

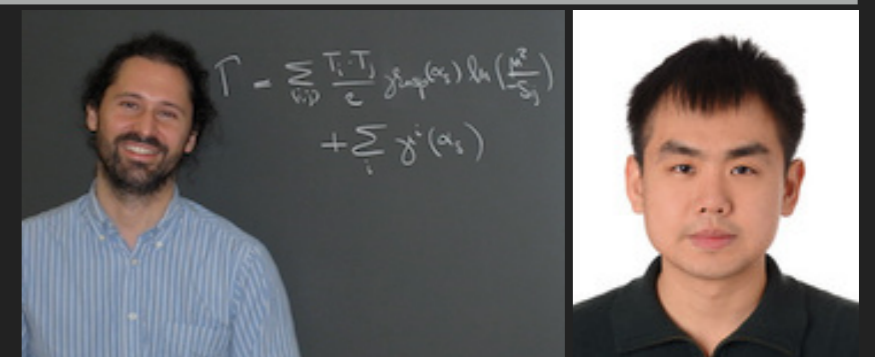
# RESUMMATION OF SUPER-LEADING LOGARITHMS ( SOLVING A 16-YEAR OLD QCD PROBLEM )

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HUMBOLDT KOLLEG — CLUES TO A MYSTERIOUS UNIVERSE

KITZBÜHEL, AUSTRIA, 27 JUNE 2022

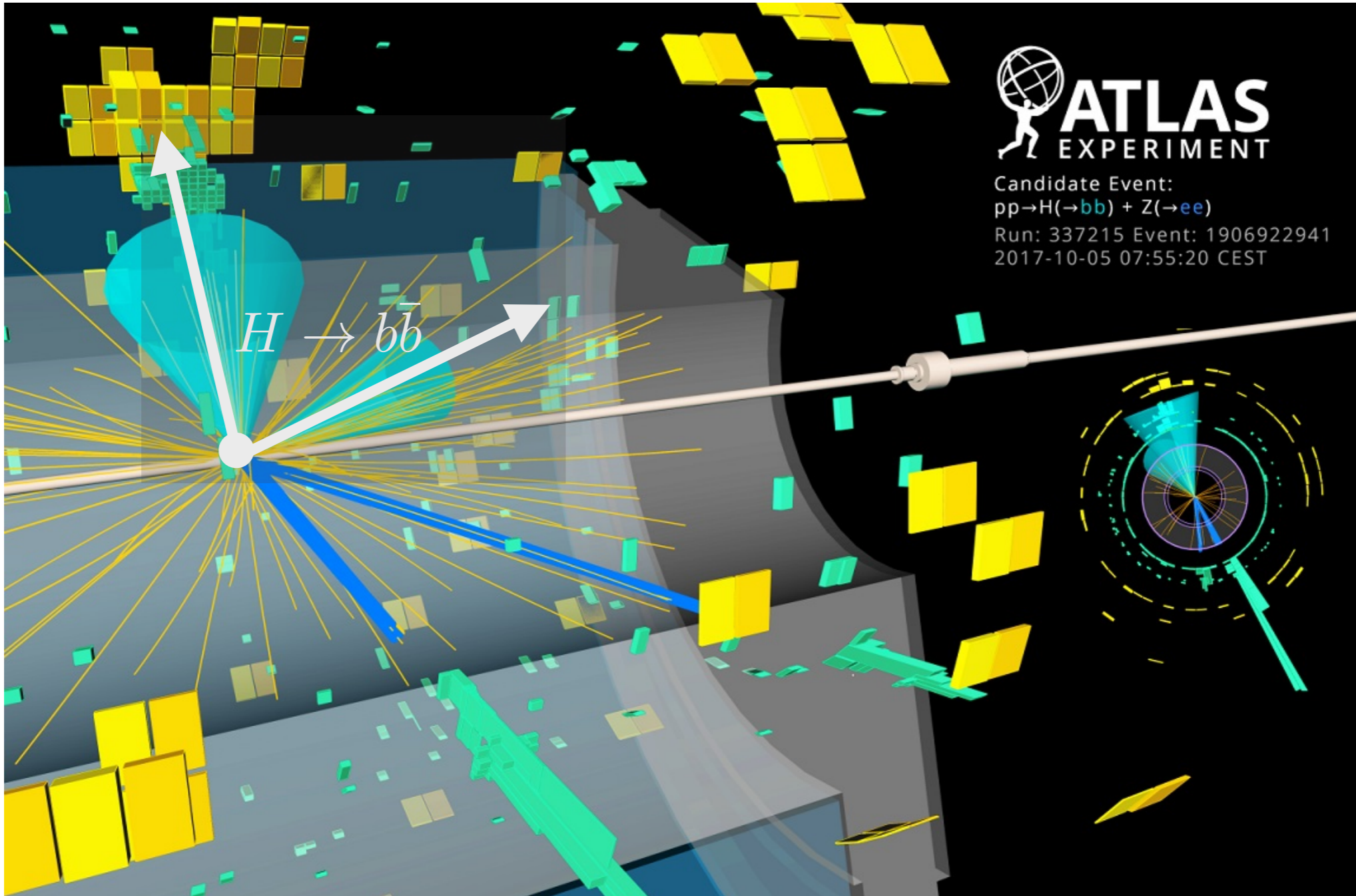
T. BECHER, MN, D.Y. SHAO, PHYS. REV. LETT. 127 (2021) 212002







