universität innsbruck

IQOQI

Simons Collaboration on Ultra-Quantum Matter

Hardware-efficient quantum simulation of non-abelian gauge theories with qudits and Rydberg atoms

arXiv:2203.15541 (2022)

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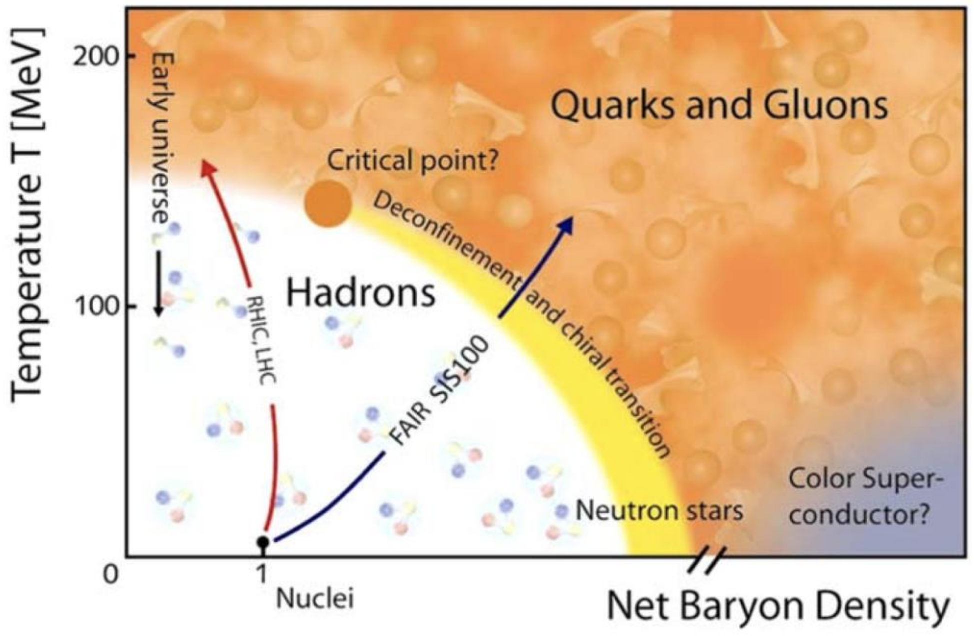
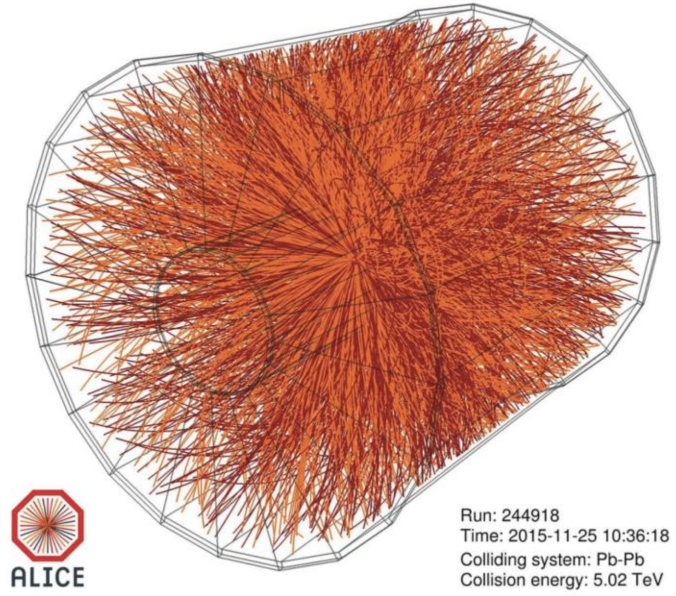
Probing the Standard Model of particle physics

Strong force: quantum chromodynamics (QCD) → heavy-ion collisions



Durante et al., Physica Scripta, 94(3), 033001 (2019)



Computational challenges:

- Real-time dynamics
- Finite baryon density

→ alternative:

