

2022 Chung-Ang University Beyond the Standard Model Workshop

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Continuum Dark Matter



Seung J. Lee

With Csaki, Hong, Kurup, Perelstein, Xue; 2105.14023 [hep-ph]

With Csaki, Hong, Kurup, Perelstein, Xue; 2105.07035 [hep-ph]

With Hong, Kurup, Lee, work in progress

With Perelstein, Ferrante, work in progress

With Csaki, Ismail, Nakli, Suzuki, work in progress

Outline

- Introduction
- Gapped Continuum
- Gapped Continuum QFT
- Equilibrium and Non-equilibrium Thermodynamics
- Z-portal Model for Gapped Continuum DM
- Summary

DM-what we don't know

- Mass of Dark Matter (range: 10^{-22} to 10^{67} eV)
- Composition of Dark Matter
- Interaction of Dark Matter

Gapped Continuum, instead of Resonances

◆ **Our Proposal:** Dark Matter is made of an ensemble of **gapped continuum states**

- It's not even clear whether the DM that provide successful explanations to the rotation curve of disk galaxies, CMB, and large structure formation is a **localized excitation of quantum field** (i.e. particle)

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- **continuum with a mass gap** is not so uncommon in **condensed matter physics**: e.g. edge state in fractional quantum hall effect, topological superfluid, 2D Ising model, 2d SU(2) Thirring model, 2d SU(N) Yang-Mills theory in large-N limit ,etc

Gapped Continuum, instead of Resonances

- ◆ The appearance of a continuum is very common in QFT's: e.g. spectrum of **CFTs** necessarily forms a continuum since the theory does not admit any mass scales (no mass gap).
- ◆ **Unparticles** (Georgi): another example of **gapless continuum**
- ◆ **String Theory** (e.g. Gubser et al, Kraus, Trivedi et al, etc): **gapped continuum** shows up when one has a large number of D3 branes distributed on a disc (which is dual to $N = 4$ SUSY broken to $N = 2$ via masses for two chiral adjoints)
- ◆ **Gapped Continuum in particle physics**: -Softwall model (Higgs with a small mass gap (before Higgs discovery) by Terning et al, Falkowski et al
-Quantum Critical Higgs (Higgs pole + gapped continuum: after Higgs discovery) by Csaki et al (SL): for off-shell form factor (by gapped continuum) for Higgs EFT
-Continuum Naturalness (for solving little hierarchy by Csaki et al (SL), and also by Quiros et al)

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So, how about DM as a gapped continuum?
(never explored before!)

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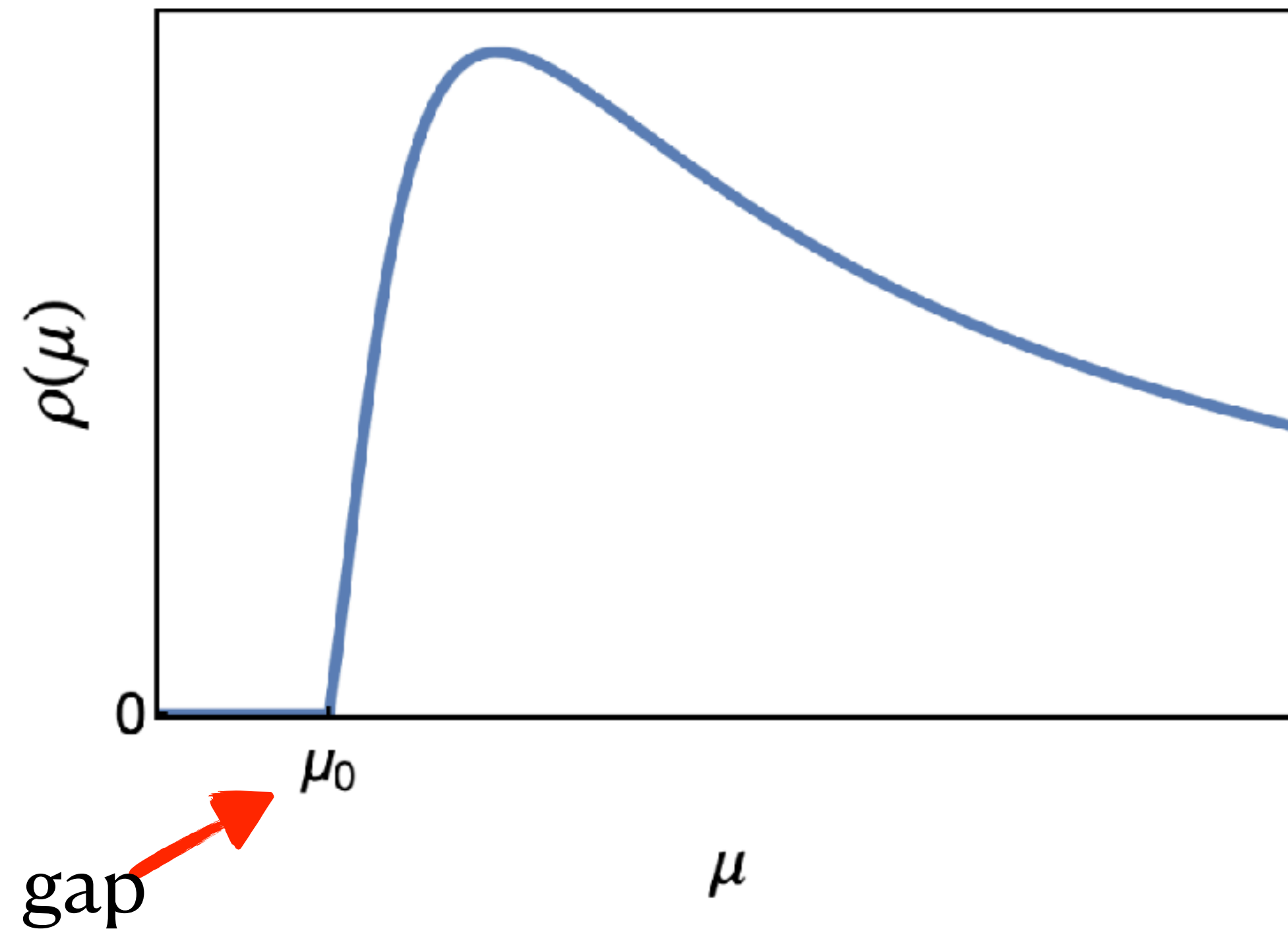
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Gapped Continuum, instead of ordinary particles

- ◆ **Continuum DM:** singly-excited states are characterized by a continuous parameter μ^2 , in addition to the usual 3-momentum p

The parameter μ^2 plays the role of mass in the kinematic relation $p^2 = \mu^2$ for each state. The number of states is proportional to $\rho(\mu^2) d\mu^2$, where ρ is the spectral density of the theory

$$\langle 0 | \Phi(p) \Phi(-p) | 0 \rangle = \int \frac{d\mu^2}{2\pi} \frac{i\rho(\mu^2)}{p^2 - \mu^2 + i\epsilon}$$

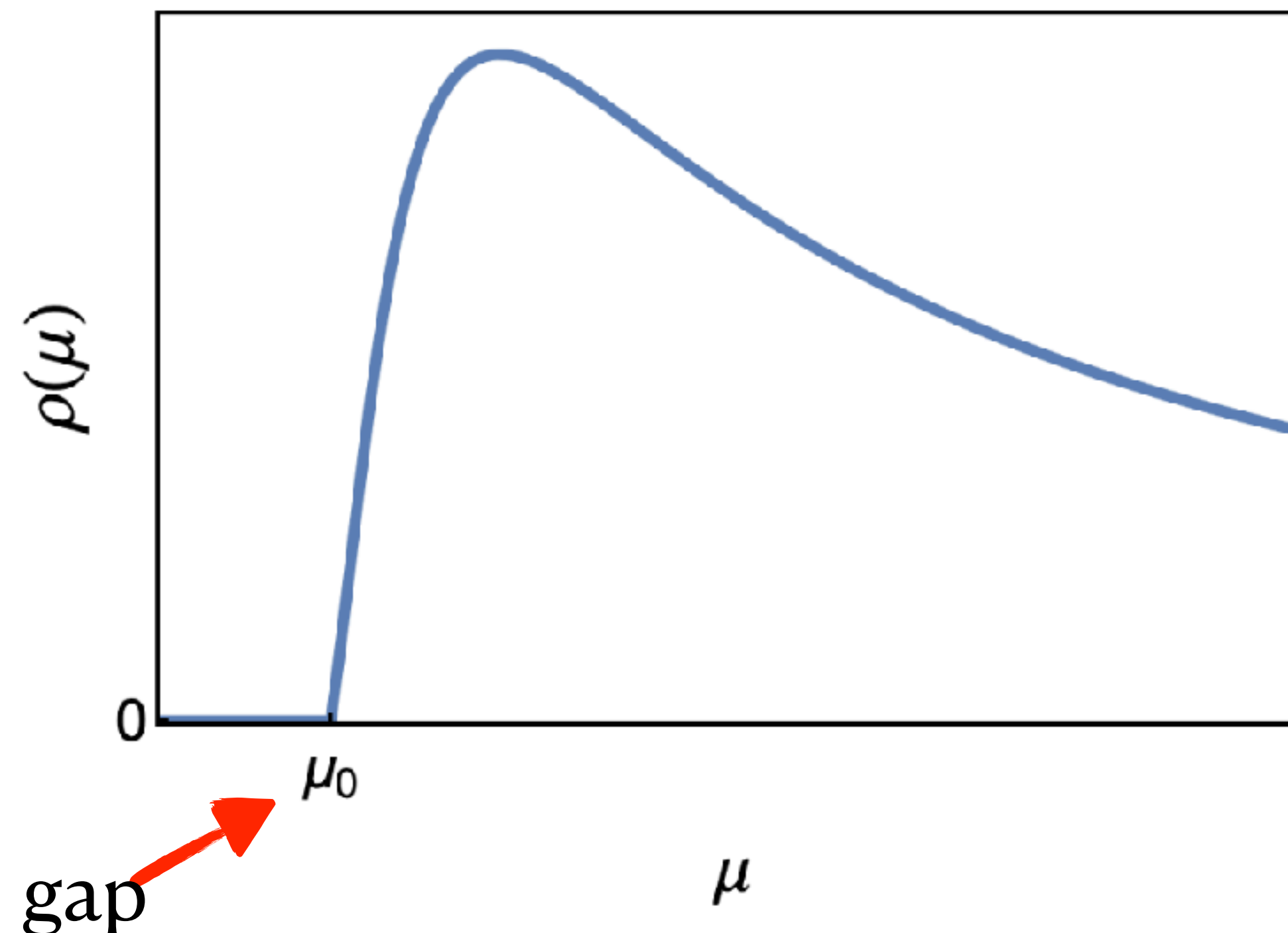


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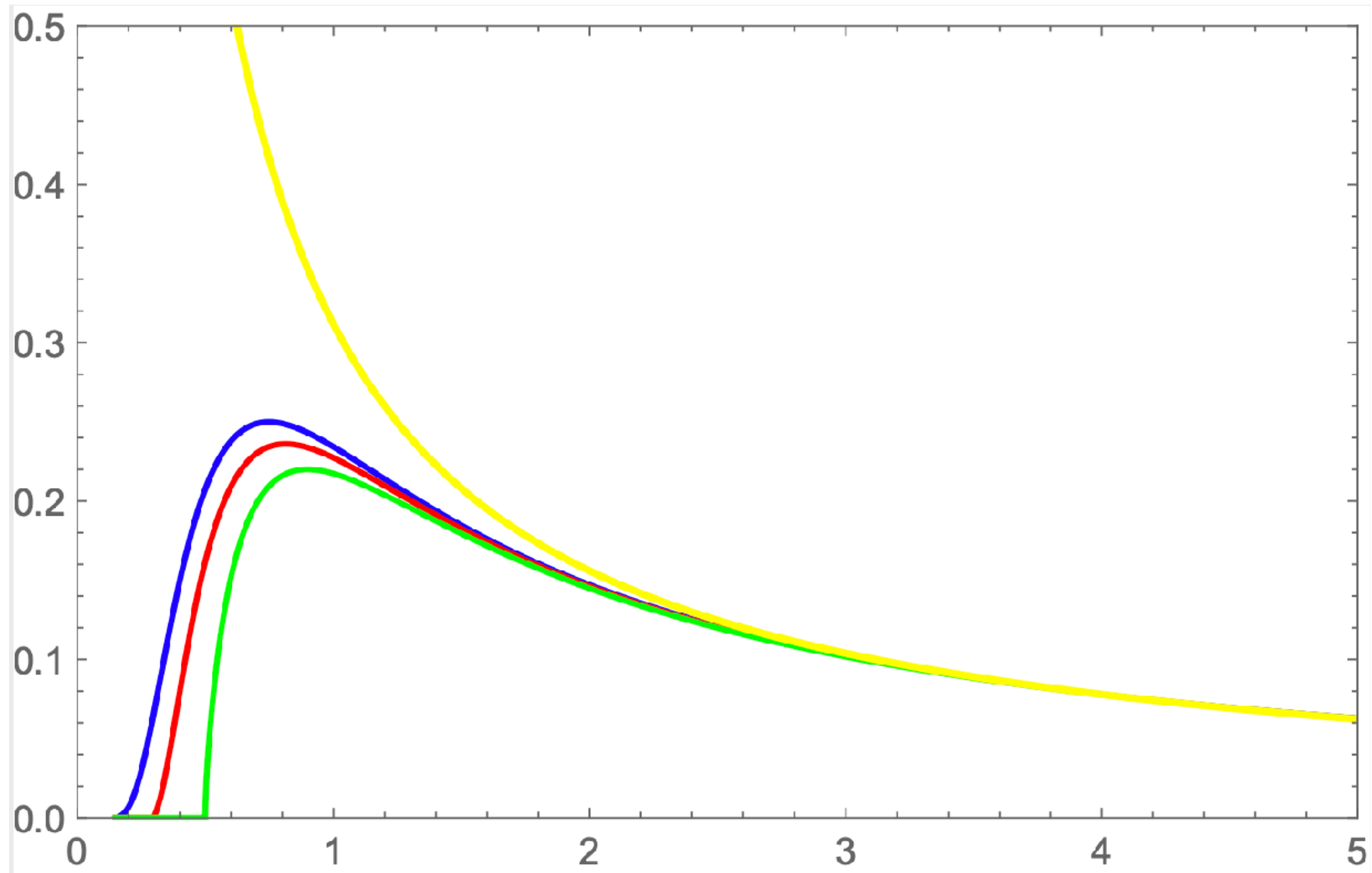
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CFT Continuum vs Gapped Continuum (IR deformation)



Physics of Gapped Continuum DM

◆ CFT continuum case:

- It's often stated that CFT's and theories with continuum spectra do not have a particle interpretation and no S-matrix can be defined: interactions leading to a non-trivial fixed point are also essential for producing the continuum spectrum of the theory
- by turning off the interactions, the spectrum changes from continuum into that of an ordinary free particle, hence the asymptotic states defined in the usual manner would not capture the physics of the system properly
- this means that one needs to find an alternative approach for defining scattering processes

◆ Our theoretic description of **gapped continuum: Generalized Free Continuum** (continuum analog of Generalized Free Fields: Greenberg 1961)

Also: Polyakov, early '70s- skeleton expansions

CFT completely specified by 2-point function-rest vanish

CFT Continuum

Generalized Free Fields Polyakov, early '70s- skeleton expansions

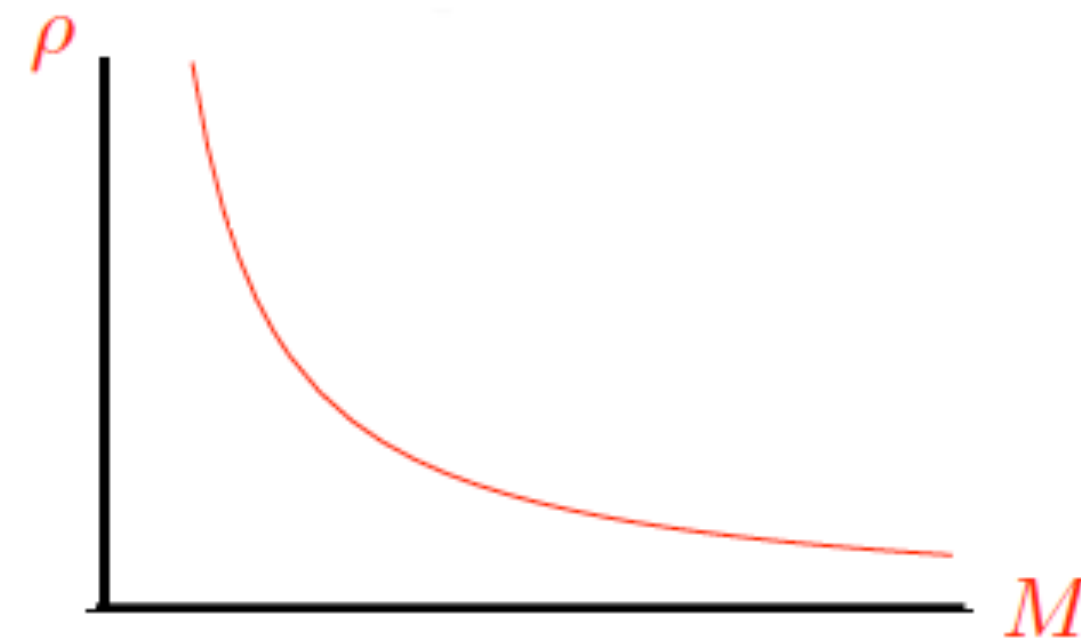
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Scaling - 2-point function: $G(p^2) = -\frac{i}{(-p^2 + i\epsilon)^{2-\Delta}}$

Can be generated from: $\mathcal{L}_{\text{GFF}} = -\hbar^\dagger (\partial^2)^{2-\Delta} \hbar$ Georgi
hep-ph/0703260

Branch cut starting at origin - spectral density purely a continuum:

$$G(p) \sim \int_{\mu^2}^{\infty} dM^2 \frac{\rho(M^2)}{p^2 - M^2}$$



Generalized Free Continuum

◆ Generalized free continuum

-consider the case that the effects of the strong interactions can be captured by the fact that there is a non-trivial continuum (with a mass gap), and described by:

$$S = \int \frac{d^4p}{(2\pi)^4} \Phi^\dagger(p) \Sigma(p^2) \Phi(p)$$

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- The above effective description is weakly coupled (reproduces the two-point function)
→ Φ corresponding to a “generalized free field”

- In addition we perturb around generalized free continuum by introducing additional weak couplings to Φ and assume that the underlying structure described by the spectral density remains unchanged, resulting in a weakly interacting continuum.

This picture is supported by the concrete extra dimensional construction!

Dark Matter Continuum Spectral Density from 5D Model

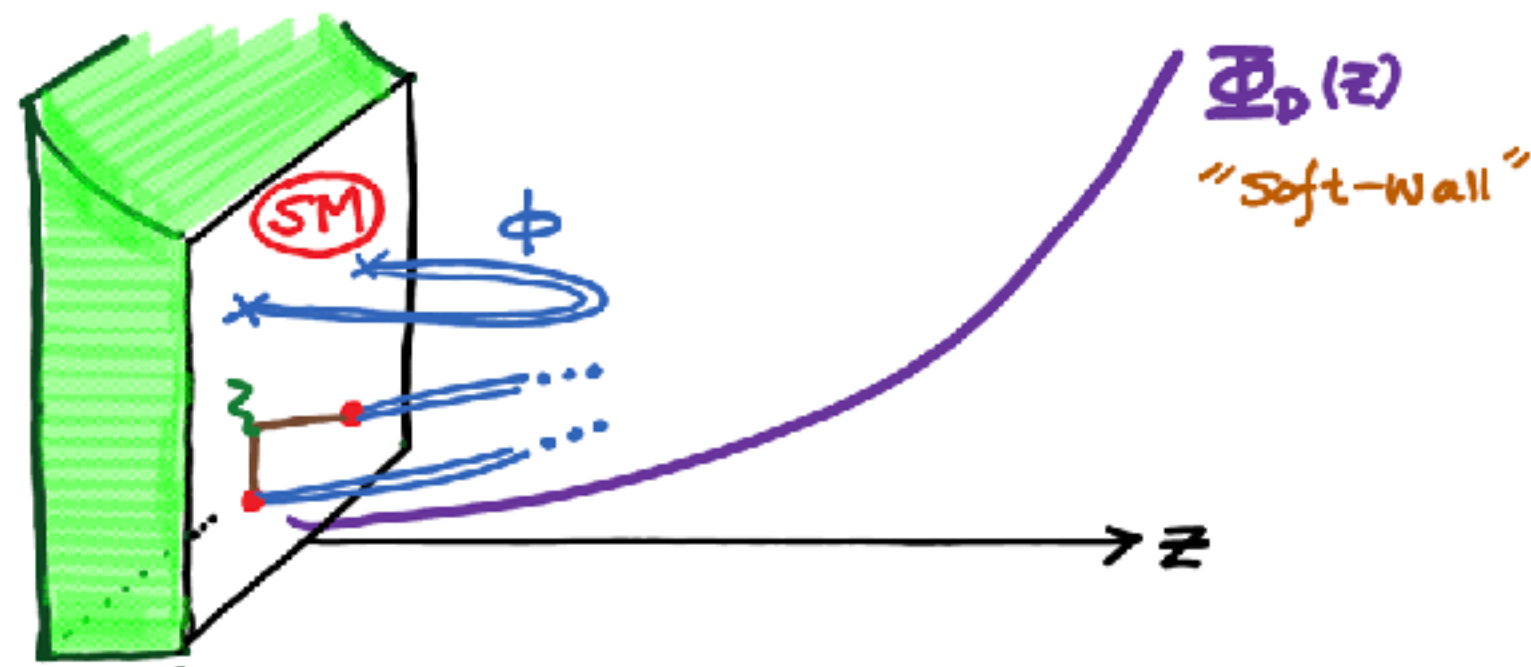
- ◆ modeling generalized free continuum by **Warped 5D model**

$$ds^2 = e^{-2A(y)} dx^2 + dy^2$$

- warped 5D setup we will have a 3-brane placed at the position $z = R$, which from the point of view of the gapped continuum field will be a UV brane cutting off the space

The 5D action of the coupled scalar-gravity system

$$S = \int d^5x \sqrt{g} \left(-M^3 R + \frac{1}{2} (\partial\phi)^2 - V(\phi) \right) - \int d^4x \sqrt{g^{ind}} V_4(\phi)$$



y_s = finite distance location of the curvature singularity where the spacetime ends in the y coordinates

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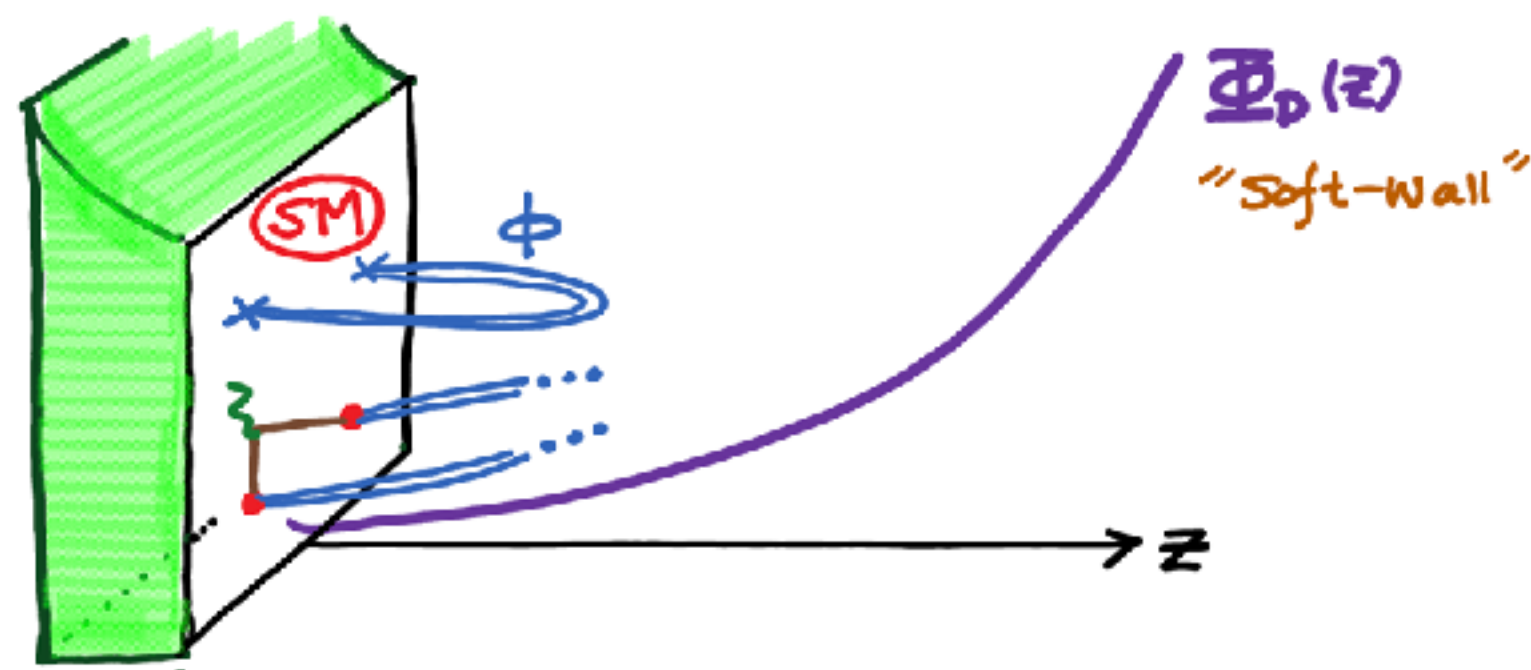
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- The superpotential (w/ relation $V = 3W'^2 - 12W^2$) leading to the desired 5D

background : $W = k(1 + e^\phi)$ (fully includes the backreaction of the metric to the presence of the scalar field)