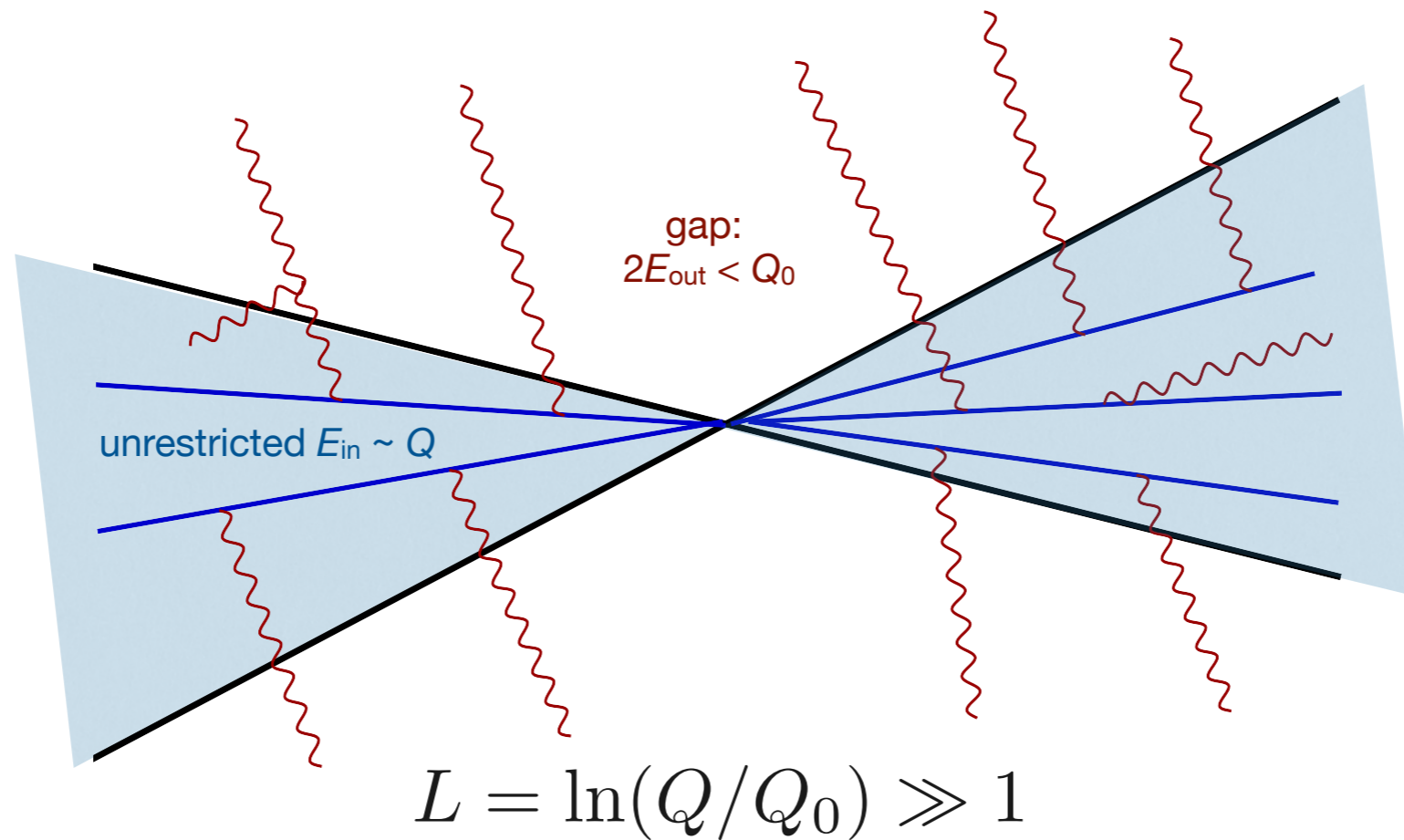
# LARGE LOGARITHMS IN JET PROCESSES



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# LARGE LOGARITHMS IN JET PROCESSES



Perturbative expansion:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 \right\}$$



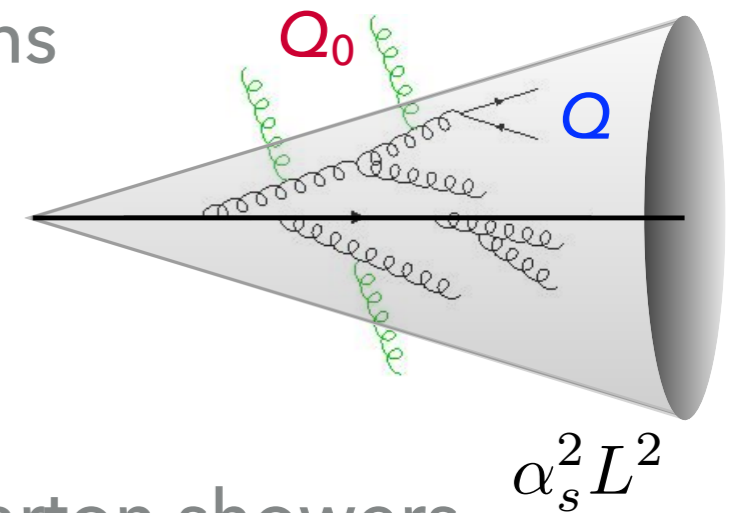
state-of-the-art: 2-loop order



# LARGE LOGARITHMS IN JET PROCESSES

## Non-global logarithms at lepton colliders

- ▶ high-energetic radiation restricted to certain regions
- ▶ soft radiation from secondary emissions inside jets leads to intricate pattern of large logarithms that do not exponentiate
- ▶ "non-global" logs not contained in conventional parton showers
- ▶ single-logarithmic effects  $\sim (\alpha_s L)^n$  at lepton colliders
- ▶ resummation in large- $N_c$  limit using BMS integral equation

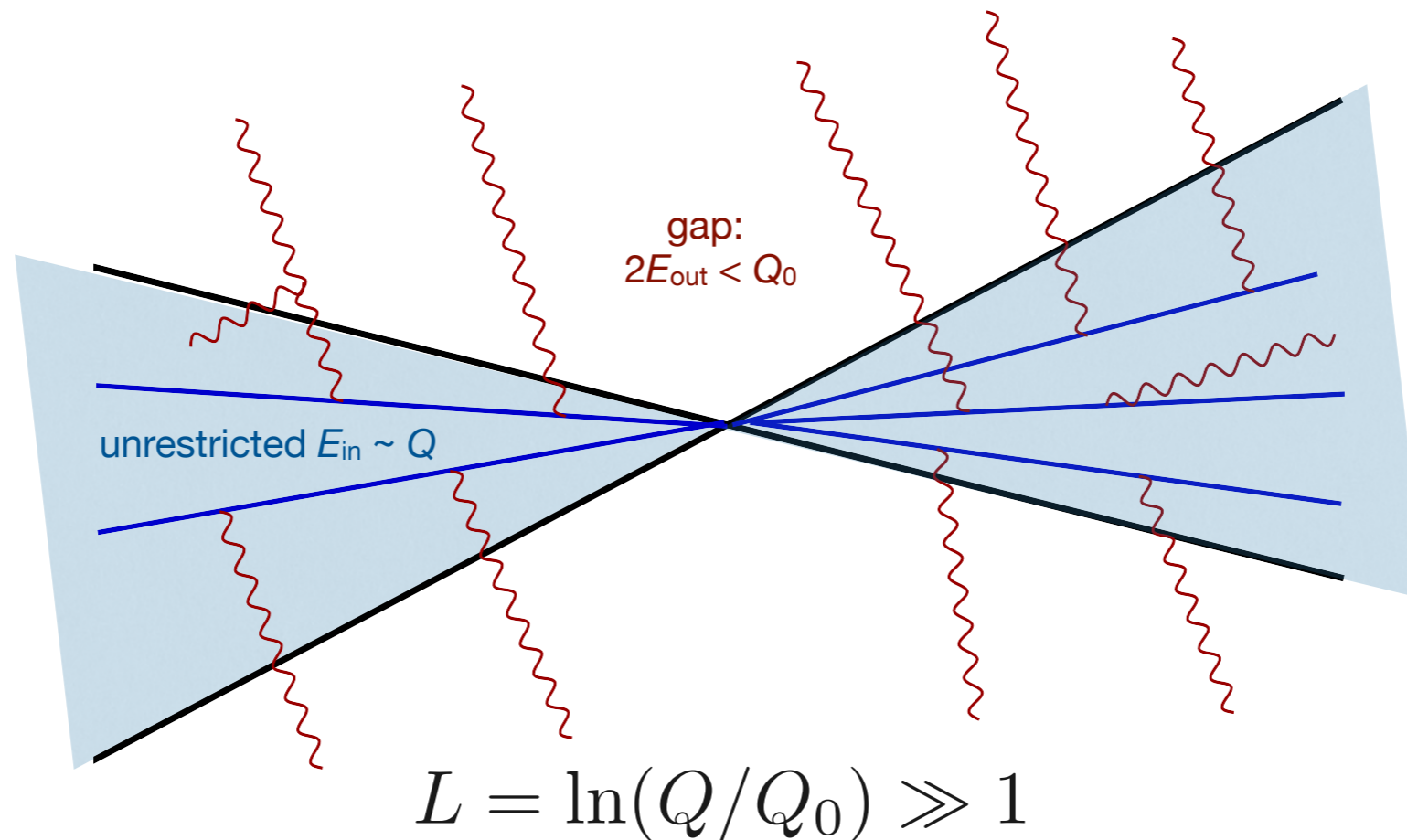


J. Banfi, G. Marchesini, G. Smye: JHEP 08 (2002) 006

**At hadron colliders, non-global logarithms take on a more intricate form, and no generalization of BMS equation exists!**



