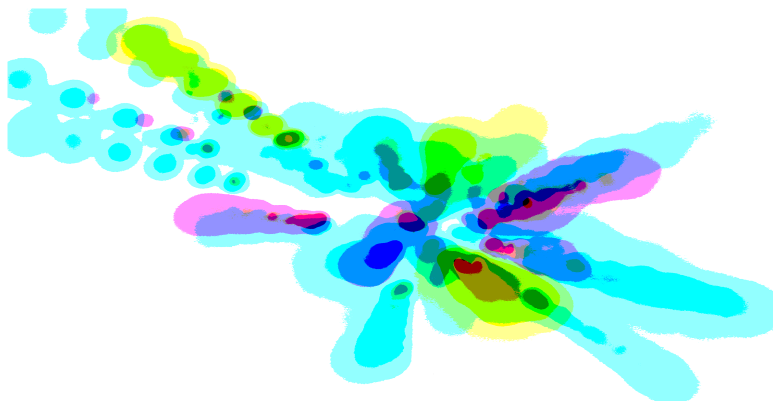
The role of the chiral anomaly in the proton's spin and sphaleron transitions at small x



Raju Venugopalan

Brookhaven National Laboratory

Humboldt Kolleg workshop, June 26-July1, Kitzbuhel

Talk outline

The proton's spin puzzle and the chiral anomaly

Fun with worldlines: the anomaly pole dominates at large and small x

WZW term for a prodigal ninth Goldstone: an axionlike effective action

Spin and the $U_A(1)$ problem: The Goldberger- Treiman relation and topological mass generation of the η'

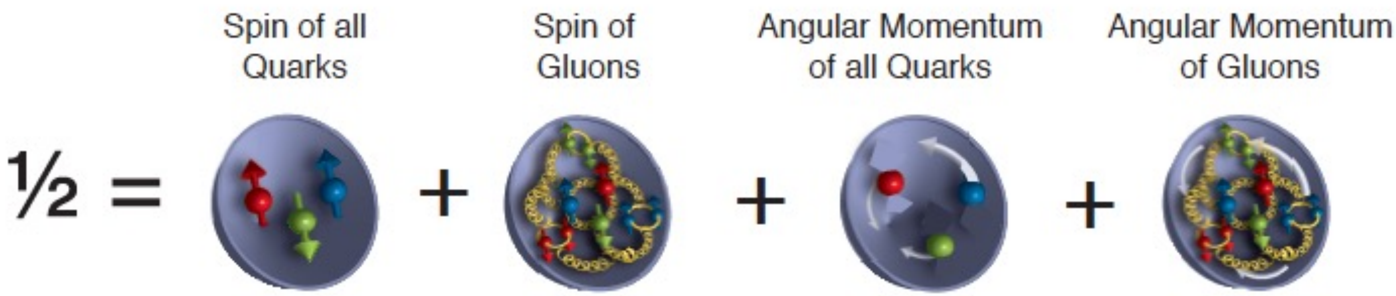
Spin damping at small x: sphaleron transitions induced by gluon saturation

Takeaway: The proton's spin is deeply influenced by the topology of the QCD vacuum
-in particular, its features that are responsible for the large mass of the η' meson

Polarized DIS at the Electron-Ion Collider can uncover evidence for sphaleron transitions resulting from this interplay

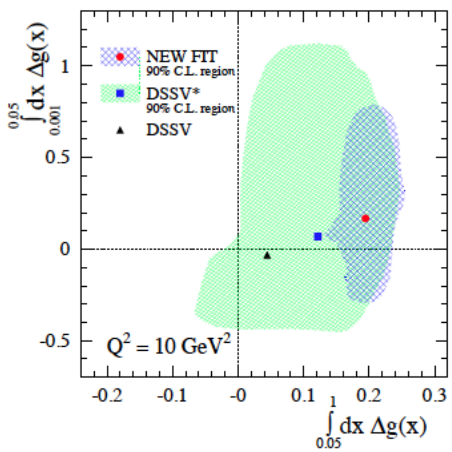
Talk based on work with A. Tarasov, arXiv: 2008.08104 (PRD 2021) and arXiv:2109.10370 (PRD 2022)

The proton's spin puzzle: a many-body picture

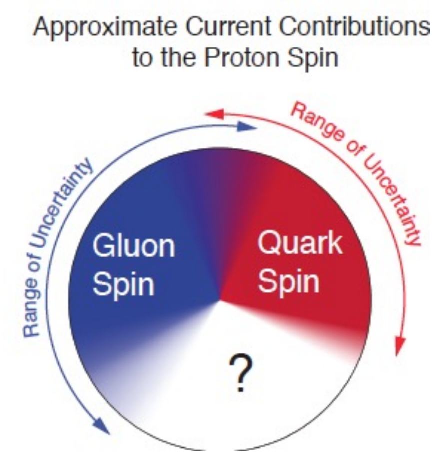


Fixed target DIS experiments (SLAC,CERN,DESY)show that quarks ($\Delta\Sigma$) carry only about 30% of the proton's spin

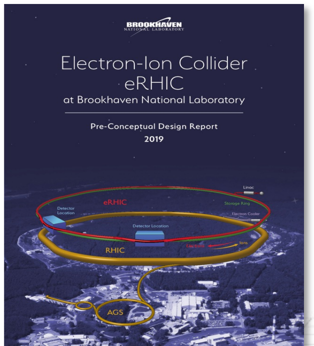
"Spin crisis": failure of the quark model ("Ellis-Jaffe sum rule") picture of relativistic "constituent" quarks



