Quarkyonic Matter

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Work in collaboration with
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Mass and radii of observed neutron stars and data from neutron star collisions give an excellent determination of the equation of state of strongly interacting matter

Such equations of state must be hard
The sound velocity squared is greater than or of the order of 1/3 at only a few times nuclear matter density
This is NOT what one expects from a phase transition
Relativistic degrees of freedom appear to be important

After a short review, will discuss a field theoretical method to include both quark and nucleon degrees of freedom in a consistent field theoretical formalism

Neutron Star Matter and Some Conjectures on Scale Invariance

From observations of neutron stars masses and radii, one gets very good information about the zero temperature equation of state of nuclear matter

One equates the outward force of matter arising from pressure inward force of gravity. This gives a general. relativistic equation of hydrostatic equilibrium.

For a specific equation of state, one obtains a relationship between radii and neutron star masses

Equations of state may be characterized by two dimensionless numbers

Sound velocity:

$$v_s^2 = \frac{dP}{de}$$

and the trace of the stress energy tensor scaled by the energy density

$$\Delta = \frac{1}{3} - \frac{P}{e}$$

P = - dE/dV

In a scale invariant theory at zero temperature:

$$E \sim (N/V)^{1/3} V \sim N^{4/3} V^{-1/3}$$

$$P = \frac{1}{3}\frac{E}{V} = \frac{1}{3} e$$

$$v_s^2 = \frac{dP}{de} = \frac{1}{3}$$

$$\Delta = \frac{1}{3} - \frac{p}{e} = 0$$

The trace of the stress energy tensor is taken to be a measure of scale invariance. It is anomalous in QCD.

$$T^\mu_\mu = \frac{\beta(g)}{g}(E^2 - B^2) + m_q(1+\gamma_q)\overline{\psi}\psi$$

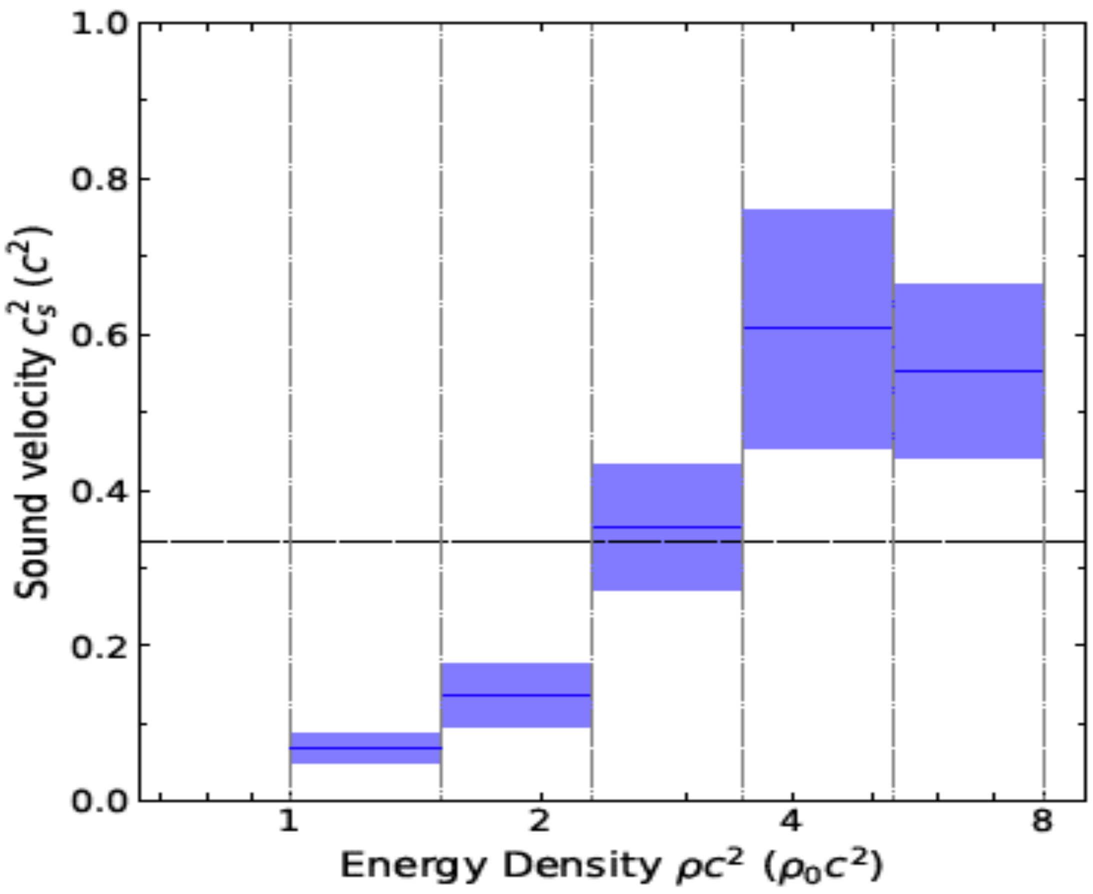
In this equation, the beta function of QCD is negative, and the fermion term is from quarks. It vanishes in the chiral limit.