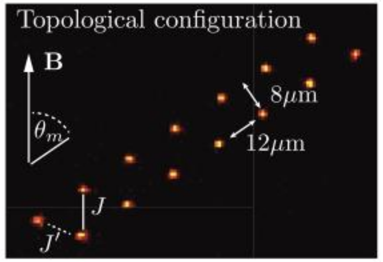
Quantum simulators

Today's quantum simulation platforms & experiments

Quantum simulator: synthetic, programmable quantum system

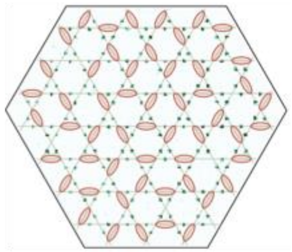
Rydberg atom arrays

SSH as SPT phase



S. Léséleuc et al., Science 2019

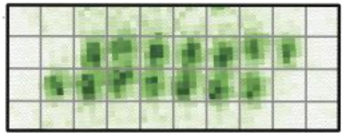
Topological Spin Liquids



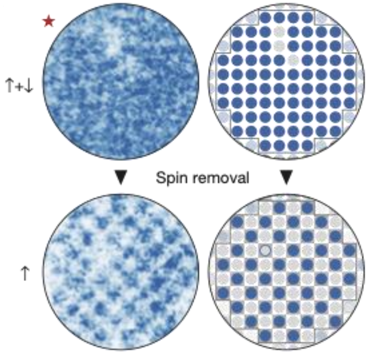
G. Semeghini et al., Nature 2022

Atoms in optical lattices

SPT Haldane Phase on Fermi Hubbard Ladder

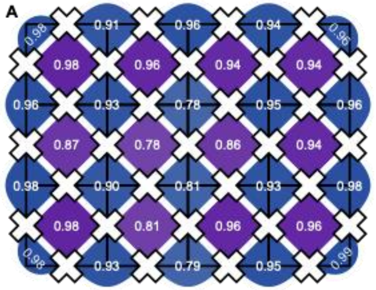


Sompet et.al. Nature 2022

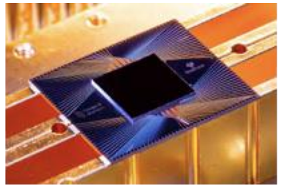


Mazurenko et.al. Nature 2017

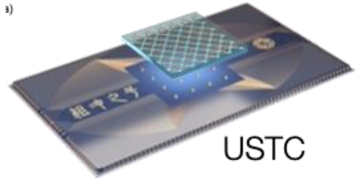
Superconducting circuits



Satzinger et.al (2021)