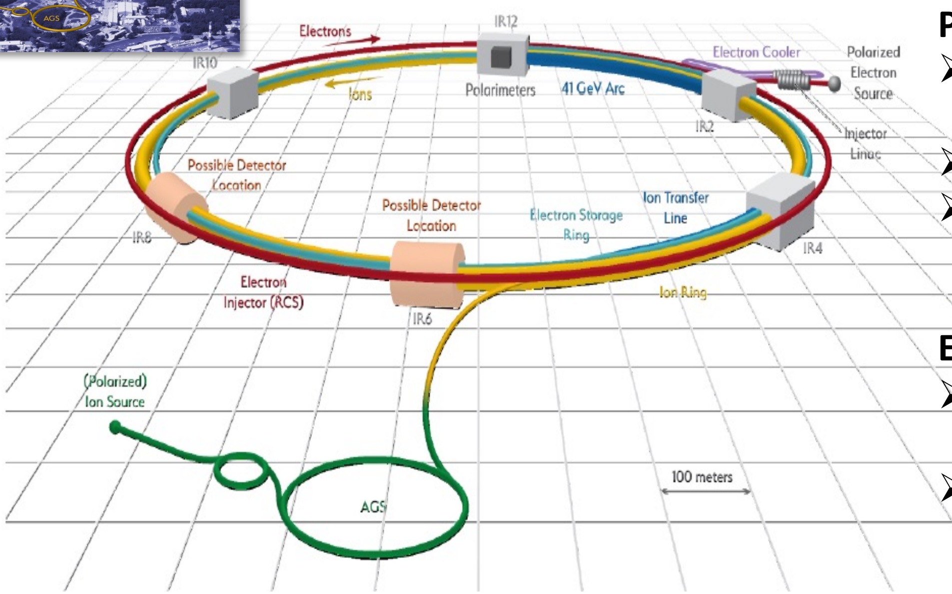
Evidence for gluon spin (ΔG) from RHIC
but large uncertainties from small x



D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)



The Electron-Ion Collider



- ❖ Electron storage ring with frequent injection of fresh polarized electron bunches
- ❖ Hadron storage ring with strong cooling or frequent injection of hadron bunches

Polarized protons up to 275 GeV; Nuclei up to ~ Z/A*275 GeV/n

- ➢ Existing RHIC complex: Storage (Yellow), injectors (source, booster, AGS)
- ➢ Need few modifications
- ➢ RHIC beam parameters fairly close to those required for EIC@BNL

Electrons up to 18 GeV

- ➢ Storage ring, provides the range sqrt(s) = 20-140 GeV. Beam current limited by RF power of 10 MW
- ➢ Electron beam with variable spin pattern (s) accelerated in on-energy, spin transparent injector (Rapid-Cycling-Synchrotron) with 1-2 Hz cycle frequency
- ➢ Polarized e-source and a 400 MeV s-band injector LINAC in the existing tunnel

Design optimized to reach 10^{34} $cm^{-2}sec^{-1}$

A long and winding road…

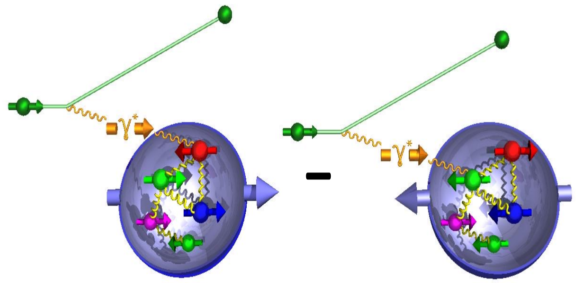
1st eRHIC workshop, BNL Dec. 3-4, 1999

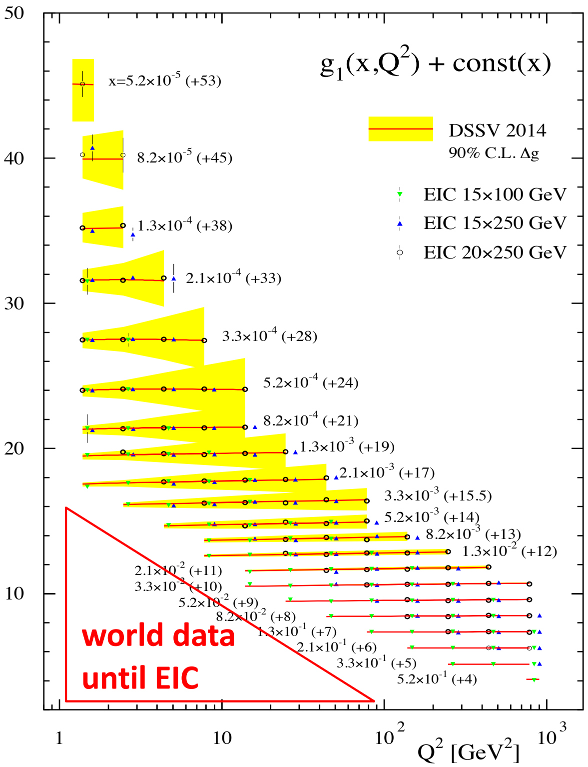
(Garvey,Ludlam,McLerran, Strikman,Venugopalan)

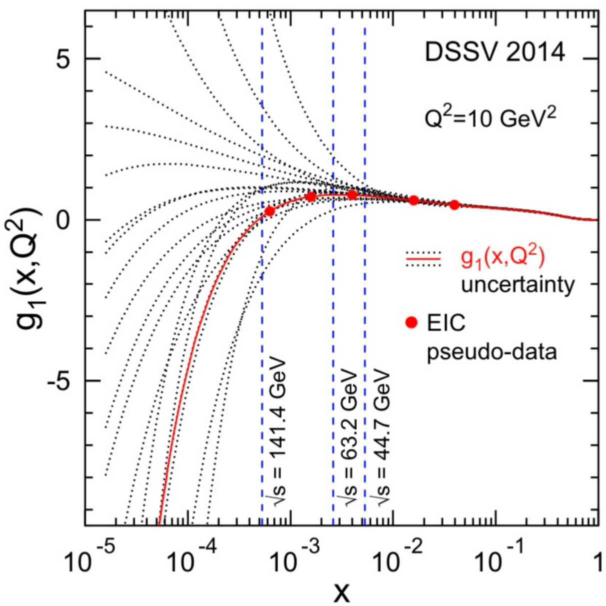
Resolving the proton's spin puzzle: the g_1 structure function

