}$$

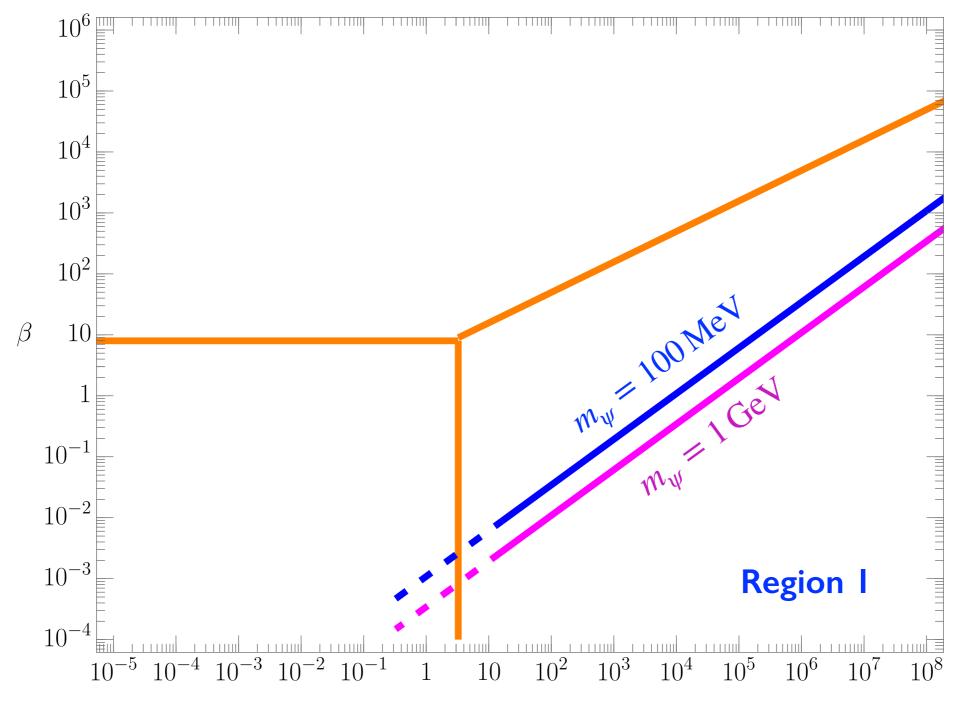
• The scalar oscillates and redshifts as matter

$$\Omega_{\phi}\big|_{0} = \frac{\rho_{\phi,0}}{\rho_{c,0}} = \frac{\frac{1}{2}m_{\phi}^{2}\phi_{\rm osc}^{2}(y_{0}/y_{\rm osc})^{3}(g_{*S}^{0}/g_{*S}^{\rm osc})}{\rho_{c,0}}$$
$$\simeq 2.2 \times 10^{5} \left(\frac{m_{\psi}}{0.1\,{\rm GeV}}\right) \frac{\beta^{2}}{\kappa^{3/2}}$$





Region I relic density
$$\Omega_{\phi}|_{0} \approx 0.3 \left(\frac{m_{\psi}}{0.1 \,\text{GeV}}\right) \left(\frac{\beta}{0.1}\right)^{2} \left(\frac{400}{\kappa}\right)^{3/2}$$



 κ

Region 2 (small β , small κ , $y_{osc} \ll 1$)

• This region is defined by $y_{\rm osc} < 1$, implying both $\beta < 3\sqrt{6\gamma}$ and $\kappa < 3\gamma$

• Initial trajectory:
$$\phi(y) \simeq \frac{\beta M_{\rm pl}}{36\gamma^2} \frac{1}{y^2}$$

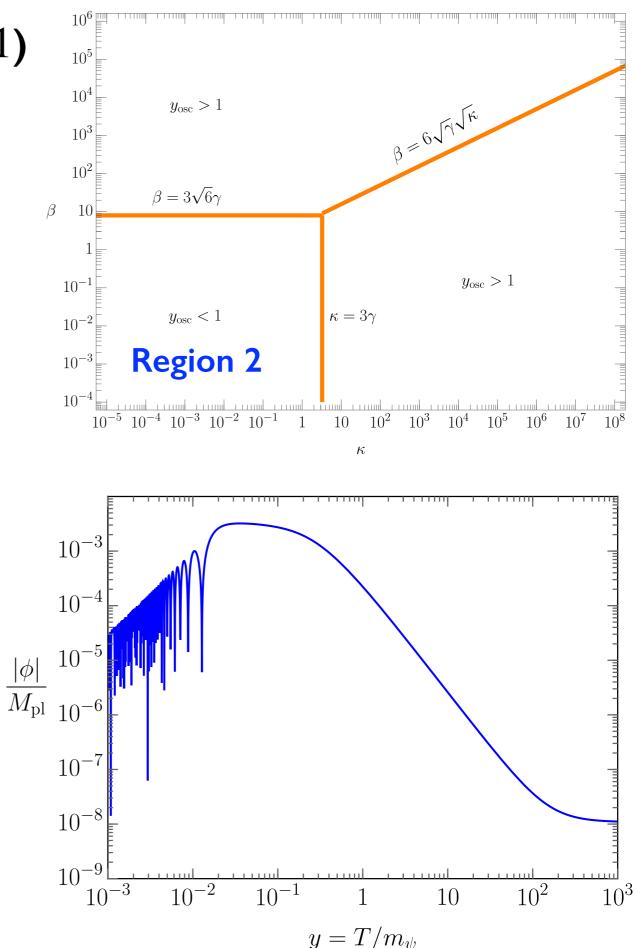
 At y ~ 1, fermion is Boltzmann suppressed and the slope vanishes. The trajectory asymptotes to a maximum value, which is also the oscillation amplitude:

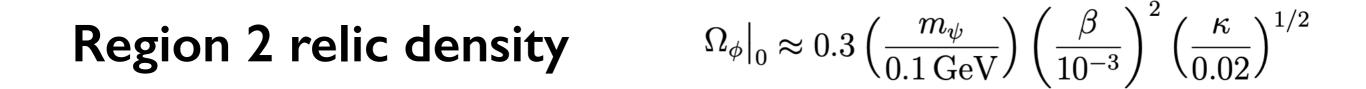
$$\phi_{
m osc} \simeq 0.27 \frac{\beta M_{
m pl}}{\gamma^2} \qquad \qquad y_{
m osc} \simeq \sqrt{\frac{\kappa}{3\gamma}}$$

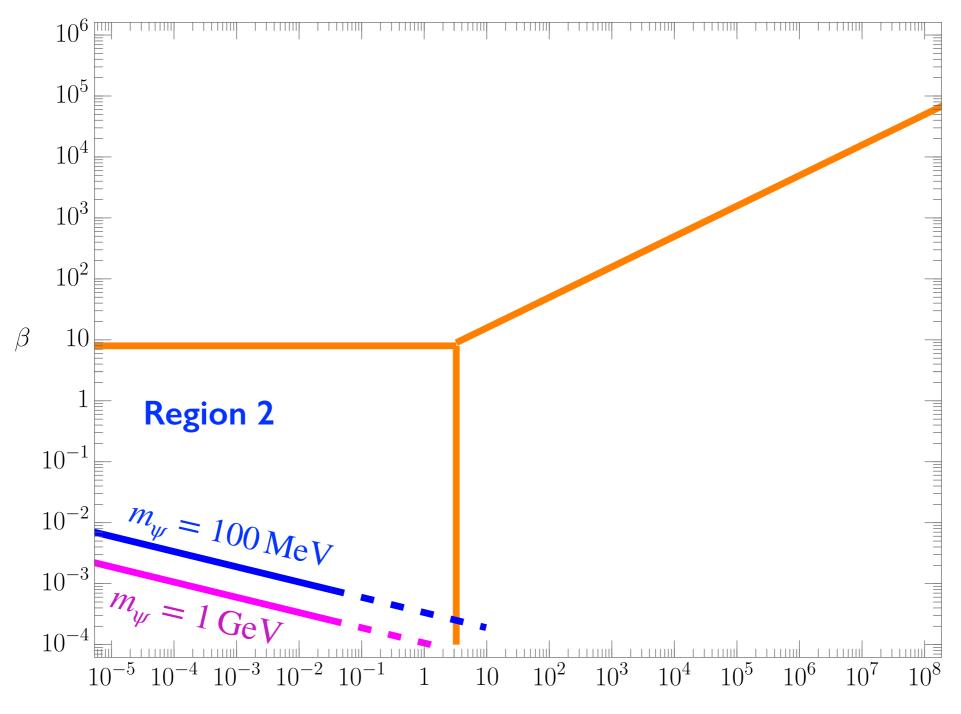
• The scalar oscillates and redshifts as matter

$$\Omega_{\phi}\big|_{0} = \frac{\rho_{\phi,0}}{\rho_{c,0}} = \frac{\frac{1}{2}m_{\phi}^{2}\phi_{\rm osc}^{2}(y_{0}/y_{\rm osc})^{3}(g_{*S}^{0}/g_{*S}^{\rm osc})}{\rho_{c,0}}$$
$$\sim 2.3 \times 10^{6} \left(\frac{m_{\psi}}{-m_{\psi}}\right) \beta^{2} \sqrt{\kappa}$$

