Higher Order Corrections to Classical Gravity

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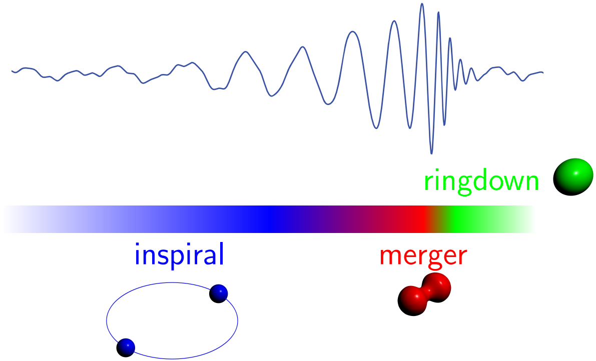
DESY

in collaboration with: A. Maier, P. Marquard, and G. Schäfer

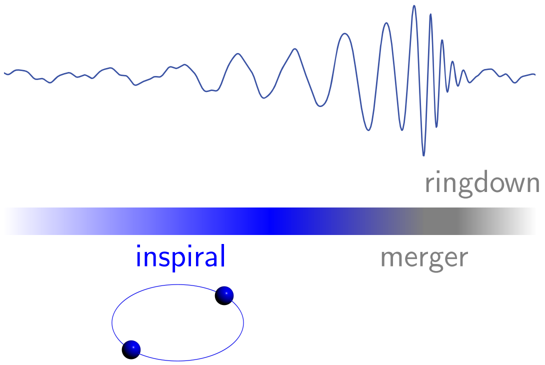
Introduction

- We consider the inspiraling phase of two massive gravitating objects (black holes and/or neutron stars) and study their Hamiltonian dynamics.
- On the basis of a Hamilitonian also their scattering can be investigated.
- While the lowest order is the Newtonian motion, the 1 PN correction to it shows the motion of the perihelion already.
- With higher oders, the motion becomes structurally more and more complicated.
- Estimates show, that future LISA measurements will require the knowledge of the dynamics at 6 PN.
- Currently the level of 4 PN is fully understood.
- The level of 5 PN is nearly, but not yet completely understood analytically and awaits a very last theoretical clarification.
- The level of 6 PN will need more theoretical e orts in the future.
- Methods developed in QFT can be applied to the classical Einstein-Hilbert Lagrangian to build an e ective field theory (EFT) to solve this ambitious problem by Feynman diagram techniques.

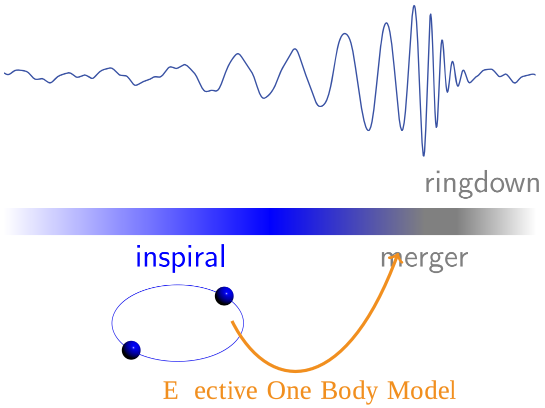
Gravitational waves from binary mergers



Gravitational waves from binary mergers



Gravitational waves from binary mergers



[Buonanno, Damour 1998]

General relativity

General relativity action:

$$S_{\mathrm{GR}}[g^{\mu\nu}] = S_{\mathrm{EH}} + S_{\mathrm{GF}} + S_{\mathrm{matter}}$$

With $\eta^{\mu\nu} = \mathrm{diag}(-1,1,1,1), g = \det(g^{\mu\nu})$:

- Einstein-Hilbert action:

$$S_{\mathrm{EH}} = \frac{1}{16G\pi}\int d^d x\sqrt{-g}R$$

- *Harmonic* gauge $\partial_\mu\sqrt{-g}g^{\mu\nu} = 0$:

$$S_{\mathrm{GF}} = -\frac{1}{32G\pi}\int d^d x\sqrt{-g}\Gamma_\mu\Gamma^\mu\,, \qquad \Gamma^\mu = g^{\alpha\beta}\Gamma^\mu{}_{\alpha\beta}$$