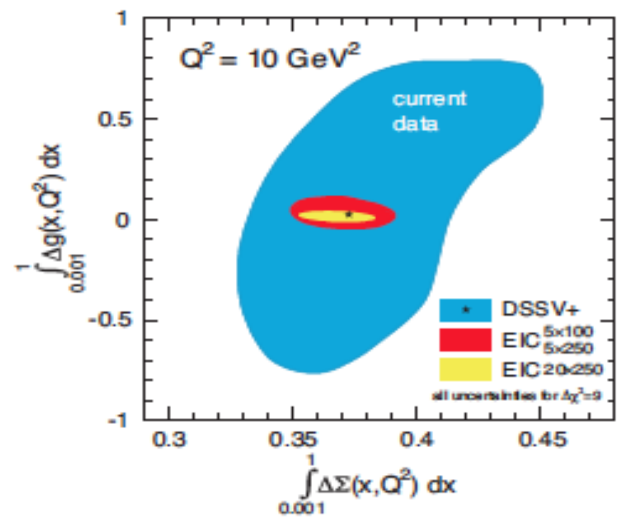
g_1 extracted from longitudinal spin asymmetry in polarized DIS

$$\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x, Q^2)$$



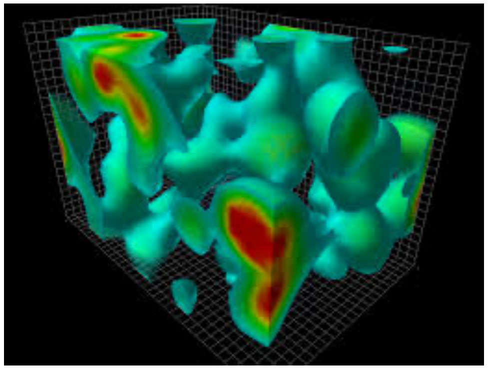






Iso-singlet axial vector current and the chiral anomaly

$$S^\mu \, \Delta\Sigma \, = \, \langle P,S \,|\, \bar{\psi}\, \gamma^\mu \gamma_5 \, \psi \,|\, P,S \rangle \, \equiv \, \langle P,S \,|\, j_5^\mu \,|\, P,S \rangle$$



$U_A(1)$ violation from the chiral anomaly:

$$\partial_\mu J_5^\mu = 2 n_f \partial_\mu K^\mu + \sum_{i=1}^{n_f} 2 i m_i \bar{q}_i \gamma_5 q_i$$

where the Chern-Simons current $K_\mu = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right]$

One suggested explanation for small $\Delta\Sigma$ *is the identification of* K_μ with ΔG

ca., 1988
Efremov, Teryaev
Altarelli, Ross
Carlitz, Collins, Mueller

However K_μ is local but not gauge-invariant (under large gauge transformations)
while ΔG is non-local but gauge-invariant - - strongly hints key role of topology

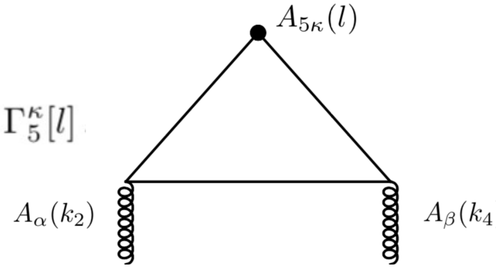
Jaffe, 2007 Varenna lectures
Review:S.D. Bass, RMP, hep-ph/0411005

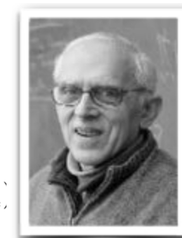
Alternative picture: topological screening of spin

Veneziano (1989); Shore,Veneziano (1990,1992)
Forte, Shuryak (1991); Zahed et al. (2016-)

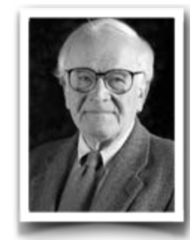
The Adler-Bell-Jackiw chiral (triangle) anomaly

$$\langle P', S | J_5^\kappa | P, S\rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)} \Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$




Steven Adler


John S. Bell


Roman Jackiw

$$= \frac{1}{4\pi^2}\frac{l^\kappa}{l^2}\int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\,\mathrm{Tr}_c F_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\,(2\pi)^4\delta^4(l+k_2+k_4)$$

Famous infrared pole of anomaly. One loop exact: Adler-Bardeen theorem


William A. Bardeen

Key insight from Fujikawa:

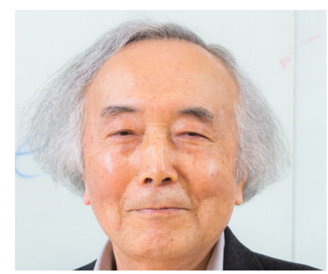
Anomaly arises from the non-invariance of the path integral measure under chiral (γ_5) rotations

$$e^{iW} = \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\ \exp\left[i\int dx(\mathcal{L}_{\rm QCD} + V_5^{\mu a}J^a_{\mu 5} + V^{\mu a}J^a_\mu + \theta Q + S_5^a\phi_5^a + S^a\phi^a)\right]$$

$$\rightarrow \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\left[\partial^\mu J^a_{\mu 5} - \sqrt{2n_f}\delta^{a0}Q - d_{abc}m^b\phi_5^c - \delta\Big(\int d^4x\mathcal{L}_{\rm QCD}\Big)\right]\exp\Big[\ldots\Big] = 0$$

Anomalous functional Ward identities from Wess-Zumino action

Wess, Zumino (1971)


Kazuo Fujikawa

Worldline formalism: box diagram for polarized DIS ($g_1(x,Q^2)$)