 $0.1 \, \text{GeV}$



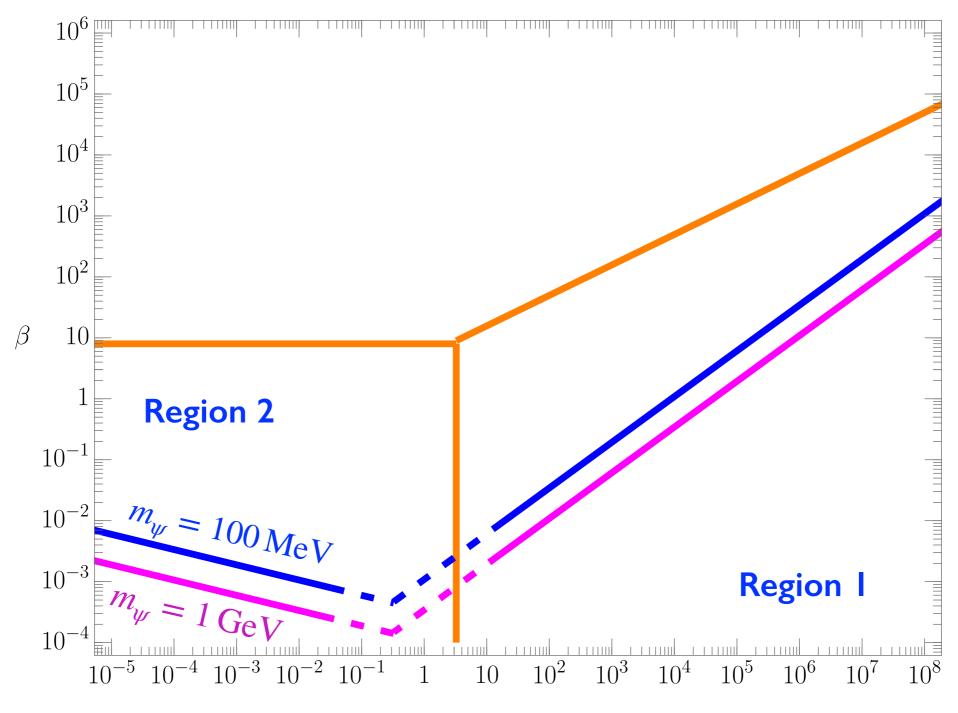




 κ

40

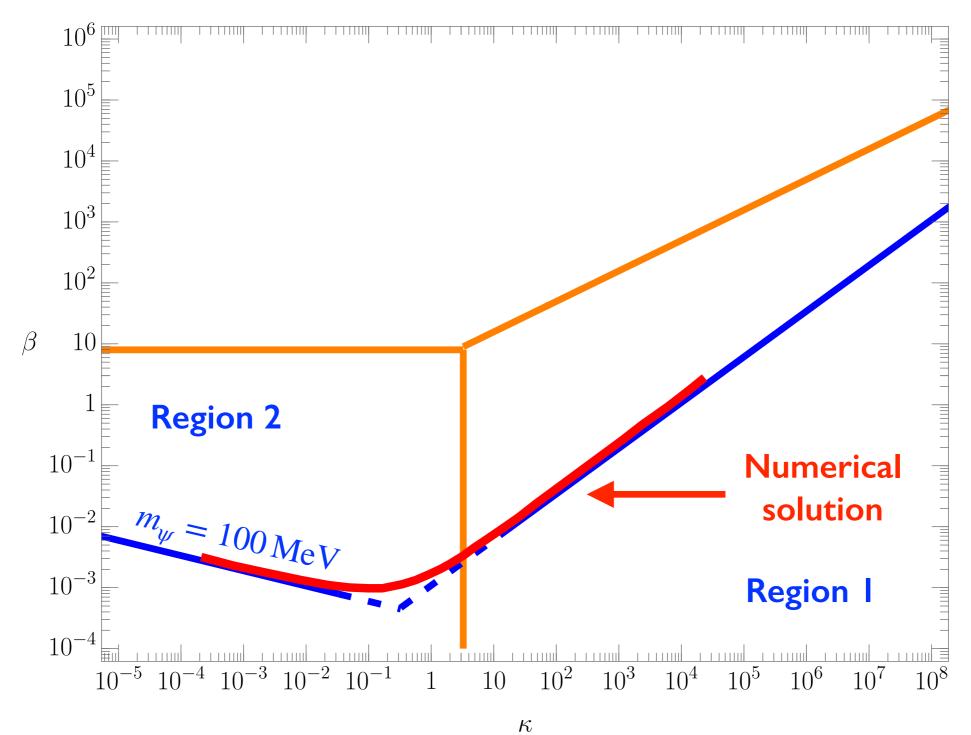
Regions I & 2 Relic Density



 κ

41

Regions I & 2 Relic Density



Initial conditions and inflation

- During inflation, the evolution of the DM scalar field involves a competition between classical rolling and quantum fluctuations
- Given a long enough period of inflation with order $N_e \sim H_I^2/m_{\phi}^2$ e- folds, the distribution of the scalar approaches a Gaussian with mean close to zero and variance $\sigma_{\phi} \sim H_I^4/m_{\phi}^2$.
- We require two conditions to be met:
 - 1. $\sigma_{\phi} \ll \phi_{\rm osc}$ in order to avoid fine tuning of initial conditions
 - 2. The reheat temperature to be greater than the oscillation temperature.
- It can be shown that these two conditions can always be satisfied for a suitable range of values for the Hubble parameter during inflation. This typically requires a low Hubble scale and long period of inflation.
- A low Hubble scale during inflation also suppresses the primoridal scalar DM fluctuations, thereby evading otherwise stringent isocurvature constraints

