The effects of magnetic fields on magnetic dipole moments

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Work done with Tuna Demircik, Gyurin Kim, Matti Järvinenar

(arXiv:2110.01169)

Anomalous magnetic moments of electrons and muons

- All massive particles with spin are known to have a magnetic dipole moment:

$$\vec{\mu} = \frac{g}{2}\frac{e}{m}\vec{S} = (a+1)\frac{e}{m}\vec{S}$$

- The magnetic moment is an intrinsic quantity of particles. Its agreement between theory and experiment has been a real triumph of QFT, since Schwinger calculated it in 1951.
- For electrons it is measured at ppb level (Haneke et. al 2008):

$$a_e^{\mathrm{exp}} = 1159652180.73(28) \times 10^{-12}$$

The discrepancy is about 2.4σ (Parker et. al 2018):

$$\Delta a_e \equiv a_e^{\mathrm{exp}} - a_e^{SM} = (-87 \pm 36) \times 10^{-14}$$

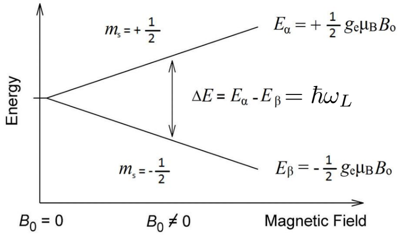
Anomalous magnetic moments of electrons and muons

- It served to test the consistency of GSW model of electroweak theory during its establishement. (Fujikawa-Lee-Sanda 1972)
- Currently it provides a test and also a hint for new physics beyond standard model.
- For muons it is measured at ppm level (FNAL 2021):

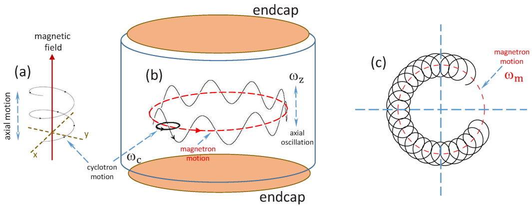
$$a_\mu^{\mathrm{exp}} = 116592061(41) \times 10^{-11}$$

with 4.2σ deviation, $\Delta a_\mu = (251 \pm 59) \times 10^{-11}$.

Measurements of a_e - Penning Trap (Old): U.W.

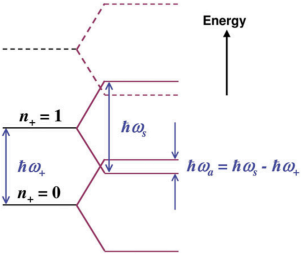


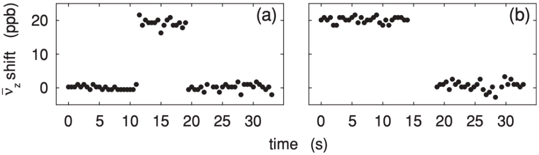
$$\Delta E = -\mu \cdot \vec{B}$$



$$\hbar\omega_c = 2\mu_B B_0$$

Measurements of a_e - Improved Penning Trap: Harvard





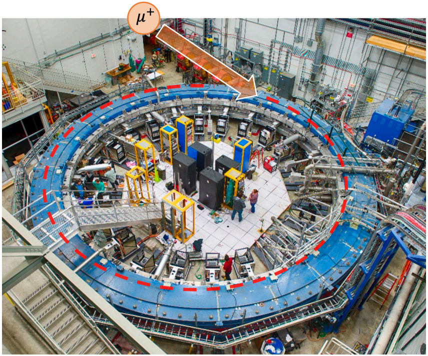
G. Gabrielse *et al.*(2004)

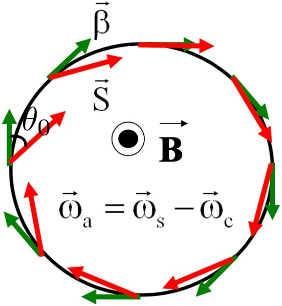
Measurements of a_μ (Fermi Lab 2021, BNL 2006)

- Muons are unstable. It decays into electron 100% with lifetime of about $2.2\mu\,\mathrm{s}$.
- The key observation is that the difference between the spin precession frequency and the (relativistic) cyclotron frequency is given as

$$\omega_a \equiv \omega_s - \omega_c = a_\mu \frac{eB}{m}\,.$$

Measurements of a_μ (Fermi Lab 2021, BNL 2006)

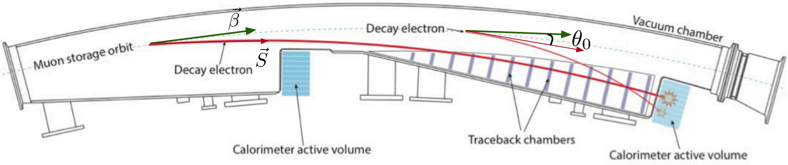




Fermi Lab Muon Storage Ring, slide from O. Kim (IBS)