DIS with worldlines:
Tarasov, RV, 1903.11624, 2008.08104, 2109.10370

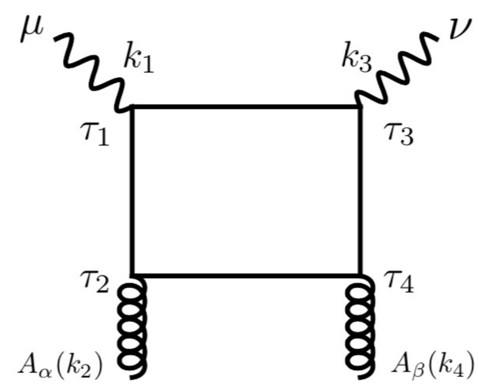
Hadron tensor in DIS: $W^{\mu\nu}(q,P,S) = \frac{1}{2\pi}\int d^4x\, e^{iqx}\langle P,S|j^\mu(x)j^\nu(0)|P,S\rangle$

Anti-symmetric part: $\tilde{W}_{\mu\nu}(q,P,S) = \frac{2M_N}{P\cdot q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\Big\{S^\beta g_1(x_B,Q^2) + \Big[S^\beta - \frac{(S\cdot q)P^\beta}{P\cdot q}\Big]g_2(x_B,Q^2)\Big\}$

$g_1 \propto \Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_4}{(2\pi)^4}\,\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\ \mathrm{Tr}_c(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$

Polarization tensor (antisymmetric piece)

Box diagram

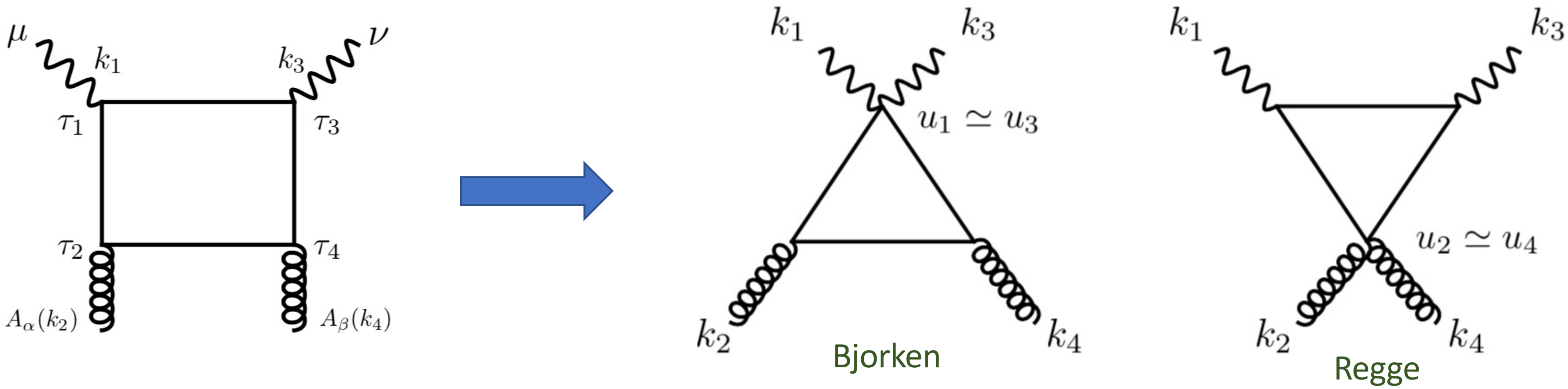


$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] = -\frac{g^2e^2e_f^2}{2}\int_0^\infty\frac{dT}{T}\int\mathcal{D}x\int\mathcal{D}\psi\ \exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\Big)\Big\}$$
$$\times\prod_{k=1}^4\int_0^T d\tau_k\ \Big[\sum_{n=1}^9 \mathcal{C}^{\mu\nu\alpha\beta}_{n;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] - (\mu\leftrightarrow\nu)\Big]e^{i\sum_{i=1}^4 k_ix_i}\,.$$

From QCD worldline path integral for spinning 0+1-D worldlines

Using worldline formulation of one loop effective action in QCD, we can compute the box diagram in exact off-forward kinematics (with no kinematic approximations of internal variables) in both Bjorken ($Q^2\to\infty, s\to\infty, x = \text{fixed}$) and Regge ($x\to 0, s\to\infty, Q^2 = \text{fixed}$) QCD asymptotics

Finding triangles in boxes in Bjorken and Regge asymptotics



Remarkably, box diagram for $g_1(x_B,Q^2)$ has same structure in both limits - dominated by the triangle anomaly !

$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int\frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole $\frac{\Lambda^2{}_{QCD}}{Q^2} << 1$

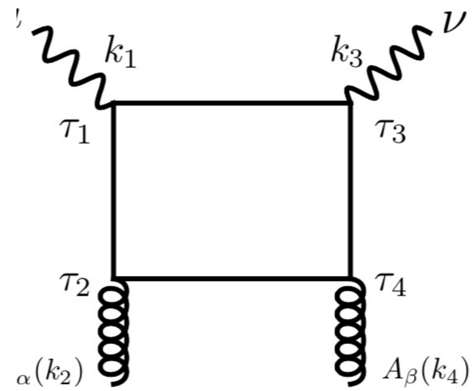
$$S^\mu g_1(x_B,Q^2)|_{x_B\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\int\frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole $\frac{x_B}{x} << 1$

