

# Cracks in LCDM

erc



**Lavinia Heisenberg**  
(ITP-Heidelberg)

[L.Heisenberg@thphys.uni-heidelberg.de](mailto:L.Heisenberg@thphys.uni-heidelberg.de)

28. June 2022, Kitzbühel Humboldt Conference, Kitzbühel/Austria

**In collaboration with**



**Hector Villarrubia Rojo**



**Jann Zosso**

# Cosmology

**Cosmology describes the Universe with 2  
fundamental pillars**

# Cosmology

**Cosmology describes the Universe with 2  
fundamental pillars**

**GR**

**General Relativity**

# Cosmology

**Cosmology describes the Universe with 2  
fundamental pillars**

**GR**

**General Relativity**

**CP**

**Cosmological Principle**

# Cosmology

Cosmology describes the Universe with 2  
fundamental pillars

GR

General Relativity

CP

Cosmological Principle



Homogeneity  
& Isotropy

# Cosmology

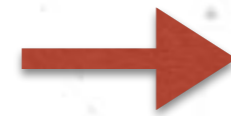
**Cosmology describes the Universe with 2  
fundamental pillars**

**GR**

**General Relativity**

**CP**

**Cosmological Principle**



**Homogeneity  
& Isotropy**

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

# Cosmology

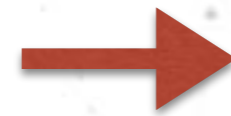
Cosmology describes the Universe with 2  
fundamental pillars

GR

General Relativity

CP

Cosmological Principle



Homogeneity  
& Isotropy

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

$$H = \frac{\dot{a}}{a}$$

expansion rate



# Cosmology

Cosmology describes the Universe with 2  
fundamental pillars

GR

General Relativity

CP

Cosmological Principle



Homogeneity  
& Isotropy

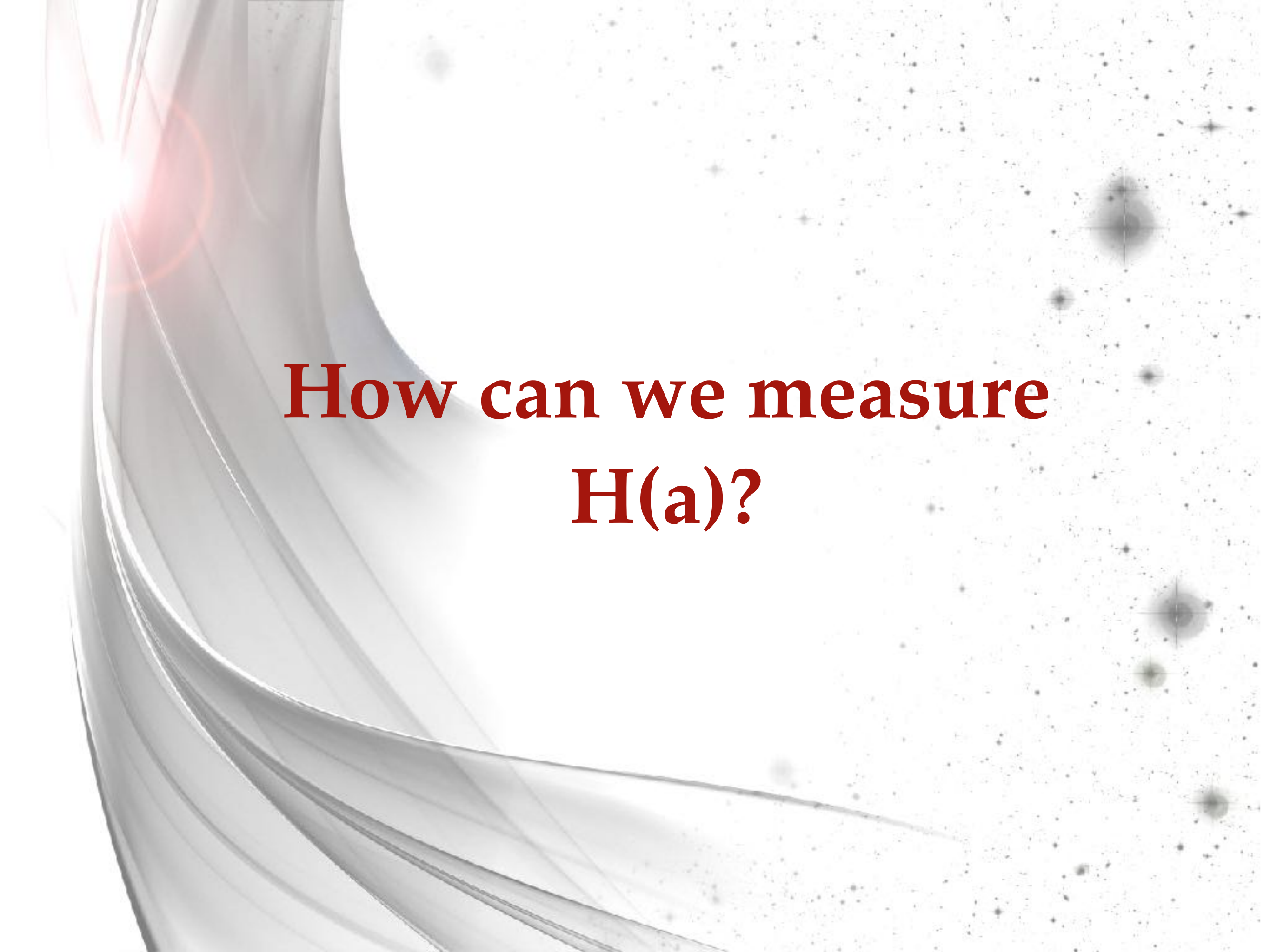
$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

$$H = \frac{\dot{a}}{a}$$

expansion rate

$$H_0 = H(\text{today})$$

$$= 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$



**How can we measure  
 $H(a)$ ?**

**How can we measure  $H(a)$ ?**

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

## How can we measure $H(a)$ ?

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

**redshift**  $a = \frac{1}{1+z}$

## How can we measure $H(a)$ ?

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

**redshift**  $a = \frac{1}{1+z}$

**comoving distance**

$$d = \int_t^{t_0} \frac{dt}{a(t)}$$
$$= \int_0^z \frac{dz}{H(z)}$$

## How can we measure $H(a)$ ?

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

**redshift**  $a = \frac{1}{1+z}$

**comoving distance**

$$d = \int_t^{t_0} \frac{dt}{a(t)}$$
$$= \int_0^z \frac{dz}{H(z)}$$

**through distance measurements we can obtain  $H$ !**

## How can we measure $H(a)$ ?

**comoving  
distance**

$$\begin{aligned} d &= \int_t^{t_0} \frac{dt}{a(t)} \\ &= \int_0^z \frac{dz}{H(z)} \end{aligned}$$

## How can we measure $H(a)$ ?

**comoving  
distance**

$$d = \int_t^{t_0} \frac{dt}{a(t)}$$
$$= \int_0^z \frac{dz}{H(z)}$$



## How can we measure $H(a)$ ?

**comoving  
distance**

$$d = \int_t^{t_0} \frac{dt}{a(t)}$$
$$= \int_0^z \frac{dz}{H(z)}$$

**for small redshift (nearby objects)**

$$d(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$$

How can we measure  $H(a)$ ?

comoving  
distance

$$d = \int_t^{t_0} \frac{dt}{a(t)}$$
$$= \int_0^z \frac{dz}{H(z)}$$

for small redshift (nearby objects)

$$d(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$$

from  $z$  and  $d(z)$  we can obtain  $H_0$

How can we measure  $H(a)$ ?

comoving  
distance

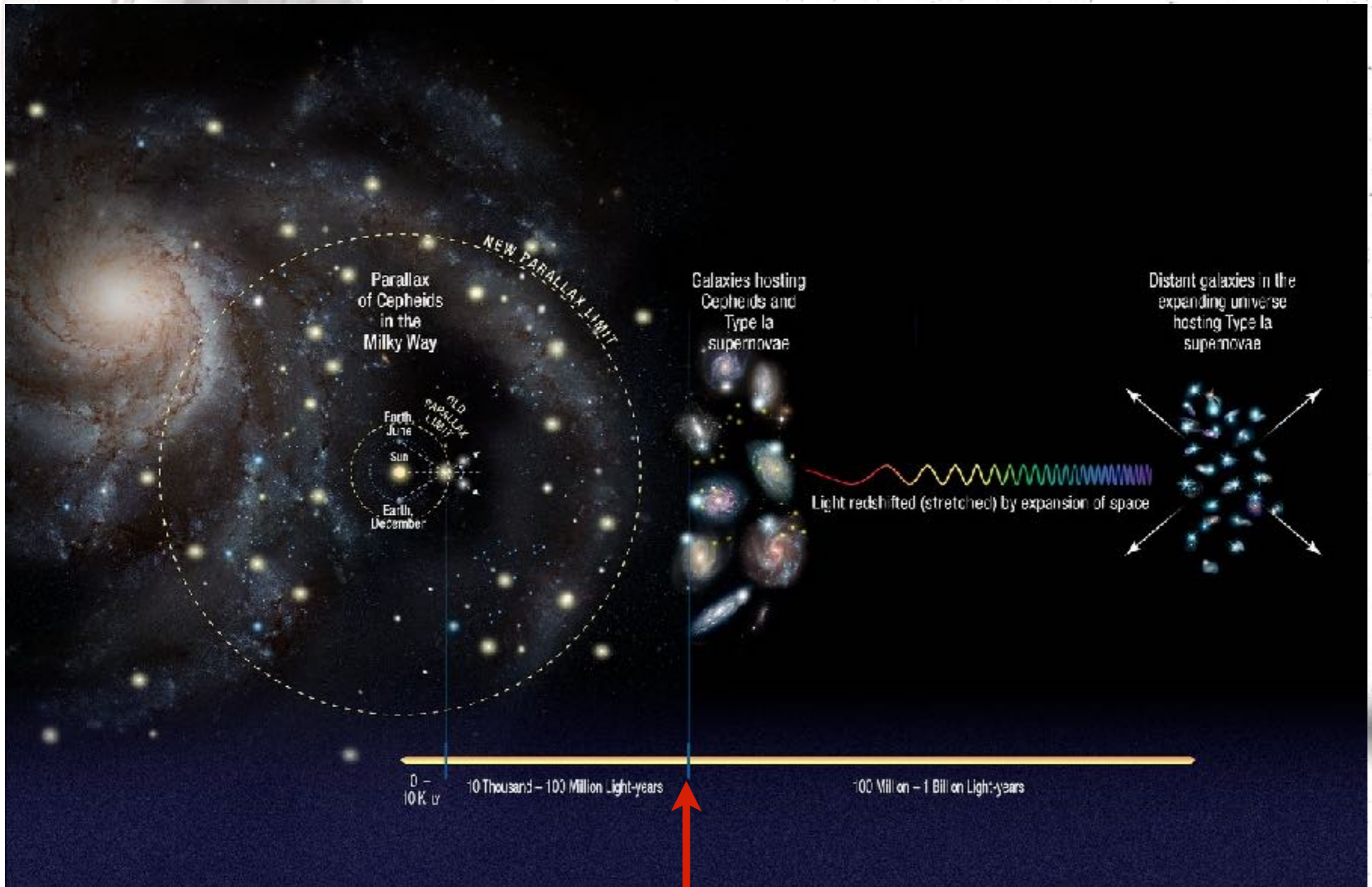
$$d = \int_t^{t_0} \frac{dt}{a(t)}$$
$$= \int_0^z \frac{dz}{H(z)}$$

for small redshift (nearby objects)

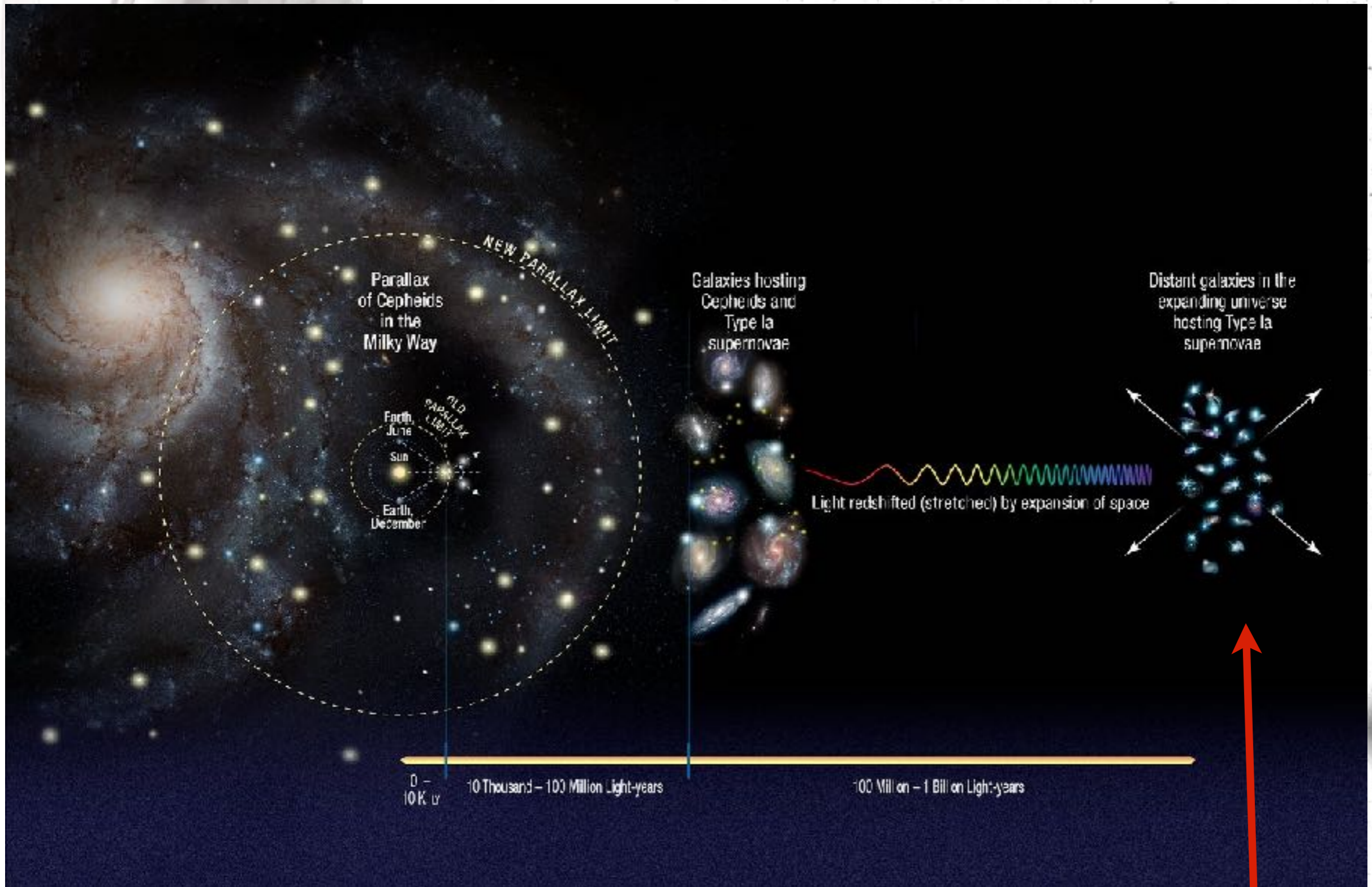
$$d(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$$

from  $z$  and  $d(z)$  we can obtain  $H_0$

(model-independent)



Cepheids



# Supernovae

How can we measure  $H(a)$ ?

comoving  
distance

$$d = \int_t^{t_0} \frac{dt}{a(t)}$$
$$= \int_0^z \frac{dz}{H(z)}$$

for small redshift (nearby objects)

$$d(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$$

from  $z$  and  $d(z)$  we can obtain  $H_0$

(model-independent)

**How can we measure  $H_0$  from CMB?**



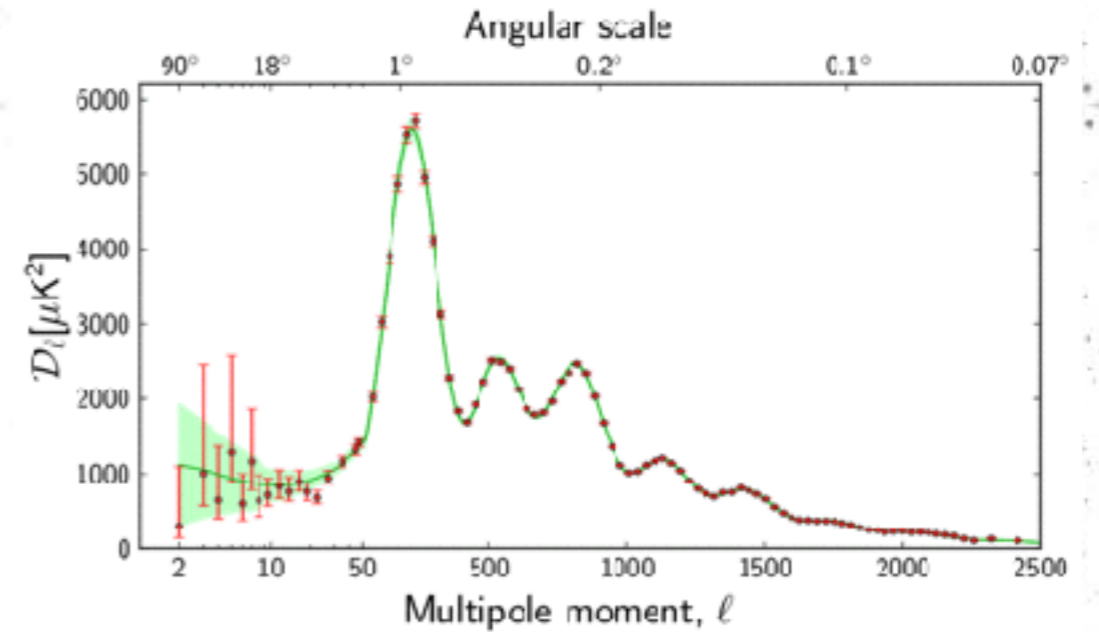
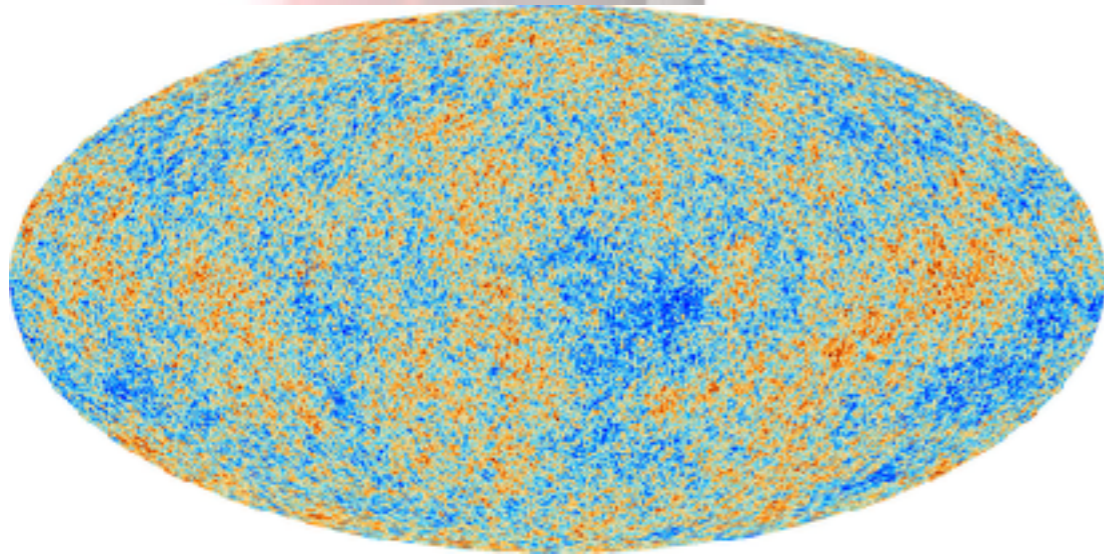
How can we measure  $H_0$  from CMB?

$\Lambda$ CDM

$\{H_0, \Omega_b, \Omega_m, A_s, n_s, \tau_{reio}\}$



# How can we measure $H_0$ from CMB?



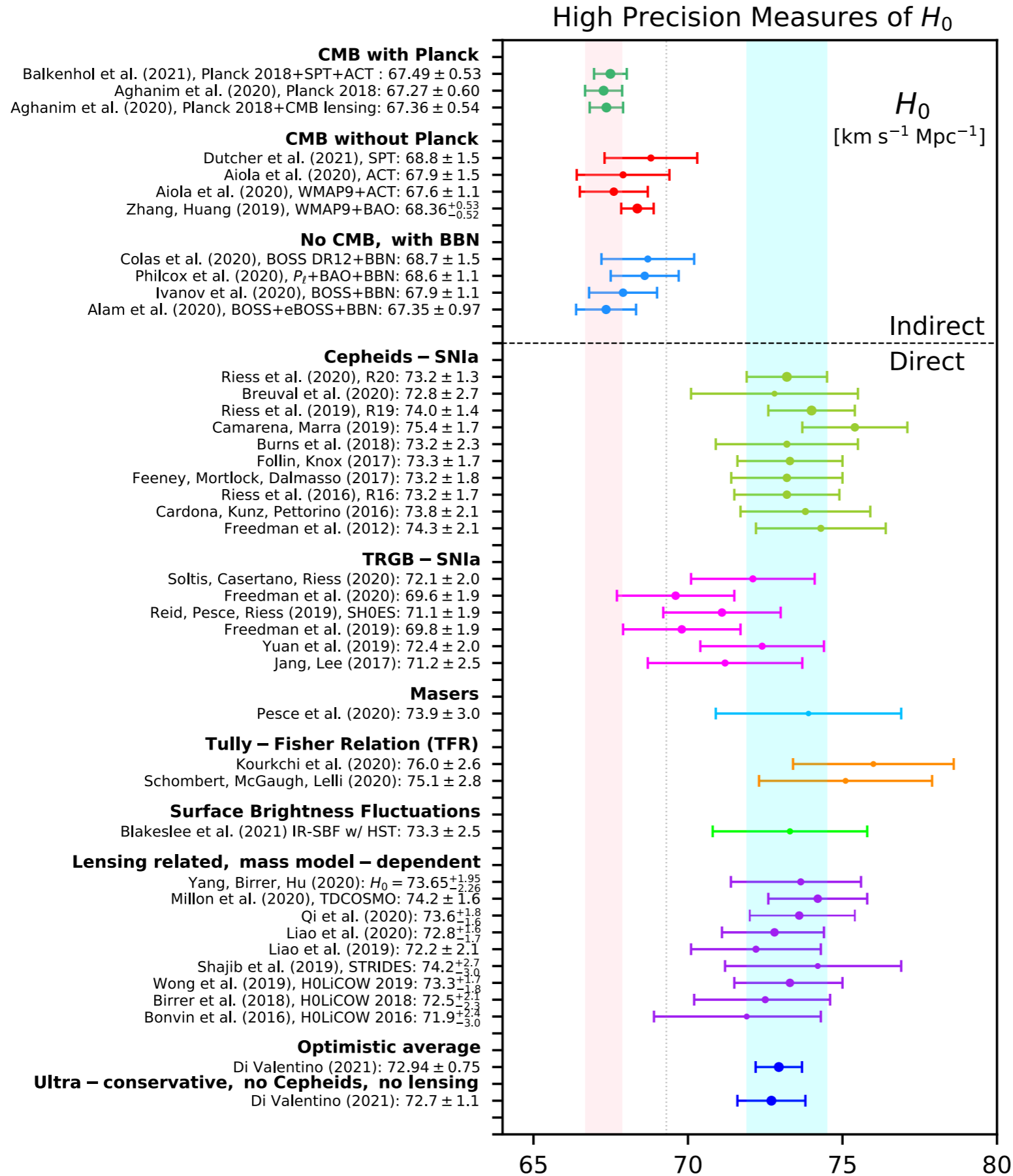
measure fluctuations

compute observables

$\Lambda$ CDM

$\{H_0, \Omega_b, \Omega_m, A_s, n_s, \tau_{reio}\}$

# H0 tension



E. Di Valentino et al.  
arXiv:2103.01183

The background features a dynamic composition of flowing, translucent grey and white shapes on the left side, resembling smoke or liquid. On the right side, there is a field of small, dark, star-like specks scattered across a light background, some with faint crosshair patterns.

**See Gia Dvali's Talk:**

**It is hard to get de Sitter space-time from  
Quantum Gravity!**



**Early versus late-time  
solutions**

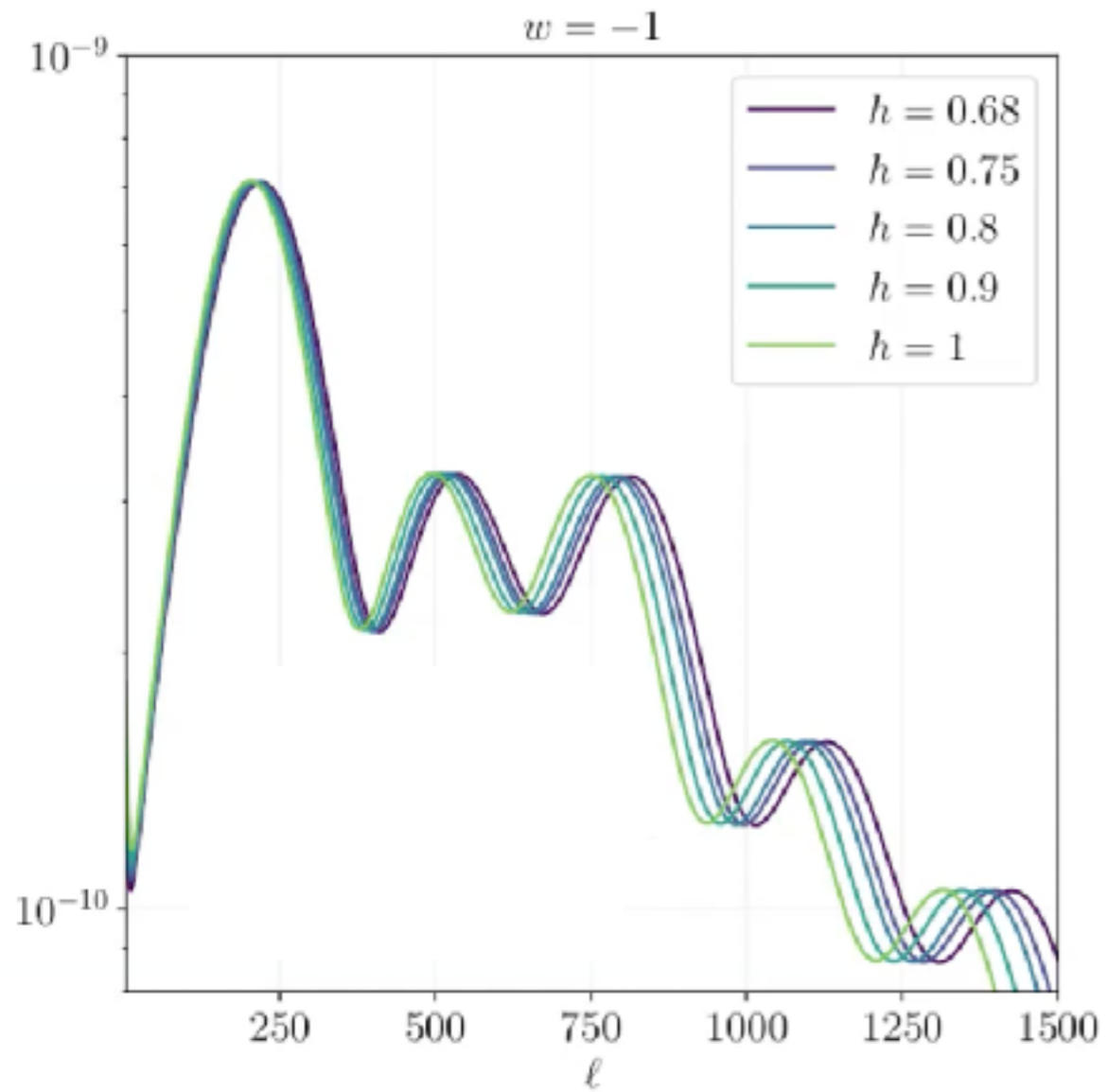


**We want  $H_0 \uparrow$**

**We want  $H_0 \uparrow$  (or  $h \uparrow$ )**

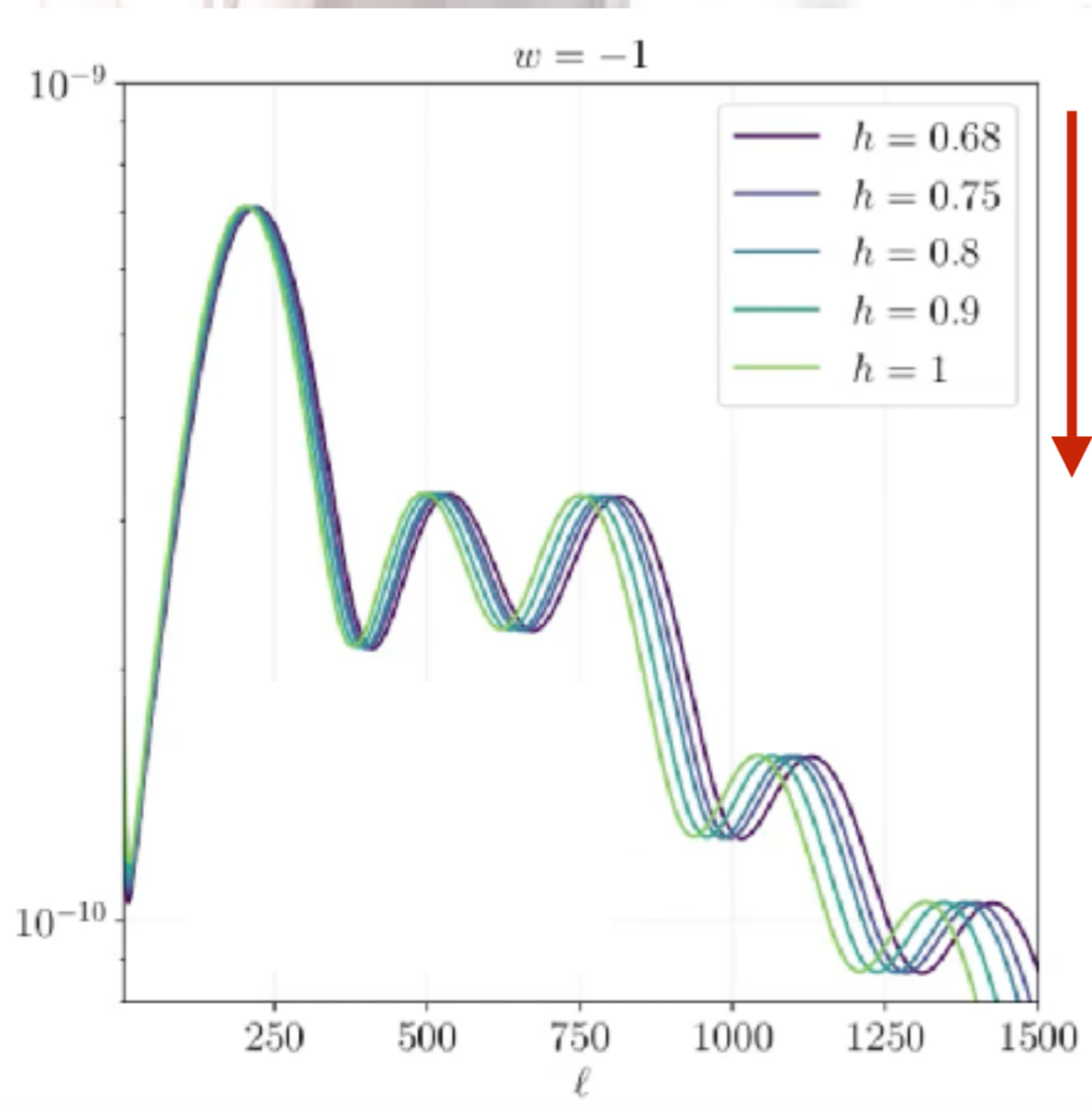
$$\begin{aligned} H_0 &= H(\text{today}) \\ &= 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \end{aligned}$$