If we take matrix elements of single particle states

$$< p \mid T^\mu_\mu \mid p > \sim p^2 = m^2 \geq= 0$$

In the chiral limit, this implies E > B, as we expect for massive quarks, except for the pion, which is very tightly. bound

For dilute systems, the trace anomaly is positive, as it is at high density for a quark gas. In general, we expect it to be positive, except possibly for small effects due to pion condensate, if they exist



Y. Fujimoto, K. Fukushima,
K. Murase

Tews, Carlso, Gandolfi and Reddy; Kojo; Anala, Gorda, Kurkela and Vorinen

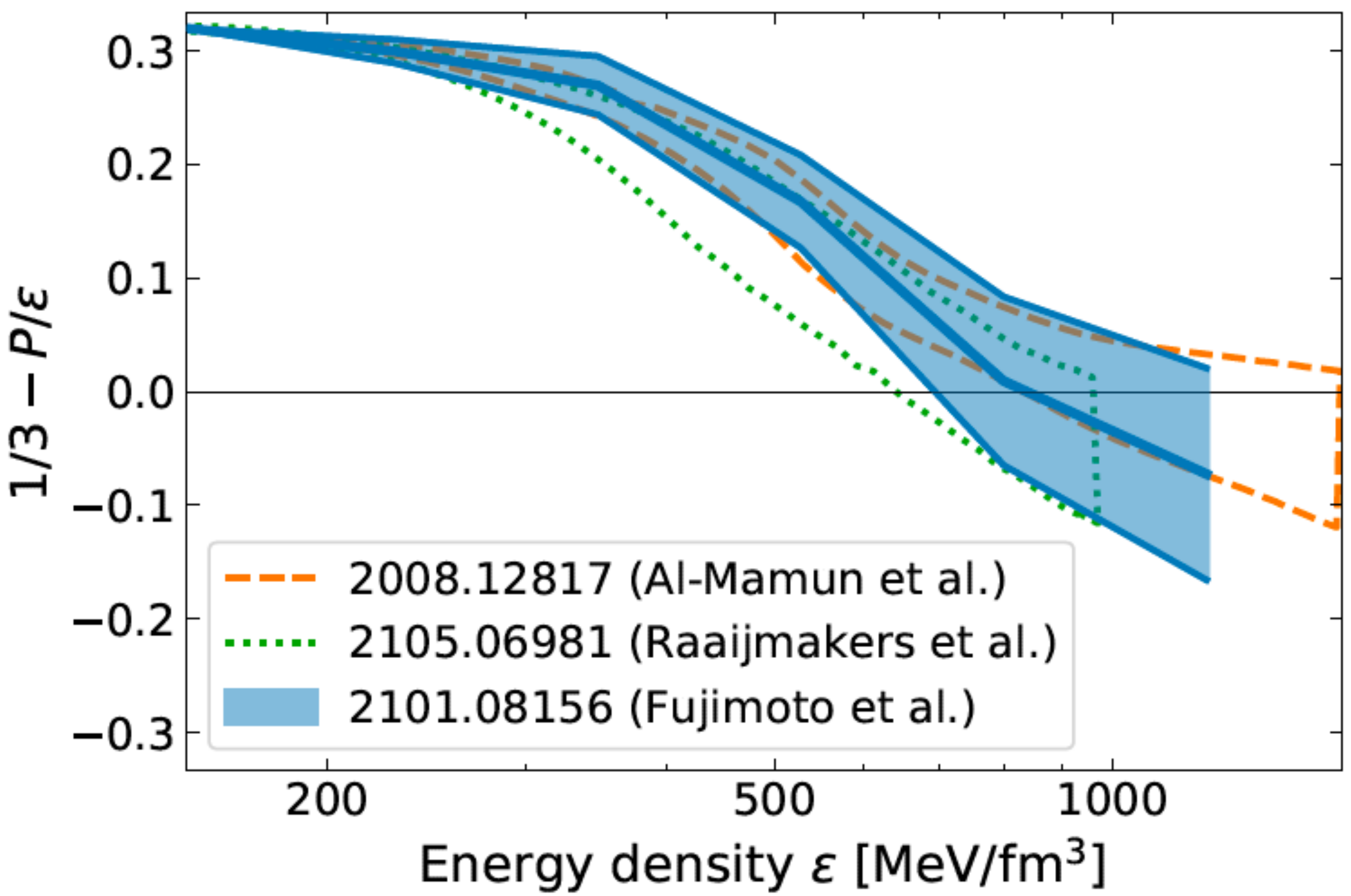
As a result of LIGO experiments, and more precise measurement of neutron star masses, the equation of state of nuclear matter at a few time nuclear matter density is tightly constrained

Sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density

Work with Y. Fujimoto, K. Fukushima and M. Praszalowicz



1/3 – P/e approaches zero from above. By about 5 times nuclear matter density, the system is approximately scale invariant.

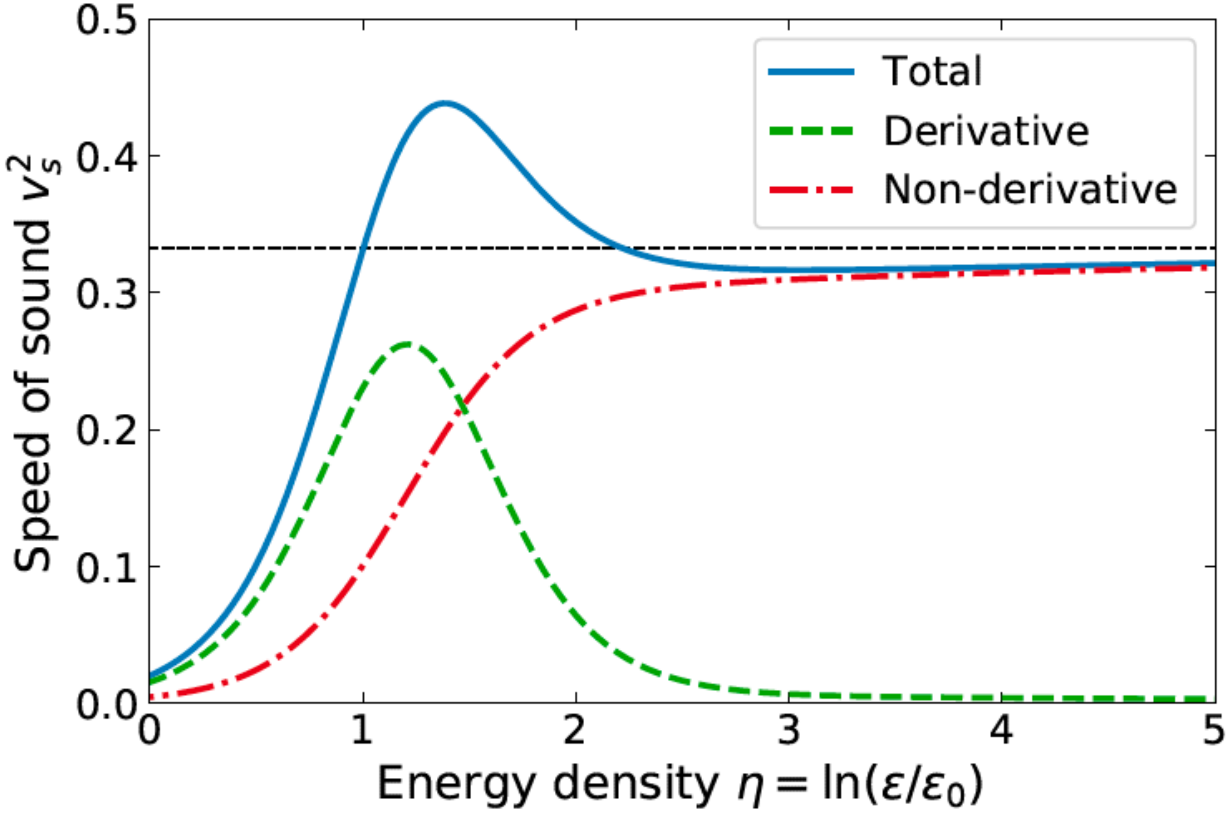
Suggests degrees of freedom are strongly interacting relativistic quarks