Hence g_1 is topological in both asymptotic limits of QCD…

Tarasov, RV, arXiv:2008.08104

The box diagram for polarized DIS ($g_1(x,Q^2)$)

$$\mathcal{C}^{\mu\nu\alpha\beta}_{1;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_3^\nu\psi_1^\mu\psi_1\cdot k_1\dot{x}_4^\beta\psi_2^\alpha\psi_2\cdot k_2;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{2;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_3^\nu\psi_1^\mu\psi_1\cdot k_1\dot{x}_2^\alpha\psi_4^\beta\psi_4\cdot k_4;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{3;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_1^\mu\psi_3^\nu\psi_3\cdot k_3\dot{x}_2^\alpha\psi_4^\beta\psi_4\cdot k_4;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{4;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_1^\mu\psi_3^\nu\psi_3\cdot k_3\dot{x}_4^\beta\psi_2^\alpha\psi_2\cdot k_2$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{5;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_3^\nu\psi_1^\mu\psi_1\cdot k_1\psi_2^\alpha\psi_2\cdot k_2\psi_4^\beta\psi_4\cdot k_4;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{6;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_1^\mu\psi_3^\nu\psi_3\cdot k_3\psi_2^\alpha\psi_2\cdot k_2\psi_4^\beta\psi_4\cdot k_4$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{7;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_4^\beta\psi_2^\alpha\psi_2\cdot k_2\psi_1^\mu\psi_1\cdot k_1\psi_3^\nu\psi_3\cdot k_3;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{8;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_2^\alpha\psi_4^\beta\psi_4\cdot k_4\psi_1^\mu\psi_1\cdot k_1\psi_3^\nu\psi_3\cdot k_3$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{9;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = 16\psi_1^\mu\psi_1\cdot k_1\psi_3^\nu\psi_3\cdot k_3\psi_2^\alpha\psi_2\cdot k_2\psi_4^\beta\psi_4\cdot k_4$$

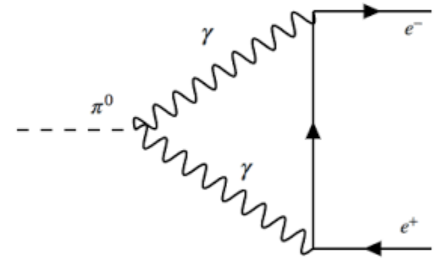


Can compute these explicitly using worldline integration techniques

Tarasov, RV, 1903.11624, 2008.08104

A big role for a phase: the WZW isosinglet term

A famous WZW term one can derive from imaginary part of the worldline action is that responsible for $\pi^0 \to 2\,\gamma$ d'Hoker,Gagne (1995,1996)



Likewise, from $-\mathcal{W}[A, B, \Phi, \Pi] = \mathrm{Ln}\,\mathrm{Det}\,[\mathcal{D}]$

with $\mathcal{D} = \not{p} - i\Phi(x) - \gamma_5\,\Pi - \not{A} - \gamma_5 \not{B}$

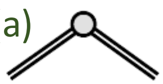
extract from phase of determinant, WZW term in the isosinglet channel $S^{\bar\eta}_{\mathrm{WZW}} = -i\frac{\sqrt{2\,n_f}}{F_{\bar\eta}} \int d^4x\, \bar\eta\,\Omega$

Ω is the topological charge density and F_η is the $\bar\eta$ decay constant

This term from the imaginary part of W_I agrees exactly with expression in nonet Ch.PT. Kaiser, Leutwler (2000)

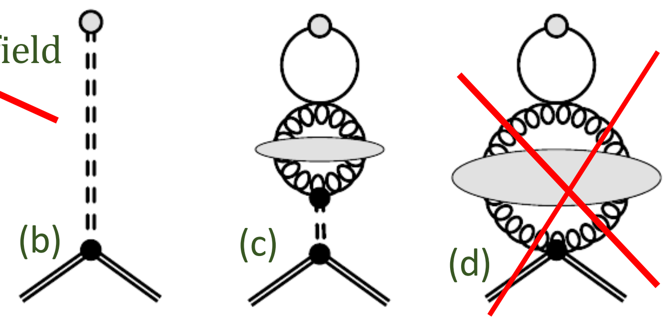
$$\langle P', S | J_5^\mu | P, S\rangle = \bar{u}(P', S)\Big[\gamma^\mu\gamma_5 G_A(l^2) + l^\mu \gamma_5 G_P(l^2)\Big] u(P, S)$$

(a)



Direct axial vector coupling of J_5^μ to polarized proton

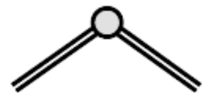
Massless $\bar\eta$ field



Pseudoscalar coupling of polarized proton to J_5^μ

We will now establish a transitive property a ↔ b ↔ c

Goldberger-Treiman relation and anomaly cancellation

I) Consider first the direct axial vector coupling: 

Trivially, $\langle P,S|J_5^\mu|P,S\rangle = \langle P,S|J_5^\mu|P,S\rangle|_{\mathrm{Fig.2a}} = 2M_N\, G_A(0)\, S^\mu$

Since G_P (0) cannot have a pole:

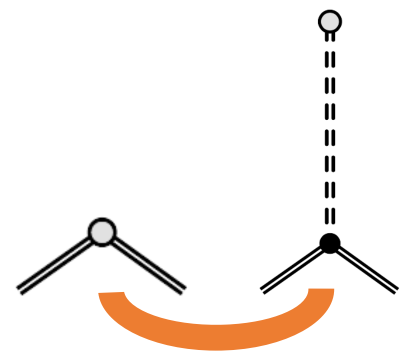
$$\lim_{l\to 0}\left[\langle P',S|J_5^\mu|P,S\rangle|_{\mathrm{Fig.2b}} + \langle P',S|J_5^\mu|P,S\rangle|_{\mathrm{Figs.2c+2d}}\right] = 0$$

$$\boxed{\Sigma(Q^2) = 2\,G_A(0)}$$

The helicity of the proton is twice its axial vector charge

II) The anomaly equation + the Dirac equation link the axial vector and pseudoscalar channels: GT relation

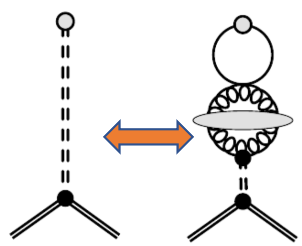
$$\boxed{G_A(0) = \frac{\sqrt{2\tilde{n}_f}}{2M_N}\, F_{\tilde{\eta}}\, g_{\eta_0 NN}}$$



Veneziano (1989)