# Early versus late-time solutions



L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

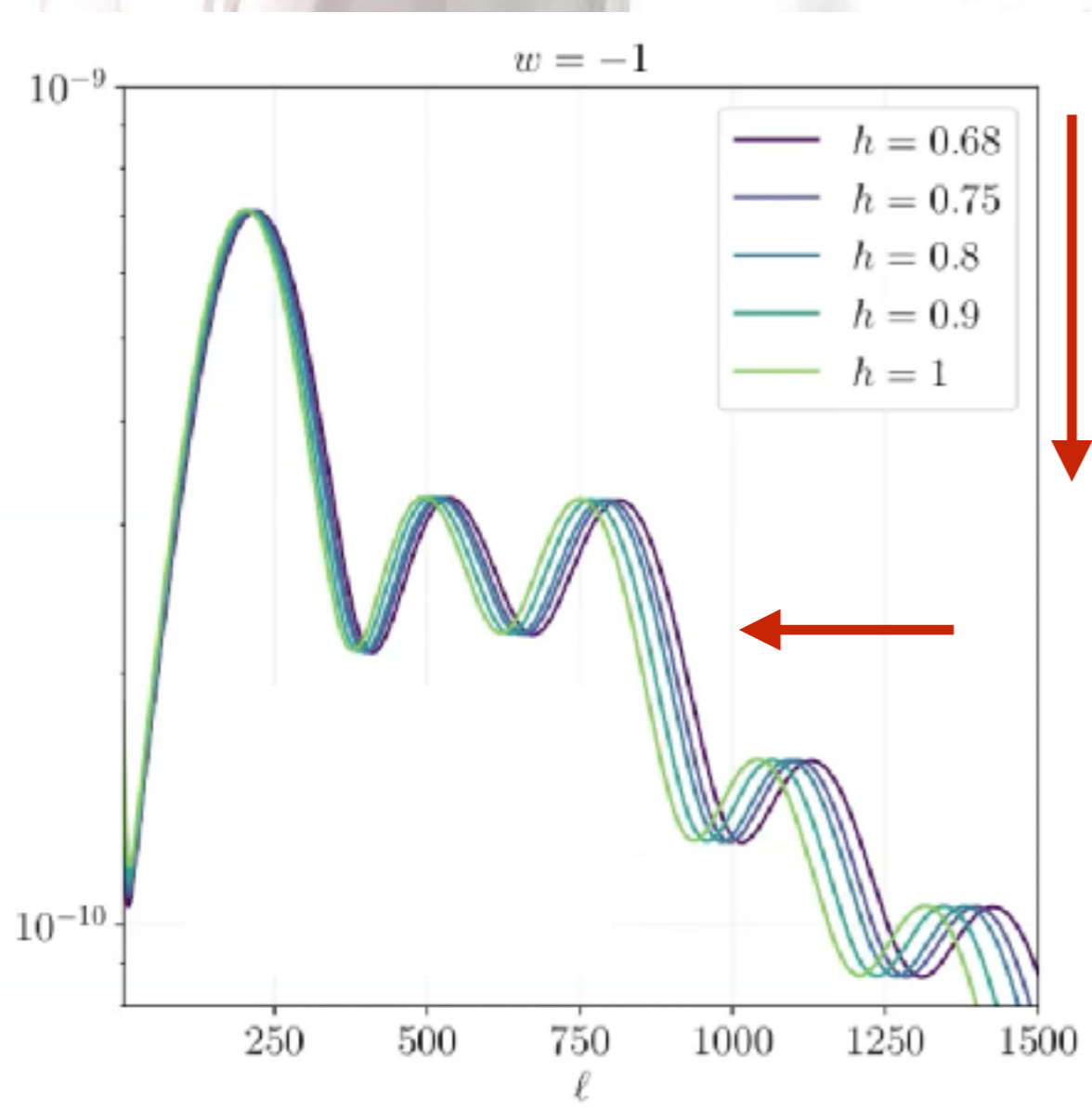
# Early versus late-time solutions



L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

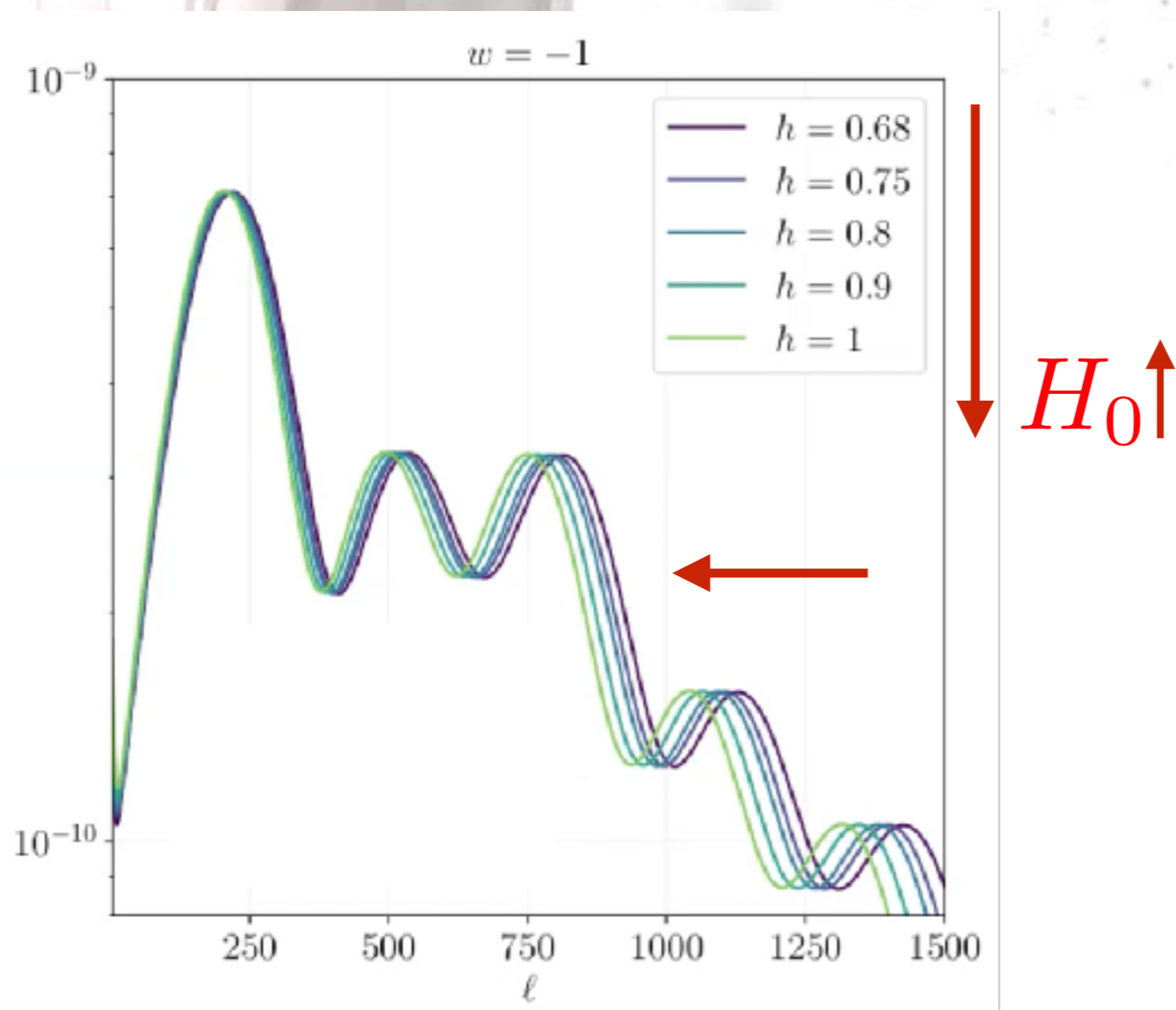


# Early versus late-time solutions



L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Early versus late-time solutions

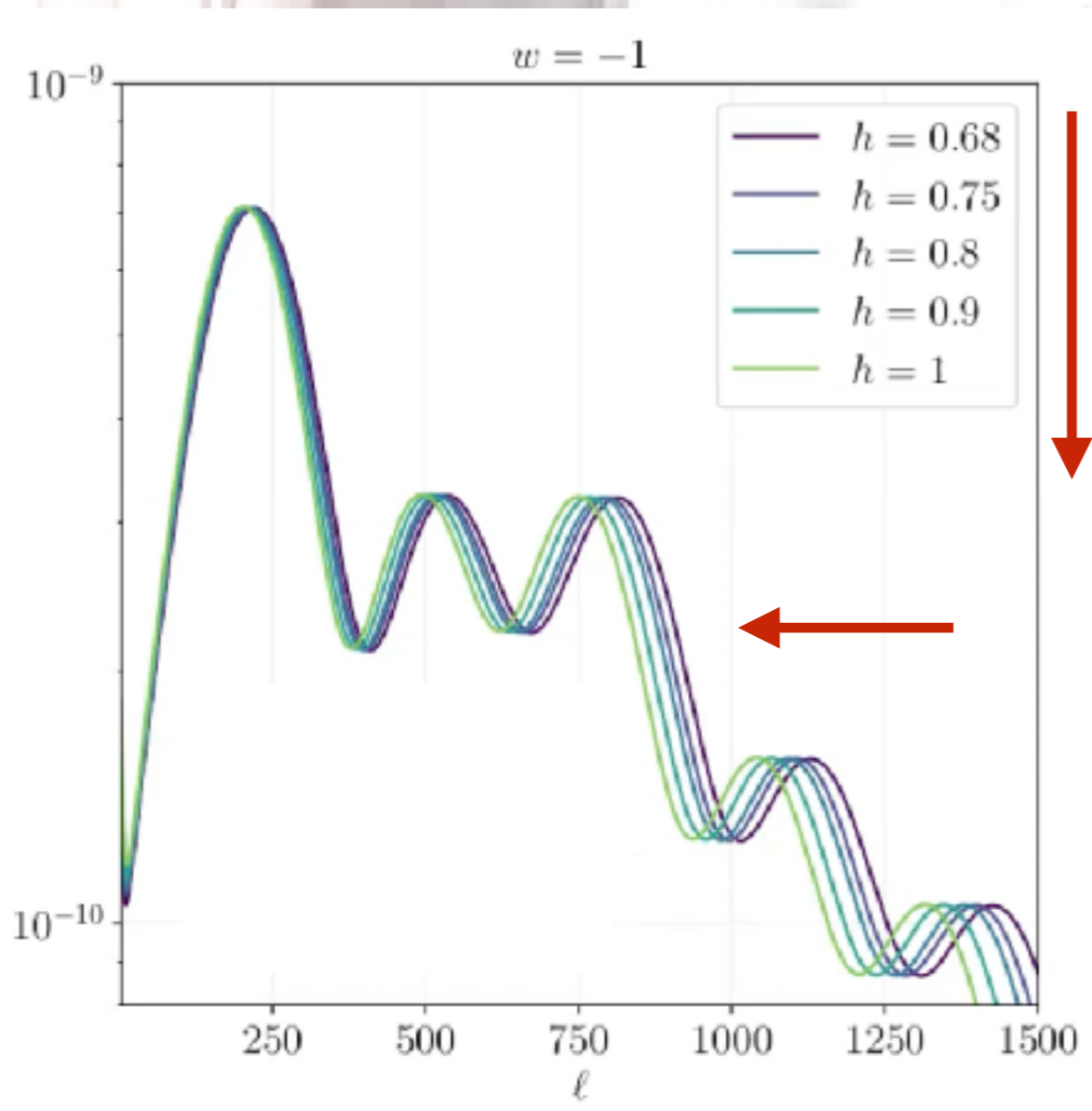


acoustic scale: overall position of the peaks

$$\theta_{\star} = \frac{r_s(z_{\star})}{d_A(z_{\star})}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Early versus late-time solutions



acoustic scale: overall position of the peaks

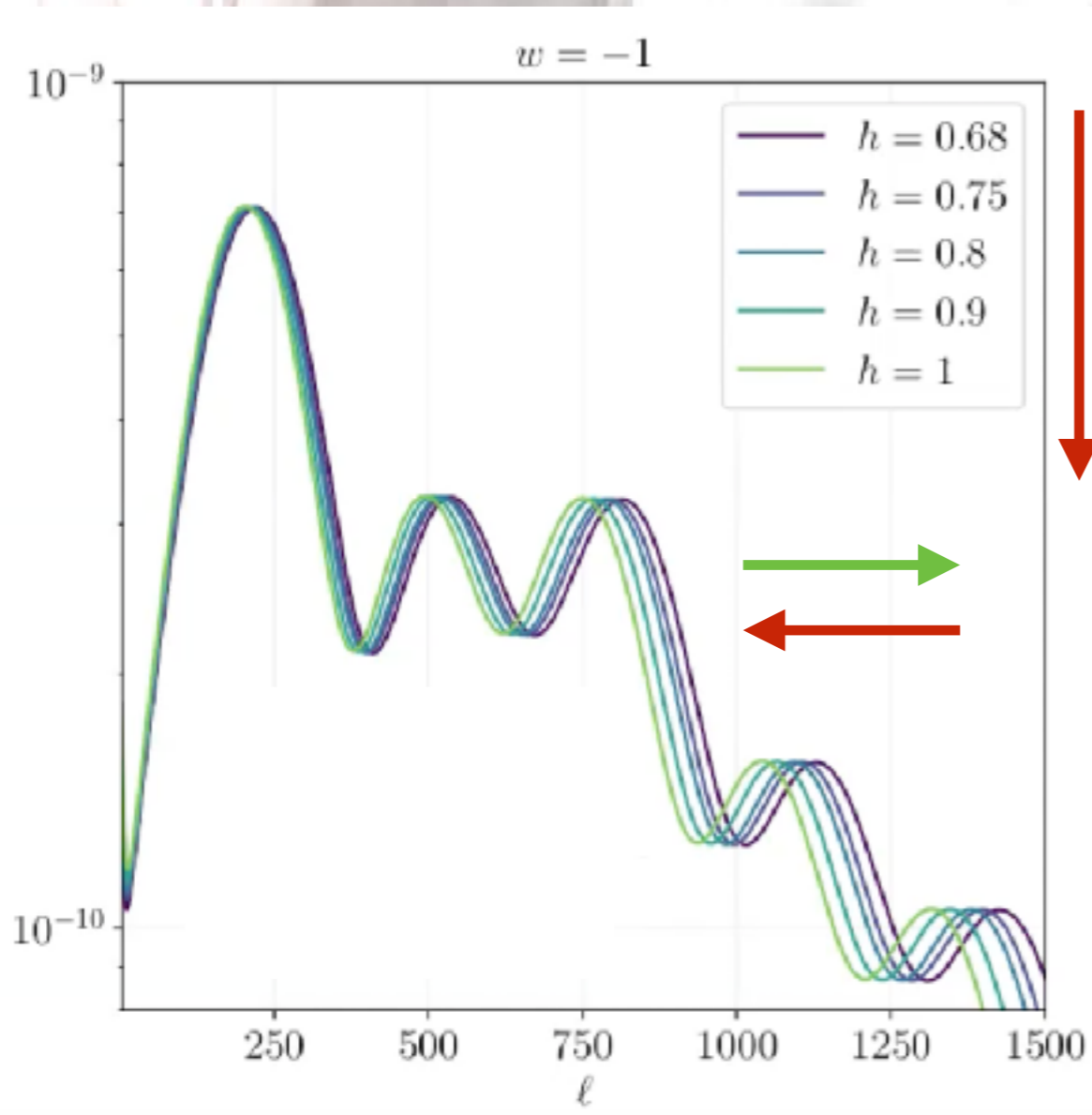
sound horizon

$$\theta_{\star} = \frac{r_s(z_{\star})}{d_A(z_{\star})}$$

angular diameter distance

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Early versus late-time solutions



acoustic scale: overall position of the peaks

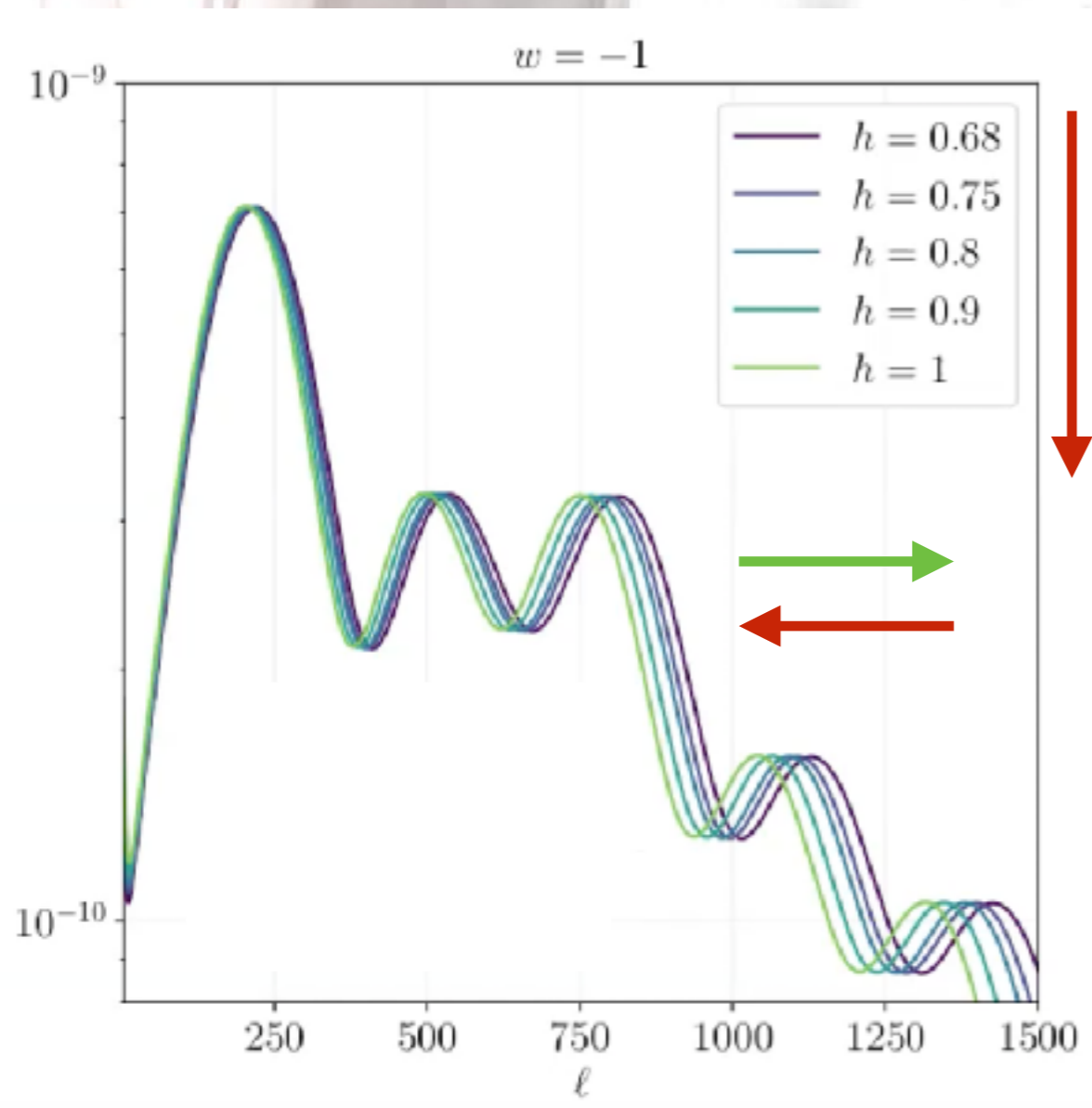
sound horizon

$$\theta_* = \frac{r_s(z_*)}{d_A(z_*)}$$

angular diameter distance

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Early versus late-time solutions



acoustic scale: overall position of the peaks

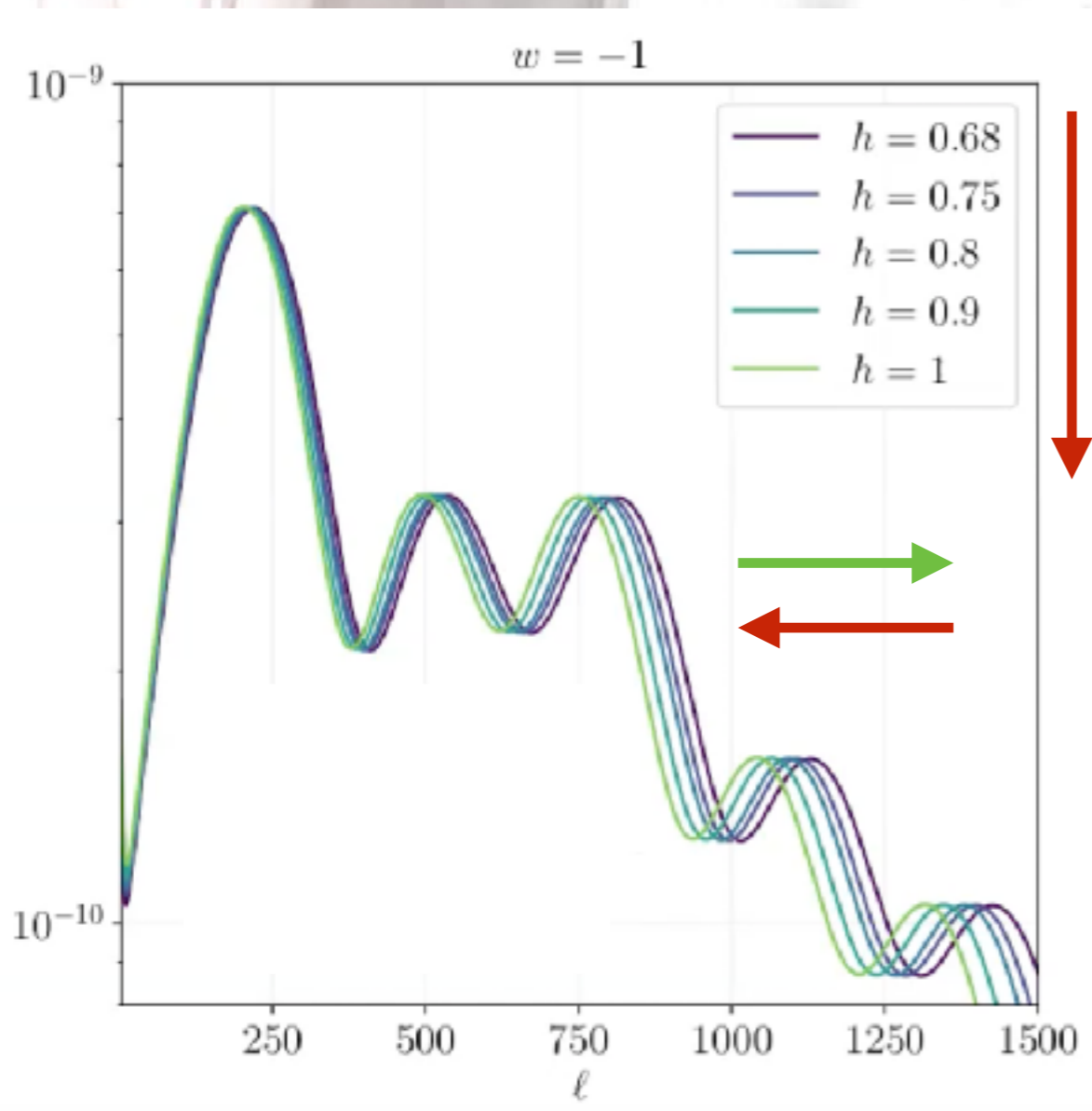
$$\theta_\star = \frac{r_s(z_\star)}{d_A(z_\star)}$$

$$r_s(z_\star) = \int_{z_\star}^{\infty} \frac{dz}{H} c_s$$

Early  $H(z)$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Early versus late-time solutions



acoustic scale: overall position of the peaks

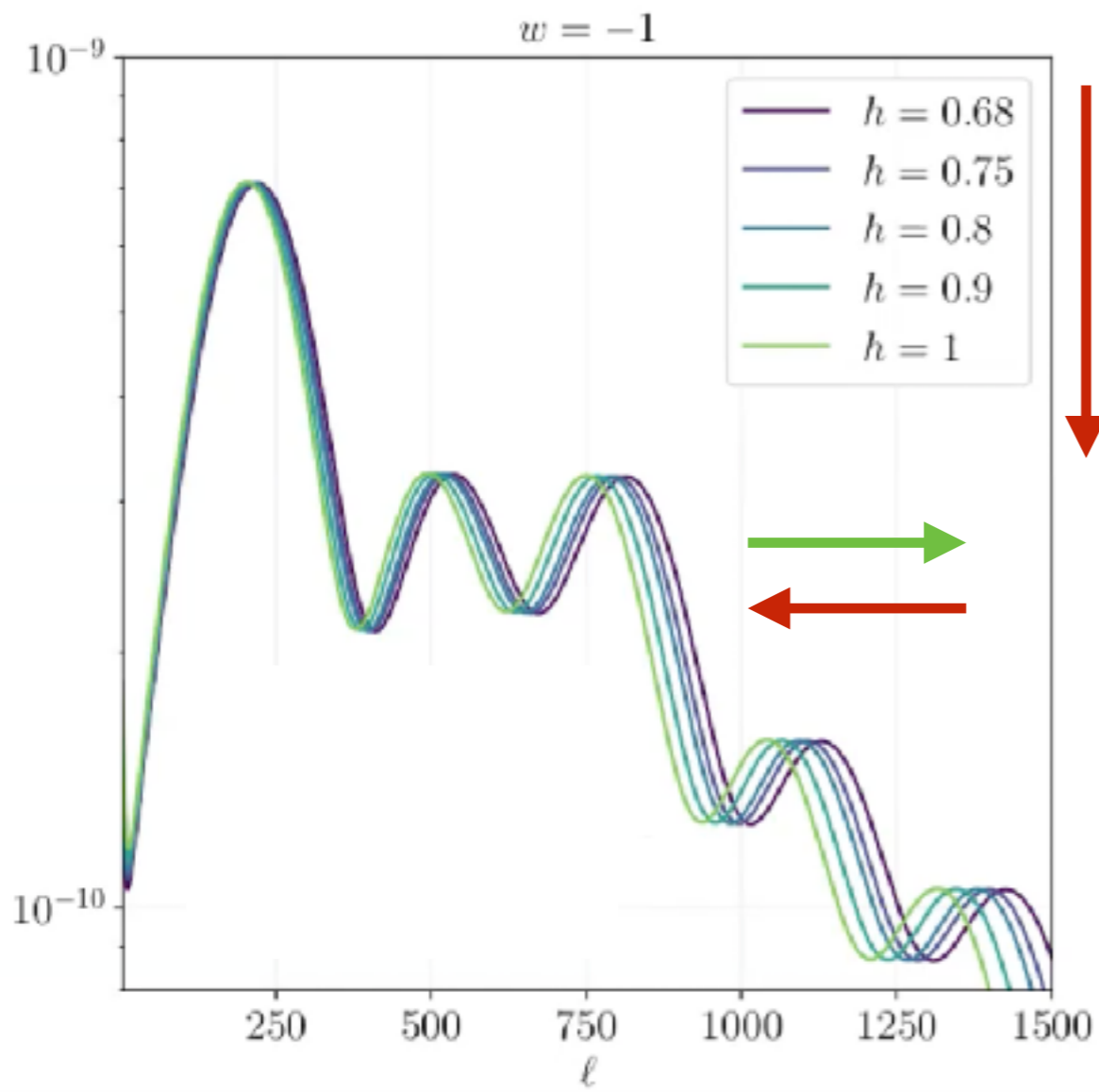
$$\theta_* = \frac{r_s(z_*)}{d_A(z_*)}$$

$$d_A(z_*) = \int_0^{z_*} \frac{dz}{H}$$

Late  $H(z)$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Early versus late-time solutions