# LARGE LOGARITHMS IN JET PROCESSES AT HADRON COLLIDERS



Perturbative expansion includes "super-leading" logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 L^5 + \alpha_s^5 L^7 + \dots}_{\text{formally larger than } O(1)} \right\}$$

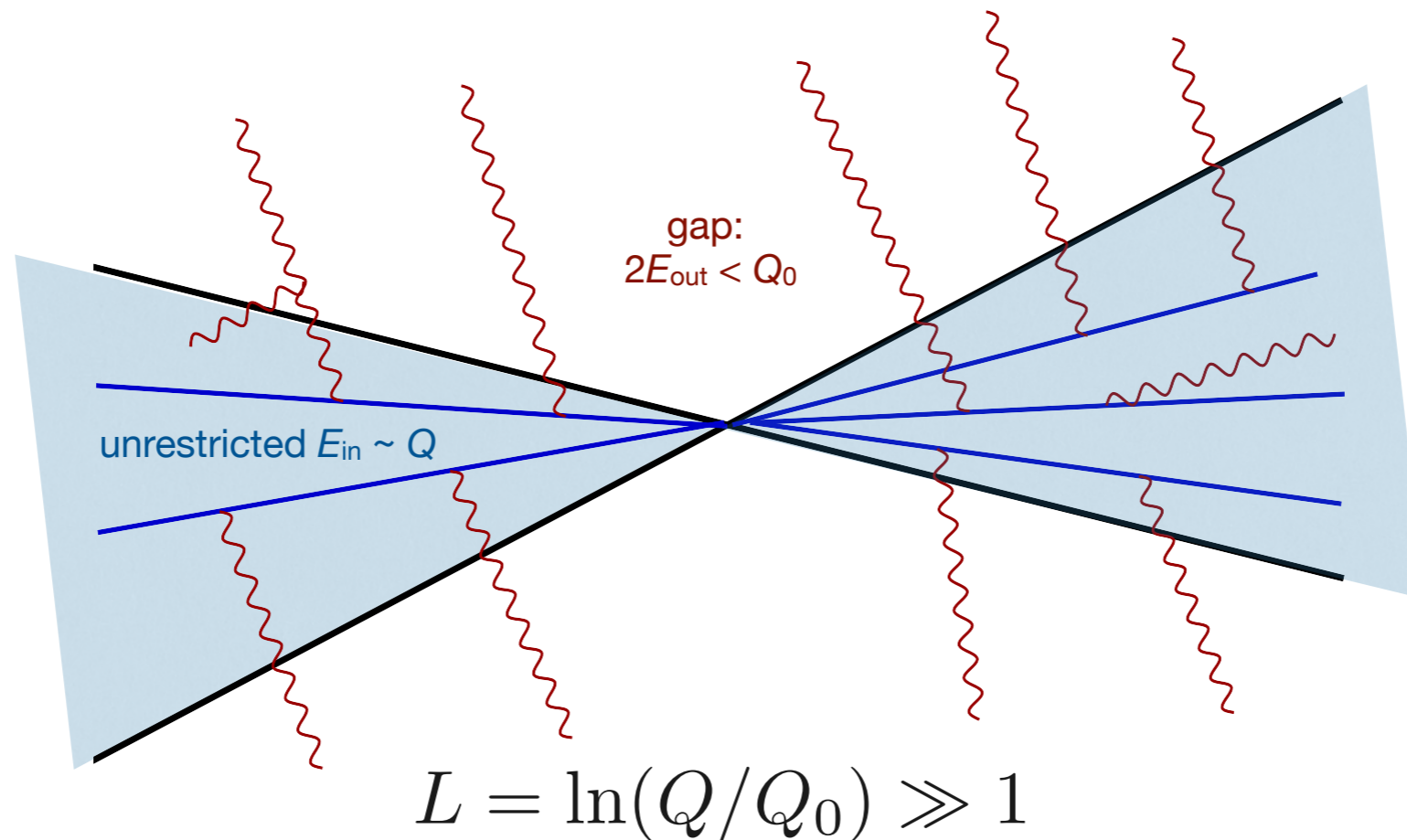
formally larger than  $O(1)$

J. R. Forshaw, A. Kyrieleis, M. H. Seymour: JHEP 08 (2006) 031





# LARGE LOGARITHMS IN JET PROCESSES AT HADRON COLLIDERS



Really, double logarithmic series starting at 3-loop order:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \pi^2) \left[ \alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots \right] \right\}$$

$(\Im m L)^2$  formally larger than  $O(1)$



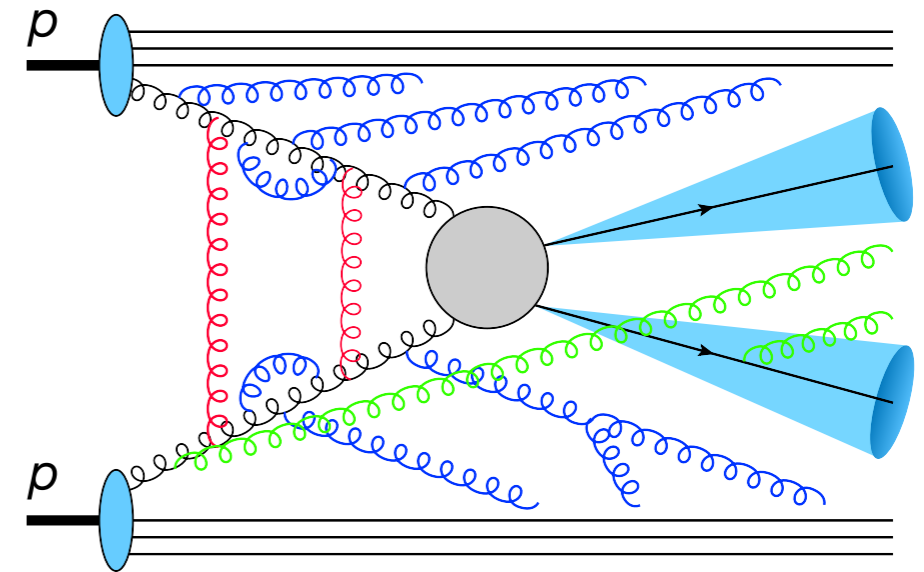
# COULOMB PHASES BREAK COLOR COHERENCE

## Super-leading logarithms

- ▶ breakdown of color coherence due to a subtle quantum effect: soft gluon exchange between initial-state partons
- ▶ soft anomalous dimension:

$$\Gamma(\{\underline{p}\}, \mu) = \sum_{(ij)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

T. Becher, M. Neubert (2009)



where  $s_{ij} > 0$  if particles  $i$  and  $j$  are both in initial or final state

- ▶ imaginary part (only at hadron colliders):

$$\text{Im } \Gamma(\{\underline{p}\}, \mu) = +2\pi \gamma_{\text{cusp}}(\alpha_s) \mathbf{T}_1 \cdot \mathbf{T}_2 + (\dots) \mathbf{1}$$

