

Lessons from Gravity about Axion

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Based on: G.D., 0507215 [hep-th]

G.D., Folkerts, Franca 1312.7273 [hep-th]

G.D., Gomez, Zell, 1811.03079 [hep-th]

G.D., 2012.02133 [hep-th]

see also: Otari Sakhelashvili, 2110.03386

Summary:

(*) S -matrix formulation of quantum gravity excludes de Sitter vacua.

(*) This excludes massive 3-forms

(*) \tilde{r} -vacua must be unphysical

(*) Axion (for each gauge) group is necessary

(*) Dual formulation

$$a \rightarrow a + \text{const} \iff B_{\mu\nu} \rightarrow B_{\mu\nu} + R_{\mu\nu}$$

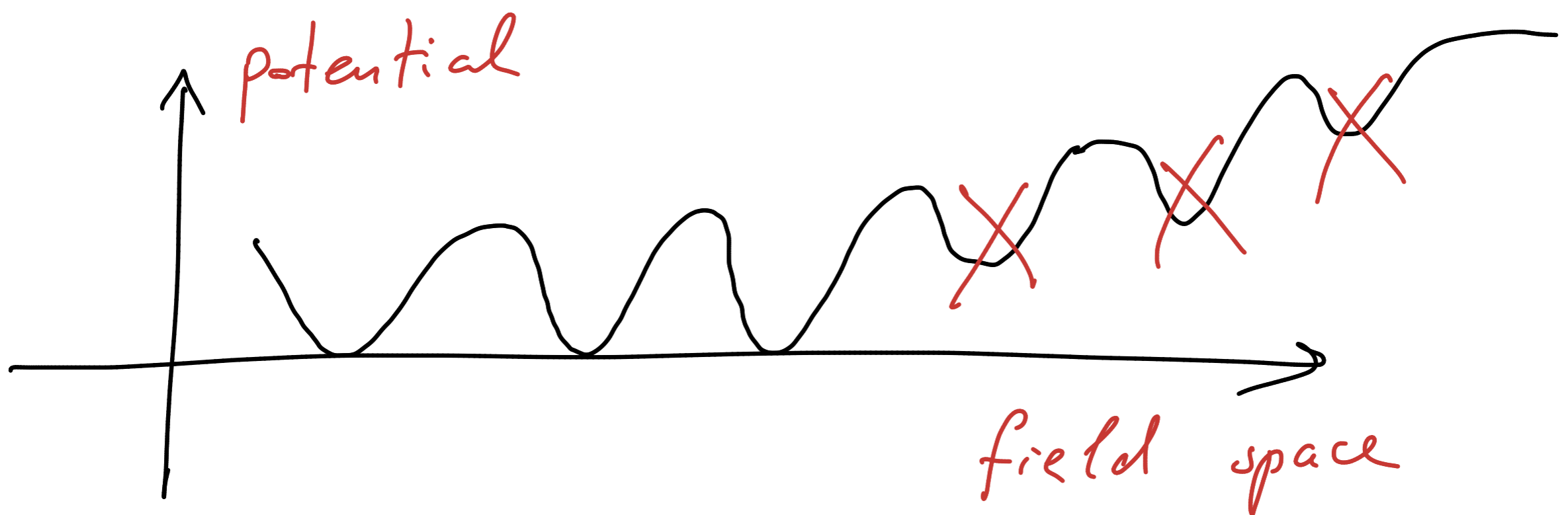
(*) Hints $f_a \rightarrow$ scale of gravity?

S -matrix is currently the only existing formulation of quantum gravity (organic to string theory).

For "no-go" scaling arguments see:

G.D., 2012.02133 [hep-th]

Our framework: 3+1 dimensional EFT of S -matrix theory defined on asymptotic S -matrix vacuum of Minkowski



This fact severely constrains the field content.

In particular, maximal 3-form fields are excluded.

$$C_{\alpha\beta\gamma} \rightarrow C_{\alpha\beta\gamma} + \partial_\alpha \Omega_{\beta\gamma}$$

Lagrangian:

$$L = E^2$$

$$E \equiv \varepsilon^{\alpha\beta\gamma\delta} \partial_\alpha C_{\beta\gamma\delta}$$

Equation

$$\partial^\mu E = 0$$

Vacua

$$E = E_0 = \text{const}$$

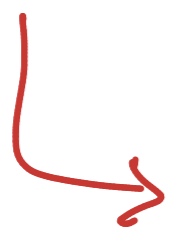
↑ arbitrary constant
"electric" field.