$H_0 \uparrow$

$\theta_* \downarrow$

acoustic scale: overall position of the peaks

$$\theta_* = \frac{r_s(z_*)}{d_A(z_*)}$$

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{dz}{H} c_s$$

$$d_A(z_*) = \int_0^{z_*} \frac{dz}{H}$$

Early

Early-time solutions

$r_s(z_*) \downarrow$

Late

Late-time solutions

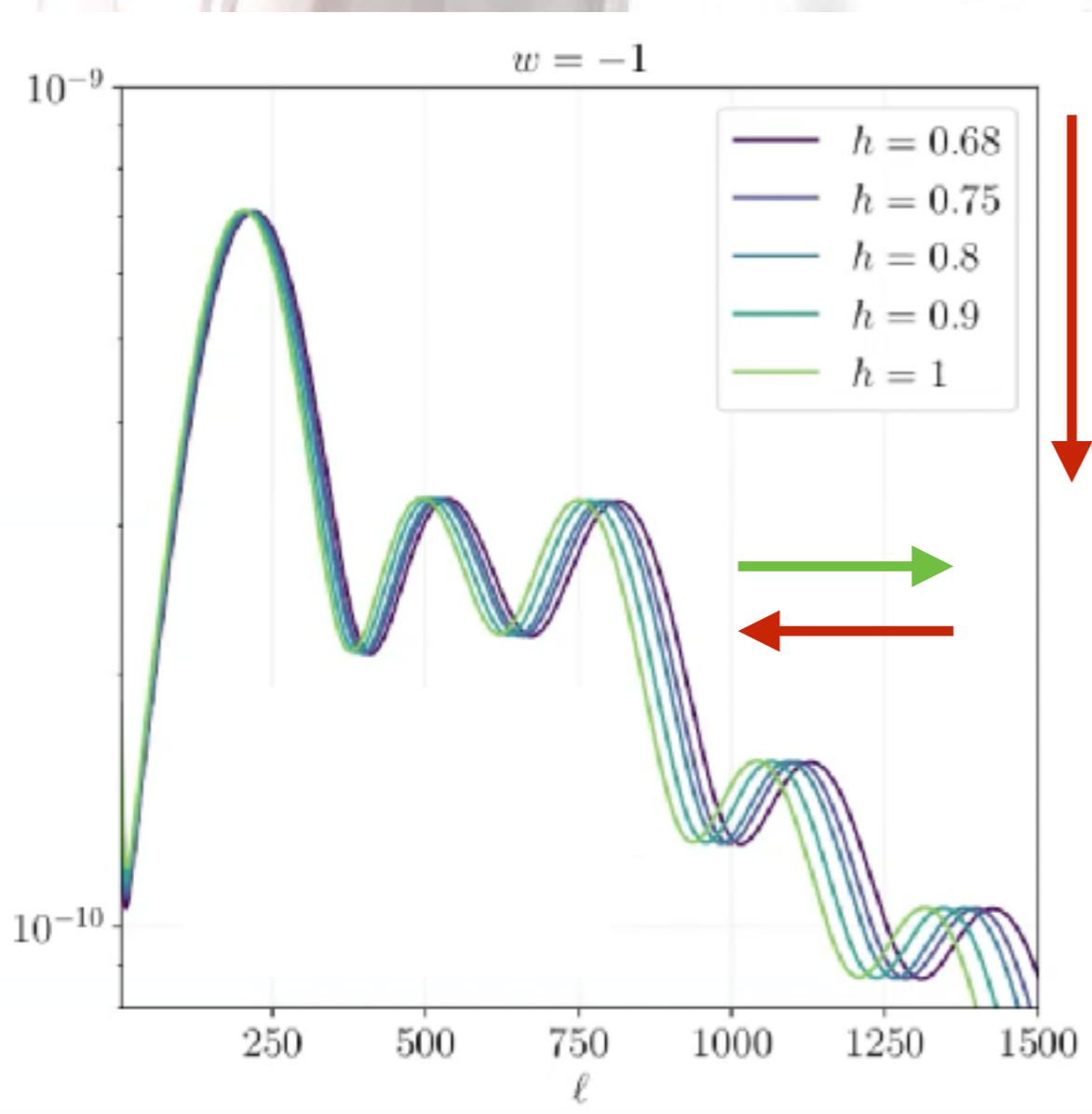
$d_A(z_*) \uparrow$

L.H & H. Villarrubia Rojo,

J. Zosso,

arxiv:2201.11623

# Early versus late-time solutions



$H_0 \uparrow$

$\theta_* \downarrow$

acoustic scale: overall position of the peaks

$$\theta_* = \frac{r_s(z_*)}{d_A(z_*)}$$

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{dz}{H} c_s$$

$$d_A(z_*) = \int_0^{z_*} \frac{dz}{H}$$

$r_s(z_*) \downarrow$

$d_A(z_*) \uparrow$

Early

Early-time solutions

Late

Late-time solutions

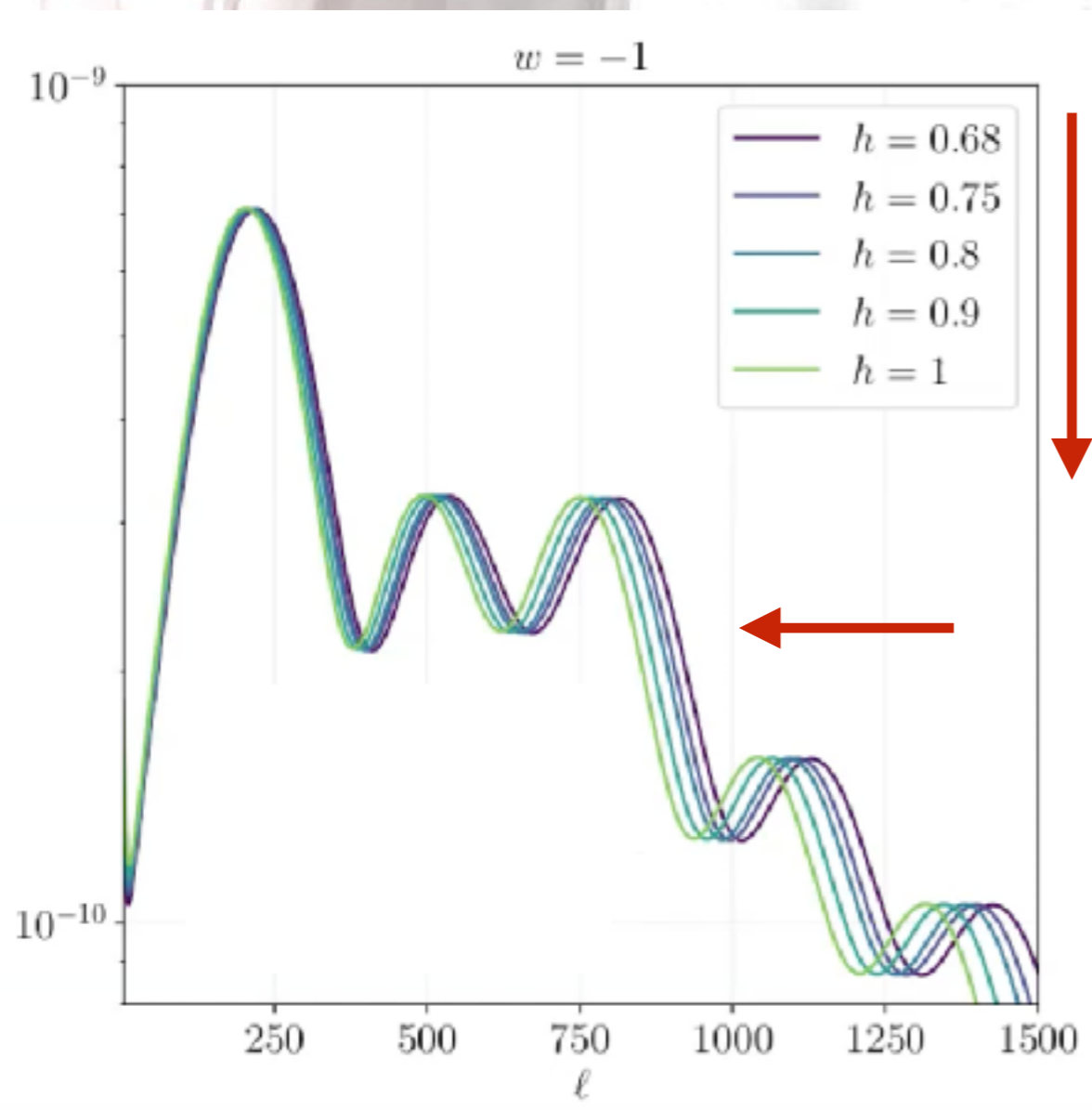
L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623





# Late-time solutions

# Late-time solutions

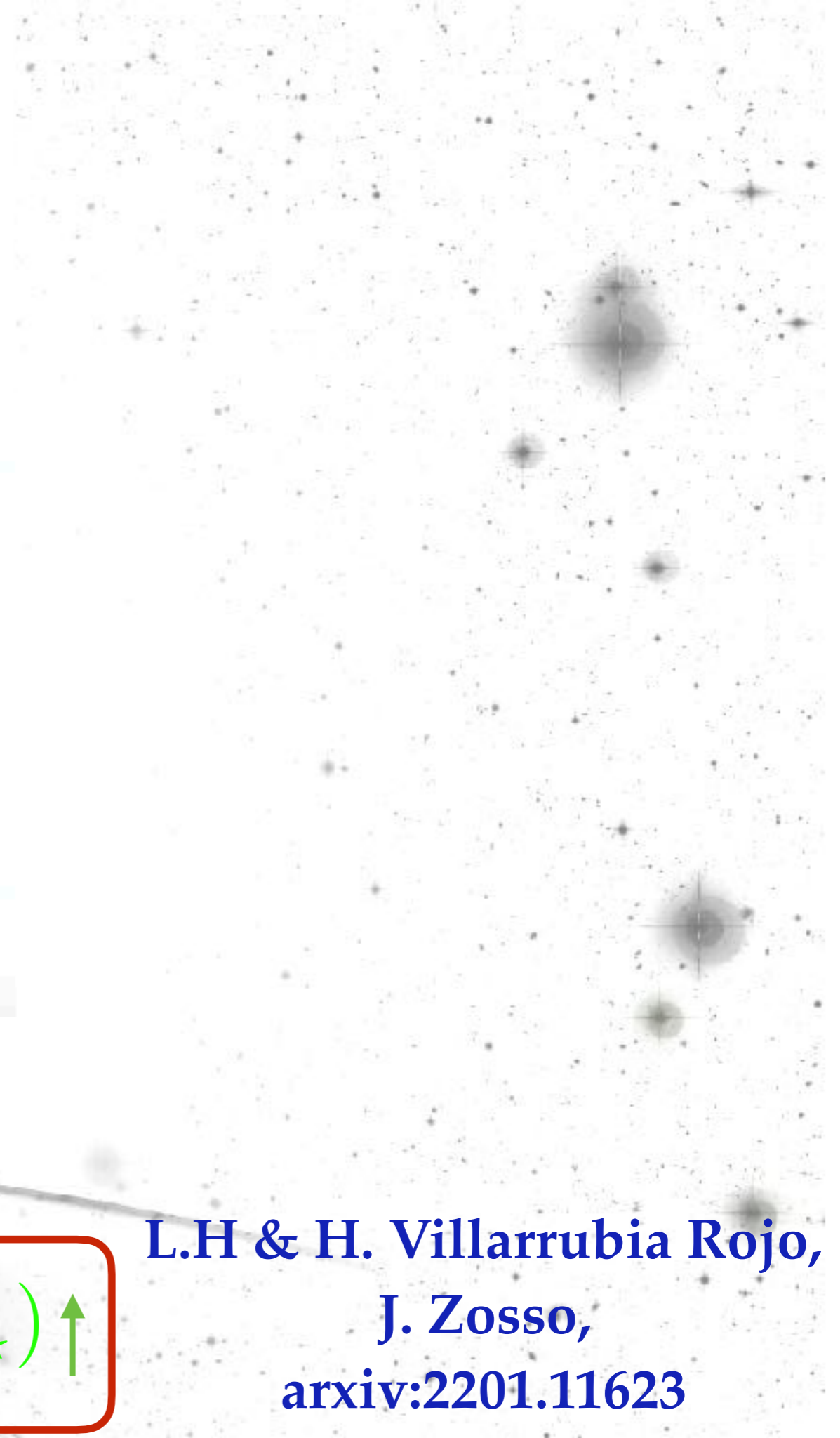
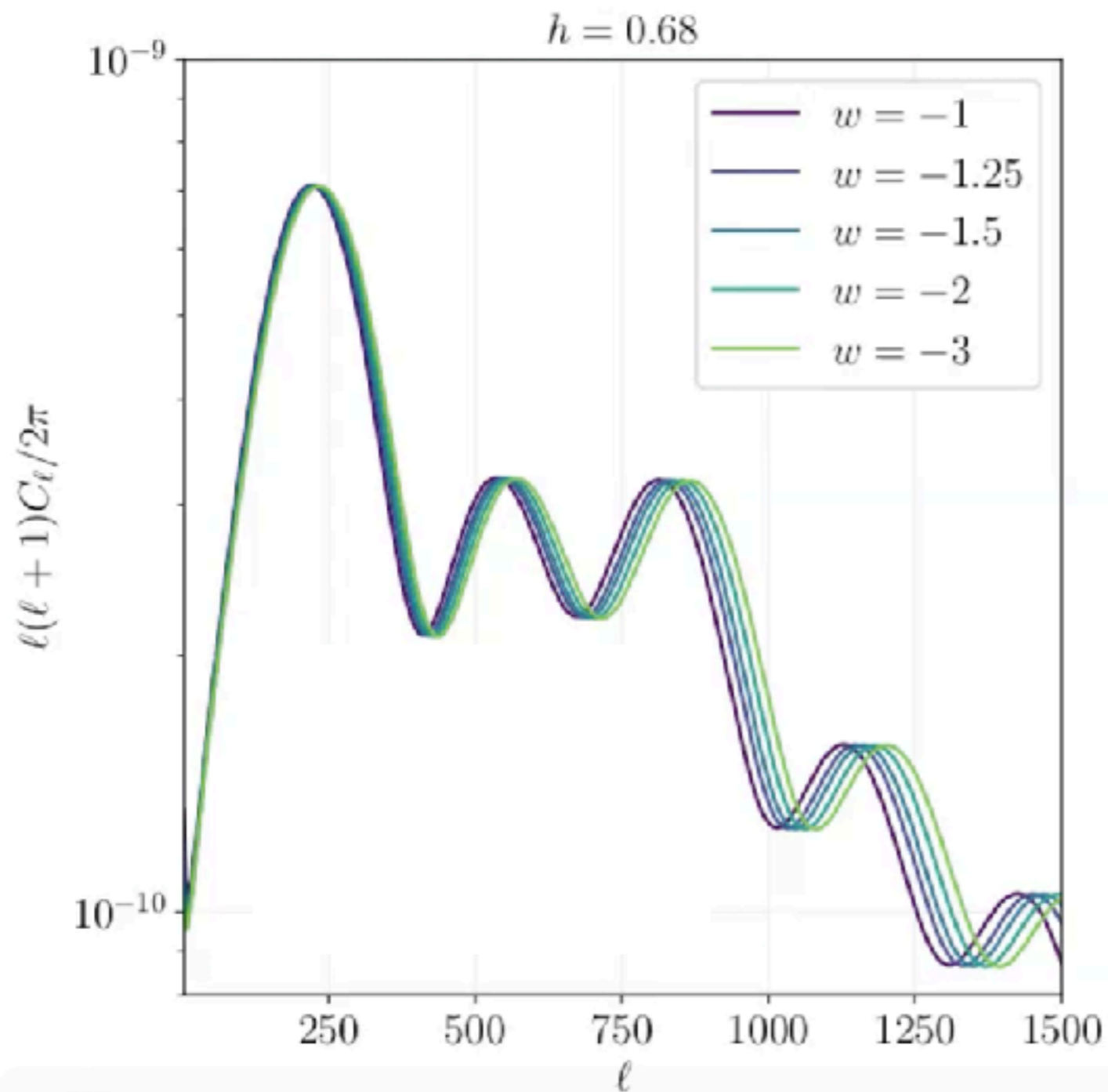


L.H. & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

Late

Late-time solutions

$d_A(z_*) \uparrow$

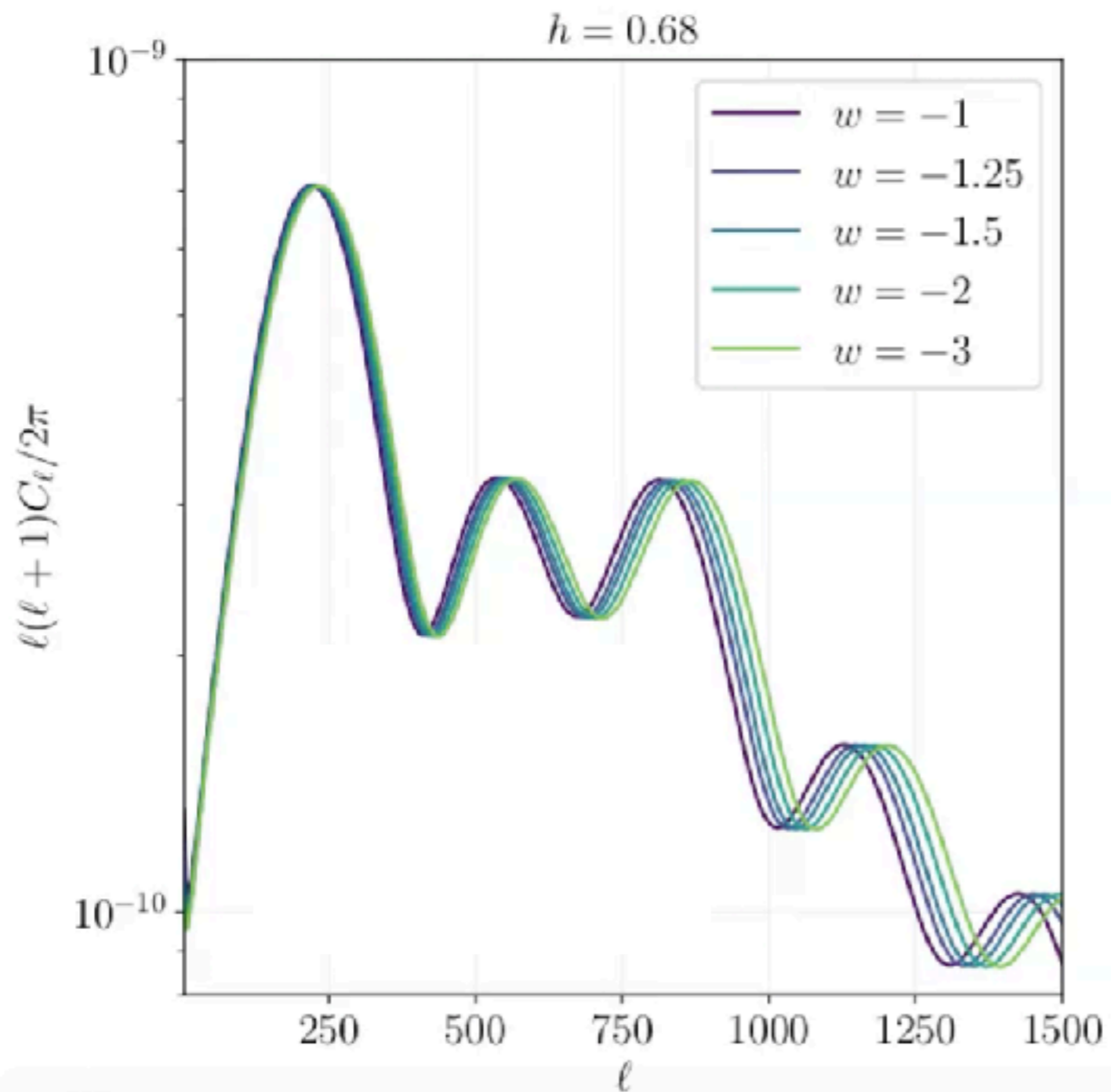


Late

Late-time solutions

$d_A(z_*) \uparrow$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

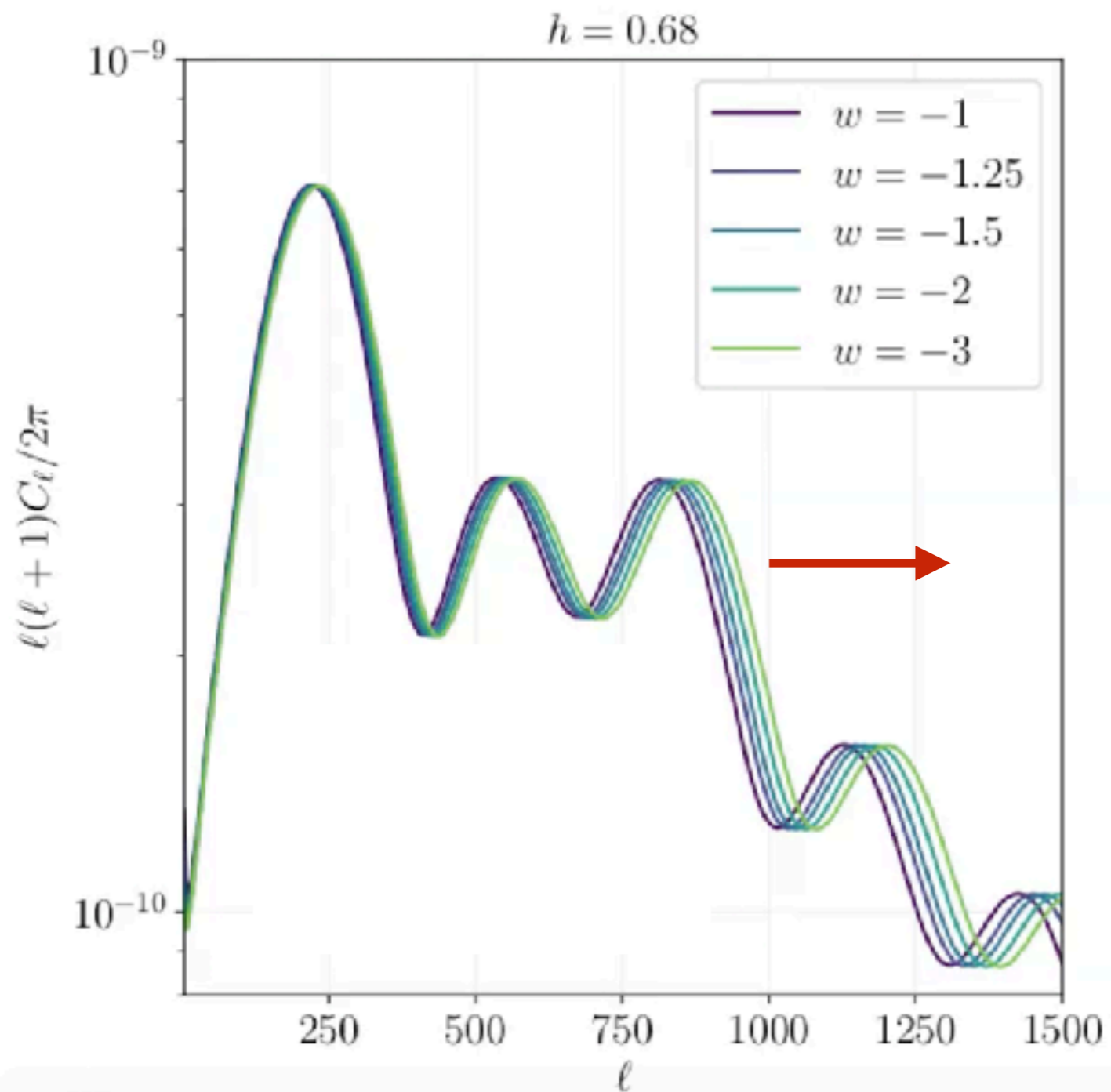


L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

Late

Late-time solutions

$d_A(z_*) \uparrow$



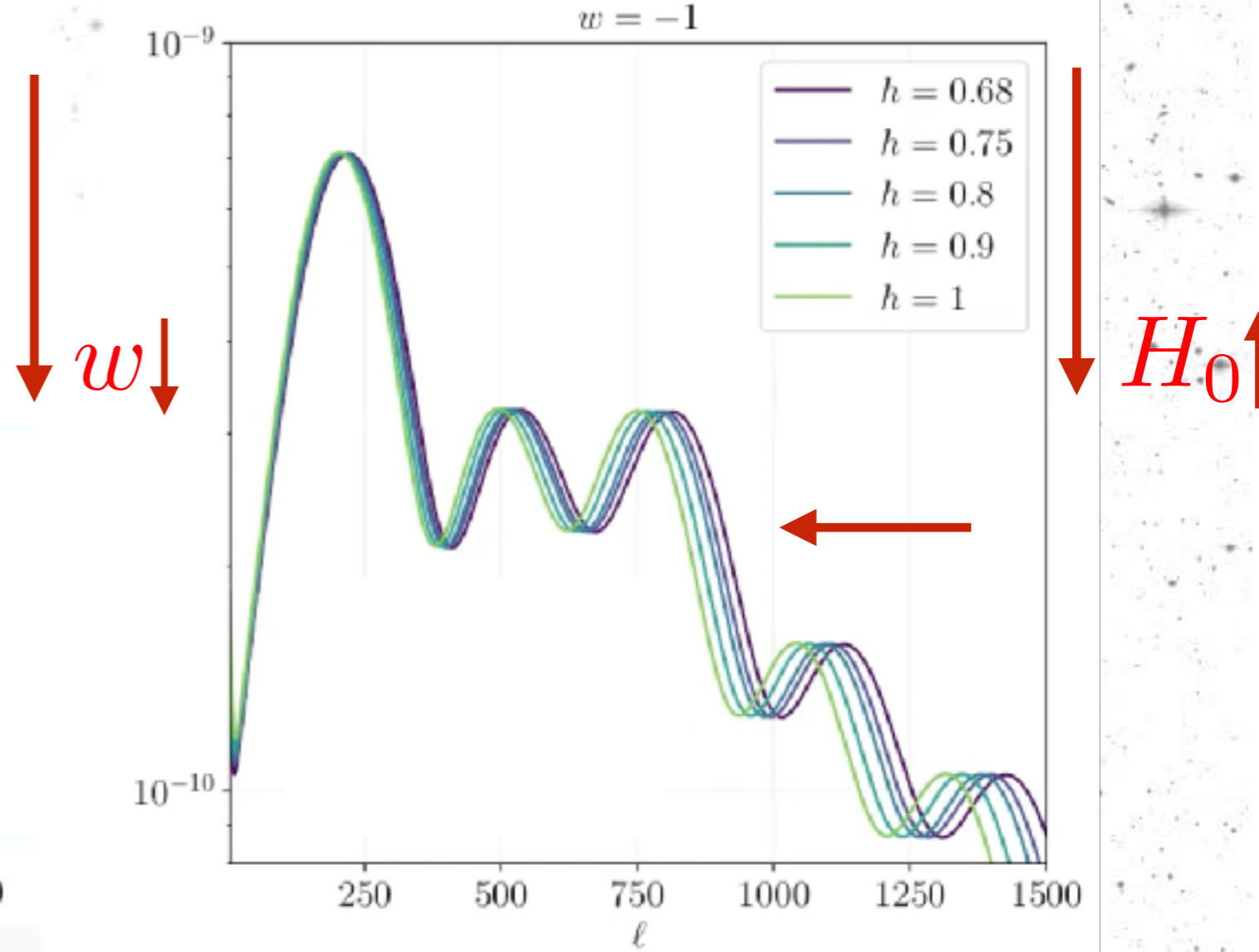
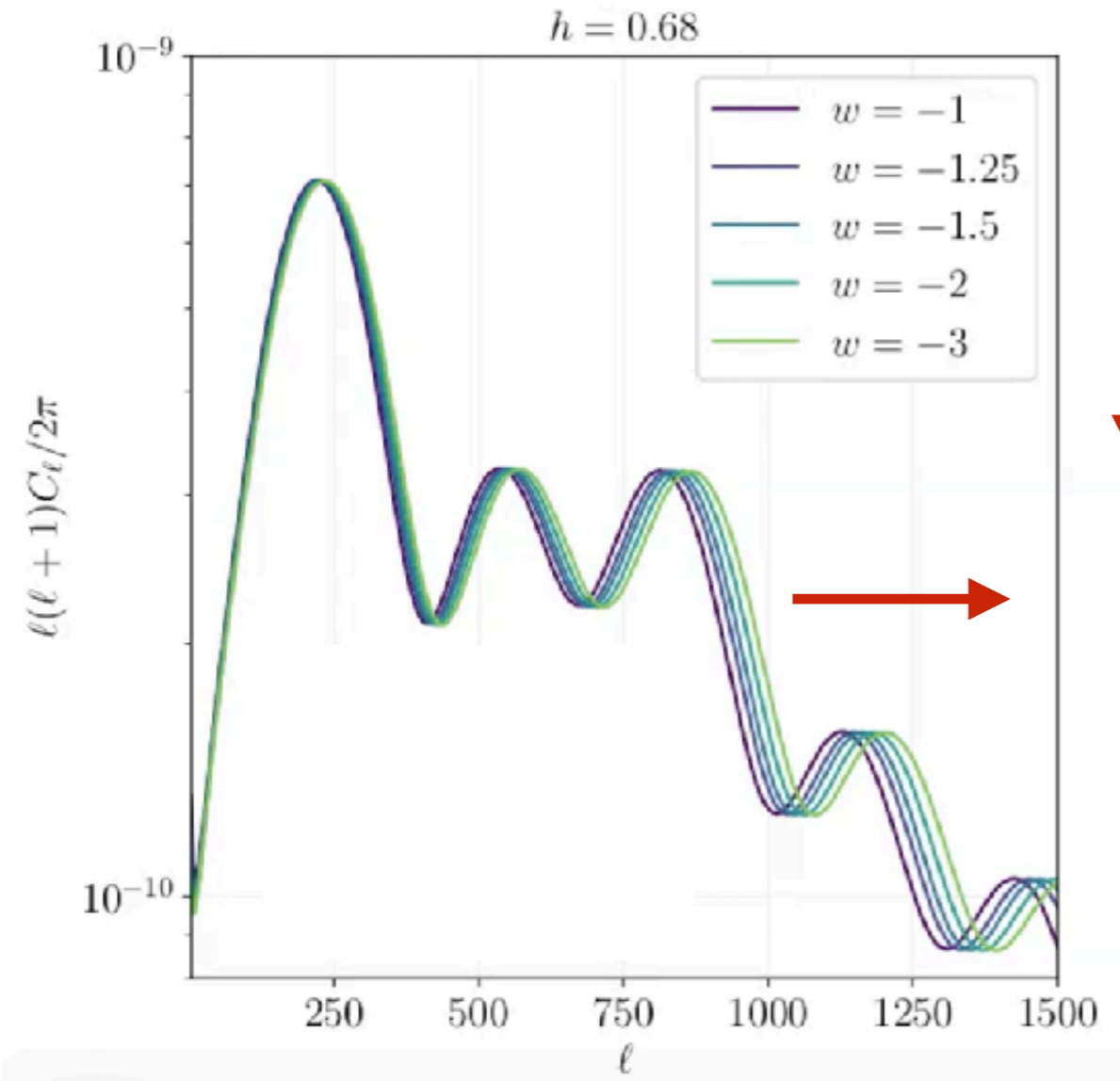
$w \downarrow$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

