Maximally entangled state at small Bjorken x

Dmitri Kharzeev

Center for Nuclear Theory

C^{2}QA Co-design Center for Quantum Advantage

U.S. DEPARTMENT OF ENERGY Office of Science

Stony Brook University

Center for Frontiers in Nuclear Science

Brookhaven National Laboratory

Based on:

Entanglement in DIS: Maximally entangled state

DK, E. Levin, Phys Rev D 95 (2017) 114008 + PRD 104 (2021) 3

DK, Phil. Trans. Royal Soc A 380 (2021) 5

Entanglement and integrability in DIS

K. Hao, DK, V. Korepin, Int J Mod Phys A34 (2019) 1950197

K. Zhang, K. Hao, DK, V. Korepin, Phys Rev D 105 (2022) 1

Entanglement in real time, quantum simulations

DK, Y. Kikuchi, Phys. Rev. Res. 2 (2020) 2, 023342

A. Florio, DK, Phys Rev D 104 (2021) 5, 056021

D. Frenklakh, A. Florio, DK, in progress

Entanglement in high energy hadron collisions

O.K. Baker, DK, Phys Rev D 98 (2018) 054007

Z. Tu, DK, T. Ullrich, Phys Rev Lett 124 (2020) 6, 062001

Related work on entanglement in DIS:

Maximally entangled state at small x, link to black holes:

G. Dvali, R. Venugopalan, Phys Rev D 105 (2022) 5, 056026
Y. Liu, M. Nowak, I. Zahed, Phys.Rev.D 105 (2022) 11, 114028; …

Maximally entangled state at small x, phenomenology:

M. Hentschinski, K. Kutak, Eur.Phys.J.C 82 (2022) 2, 111; ….

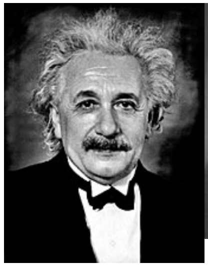
Momentum space entanglement and RG evolution:

A. Kovner, M. Lublinsky Phys Rev D 92 (2015) 3, 034016;
A. Kovner, M. Lublinsky, M. Serino, Phys Lett B 792 (2019) 4;
N. Armesto, F. Dominguez, A. Kovner, M. Lublinsky, V. Skokov, JHEP 05(2019) 025; ….

Entanglement and thermalization:

J. Berges, S. Floerchinger, R. Venugopalan, JHEP 04(2018)145; …

Quantum entanglement



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

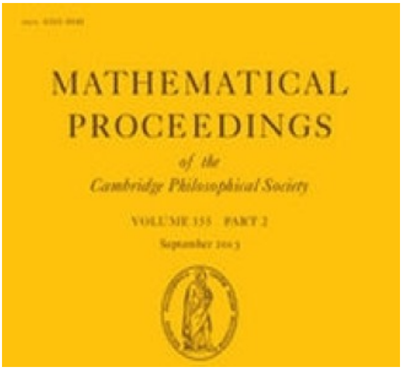
Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS



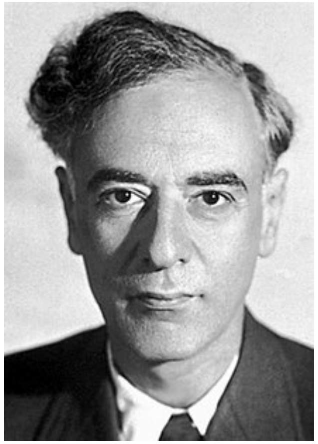
By E. SCHRÖDINGER

[Communicated by Mr M. Born]

[*Received* 14 August, *read* 28 October 1935]

1. When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. To disentangle them we must gather further information by experiment, although we knew as much as anybody could possibly know about all that happened. Of either system, taken separately, all previous knowledge may be entirely lost, leaving us but one privilege: to restrict the experiments to one only of the two systems. After re-establishing one representative by observation, the other one can be inferred simultaneously. In what follows the whole of this procedure will be called *the disentanglement*. Its sinister importance is due to its being involved in every measuring process and therefore forming the basis of the quantum theory of

Describing entanglement: the density matrix



EPR state (2 qubits):

$$\frac{|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B}{\sqrt{2}}$$

The corresponding density matrix:

$$\rho_{AB}\left(\frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\right) = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\frac{\langle 00| \pm \langle 11|}{\sqrt{2}} = \frac{|00\rangle\langle 00| \pm |00\rangle\langle 11| \pm |11\rangle\langle 00| + |11\rangle\langle 11|}{2}$$

If the state of B is unknown, A is described by the reduced density matrix:

$$\rho_A = \mathrm{tr}_B(\rho_{AB}) = \frac{|0\rangle\langle 0|\ \langle 0|0\rangle \pm |0\rangle\langle 1|\ \langle 0|1\rangle \pm |1\rangle\langle 0|\ \langle 1|0\rangle + |1\rangle\langle 1|\ \langle 1|1\rangle}{2}$$

$$= \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathbf{I}}{2}.$$

Mixed state! $\rho_A \neq \rho_A^2$

Quantifying entanglement: von Neumann entropy

$$\rho = \sum_n p_n |n\rangle\langle n|$$

Entanglement entropy:

$$S = -\mathrm{tr}\rho \ln \rho = -\mathrm{p_n} \ln \mathrm{p_n}$$

Pure states:

$$S = 0$$

e.g.