Solution $A(y) = -\log \left(1 - \frac{y}{y_s} \right) + ky,$

y_s = finite distance location of the curvature singularity where the spacetime ends in the y coordinates

$$\phi(y) = -\log(k(y_s - y)),$$

soft wall & continuum

- Scalar gapped continuum: $\mathcal{L} = \sqrt{g} \left[\frac{1}{2} g^{MN} D_M \Phi^\dagger D_N \Phi - V(\Phi) \right]$

In conformally flat coordinate, Schrödinger form of eom:

EOM: $\left(-\partial_y^2 + \hat{V}(y) \right) \Psi(p, y) = e^{2\Lambda(y)} p^2 \Psi(p, y) \quad \Psi(p, y) = e^{-2\Lambda(y)} \Phi(p, y)$

“Schrödinger Eqn” $-\ddot{\psi} + V(z)\psi = p^2\psi$

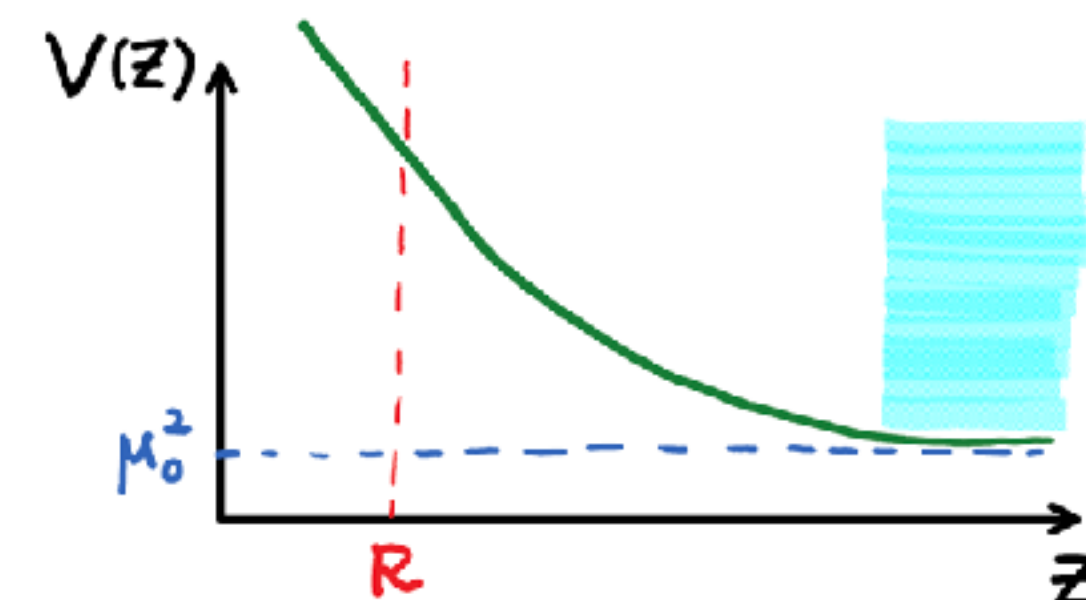
$$V(z) = \frac{e^{-2ky}}{4y_s^2} \left[4m^2(y_s - y)^2 + 15(1 + k(y_s - y))^2 - 6 \right]$$

if $V \xrightarrow{z \rightarrow \infty} \mu_0^2 = \text{finite} \Rightarrow \text{Continuum!}$

1D QM
problem

\Rightarrow continuum begins at:

$$\mu_0^2 = \frac{9}{4y_s^2} e^{-2ky_s}$$



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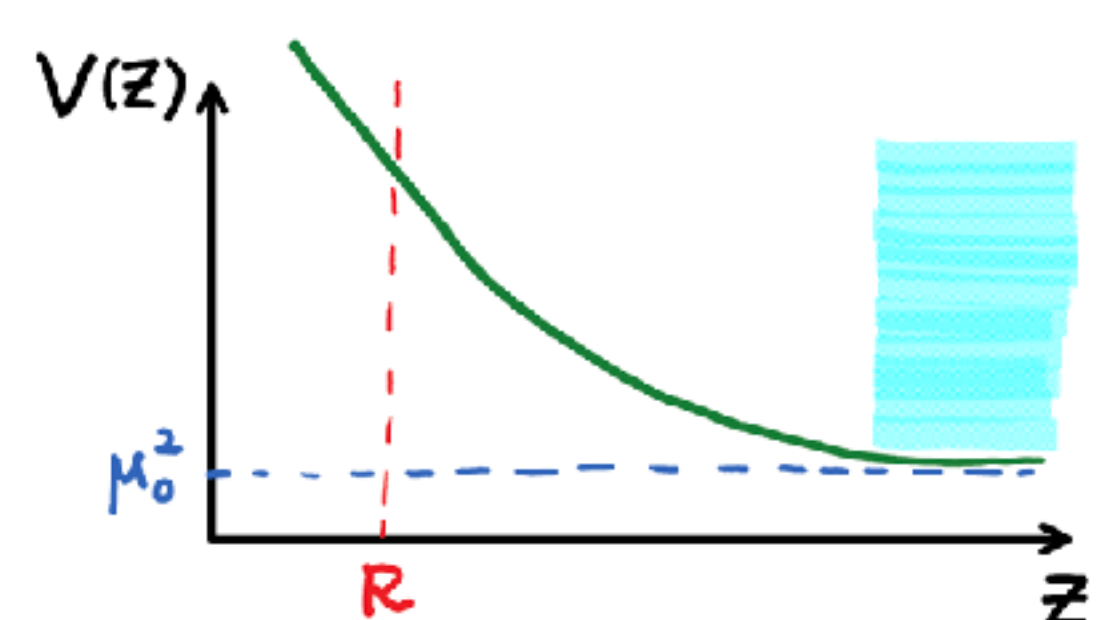
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Quantum Gravity?
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What's so new for gapped
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- One may not simply plug gapped continuum into formalism developed for particle DM: need a new theoretical framework for dealing with gapped continuum in order to calculate the relic density of DM, and to deal with the finite temperature physics necessary for describing general features of cosmological history of DM
- -requires a systematic development of theories of gapped continuum DM

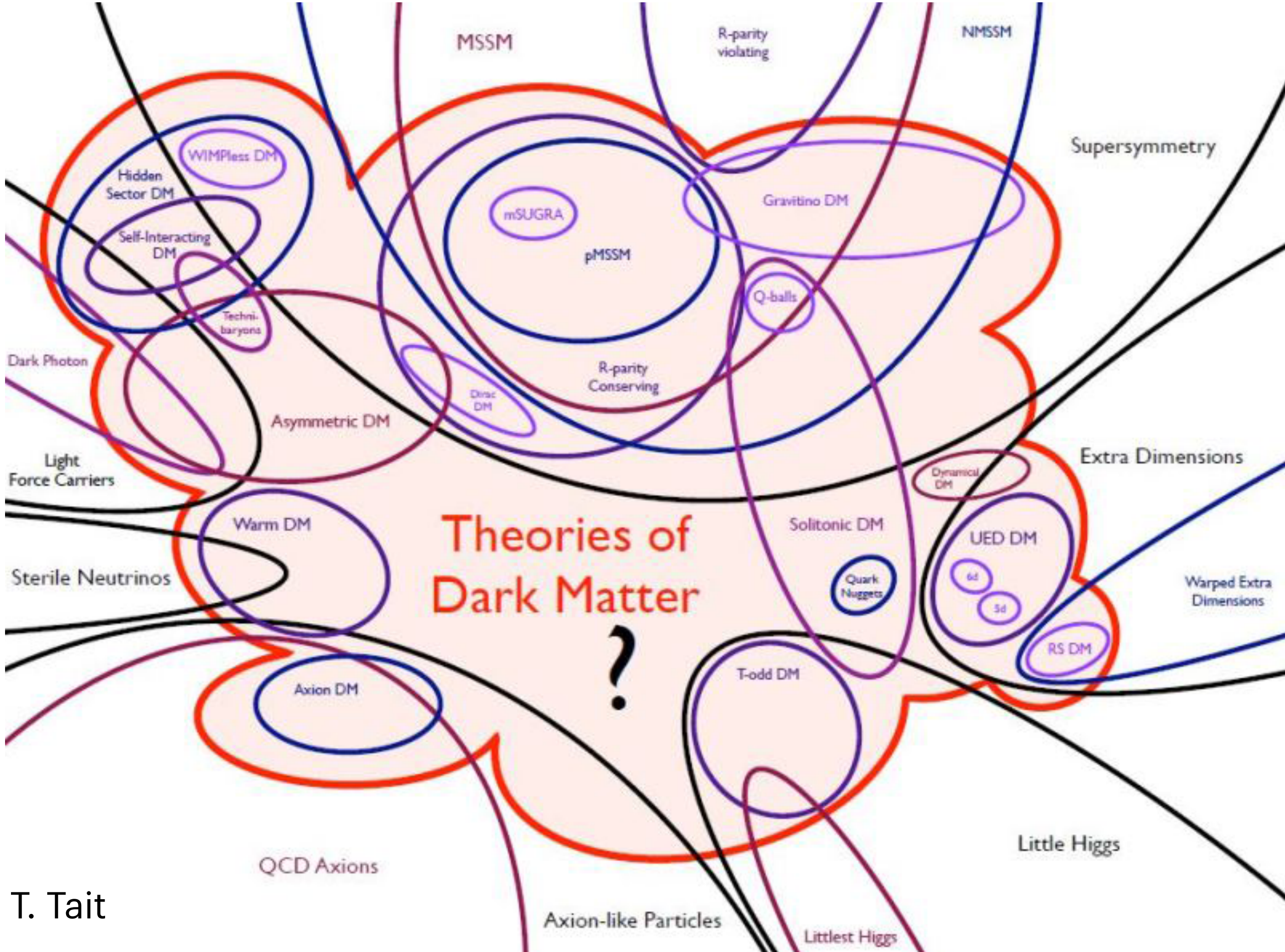
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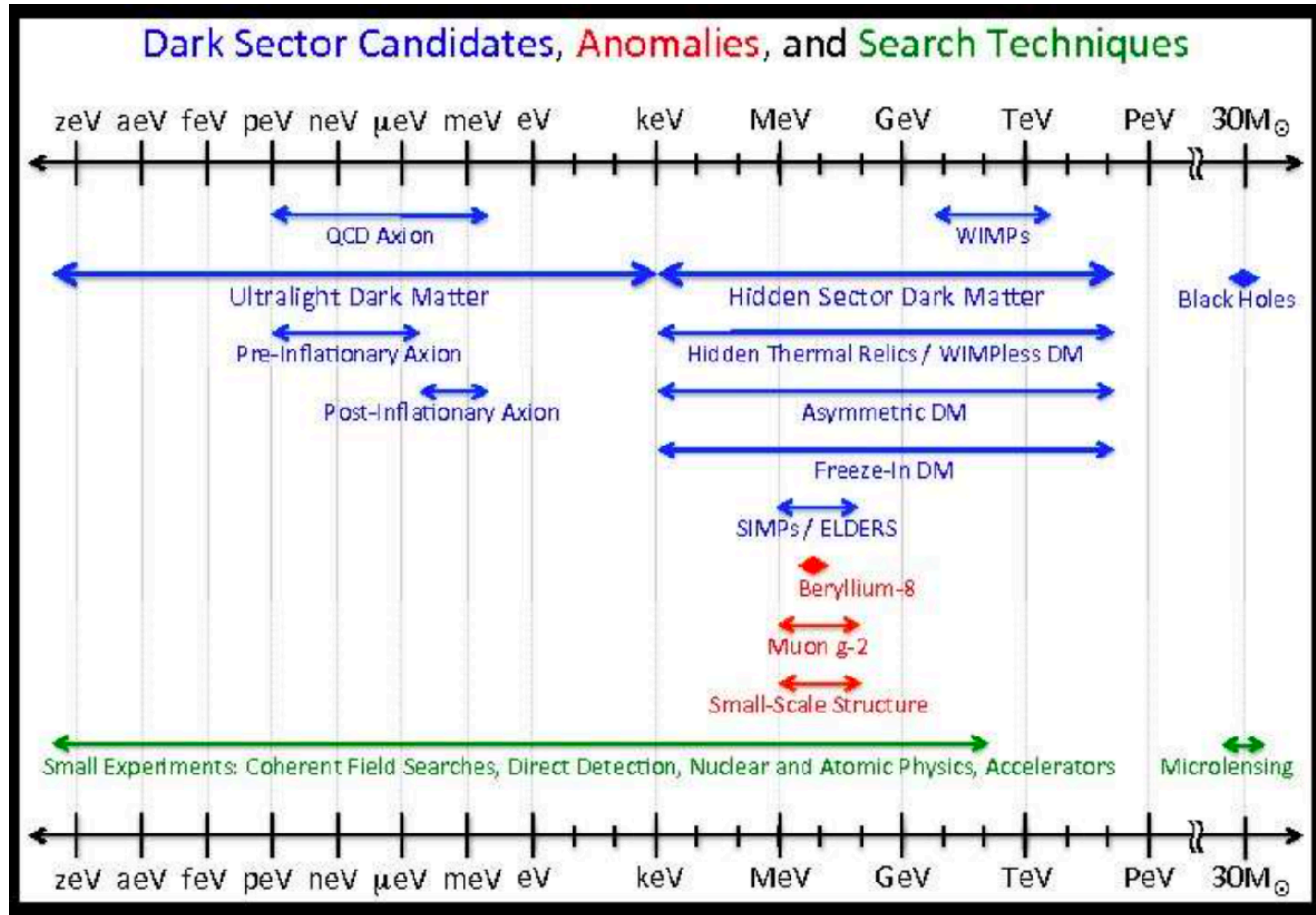
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 - -requires a systematic development of theories of gapped continuum DM
- Gapped Continuum as a DM can give striking new experimental signatures in colliders and cosmic microwave background measurements
- The strong suppression of direct detection signals (will show later) reopens the possibility of a Z-mediated dark sector again (and also other continuum version of WIMP models).

Theories of DM ?

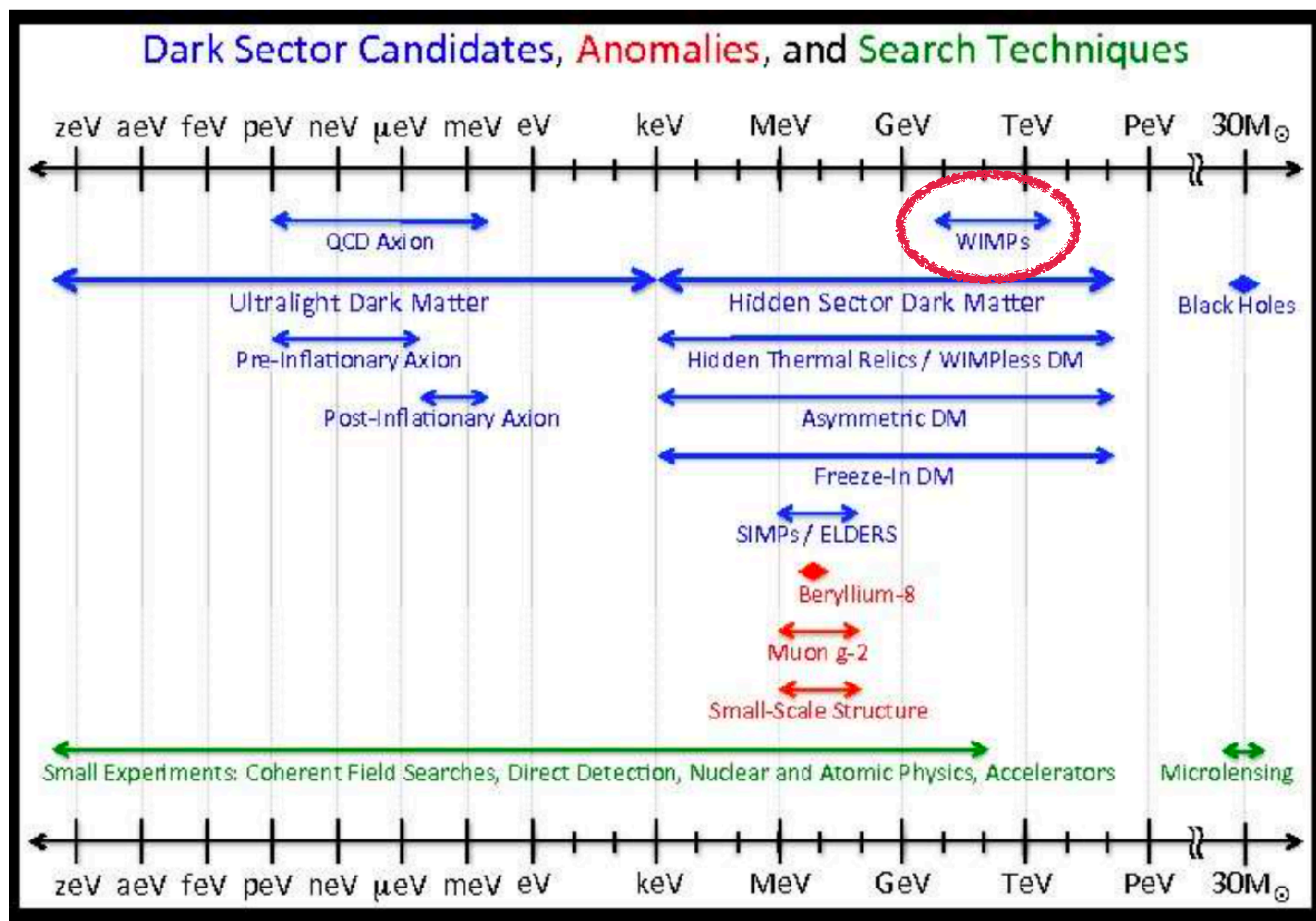


T. Tait

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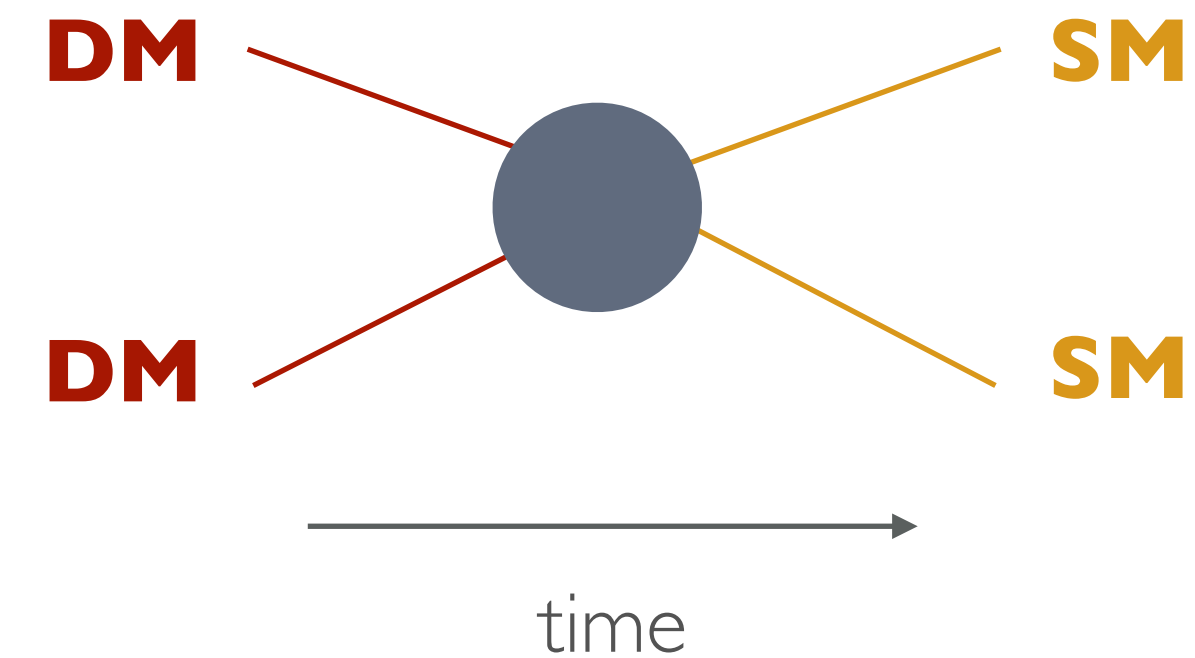


THE WIMP MIRACLE

Insensitive to the initial conditions of the Universe:

due to the thermal equilibrium between the DM and SM gases in the early Universe

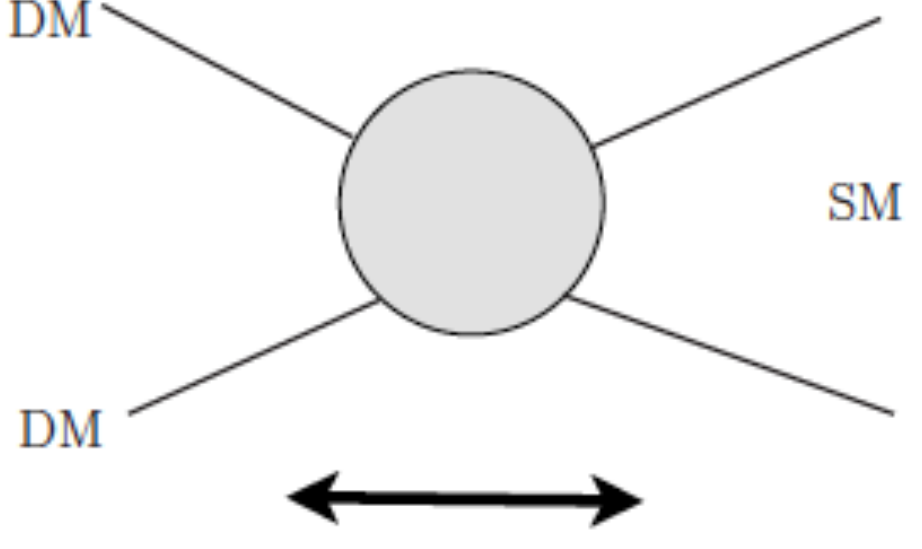
$$\text{Relic abundance} \propto \frac{1}{\text{ann. rate}}$$