Late

Late-time solutions

$d_A(z_*) \uparrow$

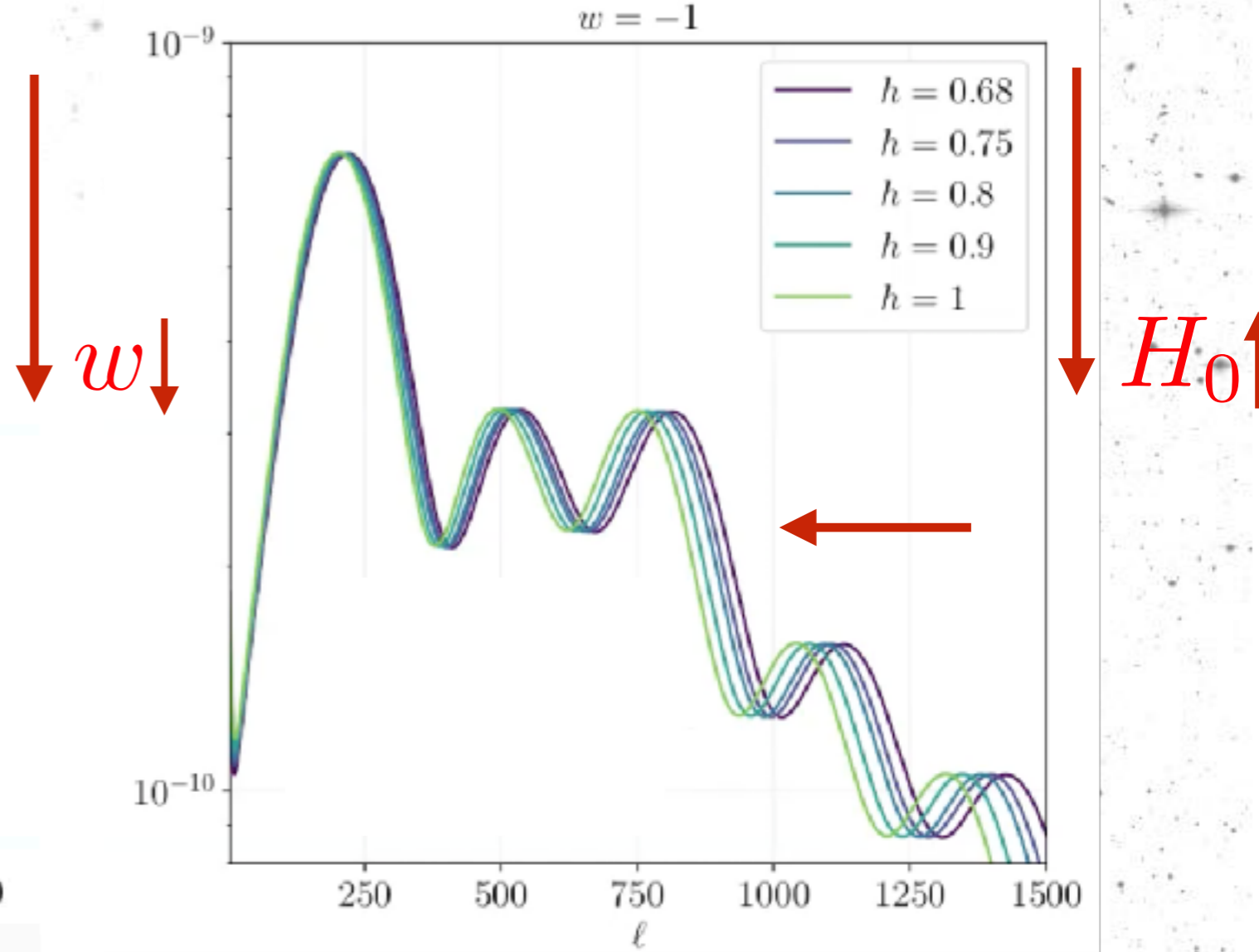
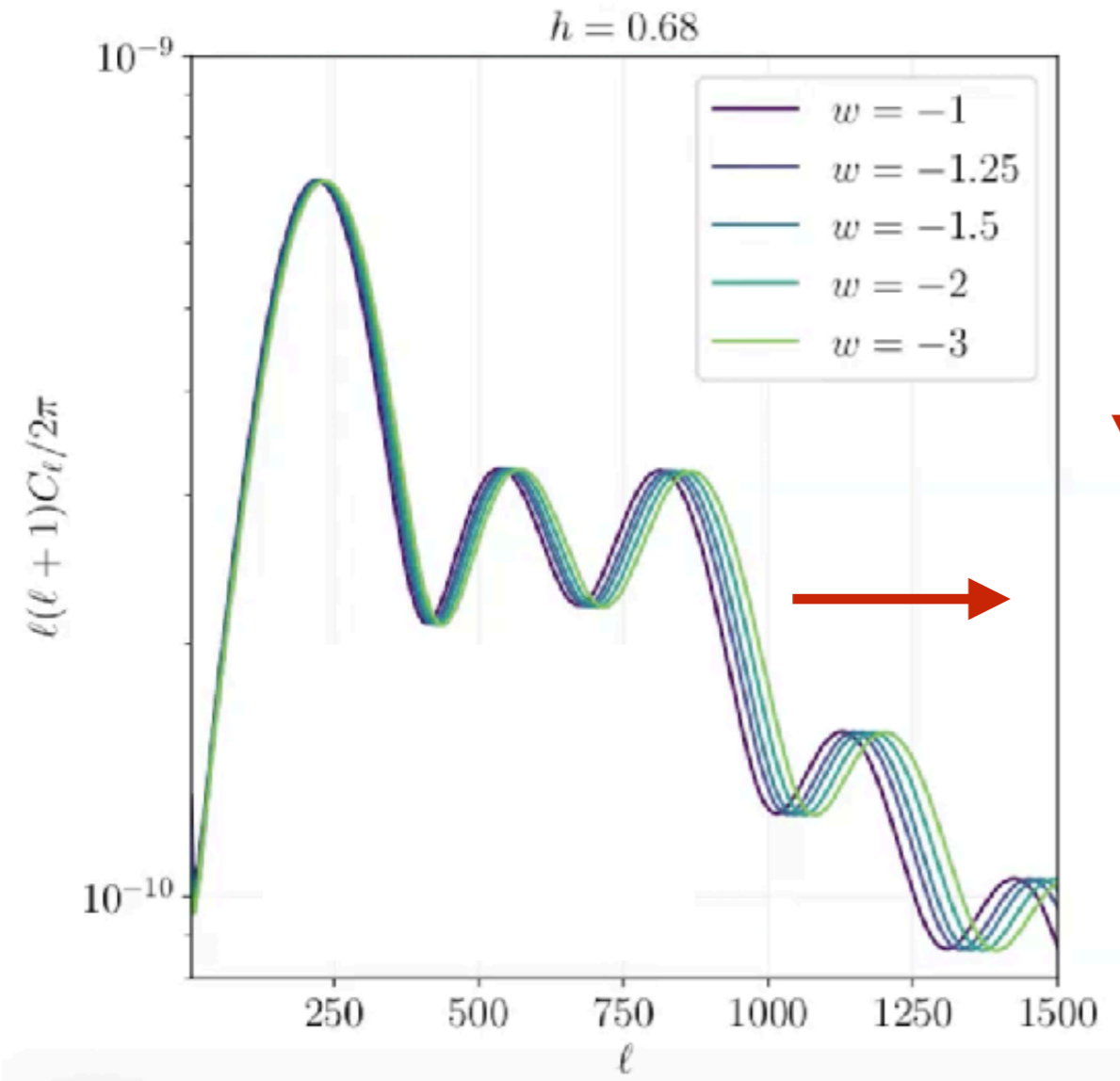


Late

Late-time solutions

$d_A(z_*) \uparrow$

L.H & H. Villarrubia Rojo,  
 J. Zosso,  
 arxiv:2201.11623



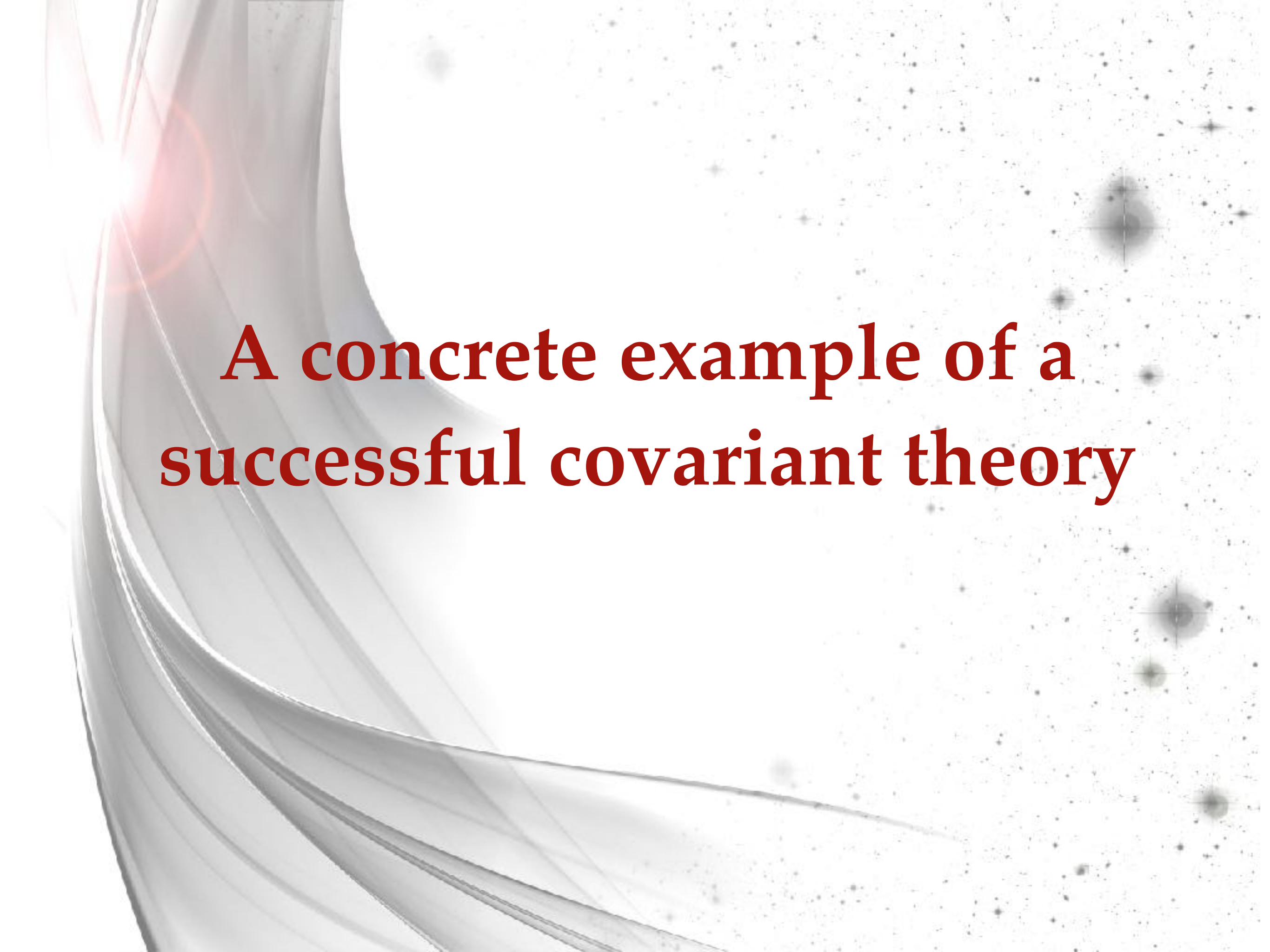
**You need phantom DE!**  $w < -1$

Late

Late-time solutions

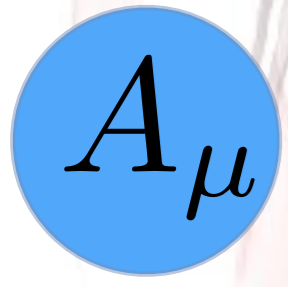
$d_A(z_*) \uparrow$

L.H & H. Villarrubia Rojo,  
 J. Zosso,  
 arxiv:2201.11623

The background features a dynamic composition of flowing, translucent grey and white lines on the left side, which curve and sweep across the frame. The right side is filled with a field of small, dark, star-like specks, some of which have a soft, glowing aura, suggesting a cosmic or digital theme.

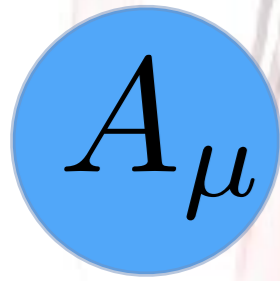
**A concrete example of a  
successful covariant theory**





# Vector Field (Generalized Proca)





# Vector Field (Generalized Proca)

- **second order equations of motion**
- **Lorentz invariant and local**
- **3 propagating degrees of freedom**

$A_\mu$ 

# Vector Field (Generalized Proca)

- second order equations of motion
- Lorentz invariant and local
- 3 propagating degrees of freedom

L. H., JCAP 1405, 015 (2014),  
arXiv:1402.7026

G.Tasinato JHEP 1404 (2014)067  
arXiv:1402.6450

Allys, Peter, Rodriguez, JCAP  
1602 (2016) 02, 004

L.H & J.Beltran,  
Phys.Lett.B757 (2016) 405-411,  
arXiv:1602.03410

$$\mathcal{L}_2 = f_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

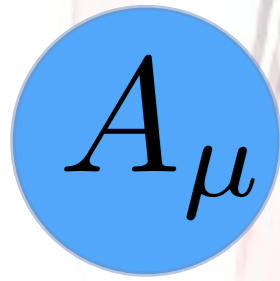
$$\mathcal{L}_3 = f_3(A^2) \partial \cdot A$$

$$\mathcal{L}_4 = f_4(A^2) [(\partial \cdot A)^2 - \partial_\rho A_\sigma \partial^\sigma A^\rho]$$

$$\mathcal{L}_5 = f_5(A^2) [(\partial \cdot A)^3 - 3(\partial \cdot A) \partial_\rho A_\sigma \partial^\sigma A^\rho$$

$$+ 2\partial_\rho A_\sigma \partial^\gamma A^\rho \partial^\sigma A_\gamma] + \tilde{f}_5(A^2) \tilde{F}^{\alpha\mu} \tilde{F}^\beta{}_\mu \partial_\alpha A_\beta$$

$$\mathcal{L}_6 = f_6(A^2) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \partial_\alpha A_\mu \partial_\beta A_\nu$$



# Vector Field (Generalized Proca)

**It was believed that a single vector field is in tension with**

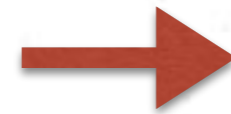
$A_\mu$

# Vector Field (Generalized Proca)

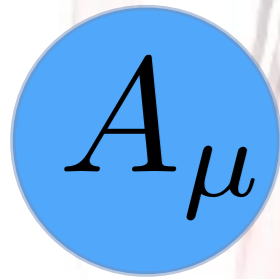
It was believed that a single vector field is in tension with

CP

Cosmological Principle



Homogeneity  
& Isotropy

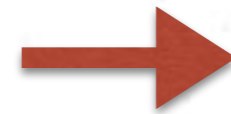


# Vector Field (Generalized Proca)

It was believed that a single vector field is in tension with



Cosmological Principle



Homogeneity  
& Isotropy

and therefore not appropriate for dark energy applications.

# Vector Field (Generalized Proca)



**Generalized Proca initiated a radical change of view!**

# Vector Field (Generalized Proca)



**Generalized Proca initiated a radical change of view!**

$$A_{\mu} = (A_0(t), 0, 0, 0)$$



# Vector Field (Generalized Proca)



Generalized Proca initiated a radical change of view!

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

# Vector Field (Generalized Proca)



Generalized Proca initiated a radical change of view!

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

● Dark Energy fixed point

G.Tasinato JHEP 1404 (2014)067

arXiv:1402.6450

L.H. & de Felice, Kase, Mukohyama,  
Tsujikawa, Zhang, JCAP

1606,2016,06,048, arXiv:1603.05806

# Vector Field (Generalized Proca)

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

# Vector Field (Generalized Proca)

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

- Reduces the  $H_0$  tension and delivers a better fit to data

L.H. & de  
Felice, Tsujikawa,  
PRD95 (2017)12, 123540,  
arXiv:1703.09573

# Vector Field (Generalized Proca)

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

Linear perturbations

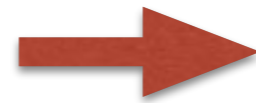
L.H & H. Villarrubia Rojo  
arxiv:2010.00513

# Vector Field (Generalized Proca)

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

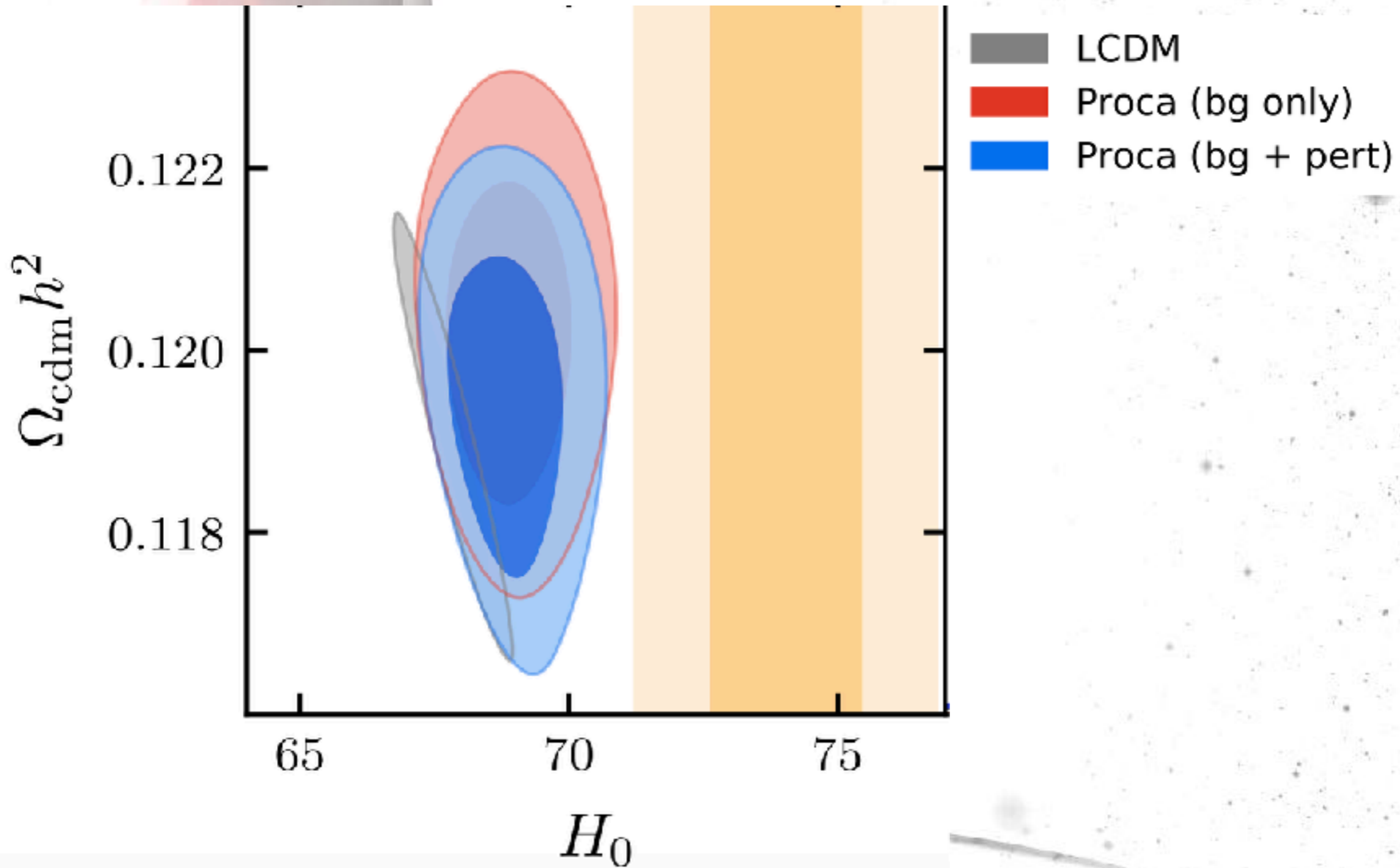
Linear perturbations



Embedding into a  
Boltzman code

L.H & H. Villarrubia Rojo  
arxiv:2010.00513

# Vector Field (Generalized Proca)



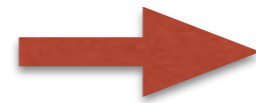
L.H & H. Villarrubia Rojo  
arxiv:2010.00513

# Vector Field (Generalized Proca)

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

Linear perturbations



Embedding into a  
Boltzman code

At the background level, it gives phantom DE  $w < -1$

At the perturbation level, non-trivial contribution from additional dof.

L.H & H. Villarrubia Rojo  
arxiv:2010.00513





$\sigma_8$

**How can we measure**

$\sigma_8$

# Cosmology

Cosmology describes the Universe with 2  
fundamental pillars

GR

General Relativity

CP

Cosmological Principle



Homogeneity  
& Isotropy

# Cosmology

Cosmology describes the Universe with 2  
fundamental pillars

GR

General Relativity

CP

Cosmological Principle



~~Homogeneity  
& Isotropy~~

+ small perturbations

**How can we measure  $\sigma_8$ ?**



# How can we measure $\sigma_8$ ?

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$



$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \quad \text{perturbations}$$

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu} \longrightarrow \{\delta\rho, \delta\vec{P}\}$$

## How can we measure $\sigma_8$ ?

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$



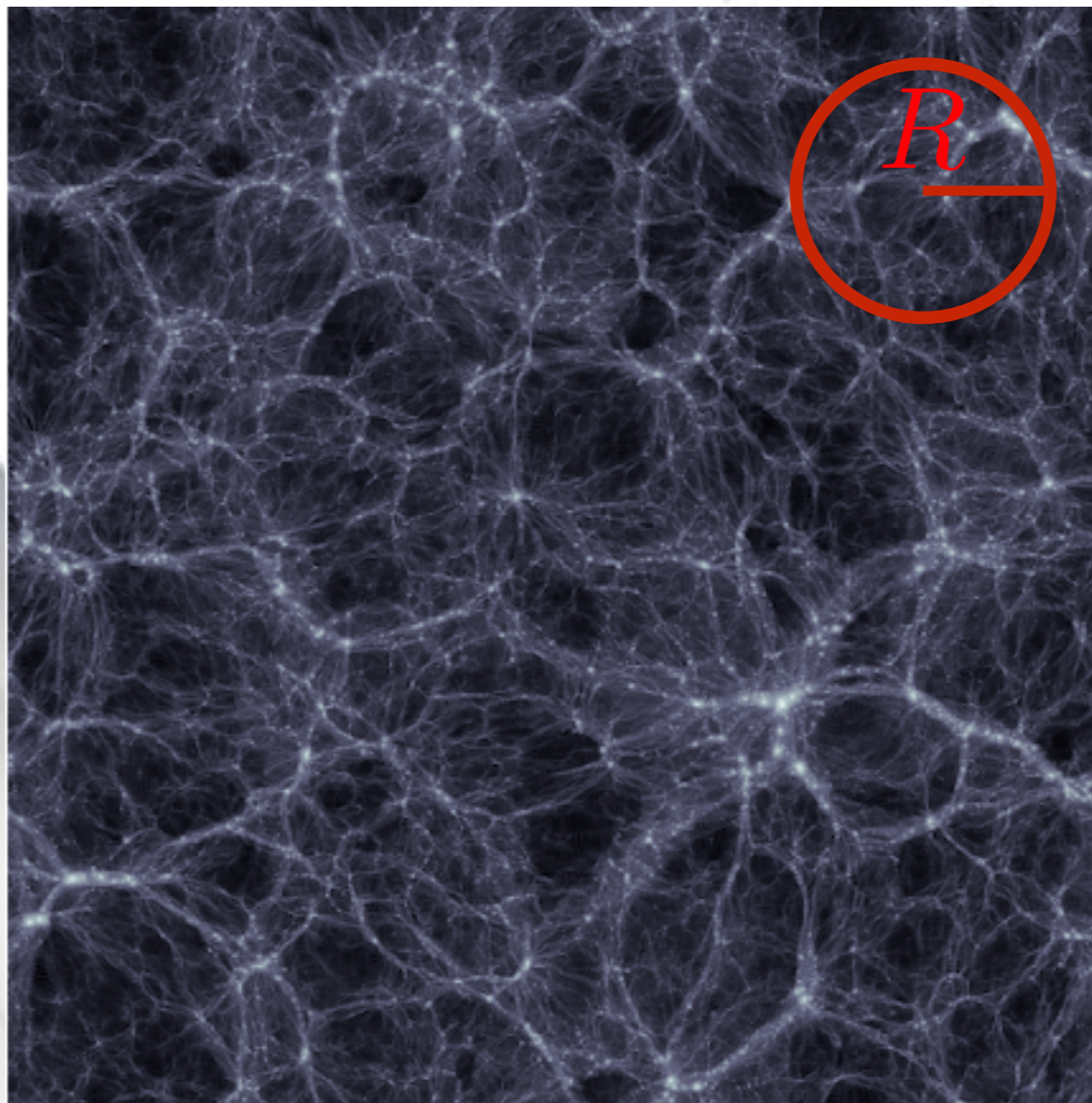
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \quad \text{perturbations}$$

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu} \longrightarrow \{\delta\rho, \delta\vec{P}\}$$

### Matter overdensity

$$\delta_m = \frac{\delta\rho_m - \bar{\rho}_m}{\bar{\rho}_m}$$

How can we measure  $\sigma_8$ ?



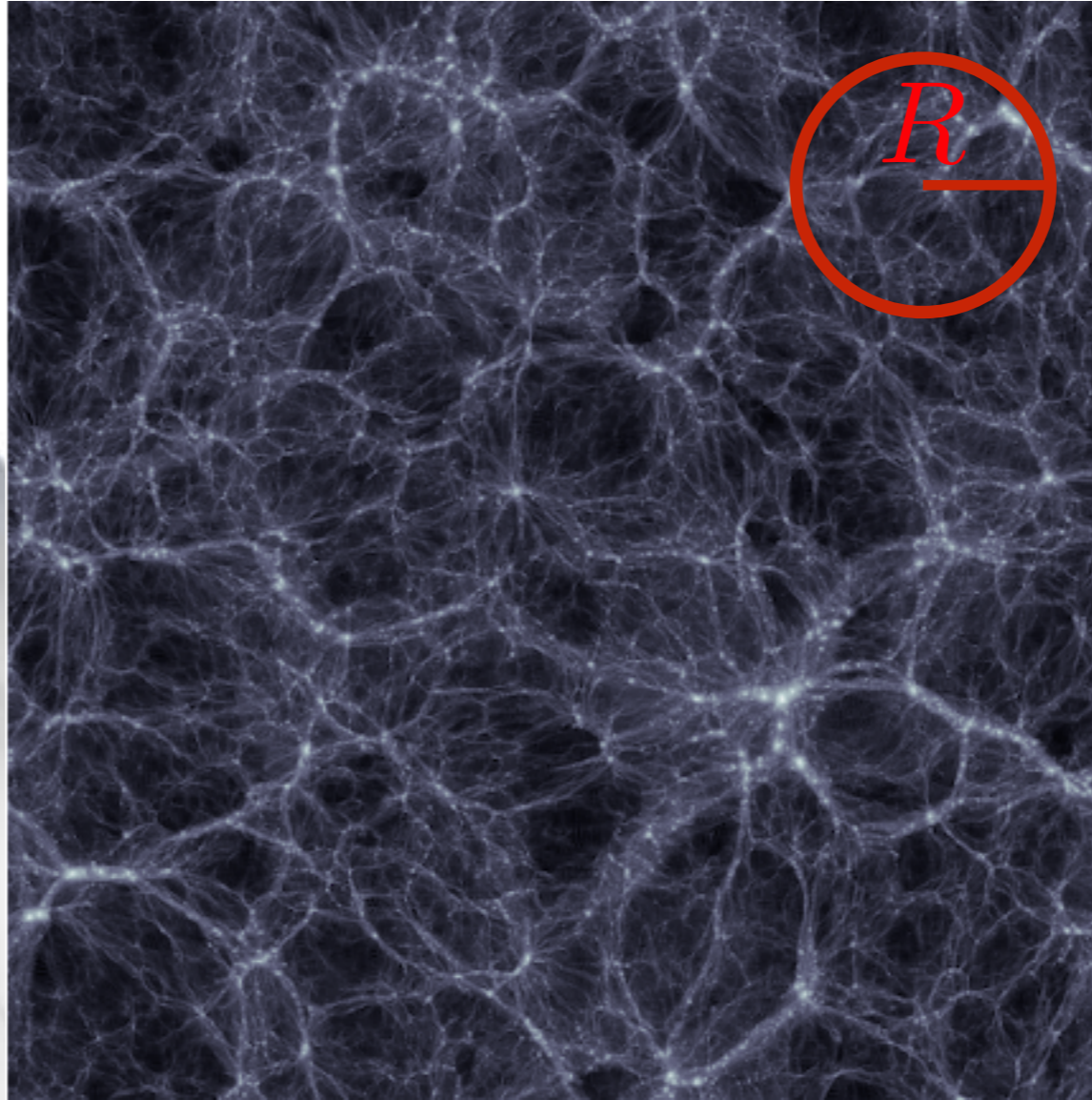
**Matter overdensity**

$$\delta_m = \frac{\rho_m - \bar{\rho}_m}{\bar{\rho}_m}$$

$$\delta_R =$$



How can we measure  $\sigma_8$ ?