Realistic example: scalar DM coupled to muon

$$-\mathcal{L} \supset rac{1}{2} \, m_{\phi}^2 \, \phi^2 + m_{\mu} \, \left(1 - rac{eta \, \phi}{M_{
m pl}}
ight) \overline{\mu} \, \mu$$

• The required coupling is obtained from the dimension-5 operator

$$\mathcal{L} \supset \frac{c_{\ell}}{M_{\mathrm{pl}}} \phi \, \overline{L}_L \, \ell_R \, H + \mathrm{h.c.} \qquad c_{\ell} = \frac{\sqrt{2} \, m_{\ell}}{v} \beta = y_{\ell} \, \beta$$

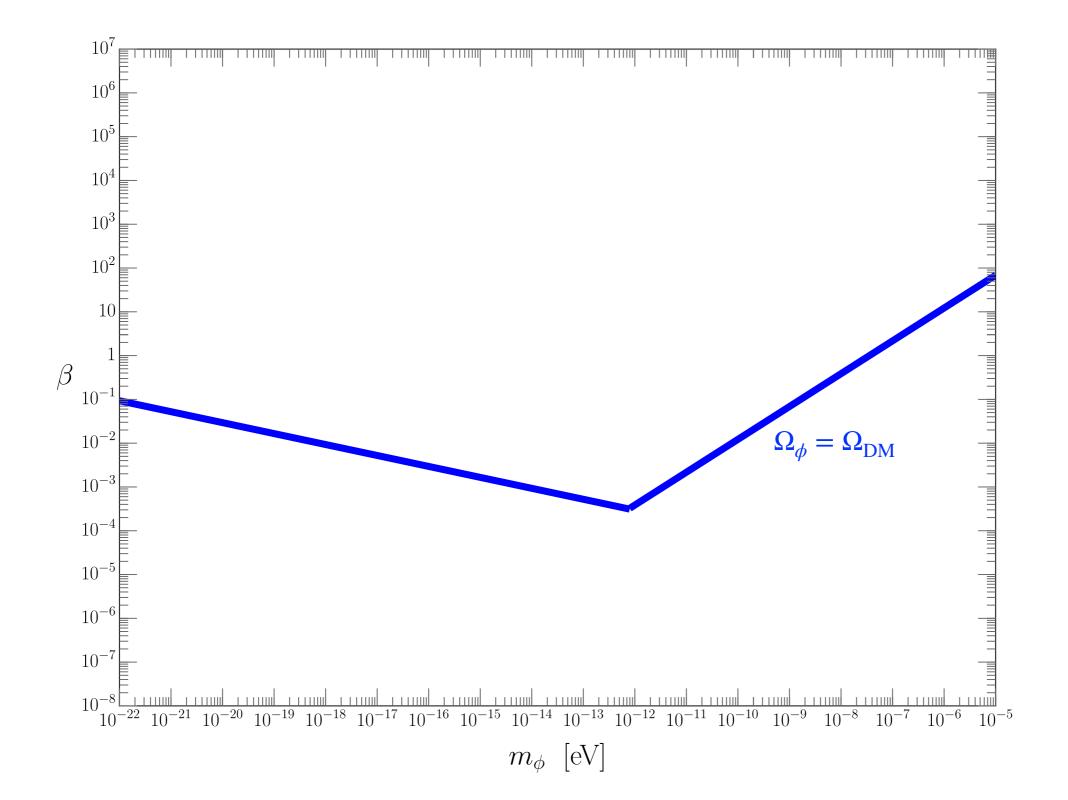
• At temperatures above the electroweak scale, the fermion feedback turns off (fermion is massless). Fermion feedback turns on at $T \lesssim v_{\rm EW}$