- Assume point-like matter, no spin:

$$S_{\mathrm{matter}} = \sum_{a=1}^{2} m_a \int d\tau_a$$

General relativity

$$
\begin{aligned}
S_{\mathrm{GR}}[\phi, A_i, \sigma_{ij}] = & \sum_{a=1}^{2} \int dt \left(m_a + \frac{1}{2} m_a v_a^2 + \mathcal{O}(v^4) \right) \\
& + \sum_{a=1}^{2} m_a \int dt \left(-\phi + v_{ai} A_i + v_{ai} v_{aj} \sigma_{ij} - \frac{1}{2}\phi^2 + \dots \right) \\
& + \int \frac{d^d x}{32\pi G} \left[-c_d (\partial_\mu \phi)^2 + (\partial_\mu A_i)^2 + \frac{1}{4}(\partial_\mu \sigma_{ii})^2 - \frac{1}{2}(\partial_\mu \sigma_{ij})^2 + \dots \right]
\end{aligned}
$$

Higher Order Corrections in Classical Gravity

Topics:

- 5 PN corrections
- Test of the PM results at 6PN
- Study the inspiraling phase of 2 massive objects
- in collaboration with: A. Maier, P. Marquard, G. Schäfer

The topic has been inspired by J. Plefka's talk at QMC in 2018. This has been the time of the 3PN / 4PN static potential corrections using e ective field-theory methods (i.e. 4PN incomplete). Fo a et al. [1612.00482]
However, the complete 4 PN corrections were known by using other technologies (ADM), Damour et al. [1401.4548]

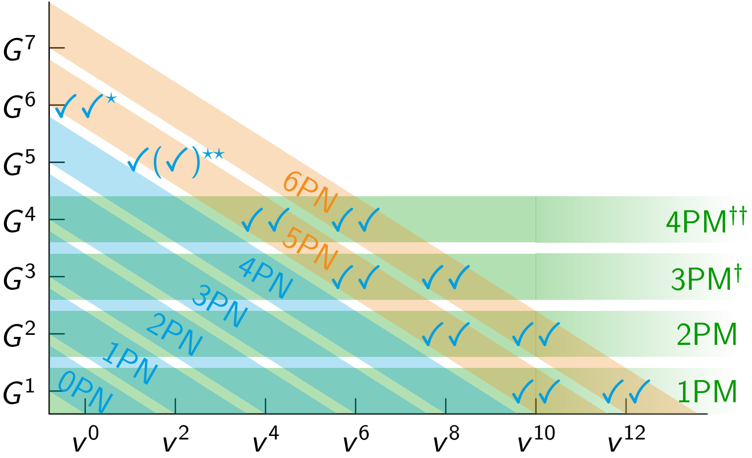
Higher Order Corrections in Classical Gravity

- Current Status:
  - Post Minkowskian approach:
    - G^4 : Bern et al. [2112.10750], Dlapa et al. [2112.11296]
    - potential contributions are checked up to 6PN in Blümlein et al. [2101.08630]
    - Blümlein et al. [2003.07145] proofed that the G^3 terms of Bern et al. [1901.04424] are correct and a hypothesis in Damour [1912.02139] does not apply.
    - Many recent research results using the post Minkowskian approach: see the extensive list of Refs. given in Blümlein et al. [2003.07145]

Higher Order Corrections in Classical Gravity

- Current Status:
  - Post Newtonian approach:
    - 4 PN
    - complete: [A lot of groups, working in at least 3 different gauges.] Canonical transformations cf.: Blümlein et al. [2003.01692]
    - 5 PN
    - partial results Bini et al. [2003.11891] tutti frutti; two constants cannot be determined
    - 5 PN potential terms Blümlein et al. [2010.13672] EFT complete
    - 5 PN tail terms through multipole expansion Blümlein et al. [2110.13822] EFT (see discussion below)
    - Bini et al. [2107.08896]: disagreement of the multipole 'tail' contributions of Foffa et al. [1907.02869] with χ_4 ν constraint.
    - 6 PN
    - partial results Bini et al. [2007.11239] tutti frutti; various more constants cannot be determined
    - However, 5 PN is not yet finished, which would be a conditio sine qua non to understand 6 PN.
  - The complete result can only be obtained by a full calculation.