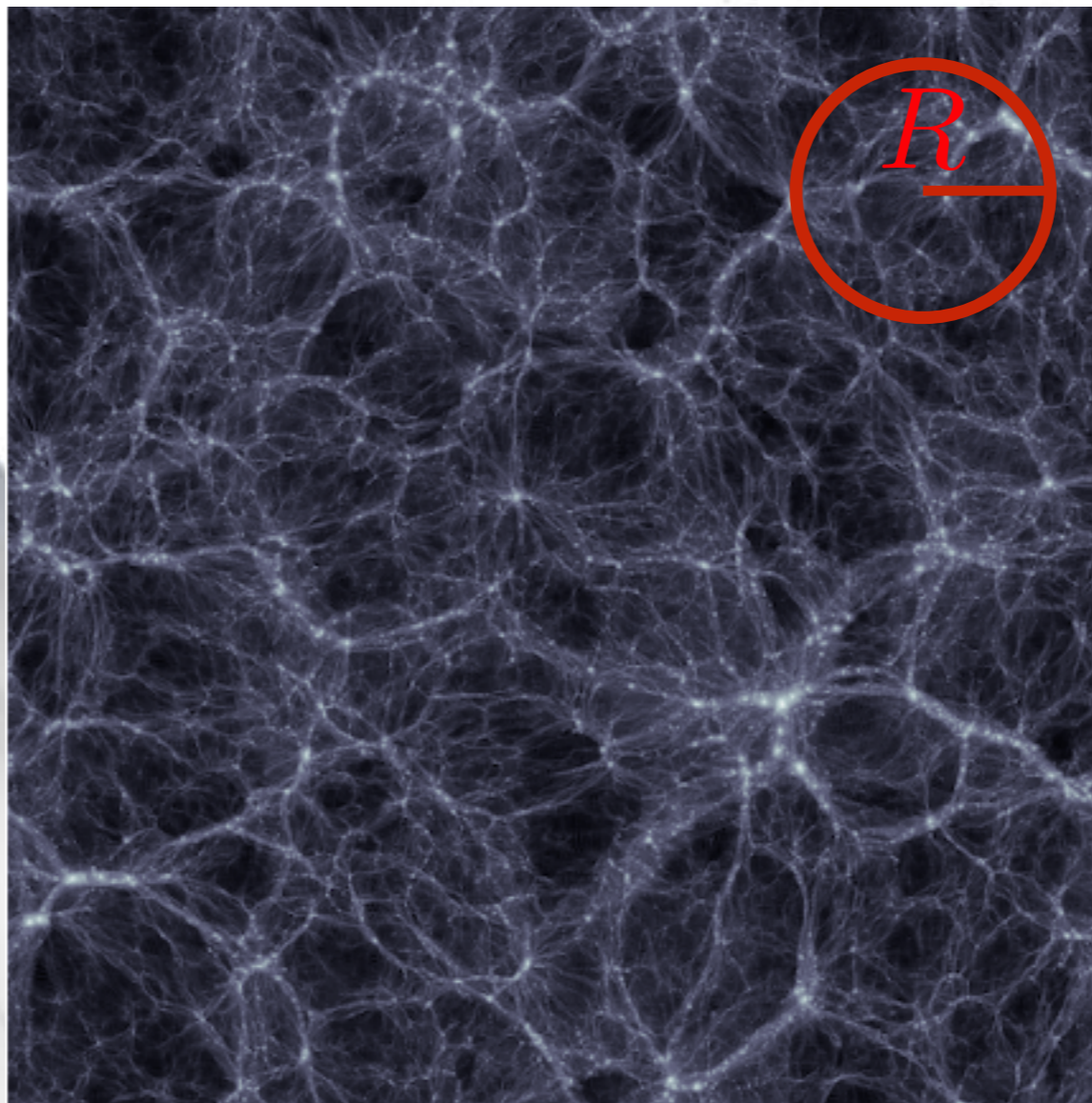


**Matter overdensity**

$$\delta_m = \frac{\delta\rho_m - \bar{\rho}_m}{\bar{\rho}_m}$$

$$\delta_R = \int_{x < R} \frac{d^3x}{V} \delta_m$$

How can we measure  $\sigma_8$ ?

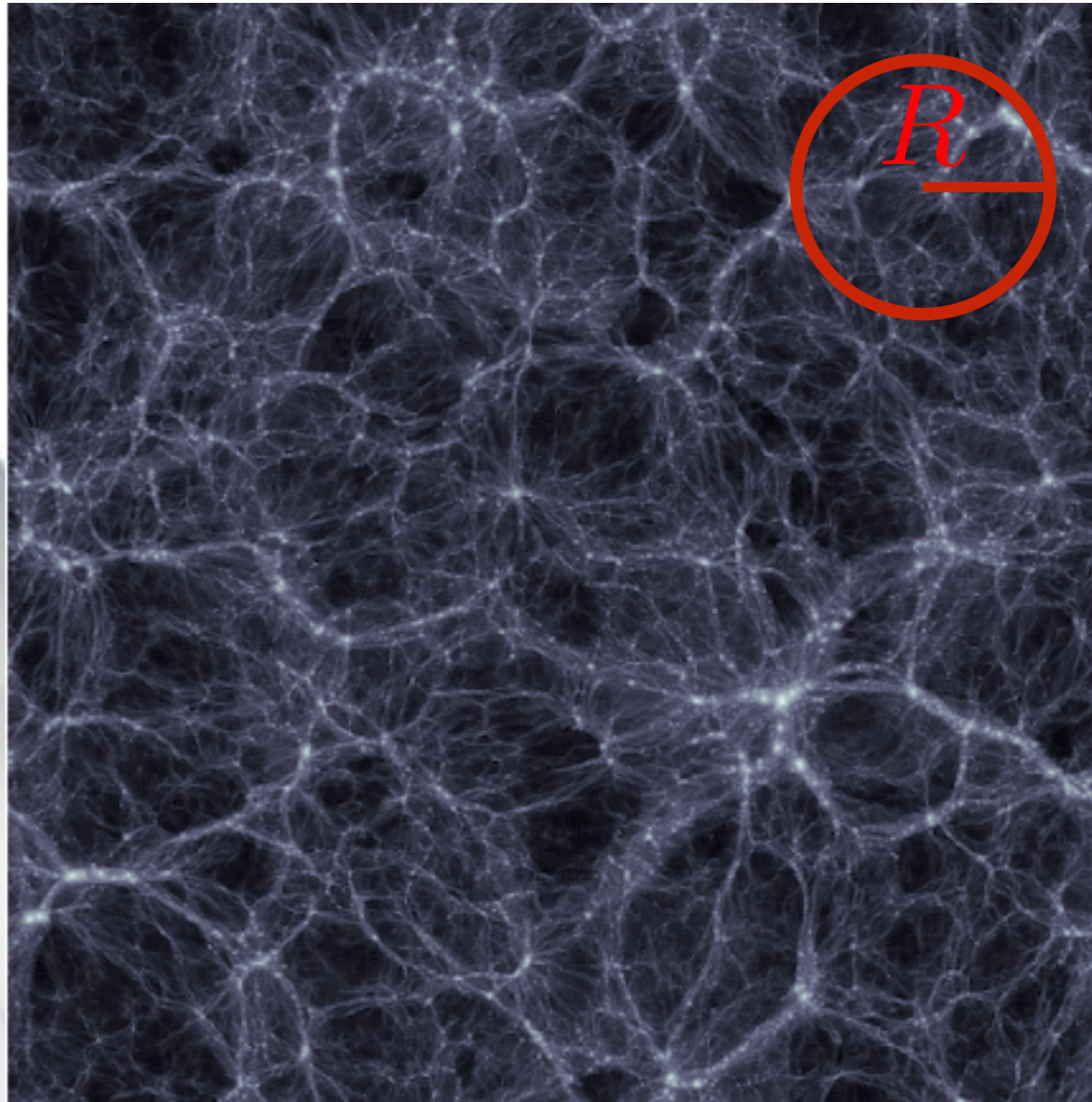


Statistics

$$\langle \delta_R^2(x) \rangle = \sigma_R^2$$

$$\delta_R = \int_{x < R} \frac{d^3 x}{V} \delta_m$$

# How can we measure $\sigma_8$ ?



## Statistics

$$\langle \delta_R^2(x) \rangle = \sigma_R^2$$

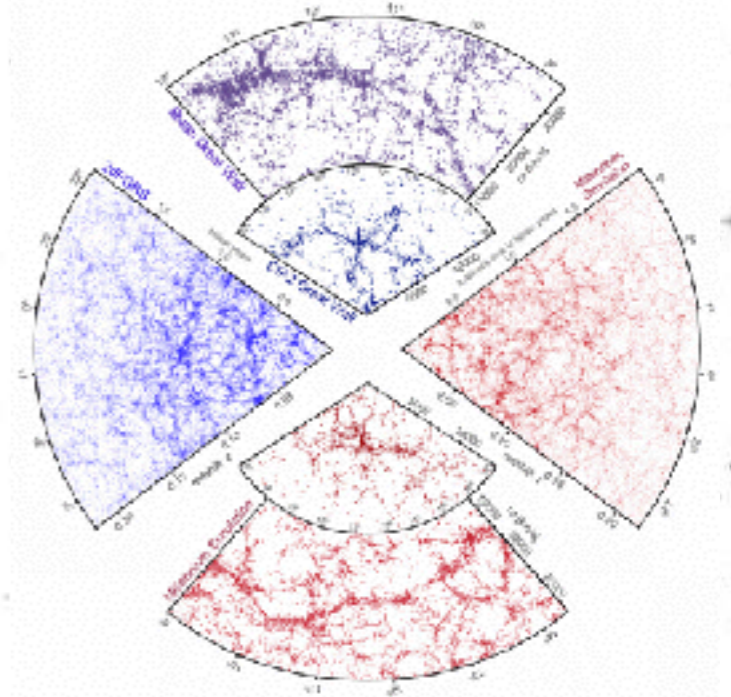
$$R = 8 \text{ Mpc } h^{-1} \longrightarrow \sigma_8$$

$$\delta_R = \int_{x < R} \frac{d^3 x}{V} \delta_m$$

# How can we measure $\sigma_8$ ?



galaxy surveys



Statistics

$$\langle \delta_R^2(x) \rangle = \sigma_R^2$$

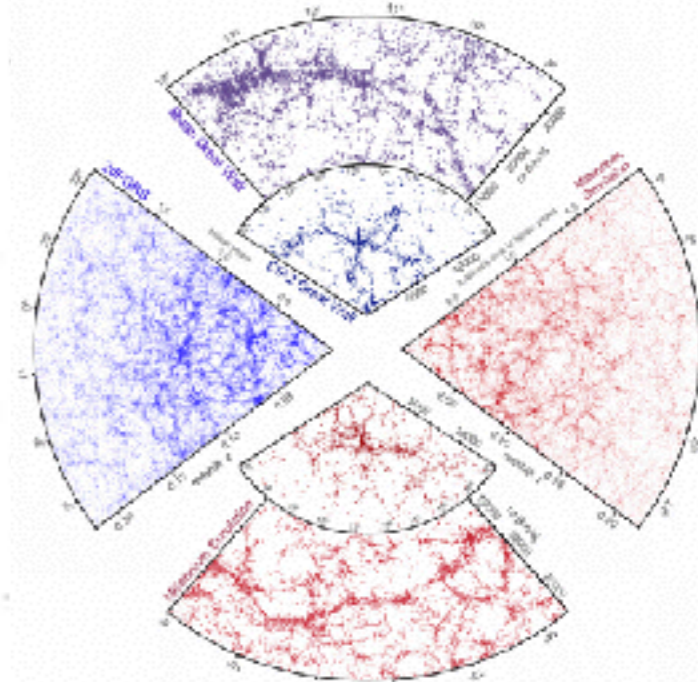
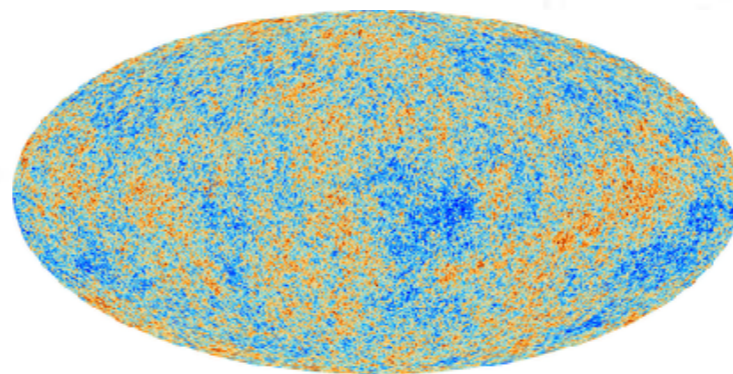
$$R = 8 \text{ Mpc } h^{-1} \longrightarrow \sigma_8$$

$$\delta_R = \int_{x < R} \frac{d^3 x}{V} \delta_m$$

# How can we measure $\sigma_8$ ?

● galaxy surveys

● CMB



$\Lambda$ CDM

$\{H_0, \Omega_b, \Omega_m, A_s, n_s, \tau_{reio}\}$

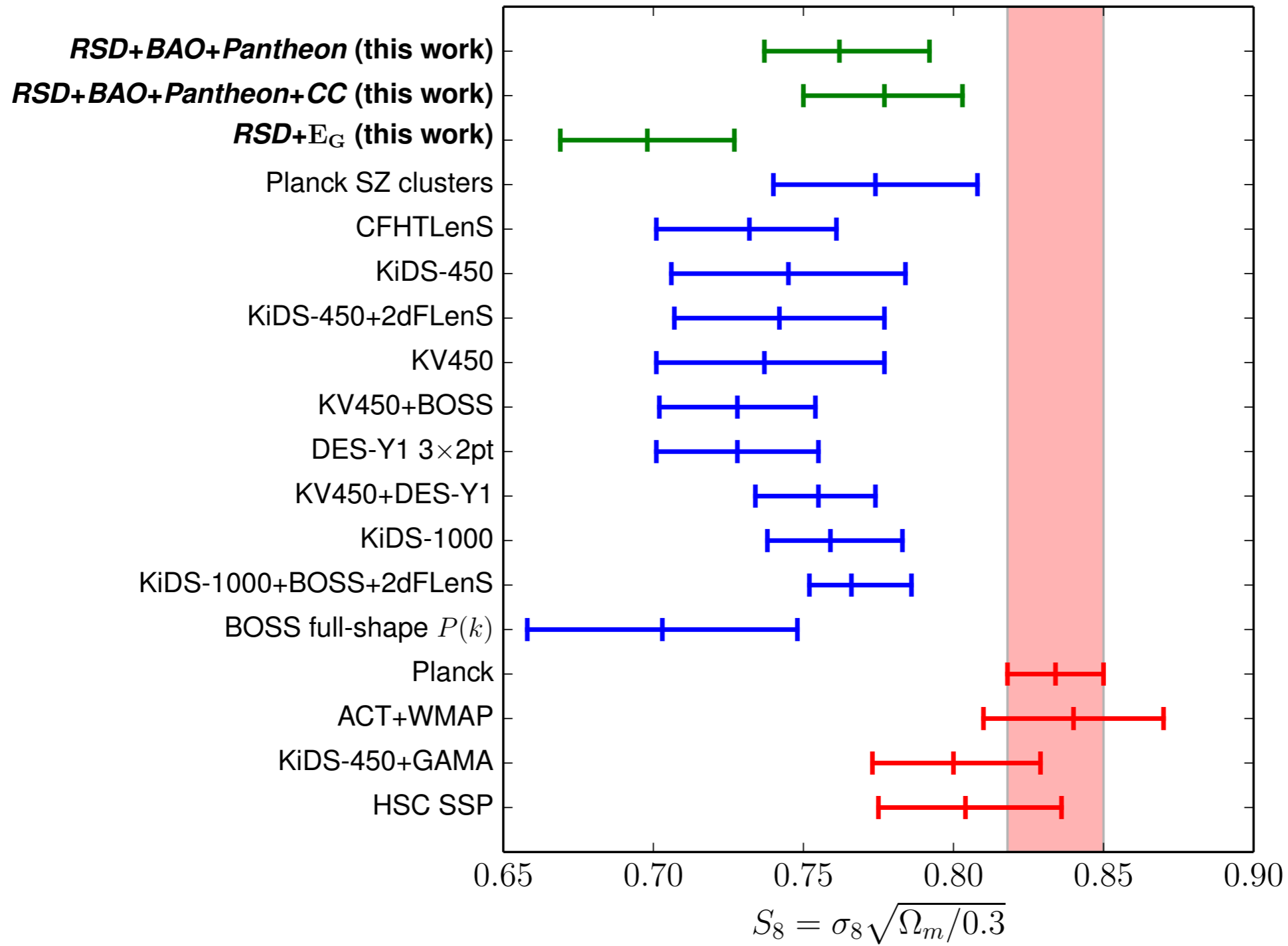
Statistics

$$\langle \delta_R^2(x) \rangle = \sigma_R^2$$

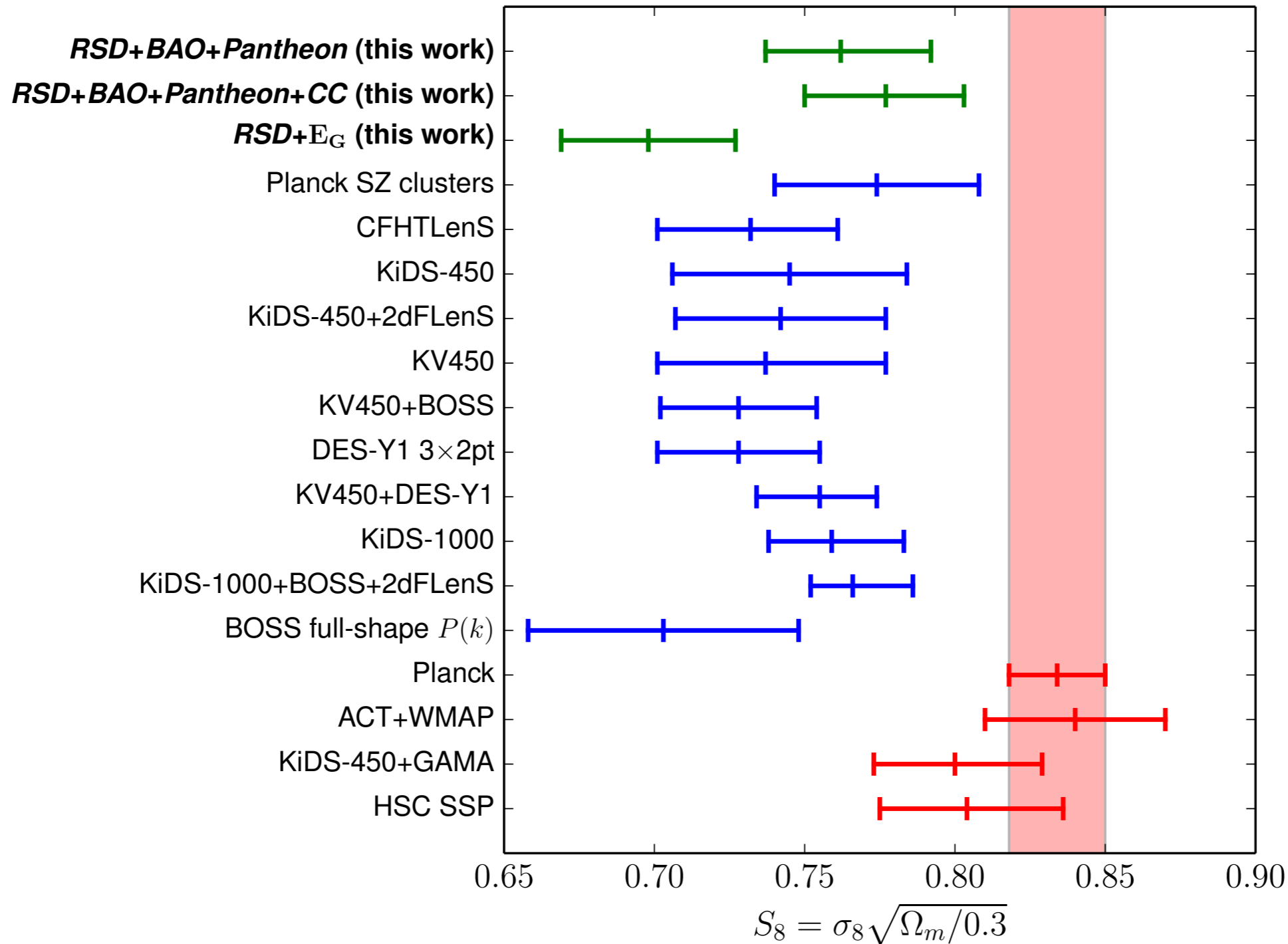
$$R = 8 \text{ Mpc } h^{-1} \longrightarrow \sigma_8$$

$$\delta_R = \int_{x < R} \frac{d^3 x}{V} \delta_m$$

# sigma8 tension



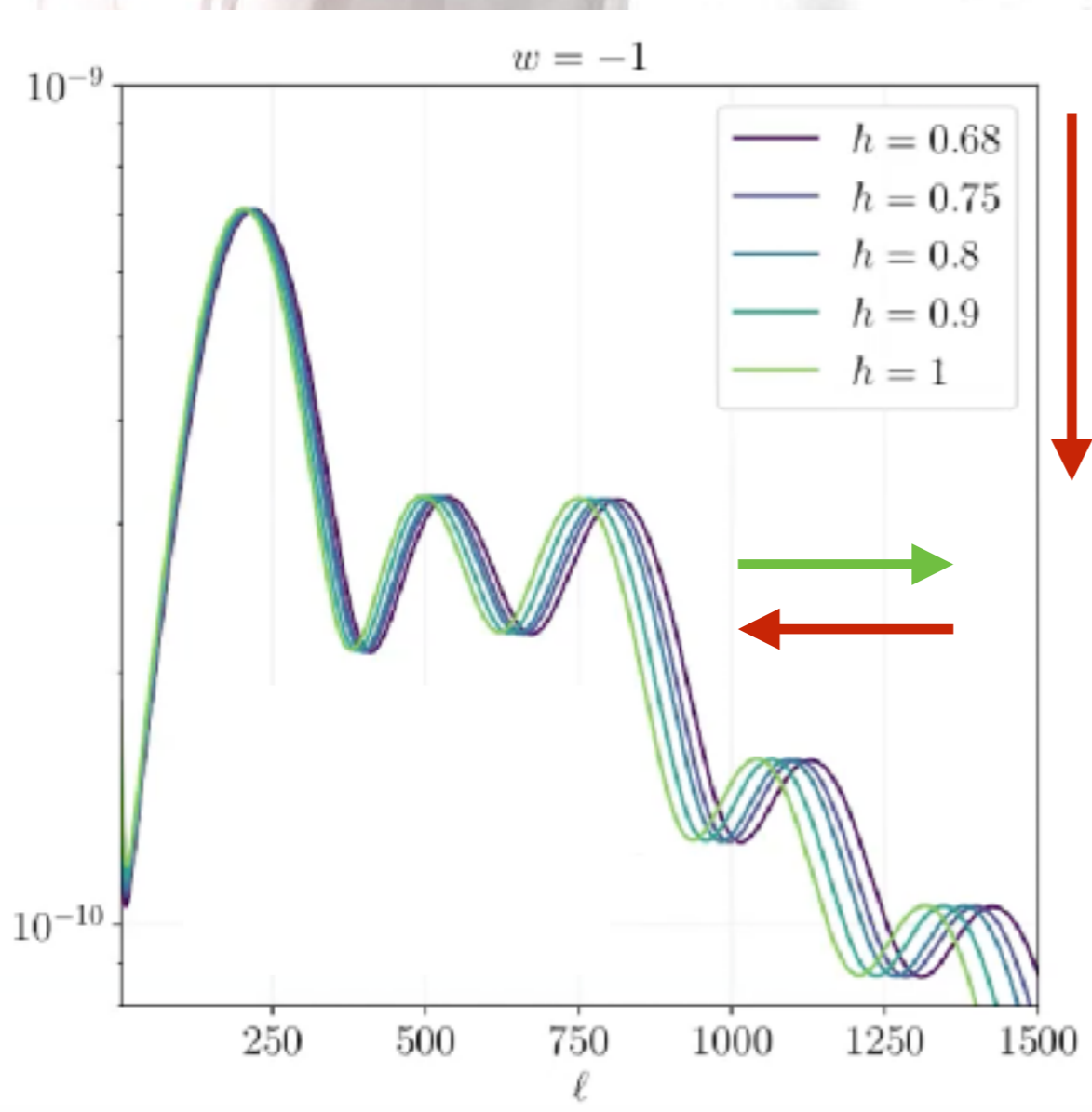
# sigma8 tension



**CMB not only yields a lower  $H_0$   
but also a higher  $\sigma_8$**

**Nunes, Vagnozzi  
arXiv:2106.01208**

# Early versus late-time solutions



$H_0 \uparrow$

$\theta_* \downarrow$

acoustic scale: overall position of the peaks

$$\theta_* = \frac{r_s(z_*)}{d_A(z_*)}$$

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{dz}{H} c_s$$

$$d_A(z_*) = \int_0^{z_*} \frac{dz}{H}$$

Early

Early-time solutions

$r_s(z_*) \downarrow$

Late

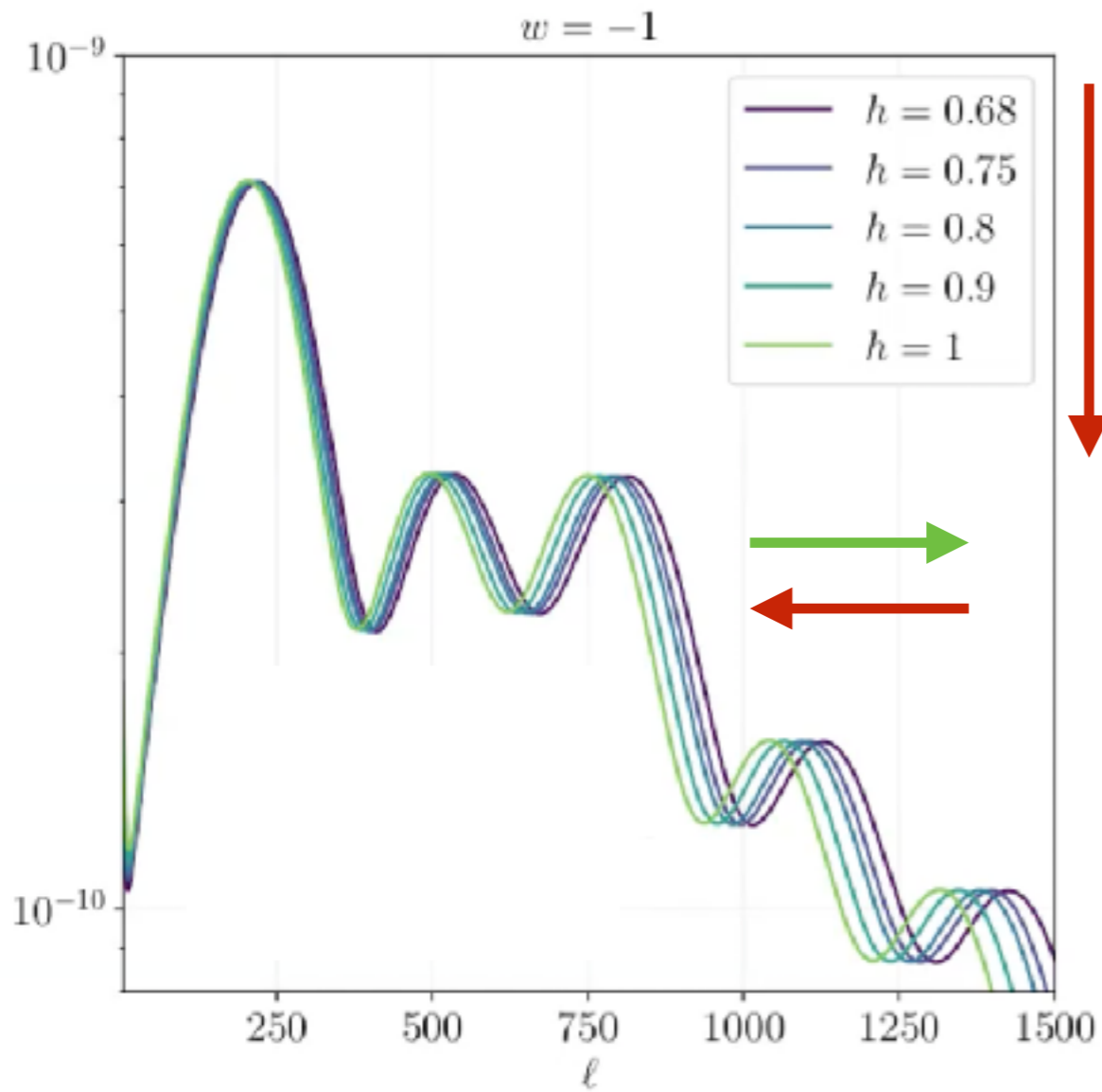
Late-time solutions

$d_A(z_*) \uparrow$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623



# Early versus late-time solutions



acoustic scale: overall position of the peaks

$$\theta_* = \frac{r_s(z_*)}{d_A(z_*)}$$

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{dz}{H} c_s$$

$$d_A(z_*) = \int_0^{z_*} \frac{dz}{H}$$

→ **sigma8 tension usually worsen!**

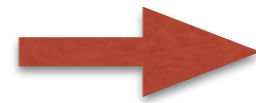
L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Vector Field (Generalized Proca)

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

Linear perturbations



Embedding into a  
Boltzman code

At the background level, it gives phantom DE  $w < -1$

At the perturbation level, non-trivial contribution from additional dof.

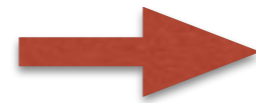
L.H & H. Villarrubia Rojo  
arxiv:2010.00513

# Vector Field (Generalized Proca)

GR

$$+ \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + G_2 \left( -\frac{1}{2} A_\mu A^\mu \right) + G_3 \left( -\frac{1}{2} A_\mu A^\mu \right) \nabla_\alpha A^\alpha \right]$$

Linear perturbations



Embedding into a  
Boltzman code

At the background level, it gives phantom DE  $w < -1$

It does not help with the sigma8  
tension but at least it does not  
worsen it.