↑  
irrelevant



# THEORY OF NON-GLOBAL LHC OBSERVABLES

## Novel factorization theorem from SCET

$$\sigma_{2 \rightarrow M}(Q, Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \langle \mathcal{H}_m^{ab}(\{\underline{n}\}, Q, \mu) \otimes \mathcal{W}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002  
 [see also: T. Becher, M. Neubert, L. Rothen, D. Y. Shao (2015, 2016)]

high scale

low scale

### Rigorous operator definition:

$$\mathcal{H}_m^{ab}(\{\underline{n}\}, Q, \mu) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^m \int \frac{dE_i E_i^{d-3}}{(2\pi)^{d-2}} |\mathcal{M}_m^{ab}(\{\underline{p}\})\rangle \langle \mathcal{M}_m^{ab}(\{\underline{p}\})| (2\pi)^d \delta\left(Q - \sum_{i=1}^m E_i\right) \delta^{(d-1)}(\vec{p}_{\text{tot}}) \Theta_{\text{in}}(\{\underline{p}\})$$

density matrix involving hard-scattering amplitude (and its conjugate) in color-space formalism





# THEORY OF NON-GLOBAL LHC OBSERVABLES

## Novel factorization theorem from SCET

$$\sigma_{2 \rightarrow M}(Q, Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \langle \mathcal{H}_m^{ab}(\{\underline{n}\}, Q, \mu) \otimes \mathcal{W}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002  
[see also: T. Becher, M. Neubert, L. Rothen, D. Y. Shao (2015, 2016)]

high scale

low scale

Renormalization-group equation:

$$\mu \frac{d}{d\mu} \mathcal{H}_l^{ab}(\{\underline{n}\}, Q, \mu) = - \sum_{m \leq l} \mathcal{H}_m^{ab}(\{\underline{n}\}, Q, \mu) \Gamma_{ml}^H(\{\underline{n}\}, Q, \mu)$$

operator in color space and in the infinite space of parton multiplicities

**All-order summation of large logarithmic corrections, including the super-leading logarithms!**



# THEORY OF NON-GLOBAL LHC OBSERVABLES

Evaluate factorization theorem at low scale  $\mu_s \sim Q_0$

- ▶ low-energy matrix element:

$$\mathcal{W}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1) f_{b/p}(x_2) \mathbf{1} + \mathcal{O}(\alpha_s)$$

- ▶ hard-scattering functions:

$$\mathcal{H}_m^{ab}(\{\underline{n}\}, Q, \mu_s) = \sum_{l \leq m} \mathcal{H}_l^{ab}(\{\underline{n}\}, Q, Q) \mathbf{P} \exp \left[ \int_{\mu_s}^Q \frac{d\mu}{\mu} \mathbf{\Gamma}^H(\{\underline{n}\}, Q, \mu) \right]_{lm}$$

- ▶ expanding the solution in a power series generates arbitrarily high parton multiplicities starting from the  $2 \rightarrow M$  Born process



# THEORY OF NON-GLOBAL LHC OBSERVABLES

Evaluate factorization theorem at low scale  $\mu_s \sim Q_0$

- ▶ anomalous-dimension matrix:

$$\mathbf{\Gamma}^H = \frac{\alpha_s}{4\pi} \begin{pmatrix} \mathbf{V}_4 & \mathbf{R}_4 & 0 & 0 & \cdots \\ 0 & \mathbf{V}_5 & \mathbf{R}_5 & 0 & \cdots \\ 0 & 0 & \mathbf{V}_6 & \mathbf{R}_6 & \cdots \\ 0 & 0 & 0 & \mathbf{V}_7 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- ▶ action on hard functions:

$$\mathcal{H}_m \mathbf{V}_m = \sum_{(ij)} \left( \text{Diagram 1} + \text{Diagram 2} \right)$$

Diagram 1: Two vertices labeled  $\mathcal{M}$  and  $\mathcal{M}^\dagger$ . The  $\mathcal{M}$  vertex has  $m$  external lines. The  $\mathcal{M}^\dagger$  vertex has  $m$  external lines. A red vertical bar connects the  $i$ -th line of  $\mathcal{M}$  to the  $j$ -th line of  $\mathcal{M}^\dagger$ .

Diagram 2: Similar to Diagram 1, but the red vertical bar connects the  $j$ -th line of  $\mathcal{M}$  to the  $i$ -th line of  $\mathcal{M}^\dagger$ .

$$\mathcal{H}_m \mathbf{R}_m = \sum_{(ij)} \text{Diagram 3}$$

Diagram 3: Two vertices labeled  $\mathcal{M}$  and  $\mathcal{M}^\dagger$ . The  $\mathcal{M}$  vertex has  $m$  external lines labeled 1, 2, ...,  $m$ . The  $\mathcal{M}^\dagger$  vertex has  $m$  external lines labeled 1, 2, ...,  $m$ . A blue double line connects the  $i$ -th line of  $\mathcal{M}$  to the  $j$ -th line of  $\mathcal{M}^\dagger$ .





# THEORY OF NON-GLOBAL LHC OBSERVABLES

## Detailed structure of the anomalous-dimension coefficients

- ▶ virtual and real contributions contain collinear singularities, which must be regularized and subtracted:

$$\left. \begin{aligned} V_m &= \bar{V}_m + V^G + \sum_{i=1,2} V_i^c \ln \frac{\mu^2}{\hat{s}} \\ R_m &= \bar{R}_m + \sum_{i=1,2} R_i^c \ln \frac{\mu^2}{\hat{s}} \end{aligned} \right\} \Gamma = \bar{\Gamma} + V^G + \Gamma^c \ln \frac{\mu^2}{\hat{s}}$$

- ▶ with:

$$\begin{aligned} V^G &= -8i\pi (\mathbf{T}_{1,L} \cdot \mathbf{T}_{2,L} - \mathbf{T}_{1,R} \cdot \mathbf{T}_{2,R}) && \text{Coulomb phase} \\ V_i^c &= 4C_i \mathbf{1} \\ R_i^c &= -4\mathbf{T}_{i,L} \circ \mathbf{T}_{i,R} \delta(n_k - n_i) \end{aligned} \left. \vphantom{\begin{aligned} V^G \\ V_i^c \\ R_i^c \end{aligned}} \right\} \text{soft \& collinear terms}$$



# THEORY OF NON-GLOBAL LHC OBSERVABLES

## Comments on notation

- ▶ color generators  $\mathbf{T}_{L,i}$  act on the amplitude (multiply hard functions from the left)
- ▶ color generators  $\mathbf{T}_{R,i}$  act on the complex conjugate amplitude (multiply hard functions from the right)
- ▶ real-emission terms take an amplitude with  $m$  partons and turn it into an amplitude with  $(m+1)$  partons:

$$\mathcal{H}_m \mathbf{T}_{i,L} \circ \mathbf{T}_{j,R} = \mathbf{T}_i^a \mathcal{H}_m \mathbf{T}_j^{\tilde{a}}$$

where  $a, \tilde{a}$  are color indices of the emitted gluon (symbol  $\circ$  indicates the additional color space of the new parton)



## THEORY OF NON-GLOBAL LHC OBSERVABLES

SLLs arise from the terms in  $\mathbf{P} \exp \left[ \int_{\mu_s}^Q \frac{d\mu}{\mu} \mathbf{\Gamma}^H(\{\underline{n}\}, Q, \mu) \right]_{lm}$  with the highest number of insertions of  $\mathbf{\Gamma}_c$

▶ three properties simplify the calculation:

- ▶ color coherence in absence of Glauber phases (sum of soft emissions off collinear partons has same effect as soft emission of parent parton):

$$\mathcal{H}_m \mathbf{\Gamma}^c \bar{\mathbf{\Gamma}} = \mathcal{H}_m \bar{\mathbf{\Gamma}} \mathbf{\Gamma}^c$$

- ▶ collinear safety (singularities from real and virtual emission cancel):

$$\langle \mathcal{H}_m \mathbf{\Gamma}^c \otimes \mathbf{1} \rangle = 0$$

- ▶ cyclicity of the trace:

$$\langle \mathcal{H}_m \mathbf{V}^G \otimes \mathbf{1} \rangle = 0$$





## THEORY OF NON-GLOBAL LHC OBSERVABLES

SLLs arise from the terms in  $\mathbf{P} \exp \left[ \int_{\mu_s}^Q \frac{d\mu}{\mu} \mathbf{\Gamma}^H(\{\underline{n}\}, Q, \mu) \right]_{lm}$  with the highest number of insertions of  $\mathbf{\Gamma}_c$

- ▶ under the color trace, insertions of  $\mathbf{\Gamma}_c$  are non-zero only if they come in conjunction with (at least) two Glauber phases and one  $\bar{\mathbf{\Gamma}}$
- ▶ relevant color traces:

$$C_{rn} = \langle \mathcal{H}_{2 \rightarrow M} (\mathbf{\Gamma}^c)^r \mathbf{V}^G (\mathbf{\Gamma}^c)^{n-r} \mathbf{V}^G \bar{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle$$



# THEORY OF NON-GLOBAL LHC OBSERVABLES

- ▶ relevant color traces:

$$C_{rn} = \langle \mathcal{H}_{2 \rightarrow M} (\mathbf{\Gamma}^c)^r \mathbf{V}^G (\mathbf{\Gamma}^c)^{n-r} \mathbf{V}^G \bar{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle$$

- ▶ extremely simple intermediate result:

$$\langle \mathcal{H} (\mathbf{\Gamma}^c)^{n-r} \mathbf{V}^G \bar{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle = -64\pi (4N_c)^{n-r} f_{abc} \sum'_{j>2} J_j \langle \mathcal{H} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_j^c \rangle$$

- ▶ kinematic information contained in  $(M + 1)$  angular integrals:

$$J_j = \int \frac{d\Omega(n_k)}{4\pi} \left( W_{1j}^k - W_{2j}^k \right) \Theta_{\text{veto}}(n_k); \quad \text{with} \quad W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k n_j \cdot n_k}$$



# RESUMMATION OF SUPER-LEADING LOGARITHMS

## General result

$$C_{rn} = -256\pi^2 (4N_c)^{n-r} \left[ \sum_{j=3}^{M+2} J_j \sum_{i=1}^4 c_i^{(r)} \langle \mathcal{H}_{2 \rightarrow M} \mathbf{O}_i^{(j)} \rangle - J_2 \sum_{i=1}^6 d_i^{(r)} \langle \mathcal{H}_{2 \rightarrow M} \mathbf{S}_i \rangle \right]$$

T. Becher, M. Neubert, D. Y. Shao: in preparation

### ► basis of 10 color structures:

$$\mathbf{O}_1^{(j)} = f_{abe} f_{cde} \mathbf{T}_2^a \{ \mathbf{T}_1^b, \mathbf{T}_1^c \} \mathbf{T}_j^d - (1 \leftrightarrow 2)$$

$$\mathbf{S}_1 = f_{abe} f_{cde} \{ \mathbf{T}_1^b, \mathbf{T}_1^c \} \{ \mathbf{T}_2^a, \mathbf{T}_2^d \}$$

$$\mathbf{O}_2^{(j)} = d_{ade} d_{bce} \mathbf{T}_2^a \{ \mathbf{T}_1^b, \mathbf{T}_1^c \} \mathbf{T}_j^d - (1 \leftrightarrow 2)$$

$$\mathbf{S}_2 = d_{ade} d_{bce} \{ \mathbf{T}_1^b, \mathbf{T}_1^c \} \{ \mathbf{T}_2^a, \mathbf{T}_2^d \}$$

$$\mathbf{O}_3^{(j)} = \mathbf{T}_2^a \{ \mathbf{T}_1^a, \mathbf{T}_1^b \} \mathbf{T}_j^b - (1 \leftrightarrow 2)$$

$$\mathbf{S}_3 = d_{ade} d_{bce} \left[ \mathbf{T}_2^a (\mathbf{T}_1^b \mathbf{T}_1^c \mathbf{T}_1^d)_+ + (1 \leftrightarrow 2) \right]$$

$$\mathbf{O}_4^{(j)} = 2C_1 \mathbf{T}_2 \cdot \mathbf{T}_j - 2C_2 \mathbf{T}_1 \cdot \mathbf{T}_j$$

$$\mathbf{S}_4 = \{ \mathbf{T}_1^a, \mathbf{T}_1^b \} \{ \mathbf{T}_2^a, \mathbf{T}_2^b \}$$

$$\mathbf{S}_5 = \mathbf{T}_1 \cdot \mathbf{T}_2$$

$$\mathbf{S}_6 = \mathbf{1}$$





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T. Becher, M. Neubert, D. Y. Shao: in preparation

### ► coefficient functions:

$$c_1^{(r)} = 2^{r-1} [(3N_c + 2)^r + (3N_c - 2)^r]$$

$$c_2^{(r)} = 2^{r-2} N_c \left[ \frac{(3N_c + 2)^r}{N_c + 2} + \frac{(3N_c - 2)^r}{N_c - 2} - \frac{(2N_c)^{r+1}}{N_c^2 - 4} \right]$$

$$c_3^{(r)} = 2^{r-1} [(3N_c + 2)^r - (3N_c - 2)^r]$$

$$c_4^{(r)} = 2^{r-1} \left[ \frac{(3N_c + 2)^r}{N_c + 1} + \frac{(3N_c - 2)^r}{N_c - 1} - \frac{2N_c^{r+1}}{N_c^2 - 1} \right]$$

$$d_1^{(r)} = 2^{3r-1} [(N_c + 1)^r + (N_c - 1)^r] - 2^{r-1} [(3N_c + 2)^r + (3N_c - 2)^r]$$

$$d_2^{(r)} = 2^{3r-2} N_c \left[ \frac{(N_c + 1)^r}{N_c + 2} + \frac{(N_c - 1)^r}{N_c - 2} \right] - 2^{r-2} N_c \left[ \frac{(3N_c + 2)^r}{N_c + 2} + \frac{(3N_c - 2)^r}{N_c - 2} \right]$$

$$d_3^{(r)} = 2^{r-1} N_c \left[ \frac{(3N_c + 2)^r}{N_c + 2} + \frac{(3N_c - 2)^r}{N_c - 2} - \frac{(2N_c)^{r+1}}{N_c^2 - 4} \right]$$

$$d_4^{(r)} = 2^{3r-1} [(N_c + 1)^r - (N_c - 1)^r] - 2^{r-1} [(3N_c + 2)^r - (3N_c - 2)^r]$$

$$d_5^{(r)} = 2^r (C_1 + C_2) \left[ \frac{N_c + 2}{N_c + 1} (3N_c + 2)^r - \frac{N_c - 2}{N_c - 1} (3N_c - 2)^r - \frac{2N_c^{r+1}}{N_c^2 - 1} \right] \\ - \frac{2^{r-1} N_c}{3} [(N_c + 4)(3N_c + 2)^r + (N_c - 4)(3N_c - 2)^r - (2N_c)^{r+1}]$$

$$d_6^{(r)} = 2^{3r+1} C_1 C_2 [(N_c + 1)^{r-1} + (N_c - 1)^{r-1}] (1 - \delta_{r0})$$

$$- 2^{r+1} C_1 C_2 \left[ \frac{(3N_c + 2)^r}{N_c + 1} + \frac{(3N_c - 2)^r}{N_c - 1} - \frac{2N_c^{r+1}}{N_c^2 - 1} \right]$$