$$p_0 = 1, \; p_{n \neq 0} = 0$$

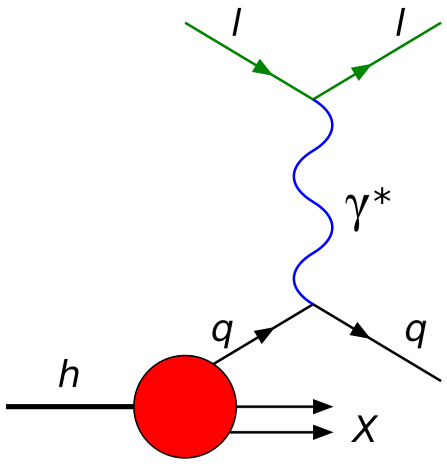
Mixed states:

$$S \neq 0$$

e.g. for EPR $\rho_A = \frac{\mathbf{I}}{2}$

$$p_0 = p_1 = \frac{1}{2} \rightarrow S = \ln 2$$

The puzzle of the parton model



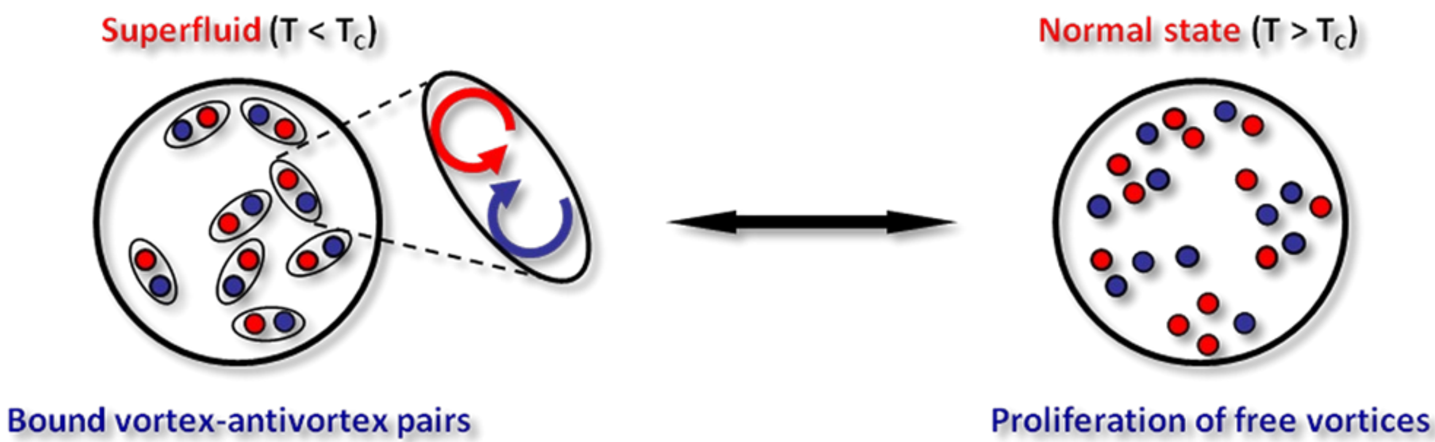
In parton model, the proton is pictured as a collection of point-like quasi-free partons that are frozen in the infinite momentum frame due to Lorentz dilation.

The DIS cross section is given by the incoherent sum of cross sections of scattering off individual partons.

How to reconcile this with quantum mechanics?

The puzzle of the parton model

In quantum mechanics, the proton is a pure state with zero entropy. Yet, a collection of free partons does possess entropy… Boosting to the infinite momentum frame does not help, as a Lorentz boost cannot transform a pure state into a mixed one.



The crucial importance of entropy in (2+1)D systems: BKT phase transition (Nobel prize 2016)

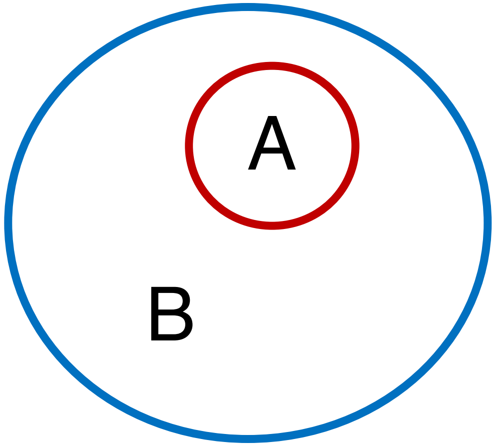
The quantum mechanics of partons and entanglement

Our proposal: the key to solving this apparent paradox is entanglement.

DK, E. Levin, arXiv:1702.03489; PRD

DIS probes only a part of the proton's wave function (region A). We sum over unobserved region B; in quantum mechanics, this corresponds to accessing the density matrix of a <u>mixed state</u>

$$\hat{\rho}_A = \mathrm{tr}_{\mathrm{B}}\hat{\rho}$$



with a non-zero <u>entanglement entropy</u>

$$S_A = -\mathrm{tr}\left[\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}\right]$$

The quantum mechanics of partons and entanglement

Another (more general?) argument:

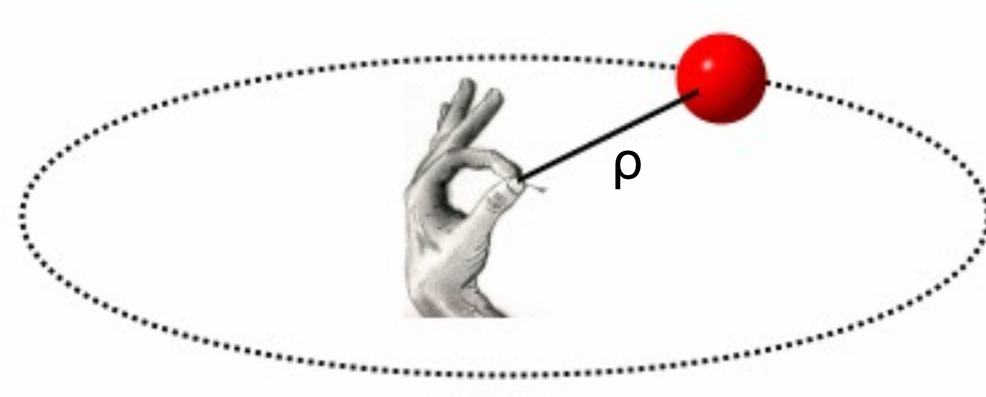
DK, Phil. Trans. Royal Soc (2022); arXiv:2108.08792

DIS takes an instant snapshot of the proton's wave function. This snapshot cannot measure the phase of the wave function.

Classical analogy:

$$z = \rho \ \exp(i\omega t)$$

Instant snapshot can measure the amplitude ρ, but not the angular velocity ω !