**Correct relic abundance for
dark matter mass around the TeV scale
and weak-force interactions**

WIMP Dark Matter

- Original idea of WIMP Miracle

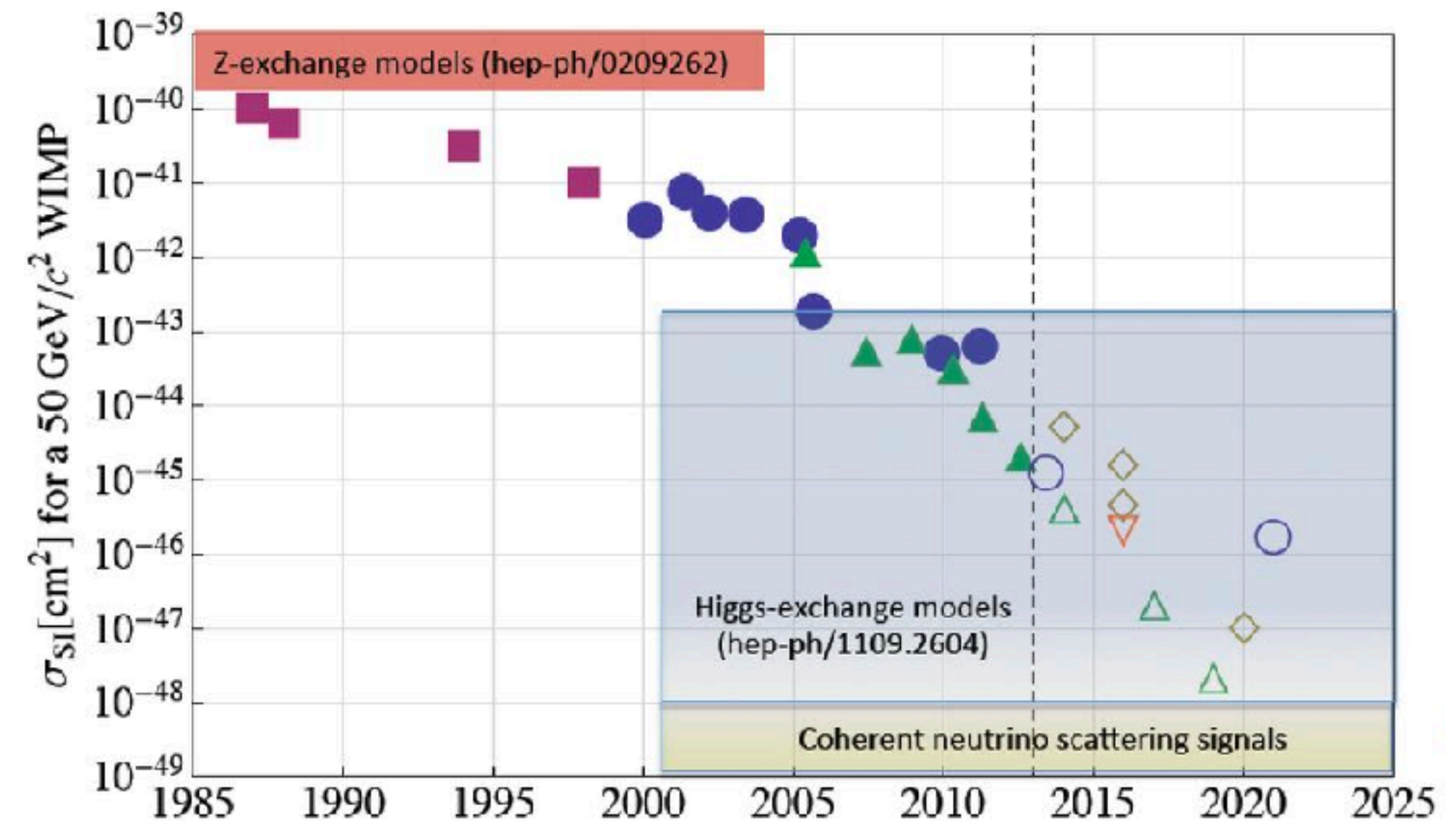
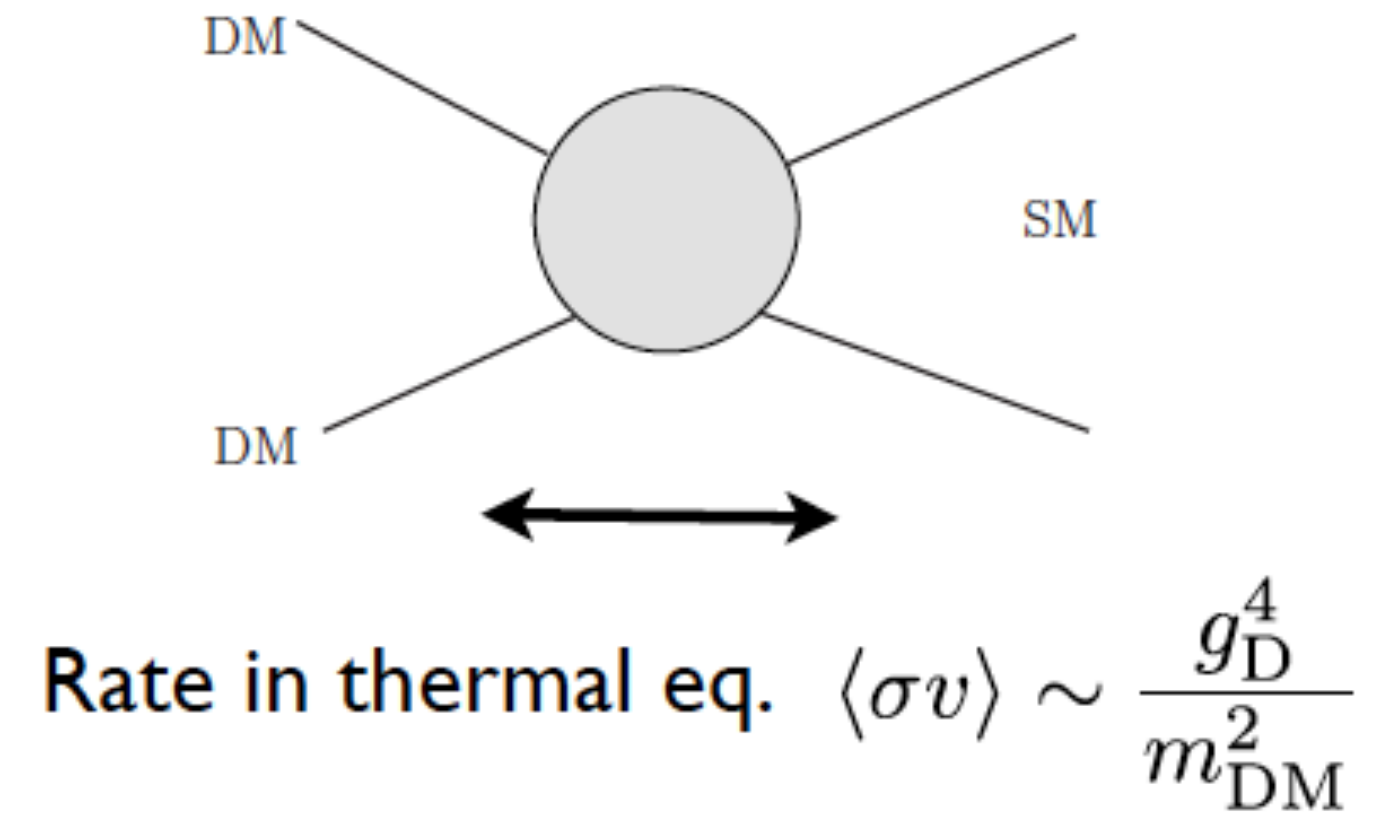


Rate in thermal eq. $\langle\sigma v\rangle \sim \frac{g_D^4}{m_{DM}^2}$

WIMP Dark Matter

- Original idea of WIMP Miracle
- => now pushed to a conner by the null results from DM direct detection experiments

Moore's Law works in DM!

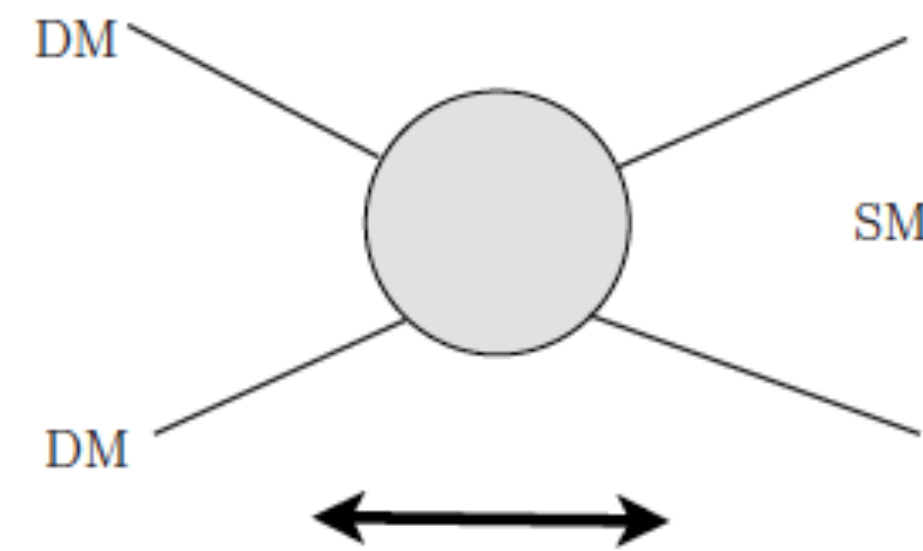


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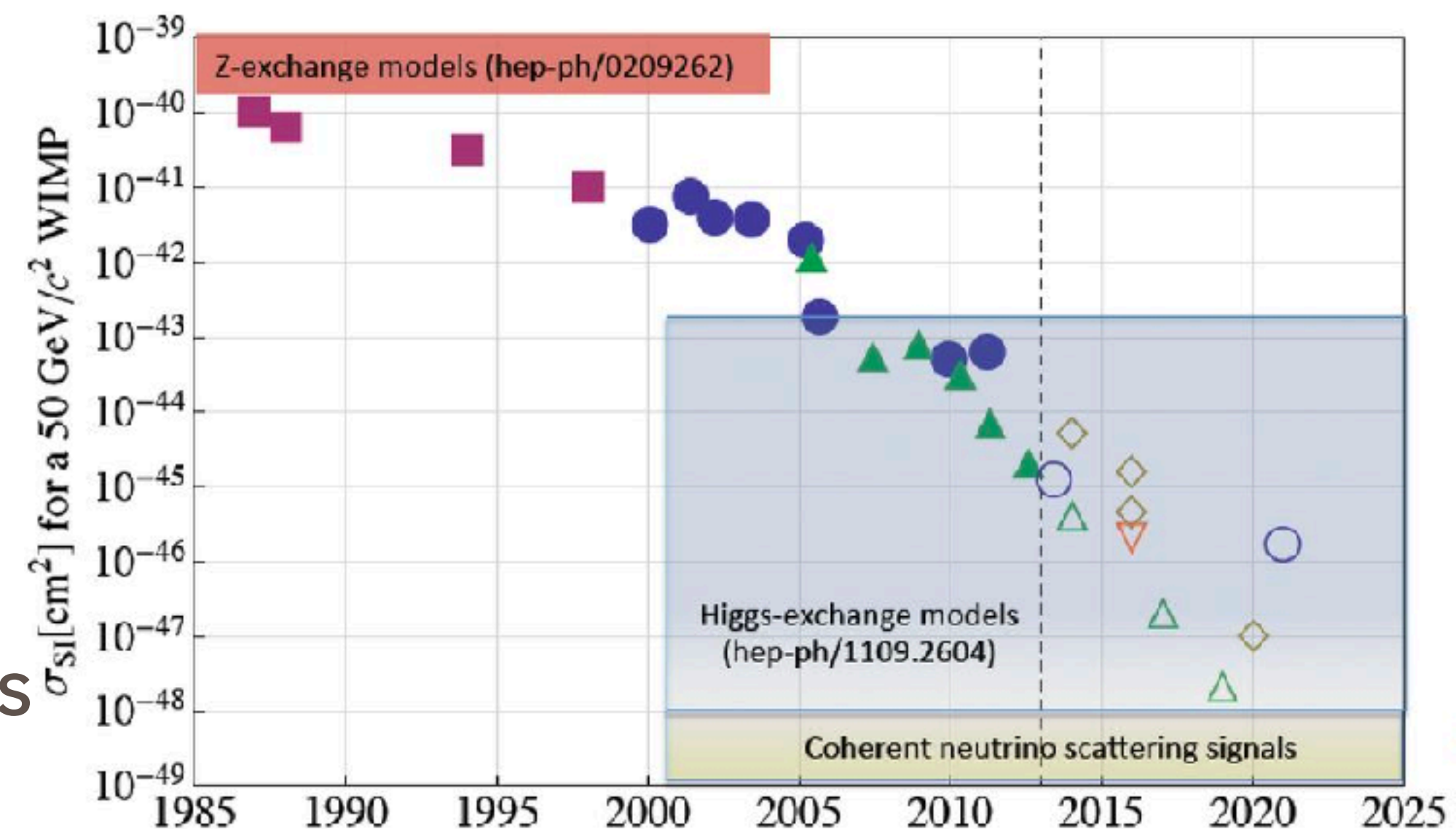
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Moore's Law works in DM!

- Z boson exchange excluded except for fine-tuned corners of parameter space, and requiring tuning for Higgs mediation as well



Rate in thermal eq. $\langle \sigma v \rangle \sim \frac{g_D^4}{m_{DM}^2}$



Been searching for WIMPs...

The dominant paradigm is being challenged.

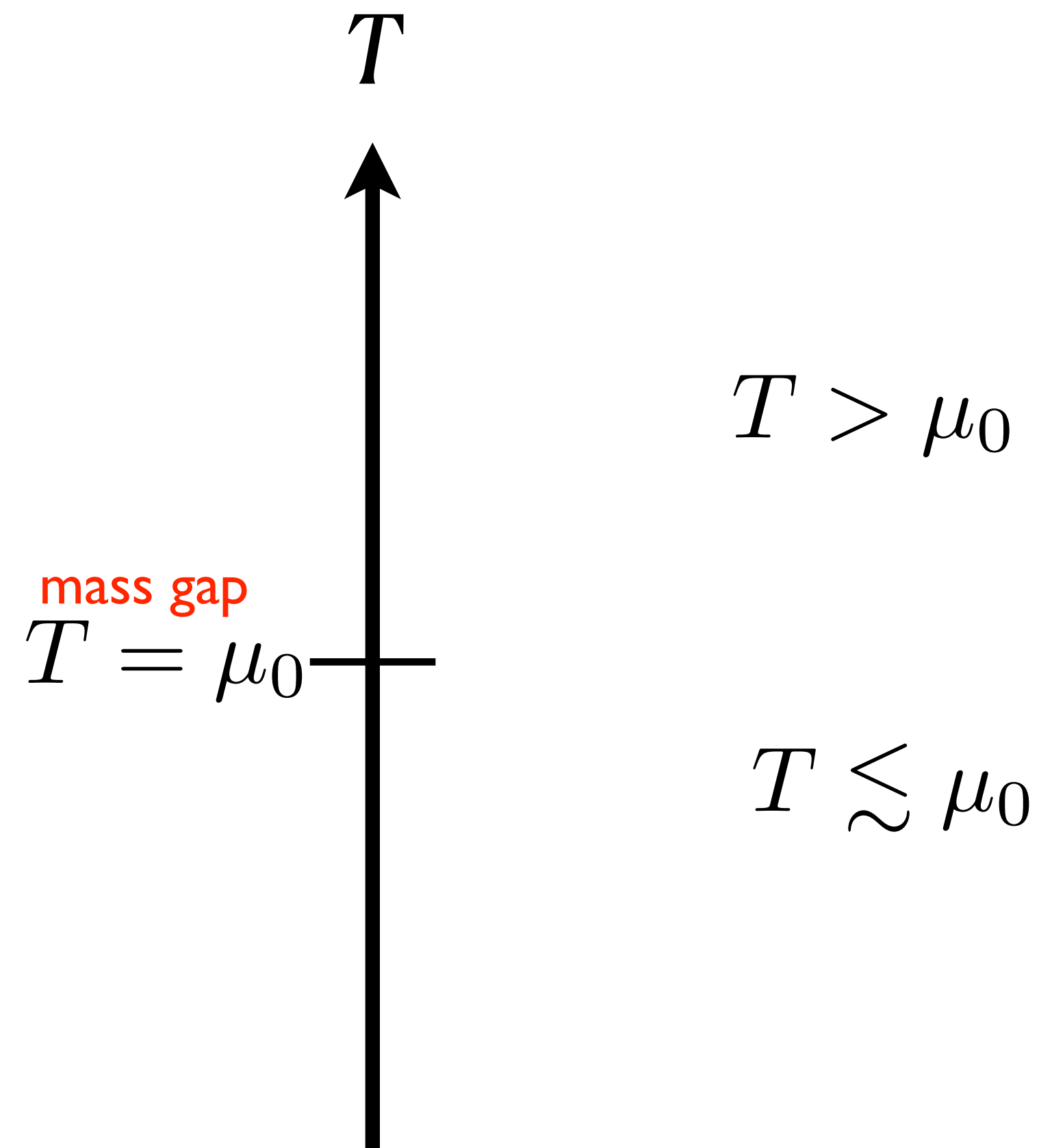
Is there another DM paradigm that gives qualitatively different signatures, but still provide the same level of simple, elegance and compelling explanation as WIMP?

Freeze-Out of Gapped Continuum DM

$$m_{\text{SM}} \ll \mu_0$$

annihilation: $\text{DM} + \text{DM} \leftrightarrow \text{SM} + \text{SM}$

quasi-elastic scattering (QES): $\text{DM} + \text{SM} \leftrightarrow \text{DM} + \text{SM}$



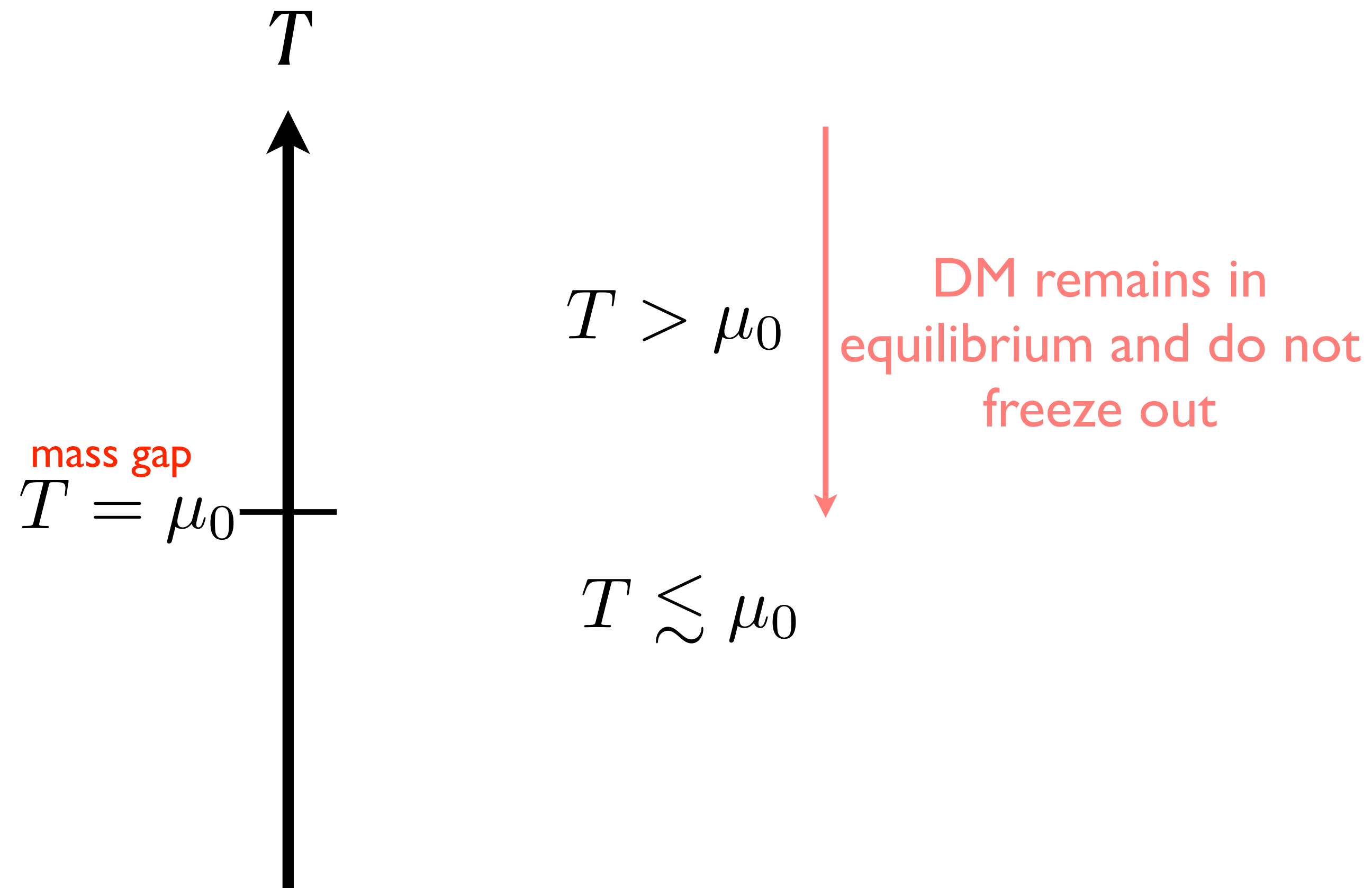
$$f_{\text{DM}} = e^{-\beta(E_{\mathbf{p}, \mu^2} - \lambda(t))}, \quad f_{\text{SM}} = e^{-\beta|\mathbf{p}|}.$$

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sufficiently strong
coupling between
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mass gap
 $T = \mu_0$

$$T > \mu_0$$

annihilation is in equilibrium, DM particles
are at the same temperature T as the SM
and is at zero chemical potential

DM remains in
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T



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DM remains in equilibrium and do not freeze out

$$T \lesssim \mu_0$$

annihilation rate drops exponentially, and annihilations decouple

“Freeze out”

rate of quasi-elastic scattering of a DM particle does not experience an exponential drop : maintain thermal equilibrium between the SM and DM (same T , and chemical)

$$f_{\text{DM}} = e^{-\beta(E_{\mathbf{p}, \mu^2} - \lambda(t))}, \quad f_{\text{SM}} = e^{-\beta|\mathbf{p}|}.$$

Gapped Continuum Z-portal DM

◆ Z-portal Model (with Z_2 symmetry)

- Consider a complex scalar field Φ with no SM gauge quantum numbers (this plays the role of DM field, and is lifted to 5D)
- Add another complex scalar field χ which is a doublet under $SU(2)_L$ and carries $U(1)_Y$ charge $-1/2$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\Phi} + \mathcal{L}_{\chi} + \mathcal{L}_{\text{int}} \quad \text{includes couplings to the SM Z and } U(1)_Y$$

$$\mathcal{L}_{\Phi} = \Phi^\dagger(p) \Sigma(p^2) \Phi(p)$$

$$\mathcal{L}_{\chi} = (D_\mu \chi)^\dagger (D^\mu \chi) - m_\chi^2 \chi^\dagger \chi$$

$$\mathcal{L}_{\text{int}} = -\lambda \Phi \chi H + \text{c.c.}$$

spectral density: $\rho(p^2) = \frac{1}{\pi} \text{Im} \Sigma^{-1}(p^2)$

- When the Higgs gets a vev, \mathcal{L}_{int} -term induces mass mixing between Φ and the neutral components of χ . The mass eigenstates are

$$\tilde{\Phi} = \cos \alpha \Phi + \sin \alpha \chi^0, \quad \tilde{\chi}^0 = -\sin \alpha \Phi + \cos \alpha \chi^0.$$

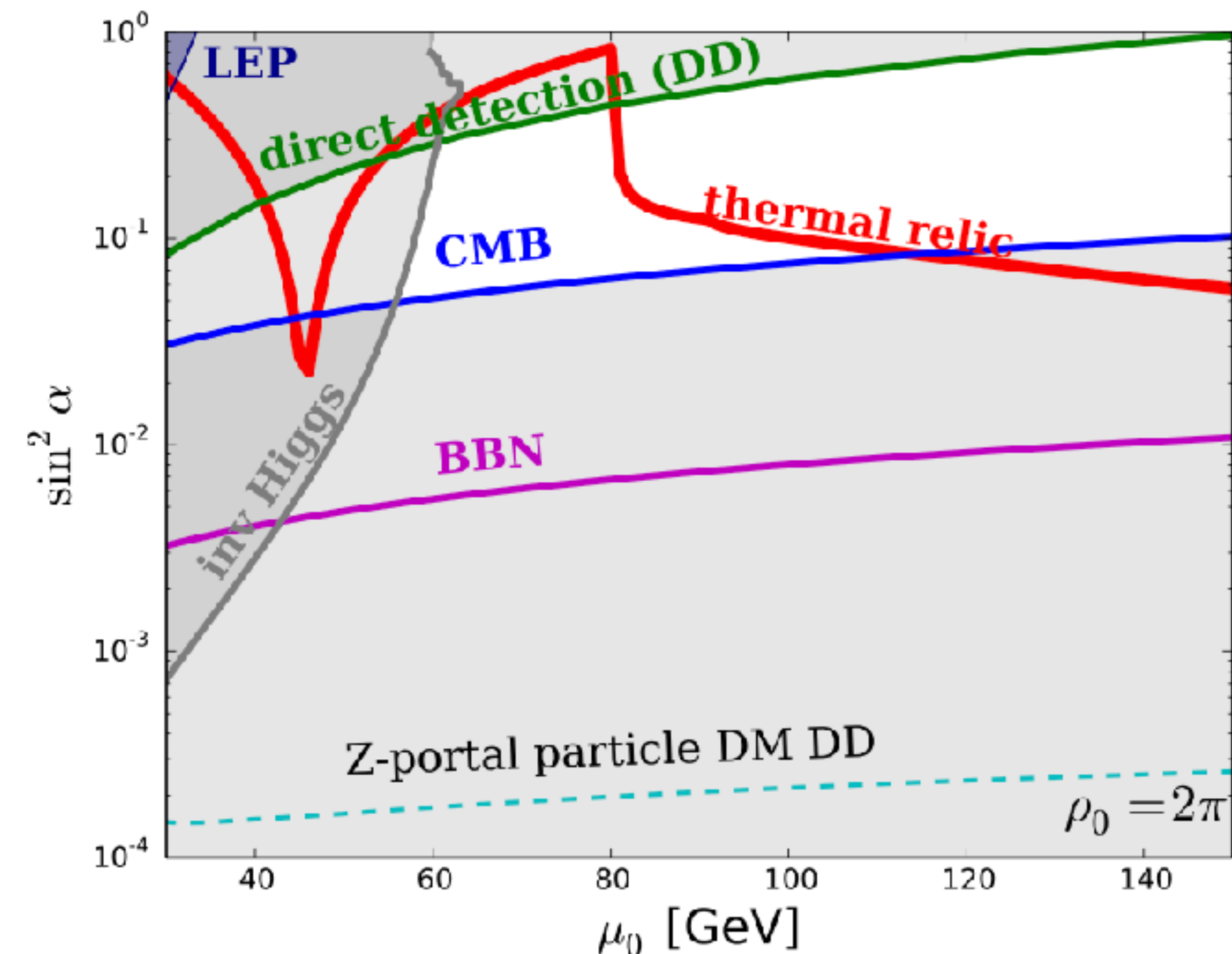
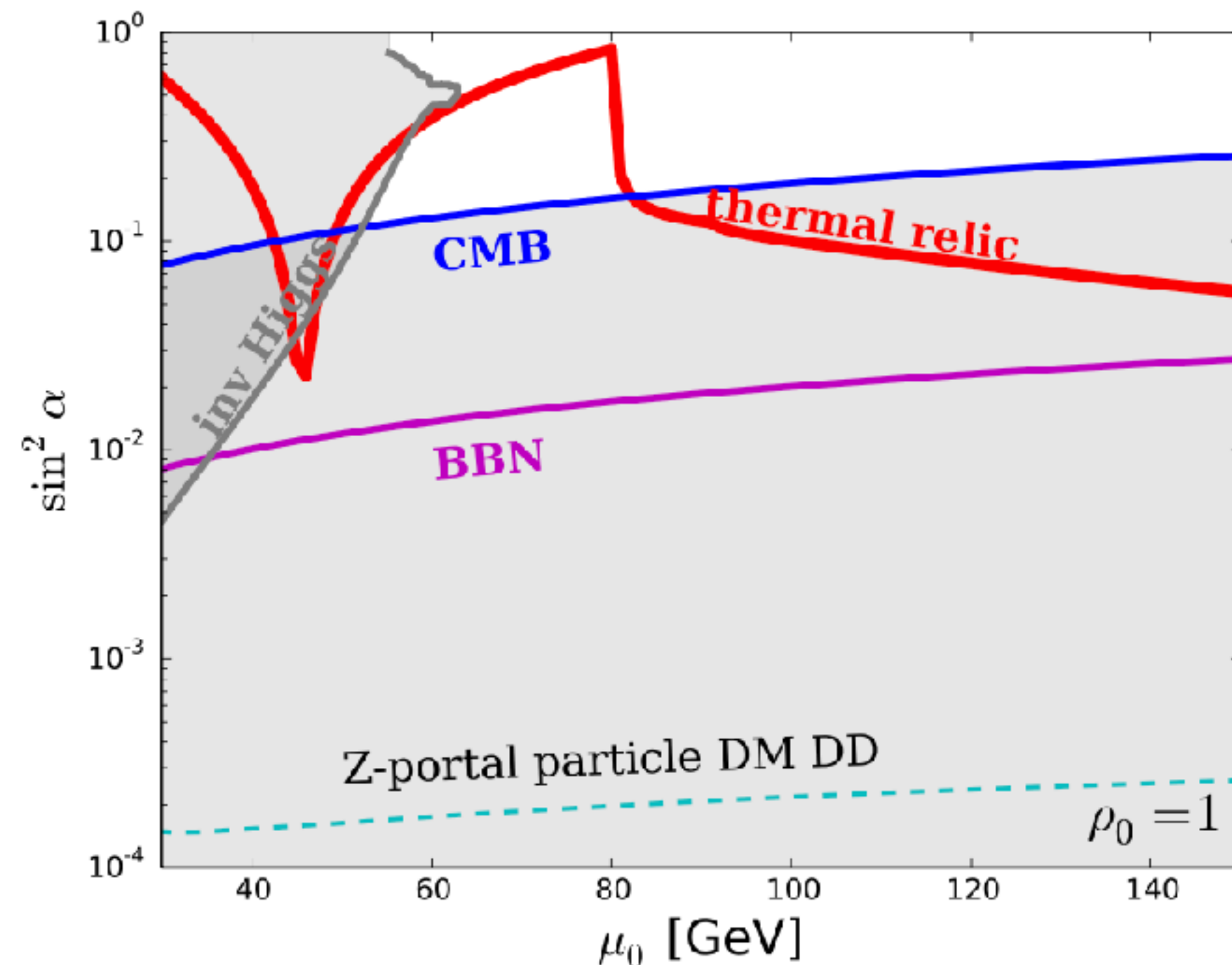
Gapped Continuum Z-portal DM