# RESUMMATION OF SUPER-LEADING LOGARITHMS

## General result

$$C_{rn} = -256\pi^2 (4N_c)^{n-r} \left[ \sum_{j=3}^{M+2} J_j \sum_{i=1}^4 c_i^{(r)} \langle \mathcal{H}_{2 \rightarrow M} \mathbf{O}_i^{(j)} \rangle - J_2 \sum_{i=1}^6 d_i^{(r)} \langle \mathcal{H}_{2 \rightarrow M} \mathbf{S}_i \rangle \right]$$

T. Becher, M. Neubert, D. Y. Shao: in preparation

- ▶ series of SLLs, starting at 3-loop order:

$$\sigma_{\text{SLL}} = \sigma_{\text{Born}} \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \frac{(-4)^n n!}{(2n+3)!} \sum_{r=0}^n \frac{(2r)!}{4^r (r!)^2} C_{rn}$$

- ▶ reproduces all that is known about SLLs (and much more...)



# RESUMMATION OF SUPER-LEADING LOGARITHMS

## Simplifications for (anti-)quark-initiated processes

- ▶ in the fundamental representation, symmetrized products of color generators can be reduced ( $\sigma_i = \pm 1$  for (anti-)quarks):

$$\{\mathbf{T}_i^a, \mathbf{T}_i^b\} = \frac{1}{N_c} \delta_{ab} + \sigma_i d_{abc} \mathbf{T}_i^c$$

- ▶ simple results in terms of three non-trivial color structures:

$$C_{rn} = -2^{8-r} \pi^2 (4N_c)^n \left\{ \sum_{j=3}^{M+2} J_j \langle \mathcal{H}_{2 \rightarrow M} [(\mathbf{T}_1 - \mathbf{T}_2) \cdot \mathbf{T}_j - 2^{r-1} N_c (\sigma_1 - \sigma_2) d_{abc} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_j^c] \rangle \right. \\ \left. - 2(1 - \delta_{r0}) J_2 \langle \mathcal{H}_{2 \rightarrow M} [C_F \mathbf{1} + (2^r - 1) \mathbf{T}_1 \cdot \mathbf{T}_2] \rangle \right\}$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002



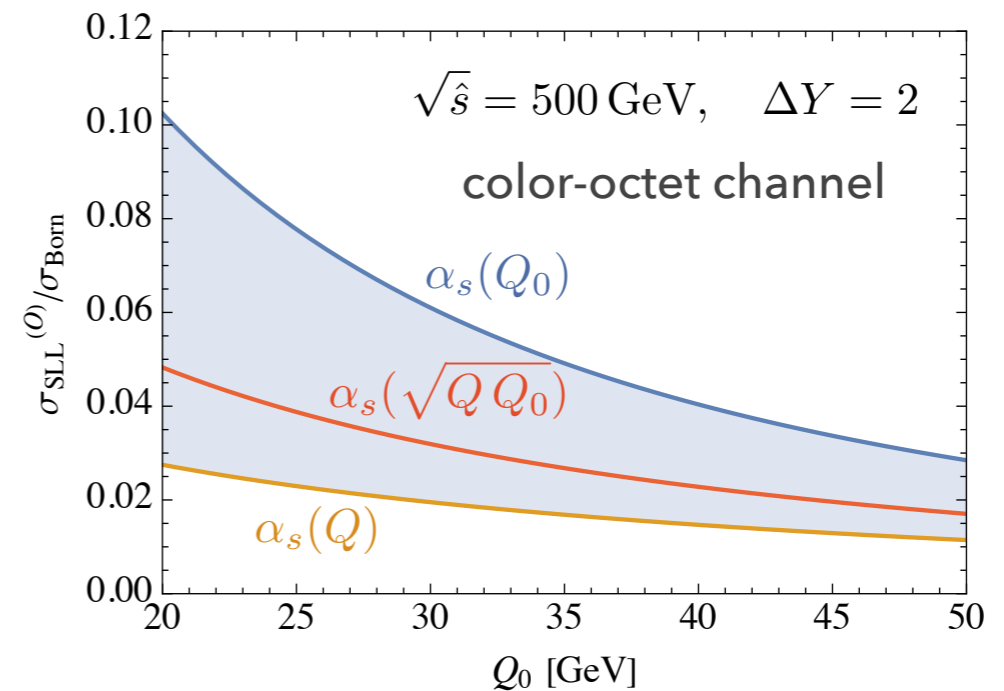
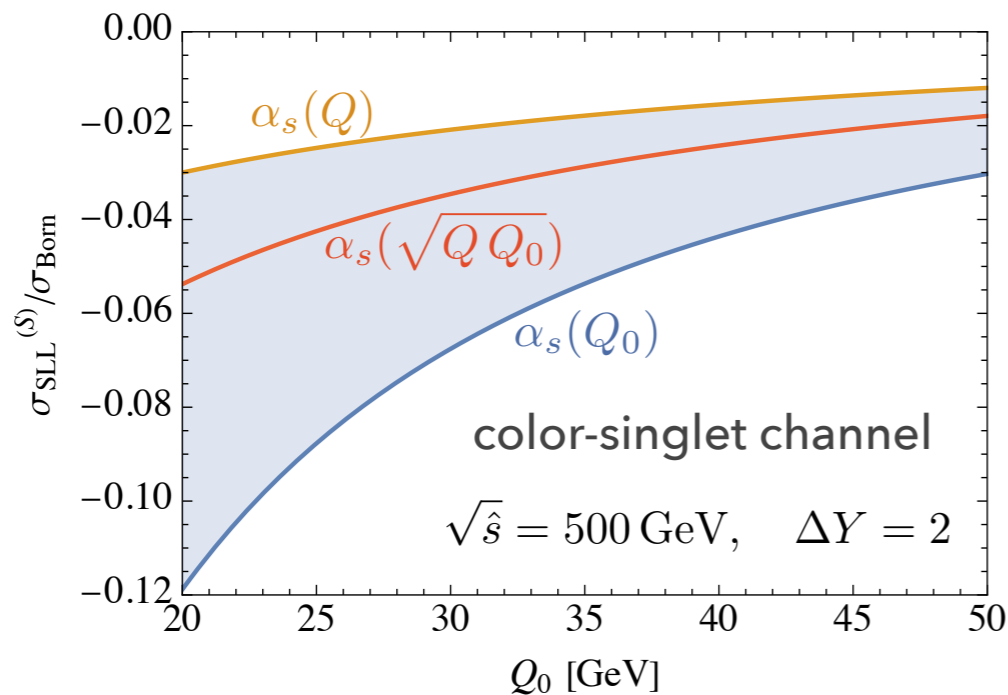
# RESUMMATION OF SUPER-LEADING LOGARITHMS