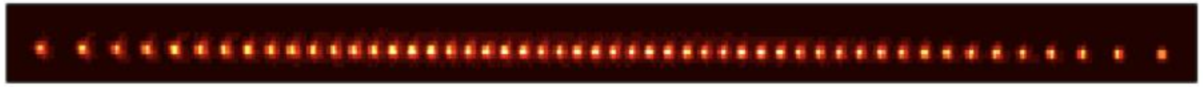


Google



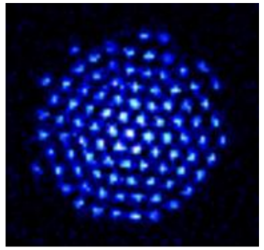
USTC

Trapped ions



51 ions Innsbruck

Innsbruck, Duke, NIST, …



scalable to many particles

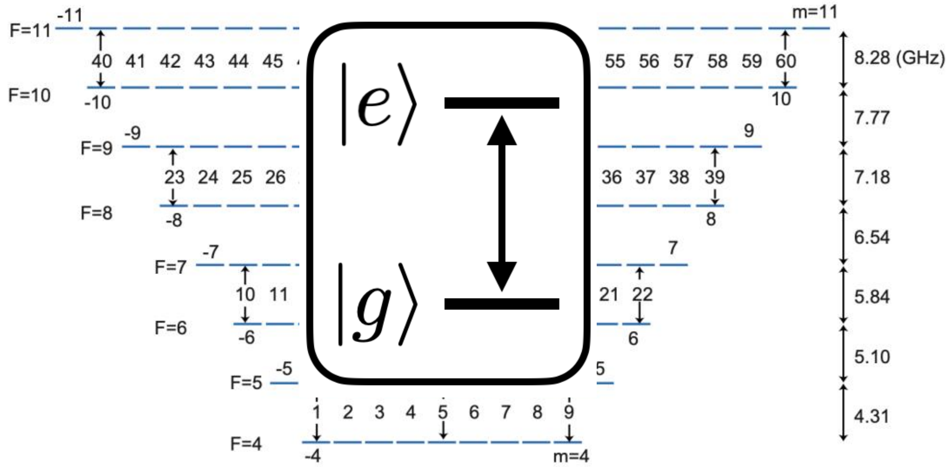
analog → digital

Quantum Information with atoms

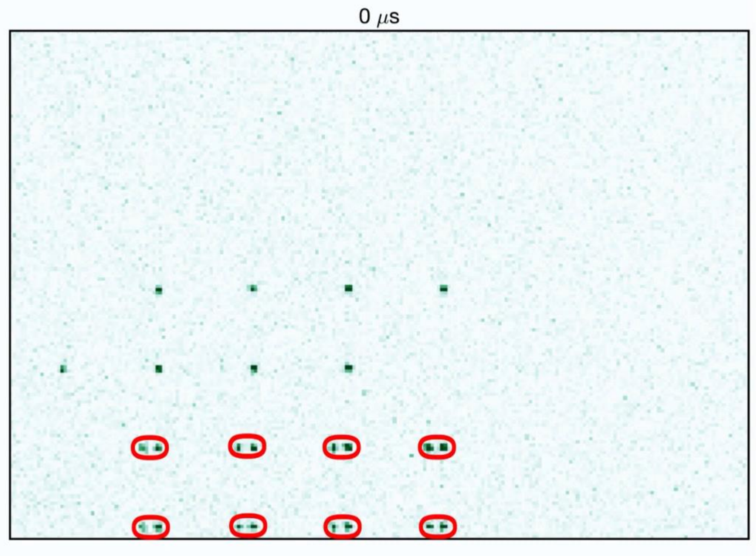
Atomic ~~two-level~~ multi-level systems = ~~qubits~~ qudits

``NISQ'' era:

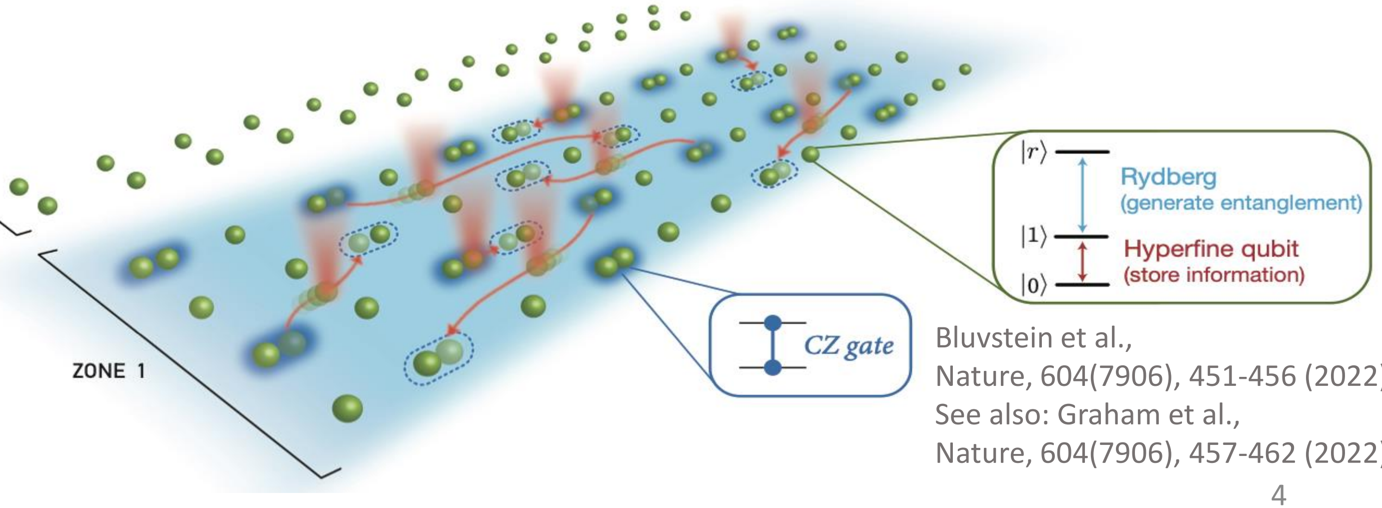
no large-scale universal quantum … ection



Saffman & Mølmer, Physical Review A, 78(1), 012336 (2008)



… e?



Bluvstein et al.,
Nature, 604(7906), 451-456 (2022)
See also: Graham et al.,
Nature, 604(7906), 457-462 (2022)