• The scalar potential is fine-tuned when

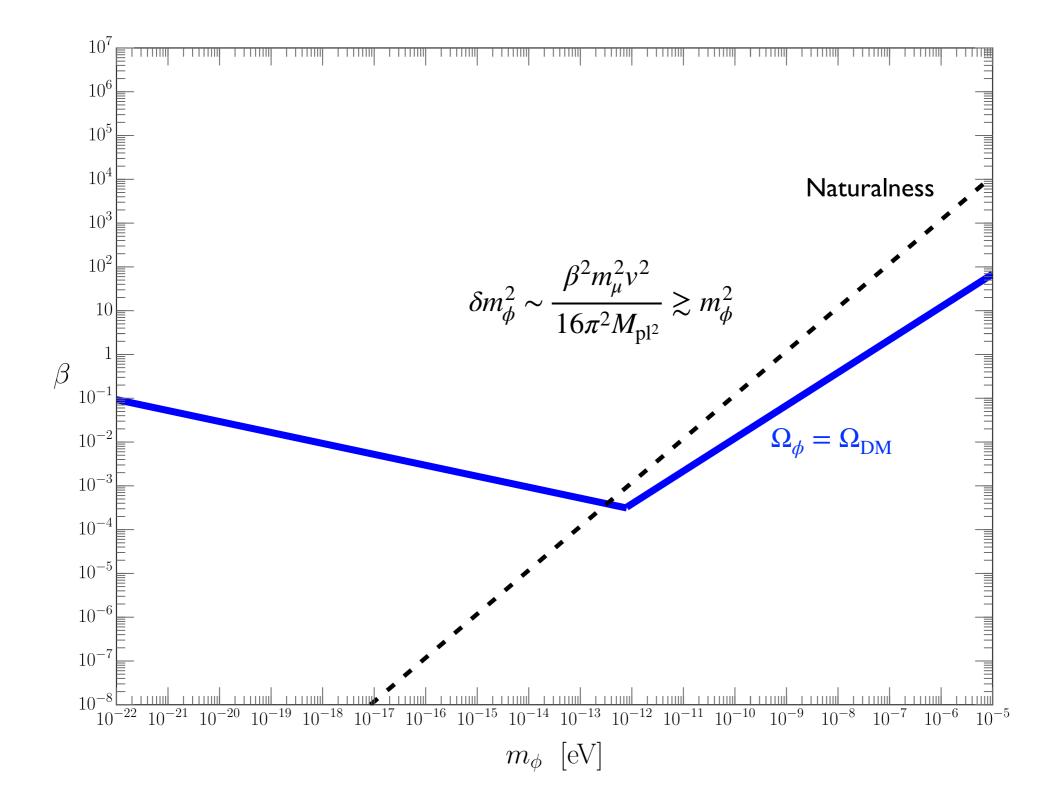
$$\delta m_{\phi}^2 \sim \frac{\beta^2 m_{\mu}^2 v^2}{16\pi^2 M_{\rm pl^2}} \gtrsim m_{\phi}^2$$

 The previous results for the relic density prediction can be applied by fixing the fermion mass to the muon mass

Relic density



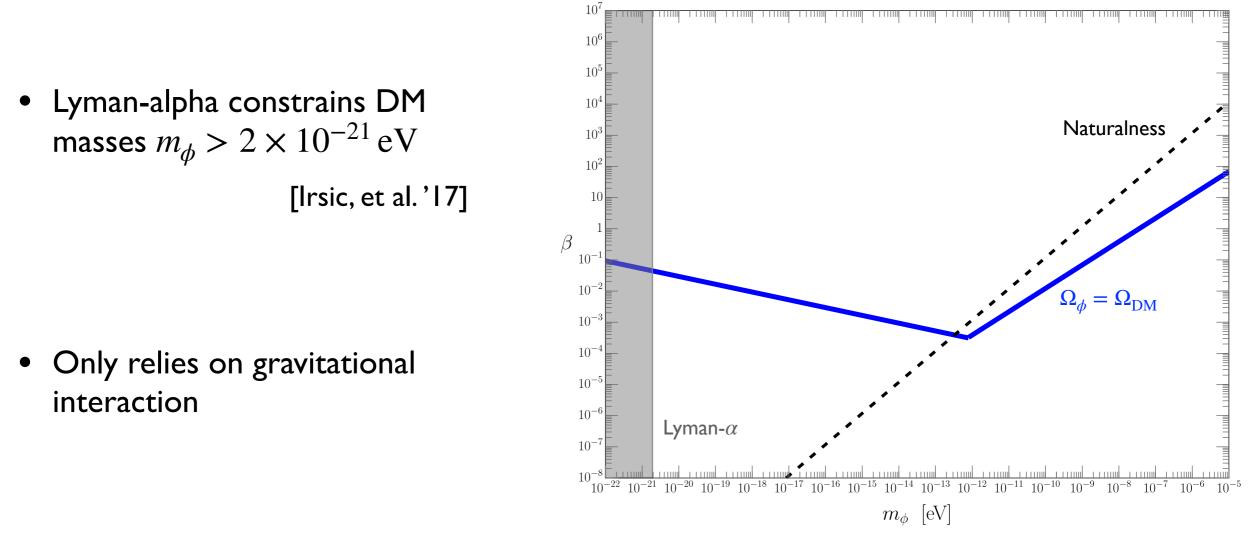
Naturalness



Experimental/observational probes

Lyman- α / Fuzzy DM constraints

- In the Fuzzy DM regime (De Broglie wavelength of ~ I kpc), matter power at small scales is suppressed
- The Lyman-alpha forest flux power spectrum provides a tracer of matter fluctuations on small scales and high-redshifts (quasi-linear regime)