L.H & H. Villarrubia Rojo  
arxiv:2010.00513

**Could late-time DE  
models solve  $H_0$  and  
 $\sigma_8$  simultaneously ?**

# Solving $H_0$ and $\sigma_8$ simultaneously

**Goal**

**Solve both tensions simultaneously!**

**Solve them without assuming any model  
nor any parametrization**

**L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623**

# Solving $H_0$ and $\sigma_8$ simultaneously

**Goal**

**Solve both tensions simultaneously!**

**Solve them without assuming any model  
nor any parametrization**

**\$**

**Embedding into a Boltzman code is very expensive!**

**Fully analytically**

**L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623**

# Solving $H_0$ and $\sigma_8$ simultaneously

**Assume:**

Small deviations from a given cosmological background (for example  $\Lambda$ CDM)

$\Lambda$ CDM

$$\{H_0, \Omega_b, \Omega_m, A_s, n_s, \tau_{reio}\}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Assume:**

Small deviations from a given cosmological background (for example  $\Lambda$ CDM)

$\Lambda$ CDM

$$\{H_0, \Omega_b, \Omega_m, A_s, n_s, \tau_{reio}\}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623



# Solving $H_0$ and $\sigma_8$ simultaneously

**Assume:**

Small deviations from a given cosmological background (for example  $\Lambda$ CDM)



$$\{H_0, \Omega_b, \Omega_m, A_s, n_s, \tau_{reio}\}$$

at the background level

$$H = H_{\Lambda\text{CDM}}(z)$$

$$G_{\text{eff}} = G$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Assume:**

Small deviations from a given cosmological background (for example  $\Lambda$ CDM)



$\{H_0, \Omega_b, \Omega_m, A_s, n_s, \tau_{\text{reio}} \dots\}$

at the background level

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

$$G_{\text{eff}} = G + \delta G(z)$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Assume:**

Small deviations from a given cosmological background (for example  $\Lambda$ CDM)



$$\{H_0 + \Delta H_0, \Omega_b + \Delta\Omega_b, \Omega_m + \Delta\Omega_m \dots\}$$

at the background level

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

$$G_{\text{eff}} = G + \delta G(z)$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Consider:** a general deviation from  $\Lambda$ CDM

$$\{H_0 + \Delta H_0, \Omega_m + \Delta\Omega_m, H(z) + \delta H(z) \dots\}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Consider:** a general deviation from  $\Lambda$ CDM

$$\{H_0 + \Delta H_0, \Omega_m + \Delta\Omega_m, H(z) + \delta H(z) \dots\}$$

**Compute:** the variations to first order in the observables

$$\Delta\theta_*, \Delta\sigma_8, \dots$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Consider:** a general deviation from  $\Lambda$ CDM

$$\{H_0 + \Delta H_0, \Omega_m + \Delta\Omega_m, H(z) + \delta H(z) \dots\}$$

**Compute:** the variations to first order in the observables

$$\Delta\theta_*, \Delta\sigma_8, \dots$$

**Relate:** the deviations of the parameters  $\Delta H_0, \Delta\Omega_m, \dots$   
with  $\delta H(z)$  using CMB priors

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Consider:** a general deviation from  $\Lambda$ CDM

$$\{H_0 + \Delta H_0, \Omega_m + \Delta\Omega_m, H(z) + \delta H(z) \dots\}$$

**Compute:** the variations to first order in the observables

$$\Delta\theta_*, \Delta\sigma_8, \dots$$

**Relate:** the deviations of the parameters  $\Delta H_0, \Delta\Omega_m, \dots$   
with  $\delta H(z)$  using CMB priors

**Generate:** response functions

$$\frac{\Delta\sigma_8(z)}{\sigma_8(z)} = \int_0^\infty \frac{dx_z}{1+x_z} \mathcal{R}_{\sigma_8}(x_z, z) \frac{\delta H(x_z)}{H(x_z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

**Consider:** a general deviation from  $\Lambda$ CDM

$$\{H_0 + \Delta H_0, \Omega_m + \Delta\Omega_m, H(z) + \delta H(z) \dots\}$$

**Compute:** the variations to first order in the observables

$$\Delta\theta_*, \Delta\sigma_8, \dots$$

**Relate:** the deviations of the parameters  $\Delta H_0, \Delta\Omega_m, \dots$   
with  $\delta H(z)$  using CMB priors

**Generate:** response functions

$$\frac{\Delta\sigma_8(z)}{\sigma_8(z)} = \int_0^\infty \frac{dx_z}{1+x_z} \mathcal{R}_{\sigma_8}(x_z, z) \frac{\delta H(x_z)}{H(x_z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623



# Solving $H_0$ and $\sigma_8$ simultaneously

**Consider:** a general deviation from  $\Lambda$ CDM

$$\{H_0 + \Delta H_0, \Omega_m + \Delta\Omega_m, H(z) + \delta H(z) \dots\}$$

**Compute:** the variations to first order in the observables

$$\Delta\theta_*, \Delta\sigma_8, \dots$$

**Relate:** the deviations of the parameters  $\Delta H_0, \Delta\Omega_m, \dots$   
with  $\delta H(z)$  using CMB priors

**Generate:** response functions

$$\frac{\Delta\sigma_8(z)}{\sigma_8(z)} = \int_0^\infty \frac{dx_z}{1+x_z} \mathcal{R}_{\sigma_8}(x_z, z) \frac{\delta G(x_z)}{G(x_z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

Observable: the comoving distance

$$d = \int_0^z \frac{dz}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

Observable: the comoving distance

$$d = \int_0^z \frac{dz}{H(z)}$$

The change in the comoving distance

$$\frac{\Delta d(z)}{d(z)} = -\frac{1}{d(z)} \int_0^z d_z \frac{\Delta H(z)}{H(z)^2}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

Observable: the comoving distance

$$d = \int_0^z \frac{dz}{H(z)}$$

The change in the comoving distance

$$\frac{\Delta d(z)}{d(z)} = -\frac{1}{d(z)} \int_0^z d_z \frac{\Delta H(z)}{H(z)^2}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

The change in any observable

$$\frac{\Delta \mathcal{O}(z)}{\mathcal{O}(z)} = I_{\mathcal{O}}(z) \frac{\delta h}{h} + \int_0^{\infty} \frac{dx_z}{1+x_z} R_{\mathcal{O}}(x_z, z) \frac{\delta H(x_z)}{H(x_z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

We need to relate  $\delta h$  with  $\delta H(z)$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

We need to relate  $\delta h$  with  $\delta H(z)$

CMB priors!  $\theta_\star = \frac{r_s(z_\star)}{d_A(z_\star)}$

In order to agree with CMB observations the acoustic scale needs to remain unchanged!

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623



# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

We need to relate  $\delta h$  with  $\delta H(z)$

CMB priors!  $\theta_\star = \frac{r_s(z_\star)}{d_A(z_\star)}$

In order to agree with CMB observations the acoustic scale needs to remain unchanged!

$$\frac{\Delta \theta_\star}{\theta_\star} = \frac{\Delta r_s(z_\star)}{r_s(z_\star)} - \frac{\Delta d_A(z_\star)}{d_A(z_\star)} = 0$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

response function relating  $\delta h$  with  $\delta H(z)$

$$\frac{\delta h}{h} = \int_0^\infty \frac{dz}{1+z} R_h(z) \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

The variation of the Hubble parameter

$$\frac{\Delta H(z)}{H(z)} = \frac{H_0^2}{H^2} \frac{\delta h}{h} + \frac{\delta H(z)}{H(z)}$$

response function relating  $\delta h$  with  $\delta H(z)$

$$\frac{\delta h}{h} = \int_0^\infty \frac{dz}{1+z} R_h(z) \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

Remember, the variation of any observable

$$\frac{\Delta \mathcal{O}(z)}{\mathcal{O}(z)} = I_{\mathcal{O}}(z) \frac{\delta h}{h} + \int_0^{\infty} \frac{dx_z}{1+x_z} R_{\mathcal{O}}(x_z, z) \frac{\delta H(x_z)}{H(x_z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving H0 and sigma8 simultaneously

Remember, the variation of any observable

$$\frac{\Delta \mathcal{O}(z)}{\mathcal{O}(z)} = I_{\mathcal{O}}(z) \frac{\delta h}{h} + \int_0^{\infty} \frac{dx_z}{1+x_z} R_{\mathcal{O}}(x_z, z) \frac{\delta H(x_z)}{H(x_z)}$$



$$\frac{\delta h}{h} = \int_0^{\infty} \frac{dz}{1+z} R_h(z) \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving H0 and sigma8 simultaneously

Remember, the variation of any observable

$$\frac{\Delta \mathcal{O}(z)}{\mathcal{O}(z)} = I_{\mathcal{O}}(z) \frac{\delta h}{h} + \int_0^{\infty} \frac{dx_z}{1+x_z} R_{\mathcal{O}}(x_z, z) \frac{\delta H(x_z)}{H(x_z)}$$



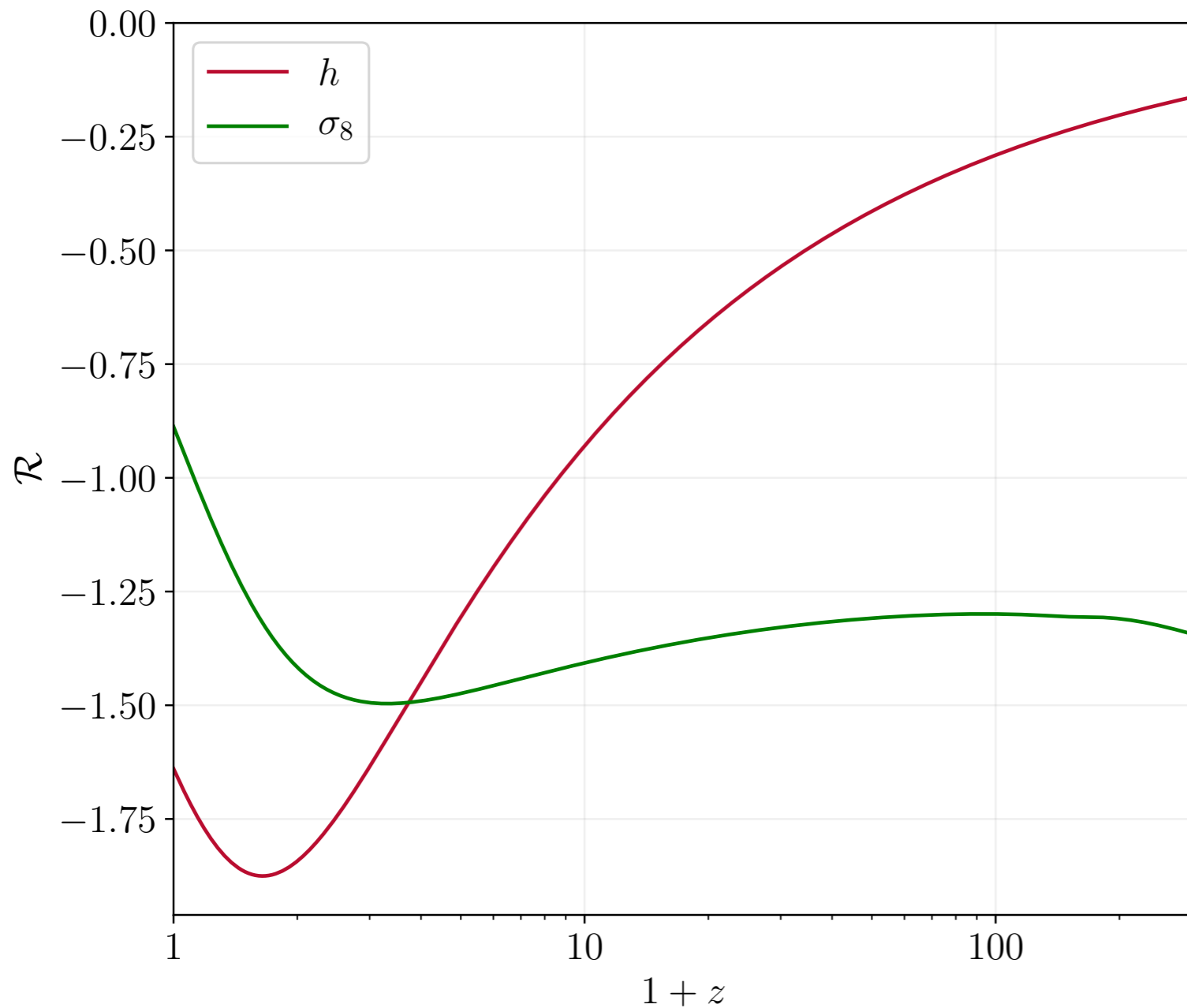
$$\frac{\delta h}{h} = \int_0^{\infty} \frac{dz}{1+z} R_h(z) \frac{\delta H(z)}{H(z)}$$

$$\frac{\Delta \mathcal{O}}{\mathcal{O}} = \int_0^{\infty} \frac{dz}{1+z} R_{\mathcal{O}}(z) \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving H0 and sigma8 simultaneously

Example: the variation of sigma8

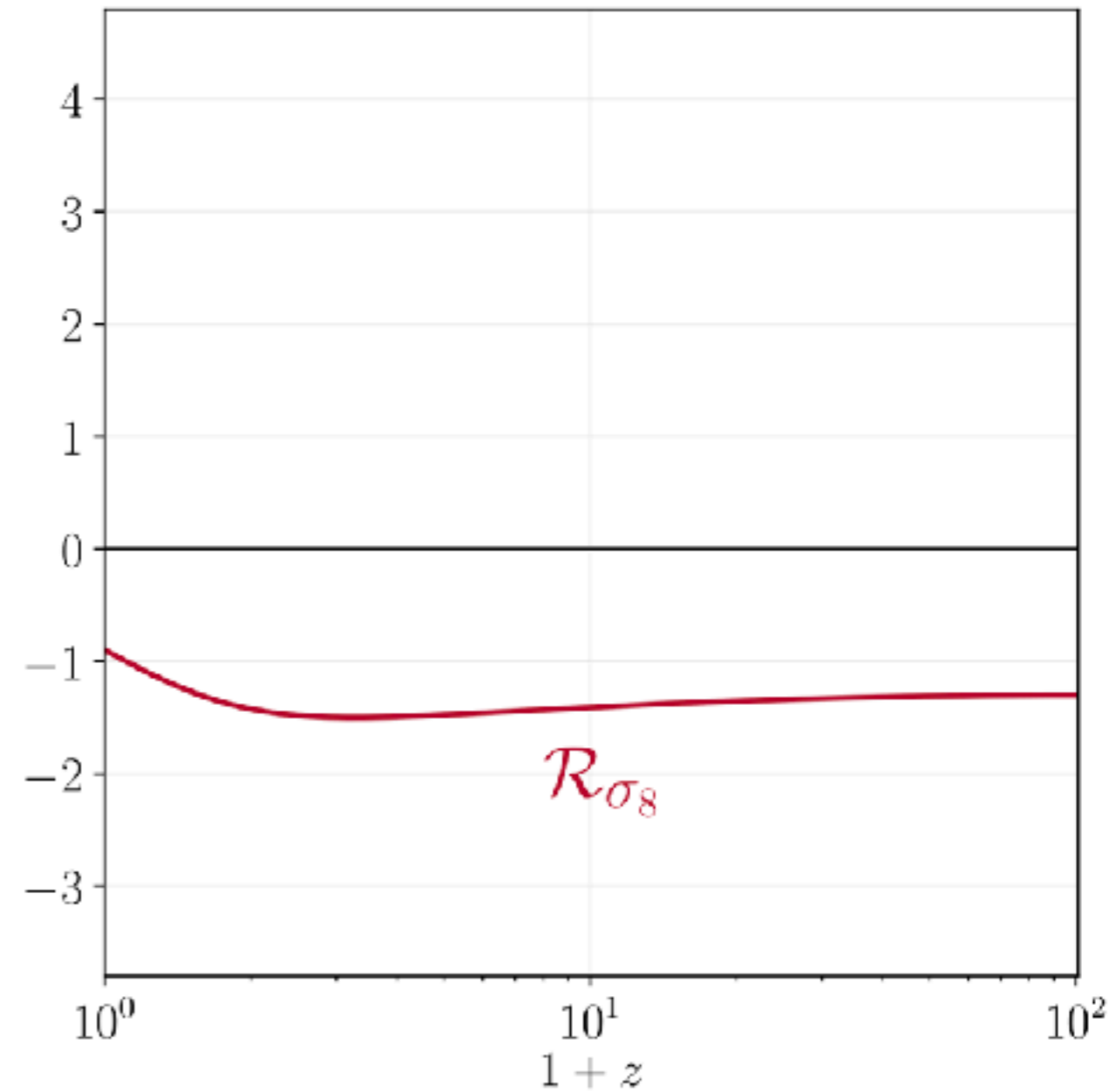
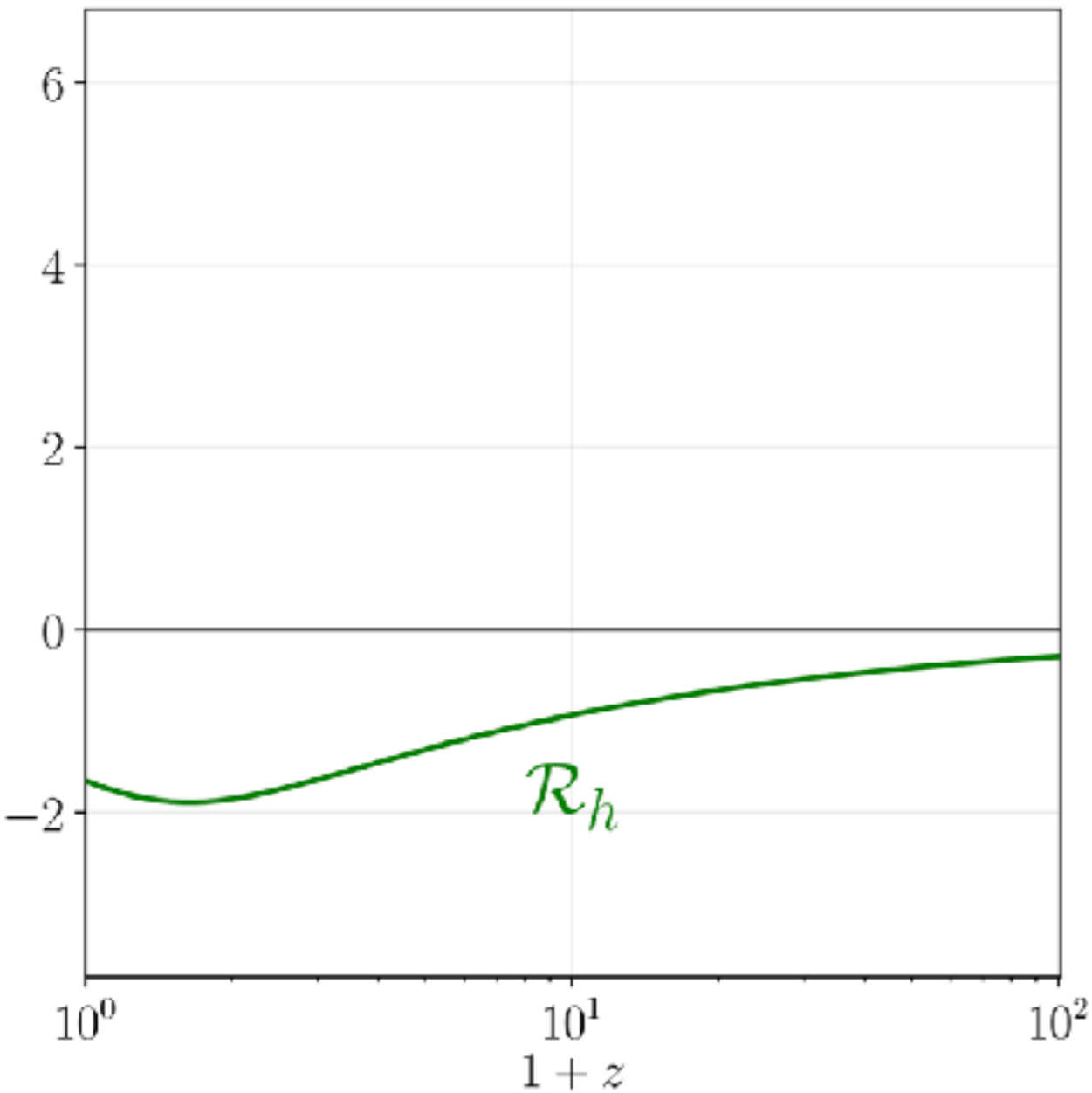


$$\frac{\delta h}{h} = \int_0^\infty \frac{dz}{1+z} R_h(z) \frac{\delta H(z)}{H(z)}$$

$$\frac{\Delta \sigma_8}{\sigma_8} = \int_0^\infty \frac{dz}{1+z} R_{\sigma_8}(z) \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously



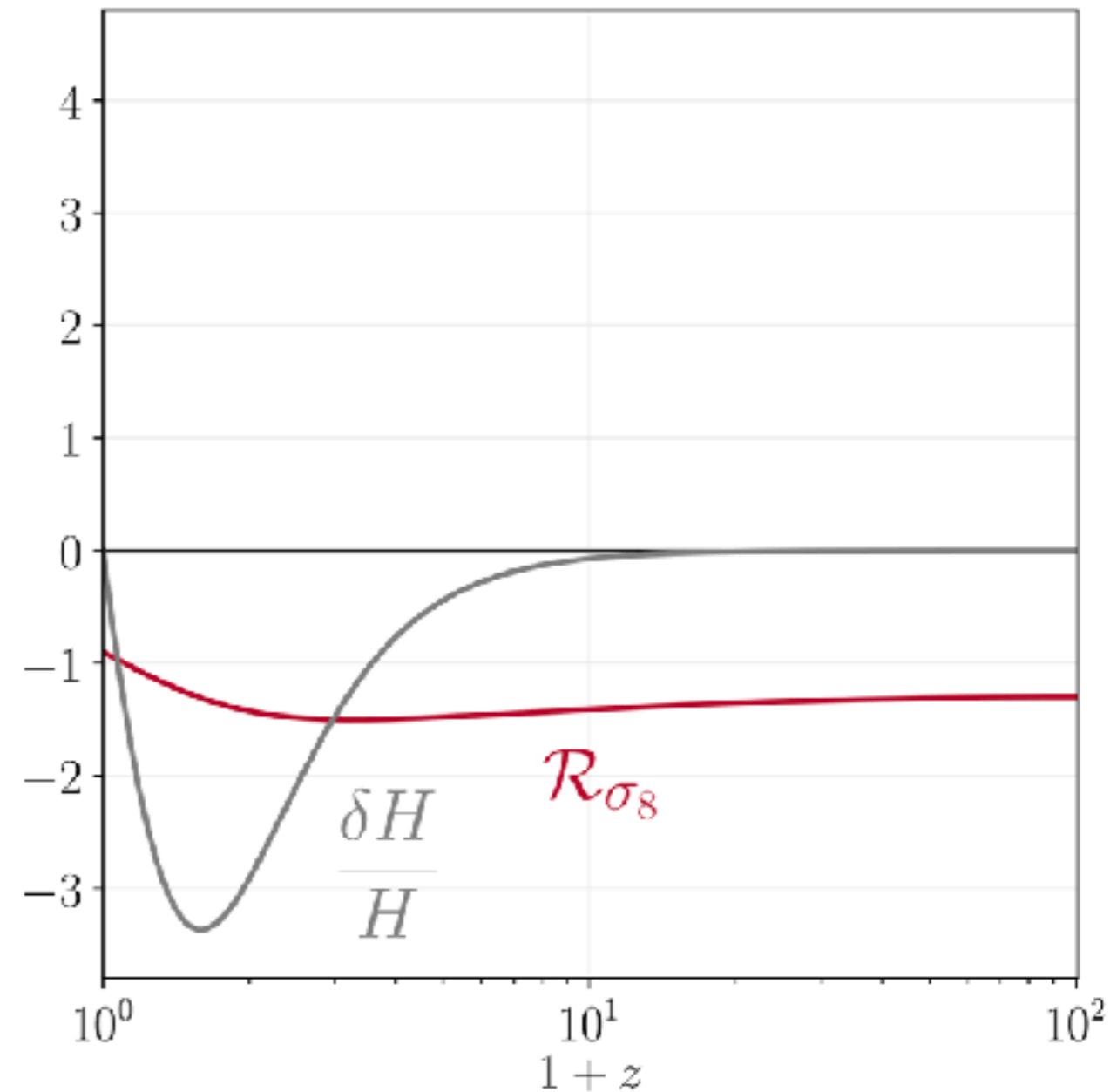
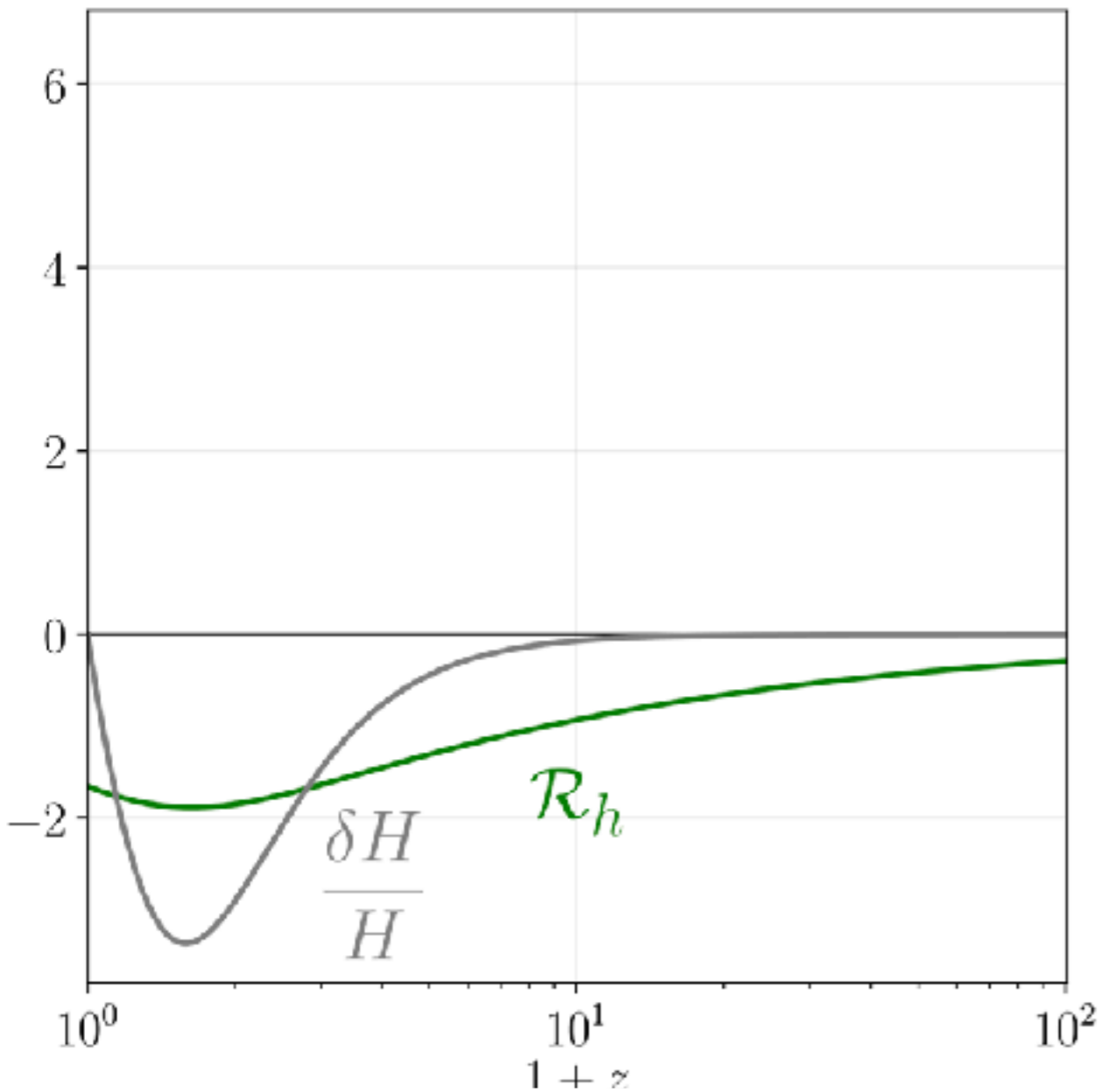
$$\mathcal{R}_h(z)$$

$$\mathcal{R}_{\sigma_8}(z)$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623