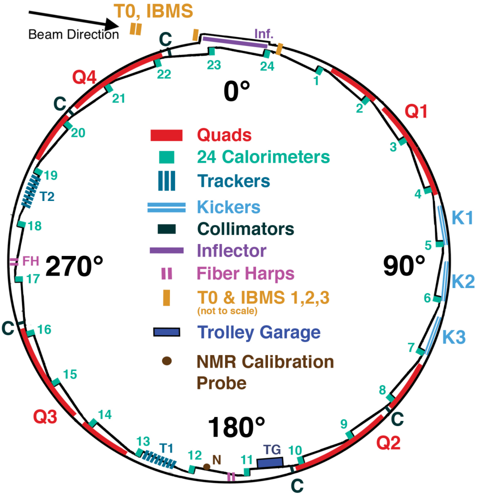
Measurements of a_μ (Fermi Lab 2021, BNL 2006)

- Because EW interaction is chiral, the electrons decay mostly along the muon spin direction with an angle $\theta_0 = \omega_a t + \phi$:



Measurements of a_μ (Fermi Lab 2021, BNL 2006)

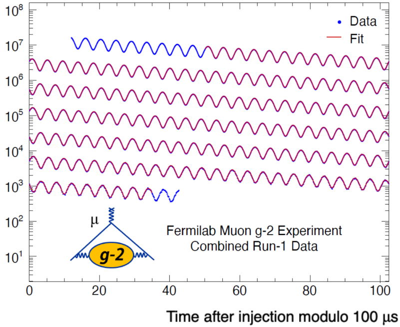
Ring layout



Measurements of a_μ (Fermi Lab 2021, BNL 2006)

- The count of positrons (electrons) at the detector as a function of time:

$$N(t) = N_0 e^{-t/\tau} \left[1 + A(E_e) \sin\left(\omega_a t + \phi\right)\right] \qquad (1)$$



The magnetic field effect

- The operational definition of magnetic dipole moment is

$$\vec{\mu} \equiv g\frac{q\vec{S}}{2m} = -\left.\frac{\partial \mathcal{E}(\vec{B})}{\partial \vec{B}}\right|_{\vec{B}=0} . \qquad (2)$$

- But in practice the magnetic fields never vanish. In fact one often needs strong magnetic fields, $B > 10\mathrm{kG}$.
- In view of current accuracy, the effect of magnetic field may not be negligible.

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The magnetic field effect

- On dimensional grounds one expects the magnetic effect to be

$$\delta(g-2) \sim \frac{\alpha}{2\pi} \cdot \frac{eB}{m^2} = 3 \times 10^{-13} \left(\frac{B}{10\,\mathrm{kG}}\right) \left(\frac{0.51\,\mathrm{MeV}}{m}\right)^2 .$$

- The magnetic effect may be compatible with experimental uncertainty for electrons, $\delta a_e^{\exp} \simeq 2.8 \times 10^{-13}$ while way small for muons $\delta a_\mu^{\exp} \simeq 54 \times 10^{-11}$
- However, since we are detecting positrons (electrons) in muon $g-2$ experiment, the magnetic field might be relevant!

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Field-dependent magnetic moment

- If $B \ll m^2/e$, one can expand the magnetic moment g factor in powers of magnetic field:

$$g_e(B) \equiv \frac{|\Delta\mathcal{E}|}{\mu_B B} = g_e(0) + \sum_{n=1}^{\infty} g_e^{(n)} \left(\frac{eB}{m^2}\right)^n .$$

QED contributions

- From the exact electron propagator to the order α under a constant magnetic the shift in the ground state energy is at one-loop :

$$\Delta\mathcal{E}^{\mathrm{QED}}(B) = \frac{\alpha}{2\pi} m \left[-\frac{eB}{2m^2} + a_2 \left(\frac{eB}{m^2}\right)^2 + a_3 \left(\frac{eB}{m^2}\right)^3 + \cdots \right],$$

where $a_2 = \left(\frac{4}{3}\ln\frac{m^2}{2eB} - \frac{13}{18}\right)$, $a_3 = \left(\frac{14}{3}\ln\frac{m^2}{2eB} - \frac{32}{5}\ln 2 + \frac{83}{90}\right)$.