The vacuum landscape with
energy density

$$P_{vac} = E_0^2$$

The same holds for

$$L = K(E)$$



$$\frac{\partial K(E)}{\partial E} = 0$$



vacua

$$E = E_0 = \text{const}$$

Superselection sectors.

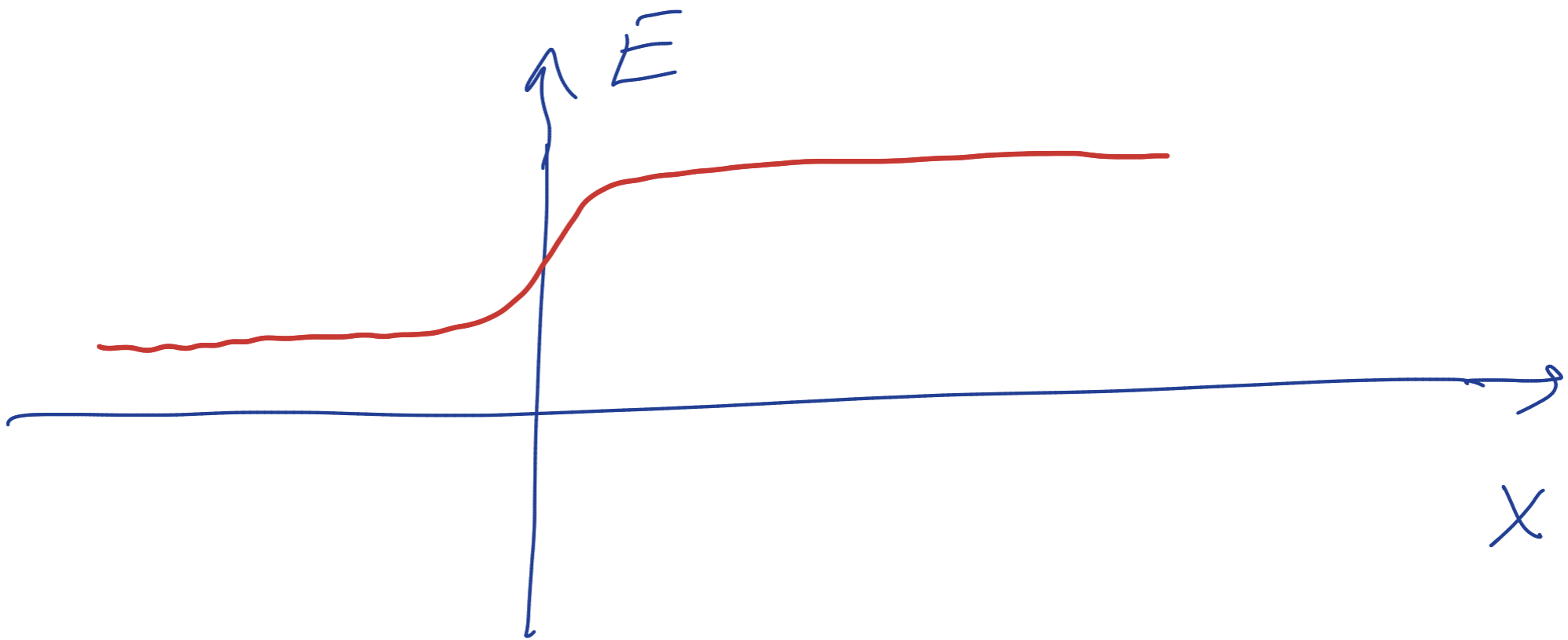
Sources allow some transitions

$$\mathcal{L} = E^2 + C_{\alpha\beta\gamma} J^{\alpha\beta\gamma}$$

$$J_{\alpha\beta\gamma} = Q \int dx_\alpha \wedge dx_\beta \wedge dx_\gamma$$

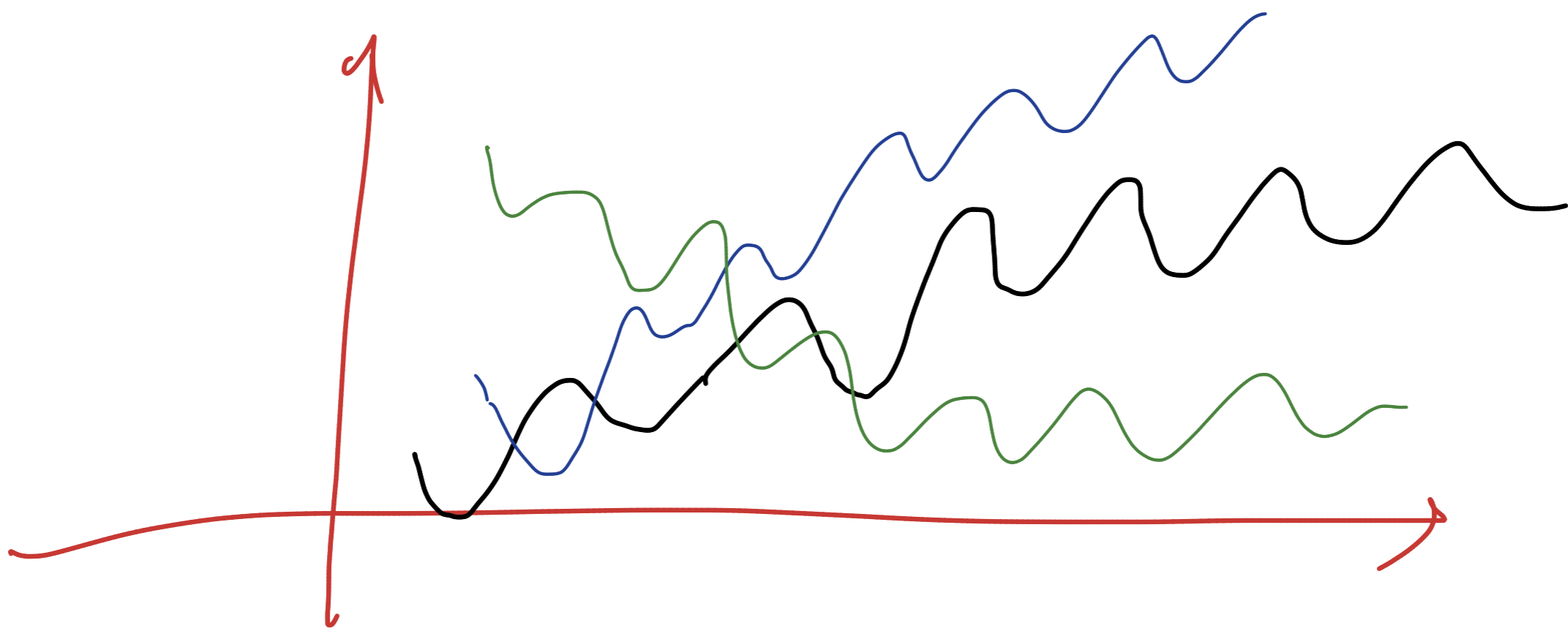
2-brane, axionic domain wall

$$\partial^\mu E = \partial^\mu * J$$



$$\Delta E = Q$$

This allows creation of
connected set of vacua



Excluded by S-matrix.

3-forms must be massive!

This requires axions.

$$\mathcal{L} = E^2 + \frac{a}{f_a} E + (\partial_\mu a)^2$$

Three form Higgs effect:

$$\partial_\mu E = \partial_\mu \frac{a}{f_a} \rightarrow E = \frac{1}{f_a} (a - a_0)$$

const

$$\square a + \frac{1}{f_a} E = 0$$

$$\left(\square + \frac{1}{f_a^2} \right) (a - a_0) = 0$$

$$M_a^2 = \frac{1}{f_a^2}$$

Vacuum: $a = a_0 \rightarrow E = 0$

This has direct relevance for Θ -vacua in gauge theories:

Θ -vacuum = 3-form vacuum.