The quantum mechanics of partons and entanglement

A simple quantum mechanical model (proton rest frame):

DK, Phil. Trans. Royal Soc (2022); arXiv:2108.08792

Expand the proton's w.f. in oscillator Fock states:

$$|n\rangle = \frac{1}{\sqrt{n!}} \prod_i^n a_i^\dagger |0\rangle,$$

$$|\Psi\rangle = \sum_n \alpha_n |n\rangle,$$

The density matrix:

$$\hat{\rho} = |\Psi\rangle\langle\Psi\rangle = \sum_{n,n'} \alpha_n \, \alpha_{n'}^* \, |n\rangle\langle n'|,$$

depends on time:

$$\hat{\rho}(t) = \sum_{n,n'} e^{i(n'-n)\omega t} \, \hat{\rho}(t=0).$$

But this time dependence cannot be measured by a light front – it crosses the hadron too fast, at time $t_{light} = R,$

The quantum mechanics of partons and entanglement

DK, Phil. Trans. Royal Soc (2022)

Therefore, the observed density matrix is a trace over an unobserved phase:

$$\hat{\rho}_{parton} = \mathrm{Tr}_{\varphi}\hat{\rho} = \sum_{n,n'} \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i(n'-n)\varphi}\, \alpha_n\, \alpha^*_{n'} |n\rangle\langle n'| = \sum_n |\alpha_n|^2\, |n\rangle\langle n|.$$

U(1) Haar measure

"Haar scrambling"
Y.Sekino, L.Susskind '08

After "Haar scrambling", the density matrix becomes diagonal in parton basis (Schmidt basis) –

Probabilistic parton model!

This is a density matrix of a mixed state, with non-zero entanglement entropy!

The quantum mechanics of partons and entanglement

DK, Phil. Trans. Royal Soc (2022); arXiv:2108.08792

The parton model density matrix:

$$\hat{\rho}_{parton} = \sum_n p_n \, |n\rangle\langle n|$$

is mixed, with purity

$$\gamma_{parton} = \mathrm{Tr}(\rho^2_{parton}) = \sum_n p_n^2 < 1.$$

entanglement entropy

$$S_E = -\sum_n p_n \ln p_n$$

Parton model expressions for expectation values of operators:

$$\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}(\hat{\mathcal{O}} \hat{\rho}_{parton}) = \sum_n p_n \langle n|\hat{\mathcal{O}}|n\rangle;$$

The quantum mechanics of partons and entanglement on the light cone

The density matrix on the light cone:

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \sum_{n,n'}^{\infty} \int d\Gamma_n \, d\Gamma_{n'} \, \Psi^*_{n'}(x_{i'}, \vec{k}_{\perp i'}) \Psi_n(x_i, \vec{k}_{\perp i}) |n\rangle\langle n'|.$$

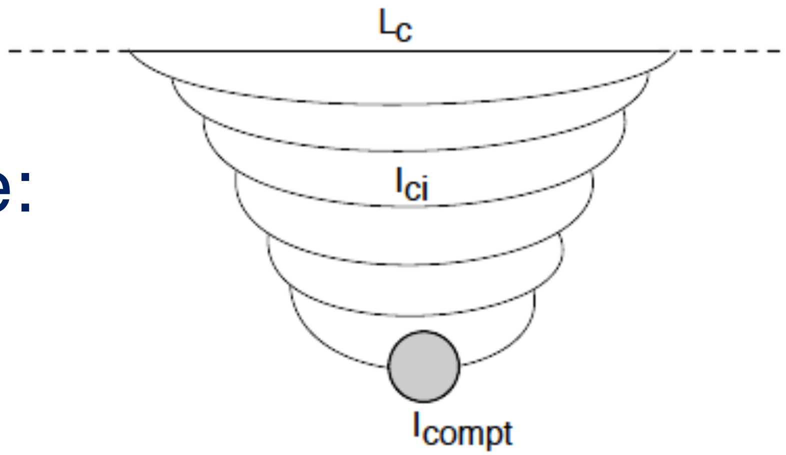
Haar scrambling: on the light cone, $t_i - z_i = x_i^- = 0,$
but t, z and $x^+ = z + t$ cannot be independently determined:

$$\int \frac{dx^+}{2\pi} e^{i(P_n^- - P_{n'}^-)x^+} = \delta(P_n^- - P_{n'}^-),$$

$$\hat{\rho}_{parton} = \mathrm{Tr}_{x^+} |\Psi\rangle\langle\Psi| = \sum_n^{\infty} \int d\Gamma_n \, |\Psi_n(x_i, \vec{k}_{\perp i})|^2 |n\rangle\langle n|,$$

The entanglement entropy from QCD evolution

Space-time picture
in the proton's rest frame:



The evolution equation:

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

The entanglement entropy from QCD evolution

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

Solve by using the generating function method

(A.H. Mueller ‘94; E. Levin, M. Lublinsky ‘04):

$$Z(Y,u) = \sum_n P_n(Y) u^n.$$

Solution:

$$P_n(Y) = e^{-\Delta Y}(1 - e^{-\Delta Y})^{n-1}.$$

The resulting von Neumann entropy is

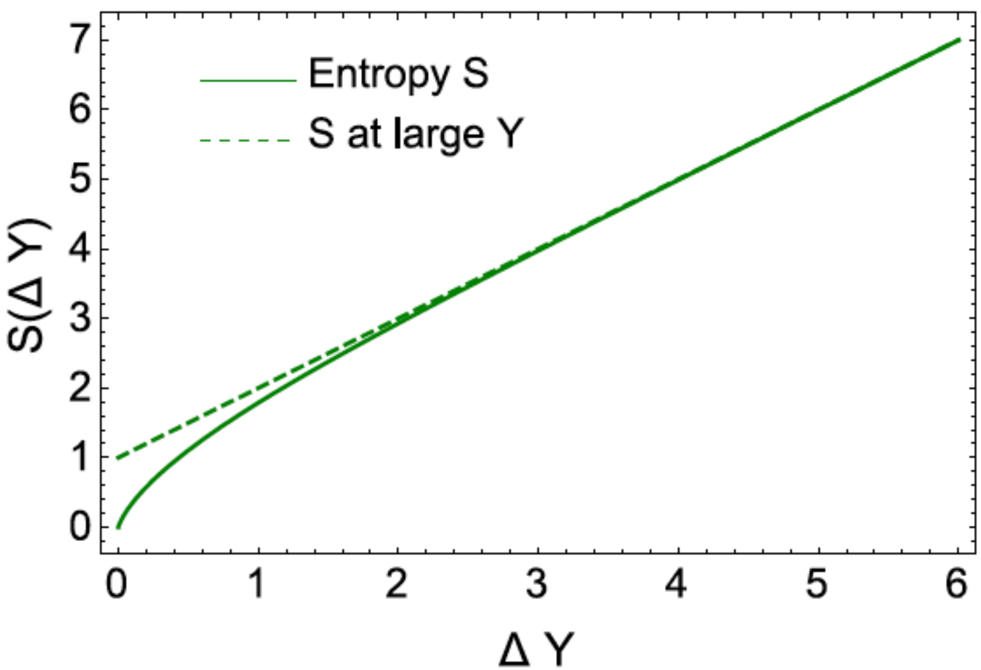
$$S(Y) = \ln(e^{\Delta Y} - 1) + e^{\Delta Y} \ln\left(\frac{1}{1 - e^{-\Delta Y}}\right)$$