# Solving $H_0$ and $\sigma_8$ simultaneously

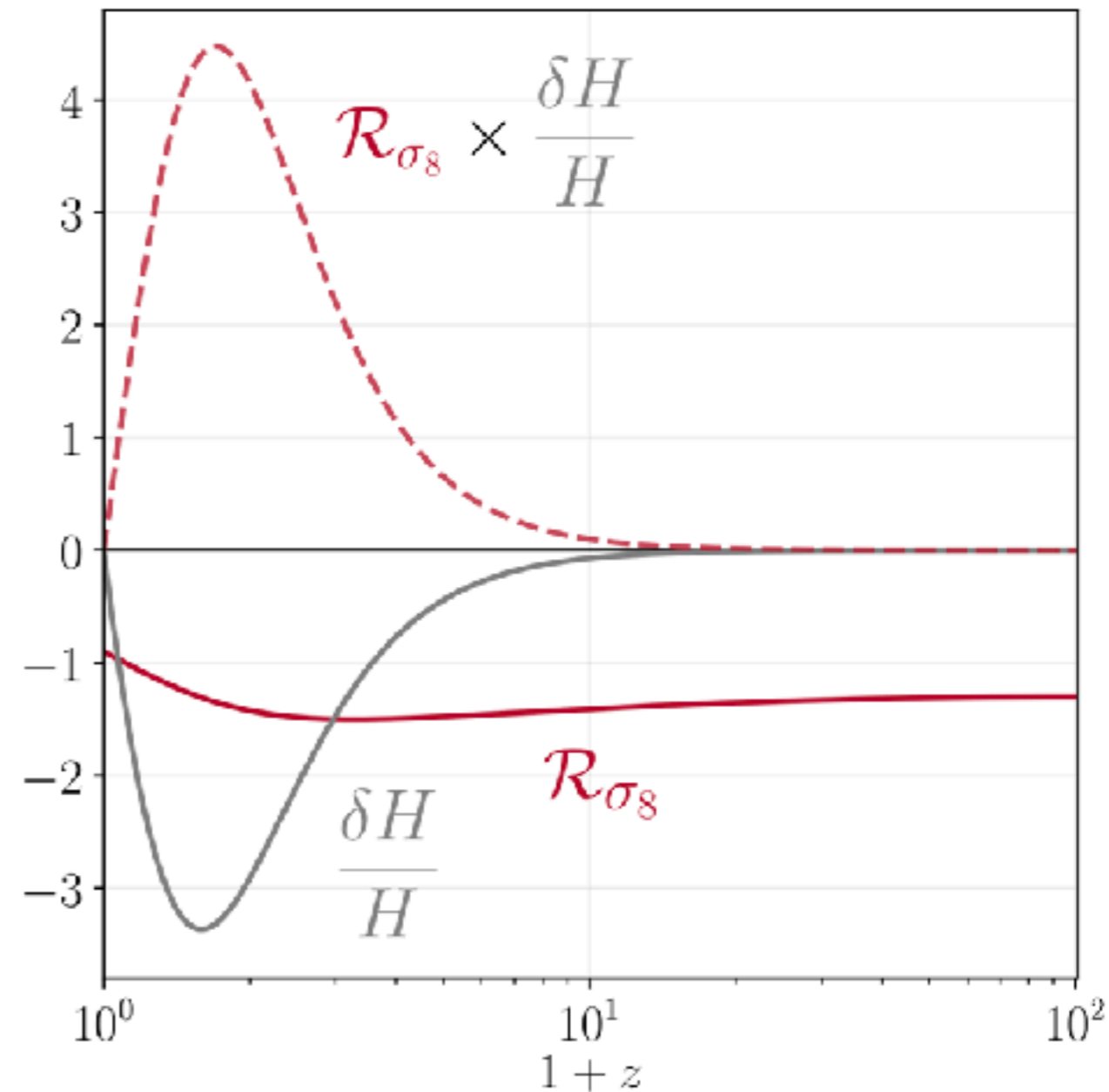
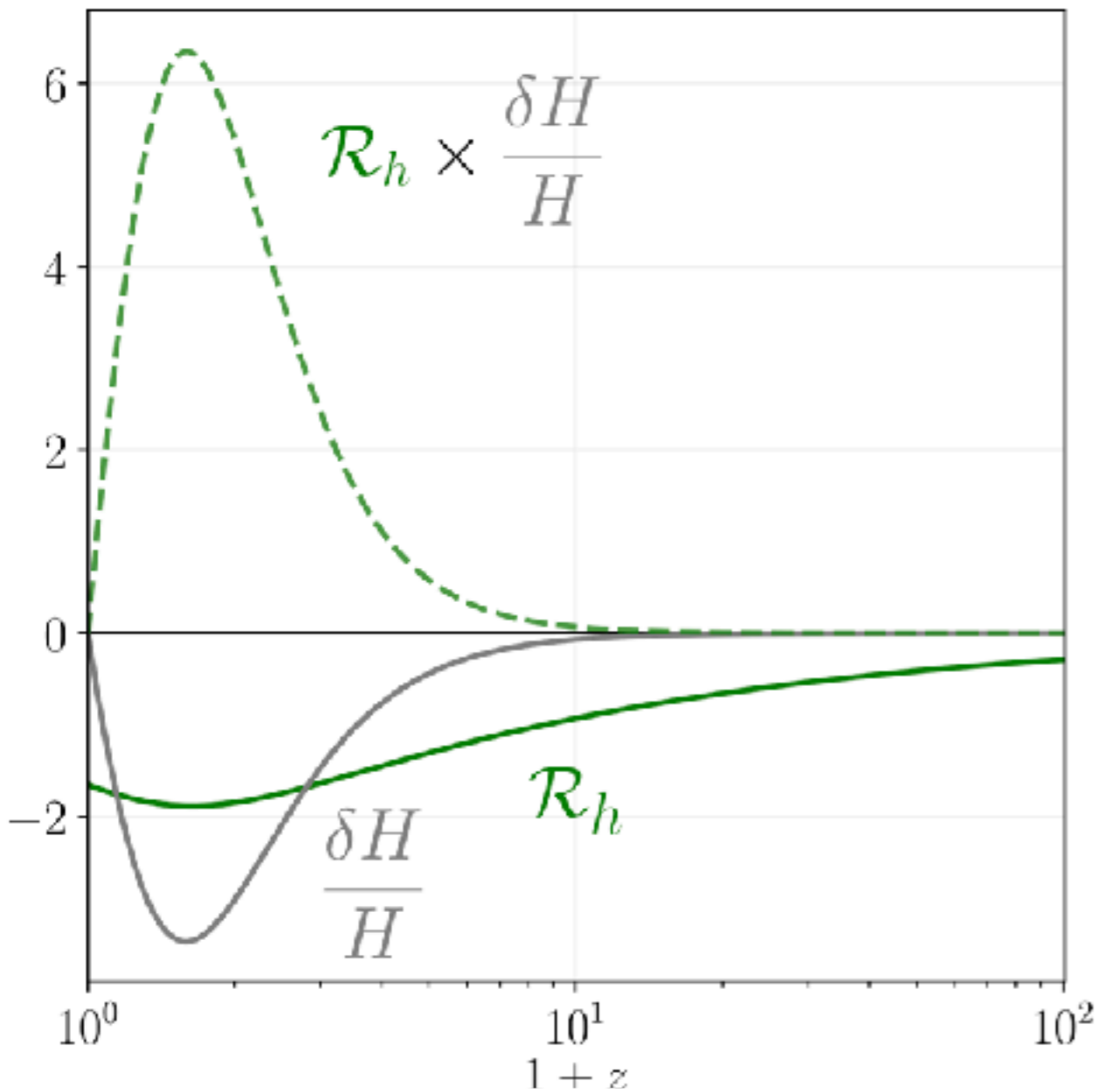


$$\boxed{R_h(z)} \frac{\delta H(z)}{H(z)}$$

$$\boxed{R_{\sigma_8}(z)} \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

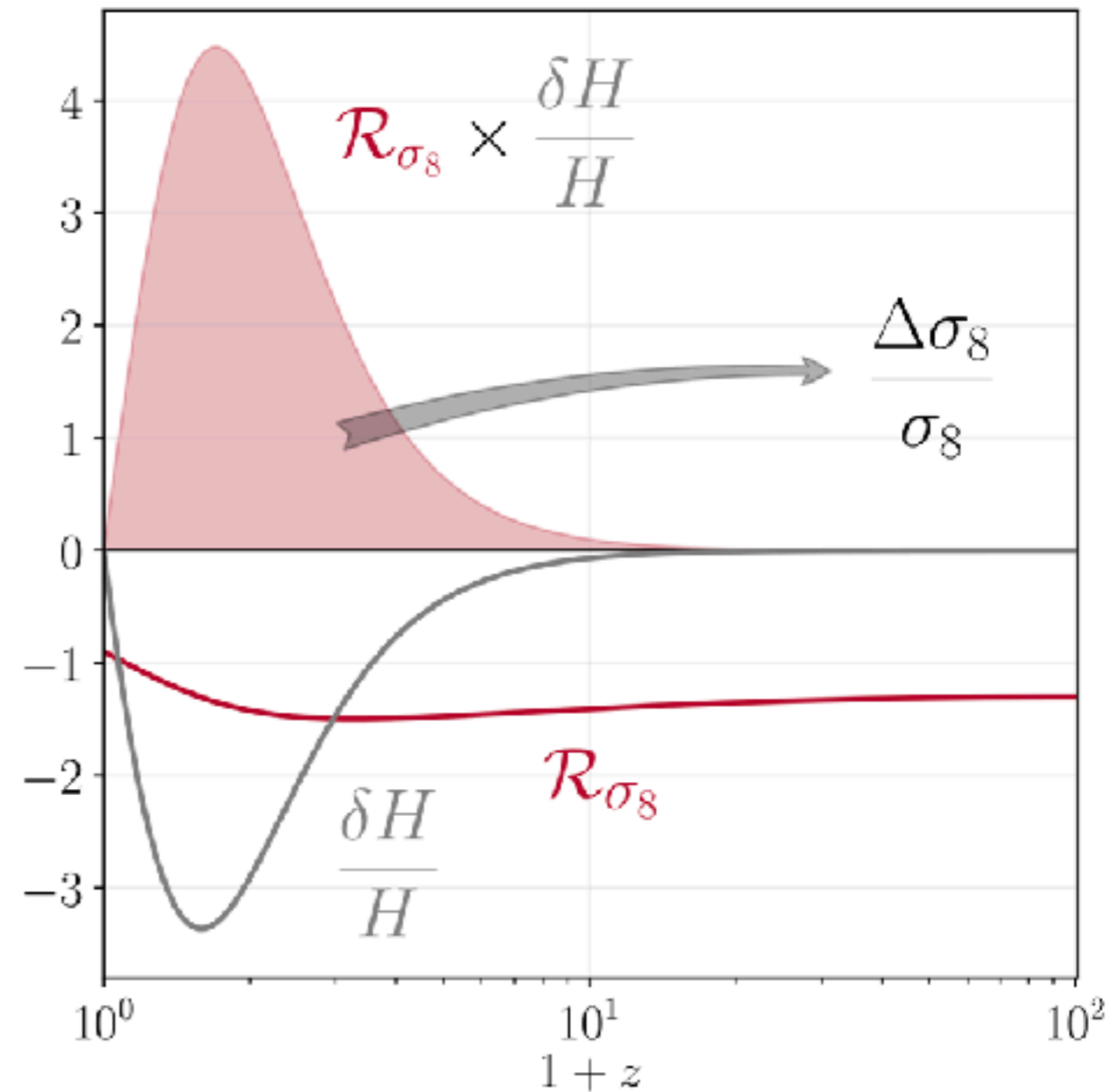
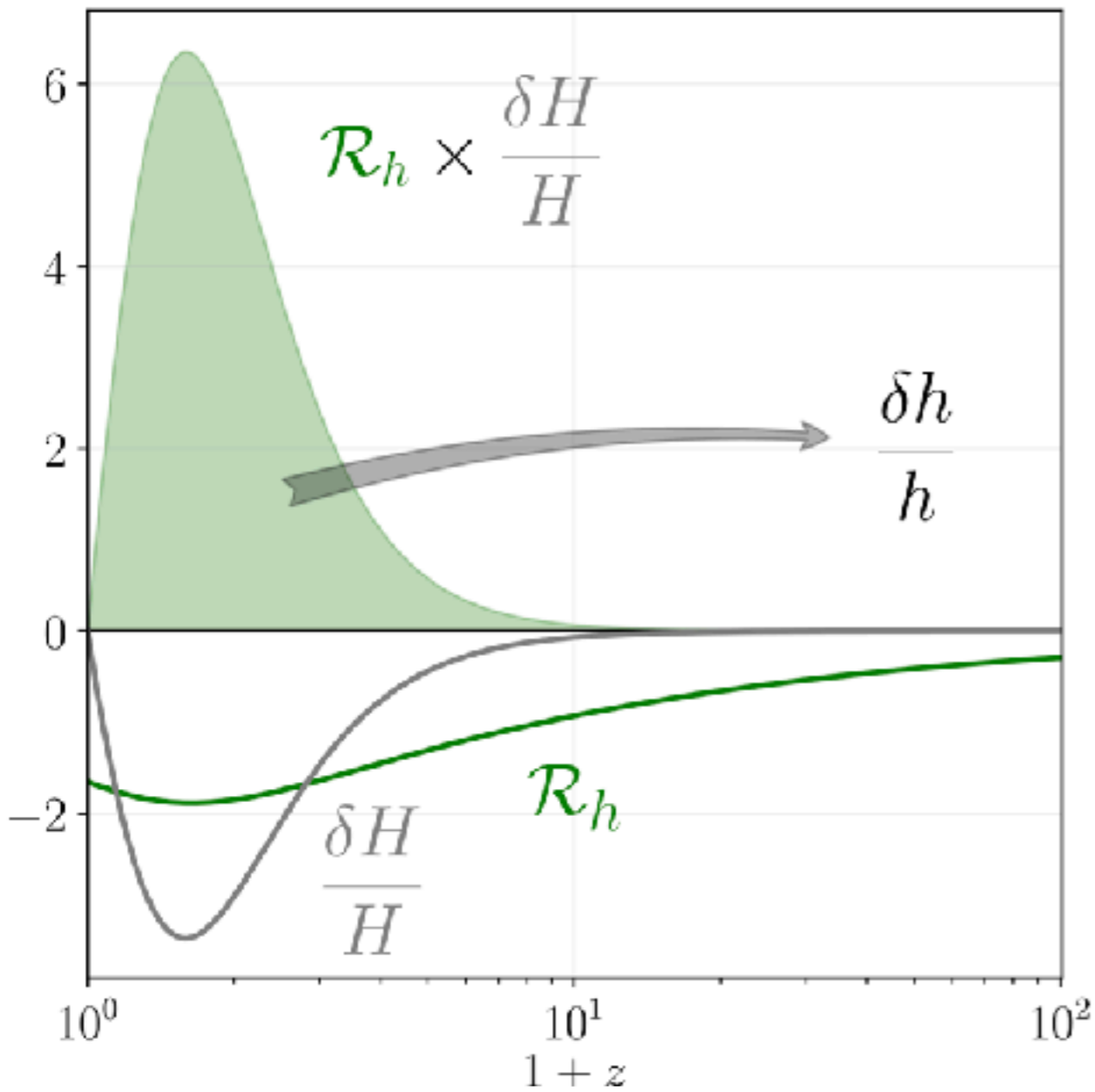


$$\boxed{\mathcal{R}_h(z)} \frac{\delta H(z)}{H(z)}$$

$$\boxed{\mathcal{R}_{\sigma_8}(z)} \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

# Solving $H_0$ and $\sigma_8$ simultaneously

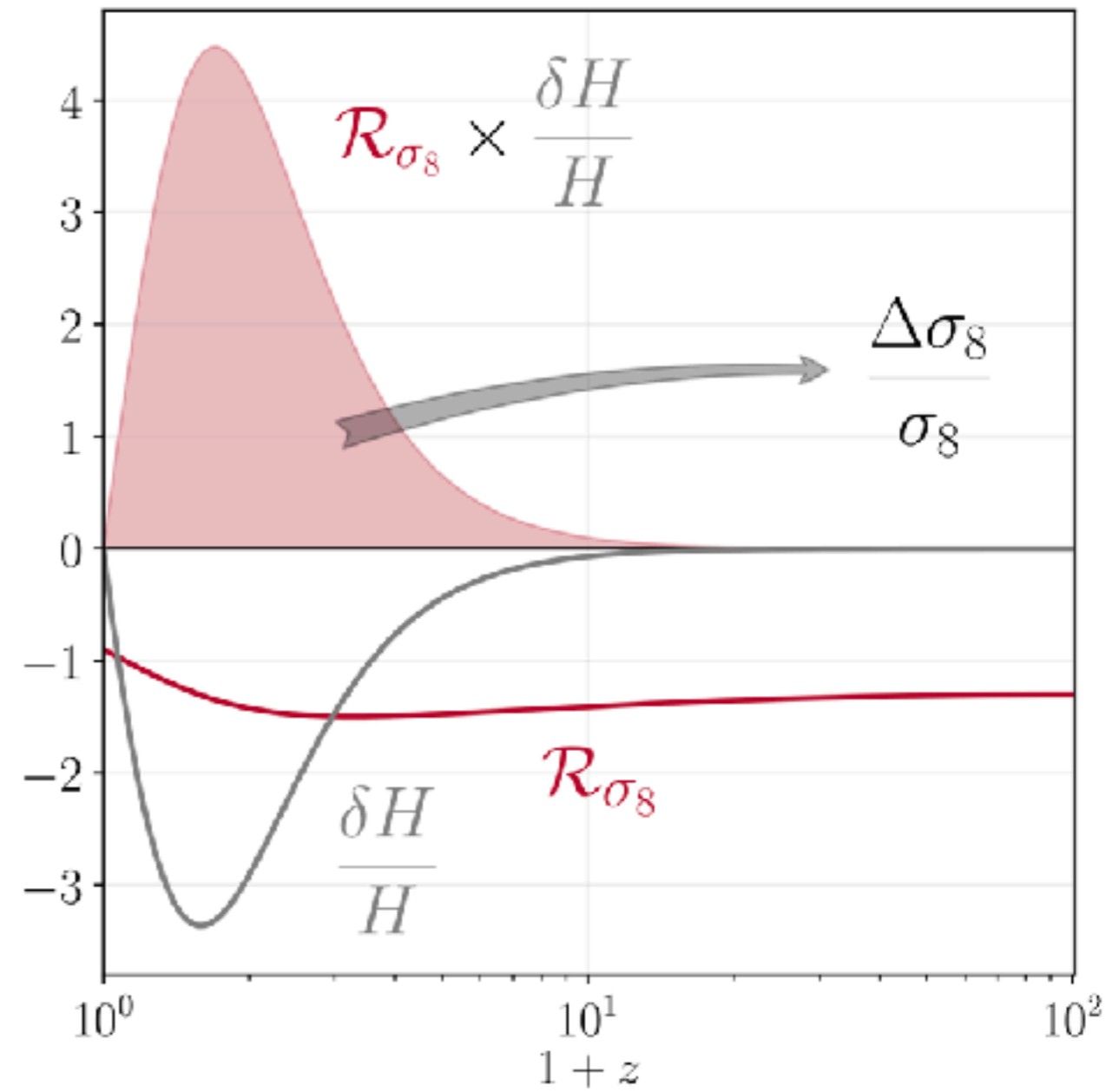
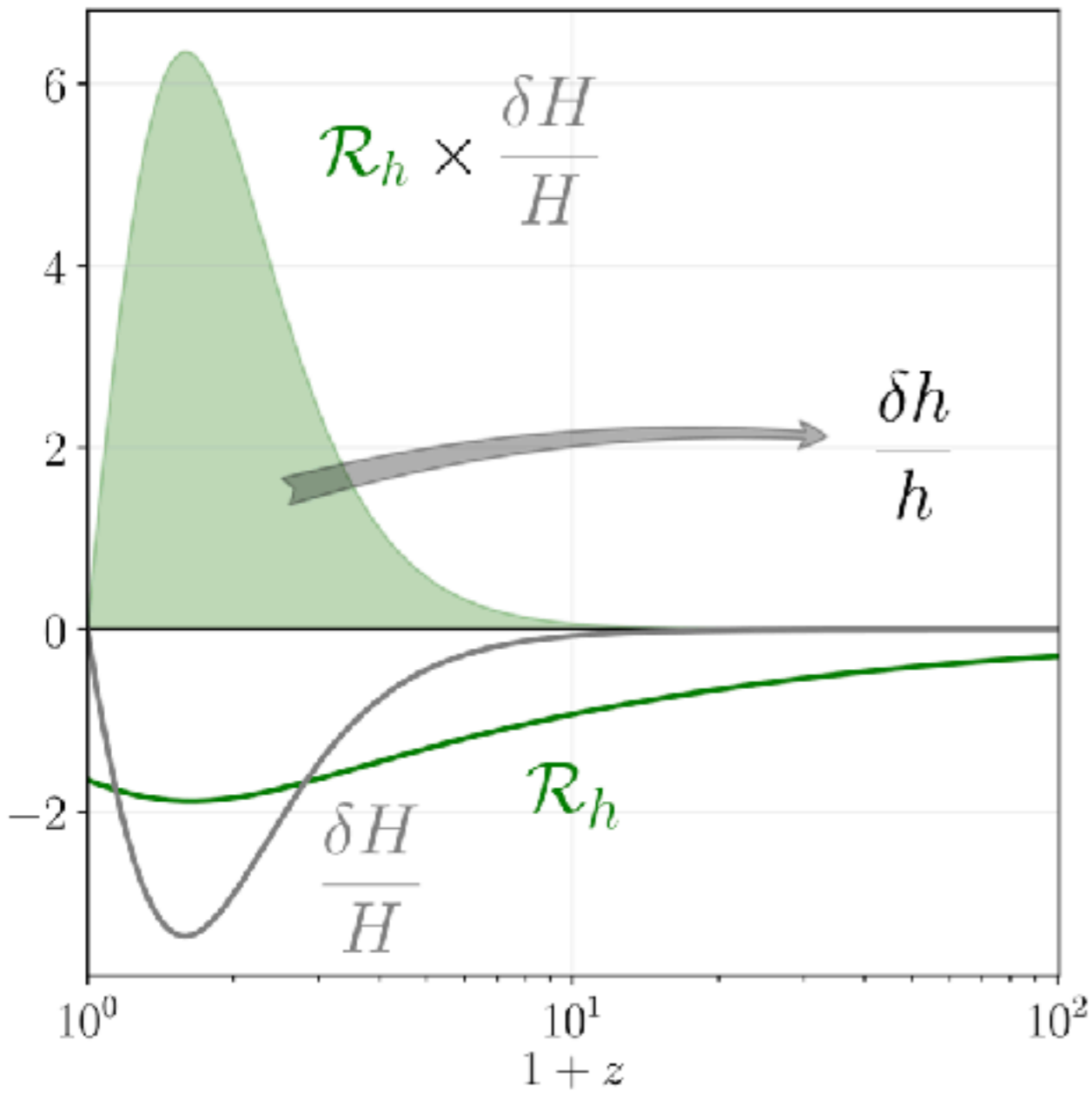


$$\frac{\delta h}{h} = \int_0^\infty \frac{dz}{1+z} \mathcal{R}_h(z) \frac{\delta H(z)}{H(z)}$$

$$\frac{\Delta \sigma_8}{\sigma_8} = \int_0^\infty \frac{dz}{1+z} \mathcal{R}_{\sigma_8}(z) \frac{\delta H(z)}{H(z)}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

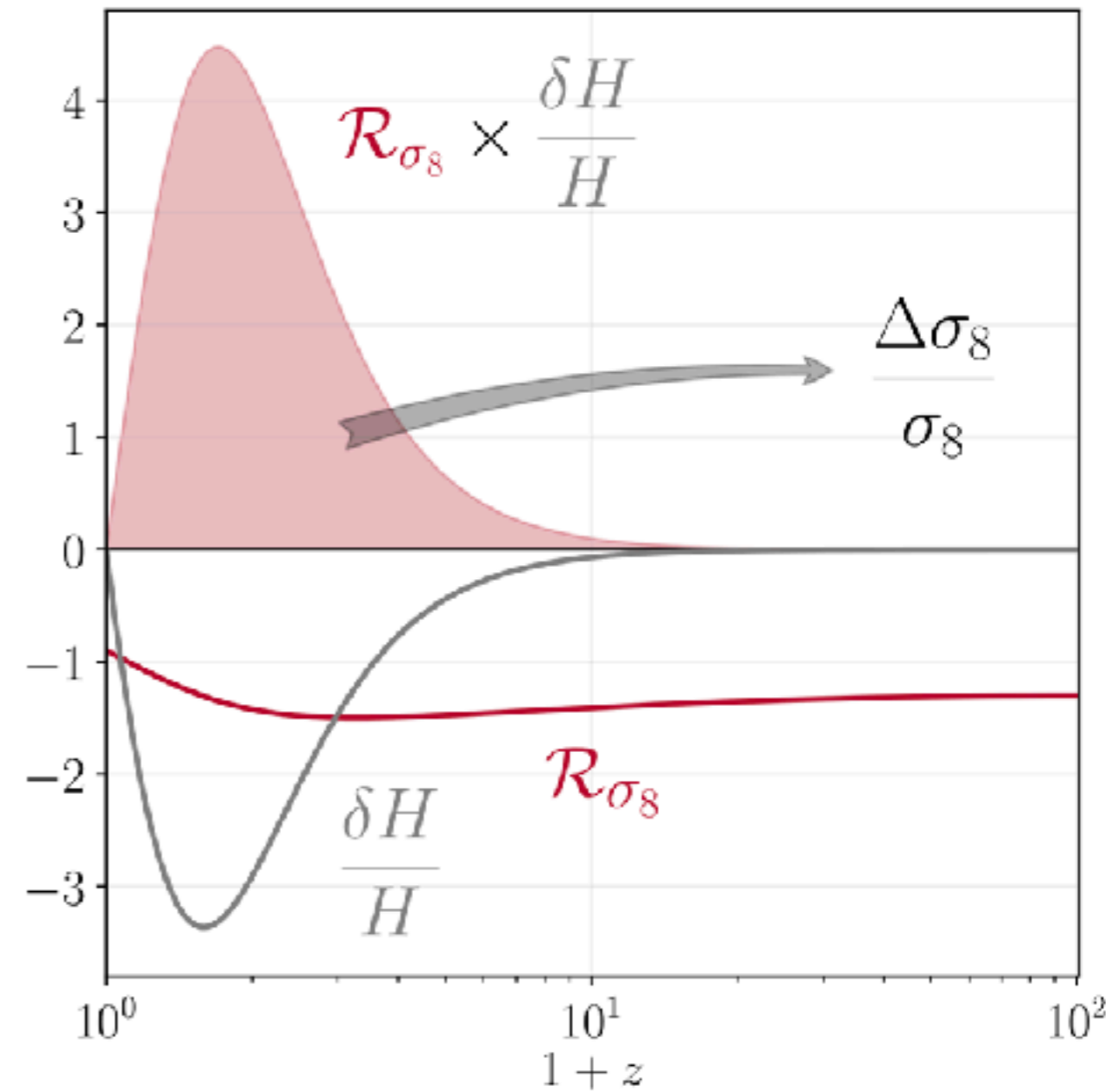
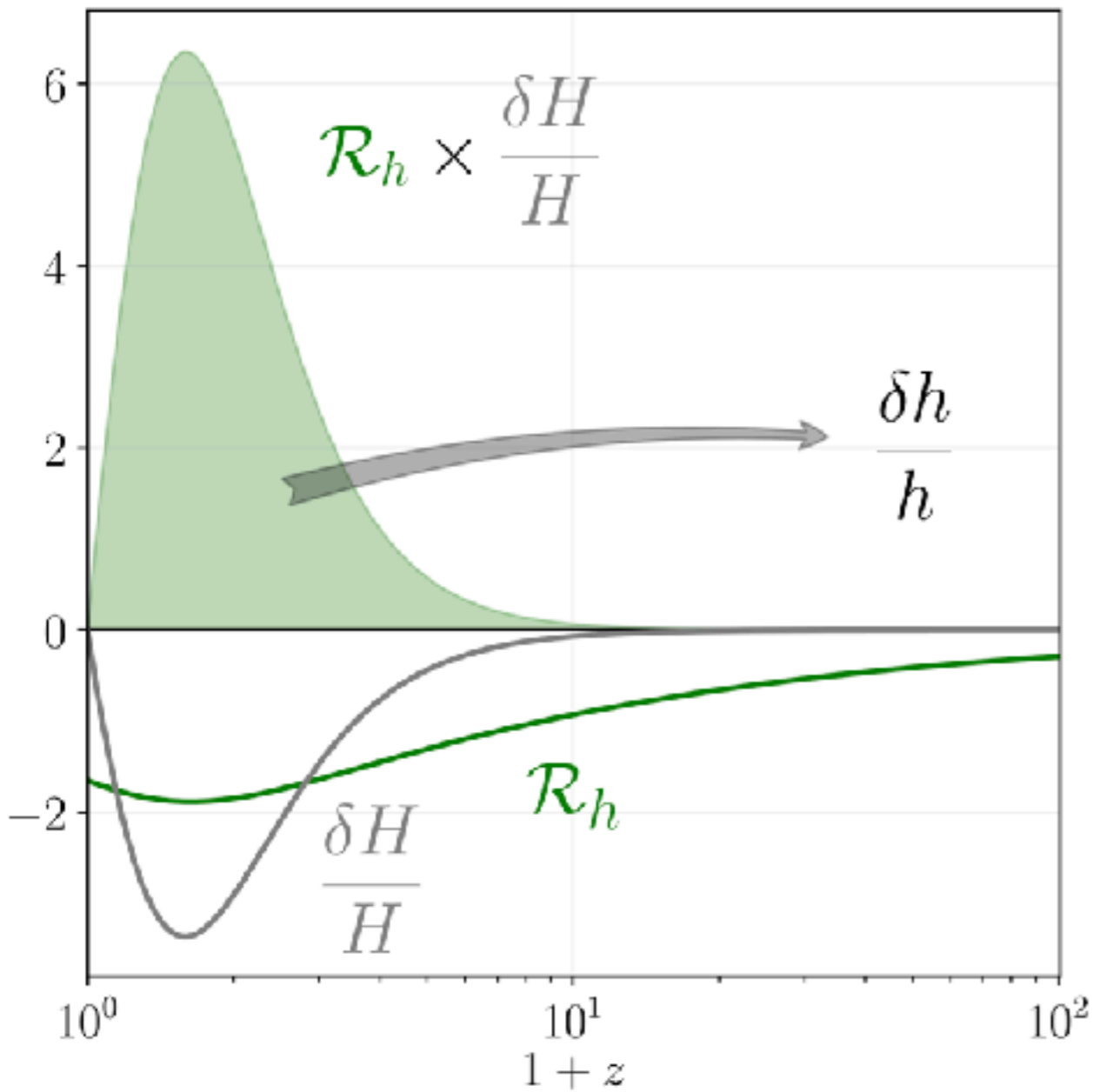
# Solving H0 and sigma8 simultaneously