$g_{\eta_0 \mathrm{NN}}$ is coupling of isosinglet field to proton

III) Absence of a pseudoscalar pole also implies…



$$\boxed{\sqrt{2\tilde{n}_f}\, F_{\tilde{\eta}} = 2n_f\, \lim_{l\to 0} i\,\langle 0|T\,\Omega\eta_0|0\rangle}$$

Goldberger-Treiman relation and anomaly cancellation

$$\chi(l^2) = i \int d^4x\, e^{ilx} \langle 0|T\,\Omega(x)\Omega(0)|0\rangle$$

1/N corrections to YM topological susceptibility induced by WZW coupling…generates QCD top. susceptibility

Resummation gives

$$\chi(l^2) = l^2 \frac{1}{l^2 - m_{\eta'}^2} \chi_{\rm YM}(l^2) \quad \text{with} \quad m_{\eta'}^2 \equiv -\frac{2\,n_f}{F_{\tilde\eta}^2} \chi_{\rm YM}(0)$$

Witten-Veneziano formula

Now $\langle 0|T\Omega\eta_0|0\rangle = -i\frac{1}{l^2}\frac{\sqrt{2\tilde n_f}}{F_{\tilde\eta}}\chi(l^2)$ Since $\chi(l^2) \to 0$ when $l^2 \to 0$, we obtain $F_{\tilde\eta}^2 = 2n_f\chi'(0)$

From the GT relation, it then follows that

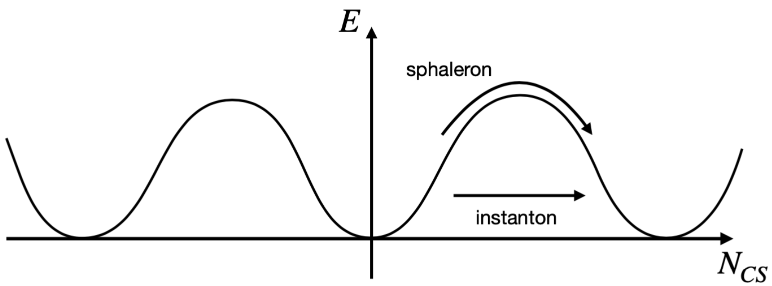
$$\Sigma(Q^2) = \sqrt{\frac{2}{3}}\frac{2n_f}{M_N} g_{\eta_0 NN}\sqrt{\chi'(0)}$$

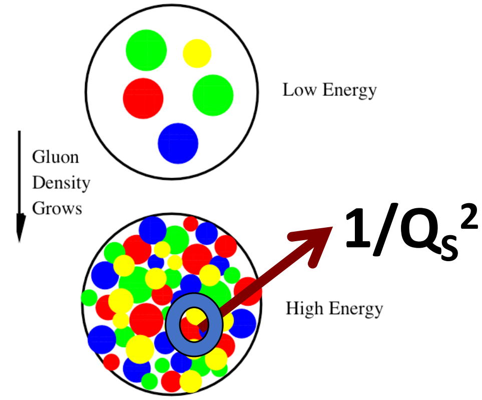
Independent derivation of a result obtained by Shore & Veneziano (1992)

The proton's helicity is determined by the QCD topological susceptibility

Comments: i) To the best of my knowledge, no derivation on the lattice yet – though should be feasible
ii) Sum rule evaluation by Shore, Narison and Veneziano compatible with COMPASS and HERMES data for Σ

What about g_1 at small x_{Bj} ?





Gluon saturation-state in protons and nuclei at small x-corresponding to maximal occupancy and characterized by a semi-hard saturation scale – many-body dynamics described by Color Glass Condensate EFT

Saturation induces over the barrier sphaleron-like transitions which can estimated by an axion-like effective action (describing the propagation of the $\bar{\eta}$ and its coupling to the CGC

Described by small x RG evolution eqns.

$$g_1^{\text{Regge}}(x_B, Q^2) = \left(\sum_f e_f^2\right) \frac{n_f \alpha_s}{\pi M_N} i \int d^4y \int_{x_B}^1 \frac{dx}{x}\left(1 - \frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \int \mathcal{D}\rho\, W_Y[\rho] \int D\bar{\eta}\, \tilde{W}_{P,S}[\bar{\eta}] \int [DA]$$

$$\times \operatorname{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0)\, \eta_0(y)\, \exp\left(iS_{\text{CGC}} + i\int d^4x \left[\frac{1}{2}(\partial_\mu \bar{\eta})(\partial^\mu \bar{\eta}) - \frac{\sqrt{2n_f}}{F_{\bar{\eta}}} \bar{\eta}\, \Omega\right]\right)$$

$$S_{\text{CGC}}[A,\rho] = -\frac{1}{4}\int d^4x F_a^{\mu\nu} F_{\mu\nu}^a + \frac{i}{N_c}\int d^2x_\perp\, \operatorname{tr}_c\left[\rho(x_\perp) \ln\left(U_{[\infty,-\infty]}(x_\perp)\right)\right]$$

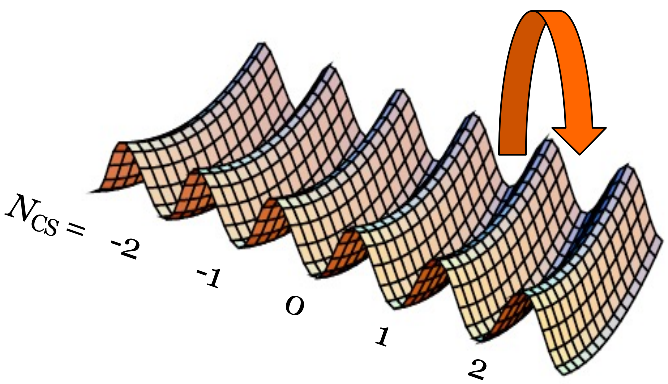
Tarasov,Venugopalan, arXiv:2109.10370

Spin diffusion via sphaleron transitions in topologically disordered media

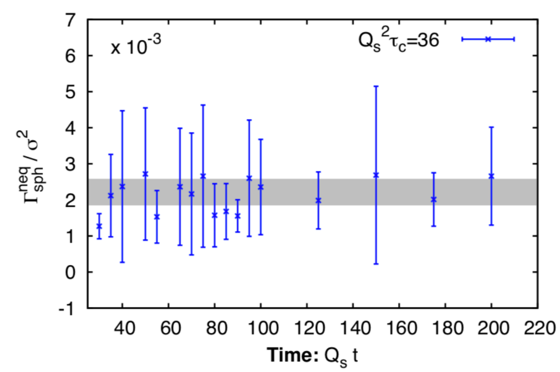
Two scales – the height of the barrier given by $m_{\eta'}^2 = 2n_f \frac{\chi_{\mathrm{YM}}}{F^2}$