DK, E. Levin, arXiv:1702.03489; PRD

The entanglement entropy from QCD evolution

At large ΔY , the entropy becomes

$$S(Y) \to \Delta Y$$



This "asymptotic" regime starts rather early, at

$$\Delta Y \simeq 2$$

DK, E. Levin, arXiv:1702.03489; PRD

The entanglement entropy from QCD evolution

At large ΔY (x ~ 10^{-3}) the relation between the entanglement entropy and the structure function

$$xG(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\Delta}$$

becomes very simple:

$$S = \ln[xG(x)]$$

DK, E. Levin, arXiv:1702.03489; PRD 95 (2017)

The entanglement entropy from QCD evolution

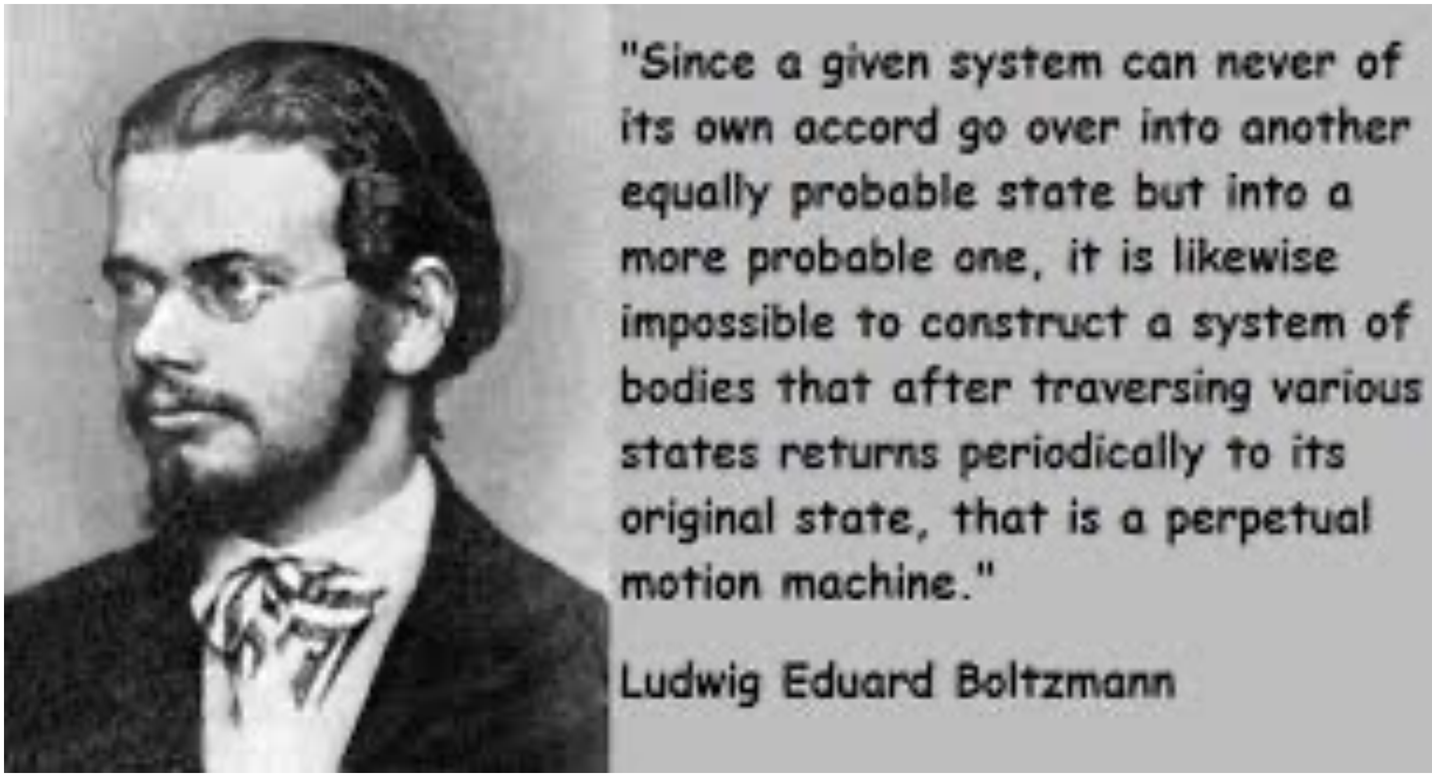
What is the physics behind this relation?

$$S = \ln[xG(x)]$$

It signals that all $\exp(\Delta Y)$ partonic states have about equal probabilities $\exp(-\Delta Y)$ – in this case the **entanglement entropy is maximal**, and the proton is a **maximally entangled state** (a new look at the parton saturation and CGC?)

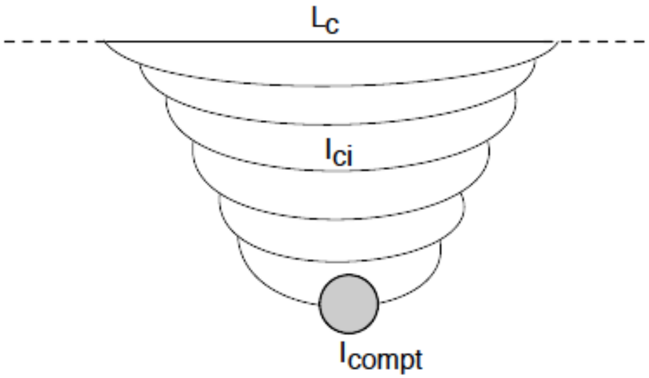
DK, E. Levin, arXiv:1702.03489; PRD 95 (2017)

L. Boltzmann:



the system is driven to the most probable state with the largest entropy

Relation to CFT?