◆ Z-portal Model

$$\mathcal{L} = \sqrt{g^2 + g'^2} \sin^2 \alpha \left(\tilde{\Phi}_2 \partial_\mu \tilde{\Phi}_1 - \tilde{\Phi}_1 \partial_\mu \tilde{\Phi}_2 \right) Z^\mu$$

The mixing angle is given by

$$\tan 2\alpha = \frac{2\lambda v}{m_\phi^2 - m_\chi^2}$$



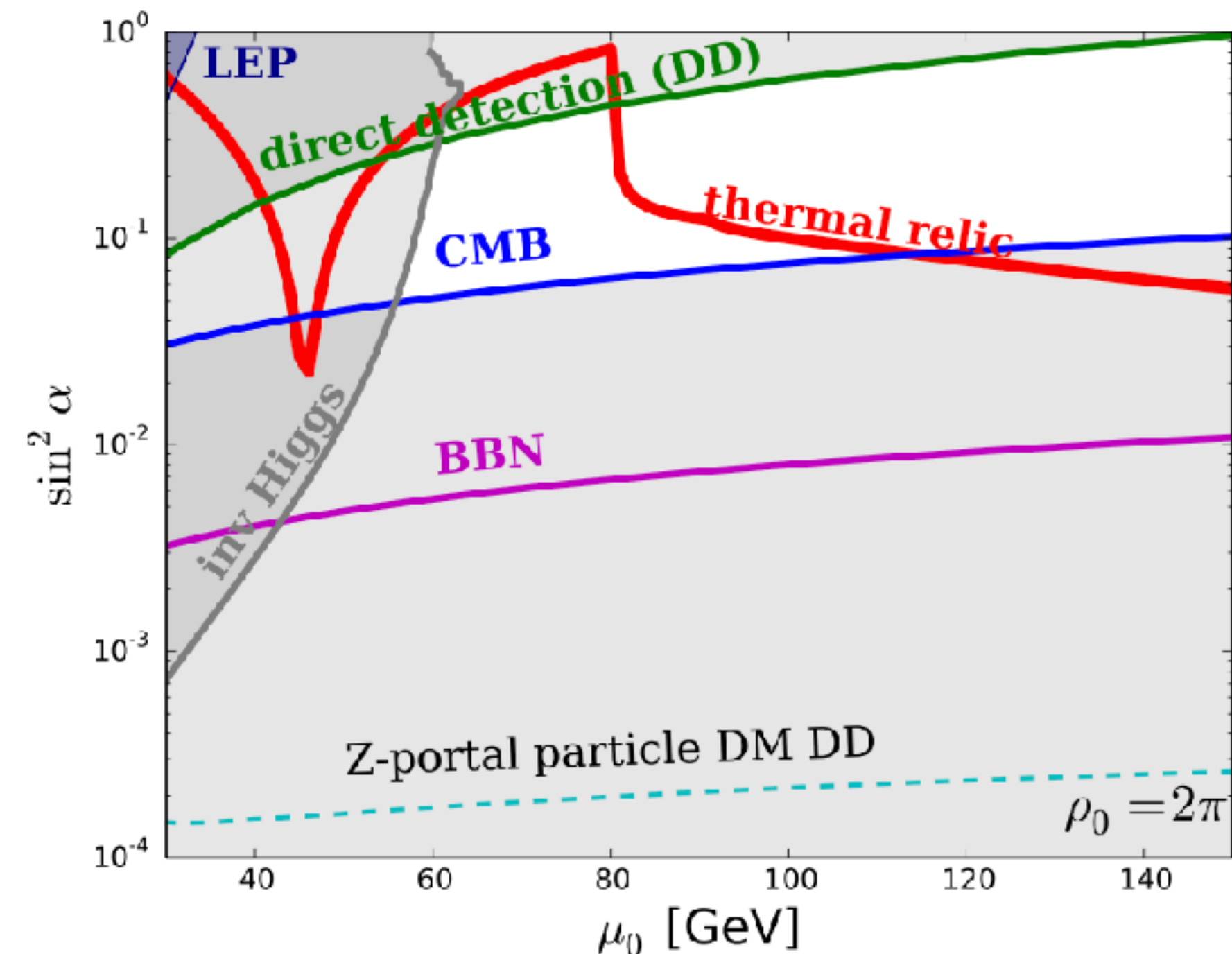
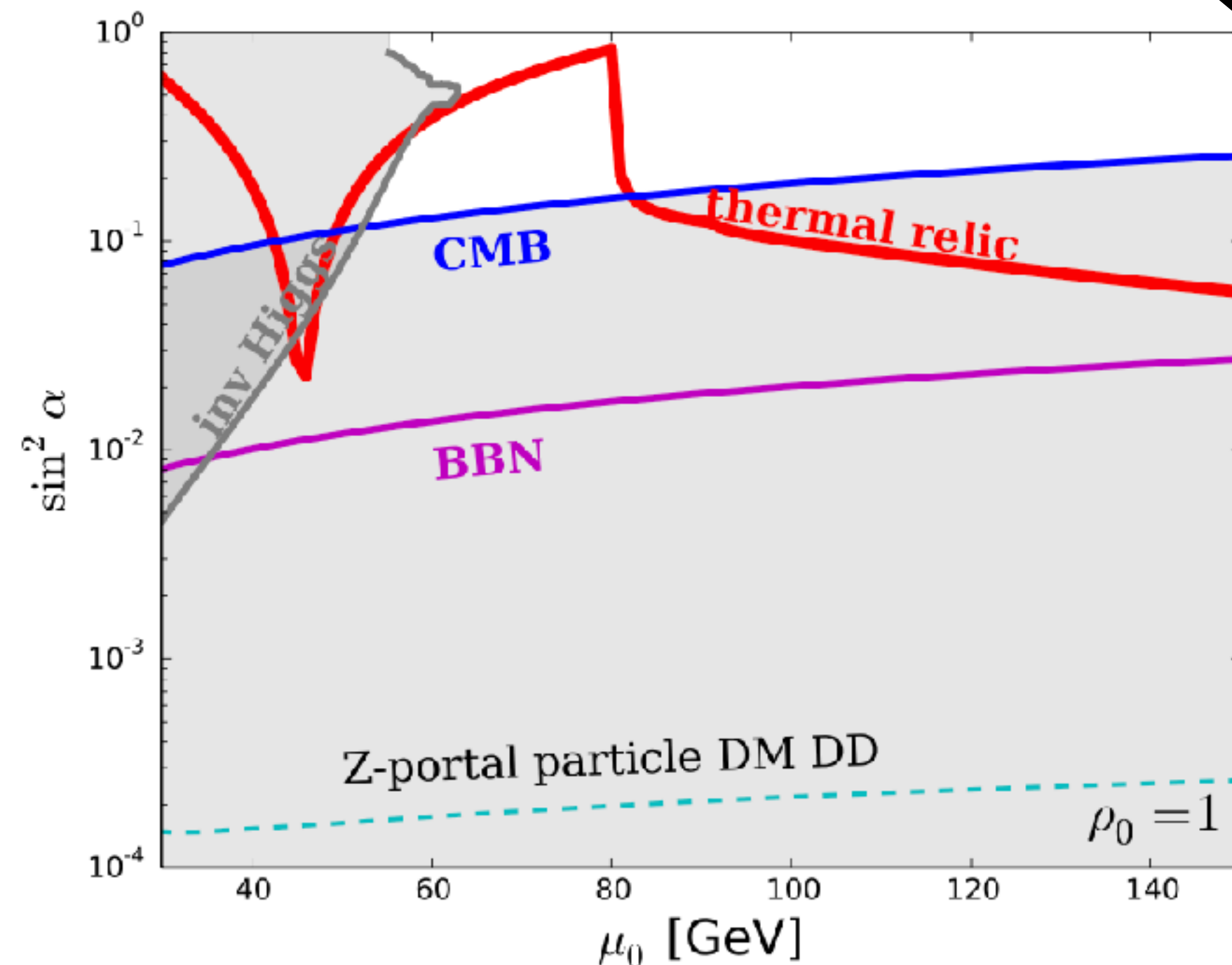
Gapped Continuum Z-portal DM

◆ Z-portal Model

$$\mathcal{L} = \sqrt{g^2 + g'^2} \sin^2 \alpha \left(\tilde{\Phi}_2 \partial_\mu \tilde{\Phi}_1 - \tilde{\Phi}_1 \partial_\mu \tilde{\Phi}_2 \right) Z^\mu$$

The salient feature of the gapped continuum DM: there is a generic rate suppression, which makes it compatible with the current null result of direct detection experiments.

$$\tan 2\alpha = \frac{2\lambda v}{m_\phi^2 - m_\chi^2}$$



Gapped Continuum Nature of DM

◆ Late decay

● decay within the continuum state: $DM(\mu_1) \rightarrow DM(\mu_2) + SM$

- Since all continuum states carry the same quantum number, such decays will necessarily occur continuously throughout the history of the universe.

-

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- In the early universe: DM in thermal and chemical equilibrium with the SM
- As temperature drops below the gap scale μ_0 , DM decouples from the SM and the total number of DM states is frozen out, just like for the usual thermal-relic particle DM
-
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Gapped Continuum Nature of DM

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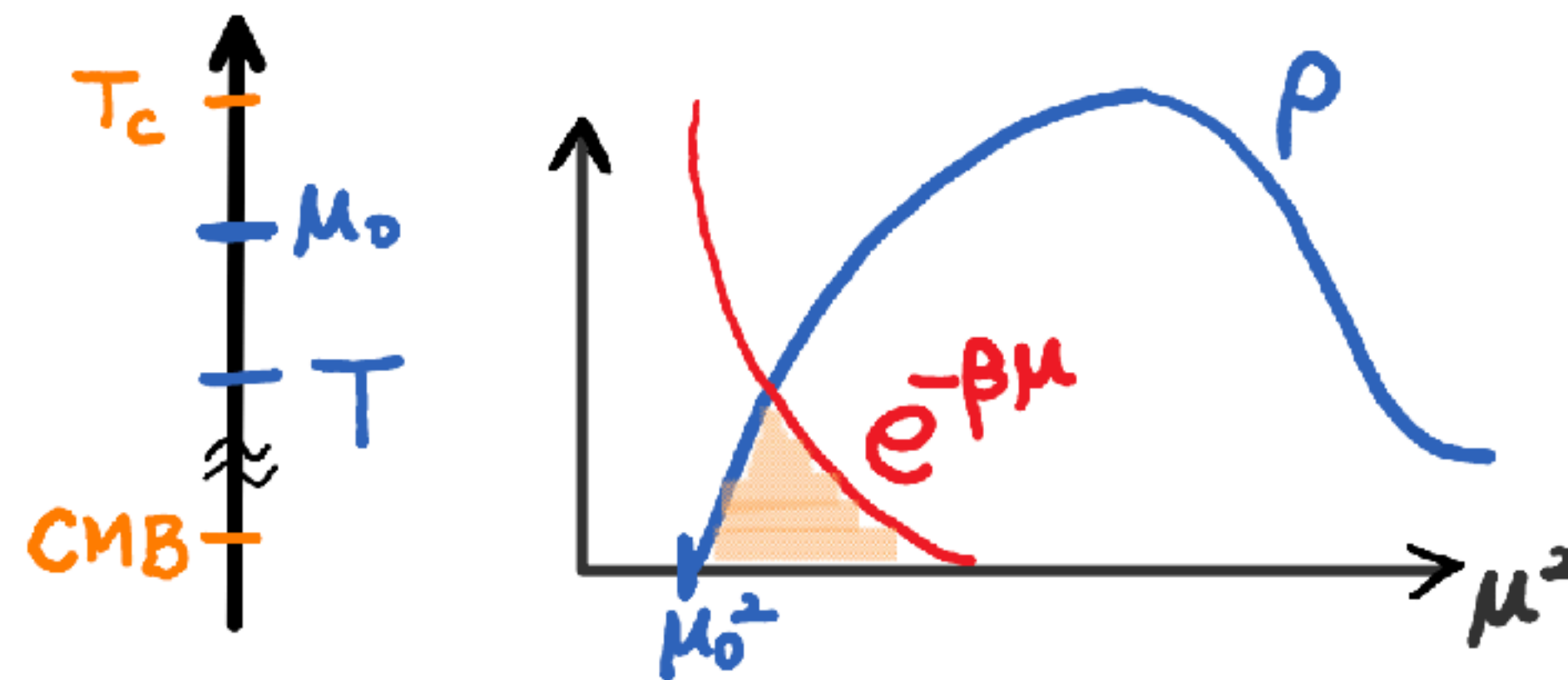
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- In the early universe: DM in thermal and chemical equilibrium with the SM
- As temperature drops below the gap scale μ_0 , DM decouples from the SM and the total number of DM states is frozen out, just like for the usual thermal-relic particle DM
- However, the mass distribution of the DM states continues to evolve, thanks to the above decays
- The decays shift the distribution towards lower masses, closer to the gap scale.
-
-

Gapped Continuum Nature of DM

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● decay within the continuum state: $\text{DM}(\mu_1) \rightarrow \text{DM}(\mu_2) + \text{SM}$



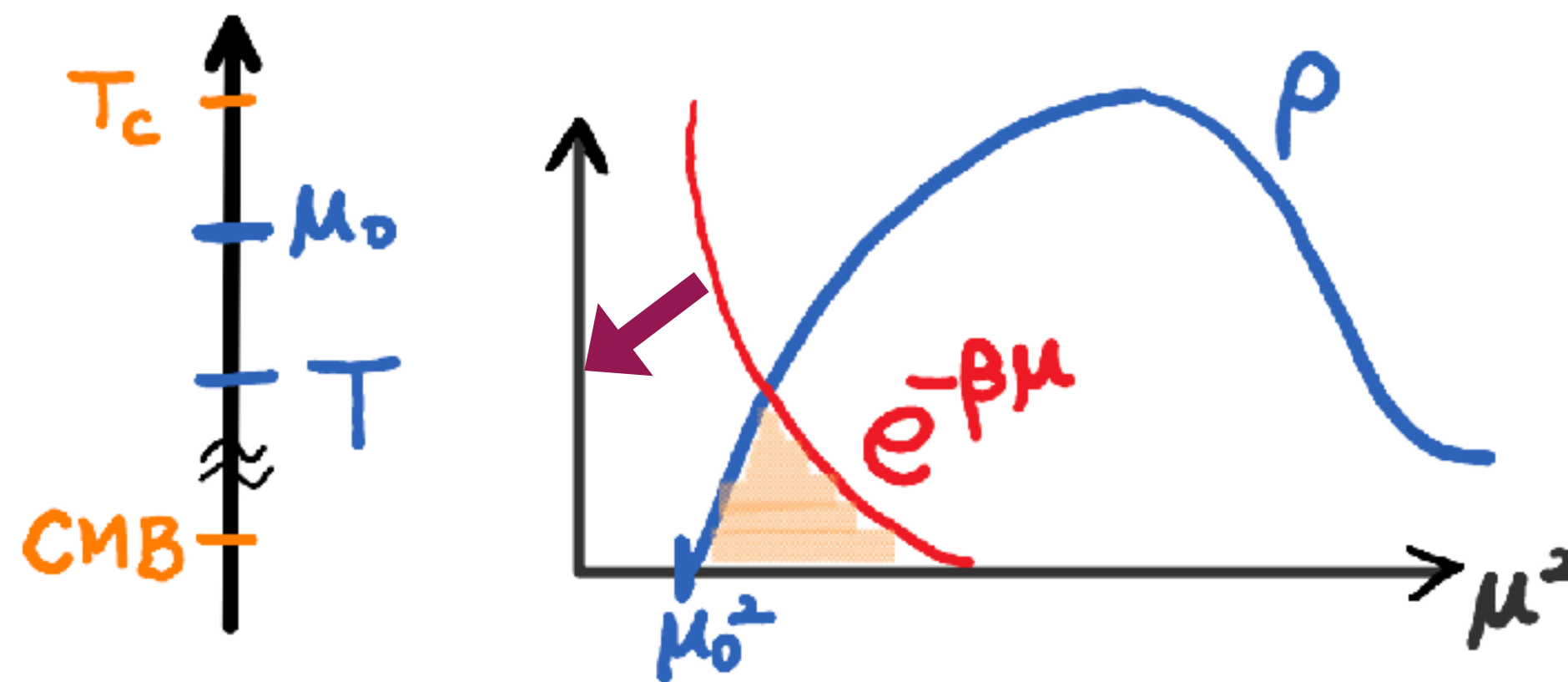
$$n = \int \frac{d\mu^2}{2\pi} \int \frac{d^3p}{(2\pi)^3} \rho(\mu^2) e^{-\beta E_\mu}$$

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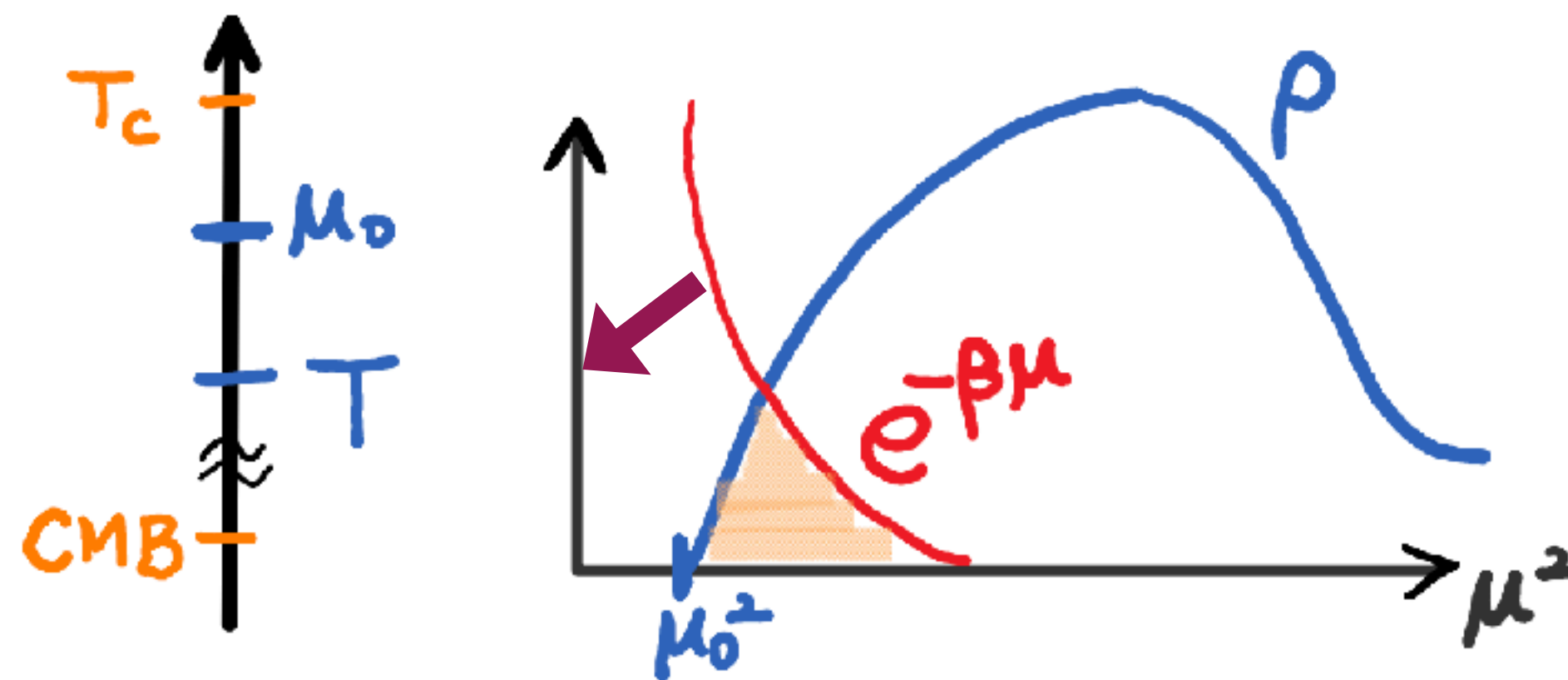
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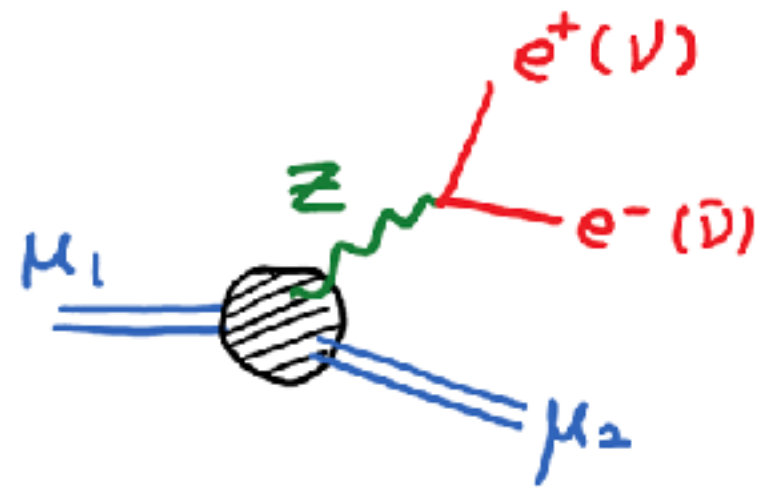


$$n = \int \frac{d\mu^2}{2\pi} \int \frac{d^3p}{(2\pi)^3} \rho(\mu^2) e^{-\beta E_\mu}$$

- The decays shift the distribution towards lower masses, closer to the gap scale.
- Lifetime of a DM state increases with decreasing mass, due to both phase-space suppression and the fact that there are fewer states for it to decay into.
- e.g. in our model, DM states are currently clustered within a few hundred keV above the gap scale (on average, each DM state undergoes roughly one decay per Hubble time)