Outline

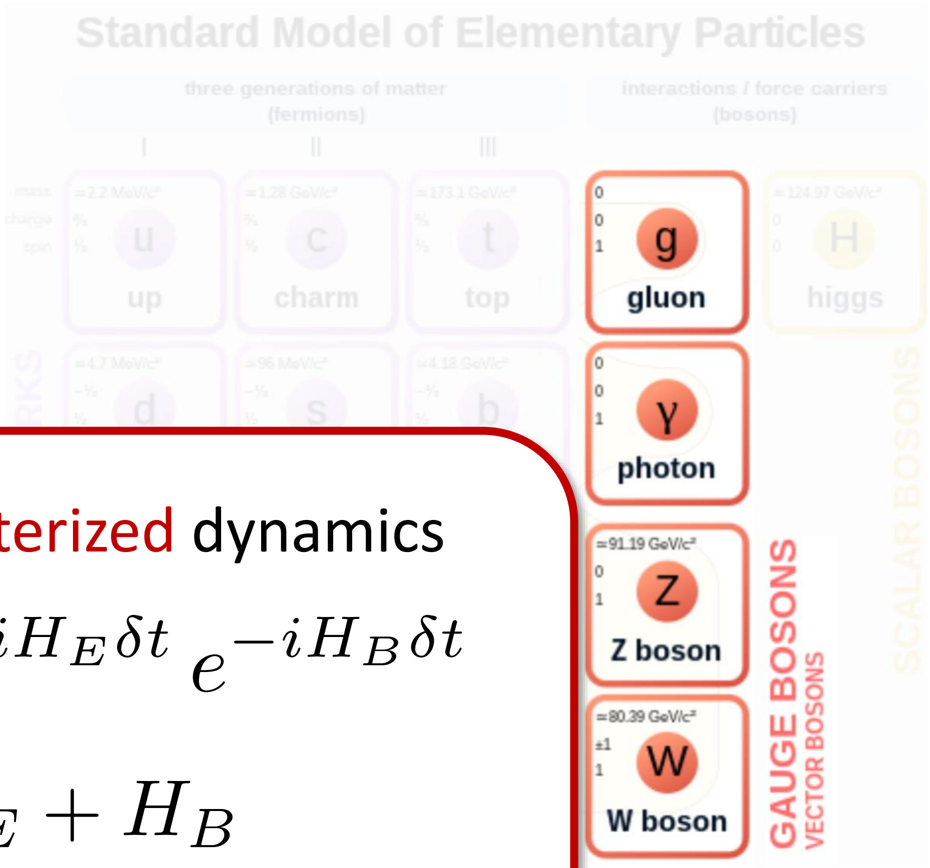
Introduction

Digitized (non-)abelian gauge-theory dynamics

Implementation with Rydberg atom arrays

Conclusion & outlook

Hamiltonian formulation of LGTs



a.org/wiki/Standard_Model

Gauge group: $G = \mathrm{U}(1), \mathrm{SU}(2), \mathrm{SU}(3), \ldots$

Hilbert space: gauge fields on li

$$|\boldsymbol{g}\rangle = \bigotimes_{\ell} |g_\ell\rangle \qquad g \in$$

Goal: realize Trotterized dynamics

$$e^{-iH\delta t} \approx e^{-iH_E\delta t}\, e^{-iH_B\delta t}$$

$$H = H_E + H_B$$

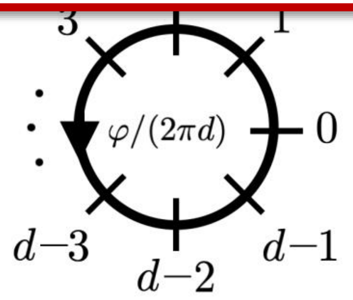
electric + magnetic

Digitization: qudits!

$$|G| = \infty \to |G| = d$$

Examples: $e^{i\varphi} \in \mathrm{U}(1) \supset \mathbb{Z}_d$

abelian

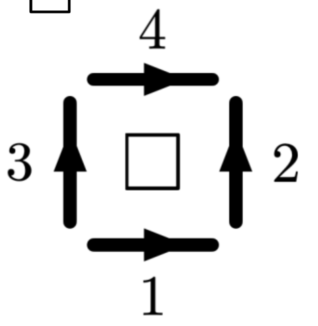


$\mathrm{SU}(2) \supset 2I, 2O, 2T, Q_8$

non-abelian

More details: Zohar et al., Physical Review A, 95(2), 023604 (2017), Lamm et al., Physical Review D, 100(3), 034518 (2019) , …, Alexandru et al.arXiv:2112.08482 (2021)
Possible alternative: Ciavarella et al., Physical Review D, 103(9), 094501 (2021), …

Decomposition of the plaquette interaction $H_B \propto \sum_{\square} \mathcal{U}_\square$

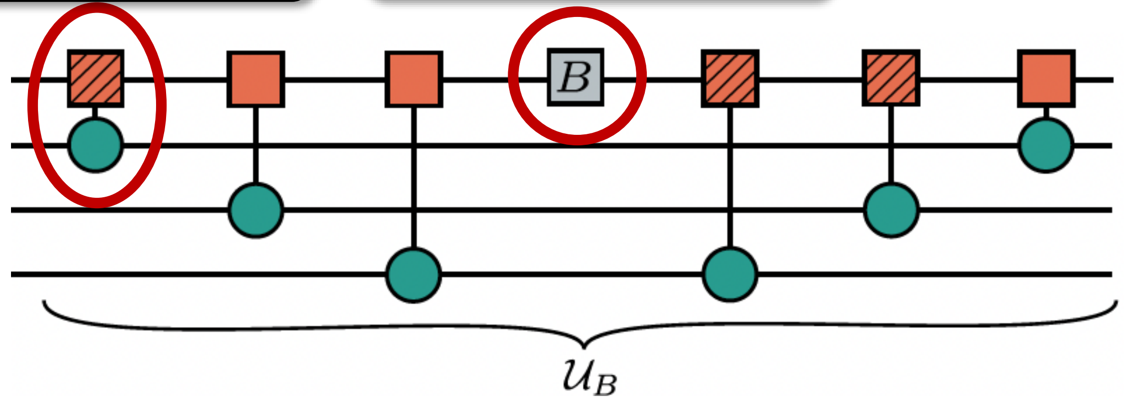


Action of plaquette operator on qudits: $e^{i\varphi} \in \mathrm{U}(1) \supset \mathbb{Z}_d$