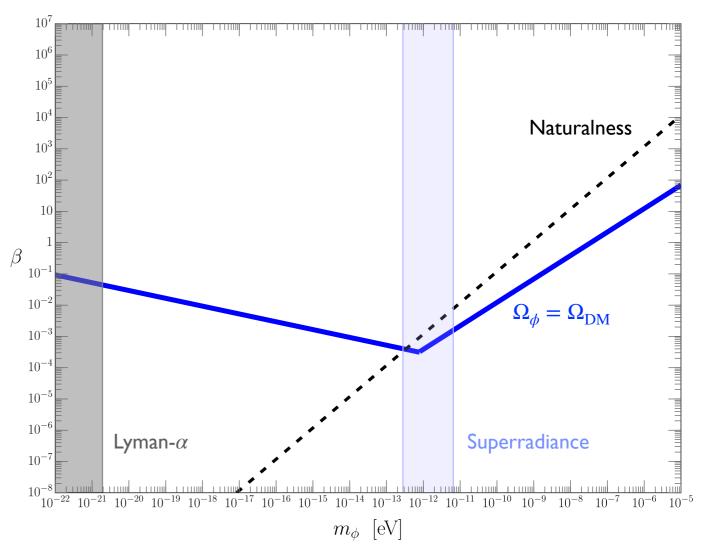
Black hole superradiance

- Light bosons with Compton wavelength of order size of spinning black hole will extract its angular momentum via the superradiance effect
- The observation of such spinning black holes therefore constrains such light bosons

• Superradiance provides strongest constraints for $m_{\phi} \sim 10^{-12} \, {\rm eV}$

[Baryakhtar, Galanis, Lasenby, Simon,'20]

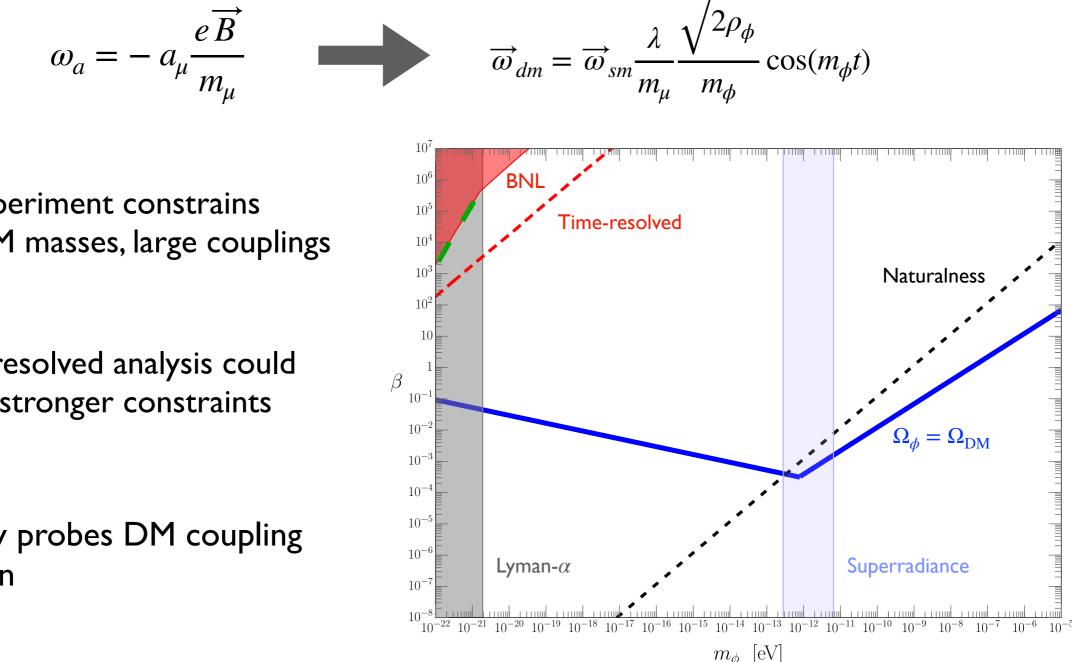
• Only relies on gravitational interaction



Muon storage ring tests and $(g-2)_{\mu}$

[Janish, Ramani, '20]

DM coupled to muons will induce an oscillating muon mass and thus alter the muon \bullet precession frequency in storage ring experiments (e.g., BNL and FNAL muon g-2)