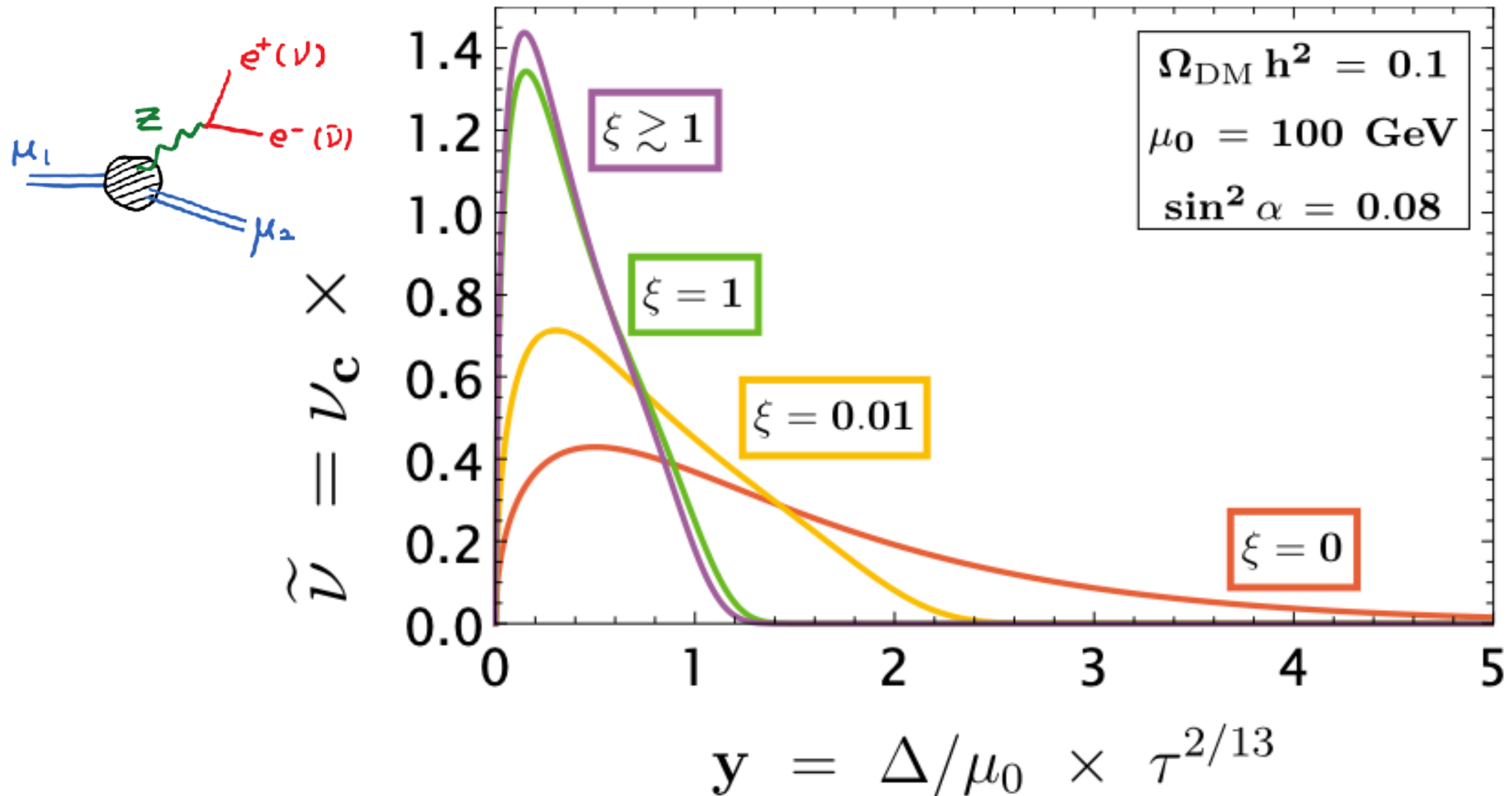
Gapped Continuum Nature of DM

- ◆ Late decay ($T > T_{\text{CMB}}$)



Gapped Continuum Nature of DM

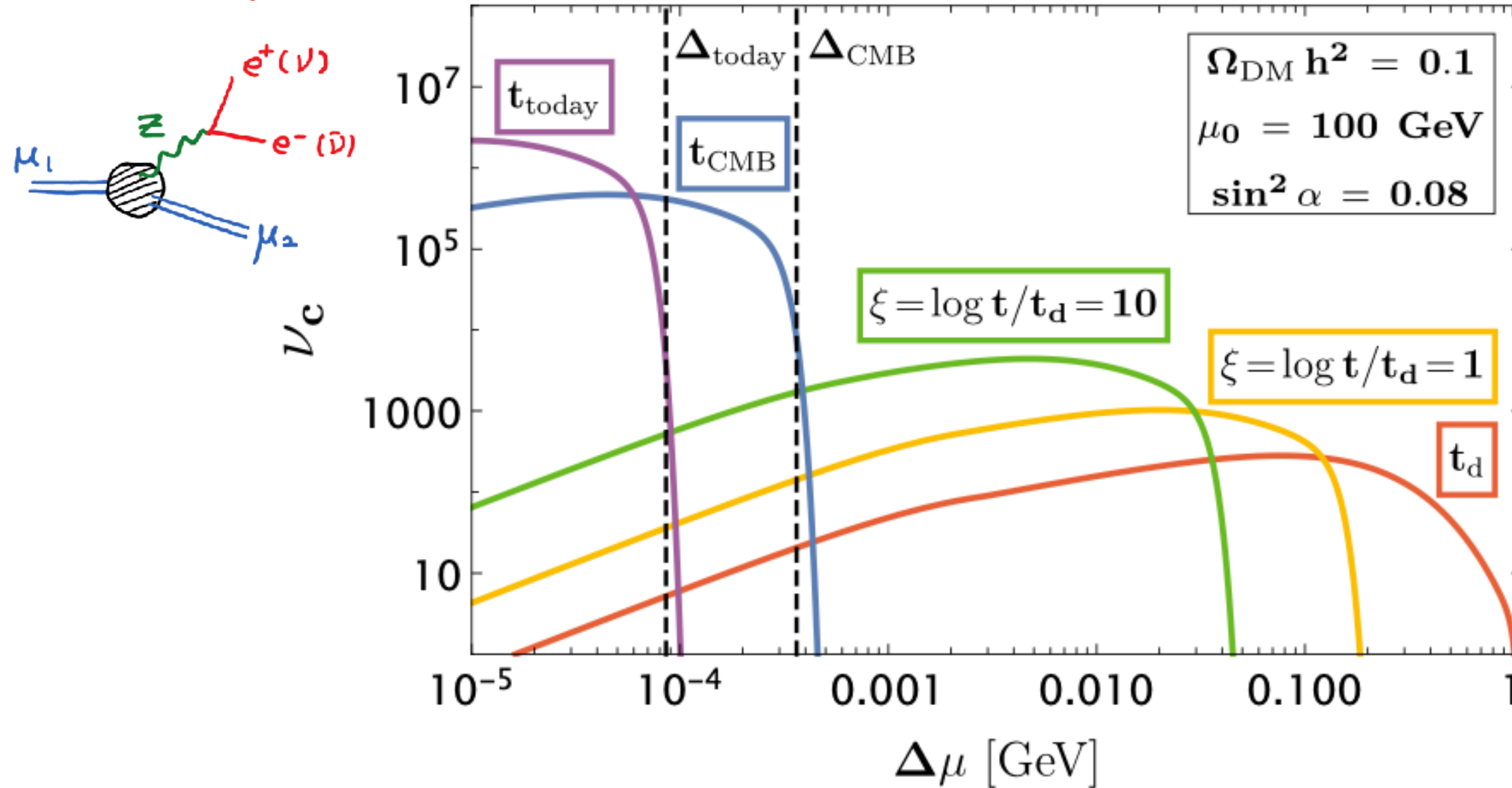
◆ Late decay ($T > T_{\text{CMB}}$)



$\xi = \log(t/t_d)$, where $\tau = \Gamma_0 t$ and t_d is the time at decoupling
 $\Delta = \mu - \mu_0$

Gapped Continuum Nature of DM

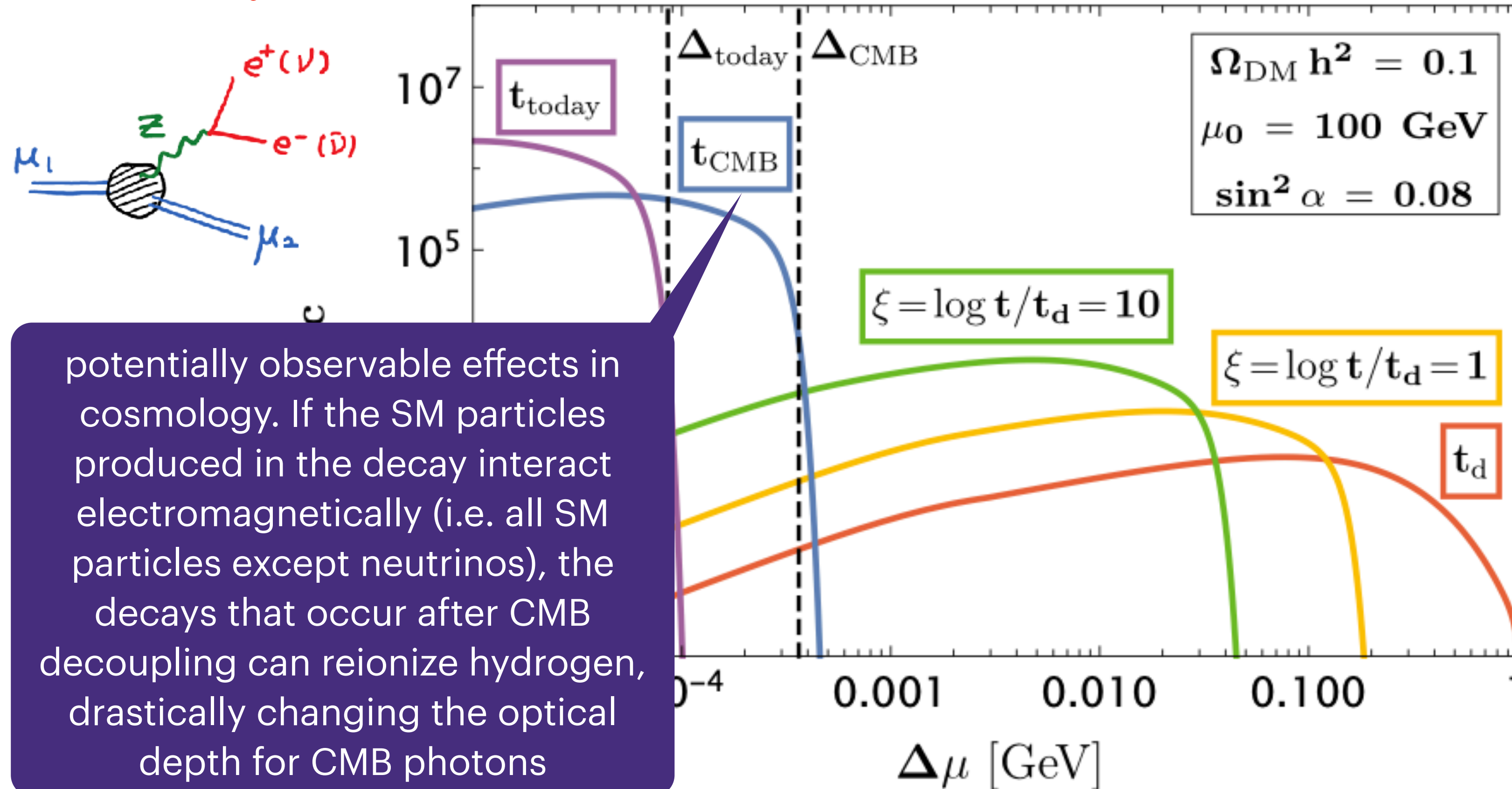
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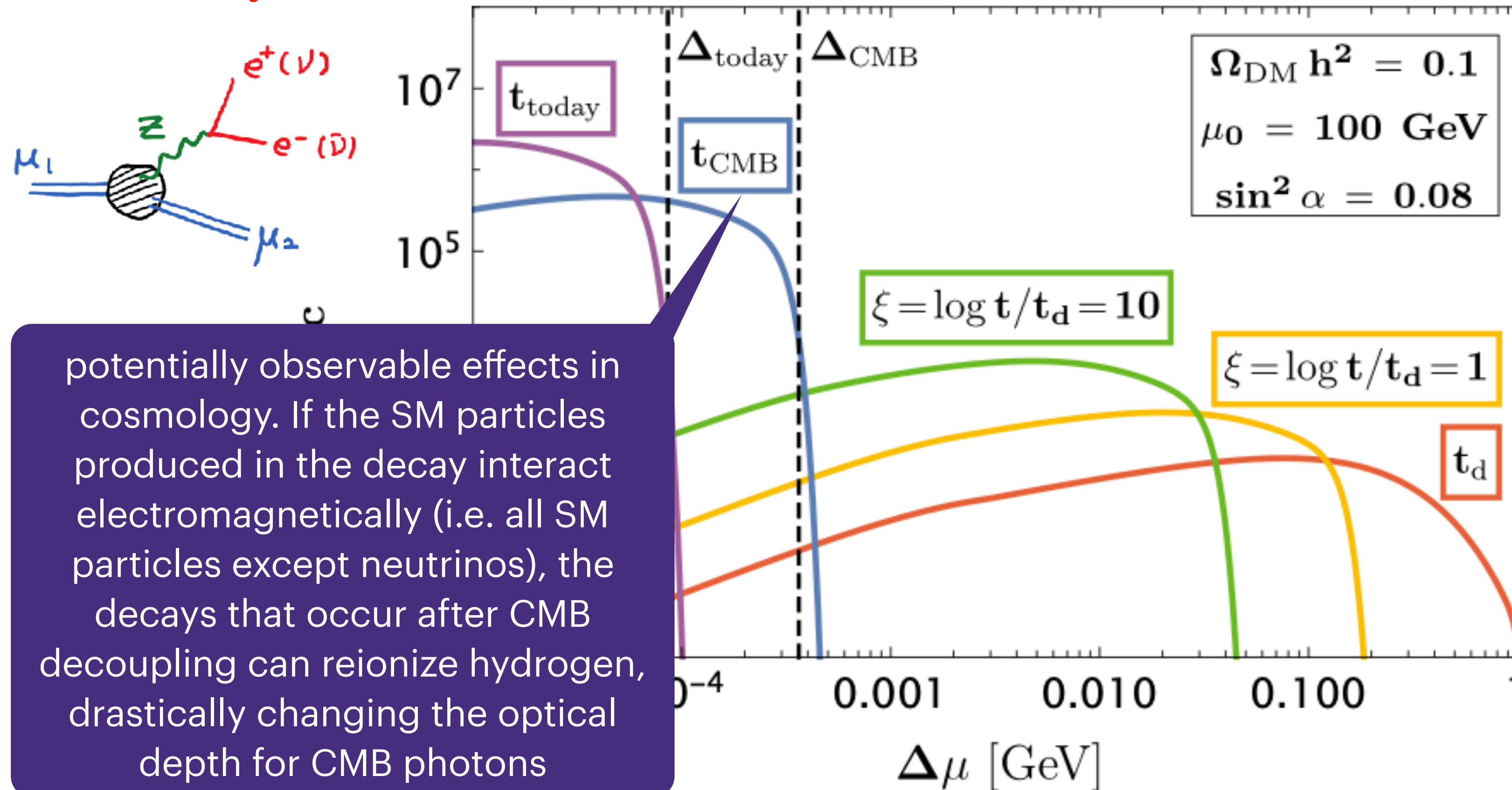
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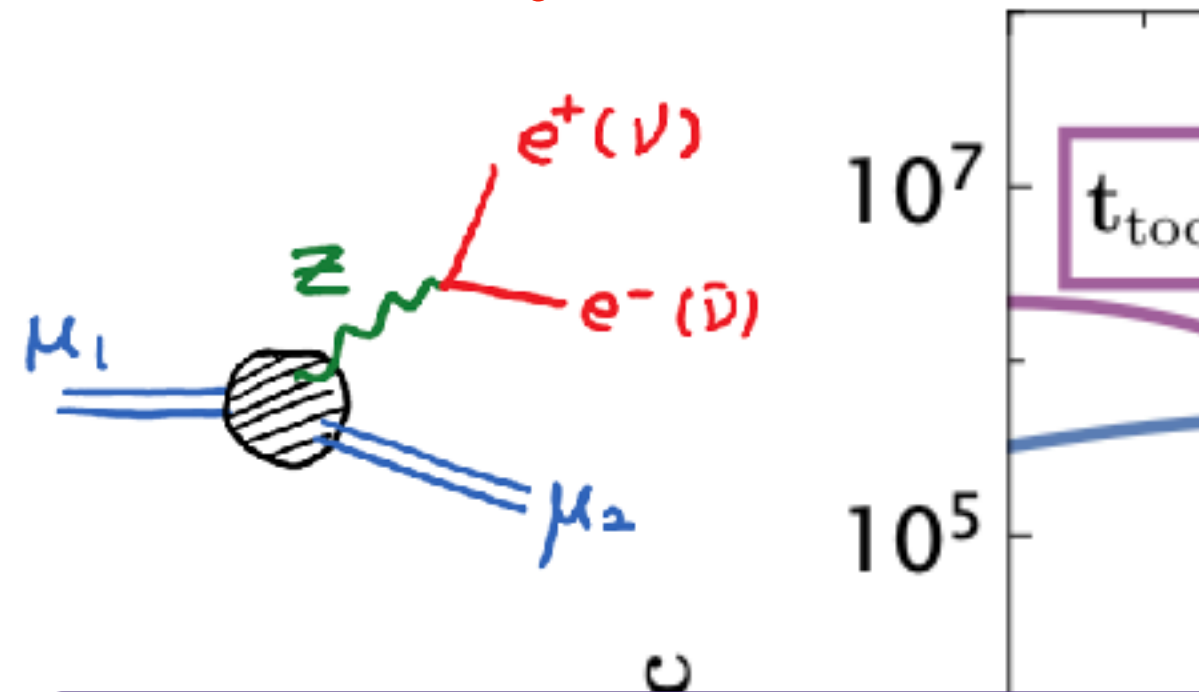
◆ Late decay ($T > T_{\text{CMB}}$)



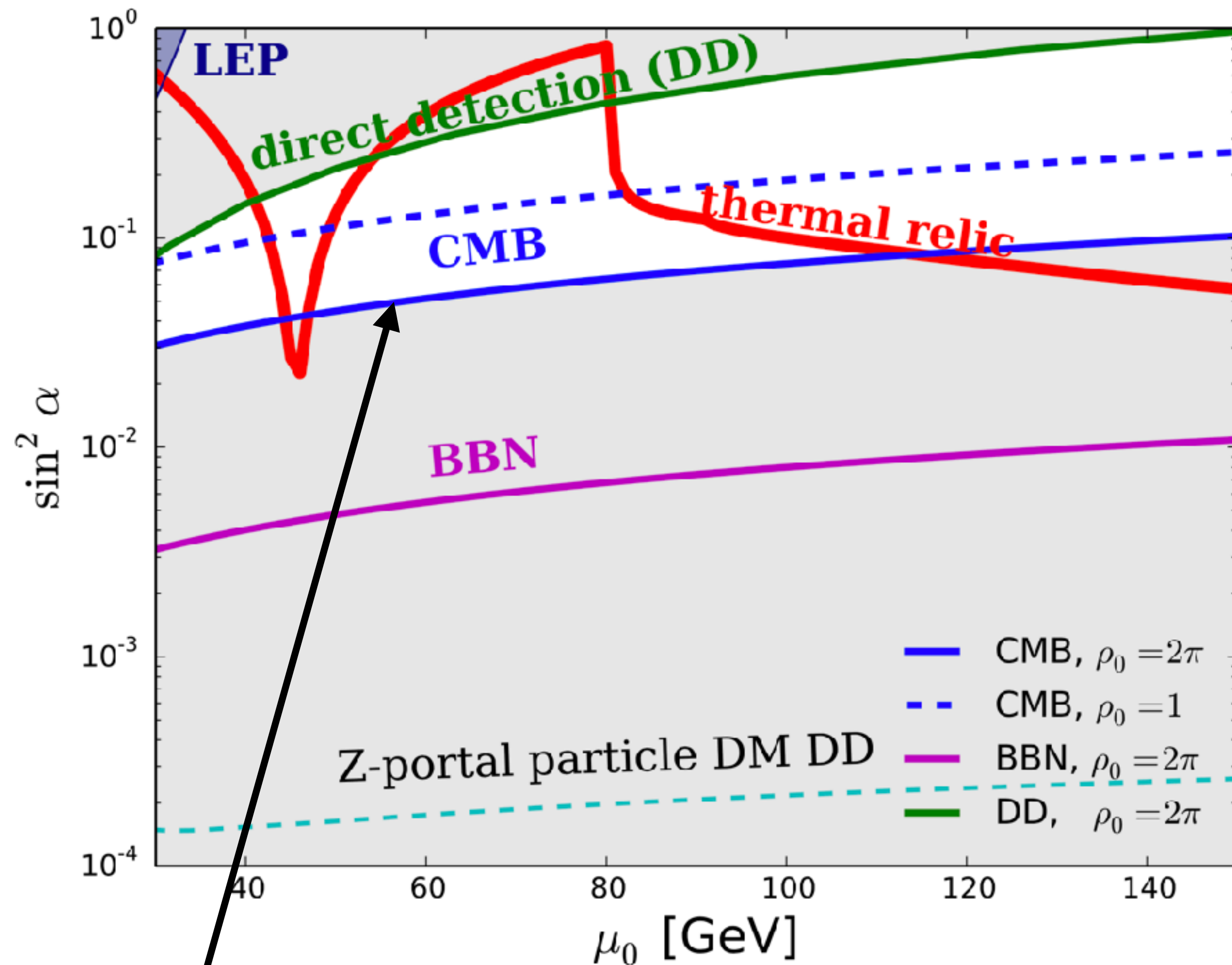
To avoid this, late decay (to e^+e^-) needs to end by $T > T_{\text{CMB}}$
=> gives a **lower bound** on the effective coupling

Gapped Continuum Nature of DM

◆ Late decay ($T > T_{\text{CMB}}$)



potentially observable effect in cosmology. If the SM particles produced in the decay interact electromagnetically (i.e. all SM particles except neutrinos), the late decays that occur after CMB decoupling can reionize hydrogens, drastically changing the optical depth for CMB photons



To avoid this, late decay (to e^+e^-) needs to end by $T > T_{\text{CMB}}$
 \Rightarrow gives a **lower bound** on the effective coupling

Gapped Continuum Nature of DM

◆ Direct detection

- quasi elastic scattering (QES): $DM(\mu_1) + SM_1 \rightarrow DM(\mu_2) + SM_2$
 - even after freeze out, distribution of DM state keeps evolving: distribution is peaked at the mass gap (μ_0) at very late time (these decays are important for CMB physics), and DM states pass through the earth with non-relativistic speed ($v \sim 10^{-3}$)

Gapped Continuum Nature of DM

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=> If incoming DM state has $\mu_1 = \mu_0 + \Delta$, accessible final continuum modes are in very narrow window $\mu_2 \in [\mu_0, \mu_0 + \Delta + Q]$. For weak scale μ_0 , this basically means that the integral appearing in the QES cross section is constrained to a tiny interval in μ , and leads to a **significant suppression** of the rate

$$\sigma \sim \int \frac{d\mu_2^2}{2\pi} \rho(\mu_2^2) \hat{\sigma}(\mu_1, \mu_2)$$

Gapped Continuum Nature of DM

◆ Direct detection

● quasi elastic scattering

- even after freeze-out
- distribution is peaked
- decays are important
- the earth with mass

Q is the kinetic energy of the collision in the center-of-mass frame

$\Delta \ll \mu_0$ in today's universe, while $Q \ll \mu_0$ as long as ambient DM is non-relativistic.

$$\sigma_{\text{cont}} \sim \left(\frac{\Delta + Q}{\mu_0} \right)^{1+r} \sigma_{\text{particle}}$$

e.g. $\Delta \sim 100$ keV at the present time, while $Q \sim 1$ keV
 μ_0 at the weak scale $\rightarrow \sim 10^9$ suppression

r is a positive number that depends on the behavior of the spectral density near the gap ($r=1/2$ for XD)

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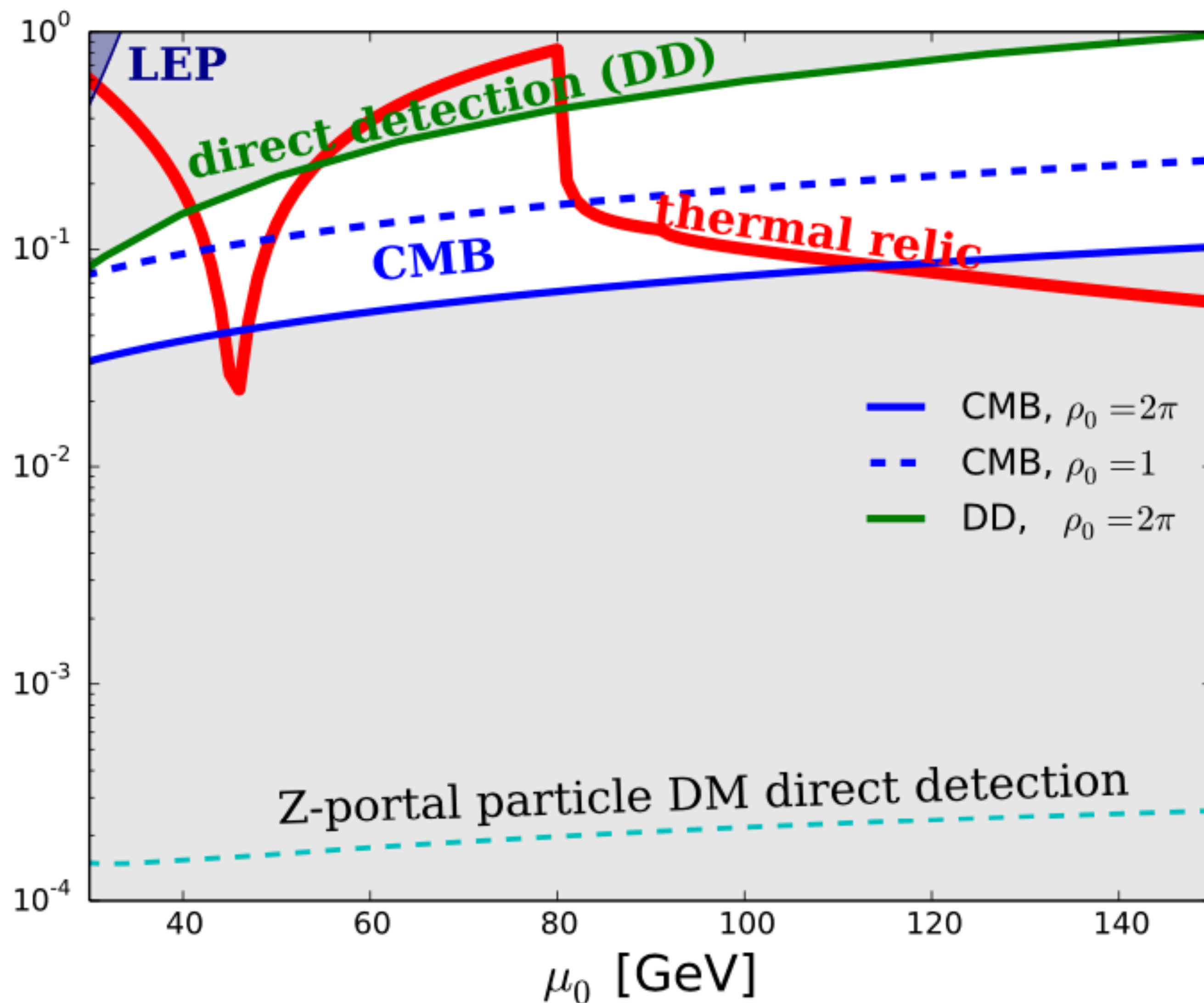
Gapped Continuum Nature of DM