$$\mathcal{U}_\square |\varphi_1\rangle|\varphi_2\rangle|\varphi_3\rangle|\varphi_4\rangle = \cos(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)|\varphi_1\rangle|\varphi_2\rangle|\varphi_3\rangle|\varphi_4\rangle$$

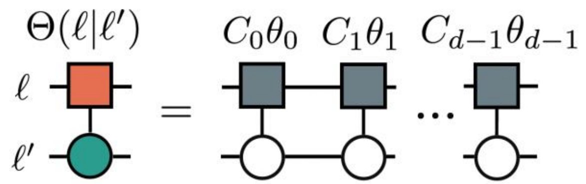
two-qudit gate

single-qudit gate



$e^{-i\ell}$ … $\ell_4)$

works for every group!

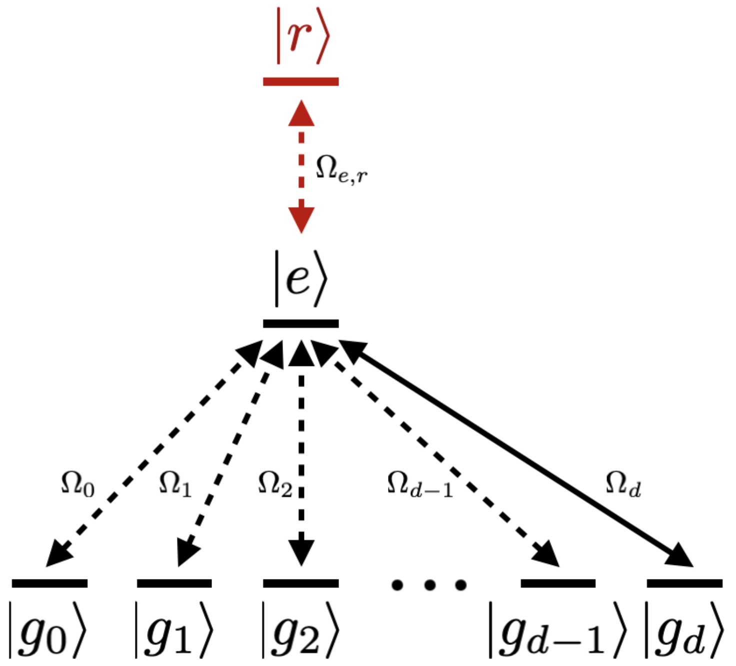


Required entangling gate:
group mulitplication

$$\Theta(\ell|\ell')|\varphi_\ell\rangle|\varphi_{\ell'}\rangle = |\varphi_\ell + \varphi_{\ell'}\rangle|\varphi_{\ell'}\rangle$$

See also: Zohar et al., Physical Review A, 95(2), 023604 (2017)
Lamm et al., Physical Review D, 100(3), 034518 (2019)

General qudit architecture



1. store qudit in manifold of long-lived states

   (e.g. dark states)

2. realize arbitrary single-qudit gates $\mathcal{U}$

   (e.g. ``holonomically'')

   Zanardi et al., Physics Letters A, 264(2-3), 94-99 (1999)

   → ``electric'' and ``magnetic'' *single-qudit gates*

3. entangling gates via Rydberg blockade

   See also: Levine et al., Physical review letters, 123(17), 170503 (2019)