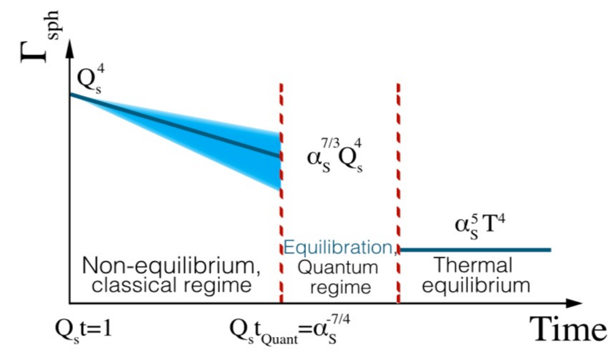
- the gluon saturation scale Q_S

When $Q_S^2 >> m_{\eta'}^2$ over the barrier gauge configurations dominate over instanton configurations



Sphaleron transition rate off-equilibrium



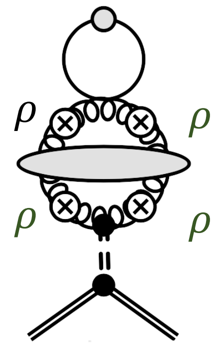


Mace,Schlichting,Venugopalan (2016)

Over the barrier (sphaleron) transitions between different
topological sectors of QCD vacuum…characterized by integer valued Chern-Simons #

g_1 at small x_{Bj} from sphaleron transitions

For $Q_S^2 < m_{\eta'}^2$
over the barrier transitions



From our small x_B effective action, $$\frac{\partial^2 \eta'}{\partial t^2} = -\gamma \frac{\partial \eta'}{\partial t} - m_{\eta'}^2 \eta'$$ $$\gamma = \frac{2n_f \, \Gamma_{sphaleron}}{F^2{}_{\eta'} \, Qs}$$

Spin diffusion due to “drag force” on “axion” propagation in the shock wave background
-drag force is proportional to sphaleron transition rate

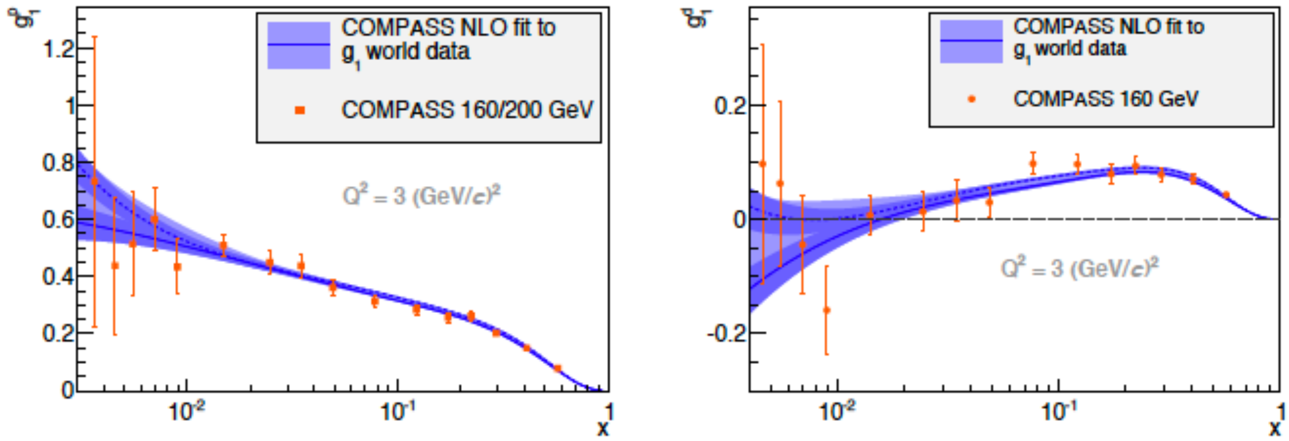
McLerran,Mottola,Shaposhnikov (1990)

$$g_1^{\mathrm{Regge}}(x_B, Q^2) \propto F(x_B) \times \frac{Q_S^2 m_{\eta'}^2}{F_{\bar\eta}^3 M_N} \exp\left(-4\, n_f C\, \frac{Q_S^2}{F_{\bar\eta}^2}\right)$$

Very rapid quenching of spin diffusion at small x_{Bj} !

g_1 at small x_{Bj} from sphaleron transitions

COMPASS: arXiv:1503.08935
arXiv: 1612.00620



The key feature of the topological screening picture is its target independence
However, as we have argued, the result is sensitive to the density of color sources, which is larger for the deuteron – one anticipates the same behavior for g_1^p as g_1^d at even smaller x_B

Other observables: semi-inclusive DIS, g_1^γ DeFlorian, Shore, Veneziano, hep-ph/9711353

Of particular interest is the g_2 structure function – in the naïve parton model, it is zero in the chiral limit. Turning on quark masses introduce non-trivial mixing between the UA(1) and SU(3) flavor sectors – which can be computed Bhattacharya, Hatta, Tarasov, RV, in progress

Thank you for your attention !