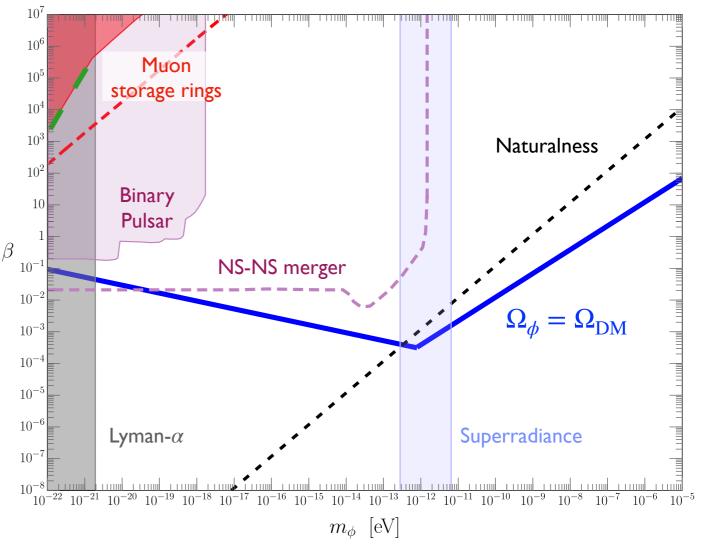
BNL experiment constrains ulletsmall DM masses, large couplings

- A time-resolved analysis could provide stronger constraints
 - Directly probes DM coupling to muon

Muonic forces in neutron star binaries

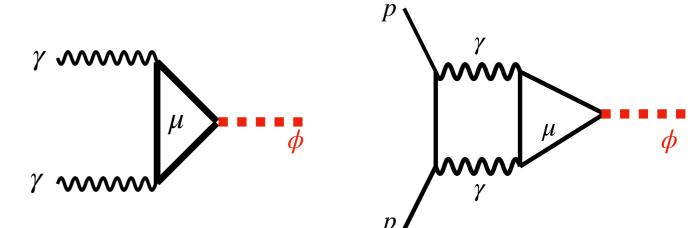
[Dror, Laha, Opferkuch '20]

- Neutron stars (NS) are muon-rich environments. The properties of NS binaries can therefore be modified if there is a long-range muonic force
- Pulsar binaries will exhibit anomalously fast decay of their orbital period due to emission of the muonic force carriers
- The additional long range force between merging NS-NS binaries modifies the inspiral phase and resulting pattern of gravitational radiation
- Probes DM coupling to muon



Equivalence principle & inverse square law tests

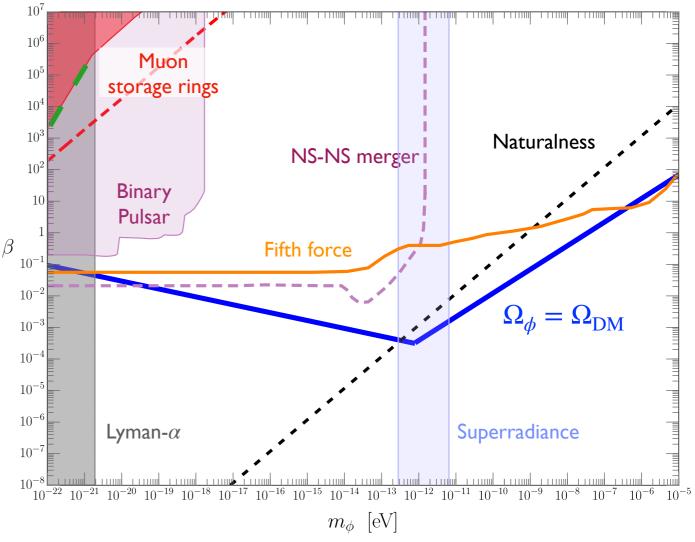
 At one (two) loop(s), DM can couple to photons (charged fermions)



 This induces a long range Yukawa force, which can be probed by equivalence principle and inverse square law tests

> [Schlamminger et al., '07] [Wagner et al., '12]