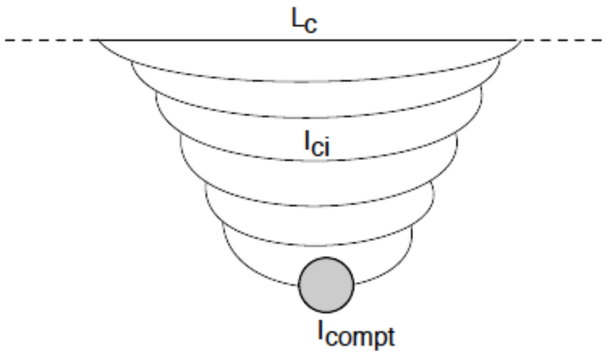


The small x formula $$S = \ln[xG(x)]$$ $$xG(x) \sim \left(\frac{1}{x}\right)^{\Delta}$$

yields $$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

where $$L = (mx)^{-1}$$

is the longitudinal length probed in DIS (in the target rest frame), and $\epsilon \equiv 1/m$ is the proton's Compton wavelength



Relation to CFT

This formula

$$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

is similar to the well-known (1+1) CFT result:

(Holzhey, Larsen, Wilczek ‘94; Vidal, Latorre, Rico, Kitaev ‘03, Korepin ‘04, Calabrese, Cardy ‘05)

$$S_E = \frac{c}{3} \ \ln \frac{L}{\epsilon}$$

where c is the central charge of the CFT, and ε is the resolution scale

Central charge at small x

L. Lipatov has discovered duality between small x effective action and an integrable XXX s=0 spin chain;

L.Lipatov, JETP Lett. 59, 596 (1994)

it was mapped to s=-1 spin chain by L. Faddeev and G. Korchemsky,

L.Faddeev and G. Korchemsky, Phys. Lett. B 342, 311 (1995)

and to Lattice non-linear Schroedinger model

K. Hao, DK, V. Korepin, IJMPA 34, 1950197 (2019)

Recently, we have computed the central charge of this model and found c = 1.

K. Zhang, K. Hao, DK, V. Korepin, Phys Rev D (2022); arXiv:2110.04881

In the maximally entangled regime, this translates into

$$xG(x) \sim \left(\frac{1}{x}\right)^{\frac{c}{3}} \sim \left(\frac{1}{x}\right)^{\frac{1}{3}}$$

Maximally entangled regime

In the maximally entangled regime at small x, it appears that the behavior of the gluon structure function becomes universal;

it is determined by the central charge of the corresponding CFT, and not by its anomalous dimension.

Analogy to statistical mechanics:

in thermal equilibrium (maximal entropy), the equation of state is determined by temperature (1/x)

and

the effective number of degrees of freedom (central charge).

Experimental tests

What is the relation between the parton and hadron multiplicity distributions?

Let us assume they are the same ("EbyE parton-hadron duality"); then the hadron multiplicity distribution should be given by

$$P_n(Y) \;=\; e^{-\Delta Y}(1 - e^{-\Delta Y})^{n-1}.$$

Consider moments

$$C_q = < n^q > / < n >^q$$

Fluctuations in hadron multiplicity

The moments can be easily computed by using the generating function

$$C_q \;=\; \left(u\frac{d}{du}\right)^q Z\left(Y,u\right)\Big|_{u=1}$$

We get

$$C_2 \;=\; 2-1/\bar{n};\qquad C_3 \;=\; \frac{6(\bar{n}-1)\bar{n}+1}{\bar{n}^2};$$

$$C_4 \;=\; \frac{(12\bar{n}(\bar{n}-1)+1)(2\bar{n}-1)}{\bar{n}^3};\qquad C_5 \;=\; \frac{(\bar{n}-1)(120\bar{n}^2(\bar{n}-1)+30\bar{n})+1}{\bar{n}^4}.$$

Fluctuations in hadron multiplicity

Numerically, for $\bar{n} = 5.8 \pm 0.1$ at $|\eta|<0.5$, E_{cm}=7 TeV we get:

theory	exp (CMS)	theory, high energy limit
$C_2 = 1.83$	$C_2 = 2.0$+-0.05	$C_2 = 2.0$
$C_3 = 5.0$	$C_3 = 5.9$+-0.6	$C_3 = 6.0$
$C_4 = 18.2$	$C_4 = 21$+-2	$C_4 = 24.0$
$C_5 = 83$	$C_5 = 90$+-19	$C_5 = 120$

It appears that the multiplicity distributions of final state hadrons are very similar to the parton multiplicity distributions – this suggests that the entropy is close to the entanglement entropy