Near-zone potential



Post Newtonian Corrections up to 5 PN

Hamiltonian and Lagrange formalism:
[applicable to the bound state and to the scattering problem]
EFT approach to Einstein gravity, cf. Kol & Smolkin [0712.4116 [hep-th]].

- 5 PN static potential
  - Foffa et al. [1902.10571] by geometric trick
  - Blümlein et al. [1902.11180] calculated within EFT ab initio
  - The papers were submitted within half a day independently.

$$\mathcal{L}^{S}_{5\text{PN}} = -\frac{G_N^6}{r^6} m_1 m_2 \left[\frac{5}{16}(m_1^5 + m_2^5) + \frac{91}{6} m_1 m_2 (m_1^3 + m_2^3) + \frac{653}{6} m_1^2 m_2^2 (m_1 + m_2)\right]$$

- 4 PN complete by EFT
  - ADM Damour et al. [1401.4548]
  - harmonic coordinates Blanchet et al. [1610.07934] Foffa & Sturani [1903.05113] Blümlein et al. [2003.01692]
  - EOB Bini et al. [2003.11891]
  - isotropic coordinates Bern et al. [2112.10750] and earlier papers

5 PN: the potential corrections

Blümlein et al. [2010.13672]:

- calculation ab initio in harmonic coordinates
- treatment of potential and singular ‘tail’ terms together in D dimensions: pole cancellation up to a canonical transformation
- pole-free Hamiltonian
- adding the non-local ‘tail’ terms [agreement with the literature]
- γ_5-like treatment of ε_{ijk} in D dimensions: leading to the correct terms $O(\nu)$; see also the later paper: Fo a et al. [2110.14146]
- obtaining all terms but the rational terms $O(\nu^2)$
- The remaining finite rational $O(\nu^2)$ terms come all from **the ‘tail’.**
- The potential terms have been mulitiply verified (static potential, up to $O(G^4)$ terms by Bern et al.)
- Blümlein et al. [2010.13672] introduced the expansion by regions to classical (EFT) gravity; only potential and ultra-soft modes contribute.

5 PN: the potential corrections

First obtained:

$$\bar{d}_5^{\pi^2\nu^2} = \frac{306545}{512}\pi^2\nu^2,$$
$$a_6^{\pi^2\nu^2} = \frac{25911}{256}\pi^2\nu^2.$$

# loops	QGRAF	source irred.	no source loops	no tadpoles	masters
0	3	3	3	3	0
1	72	72	72	72	1
2	3286	3286	3286	2702	1
3	81526	62246	60998	41676	1
4	545812	264354	234934	116498	7
5	332020	128080	101570	27582	4

Table: Numbers of contributing diagrams at the different loop levels and master integrals.

5 PN: ‘tail’ terms

There is no generally agreed field theoretic approach to the non-potential terms yet, but would be utterly needed.

Blümlein et al. [2110.13822]:

- It is assumed at present that all non-potential terms can be obtained from multi-pole insertions in the sense of an EFT approach. Foffa & Sturani et al. [1903.05113], Marchand et al. [2003.13672], Larrouturou et al. [2110.02243,2110.02240]
- Partly different propagator treatment in the literature.
- A consistent description is possible by using the in-in formalism.
- Unfortunately the ν constraint hypothesis Bini et al. [2003.11891] is not met for the finite $O(\nu^2)$ terms.
- closer analysis in the EOB representation.

5 PN: EOB representation

- Our results obtained in harmonic corrdinates can be re-parameterized in EOB form for all local terms.
- The nonlocal terms do already agree between di erent approaches.

$$
\begin{aligned}
H_{\rm EOB}^{\rm loc,eff} &= \sqrt{A(1 + AD\eta^2(p.n)^2 + \eta^2(p^2 - (p.n)^2) + Q)},\\
A &= 1 + \sum_{k=1}^{6} a_k(\nu)\eta^{2k}u^k, \quad a_2 = 0,\\
D &= 1 + \sum_{k=2}^{5} d_k(\nu)\eta^{2k}u^k,\\
Q &= \eta^4(p.n)^4[q_{42}(\nu)\eta^4u^2 + q_{43}(\nu)\eta^6u^3 + q_{44}(\nu)\eta^8u^4] + \eta^6(p.n)^6[q_{62}(\nu)\eta^4u^2 + q_{63}(\nu)\eta^6u^3]\\
&\quad +\eta^{12}(p.n)^8u^2q_{82}(\nu)\ .
\end{aligned}
$$

Here $u = 1/r$ and $\eta = 1/c$.