   → ``group-multiplication'' *two-qudit gate*

Entangling gates using the Rydberg blockade

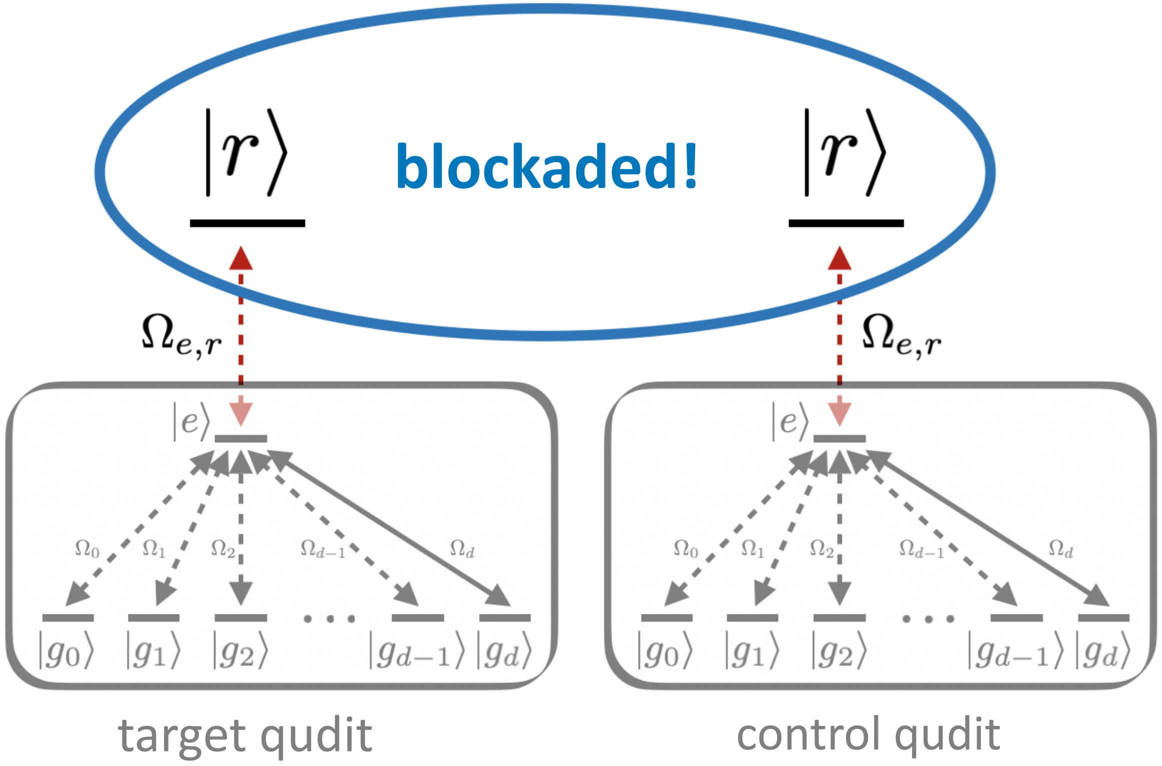
universality!

$$C_j\mathcal{U}\left(|j_1\rangle \otimes |j_2\rangle\right) = \begin{cases} (\mathcal{U}|j_1\rangle) \otimes |j_2\rangle\,, \;\; j_2 = j \\ |j_1\rangle \otimes |j_2\rangle\,, \;\; j_2 \neq j \end{cases}$$

$$\Rightarrow \{\mathcal{U}, C_j\mathcal{U}\} \; ✓$$

Implementation of $C_j\mathcal{U}$ *:*

1. Unitary $\mathcal{U}$ on target, via $|e\rangle$
2. Control to Rydberg state, $|j\rangle \rightarrow |r\rangle$
3. Inverse unitary $\mathcal{U}^\dagger$ on target, via $|r\rangle$
4. Control back from Rydberg, $|r\rangle \rightarrow |j\rangle$