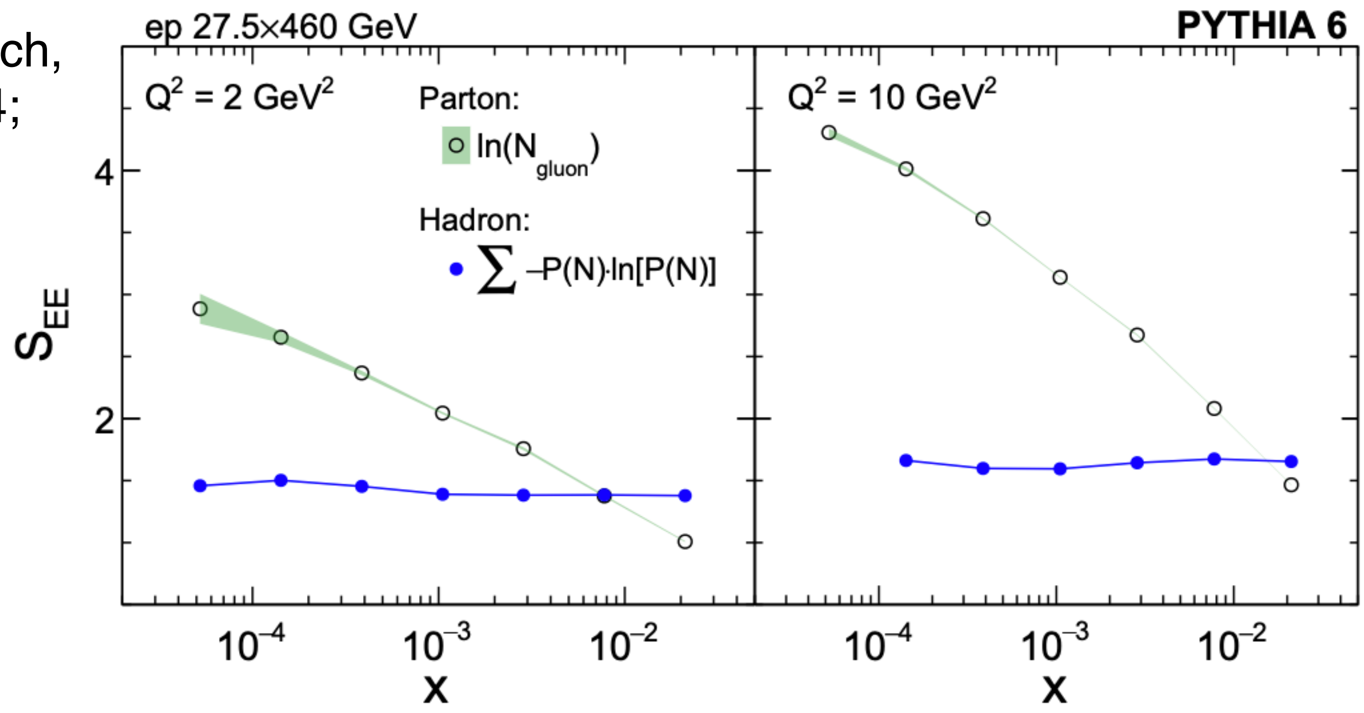
Test of the entanglement at the LHC

MC generator PYTHIA:

$$S = \ln[xG(x)]$$

is not satisfied at small x (no entanglement)

K. Tu, DK, T. Ullrich, arXiv:1904.11974; PRL (2020)



Test of the entanglement at the LHC

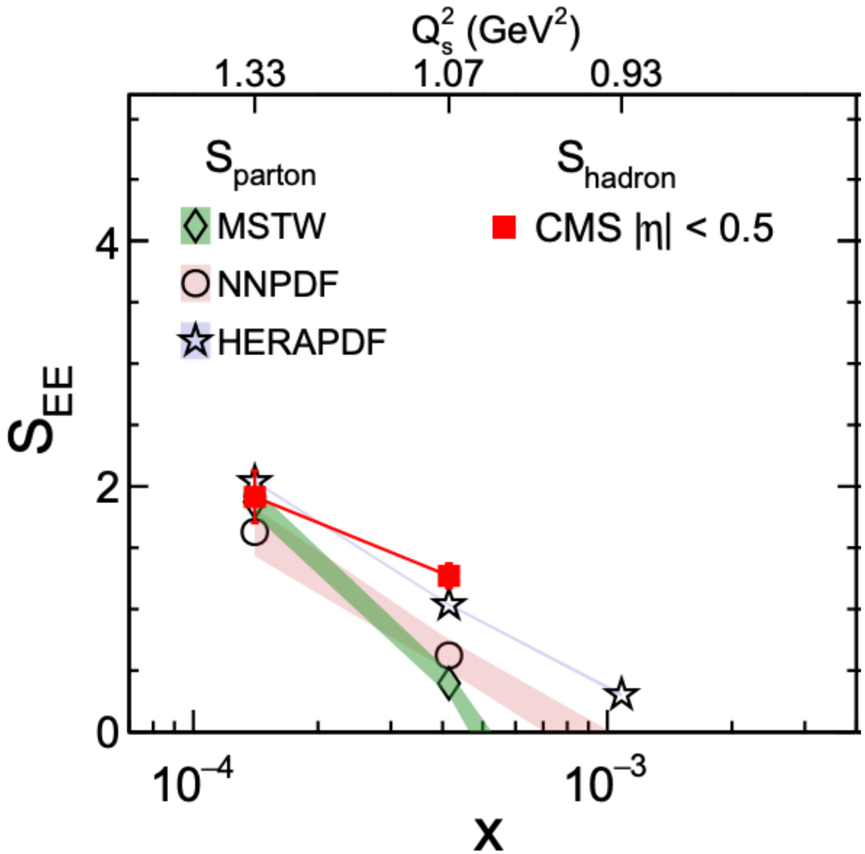
LHC data:

arXiv:1904.11974

$$S = \ln[xG(x)]$$

is satisfied at small x (**entanglement?!**)

K. Tu, DK, T. Ullrich, arXiv:1904.11974; PRL (2020)



Test of the entanglement in DIS

H1 Coll., arXiv:2011.01812; EPJC81(2021)3, 212

H1 Coll. test of

$$S = \ln[xG(x)]$$

using DIS data (current fragmentation region)