- The anomalous magnetic moment at one-loop in QED is therefore given as

$$\frac{1}{2}(g-2)^{1-\mathrm{loop}}_{\mathrm{QED}} = \frac{\alpha}{\pi} \left[\frac{1}{2} \mp a_2 \left(\frac{eB}{m^2}\right) - a_3 \left(\frac{eB}{m^2}\right)^2 + \cdots \right].$$

QED contributions

- The leading correction, linear in B, depends on the spin direction, which is coming from the $(eB)^2$ contribution to the ground state energy.
- Since the $(eB)^2$ in the energy is independent of B field direction, it contributes equally to both spin up and down.
- Therefore, the leading correction to the anomalous magnetic moment is not measurable at the current Penning trap experiment for the electron magnetic anomaly.

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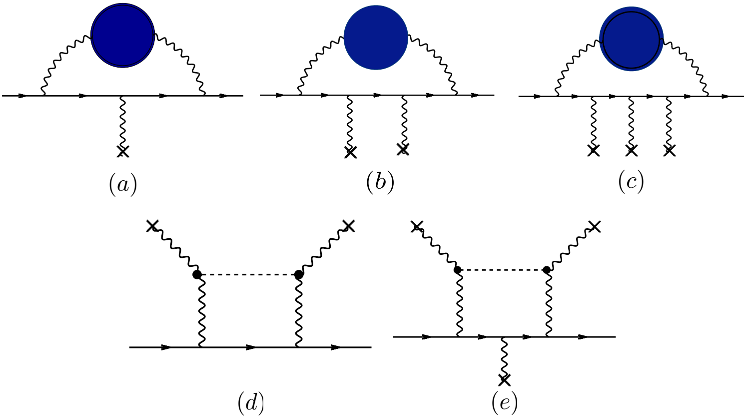
QCD contributions up to B^2



QCD contributions

- The QCD corrections to the ground state energy are :

$$\Delta\mathcal{E}^{\mathrm{QCD}}(B) = -\left(\frac{\alpha}{\pi}\right)^2 \frac{m}{2}\left[c_1\left(\frac{eB}{m^2}\right) + \bar{c}_2\left(\frac{eB}{m^2}\right)^2 + \bar{c}_3\left(\frac{eB}{m^2}\right)^3 + \cdots\right].$$

- The QCD corrections to magnetic moments are then

$$\frac{1}{2}(g-2)^{\mathrm{LO}}_{\mathrm{QCD}} = \left(\frac{\alpha}{\pi}\right)^2\left[c_1 \pm \bar{c}_2\left(\frac{eB}{m^2}\right) + \bar{c}_3\left(\frac{eB}{m^2}\right)^2 \cdots\right].$$

- From fig. (d) and (e)

$$c_2 = \frac{1}{72\pi^2}\cdot\frac{m^4}{f_\pi^2 m_\pi^2}\left(30\ln\left(\frac{m^2}{m_\pi^2}\right) + 19\right)\left[1 + \mathcal{O}\left(\frac{m^2}{m_\pi^2}\right)\right].$$

$$c_3 = -\frac{1}{72\pi^2}\cdot\frac{m^4}{f_\pi^2 m_\pi^2}\left(12\ln\left(\frac{m^2}{m_\pi^2}\right) + 17\right)\left[1 + \mathcal{O}\left(\frac{m^2}{m_\pi^2}\right)\right].$$

Direct measurements of magnetic moments

- In the current experiments of magnetic moments, the contribution linear in B is not directly measurable.
- The measurable effect is hence at B^2 order:

$$\delta g_l(B) \simeq -\frac{2\alpha}{\pi}\left[a_3 - \frac{\alpha}{\pi}\bar{c}_3\right] \cdot \left(\frac{eB}{m^2}\right)^2 \simeq 1.3 \times 10^{-20},$$

$(B = 10\mathrm{kG}$ and $m = 0.51\ \mathrm{MeV})$

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Refraction under magnetic field

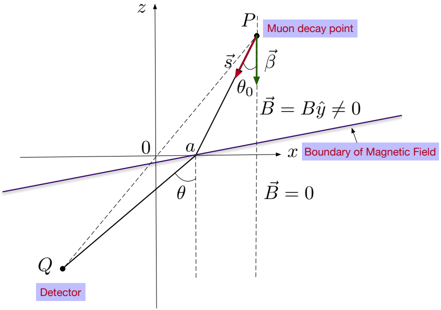
- The spin precession and cyclotron frequencies are proportional to the anomalous magnetic moment (Bargamann et al. 1959)

$$\omega_a \equiv \omega_s - \omega_c = a_\mu \frac{eB}{m} .$$

- The number of electrons $N(t)$ detected at the electromagnetic calorimeter is then

$$N(t) = N_0 e^{-t/\tau} \left[1 + A \sin\left(\omega_a t + \phi\right)\right] .$$