● Solving H0 tension requires

$$\delta h > 0$$

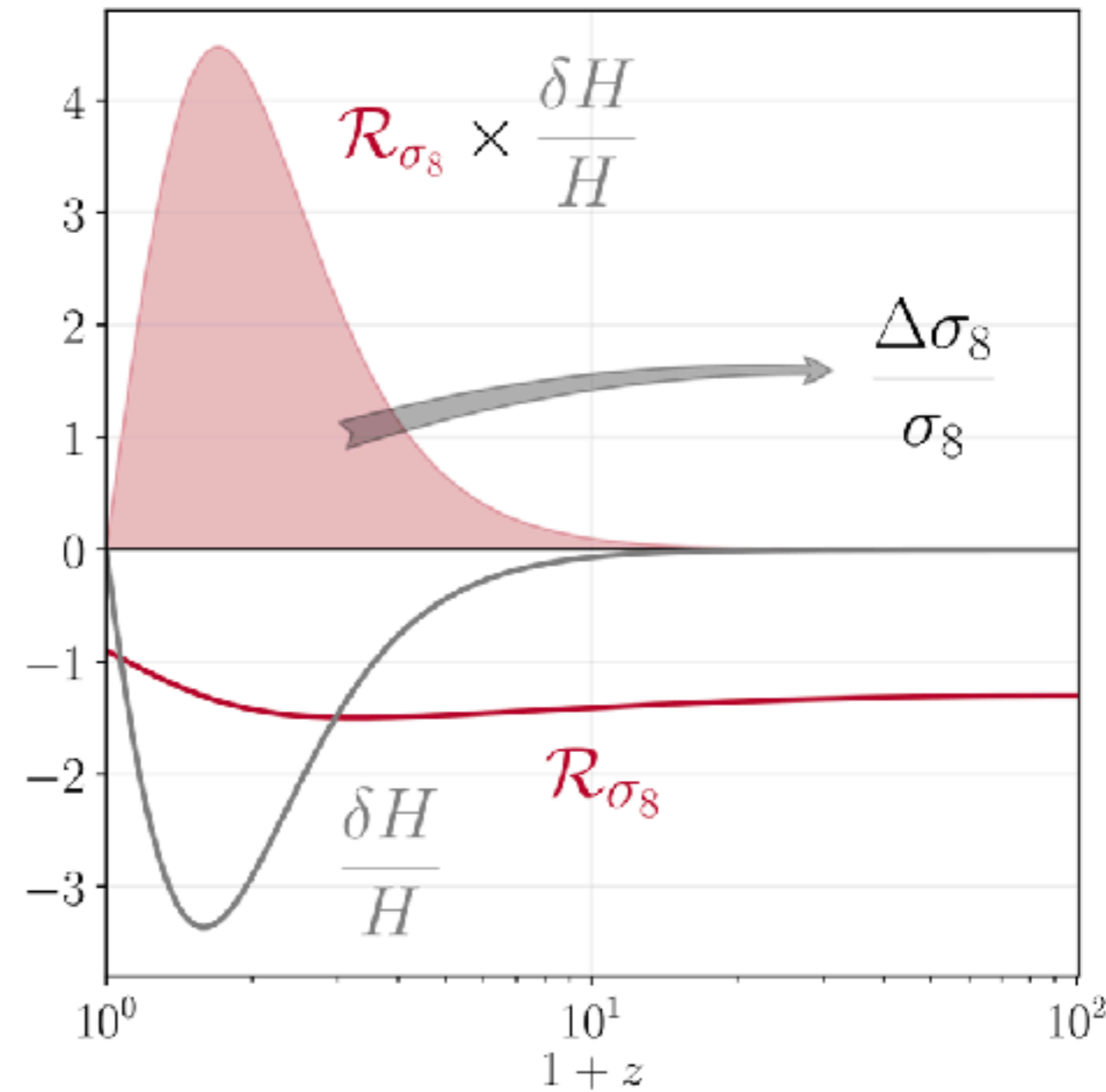
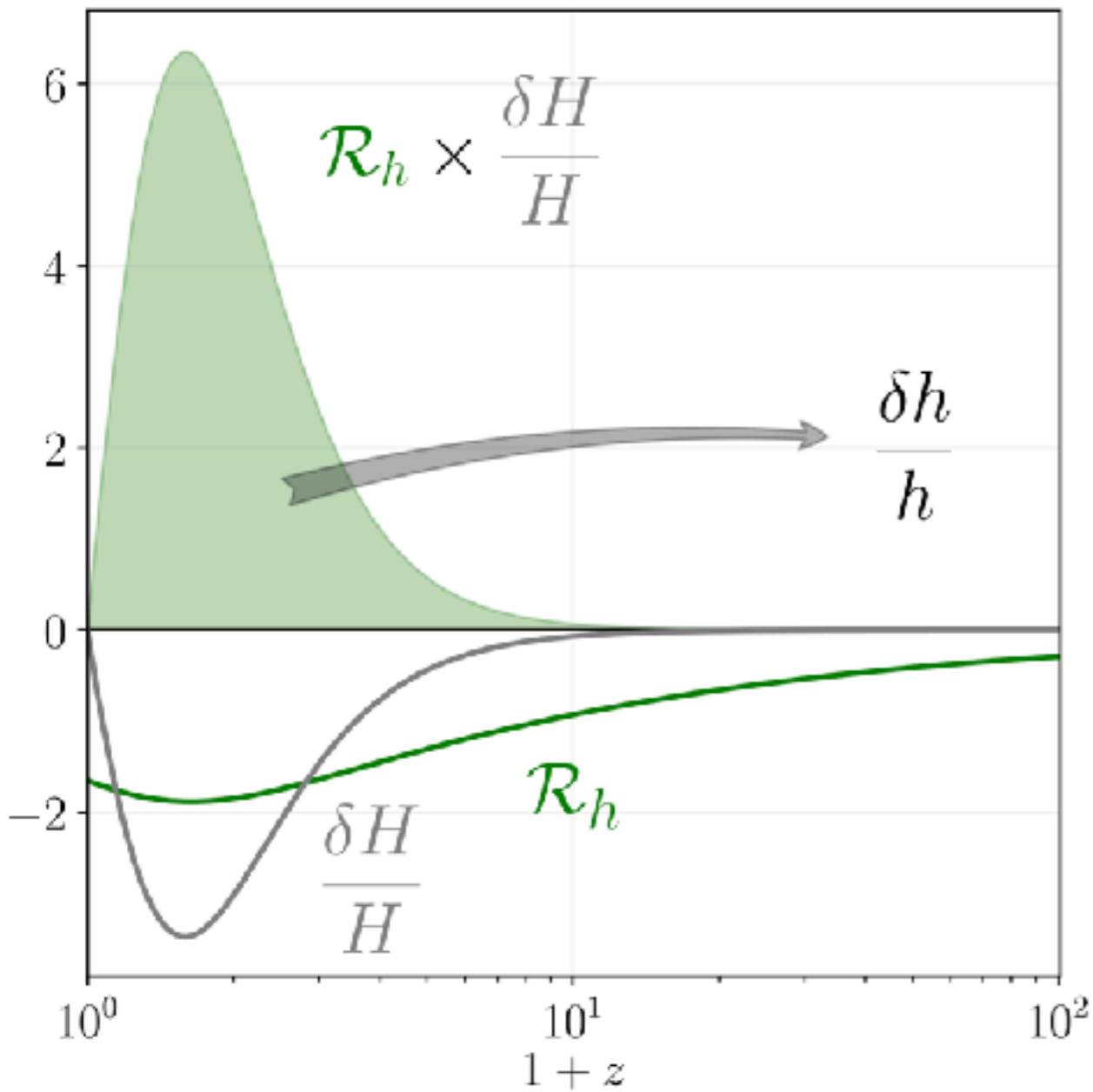
# Solving H0 and sigma8 simultaneously



Solving H0 tension requires

$$\delta h > 0 \quad \longrightarrow \quad \exists z | \delta H(z) < 0$$

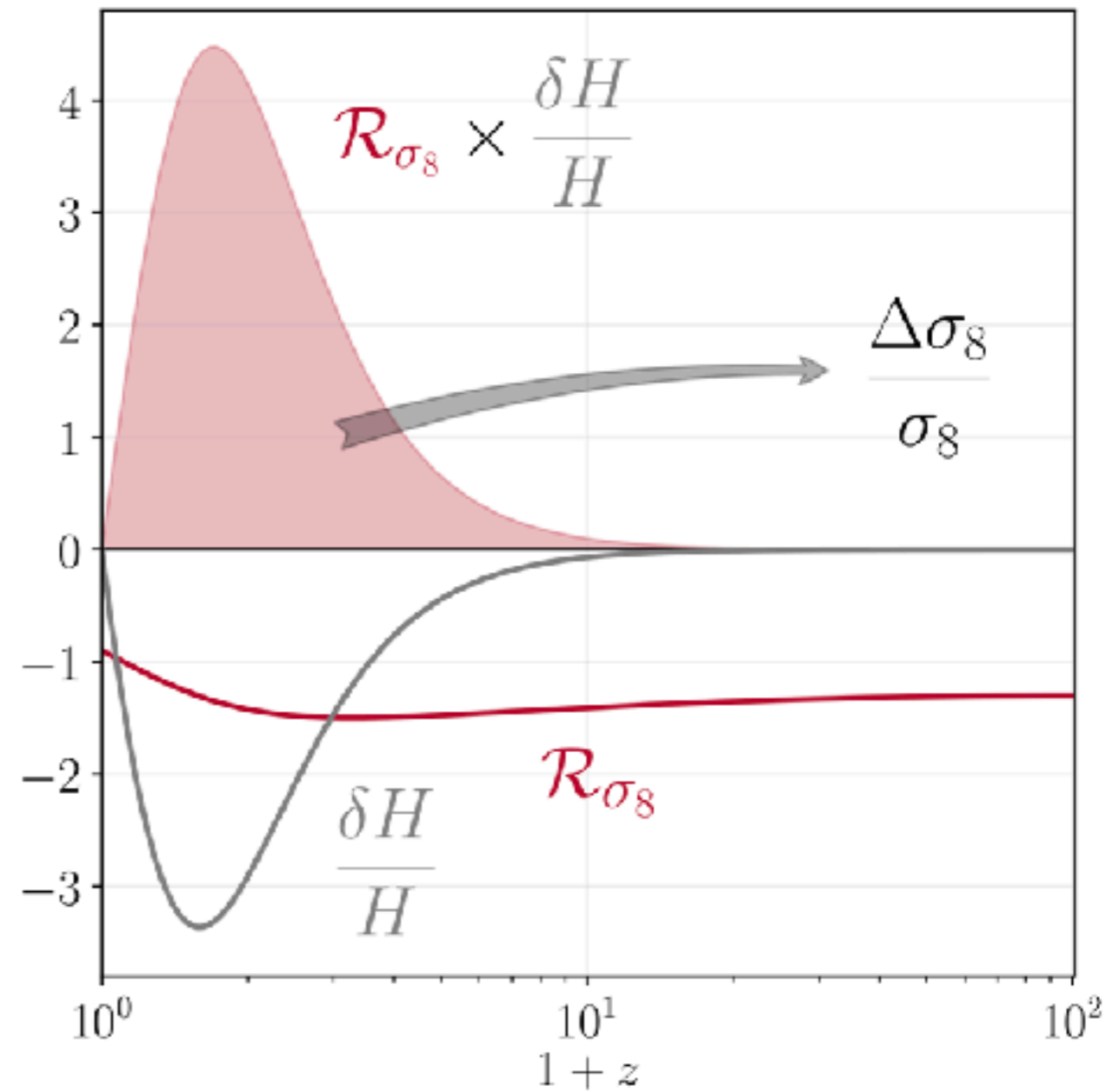
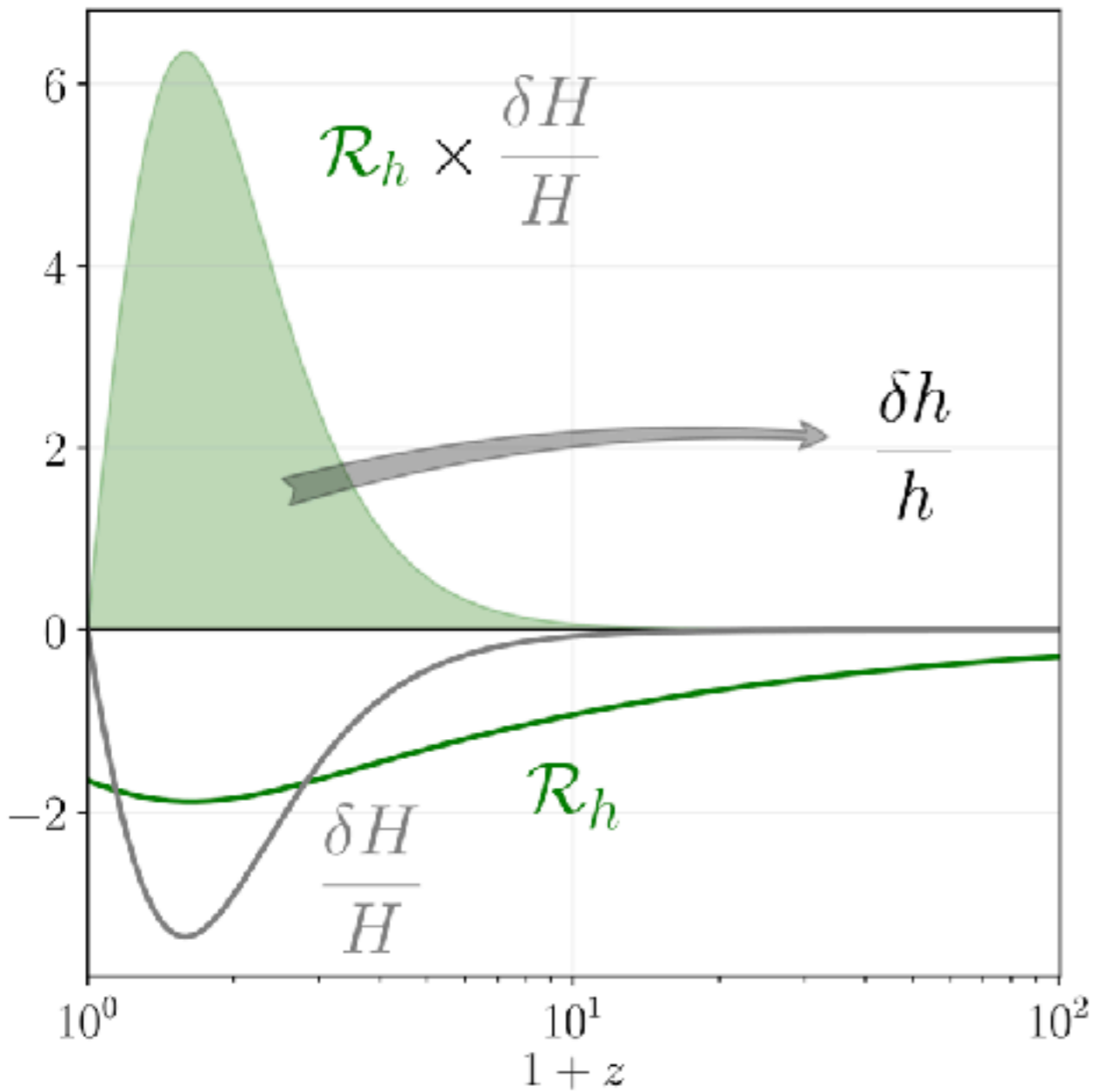
# Solving H0 and sigma8 simultaneously



● Solving H0 tension requires

- $\delta h > 0$  →  $\exists z | \delta H(z) < 0$
- $\exists z | w(z) < -1$

# Solving $H_0$ and $\sigma_8$ simultaneously

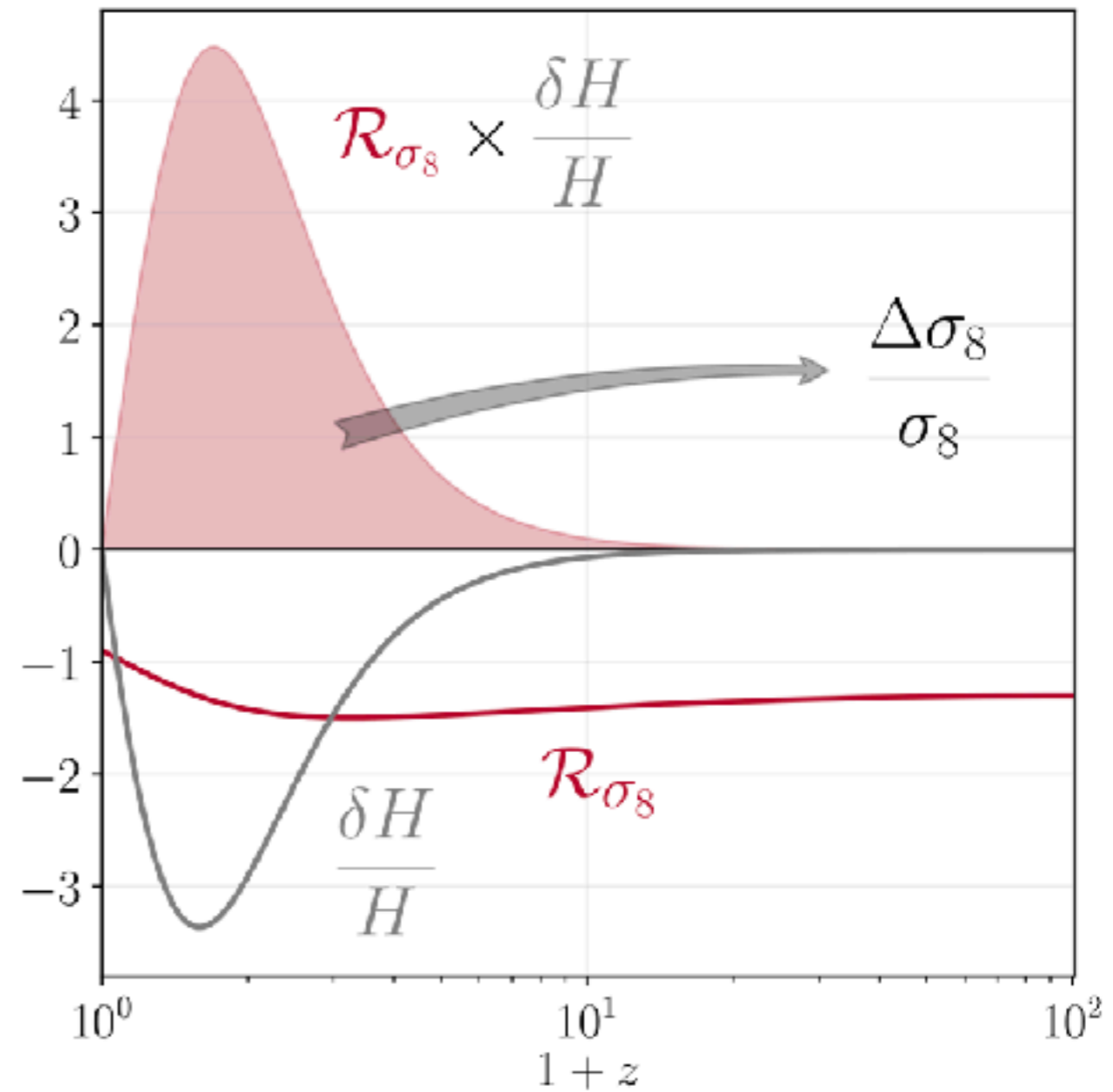
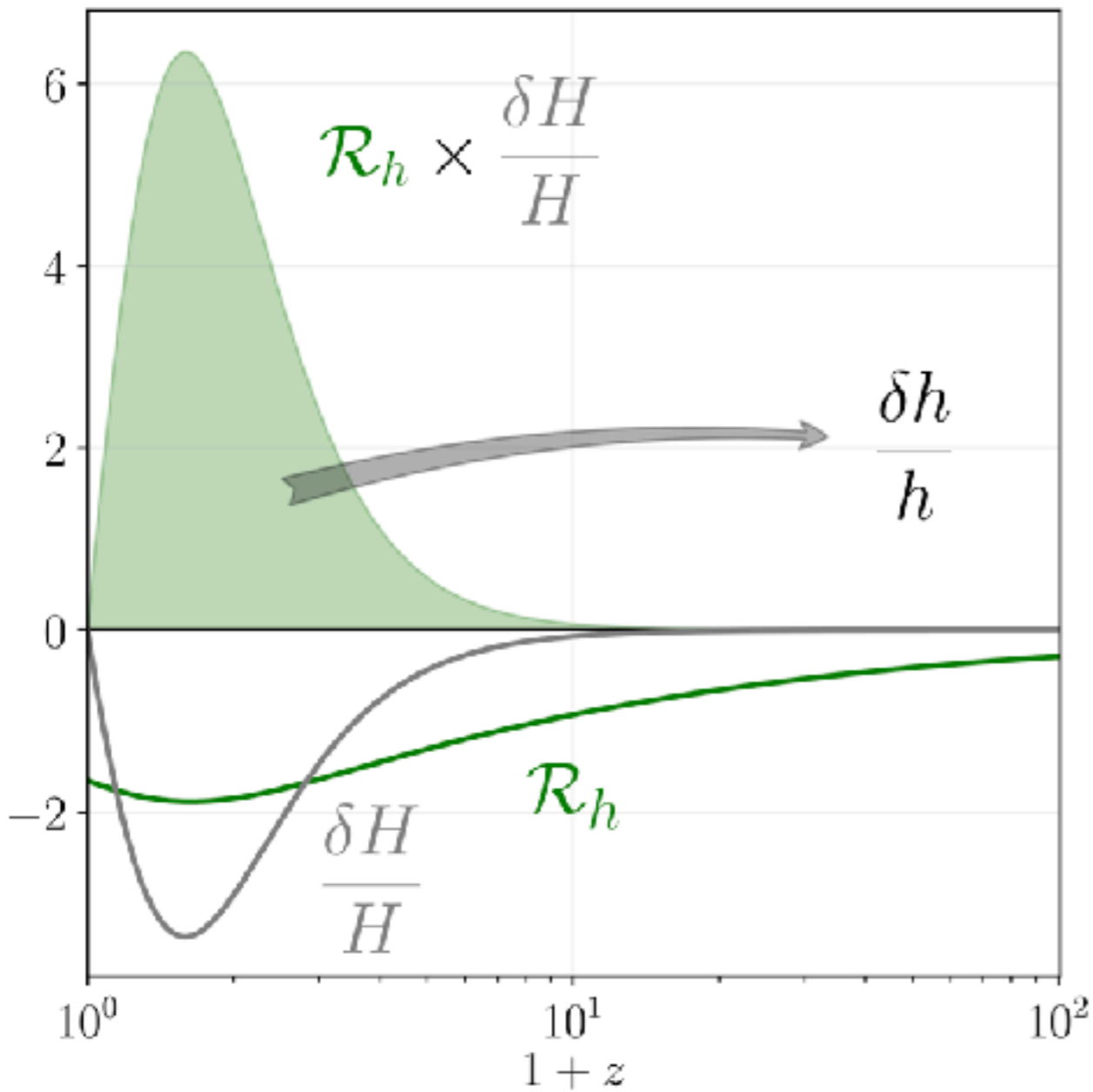


● Solving both tensions requires ( $G_{\text{eff}} = G$ )

$$\delta h > 0$$

$$\Delta\sigma_8 < 0$$

# Solving $H_0$ and $\sigma_8$ simultaneously



● Solving both tensions requires ( $G_{\text{eff}} = G$ )

$\delta h > 0$   $\longrightarrow$   $\delta H(z)$  changes sign

$\Delta\sigma_8 < 0$   $\longrightarrow$   $w(z)$  crosses  $-1$



## Solving $H_0$ and $\sigma_8$ simultaneously

- Solving both tensions requires  $(G_{\text{eff}} \neq G)$

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

## Solving $H_0$ and $\sigma_8$ simultaneously

- Solving both tensions requires  $(G_{\text{eff}} \neq G)$

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

$$G_{\text{eff}} = G + \delta G(z)$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

## Solving $H_0$ and $\sigma_8$ simultaneously

● Solving both tensions requires  $(G_{\text{eff}} \neq G)$

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

$$G_{\text{eff}} = G + \delta G(z)$$

→ 
$$\delta_m(z, k) \sim \left( D(z) + (\Delta D)|_{\delta G} \right) T(k)$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

## Solving $H_0$ and $\sigma_8$ simultaneously

● Solving both tensions requires  $(G_{\text{eff}} \neq G)$

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

$$G_{\text{eff}} = G + \delta G(z)$$

→ 
$$\delta_m(z, k) \sim \left( D(z) + (\Delta D)|_{\delta G} \right) T(k)$$

$$\frac{\Delta\sigma_8}{\sigma_8} = \int_0^\infty \frac{dz}{1+z} \boxed{R_{\sigma_8}(z)} \frac{\delta H(z)}{H(z)} + \int_0^\infty \frac{dz}{1+z} \boxed{G_{\sigma_8}(z)} \frac{\delta G}{G}$$

L.H & H. Villarrubia Rojo,  
J. Zosso,  
arxiv:2201.11623

## Solving $H_0$ and $\sigma_8$ simultaneously

● Solving both tensions requires ( $G_{\text{eff}} \neq G$ )

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

$$G_{\text{eff}} = G + \delta G(z)$$

→  $\delta_m(z, k) \sim \left( D(z) + (\Delta D)|_{\delta G} \right) T(k)$

$$\frac{\Delta\sigma_8}{\sigma_8} = \int_0^\infty \frac{dz}{1+z} \boxed{R_{\sigma_8}(z)} \frac{\delta H(z)}{H(z)} + \int_0^\infty \frac{dz}{1+z} \boxed{G_{\sigma_8}(z)} \frac{\delta G}{G}$$

$\sigma_8 \uparrow$

If  $\delta h > 0$  and  $\delta H(z) < 0$

# Solving $H_0$ and $\sigma_8$ simultaneously

● Solving both tensions requires ( $G_{\text{eff}} \neq G$ )

$$H = H_{\Lambda\text{CDM}}(z) + \delta H(z)$$

$$G_{\text{eff}} = G + \delta G(z)$$

→  $\delta_m(z, k) \sim \left( D(z) + (\Delta D)|_{\delta G} \right) T(k)$

$$\frac{\Delta\sigma_8}{\sigma_8} = \int_0^\infty \frac{dz}{1+z} \boxed{R_{\sigma_8}(z)} \frac{\delta H(z)}{H(z)} + \int_0^\infty \frac{dz}{1+z} \boxed{G_{\sigma_8}(z)} \frac{\delta G}{G}$$

$\sigma_8 \uparrow$

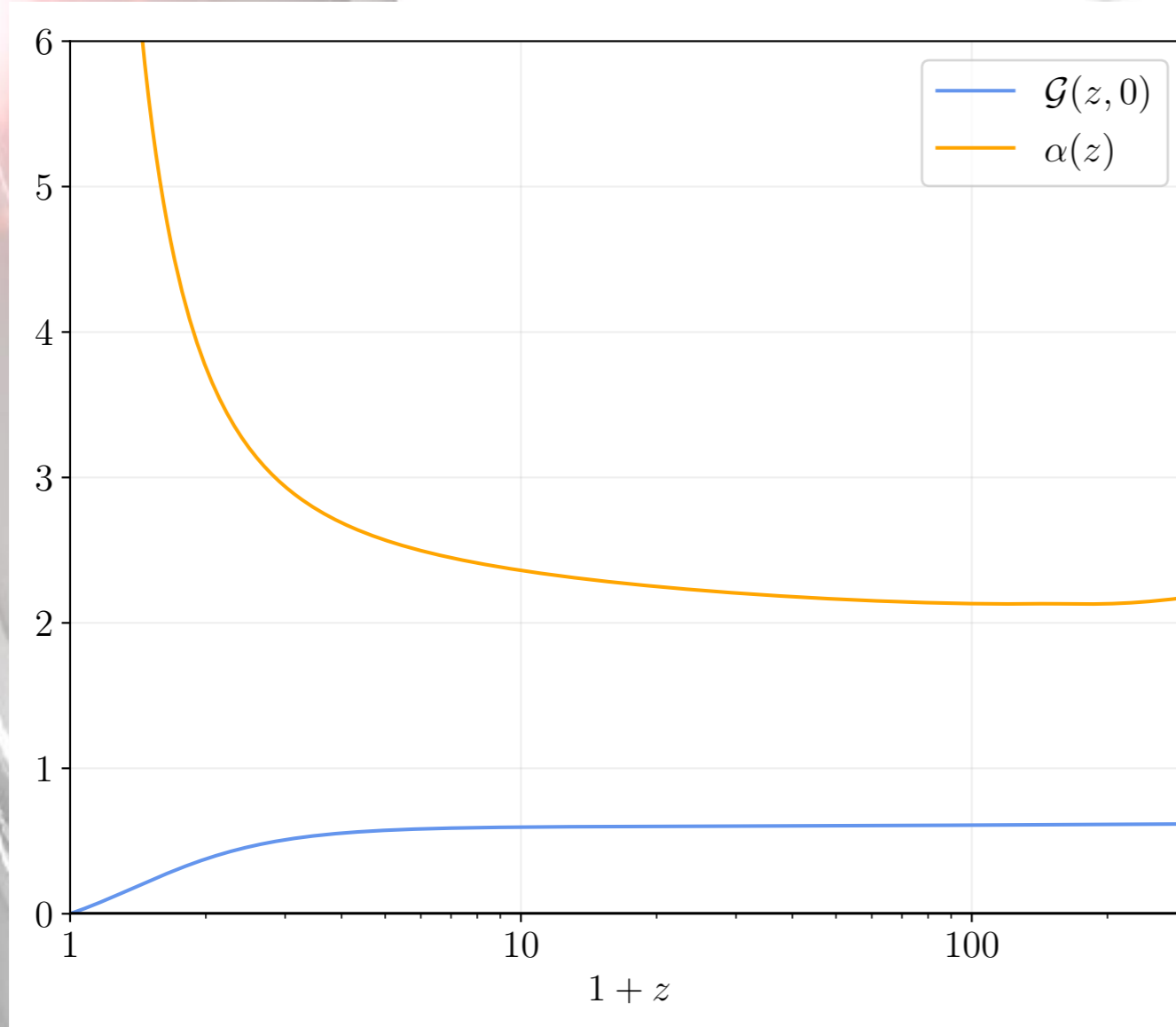
$\sigma_8 \downarrow\downarrow\downarrow$

If  $\delta h > 0$  and  $\delta H(z) < 0$

$$\frac{\delta G(z)}{G} < \alpha(z) \frac{\delta H(z)}{H(z)} < 0$$

# Solving $H_0$ and $\sigma_8$ simultaneously

● Solving both tensions requires  $(G_{\text{eff}} \neq G)$



If  $\delta h > 0$  and  $\delta H(z) < 0$

$$\frac{\delta G(z)}{G} < \alpha(z) \frac{\delta H(z)}{H(z)} < 0$$

# Summary



**Solving  $H_0$  tension**

$$\delta h > 0$$



# Summary

## ● Solving $H_0$ tension

$$\delta h > 0 \quad \longrightarrow \quad \begin{aligned} \exists z | \delta H(z) < 0 \\ \exists z | w(z) < -1 \end{aligned}$$

# Summary

## ● Solving $H_0$ tension

$$\delta h > 0 \quad \longrightarrow \quad \begin{aligned} \exists z | \delta H(z) < 0 \\ \exists z | w(z) < -1 \end{aligned}$$

## ● Solving both tensions $(G_{\text{eff}} = G)$

$$\begin{aligned} \delta h > 0 \\ \Delta\sigma_8 < 0 \end{aligned}$$

# Summary

## ● Solving H0 tension

$$\delta h > 0 \quad \longrightarrow \quad \begin{aligned} \exists z | \delta H(z) < 0 \\ \exists z | w(z) < -1 \end{aligned}$$

## ● Solving both tensions $(G_{\text{eff}} = G)$

$$\begin{aligned} \delta h > 0 \\ \Delta\sigma_8 < 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} \delta H(z) \text{ changes sign} \\ w(z) \text{ crosses } -1 \end{aligned}$$

# Summary

## ● Solving H0 tension

$$\delta h > 0 \quad \longrightarrow \quad \begin{aligned} \exists z | \delta H(z) < 0 \\ \exists z | w(z) < -1 \end{aligned}$$

## ● Solving both tensions $(G_{\text{eff}} = G)$

$$\begin{aligned} \delta h > 0 \\ \Delta\sigma_8 < 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} \delta H(z) \text{ changes sign} \\ w(z) \text{ crosses } -1 \end{aligned}$$

## ● Solving both tensions $(G_{\text{eff}} \neq G)$

$$\delta H(z) < 0$$

# Summary

## ● Solving H0 tension

$$\delta h > 0 \quad \longrightarrow \quad \begin{aligned} \exists z | \delta H(z) < 0 \\ \exists z | w(z) < -1 \end{aligned}$$

## ● Solving both tensions $(G_{\text{eff}} = G)$

$$\begin{aligned} \delta h > 0 \\ \Delta\sigma_8 < 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} \delta H(z) \text{ changes sign} \\ w(z) \text{ crosses } -1 \end{aligned}$$

## ● Solving both tensions $(G_{\text{eff}} \neq G)$

$$\delta H(z) < 0$$



$$\frac{\delta G(z)}{G} < \alpha(z) \frac{\delta H(z)}{H(z)} < 0$$