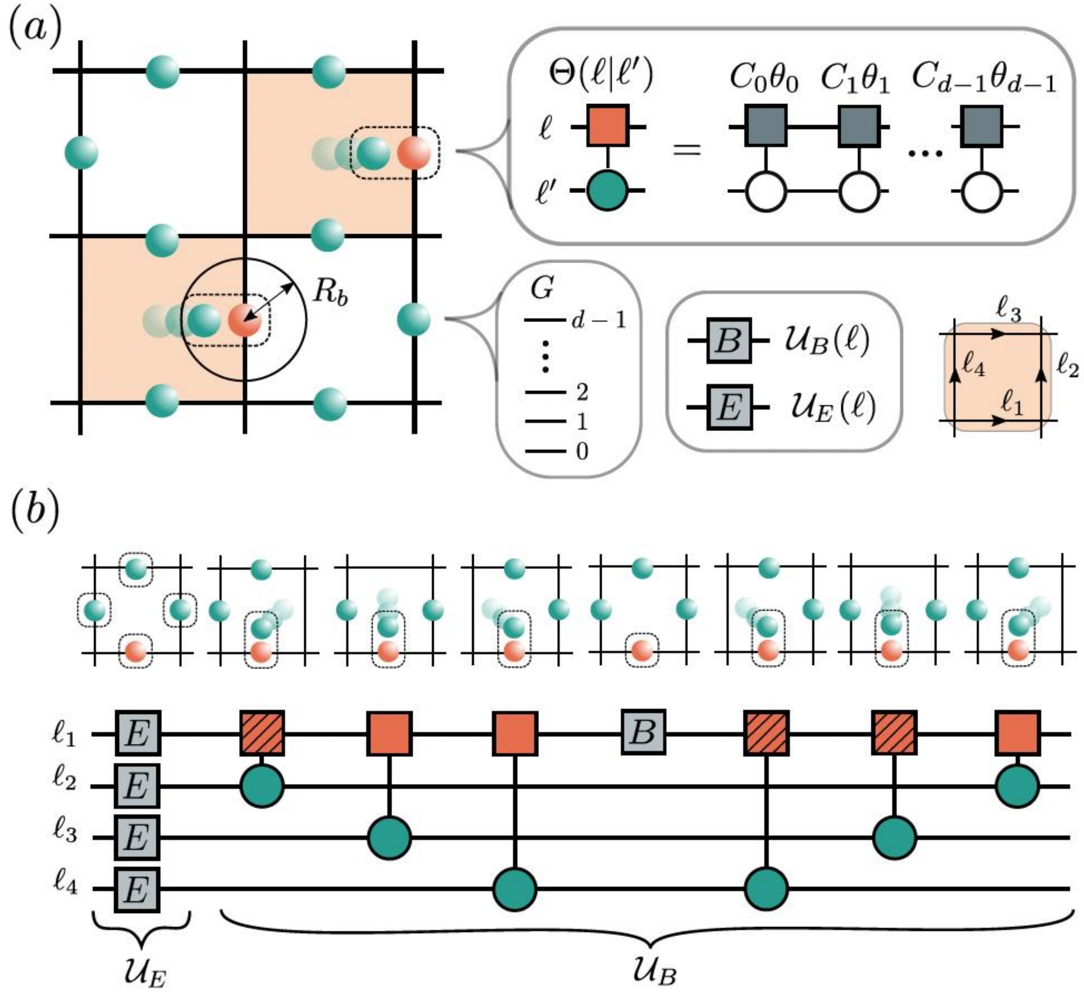


Summary of our proposal

Hardware efficiency & scalability:

- ✓ Gauge fields efficiently encoded in qudits
- ✓ Programmable array of Rydberg atoms
- ✓ Native set of gates that matches LGT gates
- ✓ Parallelization of gates

Protocol applicable to any (finite) gauge group



A minimal non-abelian example: $Q_8 \subset \mathrm{SU}(2)$

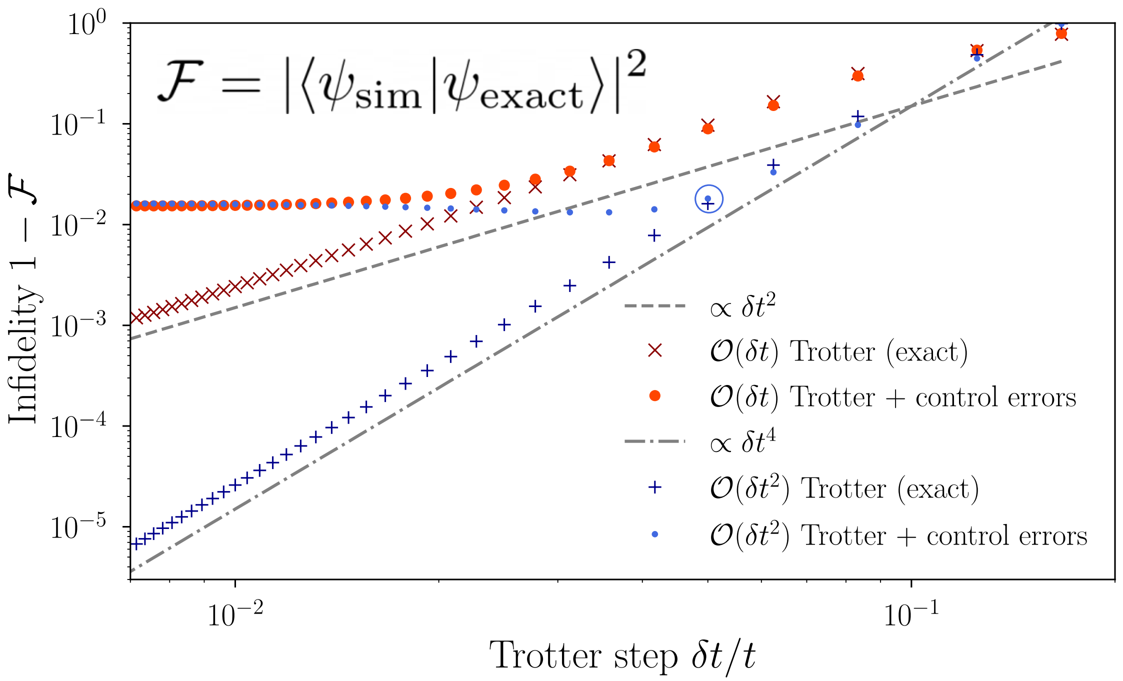
Experimental estimates:

Rydberg interaction $V \sim 2\pi \times 250\,\mathrm{MHz}$

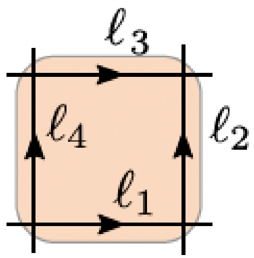
Rabi frequency $\Omega \sim 2\pi \times 50\,\mathrm{MHz}$

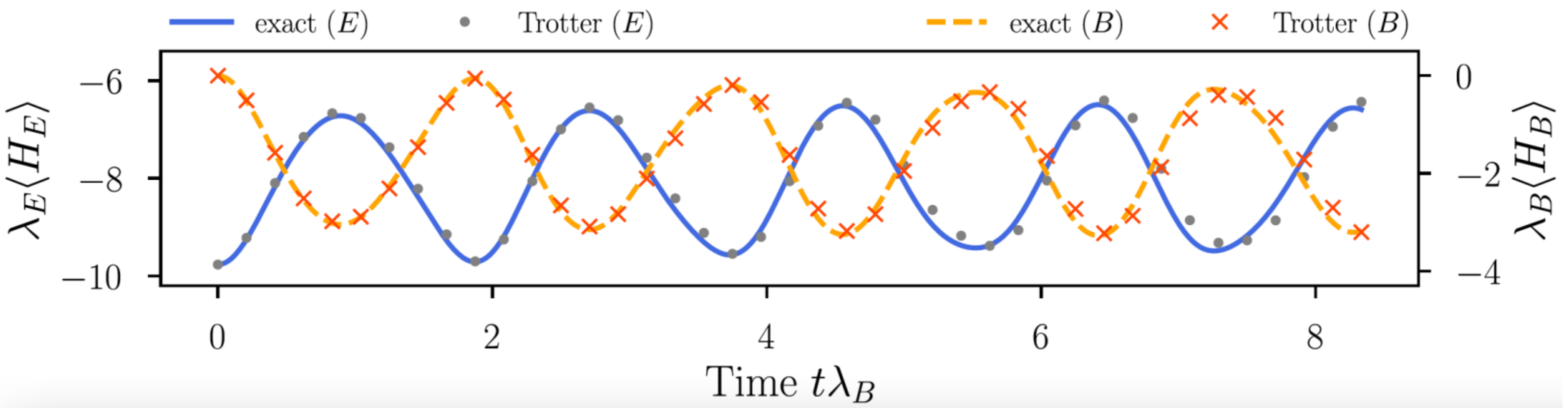
Gate error $\epsilon \sim \mathcal{O}(10^{-4})$

Trotter step time $T_{\delta t} \sim \mathcal{O}(10^3)T \sim \mathcal{O}(1)\,\mathrm{ms}$



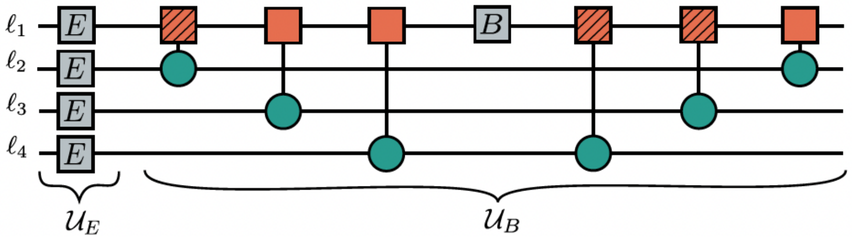
Trotter dynamics on one plaquette:



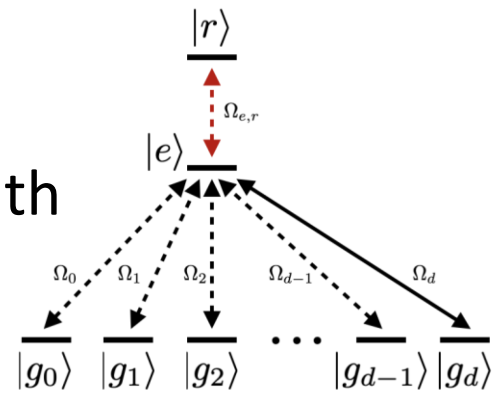


Conclusion & outlook

Natural digitization of LGTs: qudits



Hardware-efficient implementation with Rydberg atoms



- *experiment*: realize & scale-up to large 2D/3D systems!
- *theory:* include fermionic matter! improved algorithms?

Thanks for listening!