$$v_s^2 = v_{s,\,(\text{deriv})}^2 + v_{s,\,(\text{non-deriv})}^2 \equiv \varepsilon \frac{d}{d\varepsilon}\left(\frac{P}{\varepsilon}\right) + \frac{P}{\varepsilon}$$

Work with Y. Fujimoto, K. Fukushima and M. Praszalowicz

Peak in sound velocity is because system rapidly approaches scale invariant limit

The maximum measures the energy density of transition between nucleon and quark degrees of freedom



Quarkyonic matter has precisely the properties needed to describe this behaviour

Sound velocity of order one has important consequences

For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \ \frac{\delta n_B}{n_B}$$

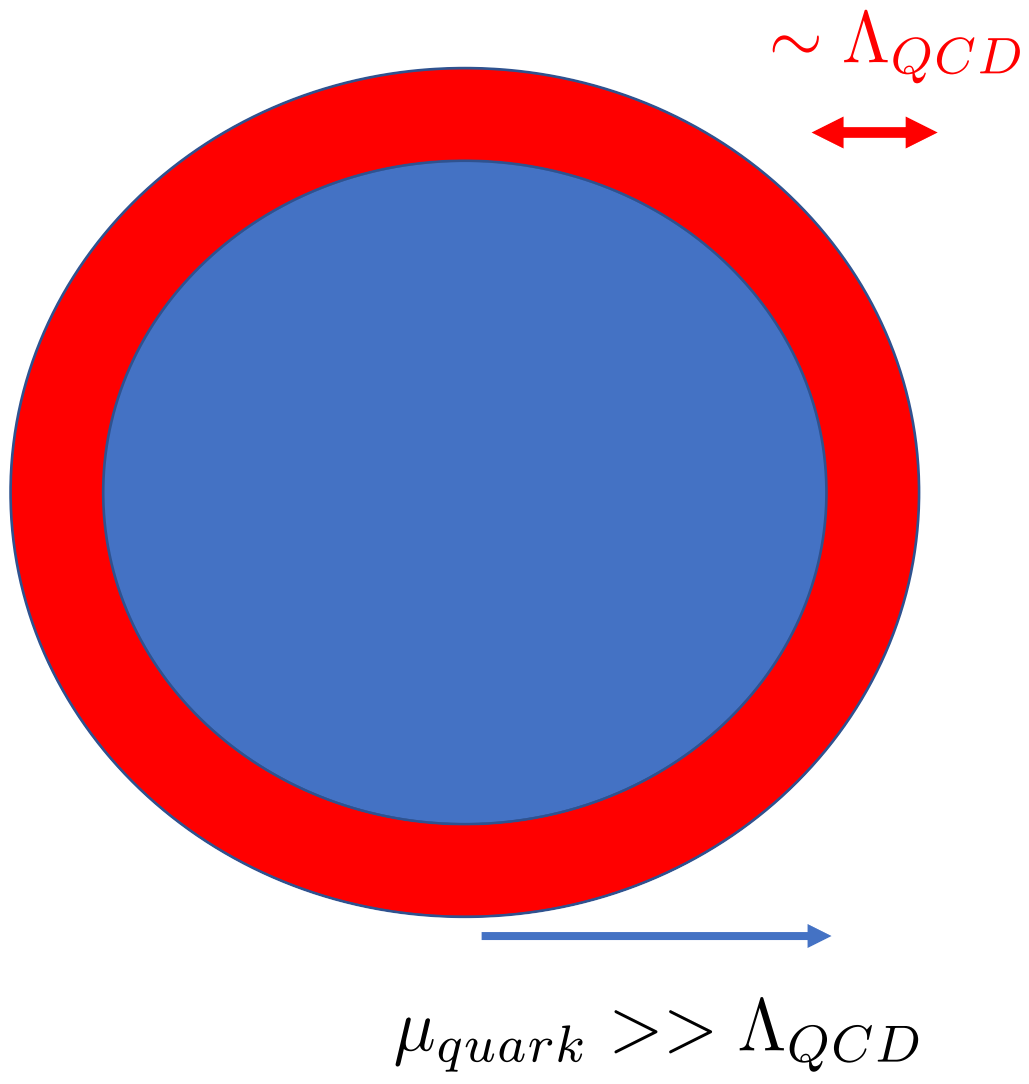
So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda_{QCD}^2}{2M} \sim 100 \ MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

Fermi Surface is Non-perturbative



Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

Relation between quark and nucleon Fermi momenta

$$m_q = m_N / N_c$$

$$\mu_q = \mu_N / N_c$$

$$k_q^2 = \mu_q^2 - m_q^2 = \mu_n^2 / N_c^2 - M_N^2 / N_c^2 = k_N^2 / N_c^2$$

For 2 flavors of nucleons

$$n_B^N = 4 \int^{k_N} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_N^3$$

For two flavors of quarks

$$n_q^N = \frac{1}{N_c} 4 N_c \int^{k_q} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_q^3$$