◆ **Direct detection**

● quasi elastic sca

— even after fi
distribution

decays are i
the earth wi $\sin^2 \alpha$



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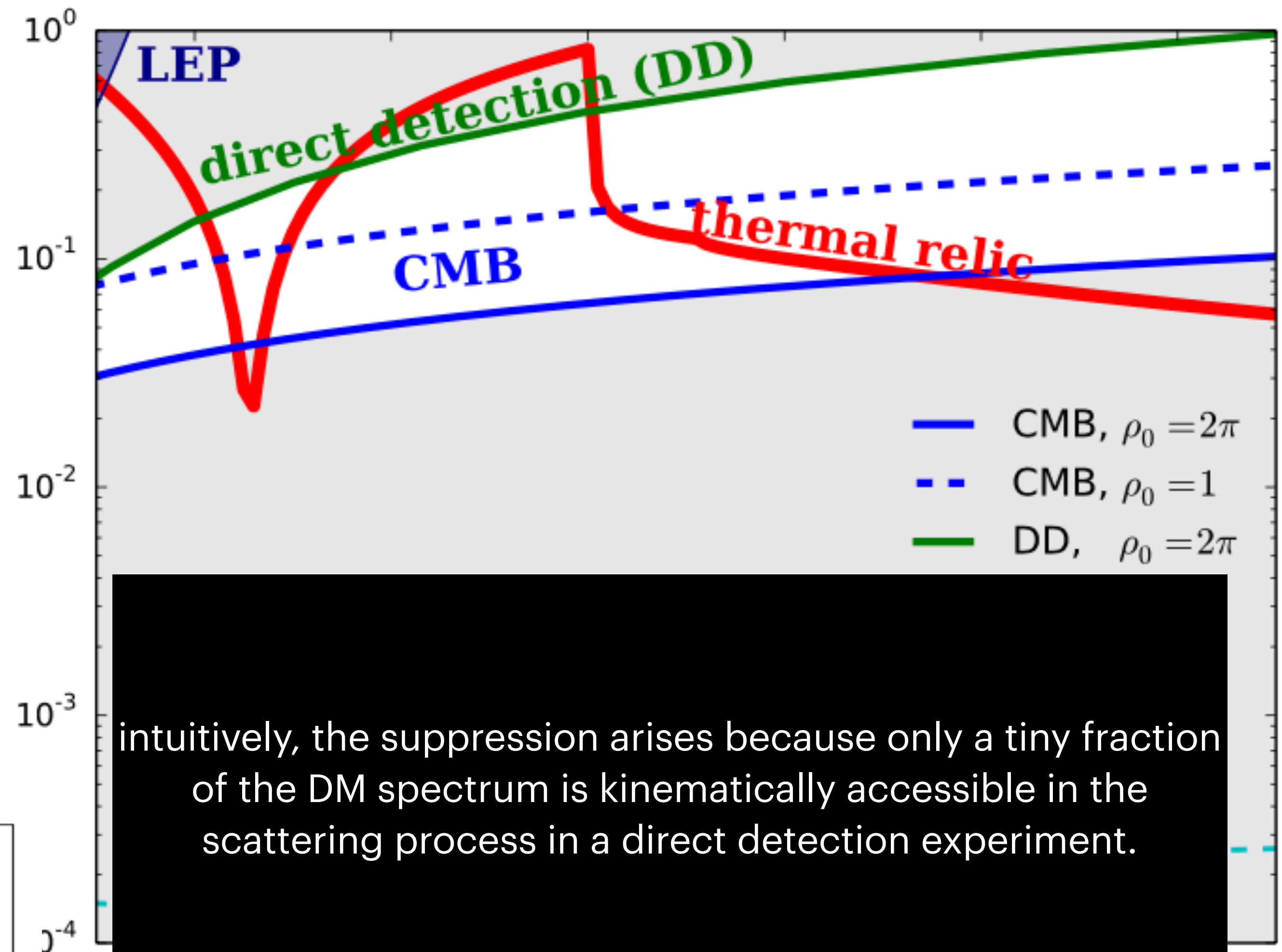
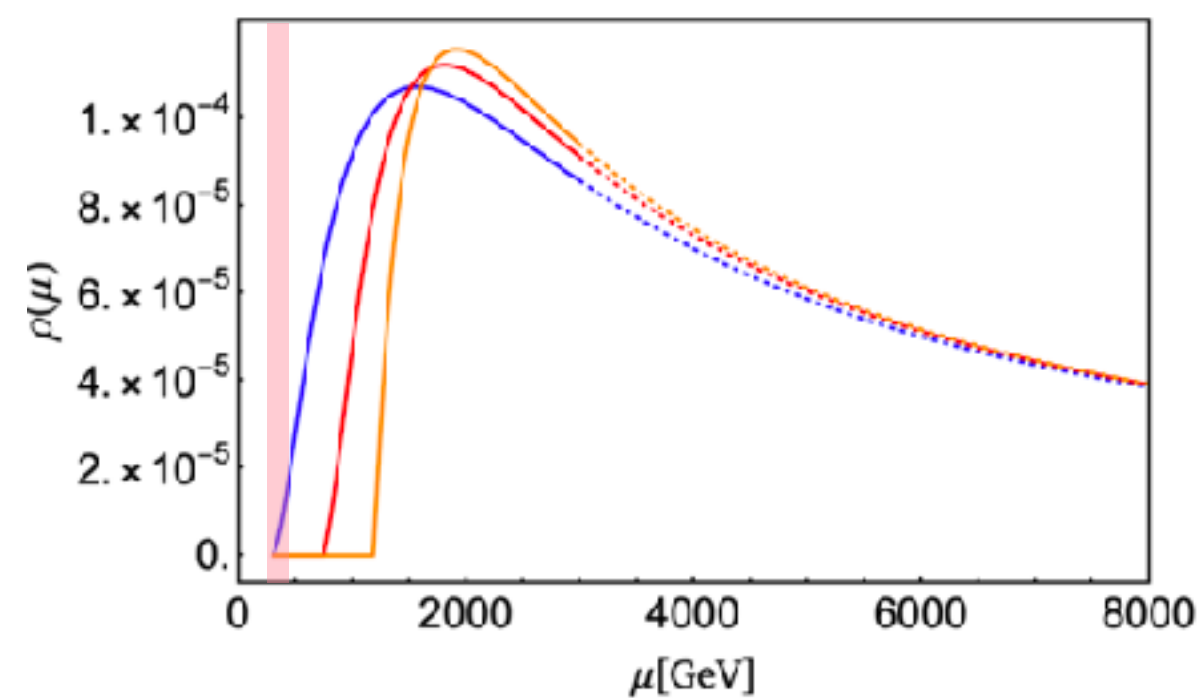
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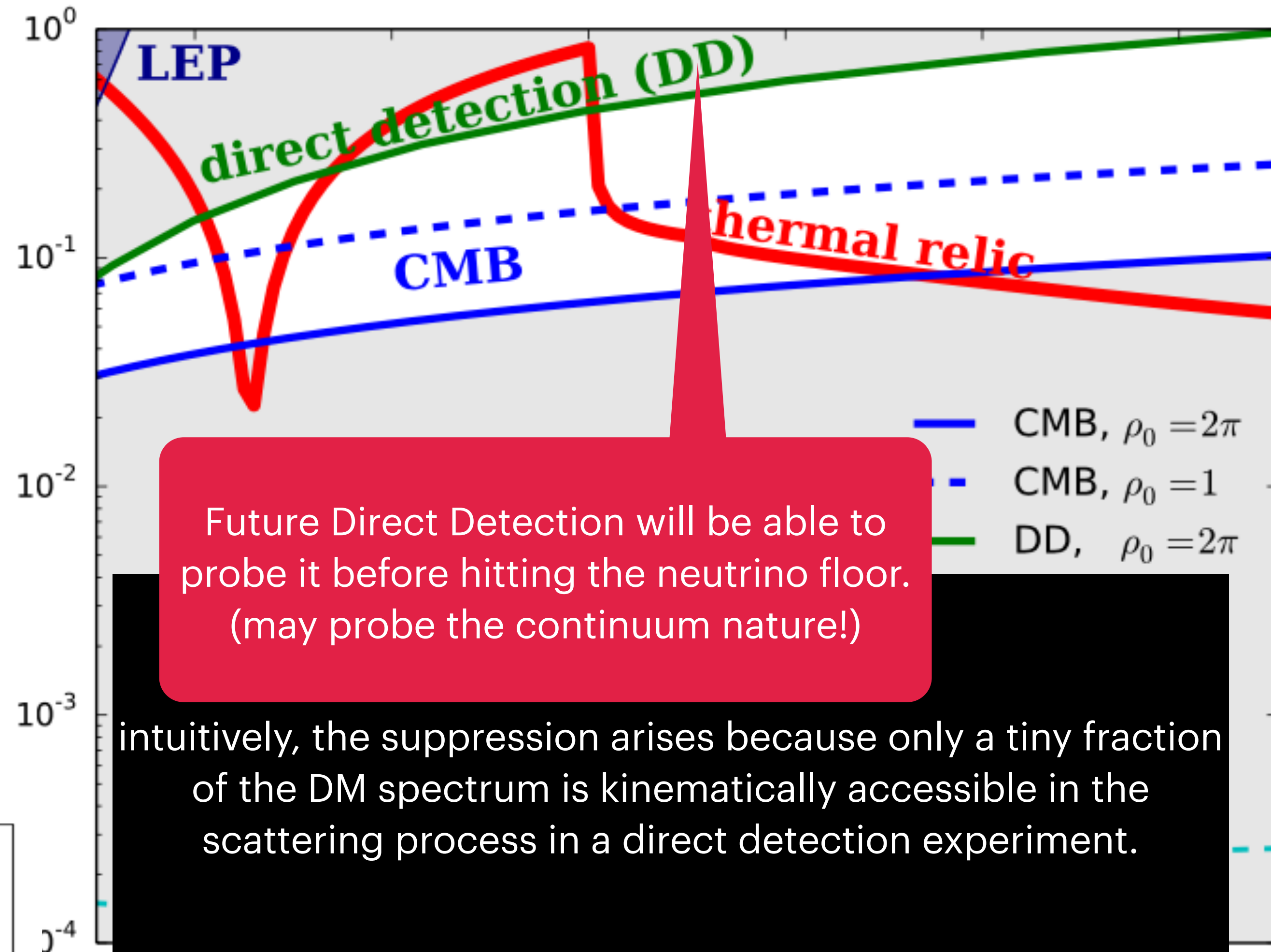
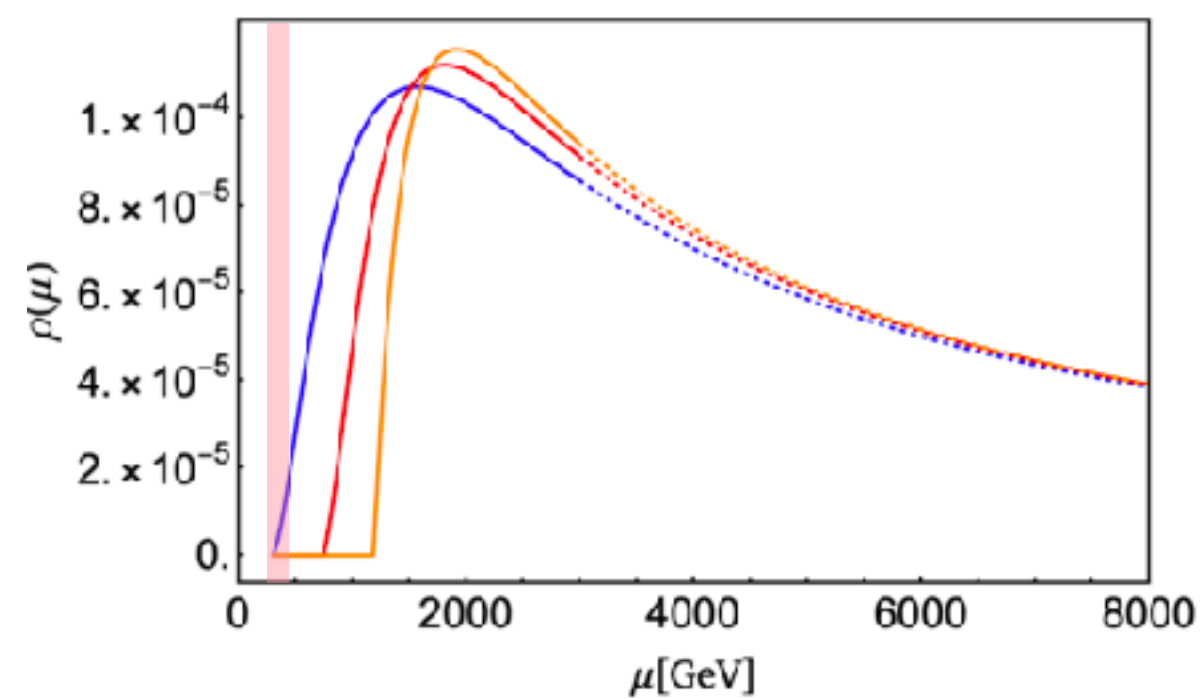
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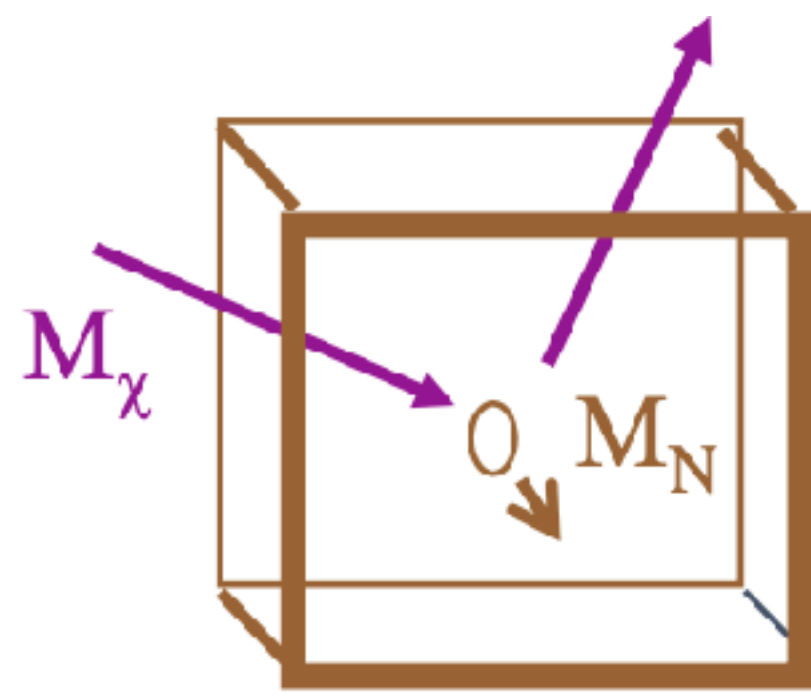
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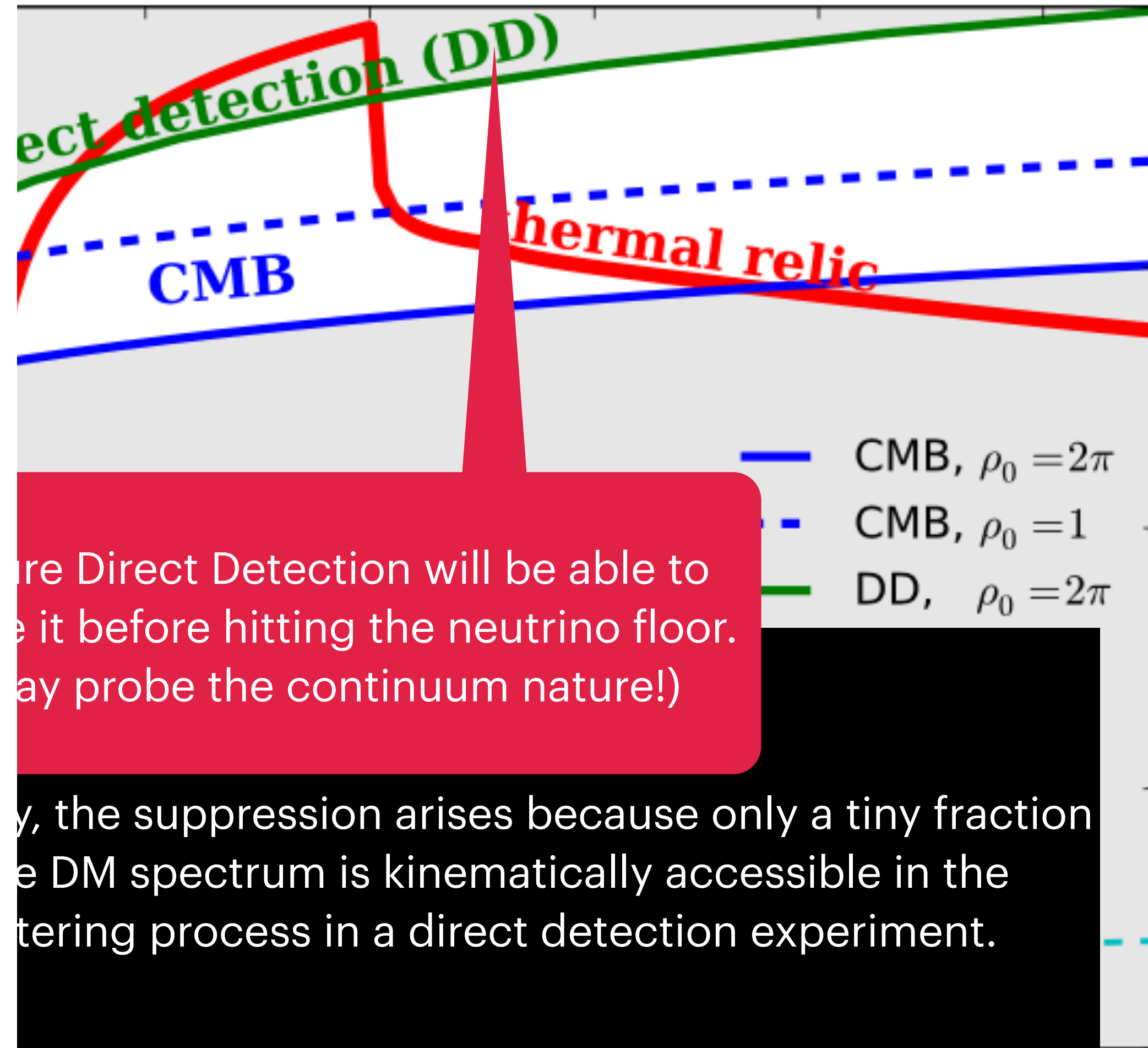
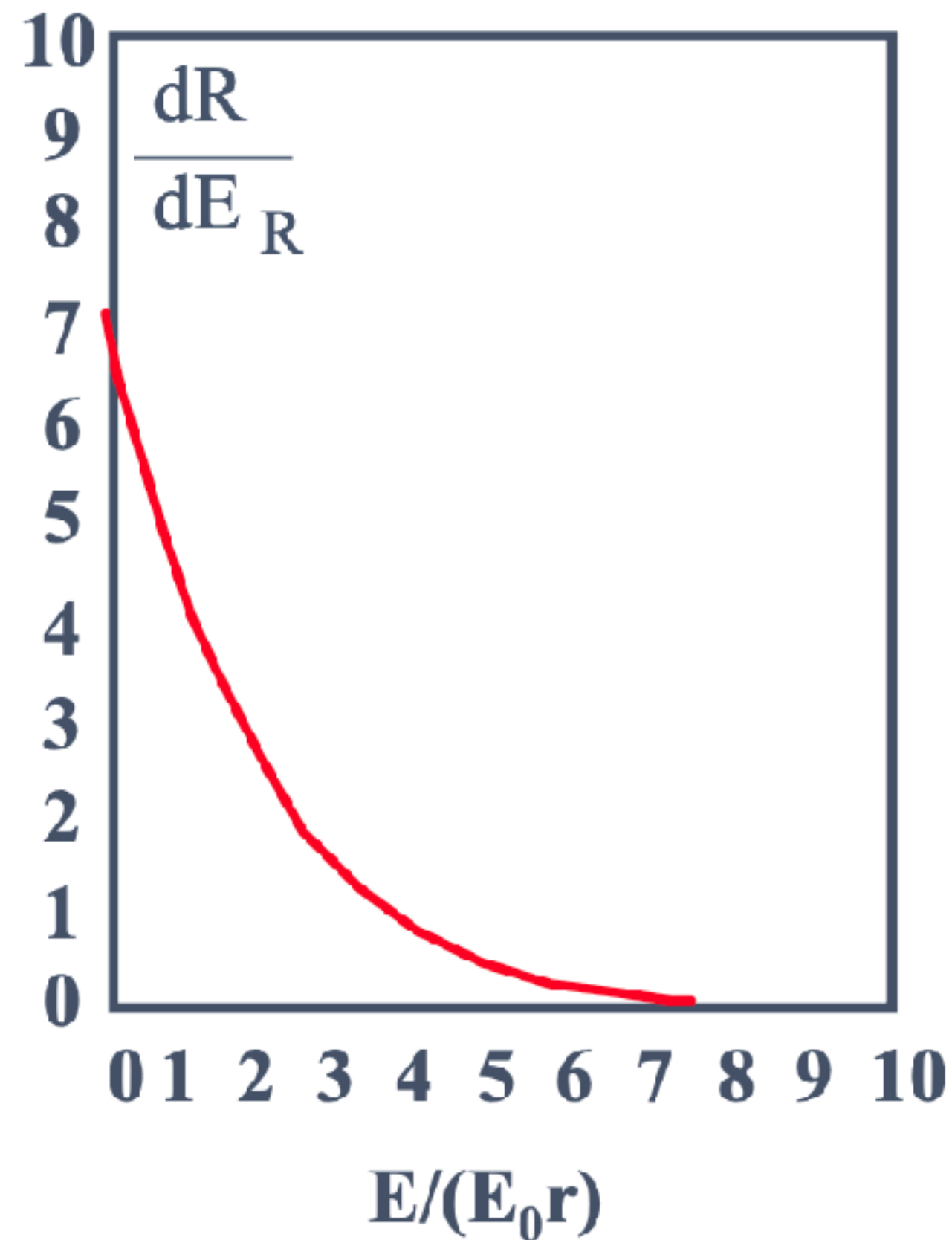


$$\sigma \sim \int \frac{d\mu_2^2}{2\pi} \rho(\mu_2^2) \hat{\sigma}(\mu_1, \mu_2)$$

Continuum Nature of DM



Ge, Si, NaI, LXe, ...

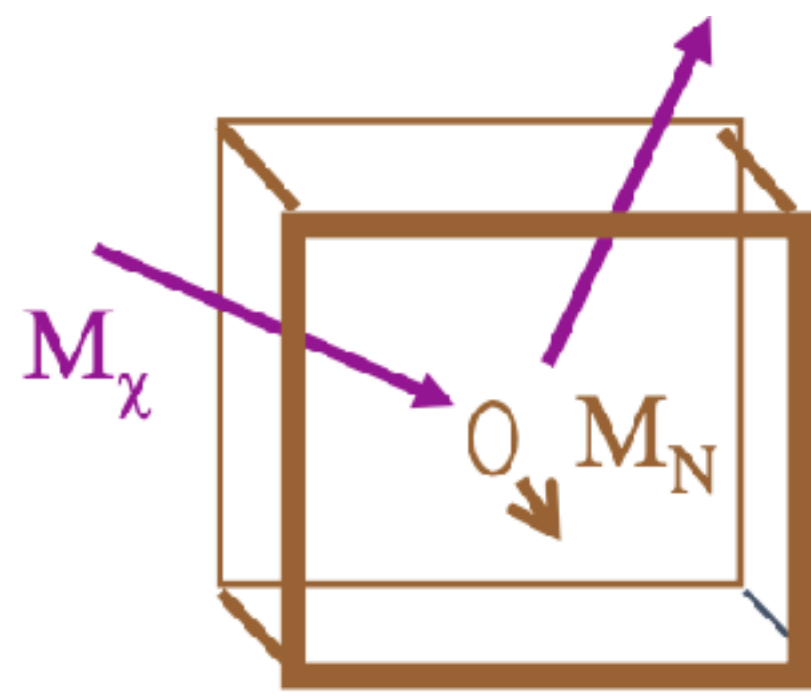


Future Direct Detection will be able to probe it before hitting the neutrino floor. (may probe the continuum nature!)