5 PN: EOB representation

$$
\begin{aligned}
5\text{PN}, u^2 \;:\; q_{82} &= \frac{6}{7}\nu + \frac{18}{7}\nu^2 + \frac{24}{7}\nu^3 - 6\nu^4, \\
u^3 \;:\; q_{63} &= \frac{123}{10}\nu - \frac{69}{5}\nu^2 + 116\nu^3 - 14\nu^4, \\
u^4 \;:\; q_{44} &= \left(\frac{1580641}{3150} - \frac{93031}{1536}\pi^2\right)\nu + \left(-\frac{3670222}{4725} + \frac{31633}{512}\pi^2\right)\nu^2 + \left(640 - \frac{615}{32}\pi^2\right)\nu^3, \\
u^5 \;:\; \bar{d}_5 &= \left(\frac{331054}{175} - \frac{63707}{512}\pi^2\right)\nu + \bar{d}_5^{\nu^2}\nu^2 + \left(\frac{1069}{3} - \frac{205}{16}\pi^2\right)\nu^3, \\
u^6 \;:\; a_6 &= \left(-\frac{1026301}{1575} + \frac{246367}{3072}\pi^2\right)\nu + a_6^{\nu^2}\nu^2 + 4\nu^3.
\end{aligned}
$$

New:

$$
\bar{d}_5^{\nu^2} = \left(-\frac{31295104}{4725} + \frac{306545}{512}\pi^2\right), \quad a_6^{\nu^2} = \left(-\frac{1749043}{1575} + \frac{25911}{256}\pi^2\right)
$$

$$
q_{44}^{\nu^2, r} = -\frac{9367}{15} \;:
$$

Bini et al. [2003.11891] refers to χ_4^{tot} as we known now.

5 PN: phenomenological results: Binding Energy

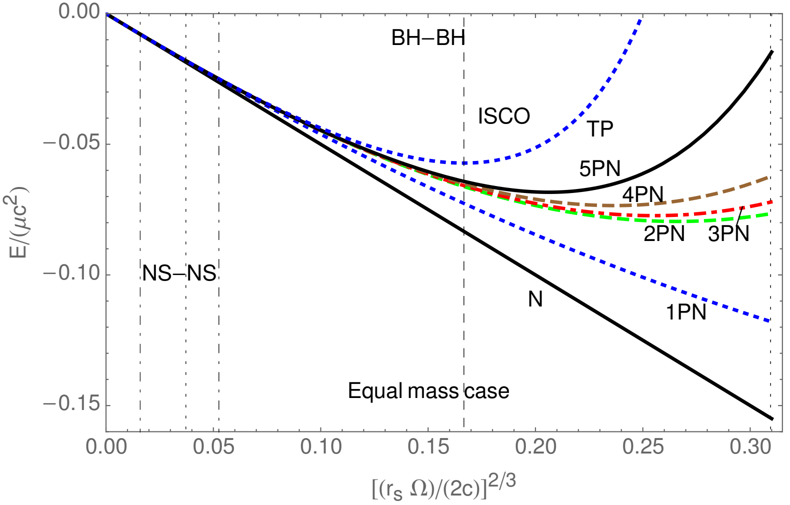
Evaluate time integral in $E_{\rm nl}$, e.g. for circular orbit:

$$\nu = \frac{\mu}{M}\,, \qquad j = \frac{J}{GM}$$

$$\frac{E^{\rm circ}_{\rm loc}(j)}{\mu} = -\frac{1}{2j^2} + \left(-\frac{\nu}{8} - \frac{9}{8}\right)\frac{1}{j^4} + \left(-\frac{\nu^2}{16} + \frac{7\nu}{16} - \frac{81}{16}\right)\frac{1}{j^6} + \left[-\frac{5\nu^3}{128} + \frac{5\nu^2}{64} + \left(\frac{8833}{384} - \frac{41\pi^2}{64}\right)\nu - \frac{3861}{128}\right]\frac{1}{j^8} + \left[-\frac{7\nu^4}{256} + \frac{3\nu^3}{128} + \left(\frac{41\pi^2}{128} - \frac{8875}{768}\right)\nu^2 + \left(\frac{989911}{3840} - \frac{6581\pi^2}{1024}\right)\nu - \frac{53703}{256}\right]\frac{1}{j^{10}} + \left[-\frac{21\nu^5}{1024} + \frac{5\nu^4}{1024} + \left(\frac{41\pi^2}{512} - \frac{3769}{3072}\right)\nu^3 + \left(-\frac{400240439}{403200} + \frac{132979\pi^2}{2048}\right)\nu^2 + \left(\frac{3747183493}{1612800} - \frac{31547\pi^2}{1536}\right)\nu - \frac{1648269}{1024}\right]\frac{1}{j^{12}} + \mathcal{O}\left(\frac{1}{j^{14}}\right)$$

$$\frac{E^{\rm circ}_{\rm nl}}{\mu} = \nu\left\{\left[-\frac{64}{5}(\ln(j) - \gamma_E) + \frac{128}{5}\ln(2)\right]\frac{1}{j^{10}} + \left[\frac{32}{5} + \frac{28484}{105}\ln(2) + \frac{243}{14}\ln(3) - \frac{15172}{105}(\ln(j) - \gamma_E) + \nu\left(\frac{32}{5} + \frac{112}{5}(\ln(j) - \gamma_E) + \frac{912}{35}\ln(2) - \frac{486}{7}\ln(3)\right)\right]\frac{1}{j^{12}} + \mathcal{O}\left(\frac{1}{j^{14}}\right)\right\}$$

5 PN: phenomenological results

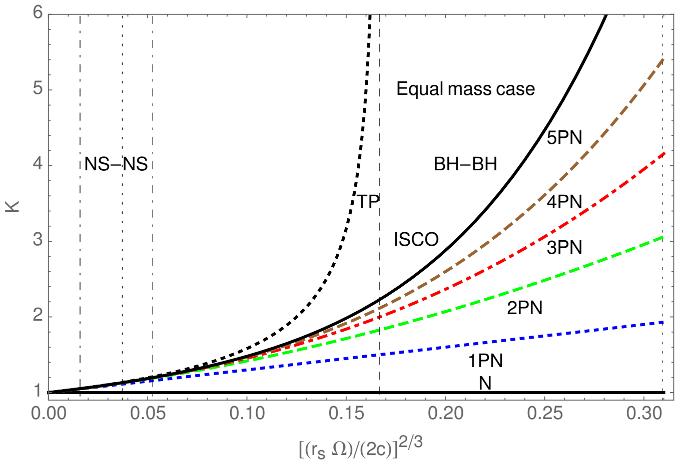


5 PN: phenomenological results: Periastron advance in the circular limit

$$K^{\text{circ}}_{\text{loc}}(j) = 1 + 3\frac{1}{j^2} + \left(\frac{45}{2} - 6\nu\right)\frac{1}{j^4} + \left[\frac{405}{2} + \left(-202 + \frac{123}{32}\pi^2\right)\nu + 3\nu^2\right]\frac{1}{j^6}$$
$$+ \left[\frac{15795}{8} + \left(\frac{185767}{3072}\pi^2 - \frac{105991}{36}\right)\nu + \left(-\frac{41}{4}\pi^2 + \frac{2479}{6}\right)\nu^2\right]\frac{1}{j^8} + \left[\frac{161109}{8}\right.$$
$$\left. + \left(-\frac{18144676}{525} + \frac{488373}{2048}\pi^2\right)\nu - \left(\frac{105496222}{4725} + \frac{1379075}{1024}\pi^2\right)\nu^2 + \left(-\frac{1627}{6} + \frac{205}{32}\pi^2\right)\nu^3\right]\frac{1}{j^{10}} + O\left(\frac{1}{j^{12}}\right)$$