To get any baryons from the quarks at the bottom of the shell of nucleons, need need the quark fermi momentum to be of the order of the QCD scale, so that the nucleon in the shell are relativistic. Naturally driven to the conformal behaviour. Also we see to absorb the baryons, that have been eaten, the chemical potential of the quarks must jump up from a small value to a typical QCD scale

If there is a continuous transition then the baryon density will have to remain fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one.

$$\epsilon_B = M_N \Lambda_{QCD}^3 \sim N_c \Lambda_{QCD}^4$$

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda_{QCD}^4$$

$$P \sim \frac{k_F}{M_B} \epsilon_N$$

The pressure on the other hand must jump by order N_c squared

$$P \sim \epsilon_q$$

Energy density and density fixed, but pressure and chemical potential jump.

A first order phase transition has pressure and chemical potential fixed, but energy density and density jump

Such matter is called quarkyonic matter:

Large N_c arguments:

Confinement disappears when the Debye screening length is important,

$$\lambda_D^{-1} \sim \mu_q/\sqrt{Nc} \sim \mu_B/N_c$$

Which is at a quark density parametrically large compared to the Debye scale!
Confinement on a Fermi surface, but “almost” deconfined quarks in a fermi sea.
Confined matter has two regions: with baryons and without since

$$n_B = \frac{1}{1 + e^{(E-\mu_B)/T}}$$

$$e^{-N_c} \; for \; \mu_B << M_N$$

$$O(1) \; for \; \mu_B >> M_N$$

Conception by Hatsuda and Fukushima