Chern-Simons 3-form:

$$C_{\alpha\beta\gamma} \equiv A_{[\alpha} A_{\beta} A_{\gamma]} - \frac{3}{2} A_{[\alpha} \partial_{\beta} A_{\gamma]}$$

gluon matrix $A_{\alpha} \equiv A_{\alpha}^a T^a$

under gauge transformation:

$$A \rightarrow e^{i\omega^a T^a} A e^{-i\omega^a T^a}$$

$$C_{\alpha\beta\gamma} \rightarrow C_{\alpha\beta\gamma} + \partial_{\alpha} [\Omega_{\beta\gamma}]$$

$$\Omega_{\beta\gamma} = A_{[\alpha} \partial_{\beta} A_{\gamma]} \omega^a$$

$$E = \epsilon^{\alpha\beta\gamma\delta} \partial_\alpha C_{\beta\gamma\delta} = F \tilde{F}$$

↑ in common notations

topological vacuum susceptibility

$$\langle E, E \rangle \equiv \lim_{q \rightarrow 0} \int d^4x e^{iqx} \langle T \{ E(x) E(0) \} \rangle =$$

$$= \text{const}(\theta) \neq 0$$

Implies

$$\langle C C \rangle = \frac{1}{q^2} + \sum_{m \neq 0} \frac{C_m}{q^2 - m^2}$$

$C =$ massless 3-form + massive modes

Theta vacua:

$$\langle E \rangle \neq 0 \longleftrightarrow \theta \neq 0$$

This is excluded by \mathcal{S} -matrix formulation of gravity.

Thus, gauge theory must come with axion, **by consistency!**

Massless quark is equivalent to axion, since for $M_{\text{quark}} = 0$,

axion is η' -meson (or the analog).

The shift symmetry

$$a \rightarrow a + \text{const}$$

must not be broken by extra source.
Otherwise, θ -vacua will be reintroduced.

What about gravity?

Can gravity be controlled?

Dual (Gauge) formulation of the axion solution

G.D., 0507215 [hep-th]

$$\mathcal{L} = K(E) + m_a^2 (C - dB)^2$$

gauge symmetry:

$$\begin{aligned} C &\rightarrow C + d\Omega, \\ B &\rightarrow B + \Omega, \end{aligned}$$

$$\begin{aligned} \Omega_{\mu\nu} &= A_{[\mu}^a \partial_{\nu]} \omega^a \\ A_\mu &\rightarrow e^{i\omega^a T^a} A_\mu e^{-i\omega^a T^a} \end{aligned}$$

Exact
dual

$$\mathcal{L} = K(E) + (\partial_\mu a)^2 - m_a a E$$

$$\mathcal{L} = (\partial_\mu a)^2 - V(a)$$

$$\hookrightarrow V(z) = m \int dz \operatorname{inv} \frac{\partial K(mz)}{\partial z}$$

At the level of EFT there is exact duality

$$a \longleftrightarrow B_{\mu\nu}$$

global symmetry

$$a \rightarrow a + \text{const}$$

Gauge symmetry

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \Omega_{\mu\nu}$$



↑
"Broken" by 3-form

Higgsed by 3-form

By power of gauge redundancy,
only way to create θ -vacua
is by introducing additional
3-form, unpaired with additional axion.

QCD vacuum must be
exact CP-conserving: θ unphysical.

What can we say about
the scale f_a ?

UV-completion of $B_{\mu\nu}$ formulation
is not possible by complex scalar
A la Peccei-Quinn.

This favors quantum gravitational
UV-completion (A la string theory).

This would hint that f_a
is connected with the scale of
gravity.

In general, since QCD axion is required by consistency of gravity, it is natural that the origin of the scale f_a is gravitational.

$$\left. \begin{array}{l} M_p \rightarrow \infty \\ f_a \rightarrow \infty \end{array} \right\} ?$$

$$L = K(E) + M_a^2 (C + \tilde{C} - dB)^2$$

$$+ \tilde{K}(\tilde{E})$$

$$\hookrightarrow \tilde{E} \equiv d\tilde{C}$$

A candidate in gravity, gravitational Chern-Simons

$$\tilde{C} \equiv \Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma$$

$$\tilde{E} \equiv d\tilde{C} = R\tilde{R} = \epsilon^{\alpha\beta\mu\nu} R_{j\alpha\beta}^i R_{i\mu\nu}^j$$

If in pure gravity $\langle R\tilde{R}, R\tilde{R} \rangle_{g \rightarrow 0} \neq 0$,

gravity must provide additional axion.

OR

anomalous chiral symmetry of
neutrino

$$\nu \rightarrow e^{i\alpha\gamma_5} \nu$$

can neutralize TVS of gravity,
resulting into $M_\nu \neq 0$.

G.P., Folkerts, Franca 1312.7273 [hep-th]

G.P., Funke, 1602.03191 [hep-ph]

Each 3-form provided by gravity
must be accompanied either by axion
or a chiral fermion.

QLD axion is safe by
consistency of S-matrix gravity.