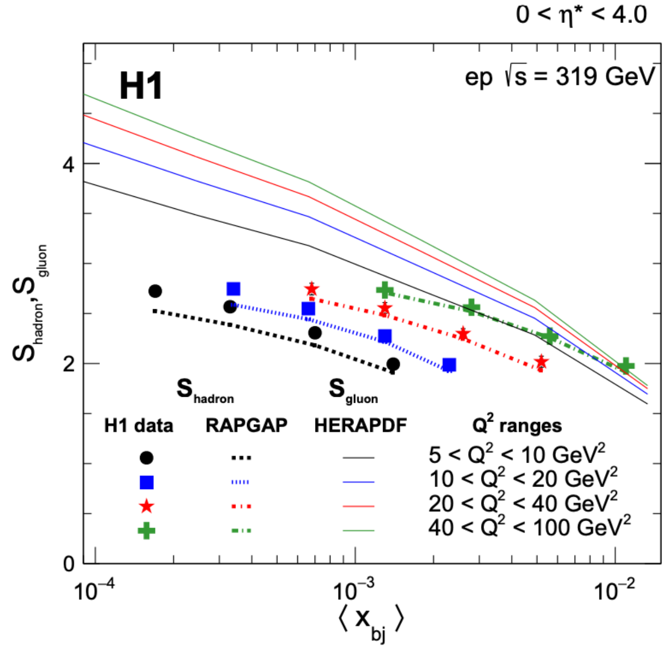


Figure 12: Hadron entropy $S_{\rm hadron}$ derived from multiplicity distributions as a function of $\langle x_{\rm bj} \rangle$ measured in different Q^2 ranges, measured in $\sqrt{s} = 319\,{\rm GeV}$ ep collisions. Here, a restriction to the current hemisphere $0 < \eta^* < 4$ is applied. Further phase space restrictions are given in Table 1. Predictions for $S_{\rm hadron}$ from the RAPGAP model and for the entanglement entropy $S_{\rm gluon}$ based on an entanglement model are shown by the dashed lines and solid lines, respectively. For each Q^2 range, the value of the lower boundary is used for predicting $S_{\rm gluon}$. The total uncertainty on the data is represented by the error bars.

Poor agreement is found!

Failure of the entanglement-based picture?

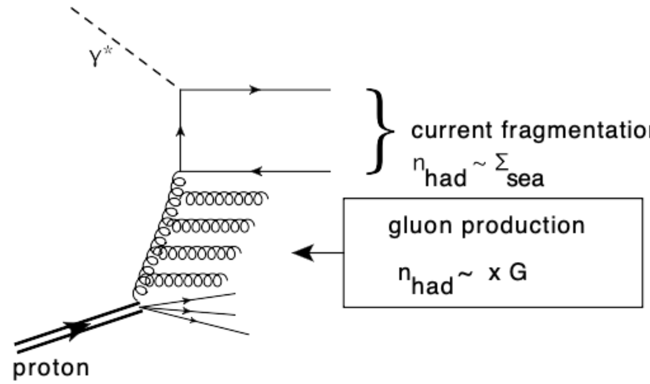
Test of the entanglement in DIS

DK, E. Levin, arXiv:2102.09773, PRD

It appears that in H1 kinematics (current fragmentation region), the assumptions used to derive the formula

$$S = \ln[xG(x)]$$

do not apply:



1. The quark structure function is not proportional to the gluon one so need to use the quark distribution explicitly

$$x\Sigma(x,Q^2) = \frac{C_F\,\alpha_s}{2\,\pi}\int_0^{\xi} d\xi' \int_x^1 dz\, P_{qG}(z)\left(\frac{x}{z}G\left(\frac{x}{z},\xi'\right)\right) \quad \textbf{with} \quad P_{qG}(z) = \frac{1+(1-z)^2}{z}$$

2. Multiplicity N is not large, so need to take into account 1/N corrections

Test of the entanglement in DIS

The result: good agreement with H1 data

DK, E. Levin, arXiv:2102.09773; PRD

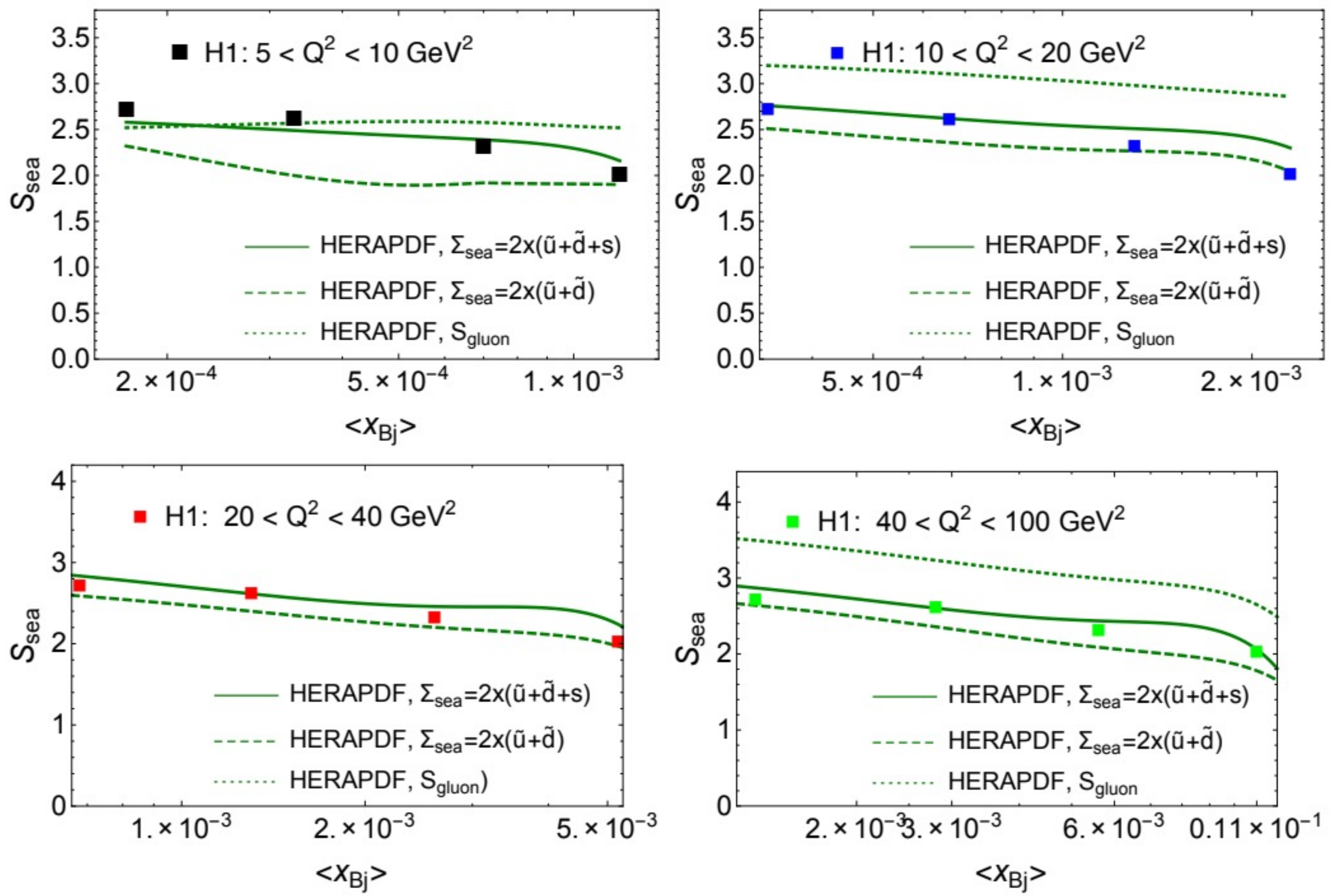


FIG. 1: Comparison of the experimental data of the H1 collaboration [6] on the entropy of produced hadrons in DIS [6] with our theoretical predictions, for which we use the sea quark distributions from the NNLO fit to the combined H1-ZEUS data.