Refraction under magnetic field



- When electrons exit the magnetic field, they are refracted, following Maupertuis' principle:

$$\delta \int_{P}^{x=a} \left(p_1 dl + \vec{A} \cdot d\vec{r} \right) + \delta \int_{x=a}^{Q} p_2 dl = 0 \, .$$

Refraction under magnetic field

- The momentum satisfies

$$p^2 + m^2 = (\mathcal{E} - \Delta\mathcal{E}(B))^2 \ .$$

- The refracted angle θ is related to the incident angle $\theta_0 = \omega_a t + \phi$ as

$$\tan\theta - \tan\theta_0 = \gamma_* \tan\theta_0 \cdot \sec^2\theta_0 \cdot \frac{\delta m}{m} \ ,$$

where $\gamma_* = m\mathcal{E}/p_2^2 \approx m/p_2$ and $\delta m = \Delta\mathcal{E}(B)$.

Refraction under magnetic field

- When $\tan\theta_0$ is small, the increment in the refraction angle

$$\delta\theta = -\gamma_* \frac{\delta m}{m} \tan\theta_0 \sim 10^{-16} \left(\frac{1\ \mathrm{GeV}}{p_2}\right) \left(\frac{B}{10\,\mathrm{kG}}\right) \tan\theta_0 \,.$$

- For a generic angle θ_0, we find

$$\sin\theta = \frac{\sin\theta_0 \left(\cos^2\theta_0 + \gamma_*\delta m/m\right)}{\sqrt{\cos^6\theta_0 + \sin^2\theta_0 \left(\cos^2\theta_0 + \gamma_*\delta m/m\right)^2}} \,.$$

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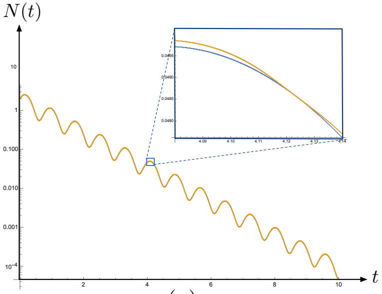
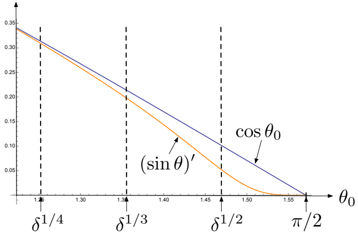
Refraction under magnetic field

- The shape of the electron distribution changes due to the refraction by the magnetic field, $\delta^n = \pi/2 - (\gamma_* \delta m/m)^n$:



(a)

$|\sin\theta - \sin\theta_0| \le (\gamma_* \delta m/m) \sim 10^{-16}$



(b)

$|(\sin\theta)' - \cos\theta_0| \le (\gamma_* \delta m/m)^{1/2} \sim 10^{-8}$

Refraction under magnetic field

- Due to the refraction, the oscillatory part of the shape of positron count changes:

$$0 \lesssim |\sin\theta - \sin\theta_0| \lesssim \gamma_* \frac{\delta m}{m} \sim 10^{-16}\,.$$

- Near the peak (valley) the new shape is very flat for the range of angles $\Delta\theta_0 \simeq (3\gamma_*\delta m/m)^{1/4} \sim 10^{-4}$. Two slopes differ by $(\gamma_*\delta m/m)^{1/2} \sim 10^{-8}$ in this region.
- Since each run subset 10^9 positrons are detected at Fermi lab experiment, one should detect 10^5 positrons in $\Delta\theta_0$, which is sufficient to distinguish two distributions.

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Conclusion

- We have calculated the effect of magnetic fields on the anomalous magnetic moment up to B^2, both QED and QCD.
- The accuracy of experiments for magnetic moments is now high enough, ppb level, to be sensitive to magnetic fields.
- Currently only the quadratic effect is measurable:

$$\delta g_e(B) \simeq -\frac{2\alpha}{\pi}\left[a_3 - \frac{\alpha}{\pi}\bar{c}_3\right]\cdot\left(\frac{eB}{m_e^2}\right)^2 \simeq 1.3\times 10^{-20}\,.$$

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Conclusion

- In the muon experiments the distribution of electron count is modified, linearly in B due to refraction.

$$0 \lesssim |\sin\theta - \sin\theta_0| \lesssim \gamma_* \frac{\delta m}{m} \sim 10^{-16}\,.$$

- The slope of the shape changes significantly near the peak (valley), $|\pi/2 - \theta| < (3\gamma_*\delta m/m)^{1/4} \sim 10^{-4}$.
- By measuring this change, especially the slope difference, $(\gamma_*\delta m/m)^{1/2} \sim 10^{-8}$, one could measure the refraction due to the magnetic field.
- Furthermore to improve the experimental uncertainties, one should consider this effect, since $\omega_a = a_\mu eB/m$ is determined by fitting the distribution shape of detected positrons.

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