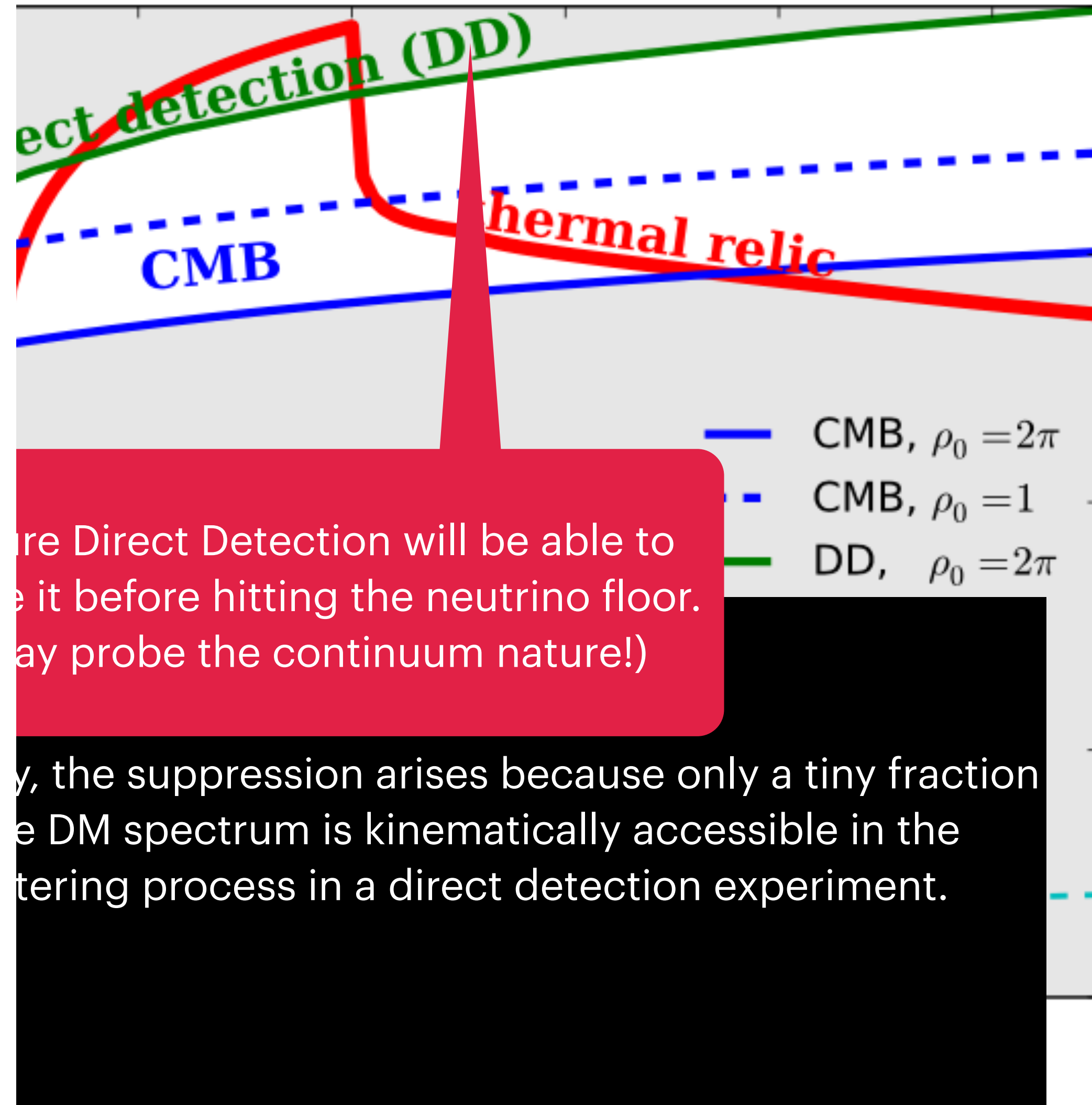
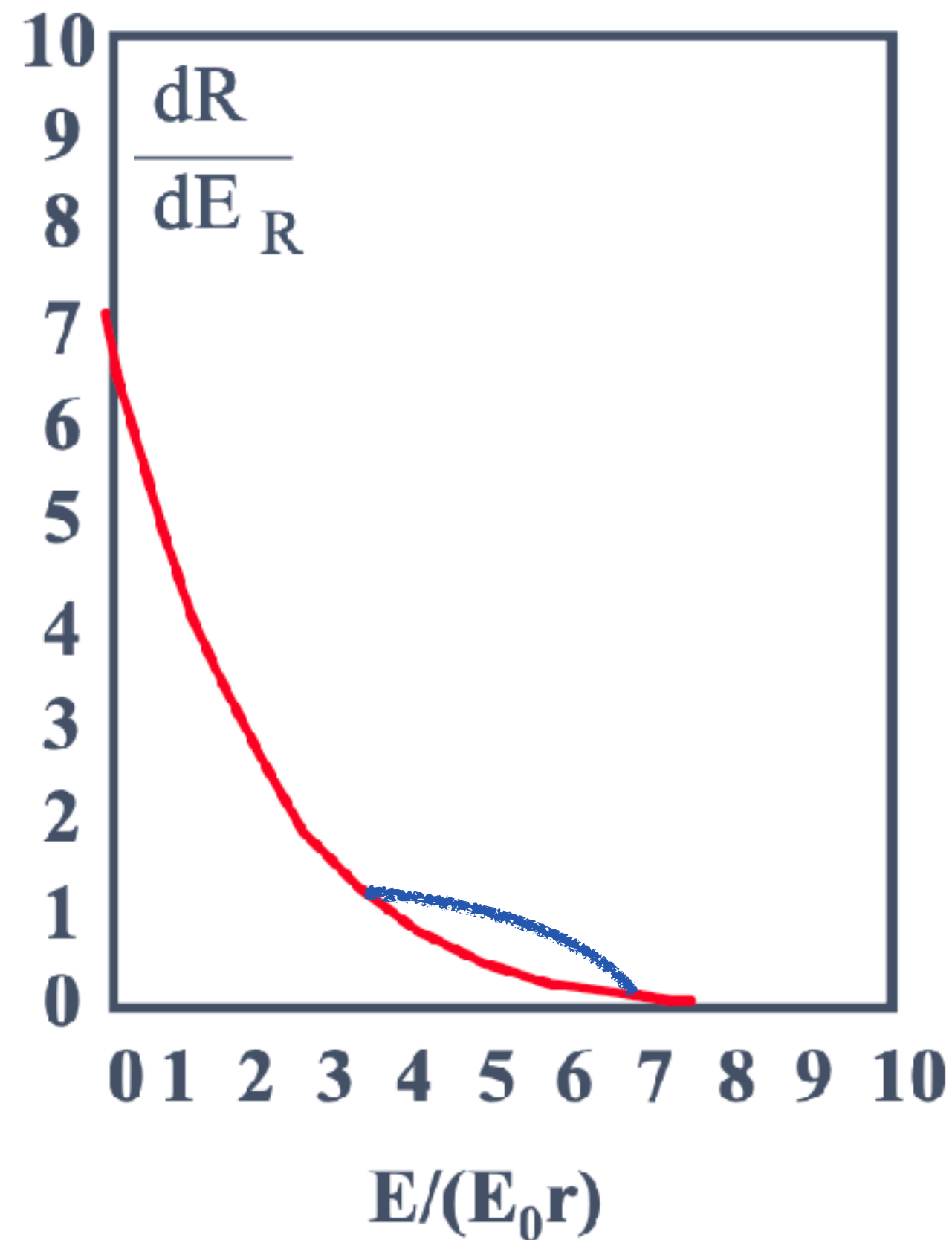
...y, the suppression arises because only a tiny fraction of the DM spectrum is kinematically accessible in the scattering process in a direct detection experiment.

$$\tau \sim \int \frac{d\mu_2^2}{2\pi} \rho(\mu_2^2) \hat{\sigma}(\mu_1, \mu_2)$$

Continuum Nature of DM



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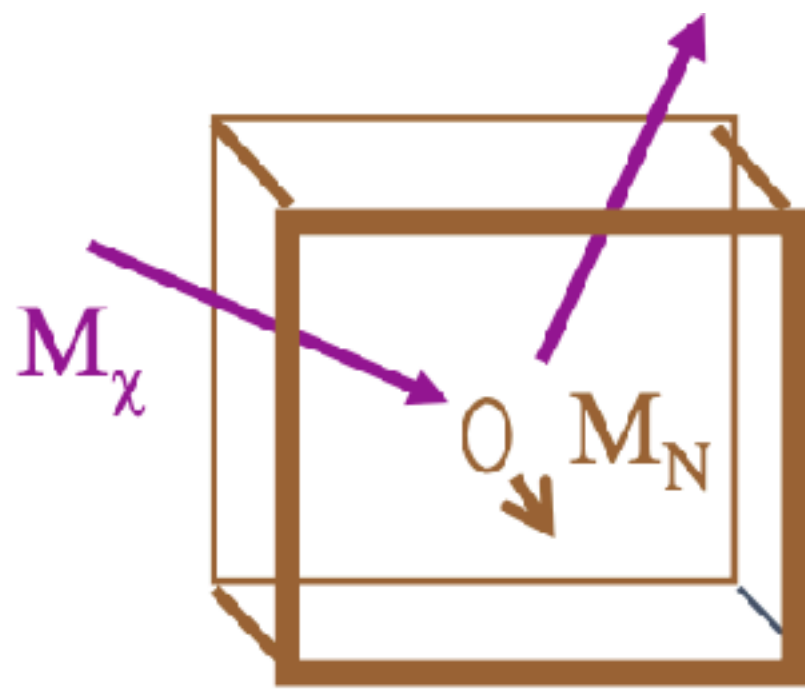


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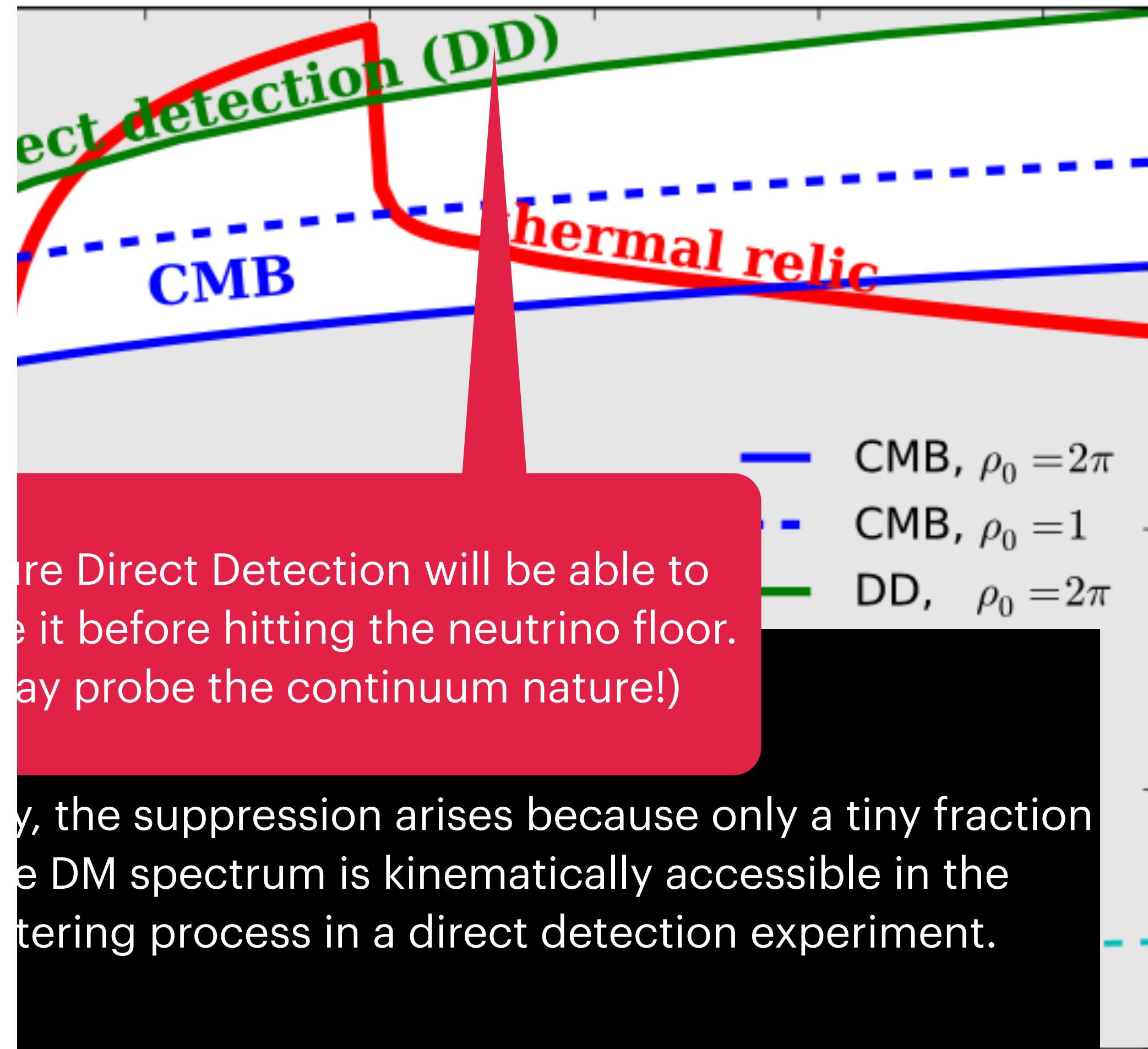
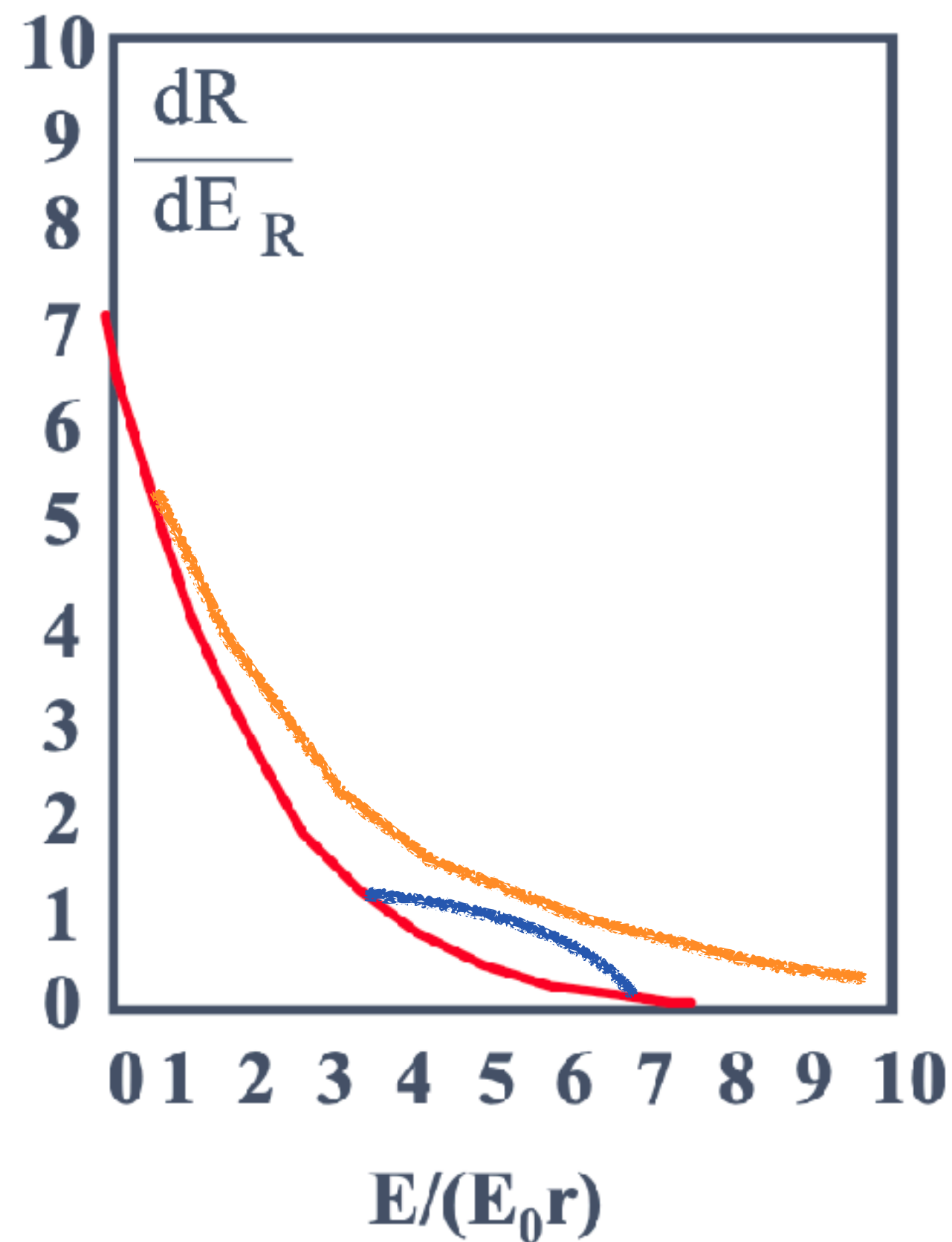
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Gapped Continuum Nature of DM

◆ Indirect Detection

$$\text{DM}(\mu_1) + \text{DM}(\mu_2) \rightarrow \text{SM}_1 + \text{SM}_2$$

- Since there is no continuum state in the final state, the rates of these processes are unsuppressed :

Gapped Continuum Nature of DM

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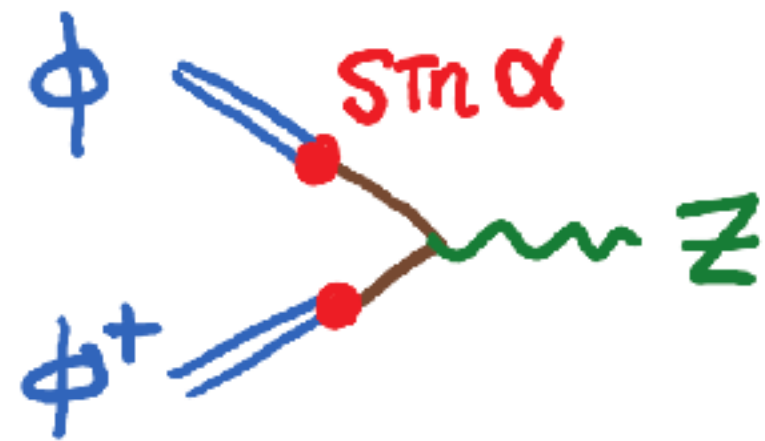
$\mu_1 \approx \mu_2 \approx \mu_0$ in the current universe \Rightarrow both rates and kinematics of annihilation in the galactic halos are basically identical to those of particle DM

Gapped Continuum Nature of DM

◆ Colliders Phenomenology

- for low energy experiments (low compared to gap scale): e.g. LEP
bound for Z-portal WIC:

Same suppression mechanism (by continuum kinematics) as in Direct Detection appl

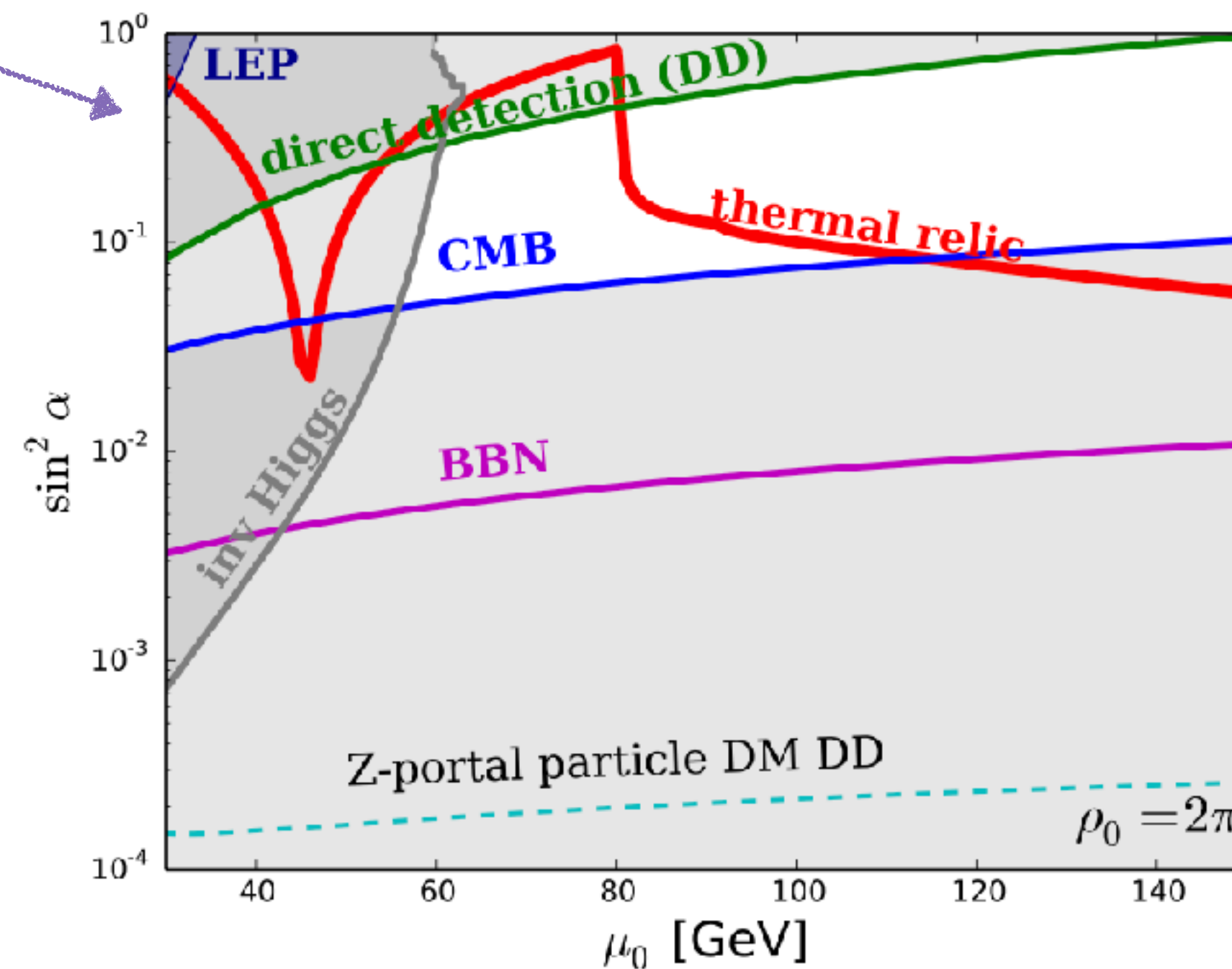
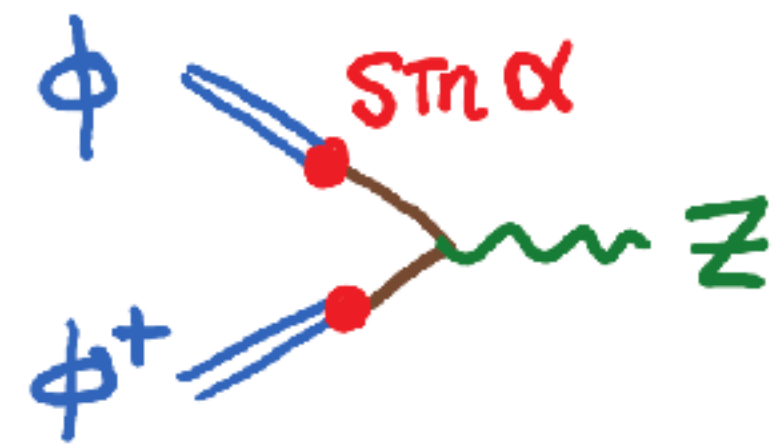


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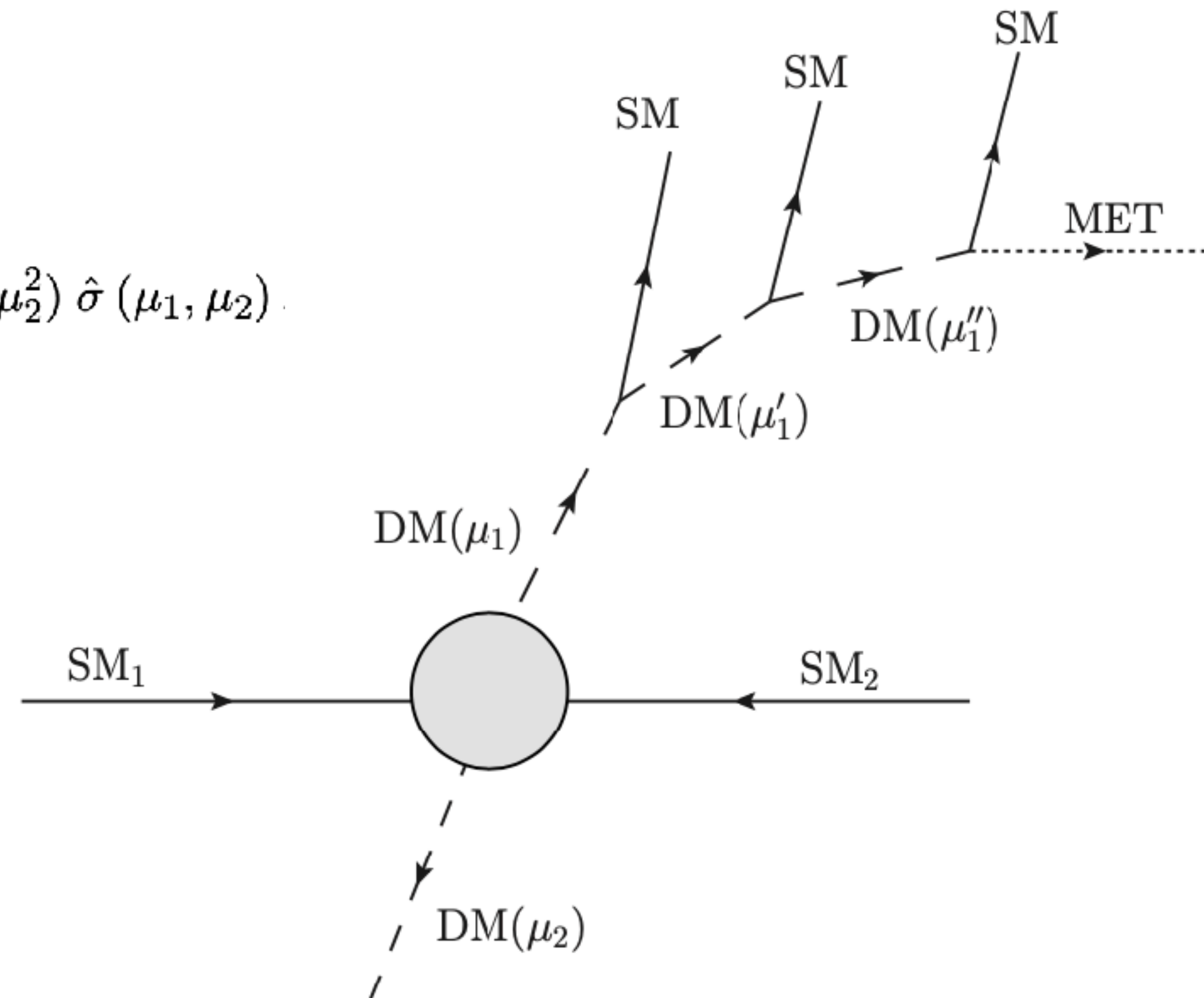