$$K^{\text{circ}}_{\text{nl}}(j) = -\frac{64}{10}\nu\left\{\frac{1}{j^8}\left[-11 - \frac{157}{6}(\ln(j) - \gamma_E) + \frac{37}{6}\ln(2) + \frac{729}{16}\ln(3)\right]\right.$$
$$+ \frac{1}{j^{10}}\left[-\frac{59723}{336} - \frac{9421}{28}[\ln(j) - \gamma_E] + \frac{7605}{28}\ln(2) + \frac{112995}{224}\ln(3)\right.$$
$$\left.\left. + \left(\frac{2227}{42} + \frac{617}{6}[\ln(j) - \gamma_E] - \frac{7105}{6}\ln(2) + \frac{54675}{112}\ln(3)\right)\nu\right] + O\left(\frac{1}{j^{12}}\right)\right.$$

5 PN: phenomenological results



A numerical remark on the scattering angle: Khalil et al. [2204.05047]
The usual scattering angle takes values of ~ 120 degrees and larger. The remaining numerical di erence is of the order of 10^{-3} degrees for velocities $< 1/2$.
Yet it has to be clarified.

Test of PM results at 6PN

- We have calculated the 6 PN contributions up to G^4 in Blümlein et al. [2003.07145], [2101.08630]
- This confirmed

$$C_B = \frac{2}{3}\gamma(14\gamma^2 + 25) + 4(4\gamma^4 - 12\gamma^2 - 3)\frac{as(\gamma)}{\sqrt{\gamma^2 - 1}}$$

  from Bern et al. [1901.04424]

- and ruled out

$$C_c = \gamma(35 + 26\gamma^2) - (18 + 96\gamma^2)\frac{as(\gamma)}{\sqrt{\gamma^2 - 1}}$$

  from Damour [1912.02139v1]

- The results also agree with Bini et al. [2004.05407]
  Here

$$as(\gamma) = arcsinh(\sqrt{(\gamma - 1)/2}), \gamma = \sqrt{p_\infty^2 + 1})$$

  and C_i contributes to $\chi_3(\gamma, \nu)$.

The 'conservative' scattering angle

- Since summer 2021 one has to distinguish between the complete scattering angle and the conservative scattering angle starting at $1/j^4$ and 5PN.
- The calculation by Bern et al. is dynamically conservative
- The ν-scaling for $\chi(j,\nu,p_\infty)$ observed by Damour implies to redefine χ to its conservative part.
- $$\begin{aligned}\frac{1}{\pi\nu}\left[\tilde{\chi}_4^{\rm tot,cons} - \chi_4^{\rm Schw}\right] &= -\frac{15}{4} + p_\infty^2\left(-\frac{557}{16} + \frac{123}{256}\pi^2\right) + p_\infty^4\left(-\frac{6113}{96} - \frac{37}{5}\ln\left(\frac{p_\infty}{2}\right) + \frac{33601}{16384}\pi^2\right)\\ &\quad + p_\infty^6\left(-\frac{615581}{19200} - \frac{1357}{280}\ln\left(\frac{p_\infty}{2}\right) + \frac{93031}{32768}\pi^2\right) + O(p_\infty^8).\end{aligned}$$
- χ and $\chi^{\rm cons}$ are di erent quantities.
- The recent results of Bern et al. refer to $\chi^{\rm cons}$. The EOB parameters have been derived from χ, on the other hand.

Conclusions

- Significant progress has been made in applying EFT methods to classical gravity during the last three years.
- Both in the post-Newtonian and the post-Minkowskian approach methods from QFT provide the only way to solve this problem to the experimental accuracy needed.
- The level of 5 PN is nearly completed and the remaing problems are expected to be solved soon, which will provide corresponding analytic expressions for the dynamics in the inspiraling phase.
- Currently the 4 PM, i.e. $O(G^4/r^4)$, level is reached in the post-Minkowskian and people work the next level for the scattering angle.
- The EOB approach allows to combine the results from both approaches in the case of the scattering process.
- The tail terms are di erent for the bound state and scattering problems.