Low energy dynamics of η' in QCD

For N_f=3, dynamical variables of effective theory are massless modes in limit $N_C \to \infty \; and \; m \to 0$

Symmetry group is G= $U_R(3) \times U_L(3)$

Spontaneous symmetry breaking: $U_R(3) \times U_L(3) \to U_V(3)$

The nine parameters of its coset space correspond to the nine pseudoscalar Goldstone bosons – including the prodigal $\eta' \to_{g_2} \eta_0$

Relative to the "standard" SU(3) framework, where $\det U(x) = e^{i\eta_0(x)}$ and η_0 transforms as

η_0'= η_0- i $\mathrm{Ln}\det V_R + i\,\mathrm{Ln}\det V_L$

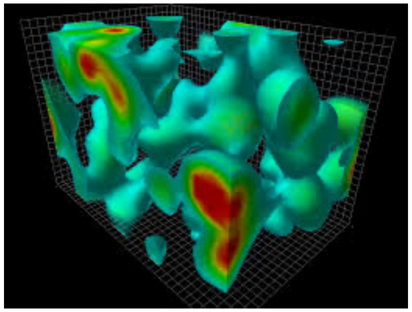
For non-zero quark masses, expansion in # of derivatives, powers of m and $1/N_c$

Wess-Zumino-Witten terms for the SU(3) and U(1) sectors correspond to the "un-natural parity"part of the effective Lagrangian

Leutwyler, hep-ph/9601234
Herrera-Siklody et al, hep-ph/9610549
Kaiser, Leutwyler, hep-ph/0007101

Iso-singlet axial vector current and the chiral anomaly

$$S^\mu \,\Delta\Sigma \;=\; \langle P,S\,|\,\bar\psi\,\gamma^\mu\gamma_5\,\psi\,|\,P,S\rangle \;\equiv\; \langle P,S\,|\,j_5^\mu\,|\,P,S\rangle$$



$U_A(1)$ violation from the anomaly:

$$\partial_\mu J_5^\mu = 2n_f\partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i\bar q_i\gamma_5 q_i$$

where the Chern-Simons current $K_\mu = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3}gf_{abc}A_b^\rho A_c^\sigma\right)\right]$

But, identification of CS charge with ΔG is intrinsically ambiguous … *the latter is gauge invariant, the former is not*

Jaffe, Manohar (1990)

$$K_\mu \to K_\mu \;+\; i\frac{g}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu\left(U^\dagger\partial^\alpha U A^\beta\right)$$
$$+\;\frac{1}{24\pi^2}\epsilon_{\mu\nu\alpha\beta}\left[(U^\dagger\partial^\nu U)(U^\dagger\partial^\alpha U)(U^\dagger\partial^\beta U)\right]$$

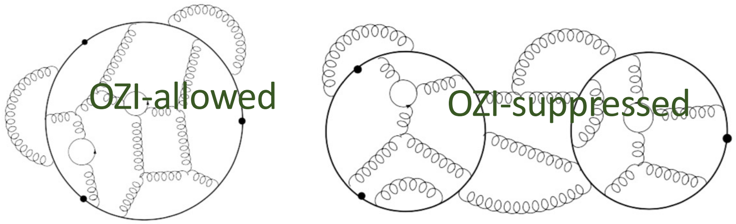
"Large gauge transformation"
- deep consequence of topology

R. Jaffe: identification of K^μ with ΔG a source of much confusion in the literature (Varenna lectures, 2007)

Anomaly cancellation and topological screeening

$$\Sigma(Q^2) = \sqrt{\frac{2}{3}}\,\frac{2n_f}{M_N}\,g_{\eta_0 NN}\sqrt{\chi'(0)}\,.$$

Magnitude of of OZI violation $\frac{a^0(Q^2)}{a^8} \simeq \frac{\sqrt{6}}{f_\pi}\sqrt{\chi'(0)}$



Computations on the lattice…

Bali et al., arXiv:2106.05398

$G_A|_{model}$ = 0.33±0.05

Sum rule analysis in good agreement with HERMES and COMPASS data

Narison,Shore,Veneziano (1998)

HERMES (Q^2= 5 GeV^2) $0.330 \pm 0.011(th) \pm 0.025(exp) \pm 0.028(evol)$

COMPASS (Q^2=3 GeV^2) $0.35 \pm 0.03(stat) \pm 0.05(syst)$

Axion-like effective action

As suggested by Shore and Veneziano, and following from our discussion as well,

$$S_{\bar\eta} = \int d^4x \left[\frac{1}{2}\left(\partial_\mu \bar\eta\right)\left(\partial^\mu \bar\eta\right) + \left(\theta - \frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\right)\Omega + \frac{\chi_{\rm YM}}{2}\,\theta^2\right]$$

Since θ *is not dynamical, can get rid of it from the equations of motion,*

$$S_{\bar\eta} = \int d^4x \left[\frac{1}{2}\left(\partial_\mu \bar\eta\right)\left(\partial^\mu \bar\eta\right) - \frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\,\Omega - \frac{\Omega^2}{2\,\chi_{\rm YM}}\right]$$

Axion-like effective action for $\bar\eta$

Defining $\eta' = \frac{F_{\eta'}}{F_{\bar\eta}}\bar\eta$ *and* $G = \Omega + \frac{\sqrt{2n_f}}{F_{\eta'}}\chi_{\rm YM}\,\eta'$

$$S_{\eta'} = \int d^4x \left[-\frac{1}{2}\eta'\left(\partial^2 + m_{\eta'}^2\right)\eta' - \frac{G^2}{2\chi_{\rm YM}}\right]$$

Re-express in terms of the η' *and a non-propagating glueball that decouples from the physical spectrum*

Shore,Veneziano (1990); Hatsuda (1990)
Dvali,Jackiw,Pi (1995)

In the instanton framework, χ_{YM} is saturated by such classical configurations

t'Hooft (1976); Schafer-Shuryak (1996)

Several Spin discussions by multiple groups in this framework:

Forte, Shuryak (1990); Qian, Zahed (2016); …