Gapped Continuum Nature of DM

◆ Colliders Phenomenology

- for high enough energy: (no suppression, an rich pheno)
 $SM_1 + SM_2 \rightarrow DM(\mu_1) + DM(\mu_2)$

$$\sigma \sim \int \frac{d\mu_1^2}{2\pi} \rho(\mu_1^2) \int \frac{d\mu_2^2}{2\pi} \rho(\mu_2^2) \hat{\sigma}(\mu_1, \mu_2)$$



Gapped Continuum Nature of DM

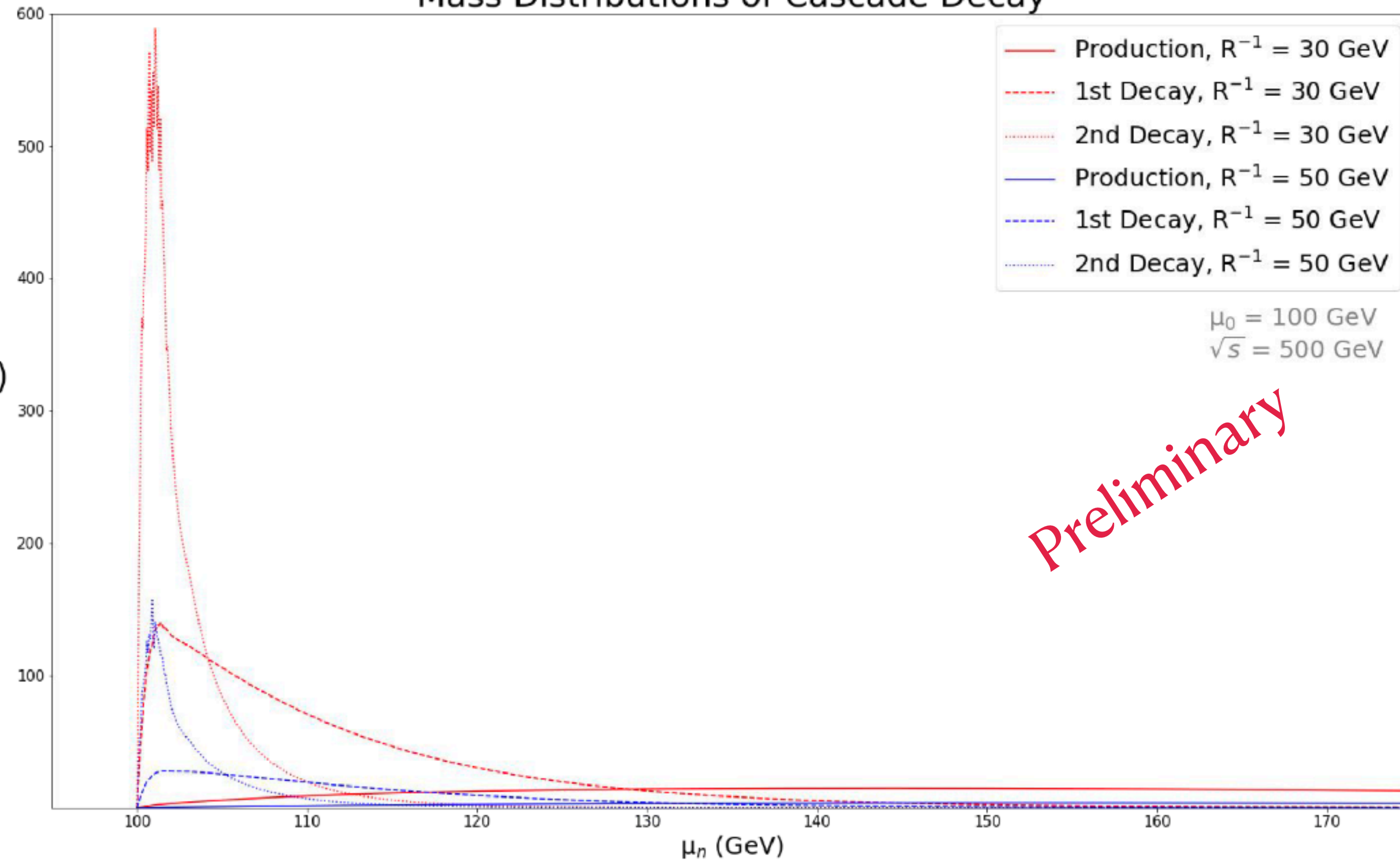
◆ Colliders Phenomenology

- for high enough energy: (no suppression, an rich pheno)
SM1+SM2 \rightarrow DM(μ_1) + DM(μ_2)

$$\sigma \sim \int \frac{d_1}{2}$$

$$\frac{d\sigma_n}{d\mu} \left(\frac{ab}{GeV} \right)$$

Mass Distributions of Cascade Decay



Dark Matter Continuum Spectral Density from 5D Model

◆ Warped 5D model

- Scalar gapped continuum: $\mathcal{L} = \sqrt{g} \left[\frac{1}{2} g^{MN} D_M \Phi^\dagger D_N \Phi - V(\Phi) \right]$

In conformally flat coordinate, Schrödinger form of eom: $\psi = e^{-\frac{3}{2}A}\Phi$

$$\left(-\partial_z^2 + \hat{V}(z) \right) \Psi(z) = p^2 \Psi(z)$$

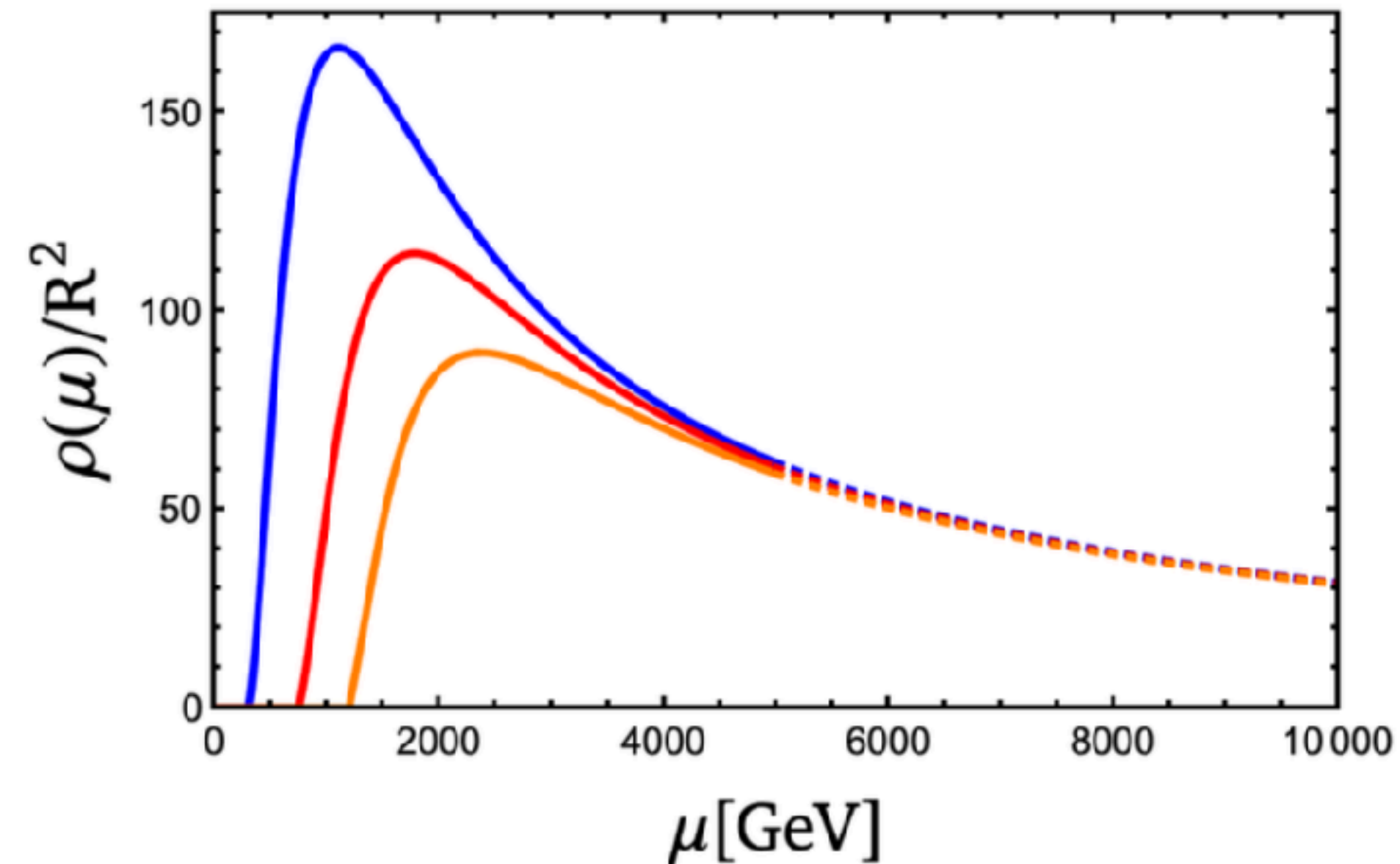
$$V(z) = \frac{e^{-2ky}}{4y_s^2} \left[4m^2 (y_s - y)^2 + 15 (1 + k(y_s - y))^2 - 6 \right]$$

after integrating out bulk:

$$S = \int \frac{d^4 p}{(2\pi)^4} \Phi^\dagger(p) \Sigma(p^2) \Phi(p)$$

$$\rho(p^2) = -2 \operatorname{Im} \frac{1}{\Sigma(p^2)}$$

$$\Sigma(p) = \frac{1}{R} \left. \frac{f'(z, p)}{f(R, p)} \right|_{z=R}$$



Dark Matter Continuum Spectral Density from 5D Model

◆ Warped 5D model

- Scalar gapped continuum **near the gap**:

In conformally flat coordinate, Schrödinger form of eom:

$$\left(-\partial_z^2 + \hat{V}(z)\right) \Psi(z) = p^2 \Psi(z)$$
$$\lim_{z \rightarrow \infty} \hat{V}(z) = \mu_0^2 \left(1 + e^{-2z(2\mu_0/3)} + \frac{8}{3} e^{-z(2\mu_0/3)}\right)$$

$$\Psi(z, \mu) = C L_m^n(3e^{-2z\mu_0/3}) \exp\left(\frac{3}{2} \sqrt{1 - \frac{\mu^2}{\mu_0^2}} \log\left(e^{-\frac{2\mu_0 z}{3}}\right) - \frac{3}{2} e^{-\frac{2\mu_0 z}{3}}\right)$$

can expand the arguments of the Laguerre polynomial around the mass gap

$$\rho(p) = \frac{1}{\pi} \text{Im} G(R, R; p).$$



$$\rho(\mu^2) \propto \left(\frac{\mu^2}{\mu_0^2} - 1\right)^{1/2}$$

Dark Matter Continuum Spectral Density from 5D Model

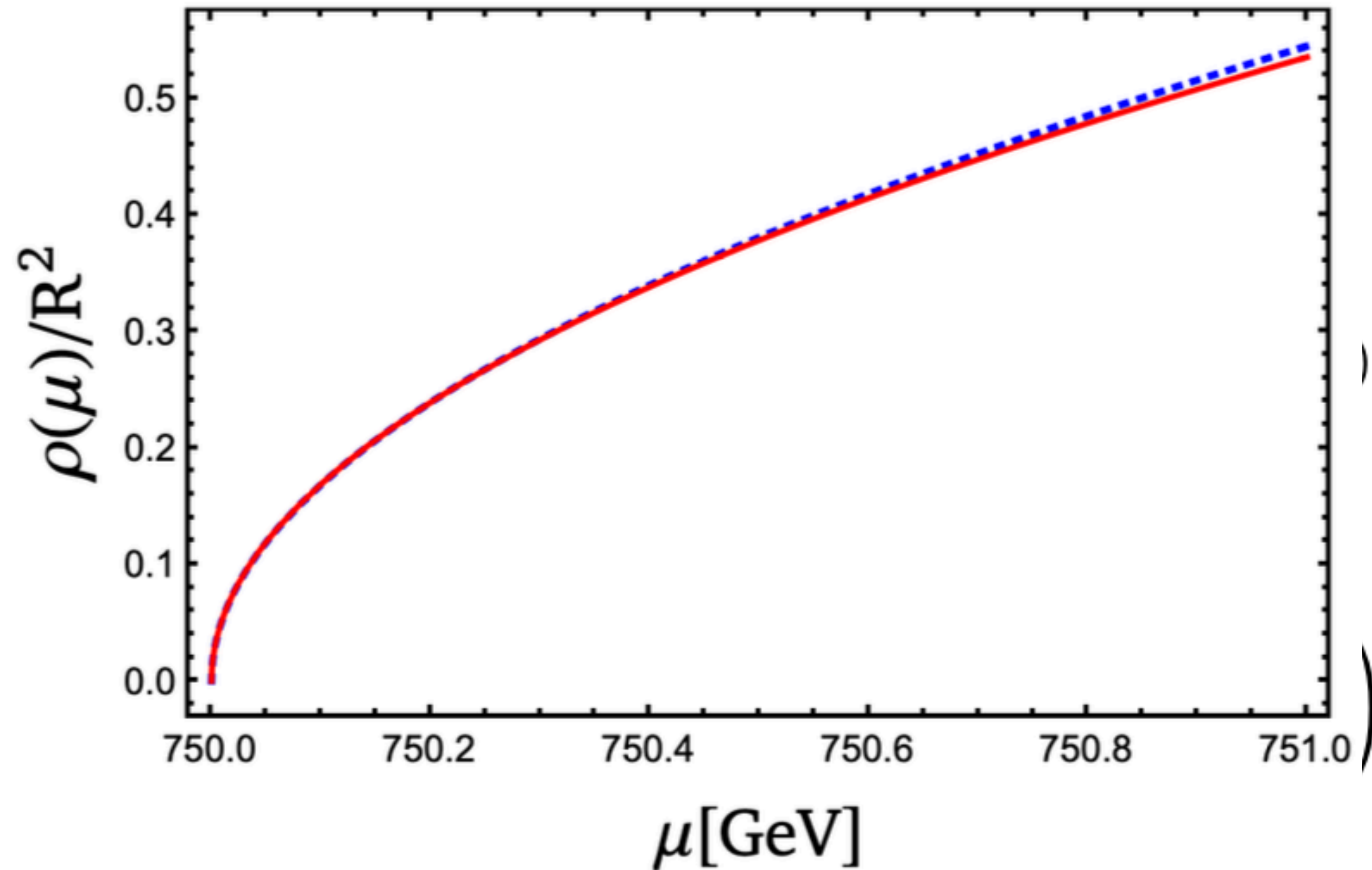
◆ Warped 5D r

- Scalar gap

In conformally

$$\lim_{z \rightarrow \infty} \hat{V}(z) = \mu$$

$$\Psi(z, \mu) = \zeta$$



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Summary

New DM Paradigm → Weakly Interacting Massive Particle → Weakly Interacting Massive Continuum

1. Gapped Continuum DM = **theoretically** and **phenomenologically** motivating!
2. **Continuum Kinematics** : late decay, relaxation of direct detection bound
3. **Revival** of **W**Weakly **I**nteracting Massive **C**ontinuum (WIC) !
4. Many possible models + many detailed pheno study to be done.
5. **Continuum Collider Physics** = totally new → **needs a systematic investigations a**
6. Many more (including continuum freeze-in DM, etc)

The background features a dark grey to black gradient. Overlaid on this are numerous white, hand-drawn style lines. These lines form a complex network of spirals, circles, and intersecting paths, creating a sense of organic movement and depth. The lines vary in thickness and density, with some areas being more chaotic and others more structured.

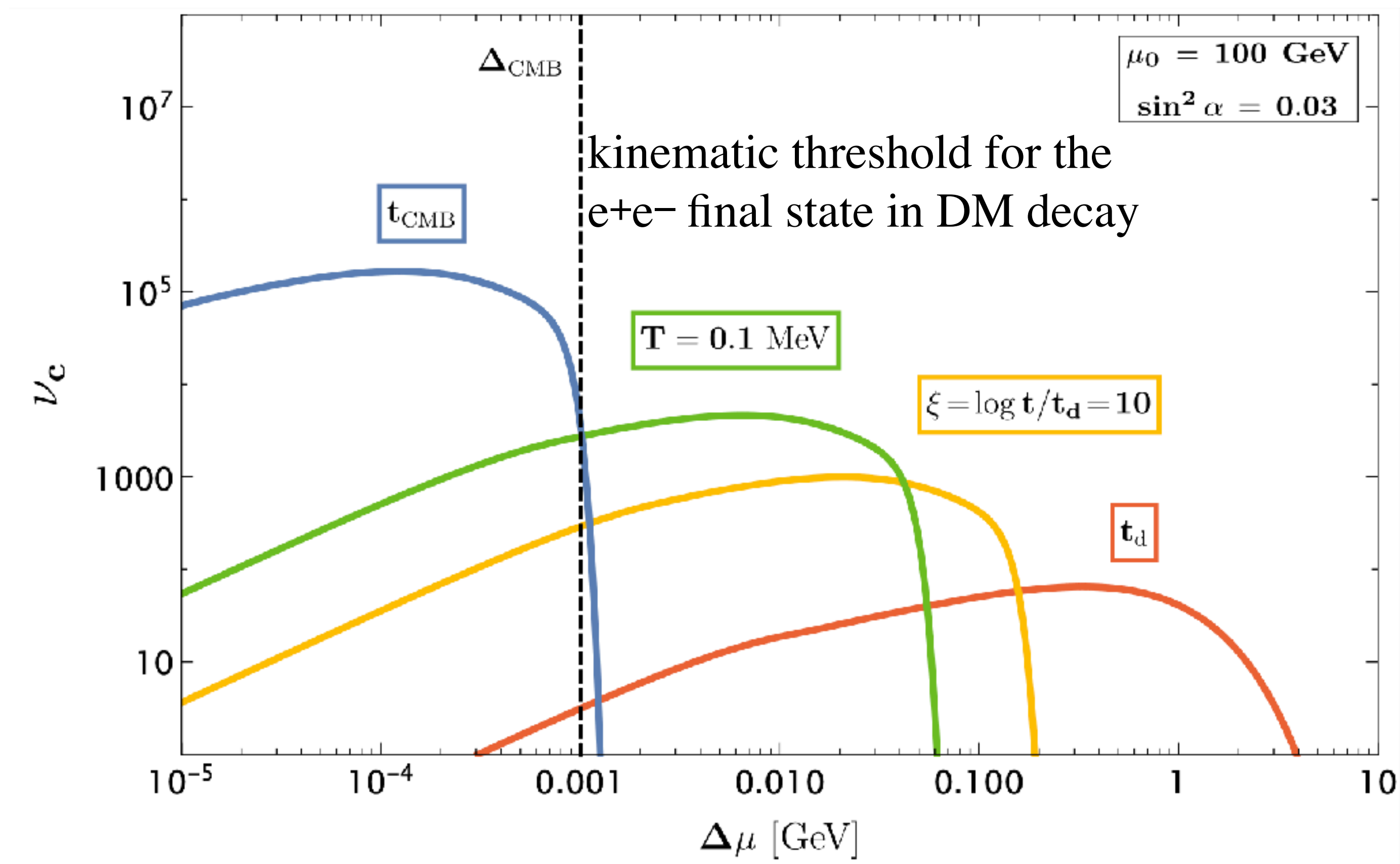
Back-up

Electromagnetic Energy Injection After Recombination

-important constraints on the model arises from re-ionization of Hydrogen by DM decays after recombination

-at most $\sim 0.1\%$ H ionization fraction for points on the boundary obtained by the simple criterion

$$\Gamma = H(t_{\text{CMB}})$$

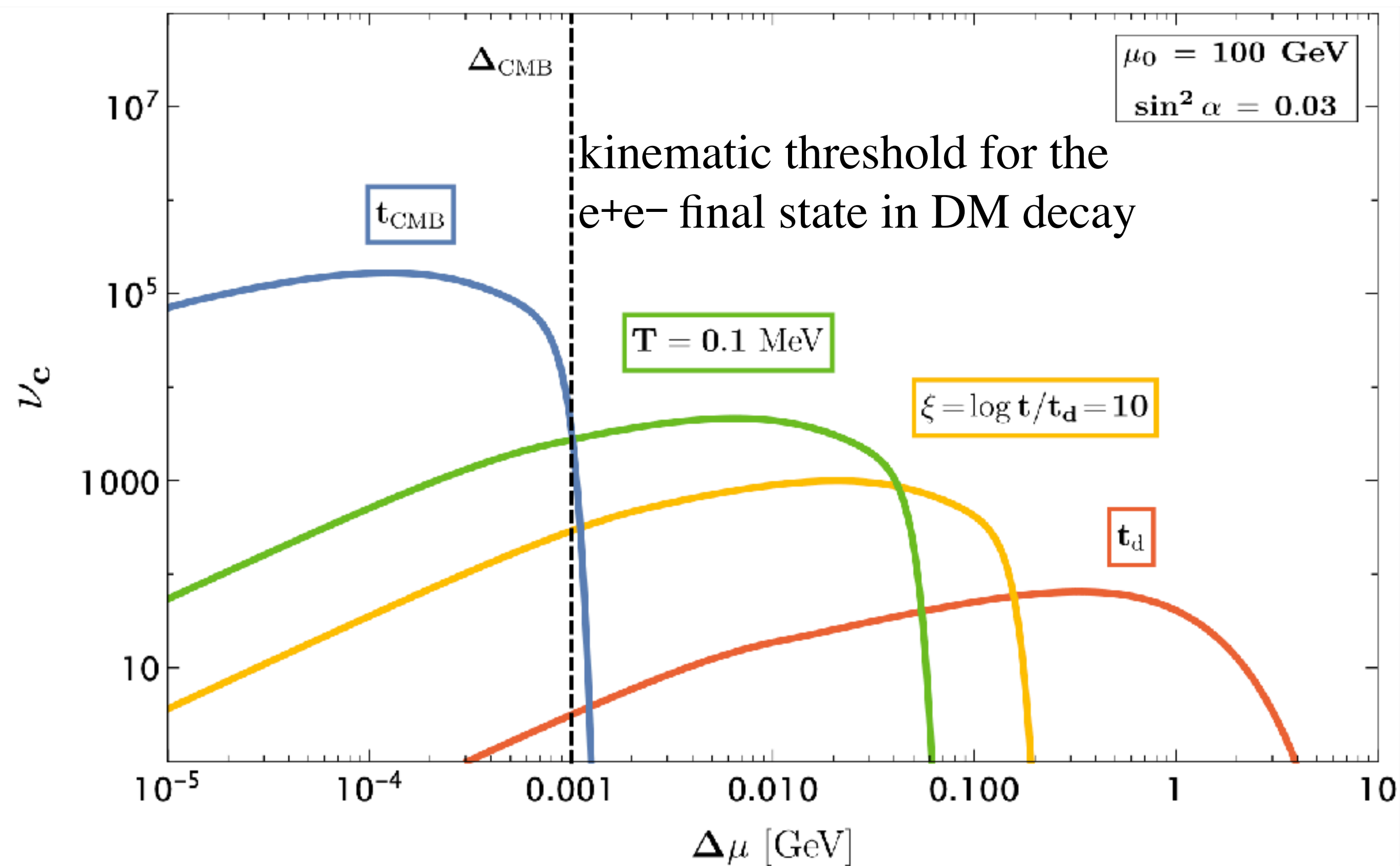


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DM mass distribution as a function of time for a sample parameter point. Only about 0.2% of DM particles lie on the tail above this threshold.

Normalization of Spectral Density

$$\begin{aligned} S_{\text{eff}} &= \int_{\text{UV}} d^4x e^{-4A} \Phi^\dagger(x, y) \partial_y \Phi(x, y) \Big|_{y=0} \\ &= \int_{\text{UV}} \frac{d^4p}{(2\pi)^4} \hat{\Phi}^\dagger(p) \left(e^{-3A(z)} \frac{f'(z, p)}{f(R, p)} \right)_{z=R} \hat{\Phi}(p) \end{aligned}$$

$$\int d^4x \sqrt{g} (D_\mu H D^\mu H e^{2A} + D_\mu \chi D^\mu \chi e^{2A} - \hat{\lambda} k^{\frac{1}{2}} \Phi \chi H + h.c.)_{z=R}$$

In order to get the proper 4D effective action

with $\lambda = \hat{\lambda} k e^{-A}$ of the order of the electroweak scale we need the field redefinitions

$$H \rightarrow H e^{-A}, \chi \rightarrow \chi e^{-A}, \Phi \rightarrow \Phi e^{-\frac{3}{2}A} \sqrt{R}.$$

$$\Sigma(p) = \frac{1}{R} \frac{f'(z, p)}{f(R, p)} \Big|_{z=R},$$