Summation of super-leading logarithms for  $qq \rightarrow qq$  scattering:

$$\sigma_{\text{SLL}}^{(S)} = -\sigma_{\text{Born}} \frac{16\alpha_s L}{27N_c\pi} \Delta Y \left( \frac{N_c\alpha_s}{\pi} \pi^2 \right) w {}_2F_2\left(1, 1; 2, \frac{5}{2}; -w\right)$$

$\uparrow$  1-loop factor
  $w = \frac{N_c\alpha_s}{\pi} L^2$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002







# RESUMMATION OF SUPER-LEADING LOGARITHMS

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↑
↑

1-loop factor

$$w = \frac{N_c\alpha_s}{\pi} L^2$$

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- ▶ asymptotic behavior for  $L \rightarrow \infty$ :

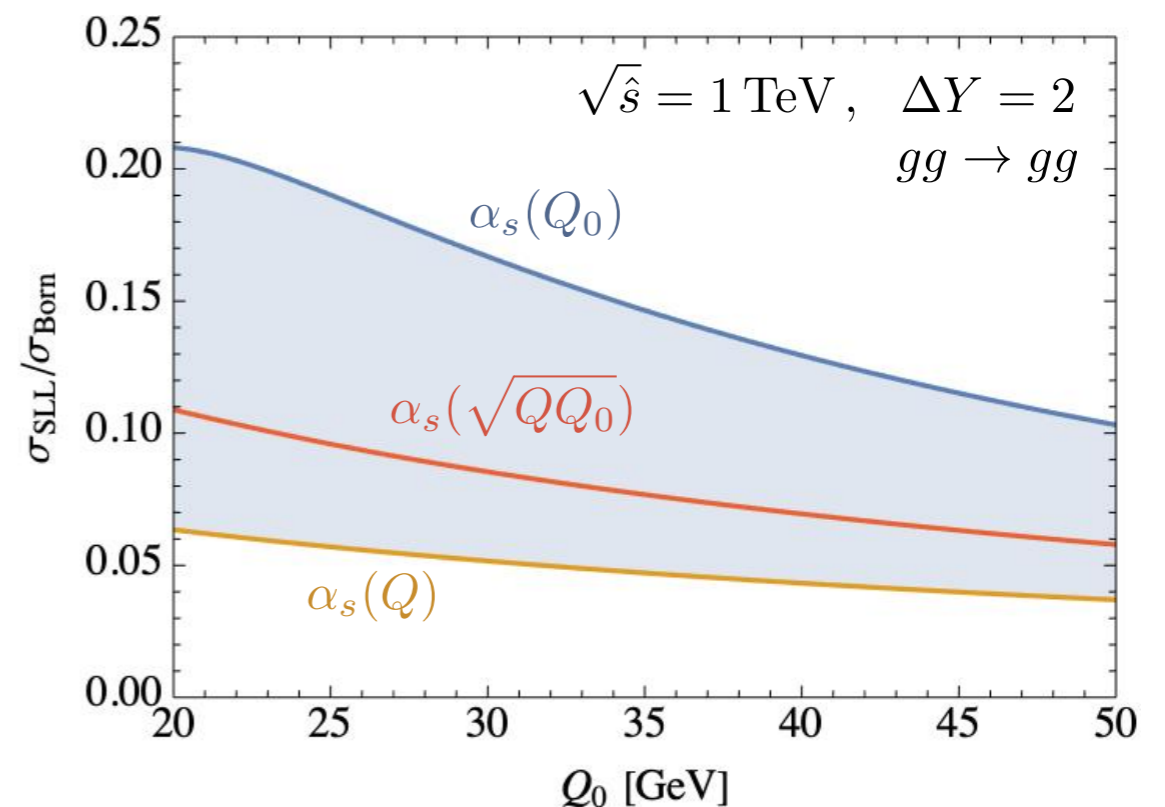
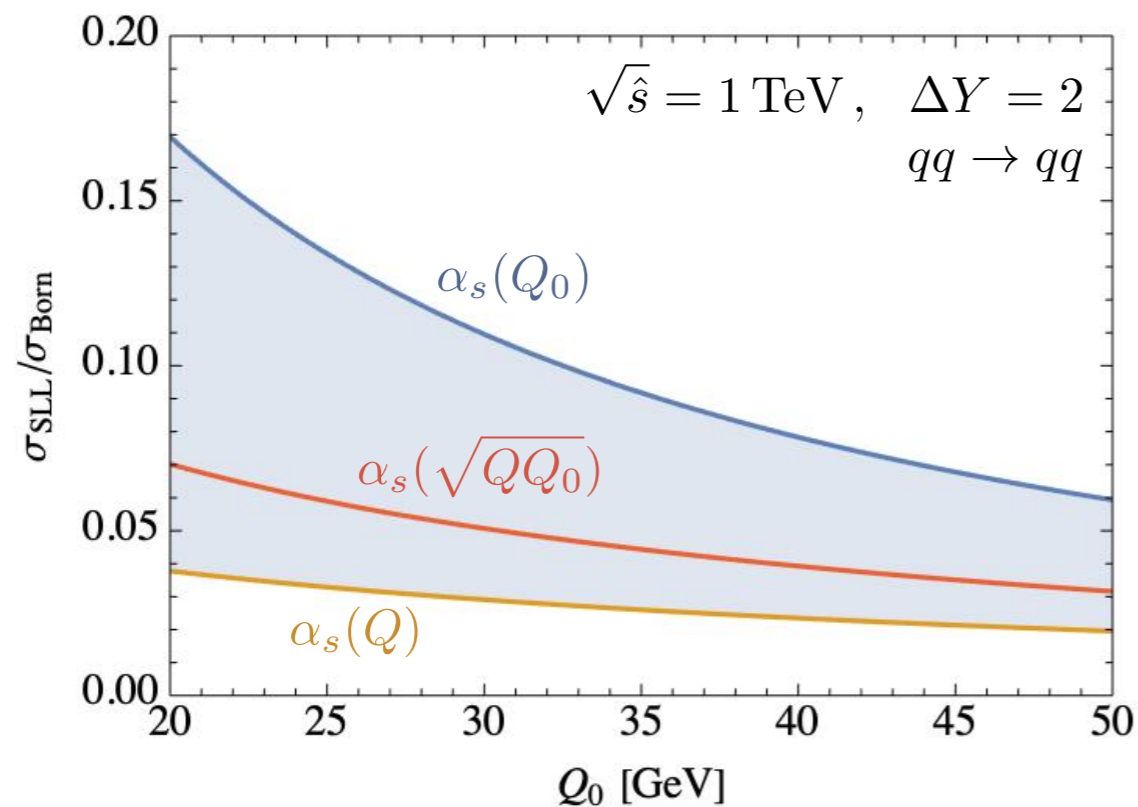
$$w {}_2F_2\left(1, 1; 2, \frac{5}{2}; -w\right) \rightarrow \frac{3}{2} [\ln(4w) + \gamma_E - 2]$$

- ▶ very different from standard Sudakov double logarithms  $\sim e^{-cW}$
- ▶ expect even larger effects for gluon-initiated processes