Evidence for the maximally entangled low x proton in Deep Inelastic Scattering from H1 data

Martin Hentschinski[1] and Krzysztof Kutak[2]

[1]Departamento de Actuaria, Física y Matemáticas, Universidad de las Americas Puebla, San Andrés Cholula, 72820 Puebla, Mexico

[2] Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342, Kraków, Poland

December 14, 2021

Abstract

We investigate the proposal by Kharzeev and Levin of a maximally entangled proton wave function in Deep Inelastic Scattering at low x and the proposed relation between parton number and final state hadron multiplicity. Contrary to the original formulation we determine partonic entropy from the sum of gluon and quark distribution functions at low x, which we obtain from an unintegrated gluon distribution subject to next-to-leading order Balitsky-Fadin-Kuraev-Lipatov evolution. We find for this framework very good agreement with H1 data. We furthermore provide a comparison based on NNPDF parton distribution functions at both next-to-next-to-leading order and next-to-next-to-leading with small x resummation, where the latter provides an acceptable description of data.

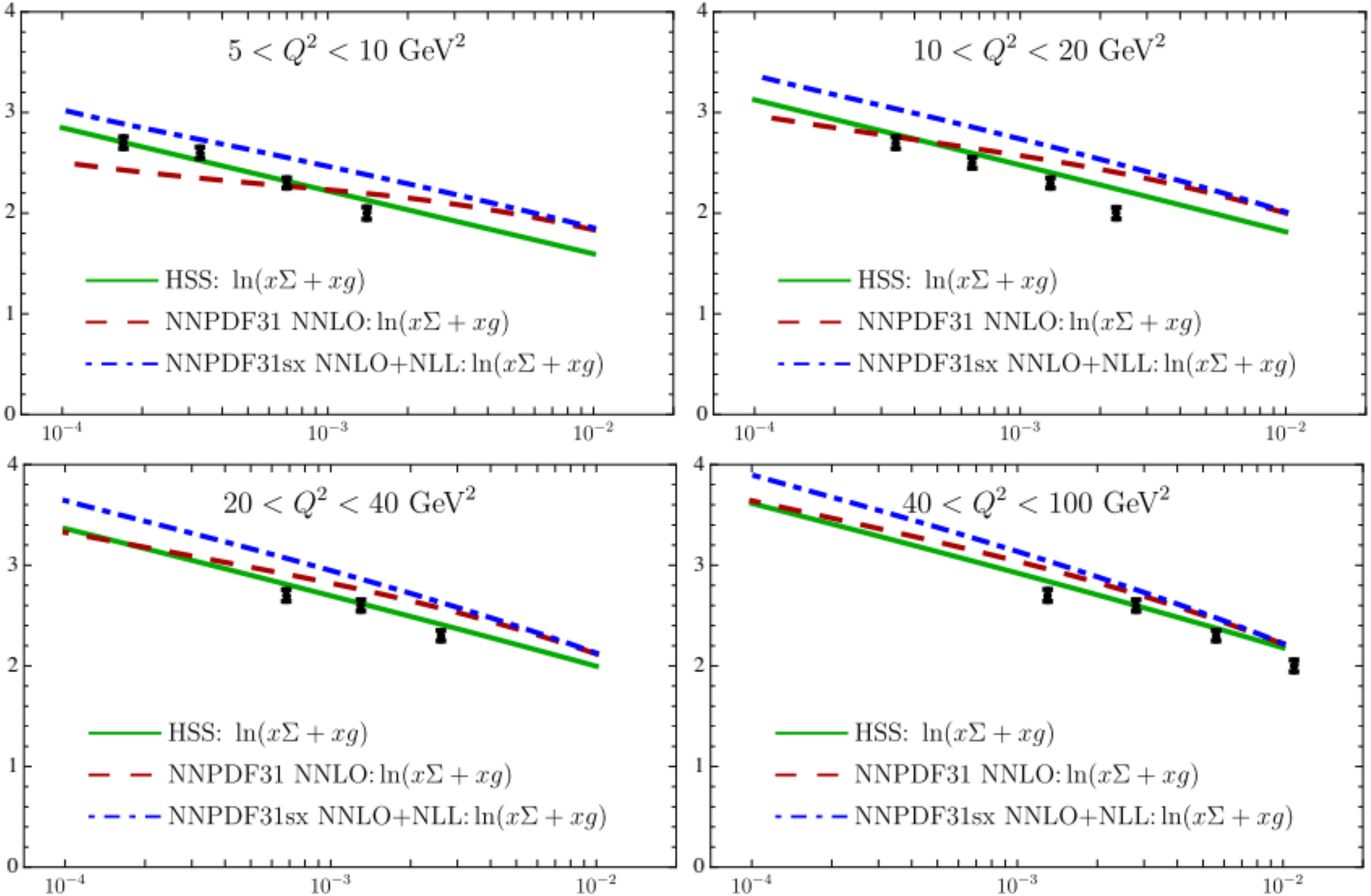


Figure 1: *Partonic entropy versus Bjorken x, as given by Eq.* (1) *and Eq.* (2). *We furter show results based on the gluon distribution only as well as a comparison to NNPDFs. Results are compared to the final state hadron entropy derived from the multiplicity distributions measured at H1* [19]

Summary

1. Entanglement entropy (EE) provides a viable solution to the apparent contradiction between the parton model and quantum mechanics.

2. Indications from experiment that the link between EE and parton distributions is real. Further tests at RHIC and EIC, requirements for detector design.

3. Entanglement may provide a mechanism for thermalization in high-energy collisions. Need for further study of real-time dynamics.