 Indirect probe of the DM-muon coupling

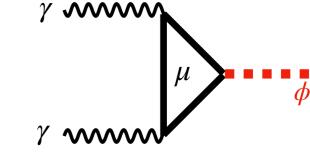


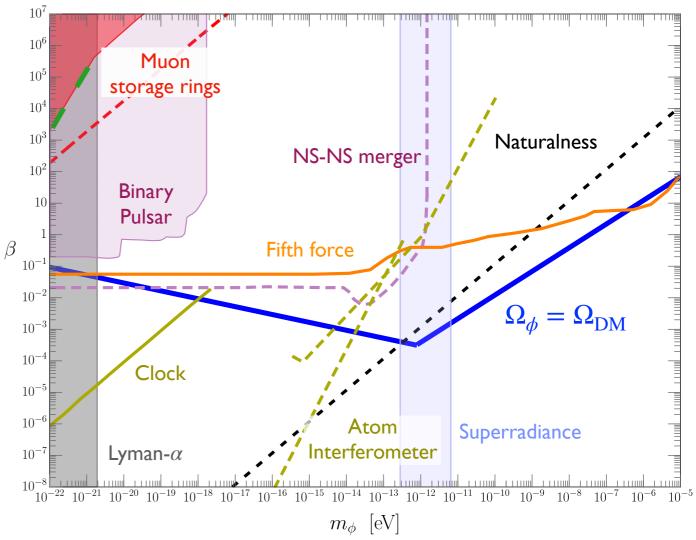
Variation in fine structure constant

- At one loop the DM can couple to photons
- This leads to temporal variations of the fine structure constant in the dark matter background
- Strong existing limits come atomic clocks measurements (e.g. Rb/Cs hyper fine transition frequency ratio)

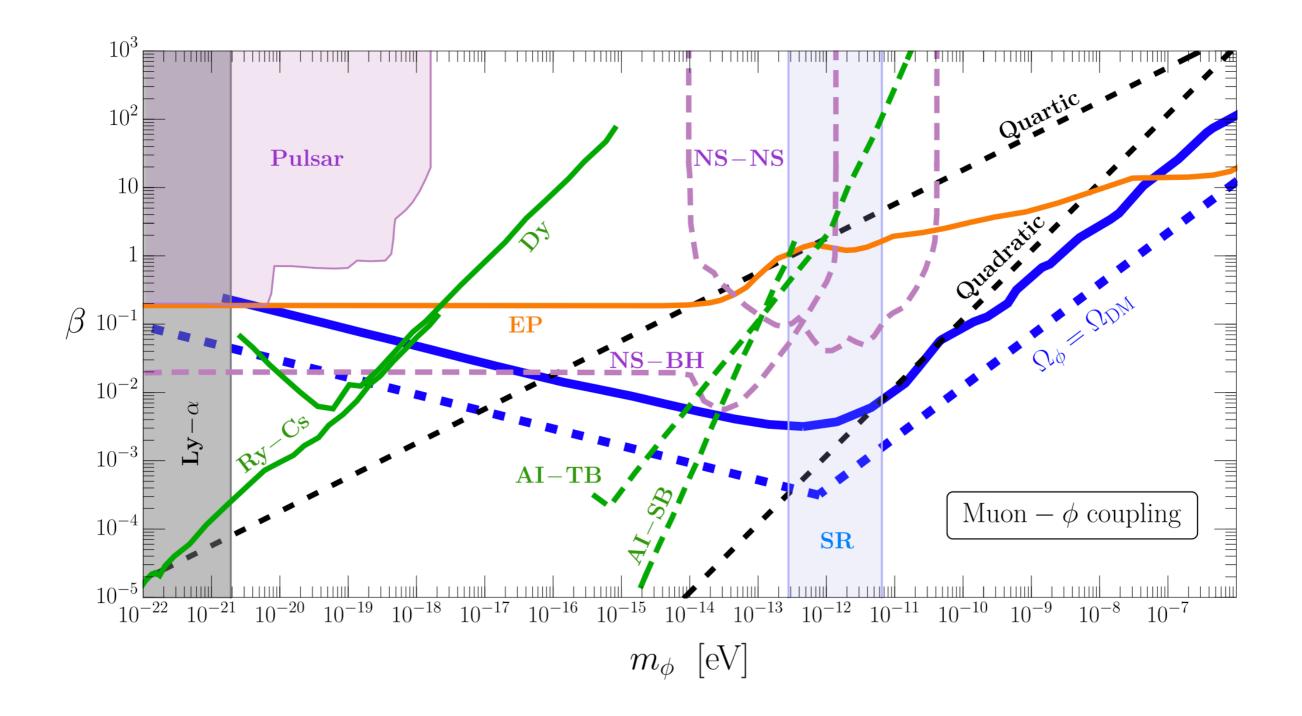
[Hees et al. '16]

- Future atom interferometry experiments can cover substantial new parameter space [Arvanitaki et al. 16]
- Indirect probe of the DM-muon coupling





Summary



Outlook

- Ultralight bosons represent a well-motivated and phenomenologically distinctive class of DM models
- We have presented *thermal misalignment* a mechanism to dynamically generate the large misalignment needed to obtain the correct DM relic abundance
- The mechanism relies on a finite temperature potential due to a coupling of DM to a fermion. It is insensitive to initial conditions and the abundance is dictated by the microphysics (couplings and masses)
- We have studied the phenomenology of a realistic scenario where the DM couples to the muon. A variety of opportunities for probing this scenario in the future exist.
- In progress/future work: couplings of DM to other fermions, couplings to bosons, Higgs portal as UV completion