# RESUMMATION OF SUPER-LEADING LOGARITHMS

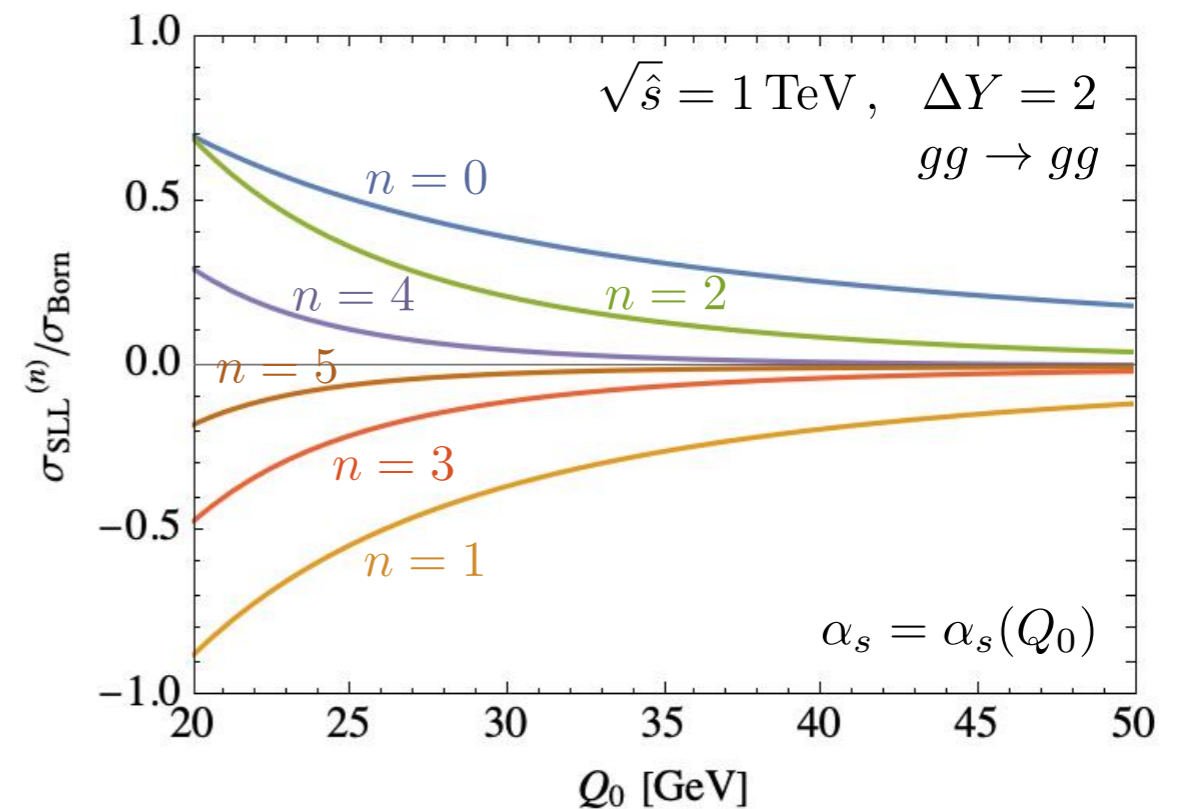
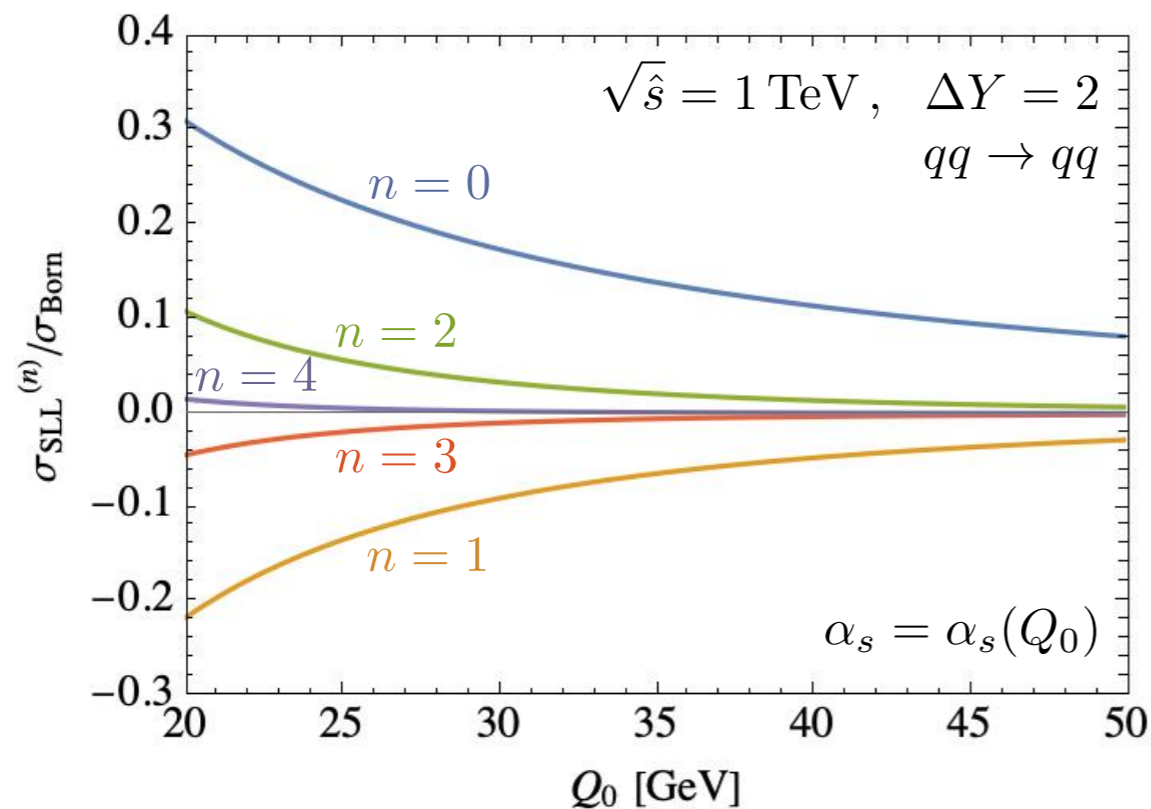
Super-leading logs for  $qq \rightarrow qq$  and  $gg \rightarrow gg$  scattering at  $\sqrt{\hat{s}} = 1$  TeV:





# RESUMMATION OF SUPER-LEADING LOGARITHMS

Super-leading logs for  $qq \rightarrow qq$  and  $gg \rightarrow gg$  scattering at  $\sqrt{\hat{s}} = 1$  TeV:



$$\sigma_{\text{SLL}} = \sigma_{\text{Born}} \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \frac{(-4)^n n!}{(2n+3)!} \sum_{r=0}^n \frac{(2r)!}{4^r (r!)^2} C_{rn}$$



## IMPORTANT REMARKS

- ▶ SCET-based approach solves 16-year old QCD problem, extending existing results to all orders of perturbation theory and to arbitrary  $2 \rightarrow M$  hard-scattering processes
- ▶ master formula also applies to cases where  $M = 1$  or even  $M = 0$ , which were not considered before (SLLs start at 4- and 5-loop order, respectively)
- ▶ relevant for both SM phenomenology (e.g.  $pp \rightarrow h + \text{jet}$ ) and New-Physics searches (e.g. WIMP searches in  $pp \rightarrow \text{jet} + \cancel{E}_T$ )





## CONCLUSIONS

### Toward a complete theory of LHC jet processes

- ▶ powerful new factorization theorem derived using SCET
- ▶ in future, extension to massive final-state partons and calculations beyond leading logarithms
- ▶ detailed study of low-energy matrix elements using SCET with Glauber gluons will offer an *ab initio* understanding of violations of conventional factorization (perturbative part of “underlying event”)
- ▶ results very relevant for future improvements of parton showers
- ▶ new levels of precision in predictions for important LHC processes